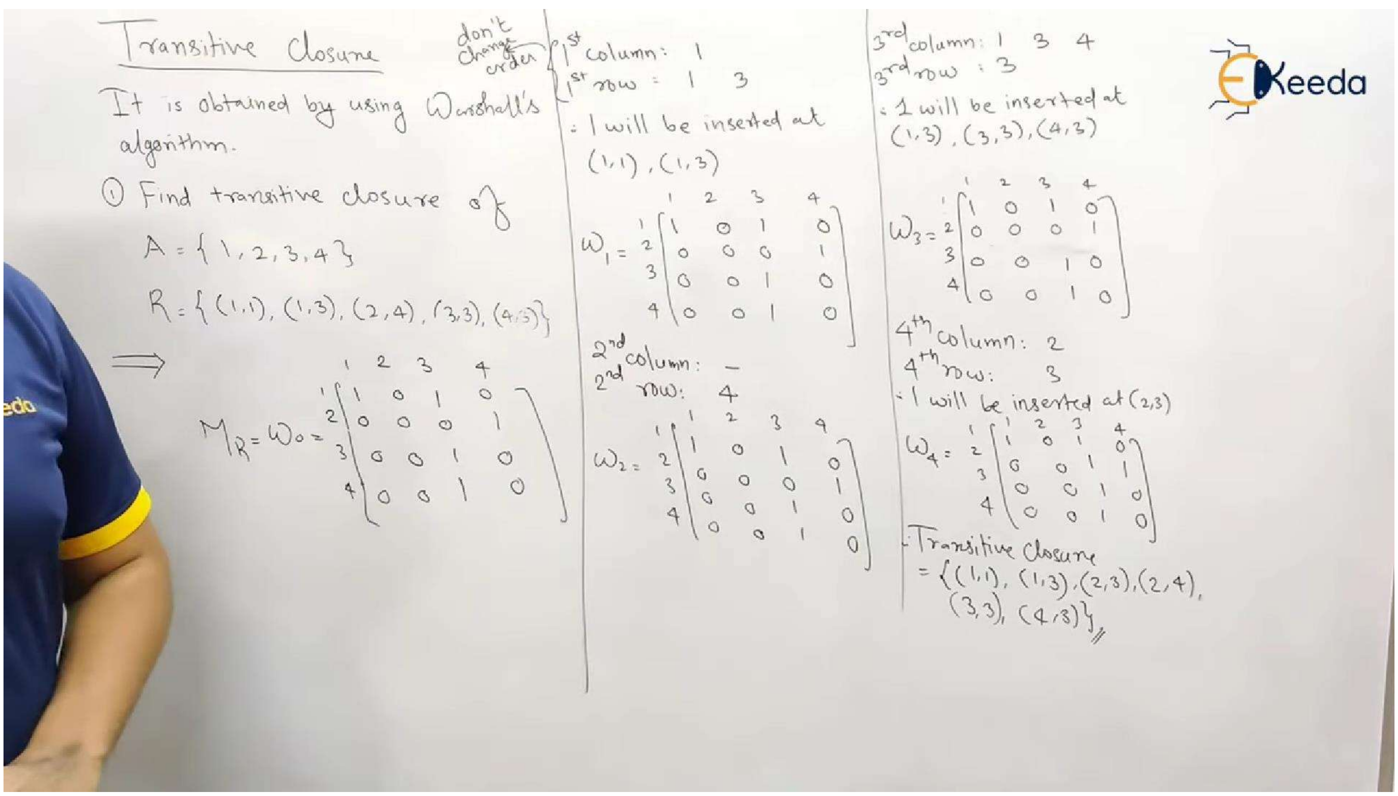
Unit test

Warshall algorithm

Warshall's Algorithm is an iterative algorithm used to compute / find the transitive closure of a relation R by successively checking whether a path from vertex i to vertex j exists through an intermediate vertex k, and updating the relation matrix accordingly.

No of steps is depends on no of elements present in set.



Warshall's Algorithm

(Finding the Transitive Closure)

Finding the transitive closure using Warshall's Algorithm

Sometimes it is difficult to find all the ordered pairs in transitive closure of a relation. Warshall's Algorithm is considered an efficient method in finding the transitive closure of a relation.

Example: By using Warshall's algorithm, find the transitive closure of the relation $R = \{(2, 1), (2, 3), (3, 1), (3, 4), (4, 1), (4, 3)\}$ on set $A = \{1, 2, 3, 4\}$.

