

Quantitative Macroeconomics.

Homework 4.

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II. Solving the ABHI model

II.1. The recursive formulation

Formulate the problem of the agent recursively, i.e. write down Bellmans equation and derive the stochastic Euler equation.

The Bellman equation

$$V(a, y) = \max_{c, a'} \left\{ U(c) + \beta EV(a', y') \right\} \quad (1)$$

Once we have discretized the idiosyncratic shocks Bellman equation can be rewritten as:

$$V(a, y) = \max_{c, a'} \left\{ U(c) + \beta \sum_{y'} p_{yy'} V(a', y') \right\} \quad (2)$$

(3)

Subject to:

$$a(1 + r) + ws = k' + c \quad (4)$$

$$c \geq 0 \quad (5)$$

$$a' \geq \bar{A} \quad (6)$$

(7)

Taking the derivative in the general form:

$$\frac{\partial}{\partial c} : \quad u'(c) + \beta EV'(a', y')(-1) = 0 \quad (8)$$

$$\frac{\partial}{\partial a} : \quad V'(a, y) = \beta EV'(a', y')(1 + r) \quad (9)$$

$$(10)$$

Substituting for the expectation in the second equation from the first one

$$V'(a, y) = u'(c)(1 + r) \quad (11)$$

$$(12)$$

Iterating forward we obtain the general form of the Euler equation:

$$u'(c) - \beta(1 + r)Eu'(c') = 0 \quad (13)$$

$$(14)$$

Which can be replaced using the discrete income process:

$$u'(c) - \beta(1 + r) \sum_{y' \in Y} \pi(y'|y)u'(c') = (\geq)0 \quad (15)$$

And thus we have obtained the stochastic Euler equation.

II.2. The infinitely-lived households economy

The solution of the infinitely-lived households economy is included in the code *II.2.Infinite.py*.

II.3. The life-cycle economy

The solution of the life-cycle economy is included in the code *II.3.Lifecycle.py*.

II.4. Partial equilibrium

II.4.1. With certainty

First, let $\gamma = 0$ and $\sigma_y = 0$, that is, there is no uncertainty.

1. For $T = \infty$ plot the consumption functions. On the x -axis should be a , on the y -axis $c(a, y_1)$ and $c(a, y_2)$ for both preference specifications. Also generate a time profile of consumption by choosing a_0 as starting assets and by using the policy functions $c(a, y)$ and $a'(a, y)$.

Answer:

The consumption policy functions for the $T = \infty$ economy are presented in Figures 3. With no uncertainty the consumption policy function is linear in assets for the CRRA utility (Figure 3a) and quadratic in assets for the quadratic utility (Figure 3c).

The generated time profiles of consumption are similar for both utility specifications (compare Figure 3b and Figure 3d). However, there is more consumption smoothing in the CRRA case. In general, we observe declining consumption profiles. This result is due to the parametric assumptions we have made for the partial equilibrium. In particular $\rho = .06 > r = .04$ the discount rate higher than the interest rate means that the rate of return on savings is not high enough to compensate for a strong time preference embedded in the discount rate. Therefore declining consumption profiles are preferred. This result will hold across the whole partial equilibrium section.

2. Do the same as in the previous question but now with $T = 45$. For the consumption function plots pick two ages, say plot $c_5(a, y)$ and $c_{40}(a, y)$.

Answer:

The consumption policy functions for the $T = 45$ economy are presented in Figures 4 for the CRRA preferences and in Figures 5 for the quadratic preferences. The finite time horizon imposes a "cake-eating" structure in the CRRA utility. The policy functions are increasing in assets but younger generations consume less for a given level of assets because they still have more periods before death and the consumption smoothing is stronger (see Figure 4a). Older households want to consume relatively more because they have fewer future periods to worry about (see Figure 4b and Figure 4c for comparison).

In the quadratic utility case the shapes of the policy function are almost the same as in the $T = \infty$ case (see Figures 5a, 5b and 5c for comparison). Younger households always consume more for a given level of assets. This is because quadratic utility does not include risk aversion but the value function problem still contains discounting. Therefore consumption earlier is strongly preferred and the utility punishment for extremely low consumption is not nearly as severe as in the CRRA case. In fact negative consumption is permitted in the quadratic utility case.

The consumption time profiles are again decreasing. However the preference for earlier consumption is much stronger for the quadratic utility than for the CRRA utility (compare Figure 4d and Figure 5d) since agents with quadratic utility are punished less for extremely low consumption in the future.

II.4.2. With uncertainty

Now let $\gamma = 0$ and $\sigma_y = 0.1$.

1. (II.4.2.1.) Plot and compare the consumption functions (for each y plot $c(a,y)$ against a) under certainty equivalence (quadratic case) with the consumption function derived in the presence of a precautionary saving motive. Are the differences more pronounced for $T = \infty$ or $T = 45$ and why?. How do they compare to what you found in the case of certainty?

Answer:

A change in the standard deviation of the income shock from $\sigma_y = 0$ to $\sigma_y = .1$ does not have a strong impact on the consumption policy functions of agents in both the $T = \infty$ and $T = 45$ economies (see Figures 6 and 7). For both types of economy the consumption policy functions are lower for $\sigma_y = 0.1$ than for $\sigma_y = 0$ because the natural borrowing constraint has become tighter as a result of an increase in σ_y . The impact is stronger for the infinite horizon case because the impact of change in the low state income is higher for the infinite sum: $-\frac{1}{r}y_{min}\Delta\eta < -\frac{(1-(1+r)^{t-45})}{r}y_{min}\Delta\eta$, than for the finite sum.

The difference between the certainty equivalent and precautionary savings consumption policy functions has become slightly more pronounced as certainty is valued more by risk averse agents when the standard deviation of the income shock is higher. This is reflected in the higher curvature of the quadratic utility function which corresponds to a larger distance from the almost linear policy function for the CRRA utility.

2. (II.4.2.2.) Present and compare representative simulated time paths of consumption for the certainty equivalence and precautionary saving economy. On the x-axis should be time, on the y-axis the income shock and the consumption realization. You may limit yourself to the $T = 45$ case.

Answer:

The simulated time paths for the life-cycle 45-period economy and both utility specifications are presented in Figure 8. We see little impact of the increase in the variance of the idiosyncratic shock on the time path in the quadratic utility case. The only main difference is that consumption fluctuates with income shock once assets are depleted (Figure 8d), which does not happen in the deterministic case (Figure 8c).

The impact of an increase in the variance of the idiosyncratic shock is more pronounced in the CRRA case. Higher risk causes agents to consume more initially and less in the later years of life than in the deterministic case (compare Figure 8a and Figure 8b). Moreover, the shocks are reflected in low frequency fluctuations which were absent in the deterministic case.

3. (II.4.2.3.) Increase prudence by increasing $\sigma = 2$ to $\sigma = 5$ and $\sigma = 20$. How much do your answers change and why?

Answer:

The RRA/IES factor σ has no impact on the results in the deterministic case (Figure 9). However, it interacts strongly with the idiosyncratic risk. The higher the parameter of risk aversion the lower the initial level of consumption and the higher the minimum level of consumption (compare Figures 9b, 9d and 9f). High risk aversion makes agents avoid lower consumption levels and this is achieved through higher initial savings which is reflected in a lower initial level of consumption.

4. (II.4.2.4.) Increase the variance of the income shock from $\sigma_y = 0.1$ to $\sigma_y = 0.5$. What happens to the consumption function in the certainty equivalence case? (a) Also plot the new consumption functions for the precautionary savings case (b). Are the differences between certainty equivalence and precautionary savings consumption functions bigger or smaller now? Explain (c). Support your explanations with simulated time paths of consumption (d). Again limit yourself to $T = 45$. How much do your answers change and why (e)?

Answer:

(a) The consumption policy function in the certainty equivalence case behaves exactly in line with the mechanics described in point (II.4.2.1.) - see Figure 10 11 and 12. That is to say an increase in the standard deviation makes the borrowing constraint tighter and increases the difference between the high and low state income. Therefore we see a shift of the consumption policy curve rightwards by about 10 on the grid of assets and the difference between consumption policies in the high and low state has increased as well compared to the $\sigma_y = 0.1$ case (see Figure 10 for cohort 5, Figure 11 for cohort 40). Moreover, an increase in σ_y has a relatively more adverse impact on the younger cohorts, which depend more on borrowing and suffer more from tightening of the borrowing conditions (see Figure 12).

(b) The consumption functions for the precautionary savings case under $\sigma_y = 0.5$ and $\sigma_y = 0$ are plotted in Figure 13a and 13c. It can be seen that the shape and age structure of the consumption function remain largely unchanged and is similar to that of the quadratic case - we can see the impact of the tightening of the borrowing conditions and an increase the difference between wages in the low and high idiosyncratic states (see Figure 13).

(c) The differences between certainty equivalence and precautionary savings case have become larger as an increase in σ_y (risk) means that agents value certain consumption relatively more

compared to the risky case. This is reflected in the increased curvature of the quadratic utility case, whereas CRRA consumption policy function is almost linear. Therefore the difference between the two has increased.

(d) The simulated time paths presented in Figures 14 support the argument presented before. The change in the simulated time path for the quadratic utility case as a result of change in σ_y is negligible (see Figure 14b vs Figure 14d). Whereas for the CRRA utility an increase in σ_y has caused a dramatic increase in initial consumption and a reduction in consumption later in life (compare Figure 14a and Figure 14c) as agents are risk-averse and the income risk has increased. The time structure of consumption retains the features characteristic of RRA/IES $\sigma = 2$ discussed in point (II.4.2.3.).

(e) Thus we can see that the differences between consumption policy functions and consumption time profiles are augmented by the differences in σ_y , σ and the choice of the utility function and these three parameters interact strongly with each other. In other words, for higher $\sigma_y = .5$ we can see even bigger differences between various utility function parametrizations.

5. (II.4.2.5.) Increase the persistence of the income shocks from $\gamma = 0$ to $\gamma = .95$ (keep $\sigma_y = .5$ as well as all other parameters constant. How much do your answers change and why?

Answer:

In the infinite time horizon $T = \infty$ the only substantial change in the results is that the difference between the consumption policy function in the high state and the low state has increased for both the CRRA and quadratic utility cases (see Figure 15a and 15b for CRRA and Figure 16a and 16b for quadratic utility). This is because the higher persistence parameter $\gamma = .95$ means that it is much more likely that you will stay in the state which you currently find yourself in. This in turn means that agents in the low state consume less as their expected lifetime income has decreased compared to the $\gamma = 0$ case and in the high state consume more as their expected lifetime income has increased compared to the $\gamma = 0$ case for both the CRRA (compare Figure 15c and 15d) and the quadratic utility (compare Figure 16c and 16d). Higher persistence is reflected in relatively longer constant consumption periods for the $\gamma = .95$ case, relative to the $\gamma = 0$ case.

The results for the finite time horizon $T = 45$ are largely the same. The only major difference that an increase in γ introduces is between the consumption policy function in different states. The consumption policy function for the high state is located relatively higher and for the low state relatively lower for the same reasons as explained in the previous paragraph for both the CRRA (see Figure 17a and 17b) and certainty equivalence cases (see Figure 18a and 18b). We also observe an increase in persistence in the simulated consumption time profiles for both the CRRA (compare Figure 17c and 17d) and quadratic utilities (compare Figure 18c and 18d).

II.5. General equilibrium

The code where all General equilibrium problems are solved: *II.5.ABHI.py*

II.5.1. The simple ABHI model

Report the endogenous distribution of consumption, income and wealth and compare them to the corresponding data distributions reported in the Handbook Chapter by Krueger, Mitman and Perri. Compare also the joint distribution of consumption and wealth in the model and the data.

