Unit - 1

* **Concept of Set Theory**

A set is a well-defined collection of objects (element). Set can be repejented by listing elements within beeces {1,2,3} or by describing their properties. Elements in a set are unique, and the order of elements does not matter. Sets are fundamental for defining relations, functions and other mathematical structures.

* **Types of Sets:**

- **Finite Set:**
  Contains a finite number of elements (e.g. vowels: {a, e, i, o, u})
- **Infinite Set:**
  Contains an infinite number of elements (e.g. natural no: {1, 2, 3, ...})
- **Empty Set: (Null)**
  contains no elements.
  denoted as ∅
- **Singleton Set:**
  It has exactly one element
  (e.g. {0})
- **Equalent Sets:**
  Two sets with same number of elements.
- **Equal Sets:**
  Two sets with the same elements (order doesn't matte)
- **Universal Set:**
  contains all objects under discussion.

* **Countable & Uncountable Sets**

countable:
A countable set is if it is ~~eg~~ either finite or countable infinite where elements can be placed in a one to one correspondence

Uncountable Sets:
A set is uncountable if it is not countable. It is an infinite set whose elements cannot be put into a one to one correspondence with the set of natural numbers.
(e.g: real numbers.)

Power Set:

The power set of a set of all possible subsets of that set including the empty set and the set itself. If set A has ~~no~~ n elements, the power set has 2^n subsets. // ⊆ = subset

e.g: $A = \{1, 2, 3\}$
$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Multiset:

- A multiset allows repeted element unlike a traditional set where objects are distinct. Useful in counting problems and computer algorithms.
- Multiset are set where an elements appear more than one

e.g $A = \{1,1,1,2,2,3\}$
$B = \{a,a,a,b,b,c,c,c\}$

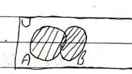
$A = \{3.1,\ 2.2,\ 1.3\}$
$B = \{3.a,\ 2.b,\ 3.c\}$

* **Operations on Sets**

1. **Union (∪)**
   The union of the two set A and B (denoted $A \cup B$), is set of all elements that are in A or in both.

   Defination: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

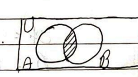
   Example: If $A = \{1,2,3\}$ and $B = \{3,4,5\}$, then
   $A \cup B = \{1,2,3,4,5\}$



2. **Intersection (∩)**
   The intersection of two sets A and B (denoted $A \cap B$), is the set of all elementes that are common to ~~bo~~ both A and B.

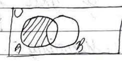
   Definition: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

   Example: If $A = \{1,2,3\}$ and $B = \{3,4,5\}$, then
   $A \cap B = \{3\}$



3. **Difference (-) or (\\)**
   The difference between set A and set B, denoted as $A - B$ or $A \setminus B$, is the set containing all elements that are ~~not~~ in set A but not in set B

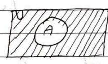
Defination : $A - B = \{x \mid x \in A$ and $x \in B\}$
Example : If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$
Then
$A - B = \{1, 2\}$



4. complement (A') or (A^c)
The complement of a set A (denoted A^c or A') is the set of all elements in the universal set (U) that are not in A. The universal set U is the set containing all possible elements under consideration.

Definition : $A^c = \{x \mid x \in U$ and $x \notin A\}$

Example : If the universal set $U = \{1, 2, 3, 4, 5\}$ and $A = \{1, 2\}$, then $A^c = \{3, 4, 5\}$.



* Principle of Inclusion and Exclusion :
The principle of Inclusion and exlusion (PIE) helps us count the number of elements in the unioan of sevral sets while avoiding double counting of element that belong to more than one set

- For two sets

If you have two sets A & B
$|A \cup B| = |A| + |B| - |A \cap B|$

Meaning :
- you fiest add all elements from both sets.
- But if an elements is in both A and B it gets counted twice - so you subtract the intersection once.

Example :
Let
$A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$
Then
$|A| = 4, |B| = 4, |A \cap B| = 2$
so,
$|A \cup B| = 4 + 4 - 2 = 6$

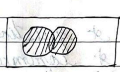
- For three sets
A, B, C
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

Meaning ;
1. add all single sets
2. subtract the overlaps pair
3. ~~Add~~ Add the overlap of all three

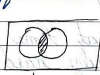
Example :
- $|A| = 10, |B| = 15, |C| = 20$
- $|A \cap B| = 5, |A \cap C| = 3, |B \cap C| = 7$
- $|A \cap B \cap C| = 2$

Then
$|A \cup B \cup C| = 10 + 15 + 20 - 5 - 3 - 7 + 2 = 32$

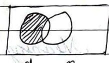
* Venn Diagram
A visual way to repesent sets, their relationships and operations. Circle show sets, their overlap shows intersections, and disjoints area show differences.



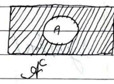
$A \cup B$



$A \cap B$



$A - B$



A^c

* Propositional Logic :
Propositional logic is the branch of logic that deals with statements (propositios) that can be either True or False but not both.
Logic is the scince of reasoning. it helps determine whether statements are true or false.

e.g. statement — truth value
1 + 1 = 2 — True
the sky is green — false

Non - Propositions

- What time is it → A question
- study hard → A command
- x + 1 = 2 → Depends on x

Symbol	Name	Meaning
$\sim p$	Negation	not p
$p \wedge q$	Conjunction	p and q
$p \vee q$	Disjunction	p or q
$p \oplus q$	Exclusive or	p or q, but not both
$p \rightarrow q$	Implication	if p then q
$p \leftrightarrow q$	Biconditional	p if and only if q

conditional →

* Teuth Table

p	q	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

p	$\sim p$
T	F
F	T

Questions "constuuction of T. Table"

1 $p \wedge (\sim q \vee q)$

p	q	$\sim q$	$\sim q \vee q$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	T	T

2. $\sim (p \vee q) \vee (\sim p \wedge \sim q)$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	T	F	F	F	F	F
T	F	T	F	F	T	F	F
F	T	T	F	T	F	F	F
F	F	F	T	T	T	T	T

3 $\sim (p \vee q) \wedge (\sim p \vee q)$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$(\sim p \vee q)$	$\sim(p \vee q) \wedge (\sim p \vee q)$
T	T	T	F	F	T	F
T	F	T	F	F	F	F
F	T	T	F	T	T	F
F	F	F	T	T	T	T

4. $p \wedge (q \vee r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

(Logical) or

* Propositional equivalence:

propositional equivalence means that two propositions (statement) are logically equivalent if they have the same teuth value in every possible situation.

Logical Equivalence is doneted by $\equiv$ or $\leftrightarrow$

e.g.:

1) $p \wedge T \equiv p$

p	T	$p \wedge T$
T	T	T
F	T	F

note : T = all values will true
p = duo to single p variable

2) $(p \equiv q) \equiv [(p \wedge q) \vee (\sim p \wedge \sim q)]$

p	q	$\sim p$	$\sim q$	$p \equiv q$	$(p \wedge q)$	$\sim p \wedge \sim q$	$a \vee b$
T	T	F	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

it ~~equ~~ equivalence ✓

3. $p \wedge (q \vee r)$ & $(p \wedge q) \vee (p \wedge r)$

p	q	r	$(q \vee r)$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

* Teanslating English Sentances into Logical Expeetion

it is a process that helps make ideas pecise, clear, and unambiguous for peoblem solving and ~~reos~~ reasoning. The process follows simple steps and uses well define rules and symbols from propositionl logic.

* Why translate Eenlish to Logic

Eenglish sentences can be ambiguous teanslating them remove confusion and making statement formal and ~~esv~~ easier to analyze.

e.g. "It is raining and cold."
translate to $p \wedge q$, if p: "it is raining"
q: "it is cold"

* Step-by-Step Translation Process

1. Identify Proposition
   Break down the sentance into simple statement / propositions.
2. Spot Logical & Connectives
   Words like "and", "or", "not", "if... then" indicates specific logic for combining propositions.
3. Assign Variables
   Assign variables for each propositions commonly P, Q, R, ...
4. Build the Expression
   Combine variables using logical symbols based on english connectives

"and": $p \wedge q$
"or": $p \vee q$
"not": $\neg p$
"if P then Q": $P \to Q$
"P if and only if Q": $P \leftrightarrow Q$

Unit - 2

* Relations and Functions

Relations:
a relation is a way of showing a connection or relationship between two sets. A relation R from a set A to set B is a subset of the Cartesian product $A \times B$

Cartesian product ($A \times B$):
The set of all possible ordered pairs (a, b), where $a \in A$ and $b \in B$

Notation:
If an ordered pair (a, b) is in the relation R, we write it as $(a, b) \in R$ or in tradition $(a R b)$.
Example:
If $R = \{(1,p), (1,q), (2,q)\}$

Domain:
The set of all first elements (input) in the ordered pair of R
domain = $\{1, 2\}$ // from above ↑

Range:
The set of all second elements (outputs) in the ordered pairs of R.
Rang = $\{p, q\}$

* Types / Properties of Relation
A Core Relations

1. Reflexive:
A Relation R on set A is said to be reflexive if every element of A is related to itself.
Example: $(x, x) \in R \ \forall x \in A$
$A = \{1, 2, 3\}$ then $(1,1), (2,2), (3,3)$
✓ $R = \{(1,1), (2,2), (3,3)\}$
× $R = \{(1,1), (2,2)\}$ // wrong
✓ $R = \{\underbrace{(1,1), (2,2), (3,3)}_{\text{imp}}, \underbrace{(1,2)}_{\text{optional}}\}$

2. Symmetric:
A relation R on set A is said to be symmetric if whenever (a, b) is in R, then (b, a) is also in R.
Example: xRy then $yRx \ \forall x, y \in A$
$A = \{1, 2, 3\}$
✓ $R = \{(1,2), (2,1), (2,3), (3,2)\}$
- $(1,2) \to (2,1)$ also exists
- $(2,3) \to (3,2)$ also exists

✓ $R_2 = \{(1,2), (1,3), (2,1), (2,2), (3,1)\}$
✓ $R_3 = \{\}$ default
× $R_4 = \{(1,2), (1,3), (3,1)\}$

