

E4-7.2_PI-NCA

January 14, 2026

1 imports

```
[8]: #@title Imports and Notebook Utilities
import os
import io
import PIL.Image, PIL.ImageDraw
import base64
import zipfile
import json
import requests
import numpy as np
import matplotlib.pyplot as pl
import glob
import random
import h5py
import os
import piq
import ast
from IPython.display import Image, HTML, Markdown, clear_output
from tqdm import tqdm_notebook, tnrange

!nvidia-smi -L
```

GPU 0: NVIDIA GeForce RTX 3080 Laptop GPU (UUID:
GPU-6cf8054c-6c04-c256-2ebb-8137cfcc2e798)

```
[9]: import torch
import torchvision.models as models
from torch.utils.data import Dataset
import torch.nn.functional as F
from torch import nn
from IPython.display import clear_output, display, HTML
import h5py
from pathlib import Path

import sys
sys.path.append("C:/Users/GAI/Desktop/Scott/NCA_Research")
```

```

from core_utils.plotting import *
from core_utils.utils_image import *
from core_utils.viz_train import *
from E4_PI_NCA.utils.helper import *
from E4_PI_NCA.utils.Dataset import *
from E4_PI_NCA.utils.PI_Metric import ScaledPhysicsMetrics

torch.set_default_device('cuda')

```

2 heliper

```

[10]: def normalize_to_01(x):
    x_min, x_max = x.amin(dim=(1,2,3), keepdim=True), x.amax(dim=(1,2,3), keepdim=True)
    return (x - x_min) / (x_max - x_min + 1e-8)

def sperate_base_channels(x, base_channel_count=6):
    """
    """
    base_channels = x[:, :base_channel_count, :, :]
    other_channels = x[:, base_channel_count:, :, :]
    return base_channels, other_channels

```

```

[11]: def to_device(data, device):
    if isinstance(data, torch.Tensor):
        return data.to(device)
    elif isinstance(data, dict):
        return {k: to_device(v, device) for k, v in data.items()}
    elif isinstance(data, list):
        return [to_device(v, device) for v in data]
    elif isinstance(data, tuple):
        return tuple(to_device(v, device) for v in data)
    else:
        return data # tensor

```

3 physics loss

```

[12]: # (Indices)
IDX_U_COMPONENTS = 6 # uped = < u >
IDX_V_COMPONENTS = 7 # vped = < v >
IDX_U_MAGNITUDE = 8 # Uped = < sqrt(u^2+v^2) >
IDX_TKE = 9 # TKEped
IDX_T_UW = 10 # Tuwped
IDX_DWDZ = 11 # dW/dz (z-gradient), " =0"

```

```

class PhysicsLoss(nn.Module):
    """
        (PINN Loss)

    V3 - L_u_bound:
    -
    <sqrt(u^2+v^2)> (U_pred) >= sqrt(<u>^2 + <v>^2) (reconstructed_mean_mag)
    - L_u_bound = relu(reconstructed_mean_mag - U_pred)

    V4 - L_continuity:
    -
    ( )
    - L_continuity = mean( (dU/dx + dV/dy + dW/dz)^2 )
    """

def __init__(self, alpha: float = 2.25, weights: dict = None):
    """
        PhysicsLoss

    :param alpha: T_uw          (alpha * TKE)
    :param weights:
    """
    super().__init__()
    self.alpha = alpha

    #
    default_weights = {
        "L_k_neg": 1.0,      # TKE
        "L_flux": 1.0,       #
        "L_u_bound": 1.0,    #
        "L_continuity": 1.0  #      ( )
    }

    if weights is None:
        self.weights = default_weights
    else:
        #      weights
        self.weights = default_weights
        self.weights.update(weights)

    print("PhysicsLoss initialized (V4 - L_continuity added)")
    print(f"Loss Weights: {self.weights}")

    def forward(self, pred: torch.Tensor, dydx: List[Tuple[float, float]] = None) -> Tuple[torch.Tensor, dict]:
        """
        """