I have run my model using the same parametrization as provided for the KS economy with idiosyncratic but without aggregate risk in the Handbook Chapter by Krueger, Mitman and Perri, i.e.:

```
sigma = 1          # inverse of the intertemporal elasticity of substitution
beta = .9899       # discount factor
gamma = 0.9695     # persistence of the process
sigma_y = np.sqrt(0.0384) # The standard deviation of the Markov process
delta = 0.025      # the depreciation rate
theta = .36        # the capital share
```

% Share held by	Assets	Consumption
Q1	0.0525273	0.26312
Q2	0.0840444	0.124745
Q3	0.151901	0.180952
Q4	0.385645	0.195282
Q5	0.27096	0.201491

Table 1: Endogenous distribution of assets and consumption in 2-state ABHI economy

Table 6 Net worth distributions: Data vs models

% Share held by:	Data		Models	
	PSID, 06	SCF, 07	Bench	KS
Q1	−0.9	−0.2	0.3	6.9
Q2	0.8	1.2	1.2	11.7
Q3	4.4	4.6	4.7	16.0
Q4	13.0	11.9	16.0	22.3
Q5	82.7	82.5	77.8	43.0
90–95	13.7	11.1	17.9	10.5
95–99	22.8	25.3	26.0	11.8
T1%	30.9	33.5	14.2	5.0
Gini	0.77	0.78	0.77	0.35

Figure 1: Table 6 Handbook of Macroeconomics

The endogenous distribution of assets which I have obtained in the solution of the ABHI economy (second column of Table 1) with idiosyncratic risk and 2 income states is quite similar to the values reported for the Krussel-Smith economy in the last column of Table 6 1. In particular both distributions are negatively-skewed with the KS being skewed more than my distribution. This is a desired property of the distribution, however the levels observed in the data are closer to the benchmark economy with aggregate risk reported in the penultimate column of Table 6 1.

Table 8 Selected variables by net worth: Data vs models
% Share of:

NW Q	% Share of:						% Expend. rate			
	Earnings		Disp. Y		Expend.		Earnings		Disp. Y	
	Data	Mod	Data	Mod	Data	Mod	Data	Mod	Data	Mod
Q1	9.8	6.5	8.7	6.0	11.3	6.6	95.1	96.5	90.0	90.4
Q2	12.9	11.8	11.2	10.5	12.4	11.3	79.3	90.3	76.4	86.9
Q3	18.0	18.2	16.7	16.6	16.8	16.6	77.5	86.0	69.8	81.1
Q4	22.3	25.5	22.1	24.3	22.4	23.6	82.3	87.3	69.6	78.5
Q5	37.0	38.0	41.2	42.7	37.2	42.0	83.0	104.5	62.5	79.6
Correlation with net worth										
	0.26	0.46	0.42	0.67	0.20	0.76				

Figure 2: Table 8 Handbook of Macroeconomics

The expenditure distribution which I have obtained is somewhat counterfactual in that the poorer households consume the most in the economy, whereas in both the data and the benchmark model considered in the Handbook approximately 40% of all expenditure are made by the top 20%. In my model the top 20% consume only 20.14% of total expenditure, while the poorest 20% consume over 25% of all expenditure. The exact shape of the asset distribution is extremely negatively skewed with the concentration of mass to the right side of the graph (Figure 19b). This particular structure is a result of the optimal decision rule of agents. In particular, agents in high idiosyncratic state prefer to accumulate assets to safeguard against future negative shocks thus moving up the asset grid. Whereas the agents in the low idiosyncratic state simply cannot afford accumulating as many assets as agents in low states. Additional contributing factor to this particular distribution is an extremely high persistence which makes agents unlikely to change their state and thus combined with the policy function mentioned before contributes to the fat tails of the distribution. This can be seen in the asset policy function in the code and in Figure 19a.

One important remark is that my code uses value function iteration method with an equispaced grid and the Howard improvement, while in the Handbook a Gaussian-Hermite quadrature is applied which may increase precision of the estimation.

II.5.2. Solve Aiyagari (1994)

Again, report the endogenous distribution of consumption, income and wealth and compare them to the corresponding data distributions reported in the Handbook Chapter by Krueger, Mitman and Perri. Compare also the joint distribution of consumption and wealth in the model and the data.

II.5.2.1. 7-state Aiyagari model with Handbook parametrization

The first version of the Aiyagari model I consider is the exact reciprocal of the 2-state simple ABHI model described in the previous section only with 7 states of income discretized using the Tauchen algorithm assuming that the unconditional mean of the income process is equal to 1. Again, the natural borrowing constraint is switched off. The parametrization is the following:

```
sigma = 1          # inverse of the intertemporal elasticity of substitution
beta = .9899       # discount factor
gamma = 0.9695     # persistence of the process
sigma_y = np.sqrt(0.0384) # The standard deviation of the Markov process
delta = 0.025      # the depreciation rate
theta = .36        # the capital share
```

% Share held by	Assets	Consumption
Q1	0.0489669	0.132161
Q2	0.107392	0.192541
Q3	0.353724	0.23525
Q4	0.254974	0.210031
Q5	0.232819	0.230017

Table 2: Endogenous distribution of assets and consumption in 7-state ABHI economy

A simple extension of the model to a larger number of states has not made the wealth and consumption distributions more similar to the ones observed in the data nor in the benchmark aggregate risk model analyzed in the Handbook (compare Table 2 against 1 and 2). The plots of the distributions are included in the Appendix (Figures 20a, 20b, 20c and 20d).

II.5.2.2. 7-state Aiyagari model with Aiyagari (1994) paper parametrization

In this subsection I provide the results from my model for the following parametrization considered in the Aiyagari paper:

```

sigma = 5      # inverse of the intertemporal elasticity of substitution
beta = .96     # discount factor
gamma = 0.6    # persistence of the process
sigma_y = .4   # The standard deviation of the Markov process
delta = 0.08   # the depreciation rate
theta = .36    # the capital share

```

% Share held by	Assets	Consumption
Q1	0.0420519	0.136521
Q2	0.0910188	0.169292
Q3	0.322587	0.198762
Q4	0.187627	0.237943
Q5	0.356565	0.257481

Table 3: Endogenous distribution of assets and consumption in 7-state ABHI economy with Aiyagari parametrization $\sigma = 5$, $\sigma_y = 0.4$ and $\gamma = 0.6$

Albeit, still far from the benchmark distribution, a change in parametrization and I suspect, most importantly, a substantial increase in the parameter of the risk aversion has brought the distribution closer to the data and the benchmark model from the Handbook (compare Table 3 against 1 and 2). Most importantly, the share of wealth owned by the top 20% has increased significantly. The figures with the asset and consumption distributions are included in the Appendix (see Figures 21a, 21b, 21c and 21d).

II.5.2.3. 20-state Aiyagari model with custom parametrization

In general it can be seen that including more risk in the model helps bring it close to the data. In the last version of the model I will test the most extreme parametrization:

```

N = 20      # the number of idiosyncratic states
sigma = 20   # inverse of the intertemporal elasticity of substitution
beta = .9    # discount factor
gamma = 0.4  # persistence of the process
sigma_y = .6 # The standard deviation of the Markov process
delta = 0.08 # the depreciation rate
theta = .36  # the capital share

```

% Share held by	Assets	Consumption
Q1	0.0286561	0.158424
Q2	0.094507	0.151006
Q3	0.177295	0.18481
Q4	0.21988	0.211902
Q5	0.479628	0.283692

Table 4: Endogenous distribution of assets and consumption in 7-state ABHI economy with Aiyagari parametrization $\sigma = 5$, $\sigma_y = 0.4$ and $\gamma = 0.6$

The most extreme parametrization has yielded the results which resemble the most those observed in the data but still fall short of some important features such as roughly 80% of wealth being held by the top 20% asset holders in the economy. Such a high value of the parameter σ is not consistent with the data. A more realistic approach would be to assume Epstein-Zin-Weil utility function to better parametrize IES when risk aversion is high.

The asset and consumption distribution figures are reported in the Appendix (Figures 22a, 22b, 22c and 22d).

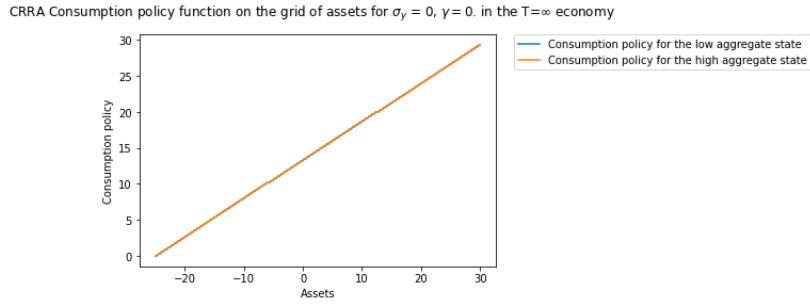
Additional remark:

I keep the natural borrowing constraint turned off in the general equilibrium section. It is included in the code and can be turned on by changing `borr=0` to `borr=1`. It may be necessary to increase the maximum point on the grid of assets to make it run. Including it does not help get better distributions.

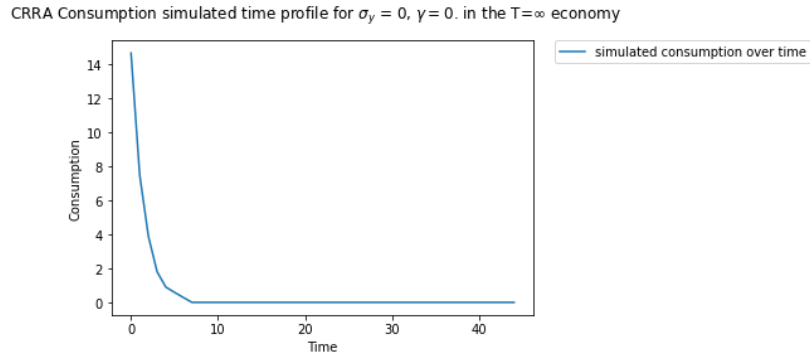
III. Appendix

II.4.1. Partial equilibrium with no uncertainty

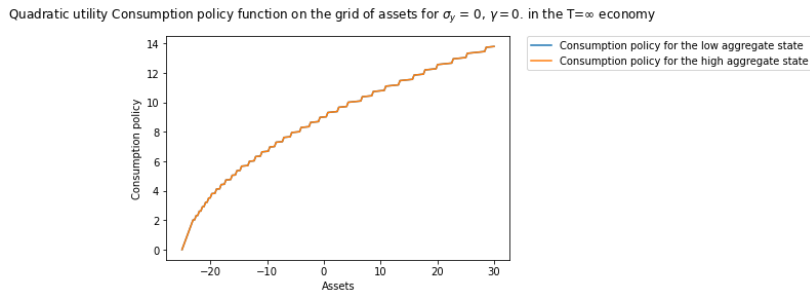
Figure 3: II.4.1. No uncertainty - $T = \infty$ - CRRA utility with $\sigma = 2$



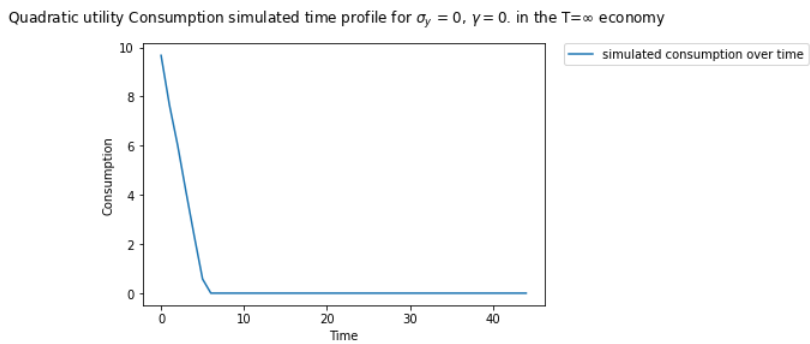
(a) (CRRA Utility) Consumption policy function