3. Transitive
A relation R on a set A is said to be transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then (a, c) must also be in R. $A \to B$ $B \to C$
Example:
$A = \{1, 2, 3\}$
✓ $R = \{(1,1), (2,2)\}$ // b not same so do not need a, c
× $R = \{(1,2), (2,3)\}$
✓ $R = \{\}$
✓ $R = \{(1,1), \underbrace{(1,2)}_{a\,b}, \underbrace{(2,1)}_{b\,c}\}$

4. Antisymmetric
An antisymmetric relation is a relation where if one element is related to another and vice versa, then both elements must be the same.
if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$
Example:
$A = \{1, 2, 3\}$
× $R = \{(1,2), (2,2), (2,1)\}$
✓ $R = \{(1,1), (2,2), (1,3)\}$

note: ~~opposite~~ opposite off symmetric

B. Special Relations

1. Equivalence :

An equivalence relation is a special type of relation that combines three important properties: Reflexive, Symmetric and Transitive.

ex:

$A = \{1, 2, 3\}$

✓ $R = \{(1,1), (2,2), (3,3)\}$ R / S / T ✓ ✓ ✓

✓ $R = \{(1,1), (2,2) (3,3), (2,1), (1,2)\}$ R / S / T ✓ ✓ ✓

× $R = \{(1,1), (2,2), (3,3), (3,2), (1,3)\}$ R / S / T ✓ × ×

× $R = \{\ \}$ R / S / T × ✓ ✓

2. Partial order : ×

Combination of Reflexive, Anti-symmetric and Transitive. Some pairs are not cmp

Example :

$A = \{1, 2, 3\}$

✓ $R = \{(1,1), (2,2), (3,3)\}$ R / A / T ✓ ✓ ✓

$R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$ R / A / T ✓ ✓ ✓

3. Total order : ×

A total order is a ordering relation where every pair of element can be compared

ex:

1) if A is any set of all positive integers [A ; ÷] is not Tos

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ... \infty\}$

$4 \div 2$ ✓
$2 \div 3$ × } Tos : total order set

2) if A is set of all real no then the poset [A ; ≤] is Tos

↖ partially ordered set

Q which of the following pairs of elemen are comparable in the poset (z ÷) ?

a) 2, 8 b) 4, 6 c) 5, 5

→

a) 2 and 8 are comparable 2/8

b) 4 and 6 are not comparable because neither 4/6 nor 6/4.

c) 5/5 is comparable.

* Representation methods of relation

1] Matrix
2) directed graph
3) tabular form
4) Arrow diagram / eg $A = \{1, 2, 3, 4\}$

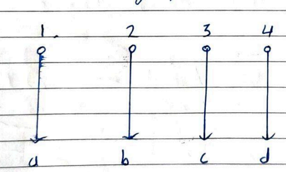
$B = \{a, b, c, d\}$

$R = \{(1,a), (2,b), (3,c), (4,d)\}$

1) Matrix Representation

$$\begin{array}{c|cccc|} & a & b & c & d \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

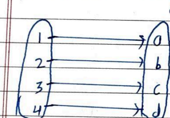
2) Directed graph (digraph)



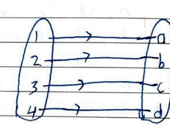
3) Tabular form

	a	b	c	d
1	✓			
2		✓		
3			✓	
4				✓

4) Arrow diagram



or

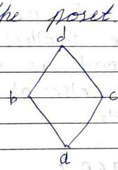


Chain & Antichain

A subset $C \subseteq P$ is called chain if every pair of elements in C is comparable.

A subset $A \subseteq P$ is called an antichain if no two distinct elements of A are comparable

so an antichain is a subset whose elements are pairwise incomparable in the poset.



$A = \{a, b, c, d\}$

$\left.\begin{array}{l}\{a, b\} \\ \{a, b, d\}\end{array}\right\}$ its chain

$\{b, c\}$ // its antichain

example:

$A = \{1, 2, 3, 4, 5, 6, 9, 12, 18\}$

$a \leq b$ if and only if a/b (a divide b)

chain	Antichain
$\{1, 2, 4\}$	$\{2, 3, 4\}$
$\{1, 2, 4, 12\}$	$\{2, 3, 4, 5\}$
$\{3, 6, 9, 18\}$	$\{3, 4\}$

* Closure and warshall Algorithm

Transitive Closure

A relation R on set A is a subset of the cartesian product $A \times A$. It consists of ordered pairs of elements from A. The closure of a relation refers to the smallest relation that contains R and satisfies certain properties.

1. Reflexive ~~closer~~ closure

The reflexive closure of a relation R on set A is the smallest relation R' that contains R and is reflexive. This means that every element in A is related to itself

formula : $R' = A \cup \{(a,a) \mid a \in A\}$

Example :

Let $A = \{1, 2\}$ and $R = \{(1,2)\}$. The reflexive closure of R is:

$R' = \{(1,2), (1,1), (2,2)\}$

2. Symmetric closure

The symmetric closure of a relation R on set A is the smallest relation R' that contains R and is symmetric. This means that if $(a,b) \in R'$, then $(b,a) \in R'$

formula :

$R' = R \cup \{(b,a) \mid (a,b) \in R\}$

Example :

Let $A = \{1, 2\}$ and $R = \{(1,2)\}$. The symmetric closure of R is : $R' = \{(1,2), (2,1)\}$

3. Transitive closure

The transitive closure of a relation R on a set A is the smallest relation R' that contains R and is transitive. this means that if $(a,b) \in R$ and $(b,c) \in R'$, then $(a,c) \in R'$

Algorithm (Warshall's Algorithm)

1. Initialize $R' = R$.
2. for each $(a,b) \in R'$ and $(b,c) \in R$, add (a,c) to R'.

ex:

Let $A = \{1, 2, 3\}$ and $R = \{(1,2), (2,3)\}$

the closure is: $R' = \{(1,2), (2,3), (1,3)\}$.

ex 2

Q. Let $A = \{1, 2, 3, 4\}$ and $R \{(1,2), (2,3), (3,4), (2,1)\}$ find the transitive closure of R using warshall's algorithm

$\rightarrow$

$A = \{1, 2, 3, 4\}$

$R = \{(1,2), (2,3), (3,4), (2,1)\}$

$|A| = 4$

$$M_R = W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

step^{1st} :

1^{st} column $= \{2\}$

1^{st} Row $= \{2\}$

$R = \{2, 2\}$

$$W = 1 \quad \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

step 2 :

2^{nd} column $= \{1, 2\}$

2^{nd} row $= \{1, 2, 3\}$

$R = (1,1), (1,2), (1,3), (2,1), (2,2), (2,3)$

$$
W_2 = \begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

step 3:

3^{rd} column $\{1, 2\}$

3^{rd} row $\{4\}$

$R = \{(1,4)\ (2,4)\}$

$$
W_3 = \begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

step 4: last step

4^{th} column $= 1, 2, 3$

4^{th} row $= -$

$R = \phi$ (Null)

$W_4 = W_3$

Transitive closure $= \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), \cancel{3,} (3,4)\}$.

UNIT 3 - Probability and Combinatorics

Introduction to Probability:

Probability means possibility. It is a branch of mathematics that deals with the occurrence of a random event or experiment. The value is expressed from zero to one. Probability has been introduced in Maths to predict how likely events are to happen.

Formula: P(E) = Number of favourable outcomes/Total Number of outcomes

Example 1:
There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?
Ans: The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows, i.e. 2/6 = 1/3.

Sample Space and Events

the sample space is the set of all possible outcomes of a random experiment. An experiment is any process that gives a result

Example:

Experiment: Spinning a European roulette wheel

Sample space: {0, 1, 2, 3, 4, 5, ..., 36}

Example 2 :

When Two Dice are Rolled

When rolling two dice, the sample space represents all the combinations of outcomes that can occur. It consists of pairs of numbers ranging from (1, 1) to (6, 6), consisting of 66= 36 pairs. It helps in calculating probabilities for various sums or events involving two dice.

The following is a table of the sample space of rolling two dice.

Dice 1 \ Dice 2	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

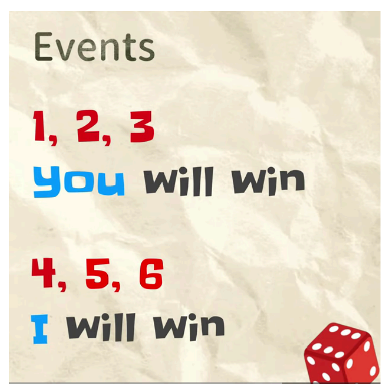
Question : How many possible outcomes are there when rolling a fair six-sided die?

Solution:

There are 6 possible outcomes when rolling a fair six-sided die.

Event:

In Probability, an event can be defined as any outcome or set of outcomes from a random experiment. In other words, an event in probability is a subset of the respective sample space.



Axioms of Probability

There are three primary axioms:

1. Non-Negativity
The Rule: For any event A, the probability P(A) must be greater than or equal to zero.
P(A) >= 0
What it means: You can't have a negative chance of something happening. The lowest possible probability is 0, which represents an impossible event.

2. Unitarity (Certainty)
The Rule: The probability of the entire sample space S is exactly 1.
P(S) = 1
What it means: In any experiment, something must happen. If you roll a die, the probability that the result is either a 1, 2, 3, 4, 5, or 6 is 100% (or 1).

3. Additivity (Mutually Exclusive Events)

The Rule: If you have a sequence of events A1, A2, A3, … that are mutually exclusive (they cannot happen at the same time), then the probability of any of them occurring is the sum of their individual probabilities.

$$P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \ldots$$

What it means: If you want to know the chance of rolling a 2 **or** a 5 on a die, you simply add P(2) + P(5). This only works because you cannot roll a 2 and a 5 at the exact same time.

Note: if the chance between 0 and 1 then how it can possible because in a dice there are 6 sample spaces.
Explanation:
The "Pizza" Analogy

In probability, the entire sample space is always equal to 1 (one whole pizza).

If you have a 6-sided die, you are essentially cutting that one pizza into 6 equal slices.

