Assignment No:	02
Assignment Title:	Inferential and Hypothesis Testing
Course:	Statistics and EDA
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Assignment Comprehension:

The pharmaceutical company Sun Pharma is manufacturing a new batch of painkiller drugs, which are due for testing. Around 80,000 new products are created and need to be tested for their time of effect (which is measured as the time taken for the drug to completely cure the pain), as well as the quality assurance (which tells you whether the drug was able to do a satisfactory job or not).

Question 1:

Since we have limited resource to conduct an experiment, we can find the probability through theoretical probability that has proved to be quite close to the experimental probability.

We will use **binomial distribution** for this purpose to calculate the probability of whether or not a drug is doing the satisfactory job among the selected 10 samples.

In **binomial distribution** type there is two possible outcomes (**SUCCESS** or **FAILURE**). There is only one outcome for each trial and each trial is mutually exclusive or independent from each other.

The formula for calculating success or failure of a single trial is following.

n is the number of trials, **p** is the probability of success, and **r** is the number of successes after **n** trials.

$$P(X = r) = {}^{n}C_{r}(p)^{r}(1-p)^{n-r}$$

The binomial distribution should meet the following conditions in order to apply the above formula:

1. The total number of trials is fixed, in this case it is at 10;
2. Each trial or observation is independent from others;
3. Probability of success or failure is exactly the same from one trial to another. In this case the probability of success is that the drug is not able to do satisfactory job with percentage of 20%

To calculate the probability with the following question "Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job."

We need to find at most 3 drugs that are not able to do a satisfactory job. This can include 0 drug or 1 drug or 2 drug or 3 drugs that are not able to do satisfactory job. We need to sum the possibility of each from 0 to 3 drugs. The probability of success is 20% while the probability of failure is 80%.

n = 10
r=3
p=0.2
q=1-0.2=0.8

P(0 drugs) = $\binom{10}{0}(0.2)^0(0.8)^{10-0}$ =1*1* $(0.8)^{10}$ = 0.107

P(1 drugs) = $\binom{10}{1}(0.2)^1(0.8)^{10-1} = 10 * 0.20 * (0.8)^9$ = 0.268

P(2 drugs) = $\binom{10}{2}(0.2)^2(0.8)^{10-2} = 45 * 0.04 * (0.8)^8$ = 0.302

P(3 drugs) = $\binom{10}{3}(0.2)^3(0.8)^{10-3} = 120 * 0.01 * (0.8)^7$ = 0.201

P(X=at most 3)=P(X=0)+P(X=1)+P(X=2)+P(X=3)

0.107 + 0.268 +0.302 + 0.201 =0.88

Answer = 0.88 or 88%

So, the sum of the probabilities is 88%. Meaning that the probability that at most 3 drugs are not doing satisfactory job is about 88%**%**.

Question 2:

With the given 100 sample drugs with mean of 207 seconds, it is almost difficult to estimate the population mean ($\boldsymbol{\mu}$) by sample mean **(** $\bar{\mathbf{x}}$**)**

To have a better estimate, we use confidence interval to calculate the range within which we expect the population parameter to be. With confidence interval we can estimate the true value lies between a range with a confidence level of 90% etc.

Since the sample is 100 and it is more than 30, we can infer it that the sampling distribution is a normal

The formula for confidence interval is as following $(\bar{X} - \frac{Z^*S}{\sqrt{n}}, \bar{X} + \frac{Z^*S}{\sqrt{n}})$,

Thus, margin of error is calculated using this formula: $Z * \frac{s}{\sqrt{n}}$

Now

n= 100 drugs

$\mu\overline{\mathrm{x}}$ = 207 seconds

SD = 65 seconds

CI= 95%

Putting it into formula we will get the following

Lower Limit: **207-(1.64*$\frac{65}{\sqrt{100}}$)=207-10.66 = 196.34** seconds

Higher Limit: **207+(1.64*$\frac{65}{\sqrt{100}}$)=207+10.66 = 217.66** seconds

So with 95% confidence we can say that the time of affect lies between the range of **196.34** seconds to **217.66** seconds.