(b) (CRRA Utility) Simulated consumption time profile



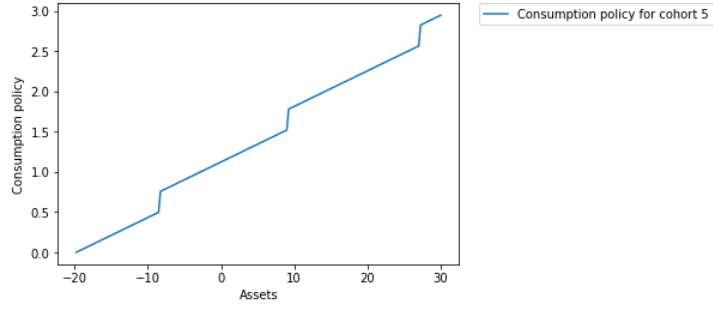
(c) (Quadratic Utility) Consumption policy function



(d) (Quadratic Utility) Simulated consumption time profile

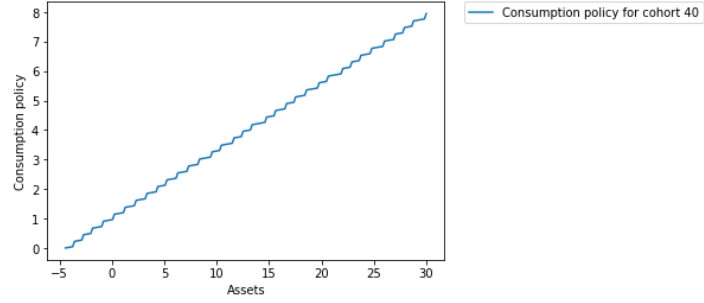
Figure 4: II.4.1. No uncertainty - $T = 45$ OLG - CRRA utility with $\sigma = 2$

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



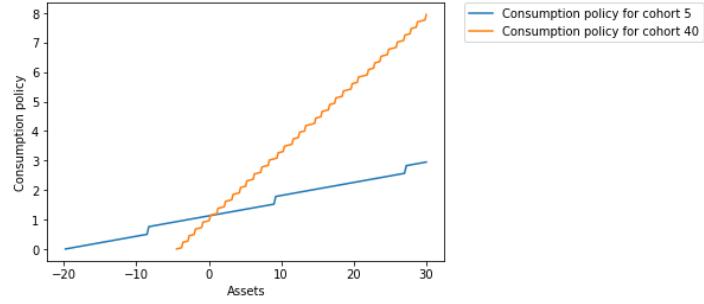
(a) (CRRA Utility) Consumption policy function for Cohort 5

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



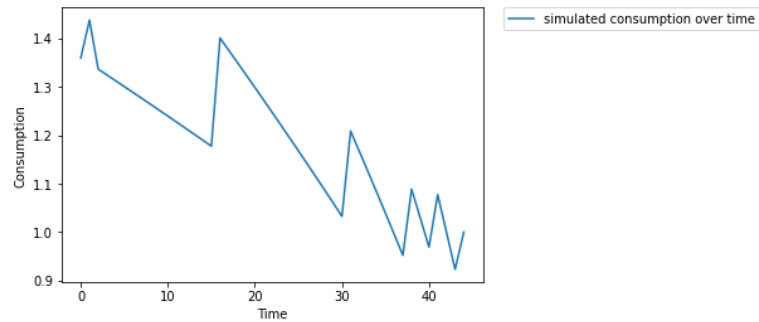
(b) (CRRA Utility) Consumption policy function for Cohort 40

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(c) (CRRA Utility) Comparison of Cohort 5 and Cohort 40 consumption policy functions

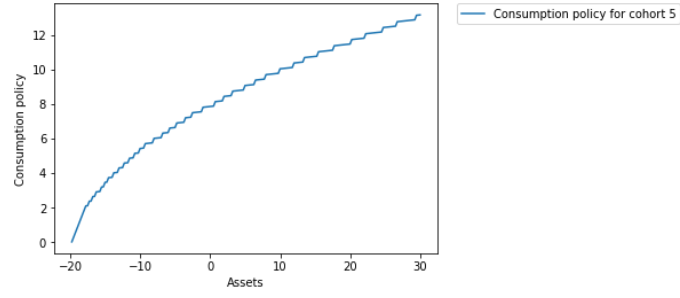
CRRA Consumption simulated time profile for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(d) (CRRA Utility) Consumption time profile

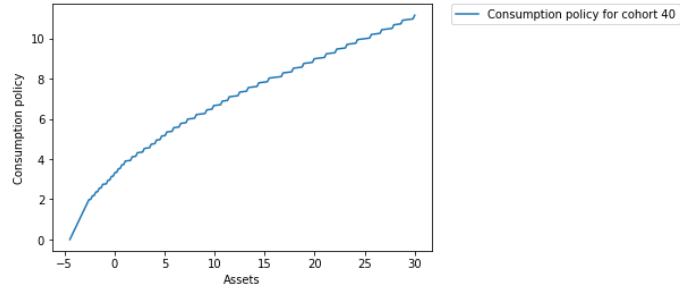
Figure 5: II.4.1. No uncertainty - $T = 45$ OLG - Quadratic utility with $\bar{c} = 100$

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



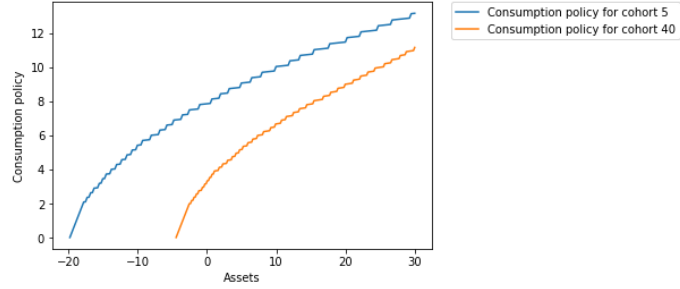
(a) (Quadratic Utility) Consumption policy function for Cohort 5

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



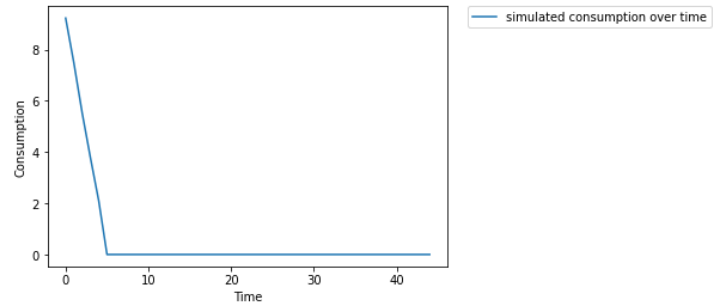
(b) (Quadratic Utility) Consumption policy function for Cohort 40

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(c) (Quadratic Utility) Comparison of Cohort 5 and Cohort 40 consumption policy functions

Quadratic utility Consumption simulated time profile for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



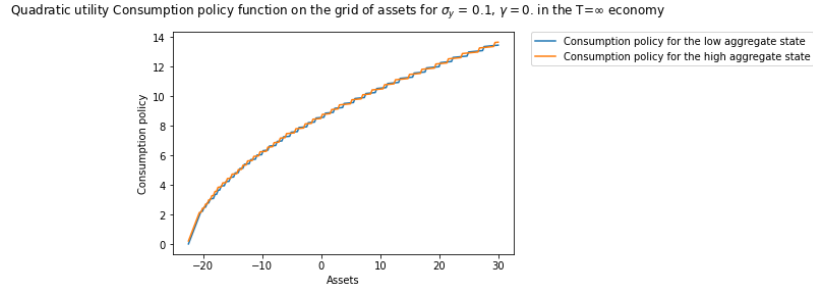
(d) (Quadratic Utility) Consumption time profile

II.4.2. Partial equilibrium with uncertainty

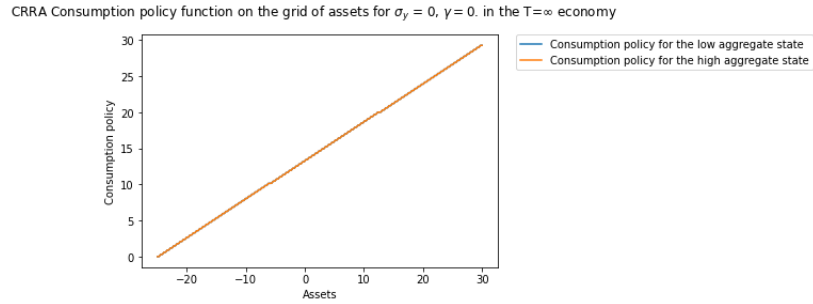
Figure 6: II.4.2.1. Uncertainty $\sigma_y = .1$ certainty equivalence vs precautionary savings in $T = \infty$



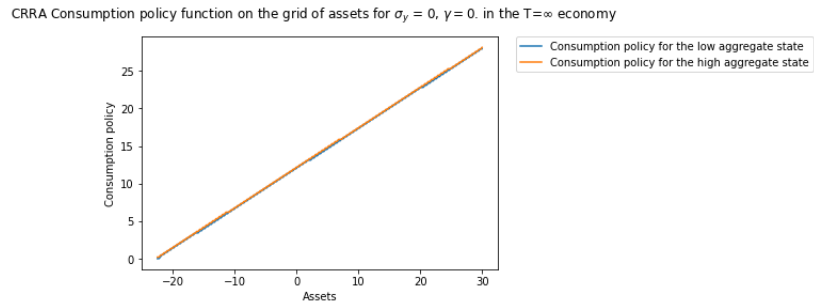
(a) (Quadratic Utility) Consumption policy function for $\sigma_y = 0$ for $T = \infty$



(b) (Quadratic Utility) Consumption policy function for $\sigma_y = .1$ for $T = \infty$



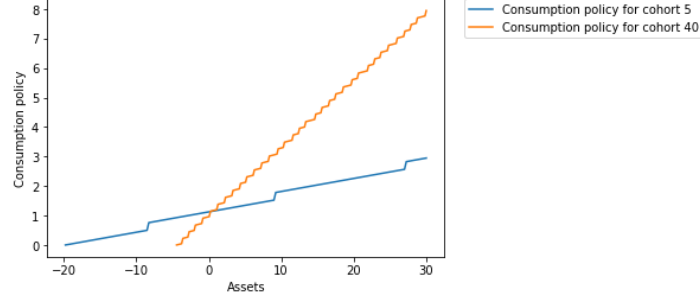
(c) (CRRA Utility) Consumption policy function for $\sigma_y = 0$ for $T = \infty$



(d) (CRRA Utility) Consumption policy function for $\sigma_y = .1$ for $T = \infty$

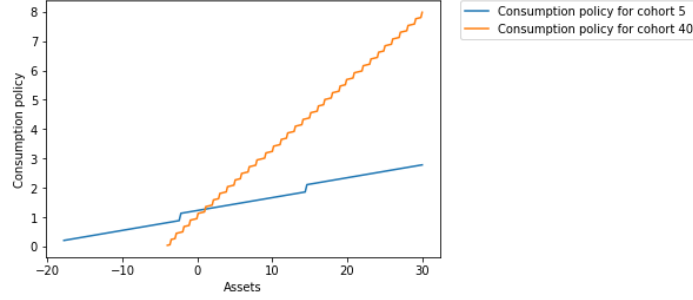
Figure 7: II.4.2.1. Uncertainty $\sigma_y = .1$ certainty equivalence vs precautionary savings in $T = 45$

