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HW-1

“Amortise, Online, Streaming”

Amortise:

1.

Suppose we perform a sequence of n operations on a data structure in which the ith operation costs i if i is an exact power of 2, and 1 otherwise. Use aggregate analysis to determine the amortized cost per operation.

Online:

2.

Imagine you're going skiing, but you don't know how many days you'll ski. You have two options: rent skis for a daily fee at cost 1 or buy them outright for a price B.

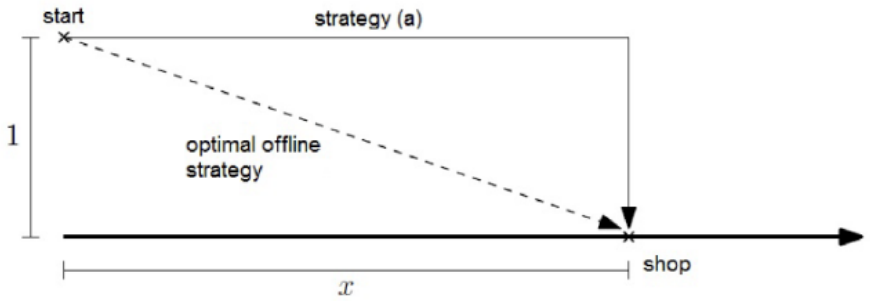
You see a deteministic algorithm with competitige-Ratio of 2-1/B at class

Suggest an algorithm with competitive-Ratio <= 15/8 = 1.85

(**hint:flip a coin for two different days to buy**)

3.

مساله‌ی پیش‌رو را در نظر بگیرید. شما در یک خیابان به دنبال یک مغازه‌ی خاص هستید. می‌دانید که در چه جهتی باید حرکت کنید اما نمی‌دانید چقدر. همچنین برای اینکه مغازه‌ی مورد نظرتان را شناسایی کنید باید دقیقا روبروی آن قرار بگیرید و می‌خواهید با طی کمترین فاصله به هدف برسید. در آغاز، در فاصله‌ی ۱ از مغازه‌ها قرار دارید (به شکل دقت کنید.)



۱. این استراتژی را در نظر بگیرید: موازی ردیف مغازه‌ها حرکت می‌کنید تا به مغازه‌ی مورد نظرتان برسید، آنگاه به سمت آن می‌روید. نسبت رقابتی این استراتژی چقدر است؟

۲. یک استراتژی با نسبت رقابتی ۱/۱ ارائه دهید.

۳.(**hard**) فرض کنید که میخواهیم روی منحنی های $1/z^x$ حرکت کنیم z ای را محاسبه کنید که نسبت رقابتی را حداقل کند(لازم نیست عددی بدست بیاورید یا انتگرال ها را حساب کنید کافیست مساله را فرموله کنید و جلو ببرید)

Hint: برای قسمت ۱و۲ هم میتوان هندسی حساب کرد هم با مساوی صفر قرار دادن مشتق و یا هر ایده ی دیگر

Streaming:

4.

یک الگوریتم جریانی (streaming) در حالت یک بارگذر، برای مساله‌ی بررسی kـراس همبند بودن گراف ارائه دهید. برای گراف $G = (V, E)$ می‌خواهیم بررسی کنیم که آیا k راس وجود دارد که با حذف آن‌ها گراف باقی‌مانده ناهمبند شود.

5, 6.

These exercises are designed to get you familiar with the very important concept of a 2-universal hash family, as well as give you constructive examples of such families.

Let X and Y be finite sets and let Y^X denote the set of all functions from X to Y. We will think of these functions as "hash" functions. [The term "hash function" has no formal meaning; strictly speaking, one should say "family of hash functions" or "hash family" as we do here.] A family $\mathscr{H} \subseteq Y^X$ is said to be 2-universal if the following property holds, with $h \in_R \mathscr{H}$ picked uniformly at random:

$$\forall x,x' \in X\ \forall y,y' \in Y \left(x \neq x' \ \Rightarrow\ \mathbb{P}_h\left\{h(x) = y \wedge h(x') = y'\right\} = \frac{1}{|Y|^2} \right).$$

We shall give two examples of 2-universal hash families from the set $X = \{0,1\}^n$ to the set $Y = \{0,1\}^k$ (with $k \leq n$).

2-1 Treat the elements of X and Y as column vectors with 0/1 entries. For a matrix $A \in \{0,1\}^{k\times n}$ and vector $b \in \{0,1\}^k$, define the function $h_{A,b} : X \to Y$ by $h_{A,b}(x) = Ax + b$, where all additions and multiplications are performed mod 2.

Prove that the family of functions $\mathscr{H} = \{h_{A,b} : A \in \{0,1\}^{k\times n}, b \in \{0,1\}^k\}$ is 2-universal.

2-2 Identify X with the finite field $\mathbb{F}_{2^n}$ using an arbitrary bijection—truly arbitrary: e.g., the bijection need not map the string 0^n to the zero element of $\mathbb{F}_{2^n}$. For elements $a,b \in X$, define the function $g_{a,b} : X \to Y$ as follows:

$$\begin{aligned} g_{a,b}(x) &= \text{rightmost } k \text{ bits of } f_{a,b}(x), \quad \text{where} \\ f_{a,b}(x) &= ax + b, \quad \text{with addition and multiplication performed in } \mathbb{F}_{2^n}. \end{aligned}$$

Prove that the family of functions $\mathscr{G} = \{g_{a,b} : a,b \in \mathbb{F}_{2^n}\}$ is 2-universal. Is the family $\mathscr{G}$ better or worse than $\mathscr{H}$ in any sense? Why?

Finding meidan in streaming data:

7.

We have stream of infinite integer between a and b from unknown distribution.

Suggest an algorithm that in each timestep of n converge to (k*n)th element of seen data with space complexity of O(1). (note: 0 <= k <= 1 | for example k=0.5 means median | k=0 means minimum |k=1 means maximum)

(**hint: start with some number, In each time step compair it with current number of stream and update its value with sum with x or -y**)

8.

We have stream of n integer between a and b from unknown distribution.

Suggest an algorithm that approximate median of the stream in space complexity of O(k)

9.

(**hard**)We have stream of n integer between a and b from unknown distribution.

Suggest an algorithm that approximate median of the stream in space complexity of O(log(n))