Com S 576 – Motion Planning: HW2

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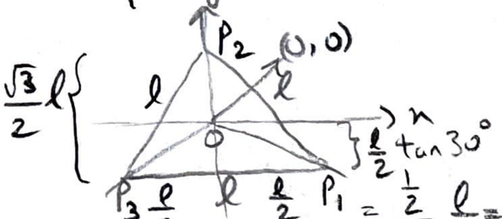
Task 1 P3.1

$H_i = \{(x,y) \in \overset{\mathbb{R}^2}{W} \mid f_i(x,y) \leq 0\}$



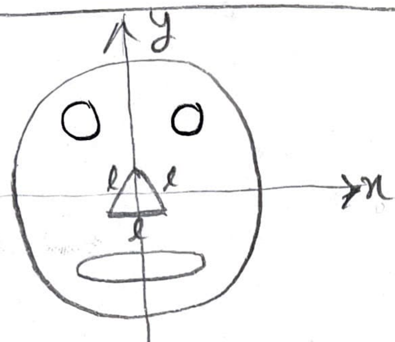


Fig. 3.4 → Amended by a triangle nose @ the center

$\frac{\frac{1}{2}\ell}{\frac{\sqrt{3}}{2}} = \frac{\ell}{2\sqrt{3}} = \frac{\sqrt{3}}{6}\ell$

The requested task of the question can be done by adding another algebraic primitive to O;

$O = H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge H_5$ ← The new one

$\boxed{H_5 = H_{51} \wedge H_{52} \wedge H_{53}}$

$P_1 = \left(\frac{\ell}{2}, \frac{-\sqrt{3}}{6}\ell\right);\ P_2 = \left(0, \frac{\sqrt{3}}{3}\ell\right);\ P_3 = \left(\frac{-\ell}{2}, \frac{-\sqrt{3}}{6}\ell\right) \Rightarrow \begin{cases} H_{51} \to P_1P_2 \to f_{51} \\ H_{52} \to P_2P_3 \to f_{52} \\ H_{53} \to P_3P_1 \to f_{53} \end{cases}$

$P_1P_2: ax + by + c = 0 \Rightarrow \begin{cases} \frac{\ell}{2}a - \frac{\sqrt{3}}{6}\ell b + c = 0 \\ \frac{\sqrt{3}}{3}\ell b + c = 0 \Rightarrow c = -\frac{\sqrt{3}}{3}\ell b \end{cases} \overset{a=1}{\Longrightarrow} \frac{\ell}{2} = \frac{\sqrt{3}}{2}\ell b \Rightarrow b = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow c = -\frac{\ell}{3}$

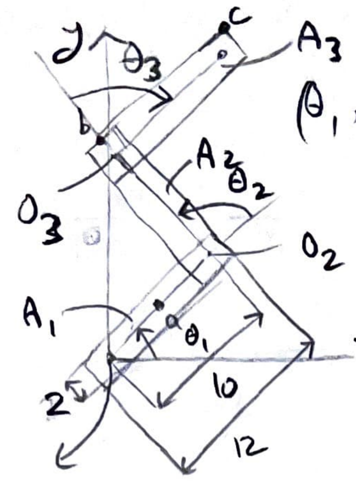
$\Rightarrow x + \frac{\sqrt{3}}{3}y - \frac{\ell}{3} = 0 \overset{O(0,0)}{\Longrightarrow} f(0,0) = -\frac{\ell}{3} < 0 \Rightarrow \boxed{f_{51}(x,y) = -\left(x + \frac{\sqrt{3}}{3}y - \frac{\ell}{3}\right)}$

$P_2P_3: a = 1 \Rightarrow \begin{cases} \frac{\sqrt{3}}{3}\ell b + c = 0 \Rightarrow c = -\frac{\sqrt{3}}{3}\ell b \\ -\frac{\ell}{2} - \frac{\sqrt{3}}{6}\ell b + c = 0 \end{cases} \Rightarrow -\frac{\sqrt{3}}{2}\ell b = \frac{\ell}{2} \Rightarrow b = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3} \Rightarrow c = \frac{\ell}{3}$

$\Rightarrow x - \frac{\sqrt{3}}{3}y + \frac{\ell}{3} = 0 \underset{O(0,0)}{\Longrightarrow} f(0,0) = \frac{\ell}{3} > 0 \Rightarrow \boxed{f_{52}(x,y) = x - \frac{\sqrt{3}}{3}y + \frac{\ell}{3}}$

$P_3P_1: y + \frac{\sqrt{3}}{6}\ell = 0 \overset{O(0,0)}{\Longrightarrow} f(x,y) = \frac{\sqrt{3}}{6}\ell > 0 \Rightarrow \boxed{f_{53}(x,y) = y + \frac{\sqrt{3}}{6}\ell}$

Task 2 | P3.7



$(\theta_1, \theta_2, \theta_3) = \left(\frac{\pi}{4}, \frac{\pi}{2}, \frac{-\pi}{4}\right)$

$a \in A_1 \xrightarrow[\text{frame}]{\text{Body}} a = (5, 0)$

$b \in A_2 \rightarrow b = (11, 0)$

$c \in A_3 \rightarrow c = (11, 1)$

a) $O = O_1$

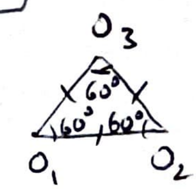
$$\Rightarrow a \in W : \underset{a \in A_1}{T_1 a} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{5\sqrt{2}}{2} \\ \frac{5\sqrt{2}}{2} \\ 1 \end{bmatrix}}_{a}$$

$$b \in W : T_1 T_2 b = T_1 \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 10 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 11 \\ 0 \\ 1 \end{bmatrix} = T_1 \begin{bmatrix} 10 \\ 11 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 11 \\ 1 \end{bmatrix} = \overbrace{\begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{21\sqrt{2}}{2} \\ 1 \end{bmatrix}}^{b}$$

$$c \in W : T_1 T_2 T_3 c = T_1 T_2 \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 10 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 11 \\ 1 \\ 1 \end{bmatrix} = T_1 T_2 \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 10 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 11 \\ 1 \\ 1 \end{bmatrix} = T_1 T_2 \begin{bmatrix} 6\sqrt{2}+10 \\ -5\sqrt{2} \\ 1 \end{bmatrix}$$

$$= T_1 \begin{bmatrix} 0 & -1 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6\sqrt{2}+10 \\ -5\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5\sqrt{2}+10 \\ 6\sqrt{2}+10 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 \\ 11+10\sqrt{2} \\ 1 \end{bmatrix}}_{c}$$

b) All such configs should form an equilateral trinangle:



So $\theta_1 \in [0, 2\pi]$; $\theta_2 \in \left\{\frac{\pi}{3}\right\}$; $\theta_3 \in \left\{\frac{\pi}{3}\right\}$