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



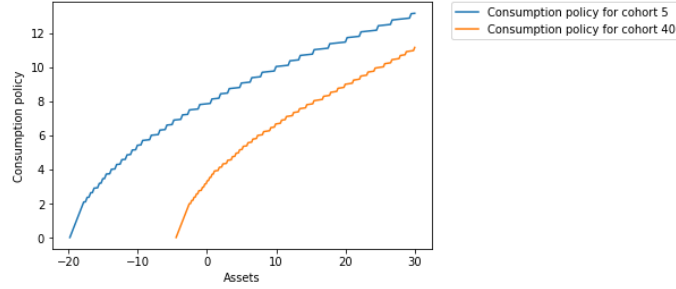
(a) (CRRA Utility) Consumption policy function for $\sigma_y = 0$ cohort comparison for $T=45$

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



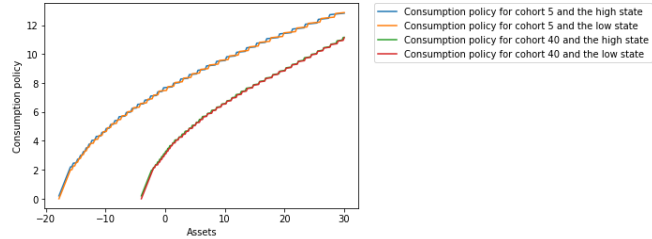
(b) (CRRA Utility) Consumption policy function for $\sigma_y = .1$ cohort comparison for $T=45$

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(c) (Quadratic Utility) Consumption policy function for $\sigma_y = 0$ cohort comparison for $T=45$

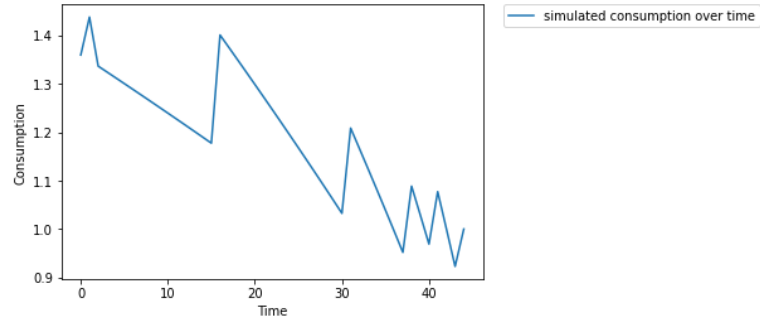
Cohort 5 and 40 Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



(d) (Quadratic Utility) Consumption policy function for $\sigma_y = .1$ cohort comparison for $T=45$

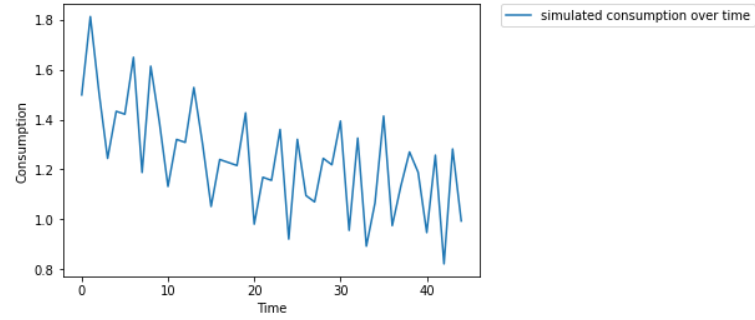
Figure 8: II.4.2.2. Uncertainty $\sigma_y = .1$ Compare time paths of consumption $T=45$

CRRA Consumption simulated time profile for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



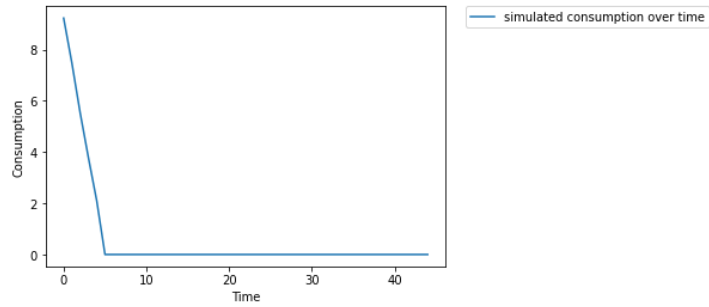
(a) (CRRA Utility) Consumption time path for $\sigma_y = 0$

CRRA Consumption simulated time profile for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



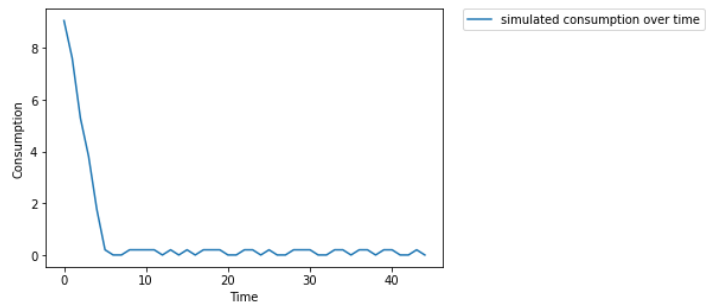
(b) (CRRA Utility) Consumption time path for $\sigma_y = 0.1$

Quadratic utility Consumption simulated time profile for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(c) (Quadratic Utility) Consumption time path for $\sigma_y = 0$

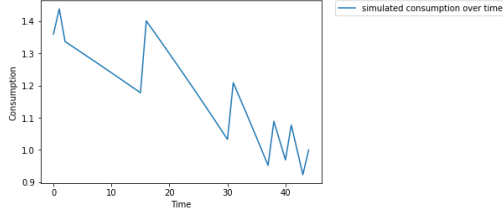
Quadratic utility Consumption simulated time profile for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



(d) (Quadratic Utility) Consumption time path for $\sigma_y = 0.1$

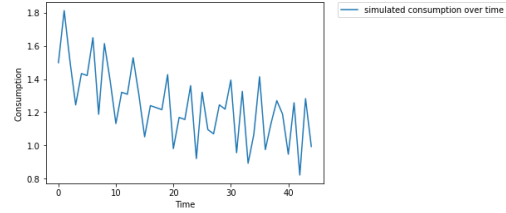
Figure 9: II.4.2.3. Uncertainty $\sigma_y = .1$ Compare time paths of consumption $T=45$ for $\sigma = 2$, $\sigma = 5$ and $\sigma = 20$

CRRA Consumption simulated time profile for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



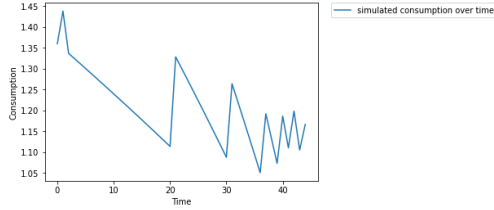
(a) (CRRA Utility) Consumption time path for $\sigma_y = 0$ and $\sigma = 2$

CRRA Consumption simulated time profile for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



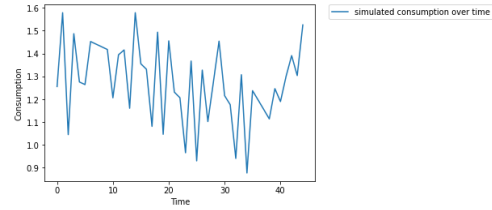
(b) (CRRA Utility) Consumption time path for $\sigma_y = 0.1$ and $\sigma = 2$

CRRA Consumption simulated time profile for $\sigma = 5$, $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



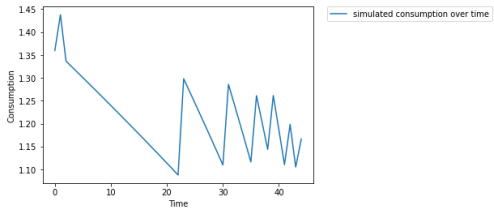
(c) (CRRA Utility) Consumption time path for $\sigma_y = 0$ and $\sigma = 5$

CRRA Consumption simulated time profile for $\sigma = 5$, $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



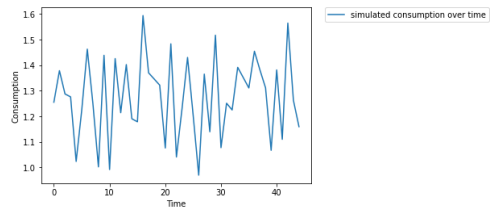
(d) (CRRA Utility) Consumption time path for $\sigma_y = 0.1$ and $\sigma = 5$

CRRA Consumption simulated time profile for $\sigma = 20$, $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(e) (CRRA Utility) Consumption time path for $\sigma_y = 0$ and $\sigma = 20$

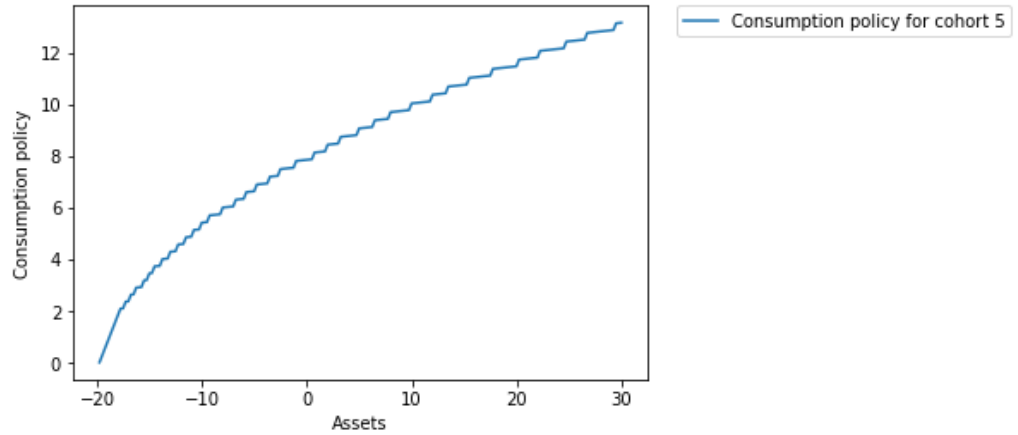
CRRA Consumption simulated time profile for $\sigma = 20$, $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



(f) (CRRA Utility) Consumption time path for $\sigma_y = 0.1$ and $\sigma = 20$

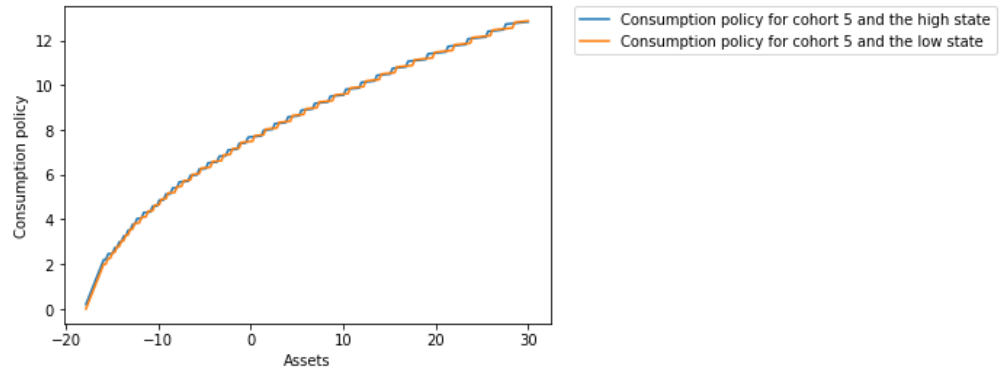
Figure 10: II.4.2.4. Uncertainty $\sigma_y = .5$ vs $\sigma_y = .1$ and $\sigma_y = 0$ for quadratic utility - cohort 5