- Each slice represents one outcome (rolling a 1, 2, 3, 4, 5, or 6).
- The "size" of each slice is 1/6 of the pizza.
- The number 1/6 is approximately **0.166**, which is between 0 and 1

Conditional Probability

Conditional probability is the likelihood of an event occurring based on the fact that another event has already happened.
In regular probability, you look at the chances of something happening out of all possible outcomes.
In conditional probability, you **narrow your focus** because you have extra information that rules out some of those outcomes.
Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Where:

- $P(A|B)$: The probability of event **A** happening, given that **B** has already occurred.
- $P(A \cap B)$: The "joint probability"—the chance that both A and B happen together.
- $P(B)$: The probability of the condition (event B) happening on its own.

Bayes' Theorem

Bayes' Theorem is a fundamental concept in probability that allows you to update the probability of a hypothesis as more evidence or information becomes available. In simple terms, it helps you figure out: "Given that this specific thing just happened, what is the likelihood that it was caused by X?

Formula:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

:

Where:

- $P(A|B)$ **(Posterior):** The probability of event A occurring given that B is true.
- $P(B|A)$ **(Likelihood):** The probability of event B occurring given that A is true.
- $P(A)$ **(Prior):** The initial probability of event A before seeing the evidence.
- $P(B)$ **(Evidence):** The total probability of event B occurring under all possible scenarios.

Example:

The Scenario:

- A rare disease affects **1%** of the population.
- A medical test for this disease is **95% accurate** (it returns a positive result for 95% of people who have it).
- The test also has a **2% false positive rate** (it returns a positive result for 2% of healthy people).

The Question: If you take the test and it comes back **positive**, what is the actual probability that you have the disease?

---

1. Identify the Variables

- $P(D)$**:** Probability of having the disease = **0.01**
- $P(H)$**:** Probability of being healthy = **0.99**
- $P(+|D)$**:** Probability of testing positive if you have it = **0.95**
- $P(+|H)$**:** Probability of testing positive if you are healthy = **0.02**

2. Calculate the Total Evidence ($P(+)$)

Before we find the final answer, we need the total probability of anyone testing positive. This happens in two ways: you have it and test positive, OR you don't have it and test positive.

$$P(+) = (P(+|D) \cdot P(D)) + (P(+|H) \cdot P(H))$$

$$P(+) = (0.95 \cdot 0.01) + (0.02 \cdot 0.99)$$

$$P(+) = 0.0095 + 0.0198 = \mathbf{0.0293}$$

3. Apply Bayes' Theorem

Now we want to find $P(D|+)$—the probability you have the disease given a positive test.

$$P(D|+) = \frac{P(+|D) \cdot P(D)}{P(+)}$$

$$P(D|+) = \frac{0.0095}{0.0293}$$

$$P(D|+) \approx \mathbf{0.324}$$

The Result

Even though the test is "95% accurate," the probability that you actually have the disease is only about **32.4%**.

Why? Because the disease is so rare (1%) that the number of "false positives" from the healthy 99% of the population outweighs the number of "true positives" from the sick 1%.

> **Key Takeaway:** Bayes' Theorem teaches us that "common things are common." New evidence must always be weighed against the original "prior" probability.

Basics of Counting(skiped)

Rule of Sum and Product(skiped)

Generalized Permutations and Combinations(skiped)

Binomial Coefficient Identities(skiped)

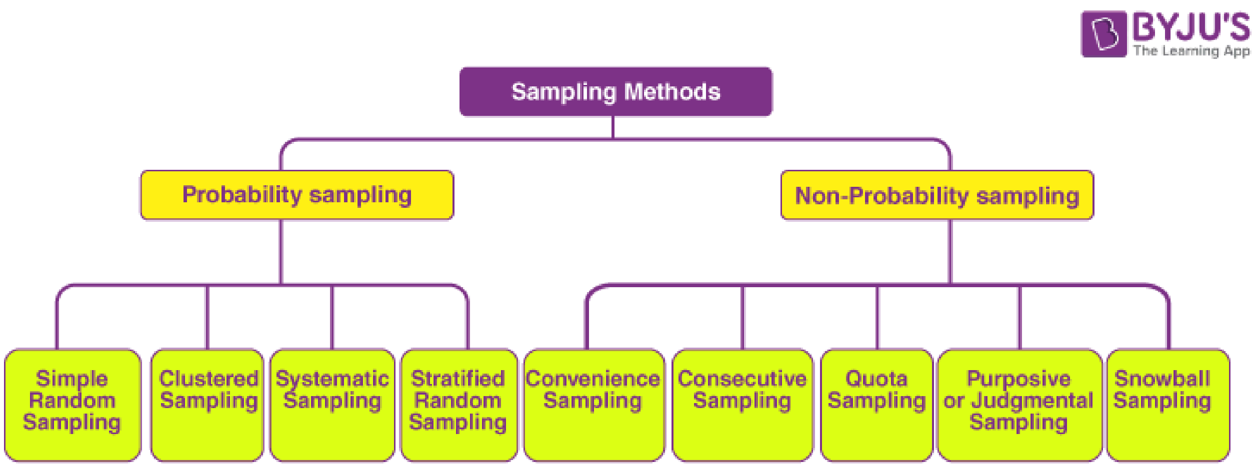
Pascal’s Triangle(skiped)

Unit 4 - Descriptive statistics

Sampling:

In Statistics, the sampling method or sampling technique is the process of studying the population by gathering information and analyzing that data. It is the basis of the data where the sample space is enormous.

Types of Sampling methods:



Probability sampling:

The probability sampling method utilizes some form of random selection. In this method, all the eligible individuals have a chance of selecting the sample from the whole sample space. This method is more time consuming and expensive than the non-probability sampling method. The benefit of using probability sampling is that it guarantees the sample that should be the representative of the population.

Simple Random Sampling

In simple random sampling technique, every item in the population has an equal and likely chance of being selected in the sample. Since the item selection entirely depends on the chance, this method is known as "**Method of chance Selection**". As the sample size is large, and the item is chosen randomly, it is known as "**Representative Sampling**".

Example:

Suppose we want to select a simple random sample of 200 students from a school. Here, we can assign a number to every student in the school database from 1 to 500 and use a random number generator to select a sample of 200 numbers.

Systematic Sampling

In the systematic sampling method, the items are selected from the target population by selecting the random selection point and selecting the other methods after a fixed sample interval. It is calculated by dividing the total population size by the desired population size.

Example:

Suppose the names of 300 students of a school are sorted in the reverse alphabetical order. To select a sample in a systematic sampling method, we have to choose some 15 students by randomly selecting a starting number, say 5. From number 5 onwards, will select every 15th person from the sorted list. Finally, we can end up with a sample of some students.

Stratified Sampling

In a stratified sampling method, the total population is divided into smaller groups to complete the sampling process. The small group is formed based on a few characteristics in the population. After separating the population into a smaller group, the statisticians randomly select the sample.

For example, there are three bags (A, B and C), each with different balls. Bag A has 50 balls, bag B has 100 balls, and bag C has 200 balls. We have to choose a sample of balls from each bag proportionally. Suppose 5 balls from bag A, 10 balls from bag B and 20 balls from bag C.

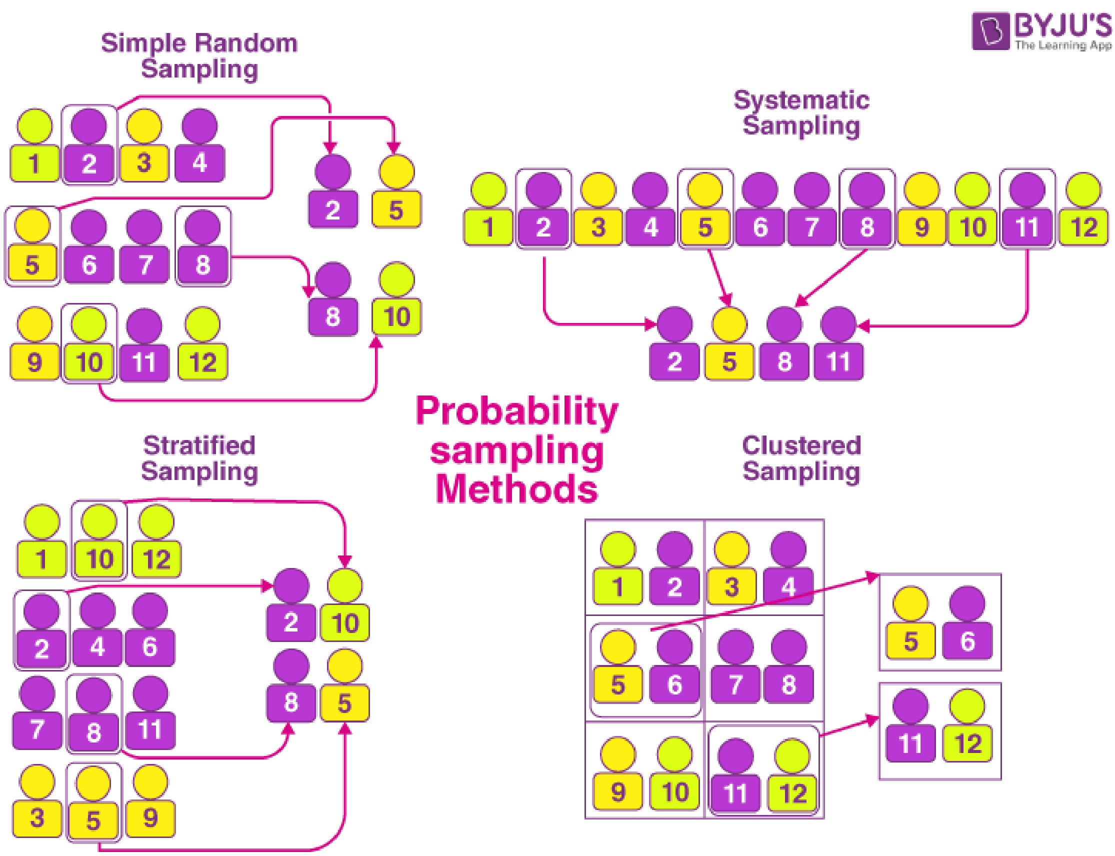
Clustered Sampling

In the clustered sampling method, the cluster or group of people are formed from the population set. The group has similar significatory characteristics. Also, they have an equal chance of being a part of the sample. This method uses simple random sampling for the cluster of population.

Example:

An educational institution has ten branches across the country with almost the number of students. If we want to collect some data regarding facilities and other things, we can't travel to every unit to collect the required data. Hence, we can use random sampling to select three or four branches as clusters.