Solution: First, we will represent the relation R in matrix form.

$$R = \begin{array}{cc} & \begin{array}{cccc}1&2&3&4\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\1&0&1&0\end{bmatrix}_{4\times4}\end{array}$$

Understand that there are 4 elements in set A. Therefore, 4 steps are required in order to find the transitive closure of relation R [According to Warshall's Algorithm].

>> In Step 1, we will consider 1st column and 1st row of the above matrix i.e., C_1 and R_1.

Write all positions where 1 is present in column 1.
$C_1 = \{2, 3, 4\}$
Also, write all position where 1 is present in row 1
$R_1 = \varnothing$

$$R = \begin{array}{cc} & \begin{array}{cccc}1&2&3&4\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}0&0&0&0\\1&0&1&0\\1&0&0&1\\1&0&1&0\end{bmatrix}_{4\times4}\end{array}$$

Now, take the cross product of C_1 and R_1.
$C_1 X\ R_1 = \varnothing$.
Therefore, no new additions.

>> In Step 2, we will consider 2nd column and 2nd row of the above matrix.

C_2	R_2
$\varnothing$	$\{1, 3\}$

$C_2 X\ R_2 = \varnothing$
Therefore, no new additions.

>> In Step 3, we will consider 3rd column and 3rd row.

C_3	R_3
$\{2, 4\}$	$\{1, 4\}$

$C_3 X\ R_3 = \{(2, 1), (2, 4), (4, 1), (4, 4)\}$

$$\begin{array}{cc} & \begin{array}{cccc}1&2&3&4\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}0&0&0&0\\1&0&1&1\\1&0&0&1\\1&0&1&1\end{bmatrix}_{4\times4}\end{array}$$

New Matrix

>> In Step 4, we will consider 4th column and 4th row of the above matrix.

C_4	R_4
$\{2, 3, 4\}$	$\{1, 3, 4\}$

$C_4 X\ R_4 = \{(2, 1), (2, 3), (2, 4), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)\}$

$$\begin{array}{cc} & \begin{array}{cccc}1&2&3&4\end{array} \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}0&0&0&0\\1&0&1&1\\1&0&1&1\\1&0&1&1\end{bmatrix}_{4\times4}\end{array}$$