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



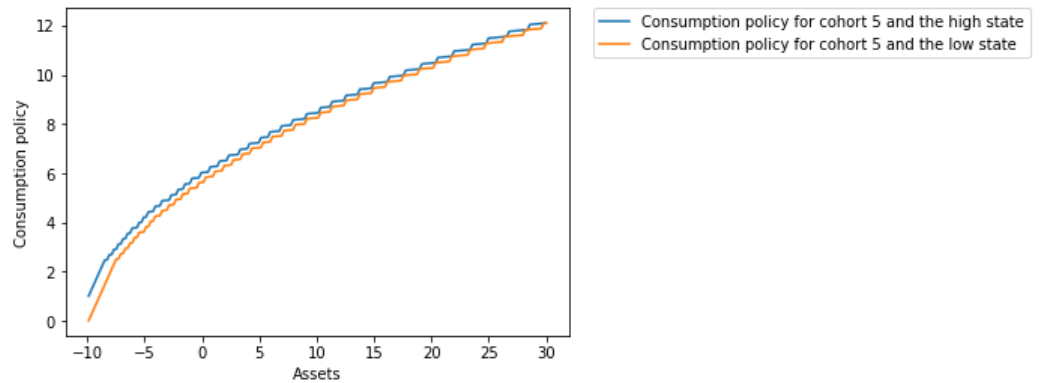
(a) (Quadratic Utility) Consumption policy function for $\sigma_y = 0$ for cohort 5

Cohort 5 Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



(b) (Quadratic Utility) Consumption policy function for $\sigma_y = 0.1$ for cohort 5

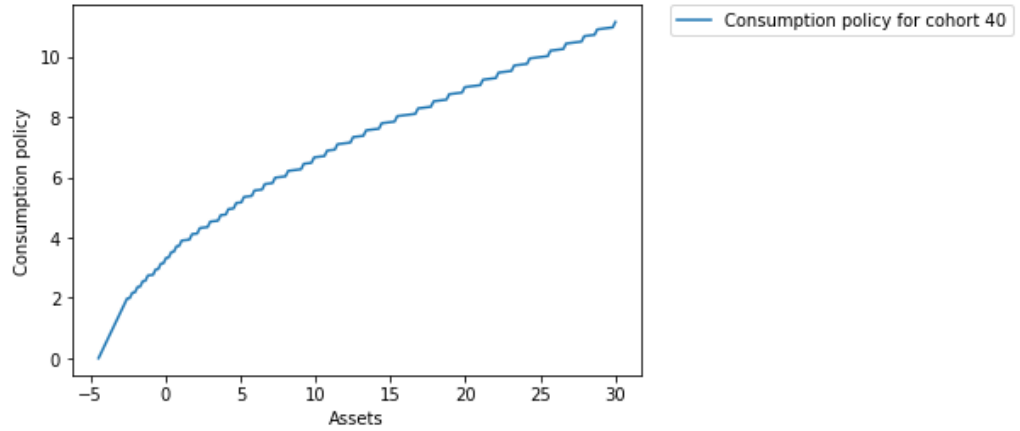
Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



(c) (Quadratic Utility) Consumption policy function for $\sigma_y = 0.5$ for cohort 5

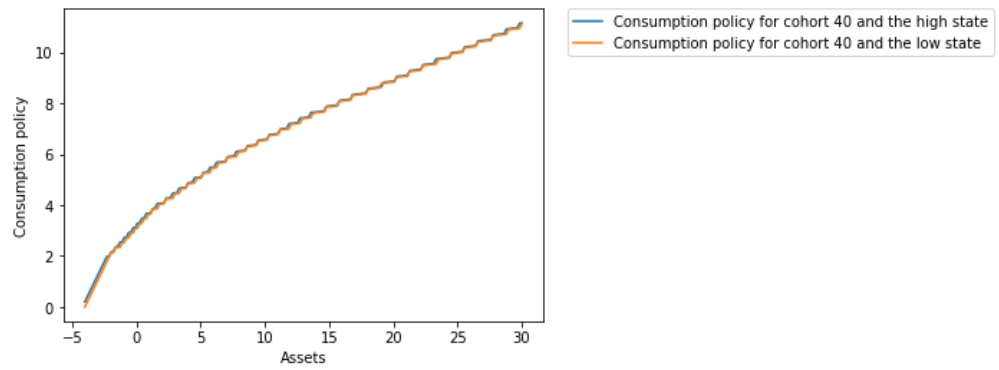
Figure 11: II.4.2.4. Uncertainty $\sigma_y = .5$ vs $\sigma_y = .1$ and $\sigma_y = 0$ for quadratic utility - cohort 40

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



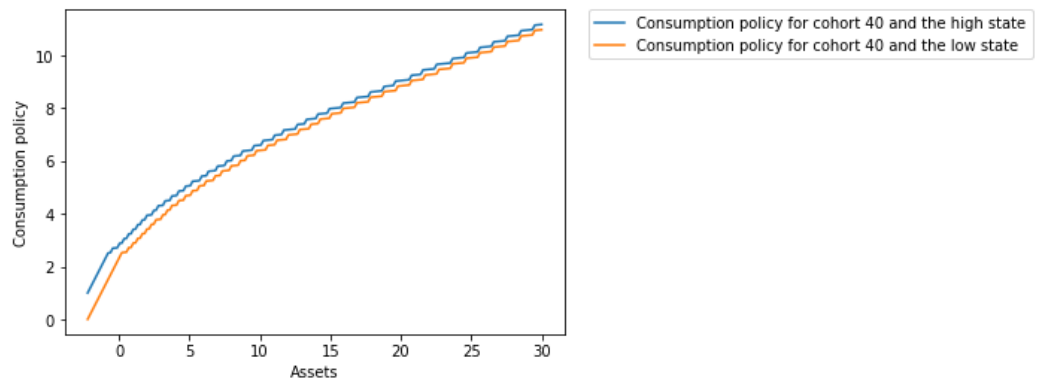
(a) (Quadratic Utility) Consumption policy function for $\sigma_y = 0$ for cohort 40

Cohort 40 Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



(b) (Quadratic Utility) Consumption policy function for $\sigma_y = 0.1$ for cohort 40

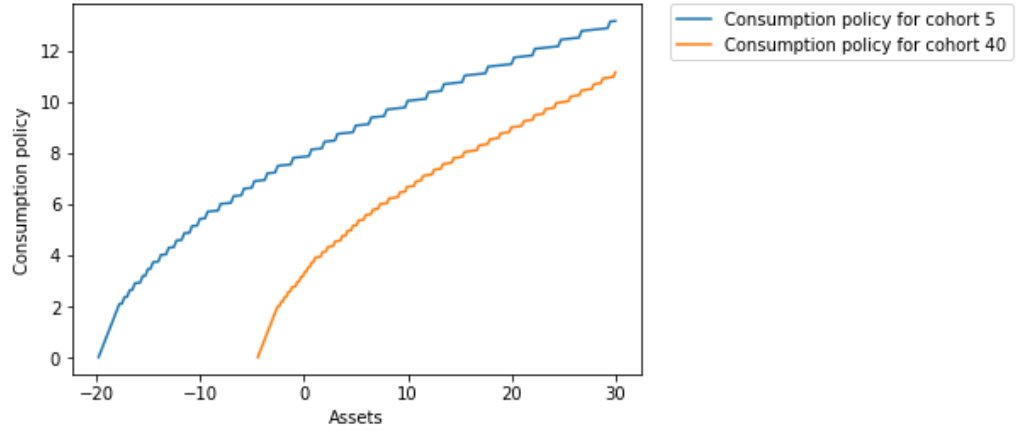
Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



(c) (Quadratic Utility) Consumption policy function for $\sigma_y = 0.5$ for cohort 5

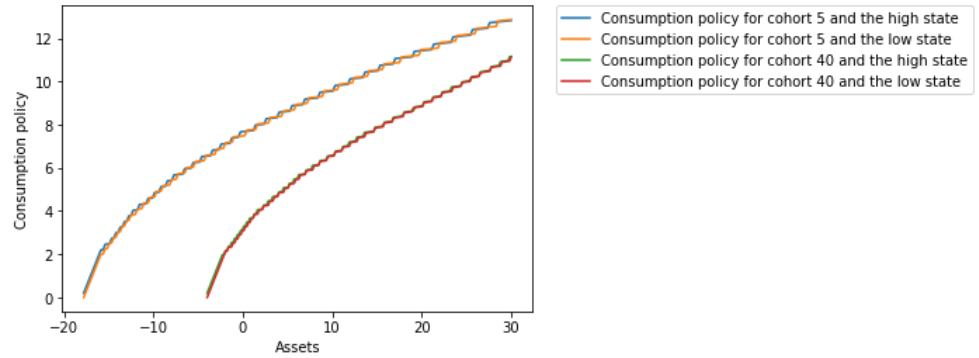
Figure 12: II.4.2.4. Uncertainty $\sigma_y = .5$ vs $\sigma_y = .1$ and $\sigma_y = 0$ for quadratic utility - cohort 5 and 40 comparison

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



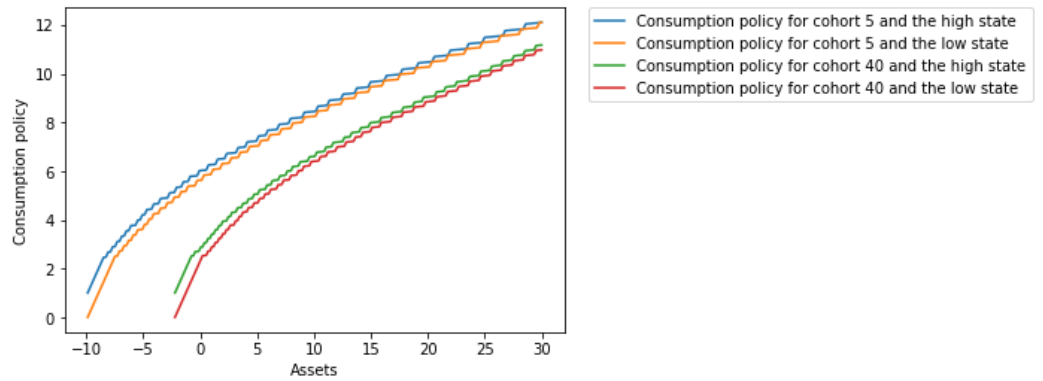
(a) (Quadratic Utility) Consumption policy function for $\sigma_y = 0$ for cohort 5

Cohort 5 and 40 Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.1$, $\gamma = 0$. in the 45 period OLG



(b) (Quadratic Utility) Consumption policy function for $\sigma_y = 0.1$ for cohort 5

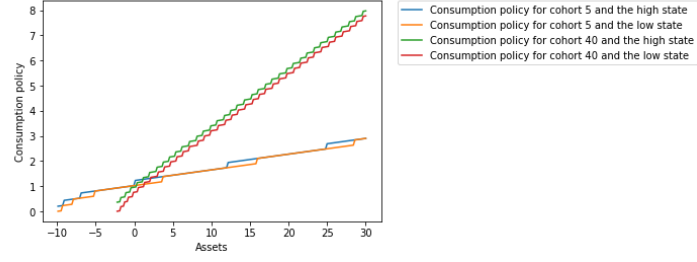
Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



(c) (Quadratic Utility) Consumption policy function for $\sigma_y = 0.5$ for cohort 5

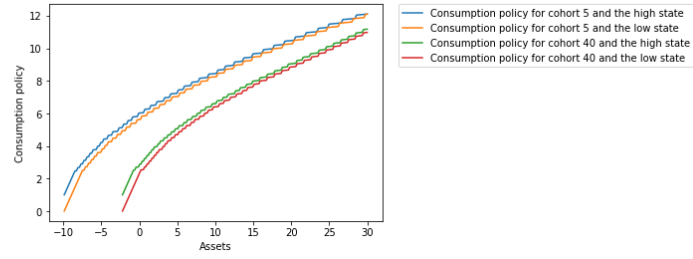
Figure 13: II.4.2.4. Uncertainty $\sigma_y = .5$ Quadratic vs CRRA