All these four methods can be understood in a better manner with the help of the figure given below. The figure contains various examples of how samples will be taken from the population using different techniques.



Non probability sampling:

The non-probability sampling method is a technique in which the researcher selects the sample based on subjective judgment rather than the random selection. In this method, not all the members of the population have a chance to participate in the study.

Convenience Sampling

In a convenience sampling method, the samples are selected from the population directly because they are conveniently available for the researcher. The samples are easy to select, and the researcher did not choose the sample that outlines the entire population.

Example:

In researching customer support services in a particular region, we ask your few customers to complete a survey on the products after the purchase. This is a convenient way to collect data. Still, as we only surveyed customers taking the same product. At the same time, the sample is not representative of all the customers in that area.

Consecutive Sampling

Consecutive sampling is similar to convenience sampling with a slight variation. The researcher picks a single person or a group of people for sampling. Then the researcher researches for a period of time to analyze the result and move to another group if needed.

Quota Sampling

In the quota sampling method, the researcher forms a sample that involves the individuals to represent the population based on specific traits or qualities. The researcher chooses the sample subsets that bring the useful collection of data that generalizes the entire population.

Learn more about quota sampling here.

Purposive or Judgmental Sampling

In purposive sampling, the samples are selected only based on the researcher's knowledge. As their knowledge is instrumental in creating the samples, there are the chances of obtaining highly accurate answers with a minimum marginal error. It is also known as judgmental sampling or authoritative sampling.

Snowball Sampling

Snowball sampling is also known as a chain-referral sampling technique. In this method, the samples have traits that are difficult to find. So, each identified member of a population is asked to find the other sampling units. Those sampling units also belong to the same targeted population.

Probability sampling vs Non-probability Sampling Methods

Probability Sampling Methods	Non-probability Sampling Methods
Probability Sampling is a sampling technique in which samples taken from a larger population are chosen based on probability theory.	Non-probability sampling method is a technique in which the researcher chooses samples based on subjective judgment, preferably random selection.
These are also known as Random sampling methods.	These are also called non-random sampling methods.
These are used for research which is conclusive.	These are used for research which is exploratory.
These involve a long time to get the data.	These are easy ways to collect the data quickly.
There is an underlying hypothesis in probability sampling before the study starts. Also, the objective of this method is to validate the defined hypothesis.	The hypothesis is derived later by conducting the research study in the case of non-probability sampling.

Frequency Distributions:

Mean:

Mean (in statistics) is the average of a set of numbers. It is one of the most important measures of central tendency in distributed data. To calculate the mean, add all the values in the data set and then divide the total by the number of values.

Class No. of defective bulbs	Class mark (x)	No. of boxes (f)	$f.x$
$0-2$	1	3	3
$2-4$	3	4	12
$4-6$	5	5	25
$6-8$	7	3	21
$8-10$	9	1	9
Total	–	$\sum f = 16$	$\sum f.x = 70$

$$Mean = \overline{X} = \frac{\sum f.x}{\sum f} = \frac{70}{16} = 4.38 \approx 4$$

Median:

The median is the middle value of the dataset when arranged in ascending or descending order. If not then ordered and apply following formula.

Median Formula

if n is odd,

$$median = \left(\frac{n+1}{2}\right)^{th}$$

if n is even,

$$median = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$$

Median for group dataset:

continuous series

arrange data in ascending form

Marks	f	cf
5-10	7	7
10-15	15	22
15-20	24	46
20-25	31	77
25-30	42	119
30-35	30	149
35-40	26	175
40-45	15	190
45-50	10	200

med = size of $\left(\frac{N}{2}\right)$ th item $= \frac{200}{2} = 100$ th item

med = 25-30

$$\text{med} = l + \frac{N/2 - cf}{f} \times i$$

$l = 25$, $N/2 = 100$ $cf = 77$, $f = 42$, $i = 5$

$$25 + \frac{100-77}{42} \times 5 = 25 + 2.74 = 27.74$$

$\therefore$ Ans is 27.74

Mode:

Marks	Frequency
0 – 10	2
10 – 20	$5 \rightarrow f_0$
20 – 30	$6 \rightarrow f_1$
30 – 40	$5 \rightarrow f_2$
40 – 50	2

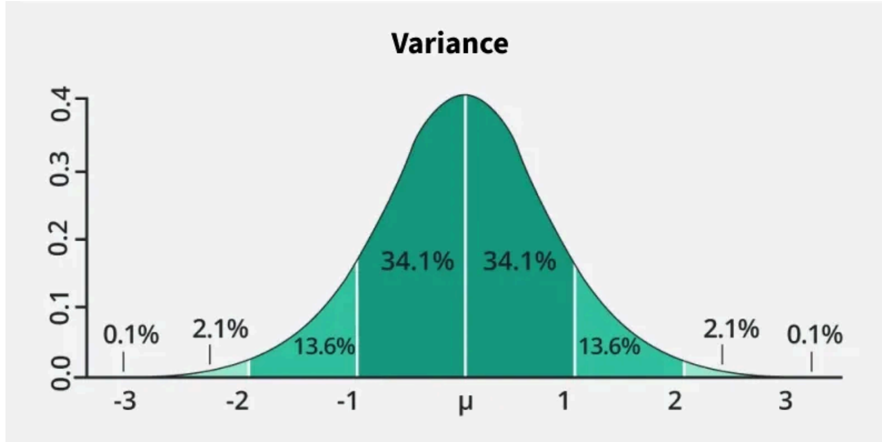
$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 20 + \left[\frac{6-5}{2 \times 6 - 5 - 5}\right] \times 10$$

$$= 25$$

Variance

Variance is a number that tells us how spread out the values in a data set are from the mean (average). It shows whether the numbers are close to the average or far away from it.



- If the variance is small, it means most numbers are close to the mean. If the variance is large, it means the numbers are spread out more widely.
- A higher variance indicates greater variability, meaning the data is spread, while a lower variance suggests the data points are closer to the mean.

How to Calculate Variance?

- **Step 1: Calculate the mean of the observation using the formula (Mean = Sum of Observations/Number of Observations)**
- **Step 2: Calculate the squared differences of the data values from the mean. (Data Value - Mean)^2**
- **Step 3: Calculate the average of the squared differences of the given values, which is called the variance of the data set**

(Variance = Sum of Squared Differences / Number of Observations

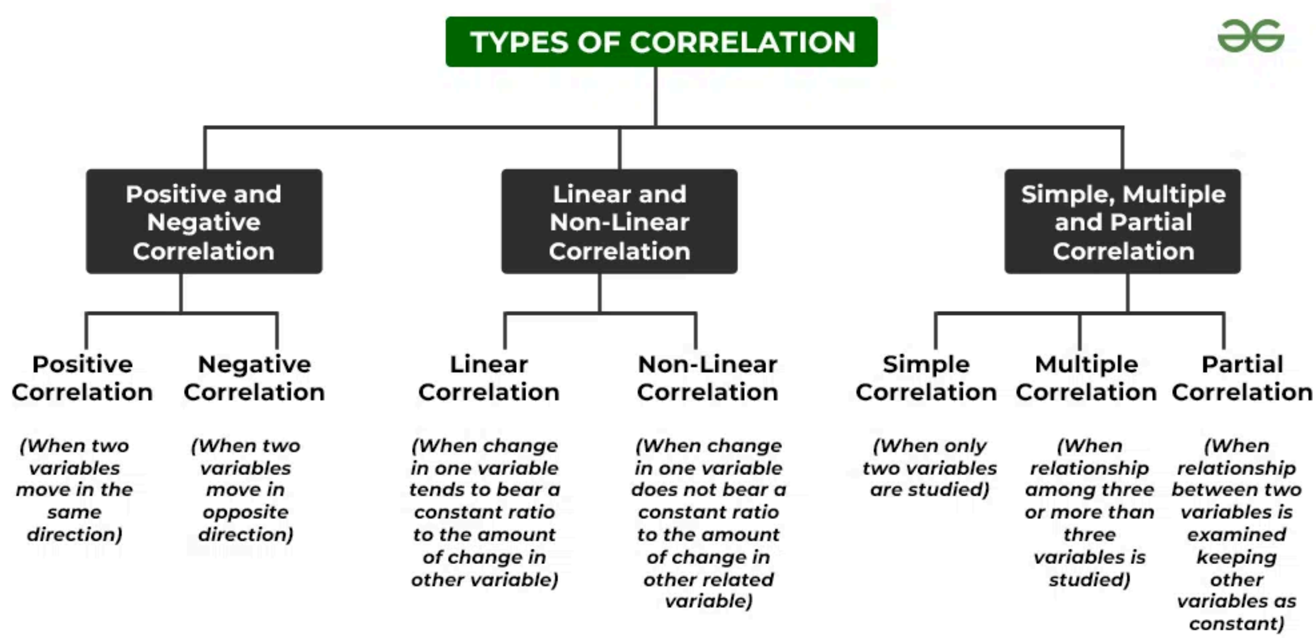
Standard Deviation for Grouped and Ungrouped Data. (Skipped)

Correlation

Correlation refers to the statistical technique used to determine the relationship between two variables. It quantifies how changes in one variable are associated with changes in another variable. The correlation coefficient, denoted as “r”

r, ranges from -1 to +1, where:

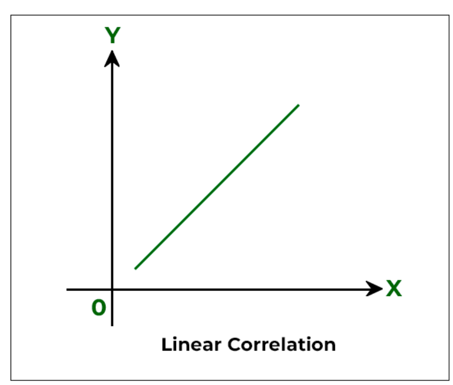
- +1 indicates a perfect positive correlation (both variables move in the same direction).
- -1 indicates a perfect negative correlation (one variable increases while the other decreases).
- 0 indicates no correlation (no relationship between the variables).



1. Linear Correlation:

When there is a constant change in the amount of one variable due to a change in another variable, it is known as Linear Correlation. This term is used when two variables change in the same ratio. If two variables that change in a fixed proportion are displayed on graph paper, a straight- line will be used to represent the relationship between them. As a result, it suggests a linear relationship.