```

```

:param pred:      (B, C, H, W)
:param dydx:     (H_orig, W_orig)      B
                  batch item      ( : m)
:return: (total_loss, loss_dict)
"""

# --- 1.      ---
#      (B, H, W)
u_pred = pred[:, IDX_U_COMPONENTS]
v_pred = pred[:, IDX_V_COMPONENTS]
U_pred = pred[:, IDX_U_MAGNITUDE]
TKE_pred = pred[:, IDX_TKE]
T_uw_pred = pred[:, IDX_T_UW]
# dW/dz    NN
dW_dz_pred = pred[:, IDX_DWDZ]

B, C, H_nn, W_nn = pred.shape

loss_dict = {}
total_loss = torch.tensor(0.0, device=pred.device)

# --- 2. L_k_neg (TKE ) ---
if self.weights.get("L_k_neg", 0.0) > 0:
    L_k_neg = torch.mean(torch.relu(-TKE_pred))
    loss_dict["L_k_neg"] = L_k_neg
    total_loss += self.weights["L_k_neg"] * L_k_neg

# --- 3. L_flux ( ) ---
if self.weights.get("L_flux", 0.0) > 0:
    TKE_non_neg = torch.relu(TKE_pred)
    flux_bound_error = torch.abs(T_uw_pred) - (self.alpha * TKE_non_neg)
    L_flux = torch.mean(torch.relu(flux_bound_error))
    loss_dict["L_flux"] = L_flux
    total_loss += self.weights["L_flux"] * L_flux

# --- 4. L_u_bound ( ) ---
if self.weights.get("L_u_bound", 0.0) > 0:
    #      : sqrt(<u>^2 + <v>^2)
    reconstructed_mean_mag = torch.sqrt(u_pred**2 + v_pred**2 + 1e-8)

    #      : U_pred >= reconstructed_mean_mag
    #      (Violation)  : reconstructed_mean_mag > U_pred
    violation = reconstructed_mean_mag - U_pred

    #

```

```

L_u_bound = torch.mean(torch.relu(violation))
loss_dict["L_u_bound"] = L_u_bound
total_loss += self.weights["L_u_bound"] * L_u_bound

# --- 5. L_continuity (      ) ---
if self.weights.get("L_continuity", 0.0) > 0:
    if dydx is None:
        #      dydx
        L_continuity = torch.tensor(0.0, device=pred.device)
        if self.training: #
            print("Warning: 'dydx' is None. Skipping L_continuity")
    else:
        #
        #      dU/dx      * *      * *
        # dU/dx = (dU / d(pixel_W)) * (d(pixel_W) / dx)
        # (d(pixel_W) / dx) = W_nn / W_orig = 1 / dx_spacing

        # 1.      (B, )
        H_orig_batch = torch.tensor([item[0] for item in dydx], device=pred.device, dtype=pred.dtype)
        W_orig_batch = torch.tensor([item[1] for item in dydx], device=pred.device, dtype=pred.dtype)

        # 2.      (spacing) (B, )
        dy_spacing = H_orig_batch / H_nn
        dx_spacing = W_orig_batch / W_nn

        # 3.      spacing (broadcast) (B, 1, 1)
        dy_b = dy_spacing.view(B, 1, 1)
        dx_b = dx_spacing.view(B, 1, 1)

        # 4.      * * (Pixel-space gradients)
        # dim=(-2, -1) (H, W)
        grad_u_y_px, grad_u_x_px = torch.gradient(u_pred, dim=(-2, -1))
        grad_v_y_px, grad_v_x_px = torch.gradient(v_pred, dim=(-2, -1))

        # 5.      * * (Physical-space gradients)
        # dU/dx = (dU / d_pixel_W) / (dx / d_pixel_W) = grad_u_x_px / dx_b
        dU_dx_pred = grad_u_x_px / dx_b

        # dV/dy = (dV / d_pixel_H) / (dy / d_pixel_H) = grad_v_y_px / dy_b
        dV_dy_pred = grad_v_y_px / dy_b

        # 6.      (Residual)

```

```

#  $R = dU/dx + dV/dy + dW/dz$ 
#  $dW/dz \quad NN$ 
physics_residual = dU_dx_pred + dV_dy_pred + dW_dz_pred

# 7. Loss (Mean Squared Error of the residual)
L_continuity = torch.mean(physics_residual**2)

loss_dict["L_continuity"] = L_continuity
total_loss += self.weights["L_continuity"] * L_continuity

# --- ---
loss_dict["L_physics_total"] = total_loss
return total_loss, loss_dict