Question 3:

Part a:

n= 100 drugs

$\bar{x}$ =207 seconds

SD = 65 seconds

μ = 200

CI= 95%

Hypothesis:

Null Hypotheses = $\boldsymbol{H_0}$ **<= 200 seconds**
Alternate Hypotheses: **H1 > 200 seconds**
It is a right side or **upper tailed test**

Need to test whether the μ is greater than 200. If greater, then 200 null hypotheses can be rejected.

Method 1: Calculation using critical value (z value and z score):

Since the sample is more than 30, the standard deviation of sample of sampling distribution can be considered as standard deviation of population.

$\sigma \bar{x} = \frac{\sigma}{\sqrt{n}} = \frac{65}{\sqrt{100}}$

Z_{value} = 1-0.05 = 0.95 (for 1-tailed upper tail)
0.95=(0.9495+0.9505)/2 which Zc = 1.645

UCV= $\mu +$ Z_{c*} $\sigma \bar{x}$ = 200+1.645*6.5 = 210.69

LCV = $\mu -$ Z_{c*} $\sigma \bar{x}$ =200- 1.645*6.5 = 189.31

The sample mean ($\bar{x}$)=207 lies between UCV (210.69) and LCV (189.31), so we cannot reject NULL hypothesis.

Method 2: Calculation using p-value:

n= 100 drugs
$\bar{x}$ =207 seconds
SD = 65 seconds
μ = 200

Significance level (α)= 5%
$\sigma \bar{x} = \frac{\sigma}{\sqrt{n}} = \frac{65}{\sqrt{100}}$ =6.5

Calculating the z score:
$Z = \frac{\bar{X}-\mu}{\sqrt{\sigma \bar{x}}}$ = $\frac{207-200}{6.5}$ = $\frac{7}{6.5}$ = 1.0769 ~ 1.08
The z-score for the sample point 1.08 is 0.8599

Since it is one tailed hypothesis test then **p = 1 - 0. 8599= 0.1401**

So, p-value is equal to 14% which is bigger than alpha (5%), we cannot reject NULL hypothesis. We could fail to reject if the p-value was less than 5%.

Part b:

Type I error ($\boldsymbol{\alpha}$): The researcher mistakenly concludes that painkiller drug time of affect is more than 200 seconds but, in fact, the time of affect less than 200 seconds.

Type II error ($\boldsymbol{\beta}$): The researcher mistakenly fails to reject that painkiller drug time of affect is more than 200 seconds.

Considering the definitions of the errors, if we apply the logic for the following two cases:

Case-1: the value of α and β come out to be 0.05 and 0.45 respectively

Case -2: the value of α and β are controlled at 0.15 each (a=b=0.15)

In first case, we can conclude that the Type II error has worse consequences that Type I error. Type I error will force the company to stop production and discard the drugs but with Type II error, the consequences is more dangerous. The it will have bad effect on the people health and the company credit will be affected and the government may revoke the license for producing the drug.

The first case is recommended for the company to test, if there is any side effect related to the time of effect of the painkiller drug, we need to keep the α error to a minimum. This will mean that we are conservative in rejecting the null hypothesis. So, if the painkiller drug has significant effect, we would like to keep the value of α and β come out to be 0.05 and 0.45 respectively.

On the other hand, to avoid or minimize the Type II error, we tend to increase the Type I error level so that the Type II error is decreased.

Question 4:

The A/B testing is a way of comparing two versions of same element against each other to evaluate which one performs better. The test is performed with application of two sample (variables) proportion test.

Step by step procedure of A/B testing:

1. Define a significance level for the test e.g. 5%;
2. Choose your target audience;
3. Apply the first tagline in the campaign and perform the advertisements;
4. Create a survey and denote positive feedback as 1, negative as 0 and launch campaign gather the information from the survey as required;
5. Apply the second tagline in the campaign and perform the advertisements;
6. Create a survey and denote positive feedback as 1, negative as 0 and launch campaign gather the information from the survey as required the same as tagline 1;
7. Compile the data from both surveys and then apply the A/B test. Find total frequency for each tagline and total samples;
8. Calculate the p-value of both the data sets with significance level of 5%.;

Note: The NULL hypothesis for AB testing is that H0 is that difference between the proportions is 0 while H1: The difference between the proportions is different from 0.

9. If the p-value is more than the significance level (=5%) the NULL hypothesis cannot be rejected. But if the p-value is less than 5% the NULL hypothesis is rejected.