CRRA Consumption policy function on the grid of assets for $\sigma = 2$, $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



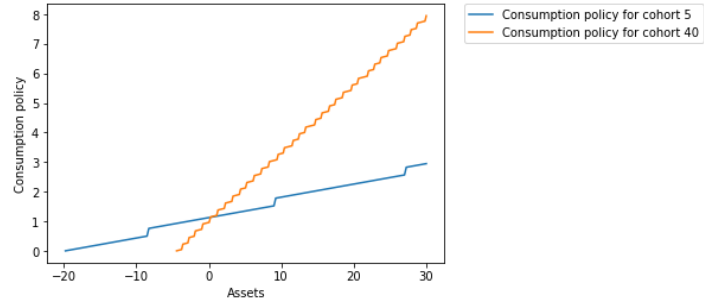
(a) (CRRA Utility) Consumption policy function for $\sigma_y = 0.5$

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



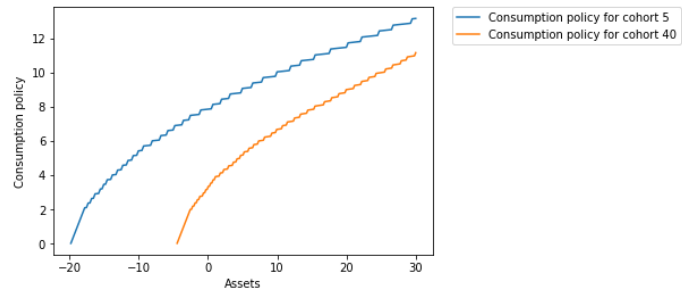
(b) (Quadratic Utility) Consumption policy function for $\sigma_y = 0.5$

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(c) (CRRA Utility) Consumption policy function for $\sigma_y = 0$

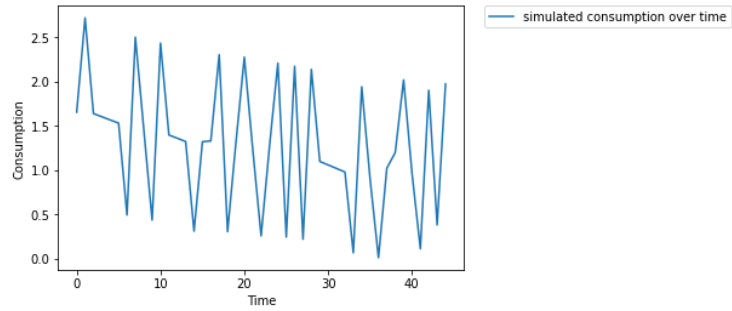
Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(d) (Quadratic Utility) Consumption policy function for $\sigma_y = 0$

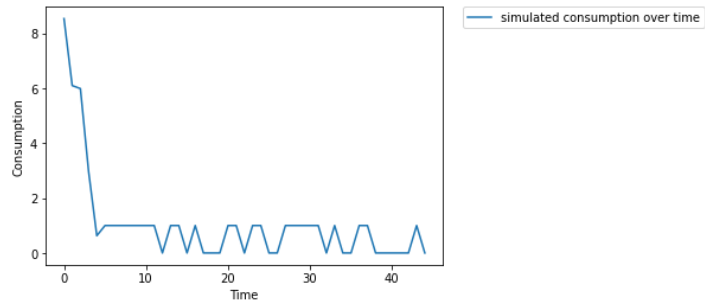
Figure 14: II.4.2.4. Uncertainty $\sigma_y = .5$ Quadratic vs CRRA - simulated consumption time paths

CRRA Consumption simulated time profile for $\sigma = 2$, $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



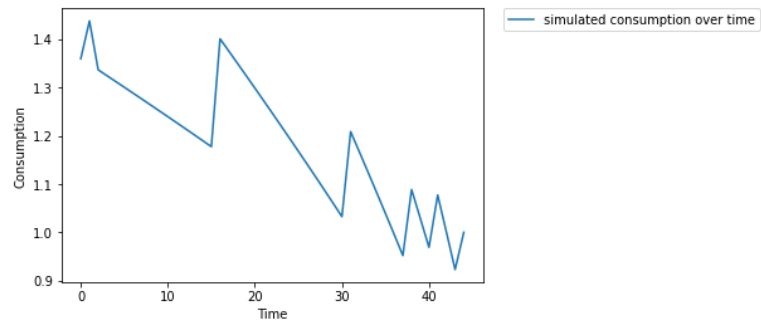
(a) (CRRA Utility) Simulated consumption time path for $\sigma_y = 0.5$

Quadratic utility Consumption simulated time profile for $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



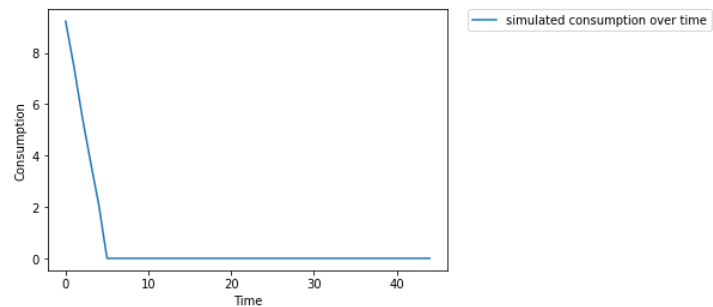
(b) (Quadratic Utility) Simulated consumption time path for $\sigma_y = 0.5$

CRRA Consumption simulated time profile for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(c) (CRRA Utility) Simulated consumption time path for $\sigma_y = 0$

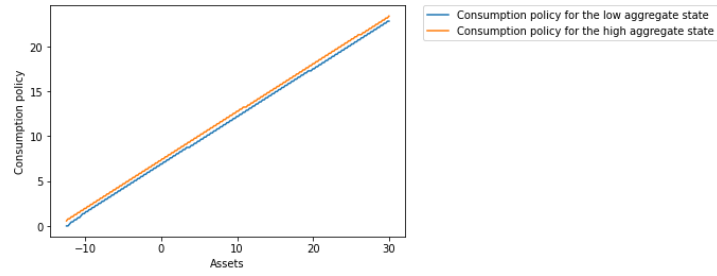
Quadratic utility Consumption simulated time profile for $\sigma_y = 0$, $\gamma = 0$. in the 45 period OLG



(d) (Quadratic Utility) Simulated consumption time path for $\sigma_y = 0$

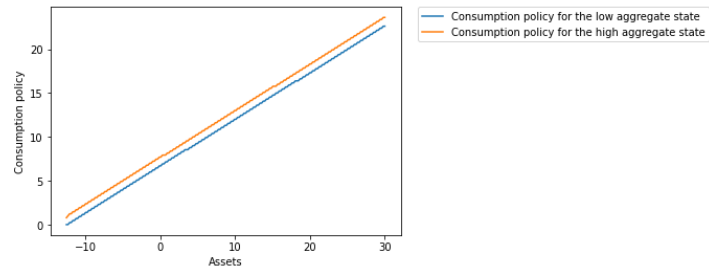
Figure 15: II.4.2.5. Uncertainty $\gamma = 0$ vs $\gamma = 0.95$ for $T = \infty$ and CRRA utility

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0$. in the $T = \infty$ economy



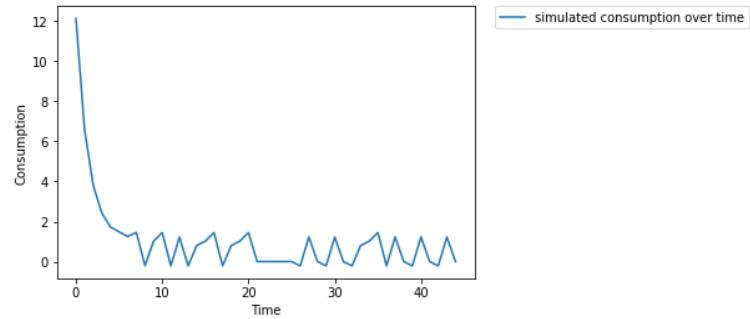
(a) (CRRA Utility) Consumption policy function for $\gamma = 0$

CRRA Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0.95$. in the $T = \infty$ economy



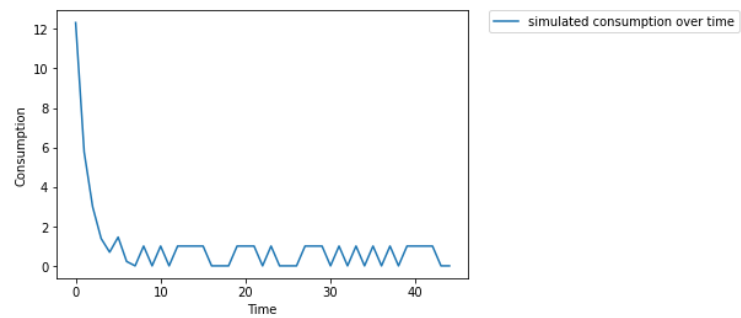
(b) (CRRA Utility) Consumption policy function for $\gamma = 0.95$

CRRA Consumption simulated time profile for $\sigma_y = 0.5$, $\gamma = 0$. in the $T = \infty$ economy



(c) (CRRA Utility) Simulated consumption time profile for $\gamma = 0$

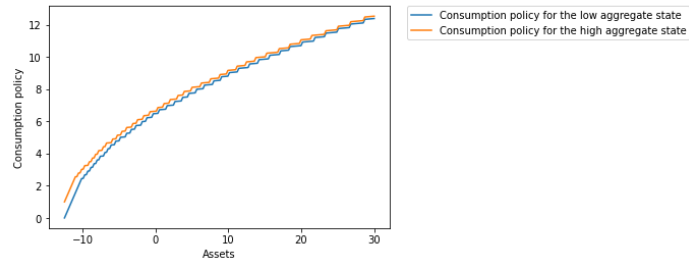
CRRA Consumption simulated time profile for $\sigma_y = 0.5$, $\gamma = 0.95$. in the $T = \infty$ economy



(d) (CRRA Utility) Simulated consumption time profile for $\gamma = 0.95$

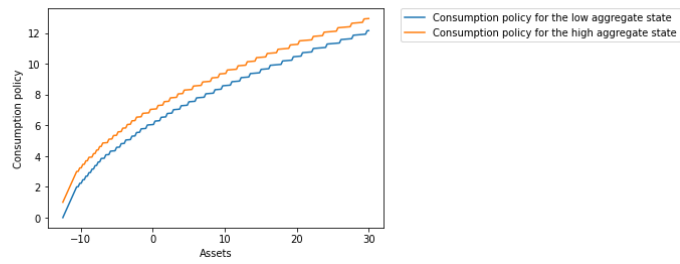
Figure 16: II.4.2.5. Uncertainty $\gamma = 0$ vs $\gamma = 0.95$ for $T = \infty$ and Quadratic utility

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0$, in the $T = \infty$ economy



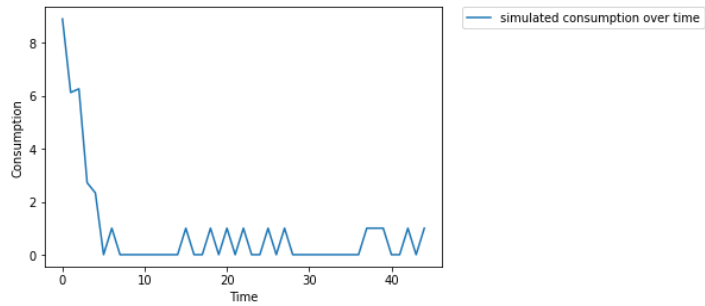
(a) (Quadratic Utility) Consumption policy function for $\gamma = 0$