```

4 NCA model

```
[13]: #@title Minimalistic Neural CA
ident = torch.tensor([[0.0,0.0,0.0],[0.0,1.0,0.0],[0.0,0.0,0.0]])
sobel_x = torch.tensor([[-1.0,0.0,1.0],[-2.0,0.0,2.0],[-1.0,0.0,1.0]])
lap = torch.tensor([[1.0,2.0,1.0],[2.0,-12,2.0],[1.0,2.0,1.0]])
IDX_GEO_MASK = 2
def perchannel_conv(x, filters):
    '''filters: [filter_n, h, w]'''
    b, ch, h, w = x.shape
    y = x.reshape(b*ch, 1, h, w)
    y = F.pad(y, [1, 1, 1, 1], 'circular')
    y = F.conv2d(y, filters[:,None])
    return y.reshape(b, -1, h, w)

def perception(x):
    filters = torch.stack([ident, sobel_x, sobel_x.T, lap])
    return perchannel_conv(x, filters)

class CA(torch.nn.Module):
    def __init__(self, chn=12, hidden_n=96):
        super().__init__()
        self.chn = chn
        self.w1 = nn.Conv2d(chn*4, hidden_n, 1)
        self.w2 = nn.Conv2d(hidden_n, chn, 1, bias=False)
        self.w2.weight.data.zero_()

    def forward(self, x, update_rate=0.5):
        y = perception(x)
        y = self.w2(torch.relu(self.w1(y)))
        b, c, h, w = y.shape
        update_mask = x[:,IDX_GEO_MASK:IDX_GEO_MASK+1]
```

```

no_change = x[:, :6, :, :]
updated = x + y * x[:, 2:3, :, :]
return torch.cat([no_change, updated[:, 6:, :, :]], dim=1)

def seed(self, n, sz=128):
    return torch.zeros(n, self.chn, sz, sz)

def to_rgb(x):
    return x[...,:3,:,:]+0.5

param_n = sum(p.numel() for p in CA().parameters())
print('CA param count:', param_n)

```

CA param count: 5856

5 setup training

```
[14]: # =====
#
# =====
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

# =====
#
# =====
DATASET_IMG_SIZE = (128,128)
dataset_dir = "../dataset_h5/train"

#      [B, C, H, W]
loaded_dataset, dataset_city_size = load_all_cases_to_BCHW(
    dataset_dir,
    target_size=DATASET_IMG_SIZE
)

# =====
#
# =====
CHANNEL_C      = 24      # CA
LEARNING_RATE   = 1e-3
POOL_SIZE       = 256     #      (replay-like)
TOTAL_EPOCH     = 5000

# =====
#
# =====
pi_module = CA(chn=CHANNEL_C).to(device)
```

```

opt = torch.optim.Adam(pi_module.parameters(), LEARNING_RATE, capturable=True)

lr_sched = torch.optim.lr_scheduler.MultiStepLR(
    opt,
    milestones=[1000, 2000],
    gamma=0.3
)

loss_log      = []
metrics_log   = []

# Gradient Checkpoint ( , 10~20%)
gradient_checkpoints = False

# =====
#
# =====
# --- Physics-informed Loss (PINN)
pinn_loss_fn = PhysicsLoss(
    alpha=2.25,
    weights={
        "L_k_neg": 1.0,          # TKE < 0
        "L_flux": 1.0,           #
        "L_u_bound": 1.0,         #
        "L_continuity": 1.0      # ( ·u = 0)
    }
).to(device)

# --- Data Loss (MSE)
mse_loss_fn = nn.MSELoss().to(device)

# --- Loss ( )
LAMBDA_MSE      = 1.0      #
LAMBDA_PINN     = 10.0     #
LAMBDA_OVERFLOW = 0.1      #

```

PhysicsLoss initialized (V4 - L_continuity added)
Loss Weights: {'L_k_neg': 1.0, 'L_flux': 1.0, 'L_u_bound': 1.0, 'L_continuity': 1.0}