X	10	15	20	25	30
Y	10	20	30	40	50

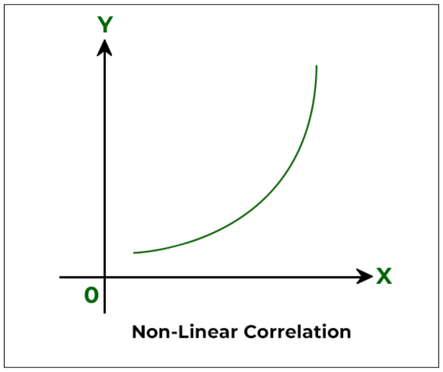


In the above graph, for every change in the variable X by 5 units there is a change of 10 units in variable Y. The ratio of change of variables X and Y in the above schedule is 1:2 and it remains the same, thus there is a linear relationship between the variables.

2. Non-Linear (Curvilinear) Correlation:

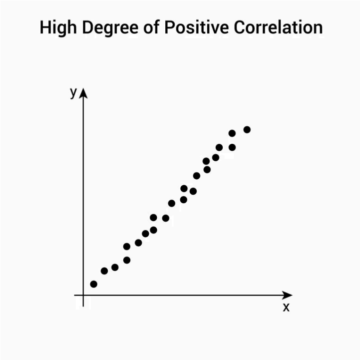
X (Fertilizers)	10	20	30	40	50
Y (Production of Grains)	7	12	19	25	35

When there is no constant change in the amount of one variable due to a change in another variable, it is known as a Non-Linear Correlation. This term is used when two variables do not change in the same ratio. This shows that it does not form a straight-line relationship. For example, the production of grains would not necessarily increase even if the use of fertilizers is doubled.

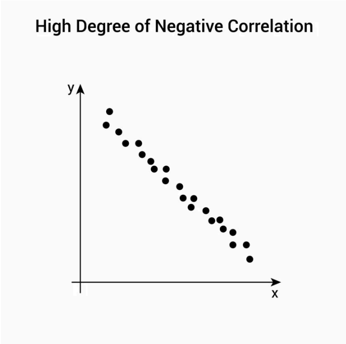


In the above schedule, there is no specific relationship between the variables. Even though both change in the same direction i.e. both are increasing, they change in different proportions. The ratio of change of variables X and Y in the above schedule is not the same, thus there is a non-linear relationship between the variables.

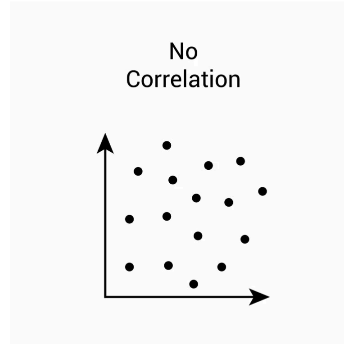
A **positive correlation** is a relationship between two variables in which both variables move in the same direction. Therefore, one variable increases as the other variable increases, or one variable decreases while the other decreases. An example of a positive correlation would be height and weight. Taller people tend to be heavier.



A **negative correlation** is a relationship between two variables in which an increase in one variable is associated with a decrease in the other. An example of a negative correlation would be the height above sea level and temperature. As you climb the mountain (increase in height), it gets colder (decrease in temperature).

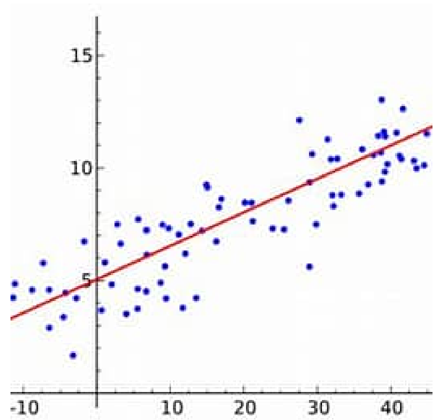


A **zero correlation** exists when there is no relationship between two variables. For example, there is no relationship between the amount of tea drunk and the level of intelligence.



Regression

Regression analysis is a set of statistical techniques used to understand the relationship between a dependent variable (the outcome or response variable) and one or more independent variables (predictors or features). The primary goal of regression is to model this relationship so that we can make predictions or understand how changes in the independent variables affect the dependent variable.



Two Regression Equations (Regression Line of X on Y and Y on X)(skipped)

Discrete Distributions(skipped)

Geometric Distribution(skipped)

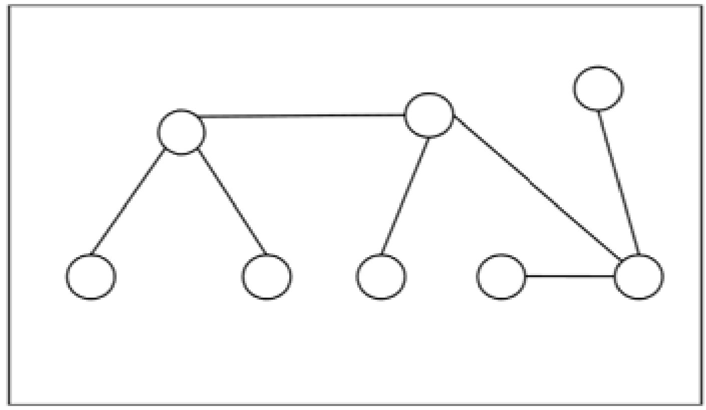
Binomial Distribution(skipped)

Poisson Distribution(skipped)

Introduction to Hypothesis Testing. (skipped)

UNIT 5 Tree and graph theory

Tree and its Properties



Definition − A Tree is a connected acyclic undirected graph. There is a unique path between every pair of vertices in G. A tree with N number of vertices contains (N-1) number of edges. The vertex which is of 0 degree is called root of the tree. The vertex which is of 1 degree is called leaf node of the tree and the degree of an internal node is at least 2.

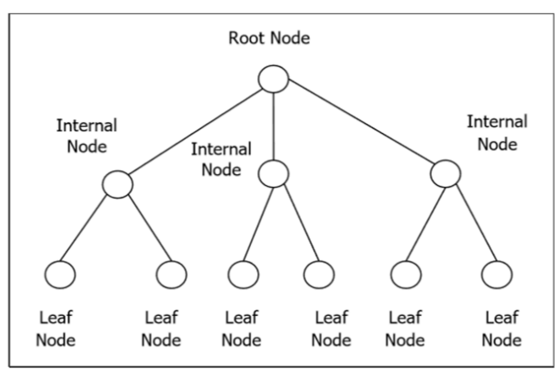
Key Terminology:

- **Node:** A fundamental part of a tree containing data.
- **Edge:** The link connecting two nodes.
- **Root:** The topmost node (no parent).
- **Leaf:** A node with no children.
- **Height/Depth:** The length of the path from the root to the deepest node.

Key Properties:

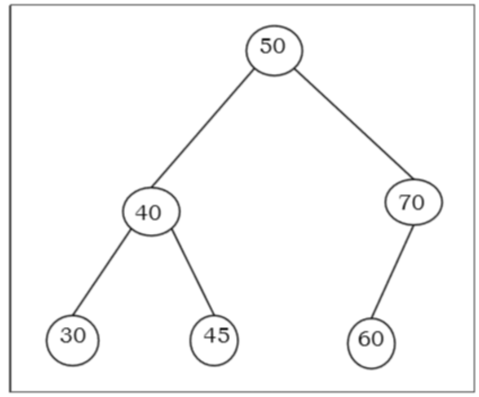
- If a tree has n vertices, it has exactly n-1 edges.
- There is exactly one path between any two vertices in a tree.
- A tree is a connected graph with no circuits.
- Adding one edge to a tree creates exactly one cycle.

Rooted Tree



A rooted tree G is a connected acyclic graph with a special node that is called the root of the tree and every edge directly or indirectly originates from the root. An ordered rooted tree is a rooted tree where the children of each internal vertex are ordered. If every internal vertex of a rooted tree has not more than m children, it is called an m-ary tree. If every internal vertex of a rooted tree has exactly m children, it is called a full m-ary tree. If m = 2, the rooted tree is called a binary tree.

Binary Search Tree



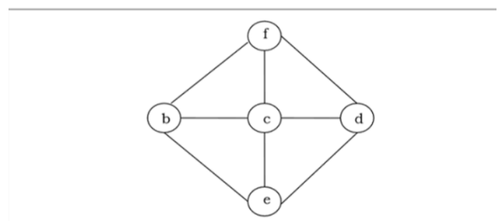
Binary Search tree is a binary tree which satisfies the following property −

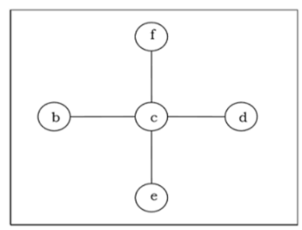
X in left sub-tree of vertex V, Value(X) ≤ Value (V)

Y in right sub-tree of vertex V, Value(Y) ≥ Value (V)

So, the value of all the vertices of the left sub-tree of an internal node V are less than or equal to V and the value of all the vertices of the right sub-tree of the internal node V are greater than or equal to V. The number of links from the root node to the deepest node is the height of the Binary Search Tree.

Spanning tree

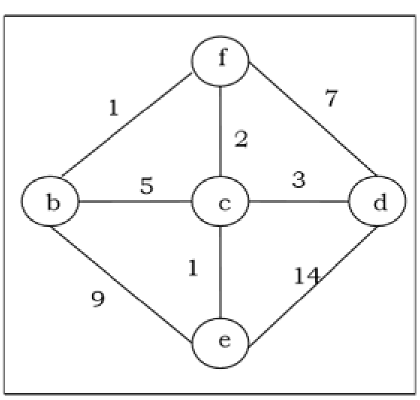


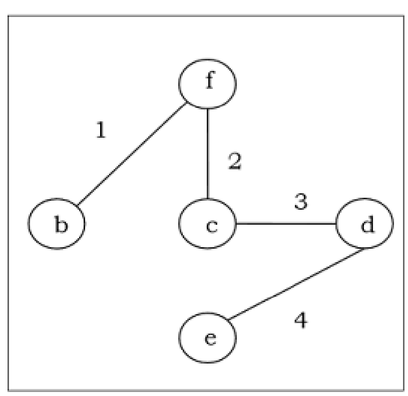


a Spanning Tree is a subset of a graph that includes (or "spans") every vertex in the graph but contains the minimum number of edges possible to keep the graph connected.