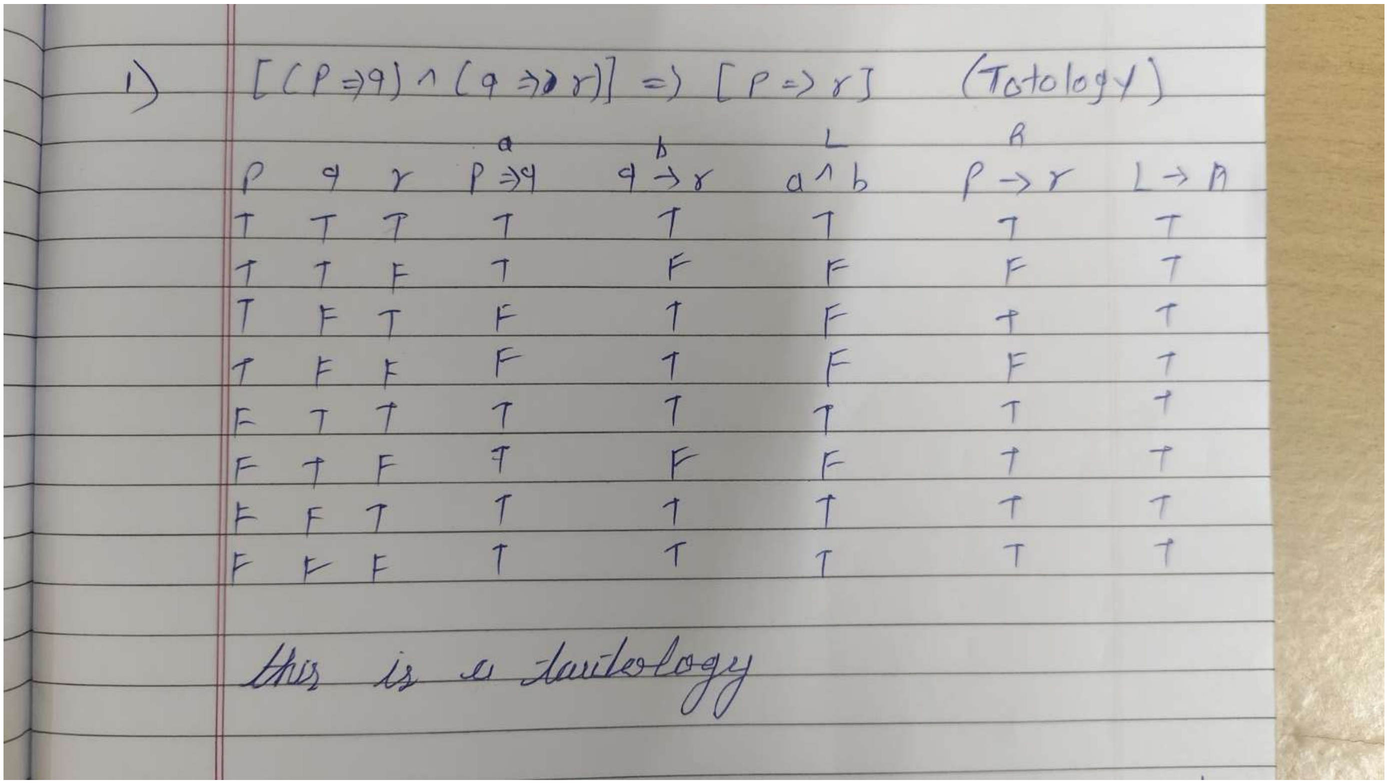
New Matrix $\quad R_t^+$

$R_t^+ = \{(2, 1), (2, 3), (2, 4), (3, 1), (3, 3), (3, 4), (4, 1), (4, 3), (4, 4)\}$

R_t^+ is the transitive closure of relation R.

Tautology

A statement which is alwayes true for all subtitution instances is called a tautology.



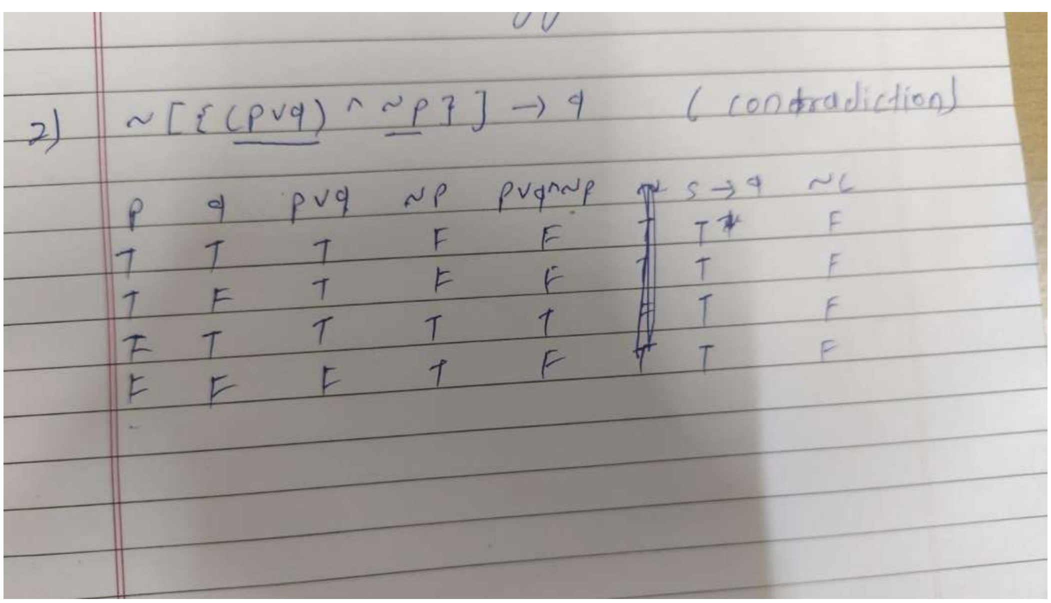
1) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow [p \Rightarrow r]$ (Totology)

p	q	r	$p \Rightarrow q$ (a)	$q \to r$ (b)	$a \wedge b$ (L)	$p \to r$ (R)	$L \to R$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

this is a tautology

Contradiction

A statement which is alwayes false for all subtitution instanes is called a contradiction.