[15]:

```

# =====
# Init ( )
# =====

#
init_module = CA(chn=CHANNEL_C)

```

```

#
CHECKPOINT_PATH = r"..\\outputs\\E4-7.1_Init-NCA_20251102-153402\\
    ↵ca_model_step_5000.pth"

#
try:
    checkpoint = torch.load(CHECKPOINT_PATH)
    init_module.load_state_dict(checkpoint["model_state_dict"])
    print(f"[Init] : {CHECKPOINT_PATH} (epoch {checkpoint['epoch']}"))

except FileNotFoundError:
    print(f"[Init] : {CHECKPOINT_PATH}")
except Exception as e:
    print(f"[Init] : {e}")

# -----
#   Init
# -----
init_module.eval()           # Dropout/BN
for p in init_module.parameters():
    p.requires_grad = False    #

#   GPU/CPU
init_module = init_module.to(device)

print("[Init] (Frozen)")

```

```

[Init] : ..\outputs\E4-7.1_Init-NCA_20251102-153402/ca_model_step_5000.pth
(epoch 5000)
[Init] (Frozen)

```

6 training loop

```

[16]: import time
# --- ( ) ---
model_save_dir = get_output_path()  #
model_name_format = "ca_model_step_{}.pth" # i

for i in range(TOTAL_EPOCH):

    # ----- Pool update -----
    t0 = time.time()
    if i % 500 == 0:
        x_pool, y_pool, pool_city_size = create_pool(
            loaded_dataset, dataset_city_size,

```

```

        pool_size=POOL_SIZE, channel_c=CHANNEL_C
    )
x_pool, y_pool = x_pool.to(device), y_pool.to(device)
metric_calculator = ScaledPhysicsMetrics(y_pool, pool_city_size)
# print(f"[{i}] pool_update: {time.time() - t0:.4f}s")

# ----- Sample batch -----
t0 = time.time()
with torch.no_grad():
    batch_idx = np.random.choice(len(x_pool), 6, replace=False)
    batch_city_size = [pool_city_size[j] for j in batch_idx.tolist()]

    x = x_pool[batch_idx].clone()
    y = y_pool[batch_idx]

    if i % 8 == 0:
        x[:1, 6:, :, :] = 0
# print(f"[{i}] sample_batch: {time.time() - t0:.4f}s")

# ----- Forward -----
t0 = time.time()
step_n = np.random.randint(32, 96)
if not gradient_checkpoints:
    x_initial = x.clone()
    for _ in range(step_n):
        with torch.no_grad():
            x = init_module(x)
            x = pi_module(x)
else:
    x.requires_grad = True
    module_list = [init_module]*step_n + [pi_module]
    x = torch.utils.checkpoint.checkpoint_sequential(module_list, 16, x)
# print(f"[{i}] forward: {time.time() - t0:.4f}s")

# ----- Loss -----
t0 = time.time()
overflow_loss = (x[:, 6:11] - x[:, 6:11].clamp(-1, 1)).abs().sum()
loss_mse = mse_loss_fn(x[:, 6:11], y[:, 6:11])
loss_pinn, pinn_loss_dict = pinn_loss_fn(x, batch_city_size)
loss = (
    LAMBDA_MSE * loss_mse +
    LAMBDA_PINN * loss_pinn +
    LAMBDA_OVERFLOW * overflow_loss
)
# print(f"[{i}] loss_calc: {time.time() - t0:.4f}s"

# ----- Backward -----