Minimum Spanning Tree

A spanning tree with assigned weight less than or equal to the weight of every possible spanning tree of a weighted, connected and undirected graph G, it is called minimum spanning tree (MST). The weight of a spanning tree is the sum of all the weights assigned to each edge of the spanning tree.





Kruskal's Algorithm

Kruskal's algorithm is a greedy algorithm that finds a minimum spanning tree for a connected weighted graph. It finds a tree of that graph which includes every vertex and the total weight of all the edges in the tree is less than or equal to every possible spanning tree.

Algorithm

Step 1 − Arrange all the edges of the given graph G(V,E) in ascending order as per their edge weight.

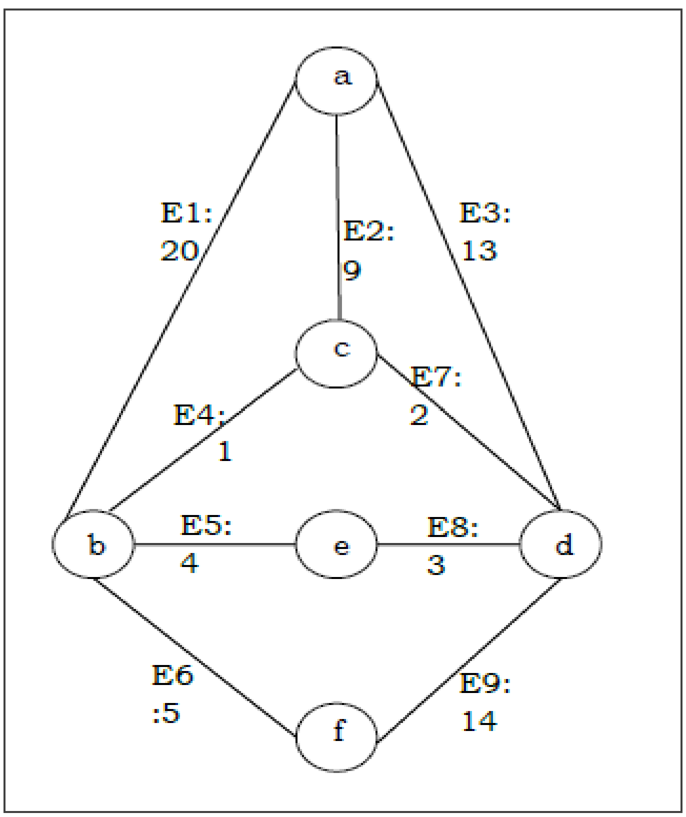
Step 2 − Choose the smallest weighted edge from the graph and check if it forms a cycle with the spanning tree formed so far.

Step 3 − If there is no cycle, include this edge to the spanning tree else discard it.

Step 4 − Repeat Step 2 and Step 3 until (V−1)number of edges are left in the spanning tree.

Problem: Suppose we want to find minimum spanning tree for the following graph G using Kruskals algorithm.

Problem:

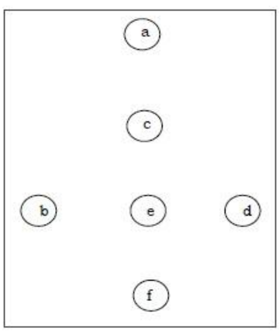


Solution:

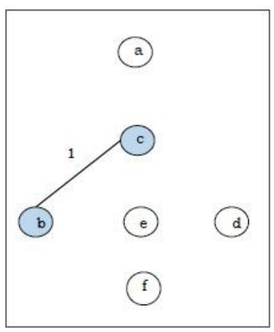
Edge No.	Vertex Pair	Edge Weight
E1	(a, b)	20
E2	(a, c)	9
E3	(a, d)	13
E4	(b, c)	1
E5	(b, e)	4
E6	(b, f)	5
E7	(c, d)	2
E8	(d, e)	3
E9	(d, f)	14

Now we will rearrange the table in ascending order with respect to Edge weight –

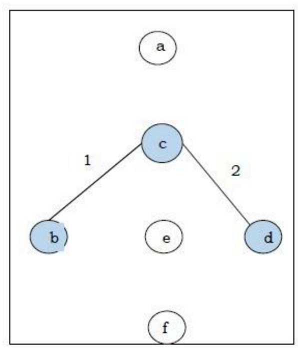
Edge No.	Vertex Pair	Edge Weight
E4	(b, c)	1
E7	(c, d)	2
E8	(d, e)	3
E5	(b, e)	4
E6	(b, f)	5
E2	(a, c)	9
E3	(a, d)	13
E9	(d, f)	14
E1	(a, b)	20



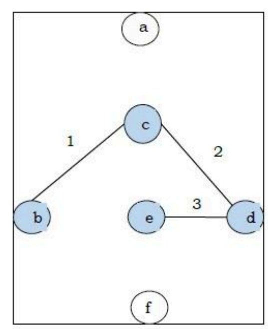
After adding vertices



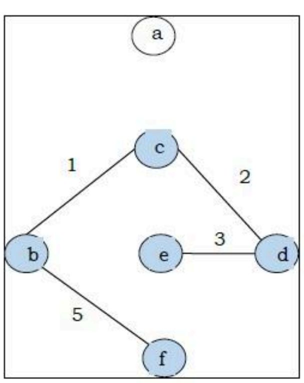
After adding edge E4



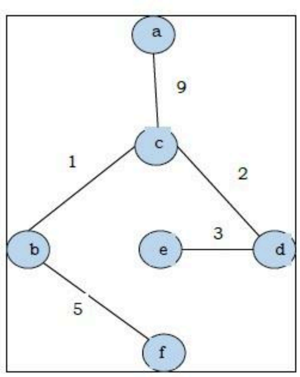
After adding edge E7



After adding edge E8



After adding edge E6
(don't add E5 since it forms cycle)



After adding edge E2

Since we got all the 5 edges in the last figure, we stop the algorithm and this is the minimal spanning tree and its total weight is :

(1+2+3+5+9)=20

Prim's Algorithm

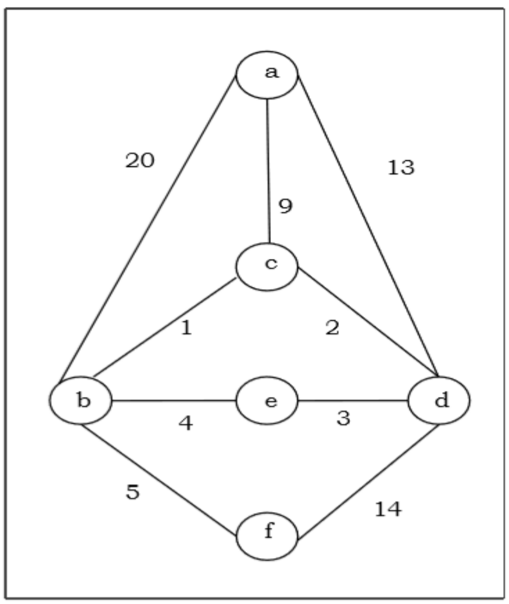
Prim's algorithm, discovered in 1930 by mathematicians, Vojtech Jarnik and Robert C. Prim, is a greedy algorithm that finds a minimum spanning tree for a connected weighted graph. It finds a tree of that graph which includes every vertex and the total weight of all the edges in the tree is less than or equal to every possible spanning tree. Prims algorithm is faster on dense graphs.

Algorithm

Initialize the minimal spanning tree with a single vertex, randomly chosen from the graph.
Repeat steps 3 and 4 until all the vertices are included in the tree.
Select an edge that connects the tree with a vertex not yet in the tree, so that the weight of the edge is minimal and inclusion of the edge does not form a cycle.
Add the selected edge and the vertex that it connects to the tree.

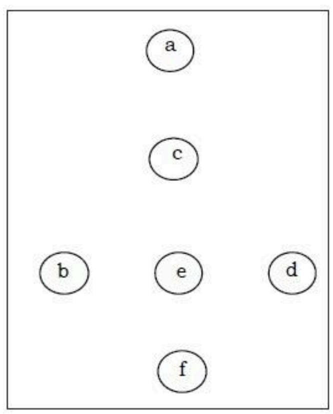
Problem

Suppose we want to find minimum spanning tree for the following graph G using Prims algorithm.

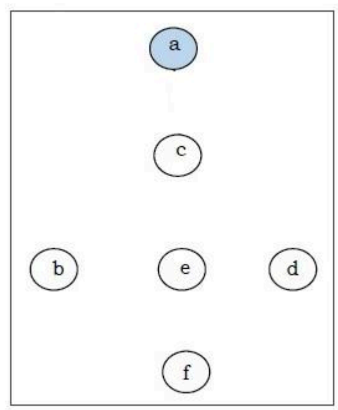


Solution:

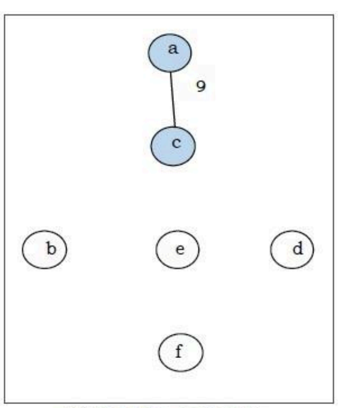
Here we start with the vertex a and proceed.