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0.95$, in the $T = \infty$ economy



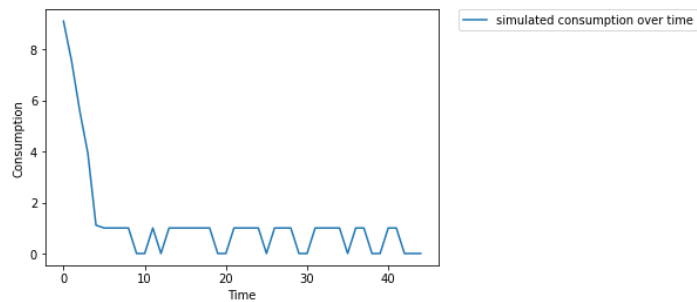
(b) (Quadratic Utility) Consumption policy function for $\gamma = 0.95$

Quadratic utility Consumption simulated time profile for $\sigma_y = 0.5$, $\gamma = 0$, in the $T = \infty$ economy



(c) (Quadratic Utility) Simulated consumption time profile for $\gamma = 0$

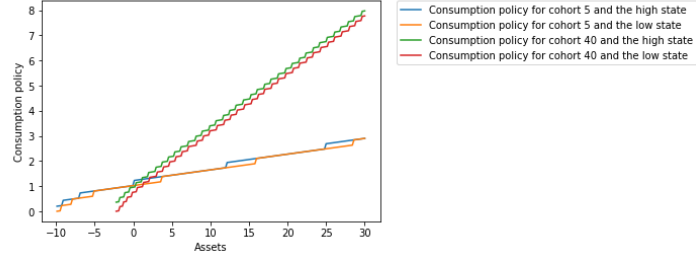
Quadratic utility Consumption simulated time profile for $\sigma_y = 0.5$, $\gamma = 0.95$, in the $T = \infty$ economy



(d) (Quadratic Utility) Simulated consumption time profile for $\gamma = 0.95$

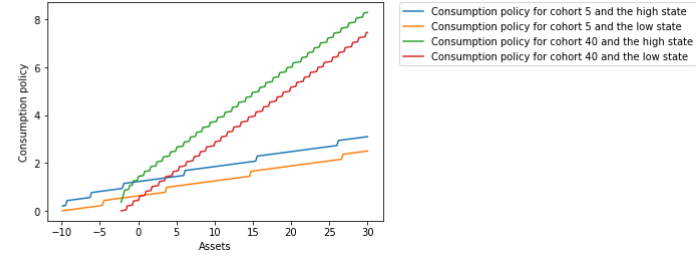
Figure 17: II.4.2.5. Uncertainty $\gamma = 0$ vs $\gamma = 0.95$ for $T=45$ and CRRA utility

CRRA Consumption policy function on the grid of assets for $\sigma=2$, $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



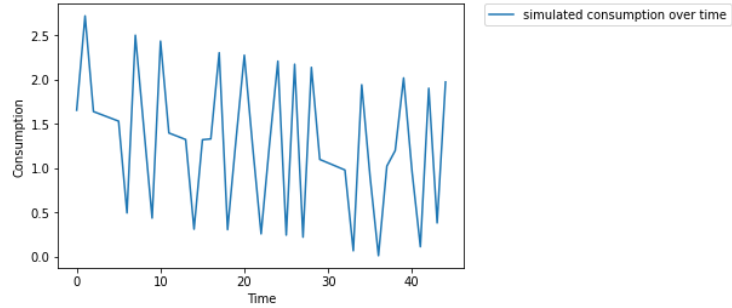
(a) (CRRA Utility) Consumption policy function for $\gamma = 0$

CRRA Consumption policy function on the grid of assets for $\sigma=2$, $\sigma_y = 0.5$, $\gamma = .95$. in the 45 period OLG



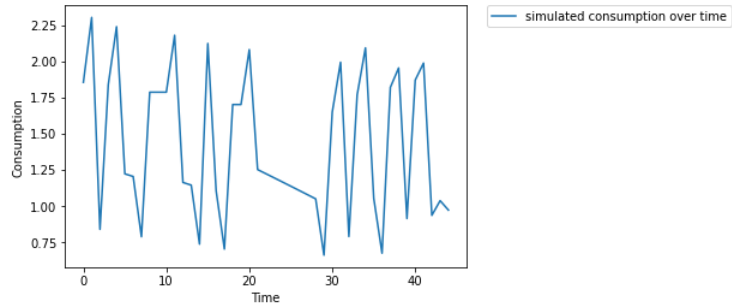
(b) (CRRA Utility) Consumption policy function for $\gamma = 0.95$

CRRA Consumption simulated time profile for $\sigma=2$, $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



(c) (CRRA Utility) Simulated consumption time profile for $\gamma = 0$

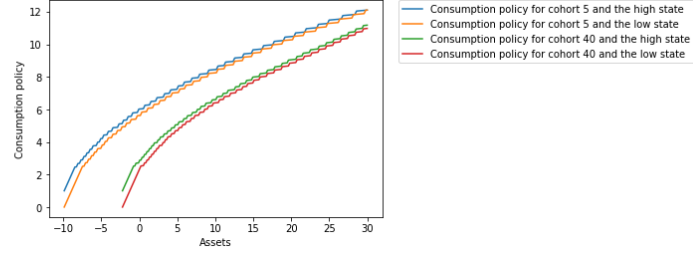
CRRA Consumption simulated time profile for $\sigma=2$, $\sigma_y = 0.5$, $\gamma = .95$. in the 45 period OLG



(d) (CRRA Utility) Simulated consumption time profile for $\gamma = 0.95$

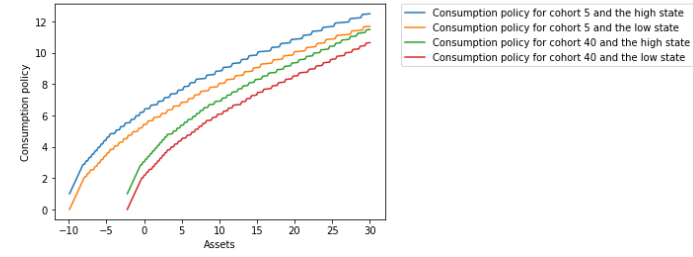
Figure 18: II.4.2.5. Uncertainty $\gamma = 0$ vs $\gamma = 0.95$ for $T=45$ and Quadratic utility

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



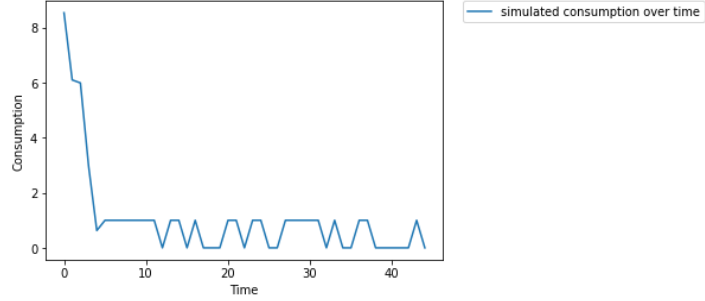
(a) (Quadratic Utility) Consumption policy function for $\gamma = 0$

Quadratic utility Consumption policy function on the grid of assets for $\sigma_y = 0.5$, $\gamma = .95$. in the 45 period OLG



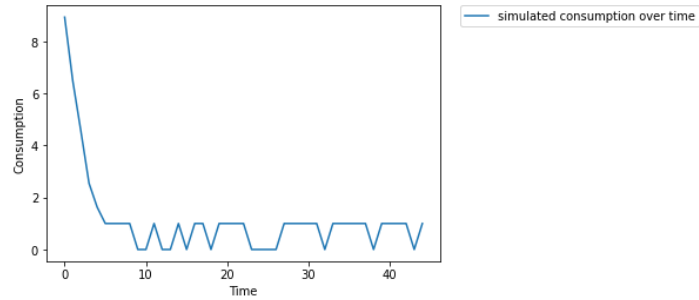
(b) (Quadratic Utility) Consumption policy function for $\gamma = 0.95$

Quadratic utility Consumption simulated time profile for $\sigma_y = 0.5$, $\gamma = 0$. in the 45 period OLG



(c) (Quadratic Utility) Simulated consumption time profile for $\gamma = 0$

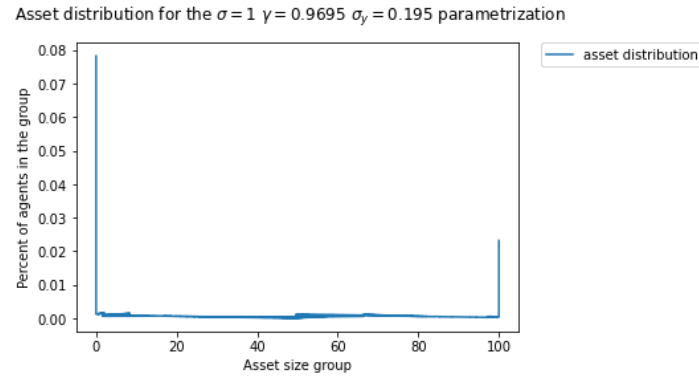
Quadratic utility Consumption simulated time profile for $\sigma_y = 0.5$, $\gamma = .95$. in the 45 period OLG



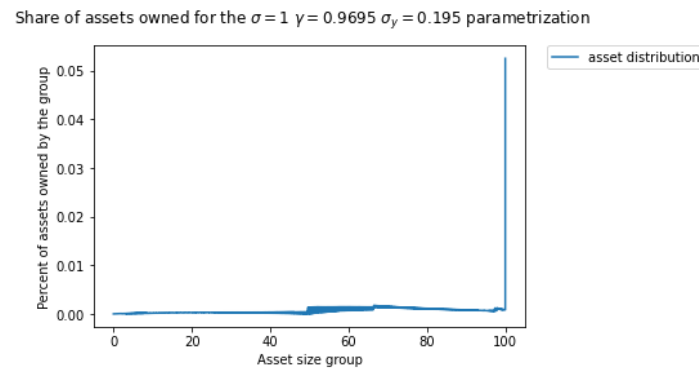
(d) (Quadratic Utility) Simulated consumption time profile for $\gamma = 0.95$

II.5. General Equilibrium

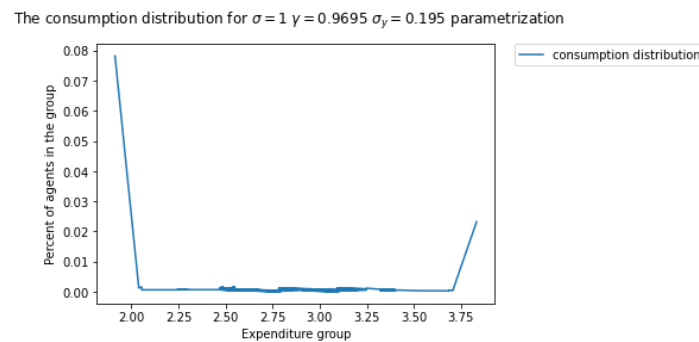
Figure 19: II.5.1. The asset and consumption distribution in the simple ABHI model for the $\sigma = 1$ $\gamma = 0.9695$ $\sigma_y = 0.195$ parametrization