```

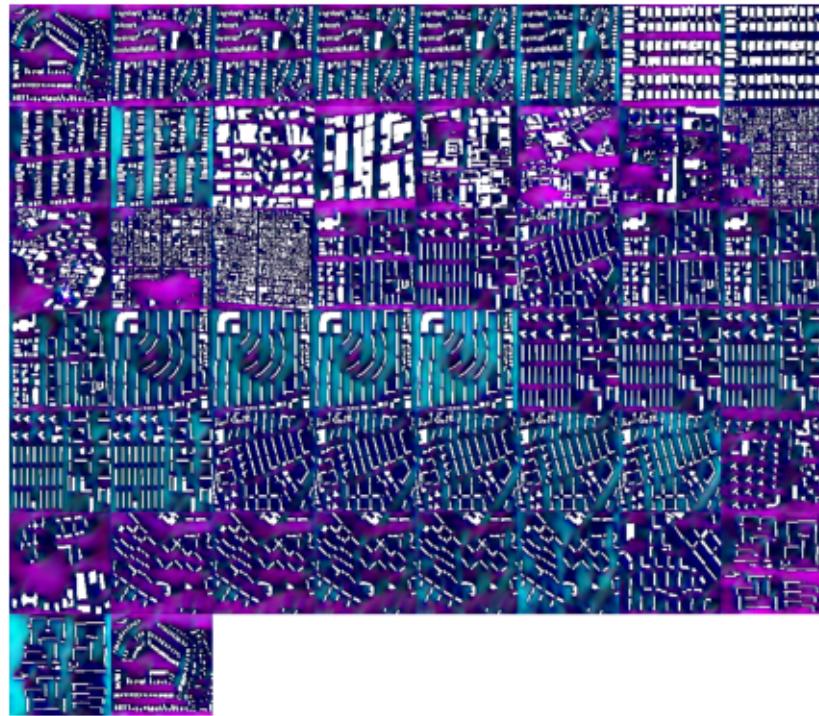
```

t0 = time.time()
opt.zero_grad()
loss.backward()
with torch.no_grad():
    for p in pi_module.parameters():
        if p.grad is not None:
            p.grad /= (p.grad.norm() + 1e-8)
    opt.step()
    lr_sched.step()
    x_pool[batch_idx] = x.detach()
# print(f"[{i}] backward: {time.time() - t0:.4f}s")

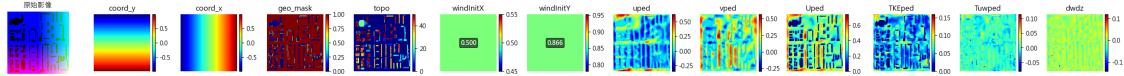
# ----- Metrics -----
t0 = time.time()
loss_log.append(loss.item())
with torch.no_grad():
    metrics_result = metric_calculator(x, y, batch_city_size)
    metrics_log.append(to_device(metrics_result, "cpu"))
# print(f"[{i}] metrics: {time.time() - t0:.4f}s")

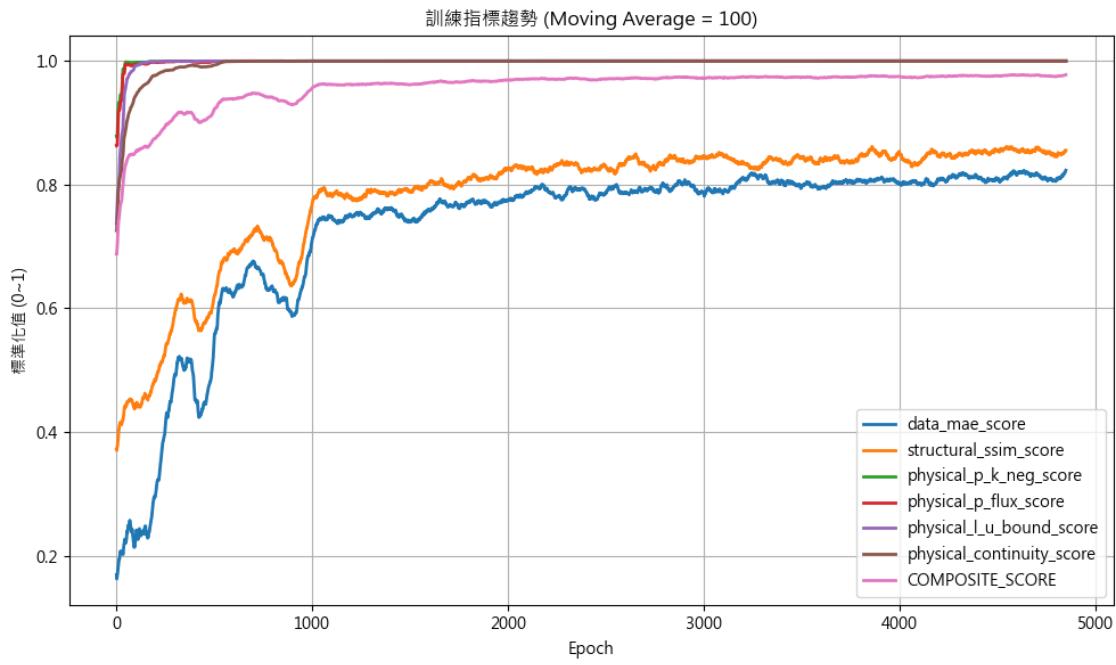
# ----- Visual & save -----
t0 = time.time()
if i % 50 == 0:
    clear_output(wait=True)
    viz_pool(x_pool, step_i=i)
    plt_HWC_split_channels(to_HWC(x[0, :12]))
    metrics_log = to_device(metrics_log, "cpu")
    plt_metrics_dict(metrics_log, window=100)
    pl.plot(loss_log, '.', alpha=0.1)
    pl.yscale('log')
    pl.ylim(min(loss_log), loss_log[0])
    pl.tight_layout()
    imshow(grab_plot(), id='log')
    imshow(np.hstack(to_rgb(x[:, 6:, :, :]).permute(0, 2, 3, 1).cpu().detach().
    numpy()), id='batch')
    imshow(np.hstack(to_rgb(y[:, 6:, :, :]).permute(0, 2, 3, 1).cpu().detach().
    numpy()), id='target')
    save_checkpoint(pi_module, opt, i, f"{model_save_dir}/
    {model_name_format.format(i)}")
# print(f"[{i}] viz: {time.time() - t0:.4f}s")
save_checkpoint(pi_module, opt, TOTAL_EPOCH, f"{model_save_dir}/
{model_name_format.format(TOTAL_EPOCH)}")