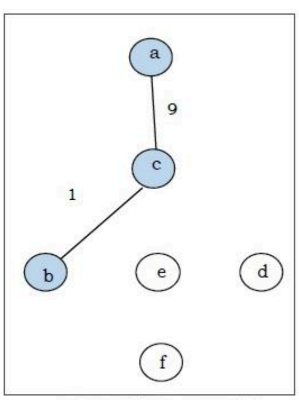
No vertices added



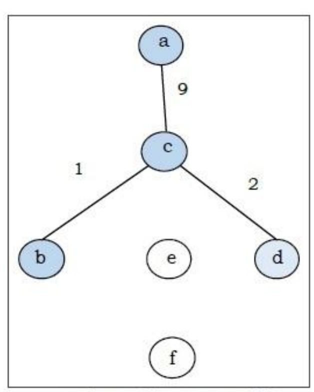
After adding vertex 'a'



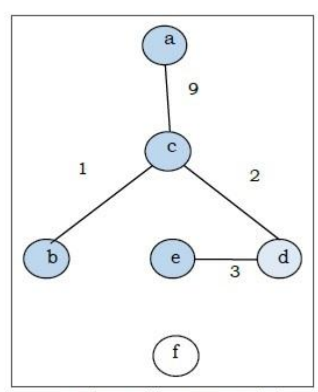
After adding vertex 'c'



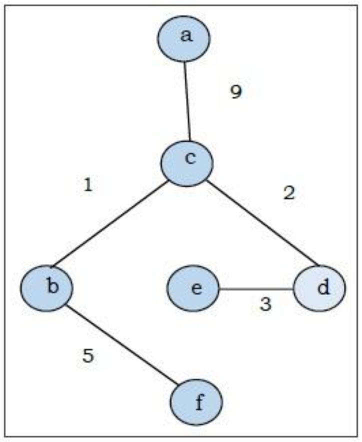
After adding vertex 'b'



After adding vertex 'd'



After adding vertex 'e'



After adding vertex 'f'

This is the minimal spanning tree and its total weight is

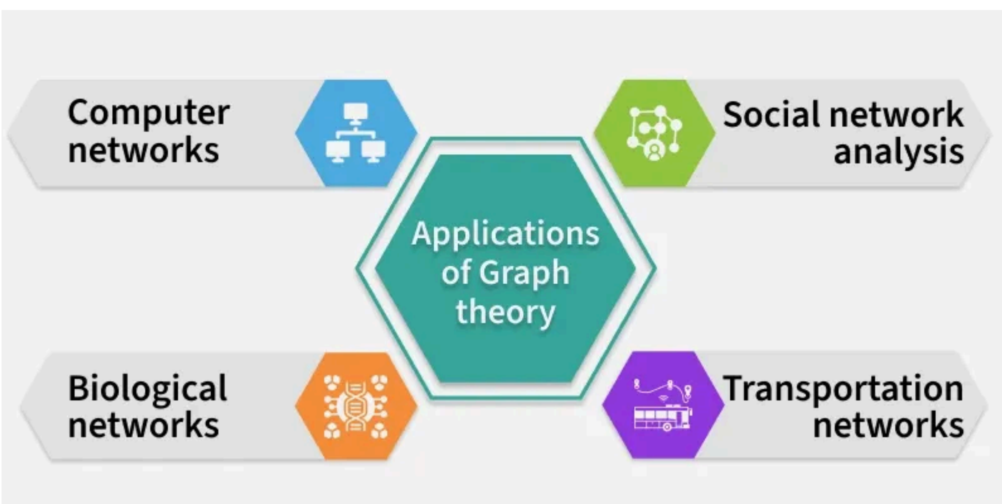
$(1+2+3+5+9)=20$

Graph theory

Applications of Graph Theory

Graph theory is a branch of mathematics that deals with the study of graphs, which are mathematical structures used to represent pairwise relationships between objects. A graph consists of two main components: vertices (also called nodes) and edges.

> Graph theory finds applications in diverse fields such as computer science, biology, sociology, and transportation, among others.
> Its versatility lies in its ability to model and analyze complex relationships and systems using graph-based representations.



1. Computer Networks (Digital Infrastructure)

Graph theory acts as the mathematical backbone for how data moves across the internet and private networks.

- **Designing Topologies:** It provides the blueprint for connecting devices. Whether using a **Star**, **Mesh**, or **Ring** configuration, graph theory ensures the layout is optimized for both performance and redundancy.
- **Routing Algorithms:** Much like a GPS for data, it determines the most efficient "roads" for packets to travel. This ensures that information reaches its destination with minimal delay (latency).
- **Traffic Optimization:** By analyzing the capacity of connections, the network can reroute data in real-time to avoid "digital traffic jams" or congestion.

2. Transportation Networks (Physical Infrastructure)

Similar to computer networks, transportation systems use graphs to map the movement of physical objects and people.

- **Modeling Infrastructure:** Intersections, airports, or train stations are treated as **nodes**, while roads, flight paths, or tracks are the **edges** connecting them.
- **Route Planning:** This is the core technology behind navigation apps. Algorithms calculate the **shortest or fastest path** between two points based on distance and real-time conditions.
- **Traffic & Resource Management:** City planners use graph analysis to identify bottlenecks and decide where to deploy resources, such as adding a new bus lane or adjusting smart traffic lights to improve flow.

3. Social Network Analysis (Human Interaction)

Beyond physical wires and roads, graph theory models the abstract connections between people and organizations.

- **Modeling Social Structures:** Individuals are represented as nodes, and their relationships (friendships, professional ties, or follows) are the edges.
- **Identifying Influencers:** Algorithms (like *Centrality*) help find the "hubs"—the key individuals who act as bridge-builders or major connectors within a community.
- **Predicting Information Spread:** By studying the network's density, analysts can forecast how quickly a viral trend, a piece of news, or even a behavior will propagate through a population.

4. Biological Networks (Life Sciences)

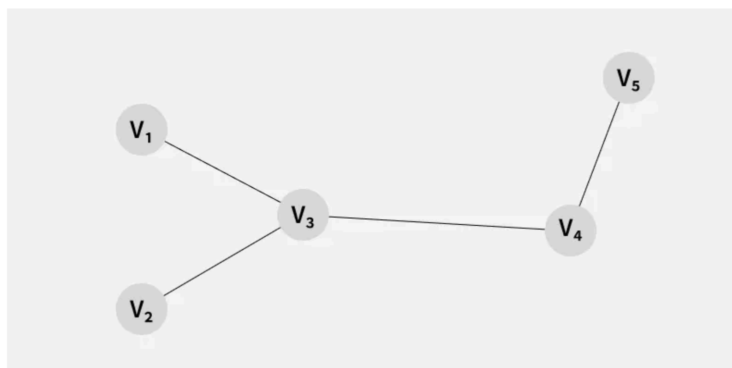
At the most granular level, graph theory helps scientists decode the complex "wiring" of life itself.

- **Modeling Interactions:** Genes, proteins, and metabolites are mapped as nodes. Their chemical reactions or functional relationships are the edges.
- **Protein-Protein Interaction (PPI):** Since proteins work in teams, graph theory identifies "protein complexes"—groups of nodes that work together to perform vital cellular functions.
- **Gene Function & Drug Discovery:**
  - **Predicting Function:** If an unknown gene is closely connected to a cluster of known genes, scientists can infer its purpose.
  - **Targeted Therapy:** By identifying "critical nodes" in a disease pathway, researchers can design drugs to disrupt those specific points, leading to more effective medical treatments.

Types of Graphs

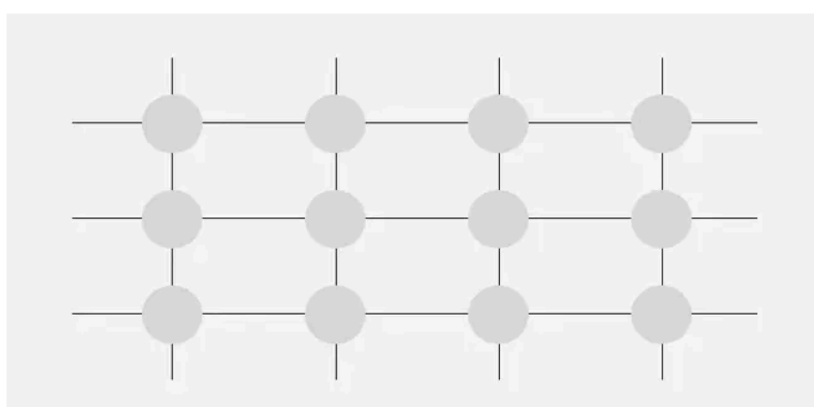
Finite Graphs

A finite graph is a graph with a finite number of vertices and edges. In other words, both the number of vertices and the number of edges in a finite graph are limited and can be counted. Finite graphs are used to represent real-world situations where there is a limited number of objects and their connections. They help in organizing, analyzing, and optimizing relationships in different applications.



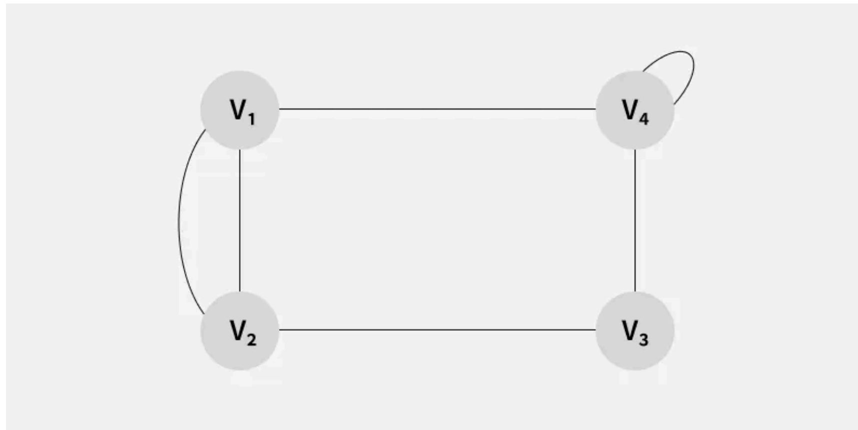
Infinite Graph:

A graph is called an infinite graph if it has an infinite number of vertices and an infinite number of edges. Unlike finite graphs, which have a fixed number of nodes and connections, infinite graphs extend indefinitely.