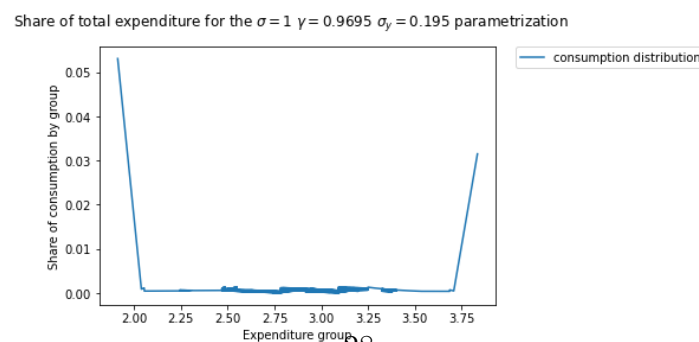
(a) Asset distribution in absolute values



(b) Asset distribution as % share of all assets

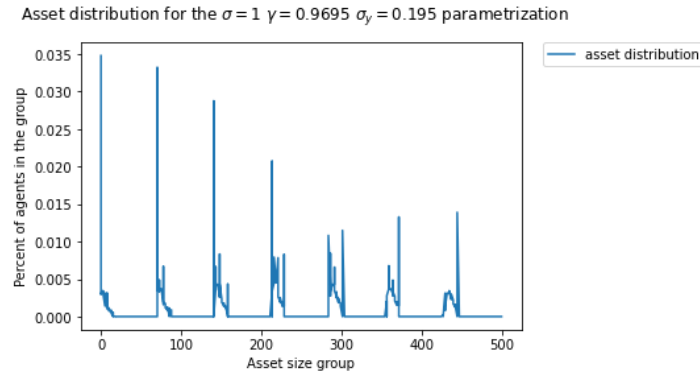


(c) Consumption distribution in absolute value

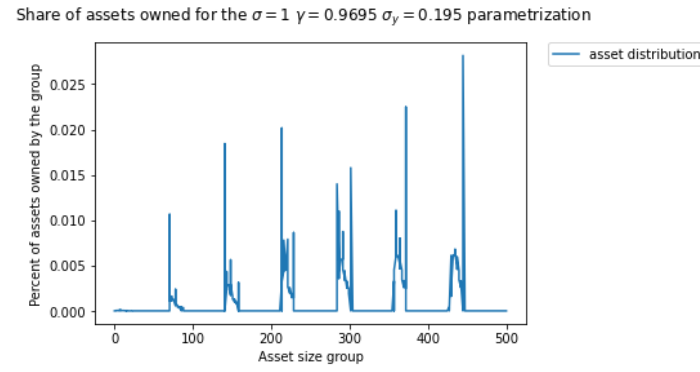


(d) Consumption distribution as share of total expenditure

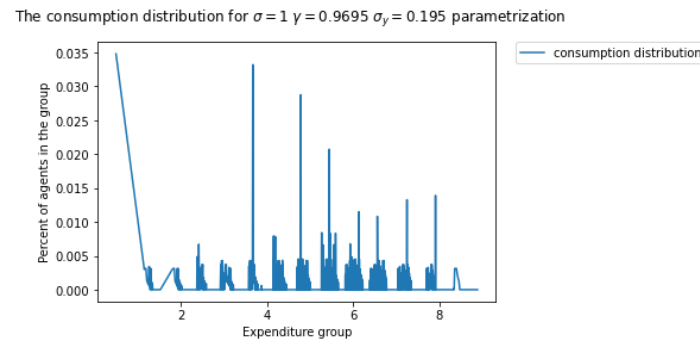
Figure 20: II.5.2.1. The asset and consumption distribution in the Aiyagari model with 7 states for the $\sigma = 1$ $\gamma = 0.9695$ $\sigma_y = 0.195$ parametrization



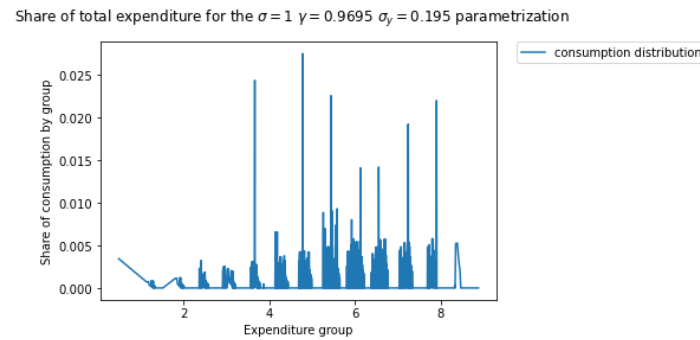
(a) Asset distribution in absolute values



(b) Asset distribution as % share of all assets

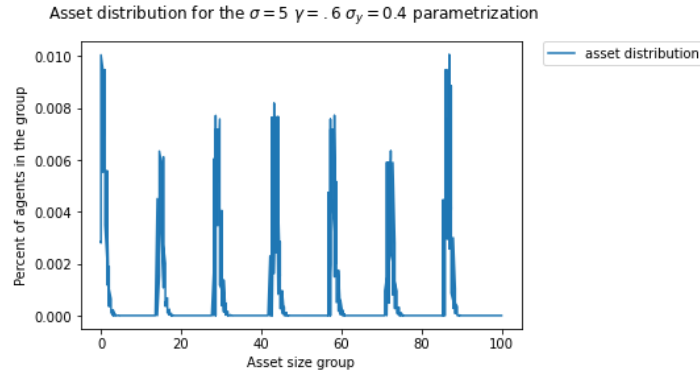


(c) Consumption distribution in absolute value

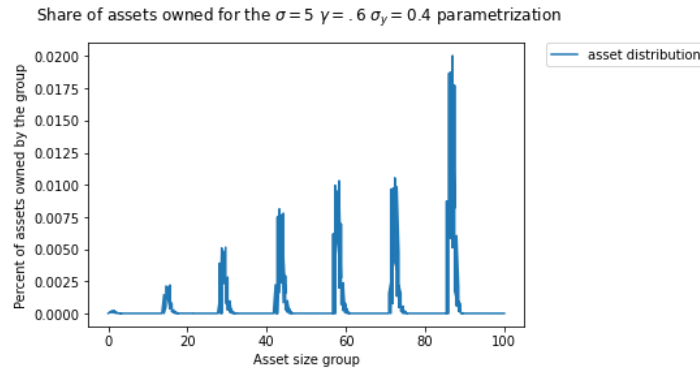


(d) Consumption distribution as share of total expenditure

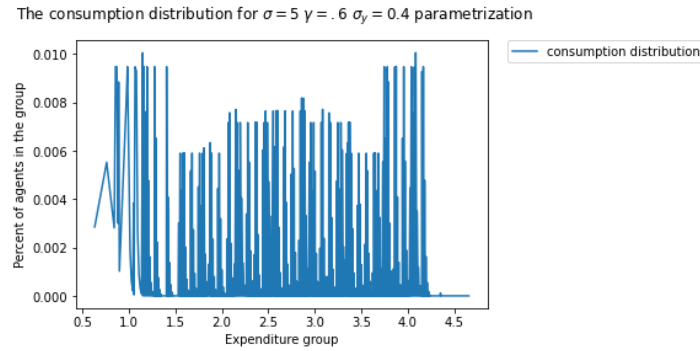
Figure 21: II.5.2.2. The asset and consumption distribution in the Aiyagari model with 7 states for the $\sigma = 5$ $\gamma = 0.6$ $\sigma_y = 0.4$ parametrization



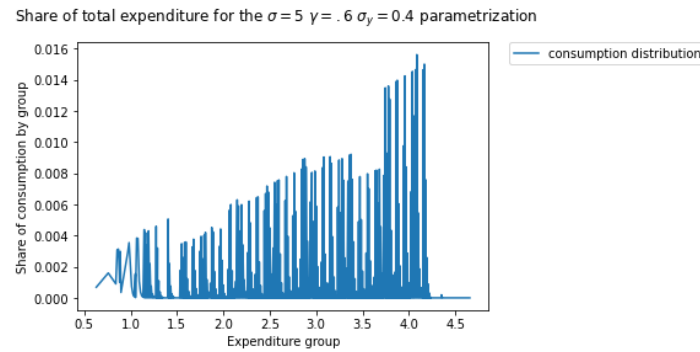
(a) Asset distribution in absolute values



(b) Asset distribution as % share of all assets

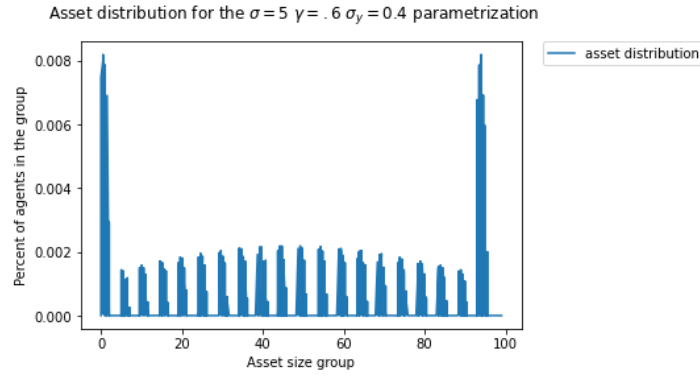


(c) Consumption distribution in absolute value

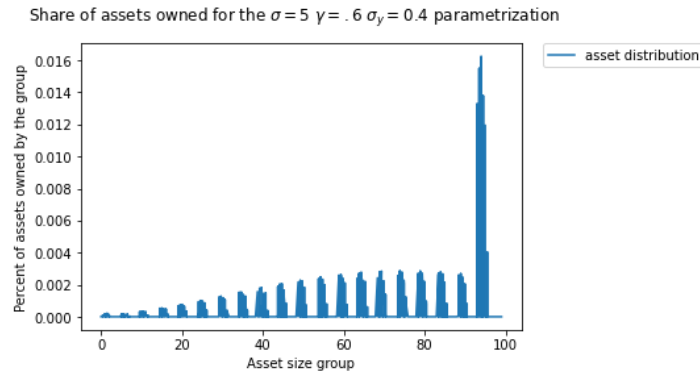


(d) Consumption distribution as share of total expenditure

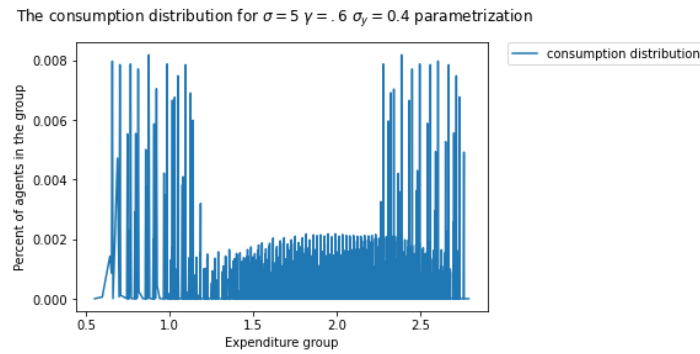
Figure 22: II.5.2.3. The asset and consumption distribution in the Aiyagari model with 20 states for the $\sigma = 20$ $\gamma = 0.4$ $\sigma_y = 0.6$ parametrization



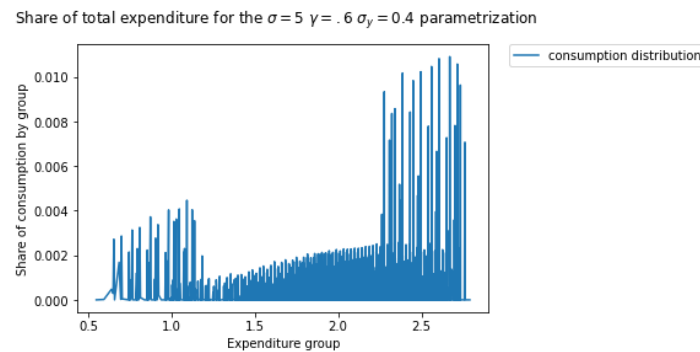
(a) Asset distribution in absolute values



(b) Asset distribution as % share of all assets



(c) Consumption distribution in absolute value



(d) Consumption distribution as share of total expenditure