```



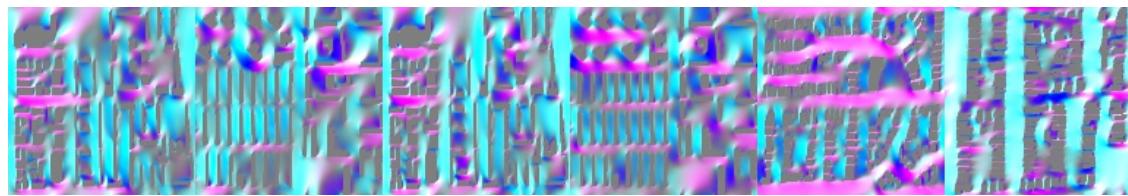
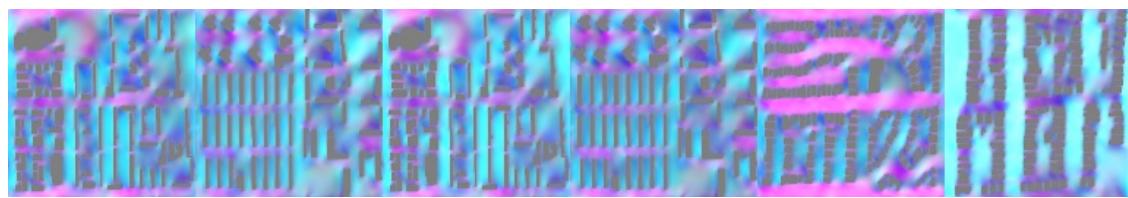
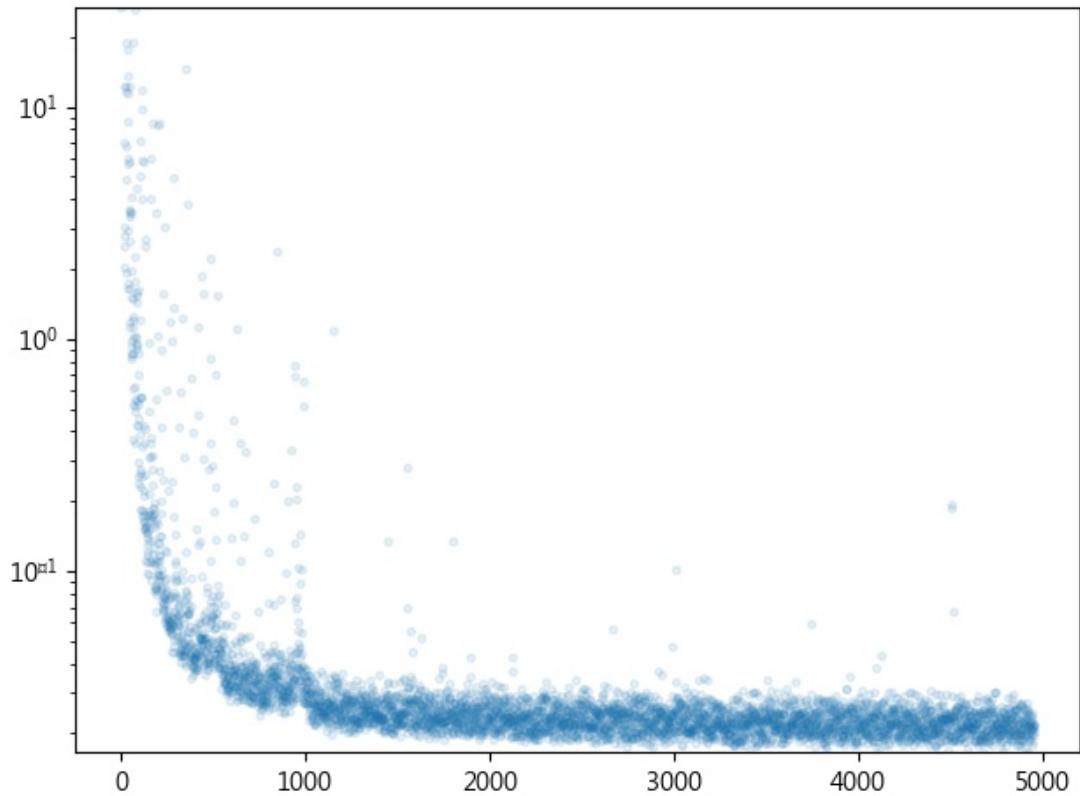
Clipping input data to the valid range for imshow with RGB data ([0..1] for floats or [0..255] for integers). Got range [-0.99383545..1.0].





