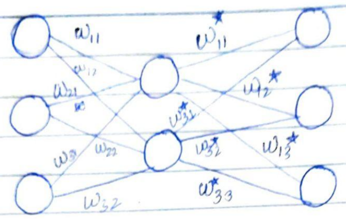
The input layer will have three nodes, the hidden layer will have 2 nodes and the output layer will have 3 nodes(as autoencoder)
I have used matrices to keep track of Weights and biases,
The derivation for backpropagation is done down below:

Ques 2 Part C.



$$W1 = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{bmatrix}$$

Input in hidden layer

$$w_1^T x + B_1$$

$$\begin{bmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$I_1 = \begin{bmatrix} w_{11}x_1 + w_{21}x_2 + w_{31}x_3 + b_1 \\ w_{12}x_1 + w_{22}x_2 + w_{32}x_3 + b_2 \end{bmatrix} \Bigg\}\ \text{Input to hidden layer.}$$

$$\frac{\partial E}{\partial W1} = \overbrace{\underbrace{\frac{\partial L}{\partial O2}\ \frac{\partial O2}{\partial I2}}_{\text{found in previous step.}}\ \frac{\partial I2}{\partial O1}\ \frac{\partial O1}{\partial I1}}^{\alpha}\ \frac{\partial I1}{\partial W1}$$

$I2 = W2 * O1 + B2$

$\frac{\partial I2}{\partial O1} = W2.$

↳ Weight matrix of second layer.

$\frac{\partial O1}{\partial I1} = \sigma'(I1)$ $\qquad \because O1 = \sigma(I1)$

$\frac{\partial I1}{\partial W1} = x$

↳ Input.

$\underbrace{w_1}_{\text{Weight matrix of first layer}} x + B_1$

$$\underbrace{\frac{\partial E}{\partial B2}}_{\text{Bias b/w hidden \& output layer.}} = \underbrace{\frac{\partial L}{\partial O2}\ \frac{\partial O2}{\partial I2}}_{\text{Calculated}}\ \frac{\partial I2}{\partial B2}$$

↳ ~~I~~

↳ ~~Indentity~~ matrix of B2 dimension having all 1.

$$\frac{\partial E}{\partial B1} = \alpha \times \underbrace{\frac{\partial I1}{\partial B1}}$$

↳ matrix of B1 dimension with all 1.

Output of Hidden layer

$$O1 = \sigma(I_1) = \begin{bmatrix} \sigma(W_{11}x_1 + W_{21}X_2 + W_{31}X_3 + b_1) \\ \sigma(W_{12}X_1 + W_{22}X_2 + W_{32}X_3 + b_2) \end{bmatrix}$$

$\hookrightarrow \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}$

Input to Output layer $\longrightarrow W^{*T}O_1 + B_2 = I2$

O_2 = Output of Output layer $= \sigma\big(\underbrace{(W^*)^T}_{3\times 2}O_1 + B_2\big)$

$$W^* = \begin{bmatrix} W^*_{11} & W^*_{12} & W^*_{13} \\ W^*_{21} & W^*_{22} & W^*_{23} \end{bmatrix} \qquad B_2 = \begin{bmatrix} b_1^* \\ b_2^* \\ b_3^* \end{bmatrix}$$

Upon Backpropagation $\quad W2 = W^*$

$E = (\hat{X} - X)^2$.

$O2 = \sigma(I2)$

$$\frac{\partial E}{\partial W2} = \frac{\partial E}{\partial O2}\,\frac{\partial O2}{\partial I2}\,\frac{\partial I2}{\partial W2}$$

$\frac{\partial E}{\partial O2} \hookrightarrow 2(\hat{X} - X)$

$$\frac{\partial O2}{\partial I2} = \sigma'(I2) \qquad \frac{\partial I2}{\partial W2} = O_1$$

Updation Rule.

$$W1\_new = W1 - \text{learning rate} * \frac{\partial E}{\partial W1}$$

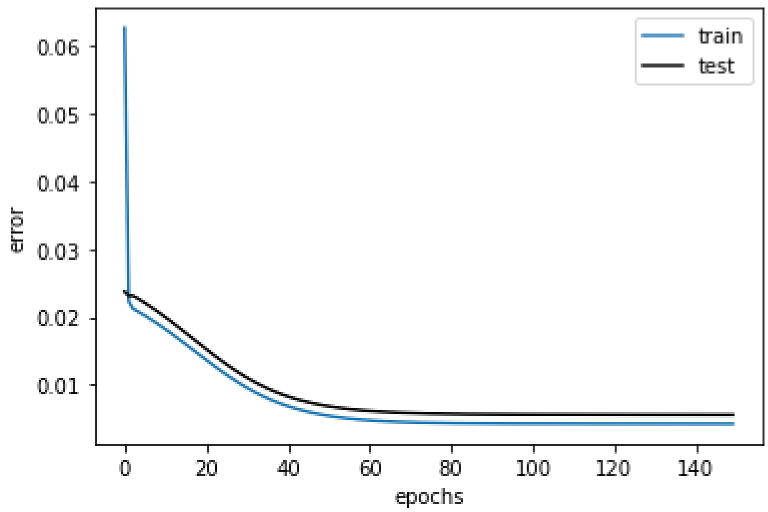
$$W2\_new = W2 - \text{learning\_rate}\ \frac{\partial E}{\partial W2}.$$

$$B1\_new = B1 - \text{learning-rate}\ \frac{\partial E}{\partial B1}$$

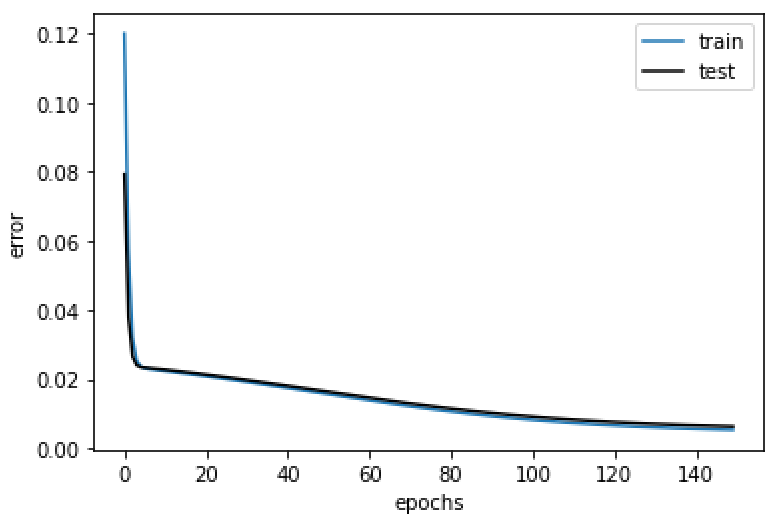
$$B2\_new = B2 - \text{learning-rate}\ \frac{\partial E}{\partial B2}.$$

Question 3)

Backpropagation which is implemented from scratch



For the autograd part:



As can be seen from both graphs that error decreases as epochs increase.