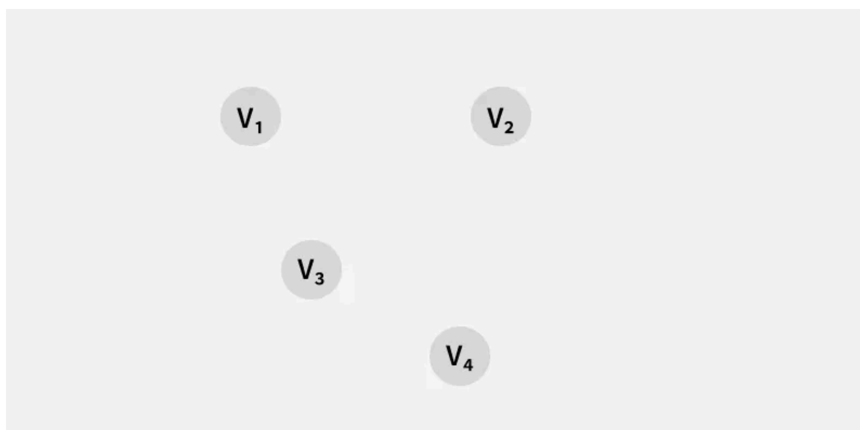
Pseudo Graph

A pseudograph is a type of graph that allows for the existence of self-loops (edges that connect a vertex to itself) and multiple edges (more than one edge connecting two vertices). In contrast, a simple graph is a graph that does not allow for loops or multiple edges.



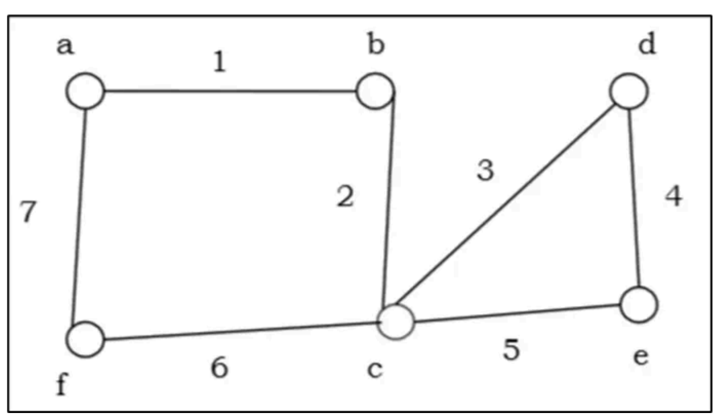
Null graph

A graph of order n and size zero is a graph where there are only isolated vertices with no edges connecting any pair of vertices.A null graph is a graph with no edges. In other words, it is a graph with only vertices and no connections between them. A null graph can also be referred to as an edgeless graph, an isolated graph, or a discrete graph

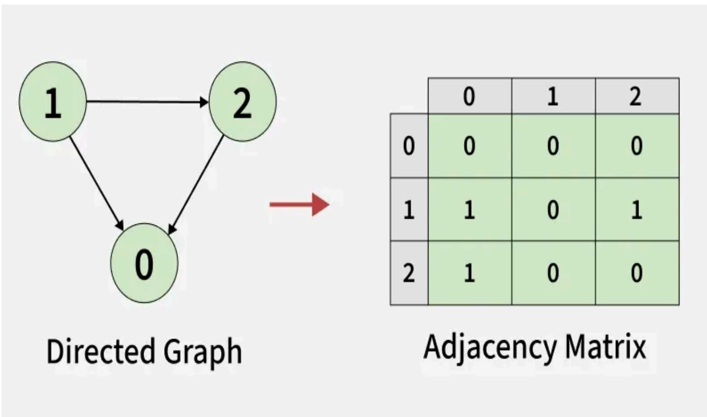


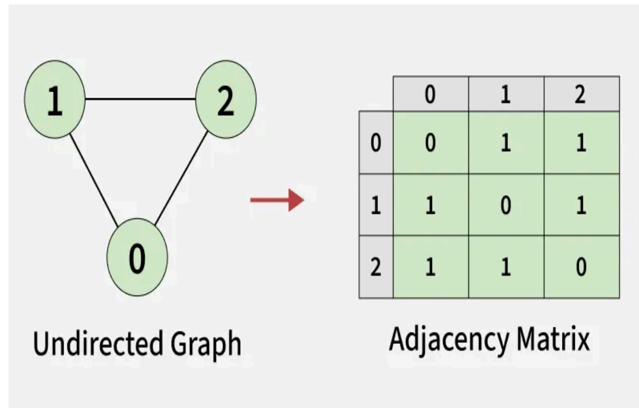
Euler Graphs

An Euler graph is a graph in which every vertex has an even degree. It was first introduced by the Swiss mathematician Leonhard Euler in 1736. Euler solved the famous problem of the Seven Bridges of Königsberg, which was to find a path through the city that would cross each of the seven bridges once and only once. Euler proved that such a path did not exist, and in doing so, he developed the concept of an Euler graph.



Representation of graph
A Graph is a non-linear data structure consisting of vertices and edges. The vertices are sometimes also referred to as nodes and the edges are lines or arcs that connect any two nodes in the graph. More formally a Graph is composed of a set of vertices(V) and a set of edges(E). The graph is denoted by G(V, E).





Pendent Vertex

By using degree of a vertex, we have a two special types of vertices. A vertex with degree one is called a pendent vertex.

Example

Here, in this example, vertex 'a' and vertex 'b' have a connected edge 'ab'. So with respect to the vertex 'a', there is only one edge towards vertex 'b' and similarly with respect to the vertex 'b', there is only one edge towards vertex 'a'. Finally, vertex 'a' and vertex 'b' has degree as one which are also called as the pendent vertex.

Isolated Vertex

A vertex with degree zero is called an isolated vertex.

Example

Here, the vertex 'a' and vertex 'b' has a no connectivity between each other and also to any other vertices. So the degree of both the vertices 'a' and 'b' are zero. These are also called as isolated vertices.

isomorphism

A graph isomorphism is a bijection between the vertex sets of two graphs that preserves the adjacency relationship. In other words, two graphs G and H are isomorphic if there is a one-to-one correspondence between their vertices such that two vertices are adjacent in G if and only if their corresponding vertices are adjacent in H.

Example:

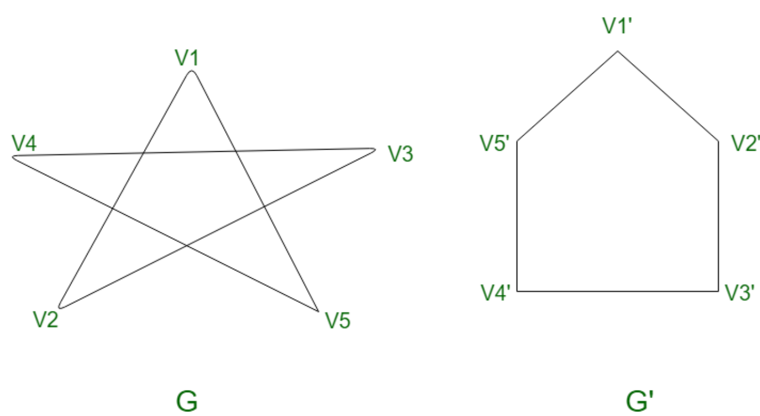
Consider the following two graphs G and H:

Graph G:

```
A -- B
|        |
C -- D
```

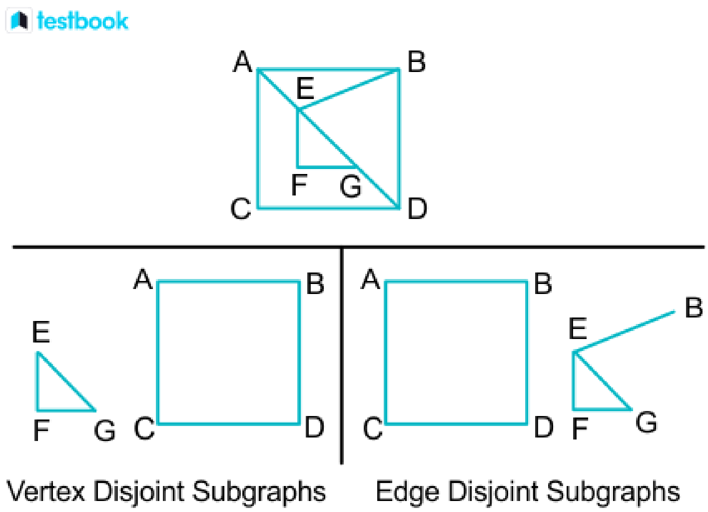
Graph H:

```
1 -- 2
|       |
3 -- 4
```



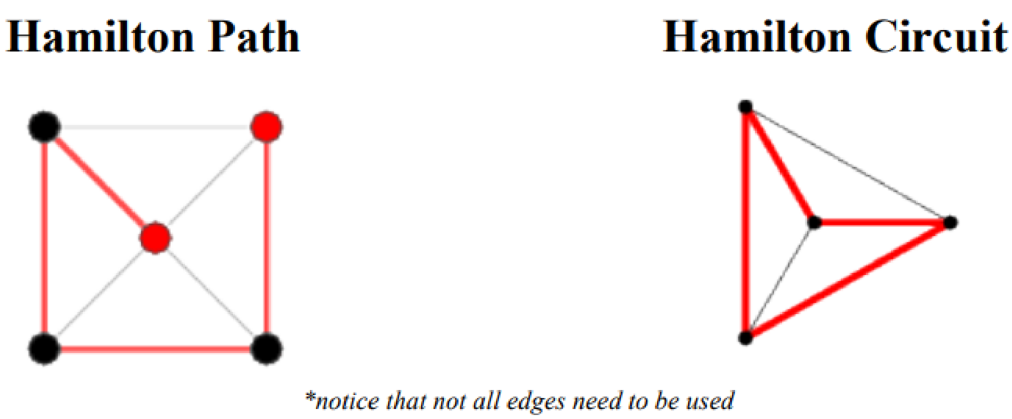
f(B)=2, fThe graphs G and H are isomorphic because there is a bijective function f such that f(A) = 1, (C) = 3, and f(D) = 4 preserving adjacency.

subgraphs
a subgraph is a graph formed from a subset of the vertices and edges of another graph. Subgraphs plays an important role in understanding the structure and properties of larger graphs by examining their smaller, constituent parts.



Hamilton Paths and Hamilton Circuits

A Hamilton Path is a path that goes through every Vertex of a graph exactly once. A Hamilton Circuit is a Hamilton Path that begins and ends at the same vertex.



*Unlike Euler Paths and Circuits, there is no trick to tell if a graph has a Hamilton Path or Circuit. A Complete Graph is a graph where every pair of vertices is joined by an edge. The number of Hamilton circuits in a complete graph with n vertices, including reversals, is equal to $(n-1)!$ If reversals are not included, the number of Hamilton circuits becomes $(n-1)!\ 2$