Font 'default' does not have a glyph for '\u2212' [U+2212], substituting with a dummy symbol.

Font 'default' does not have a glyph for '\u2212' [U+2212], substituting with a dummy symbol.



```
..\outputs\E4-7.2_PI-NCA_20251102-162009\ca_model_step_4950.pth  
..\outputs\E4-7.2_PI-NCA_20251102-162009\ca_model_step_4800.pth  
3  
..\outputs\E4-7.2_PI-NCA_20251102-162009\ca_model_step_5000.pth
```

```
..\outputs\E4-7.2_PI-NCA_20251102-162009\ca_model_step_4850.pth
3
```

7 output video

```
[17]: test_x_pool, test_y_pool = create_pool.loaded_dataset, pool_size=256, channel_c=24)
test_x_pool, test_y_pool = test_x_pool.to(device), test_y_pool.to(device)
metric_calculator = ScaledPhysicsMetrics(test_y_pool)

with VideoWriter() as vid, torch.no_grad():
    batch_idx = np.random.choice(len(test_x_pool), 4, replace=False)
    test_x = test_x_pool[batch_idx].clone() # clone in-place
    test_y = test_y_pool[batch_idx]
    plt_HWC_split_channels(to_HWC(test_x[0,:11]))

    for k in tnrange(300, leave=False):
        step_n = min(2**((k//30), 8)
        for i in range(step_n):
            test_x[:] = ca(test_x)
            img = to_rgb(test_x[0,6:]).permute(1, 2, 0).cpu()
            vid.add(zoom(img, 2))
```

```
-----  
TypeError                                         Traceback (most recent call last)  
Cell In[17], line 1  
----> 1 test_x_pool, test_y_pool = create_pool.loaded_dataset, pool_size=256, channel_c=24)  
      2 test_x_pool, test_y_pool = test_x_pool.to(device), test_y_pool.to(device)  
      3 metric_calculator = ScaledPhysicsMetrics(test_y_pool)  
  
TypeError: create_pool() missing 1 required positional argument: 'city_size'
```