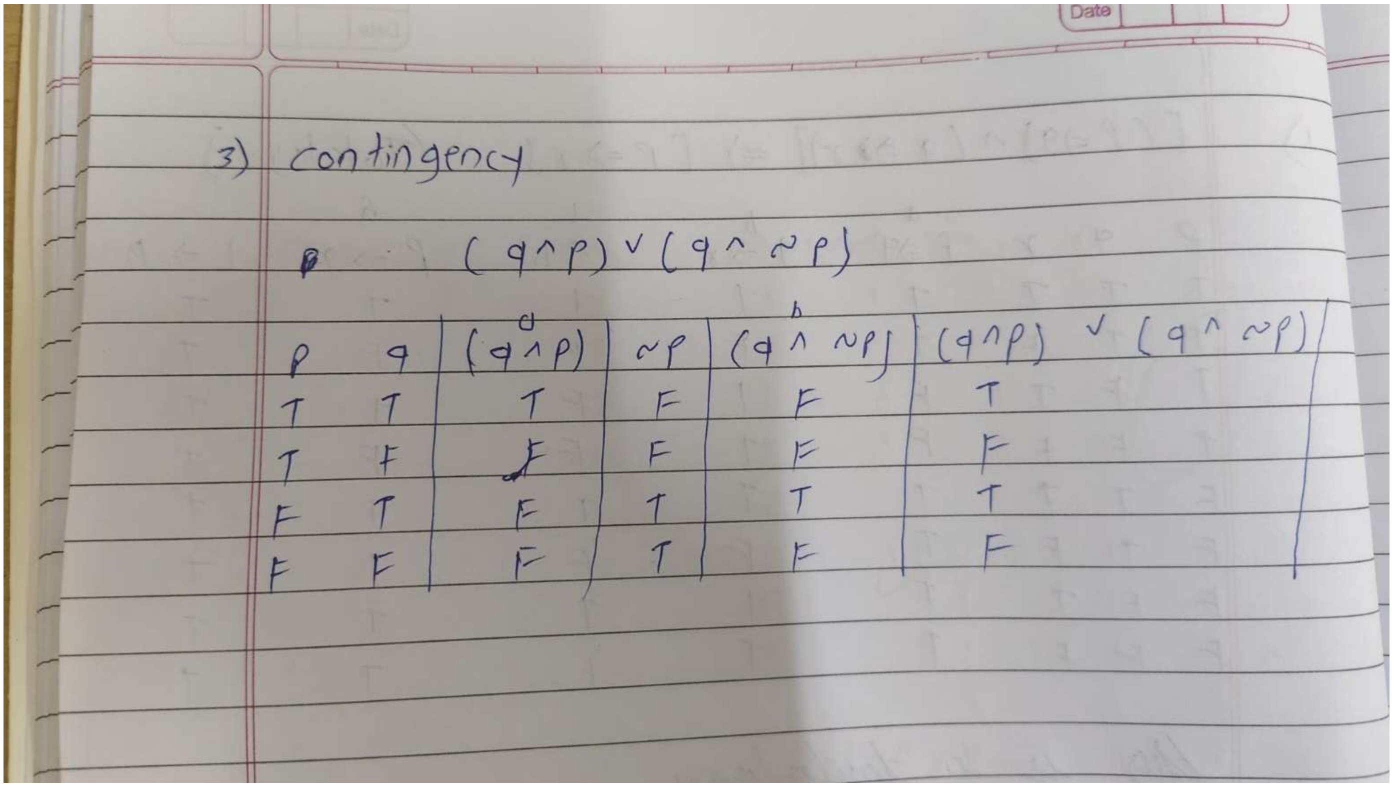
2) $\sim[\{(p \vee q) \wedge \sim p\}] \to q$ (contradiction)

p	q	$p \vee q$	$\sim p$	$p \vee q \wedge \sim p$	$s \to q$	$\sim L$
T	T	T	F	F	T	F
T	F	T	F	F	T	F
F	T	T	T	T	T	F
F	F	F	T	F	T	F

Note : ~ it can be use only proposion so it will always consider last as you made mistake.

Contingency

A statement which is naither true nor false is called contigency.



3) contingency

$p \quad (q \wedge p) \vee (q \wedge \sim p)$

p	q	$(q \wedge p)$	$\sim p$	$(q \wedge \sim p)$	$(q \wedge p) \vee (q \wedge \sim p)$
T	T	T	F	F	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	F	F

Step-by-Step: How to Find a Relation Between Two Sets

Suppose you have two sets:

A = {1, 2}
B = {3, 4}

✔ Step 1: Find Cartesian Product (A × B)

Take each element of A and pair it with each element of B:

A × B =
{ (1,3), (1,4), (2,3), (2,4) }

Step 2: Select ordered pairs according to the condition

A relation is ANY selection of these pairs.

Example 1:

Condition → x < y

Check each pair:

(1,3) → 1 < 3 ✓

(1,4) → 1 < 4 ✓

(2,3) → 2 < 3 ✓

(2,4) → 2 < 4 ✓

⭐ Relation R =

{ (1,3), (1,4), (2,3), (2,4) }

Example 2:

Condition → x + y is even

Check each pair:

(1,3) → 4 (even) ✓

(1,4) → 5 (odd) ✕

(2,3) → 5 (odd) ✕

(2,4) → 6 (even) ✓

⭐ Relation R =

{ (1,3), (2,4) }

Example 3:

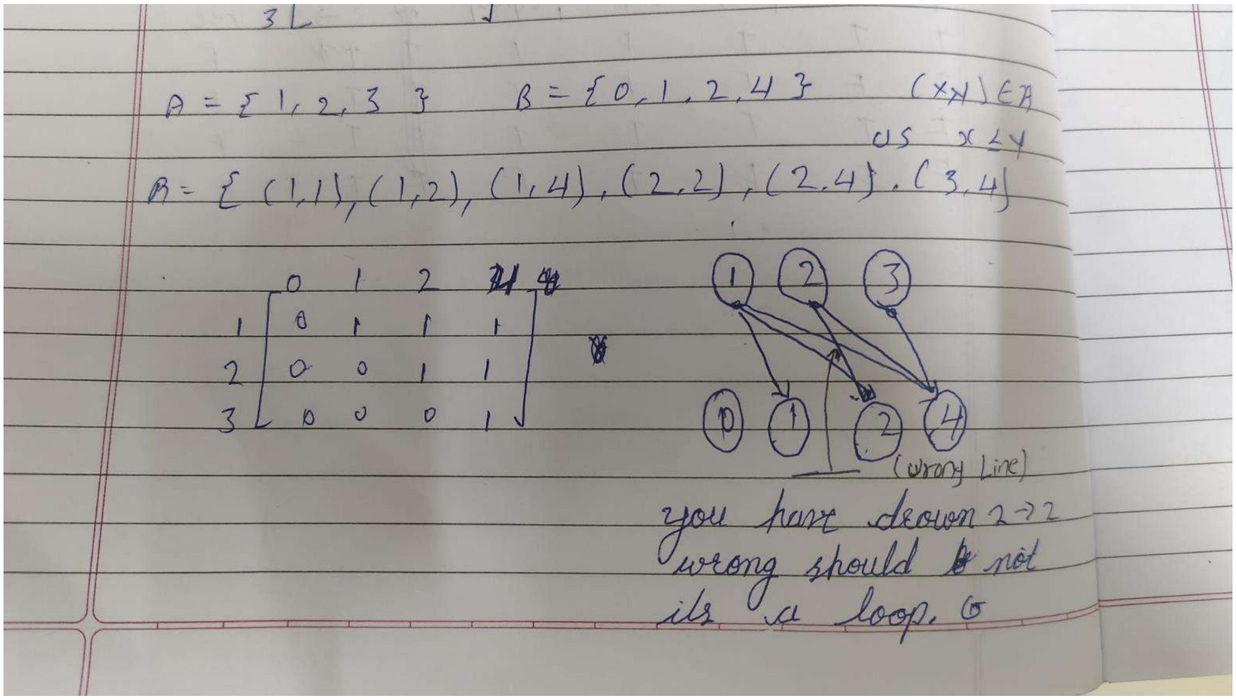
If teacher says "define any relation",
you can choose any subset.

Example:

R = { (1,4), (2,3) }
This is also a relation.

Relation matrix and draw its diagraph

A relation is a set of ordered pairs formed from two sets.
It shows how elements of one set are related to elements of another set.



Example based on inclusion and exclusion principle

The Principle of Inclusion and Exclusion is used to find the number of elements in the union of sets while avoiding double counting.

The principle of inclusion and exclusion helps us to count the number of elements in the union of several sets while avoiding double counting of element that belong to more than one set

Formula:

$|A \cup B| = |A| + |B| - |A \cap B|$

⭐ Example 1 (Very Easy)

Students who like Maths = 25

Students who like Science = 20

Students who like both = 10

Find students who like Maths OR Science.

Solution:

Using Inclusion–Exclusion:

$|M \cup S| = |M| + |S| - |M \cap S|$

$= 25 + 20 - 10 = 35$

✓ Answer: 35 students