Gain driven breathers in $\mathcal{PT}$-symmetric metamaterials

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CAPACITIES

Contents

- Introduction
- Discreteness and nonlinearity in metamaterials
- Parity-Time (PT)-symmetric quantum mechanics and optics
- PT-symmetric metamaterials
- PT symmetry in zero dimensions
- Conclusions

Introduction: Left handed metamaterials

Maxwell equations

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times H = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Constitutive relations

$$\mathbf{D} = \varepsilon_0 \varepsilon(\omega) \mathbf{E}$$

$$\mathbf{B} = \mu_0 \mu(\omega) \mathbf{H}$$

Wave equations

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{c^2}{\varepsilon\mu} \nabla^2 \mathbf{E}, \quad \frac{\partial^2 \mathbf{B}}{\partial t^2} = \frac{c^2}{\varepsilon\mu} \nabla^2 \mathbf{B}$$

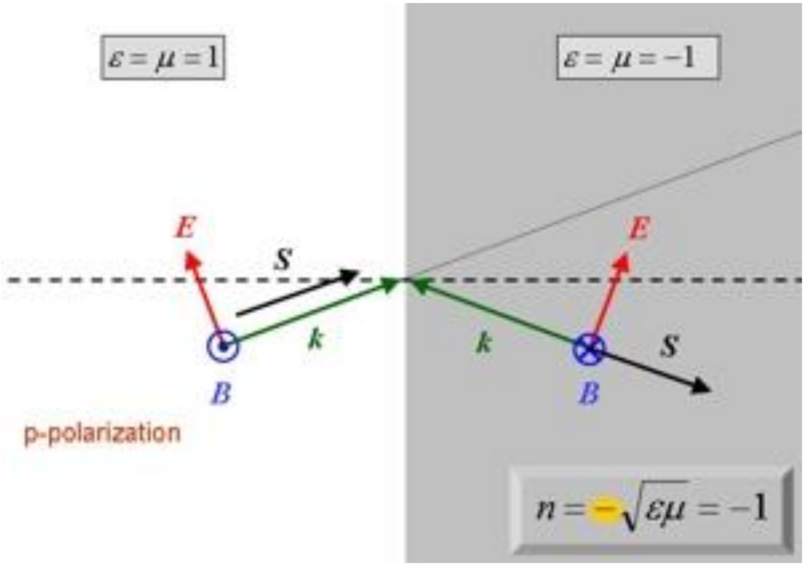
Plane waves

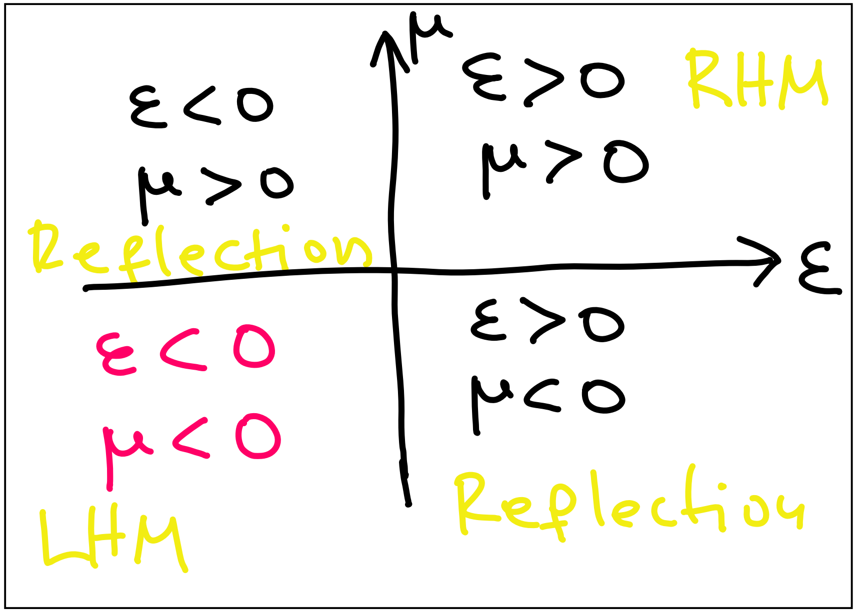
$$\mathbf{E} \approx e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

$$\boxed{k^2 = \frac{\varepsilon\mu}{c^2} \omega^2}$$

There is propagation for: $\varepsilon > 0, \mu > 0$ and $\boxed{\varepsilon < 0, \mu < 0}$

Left handed metamaterials



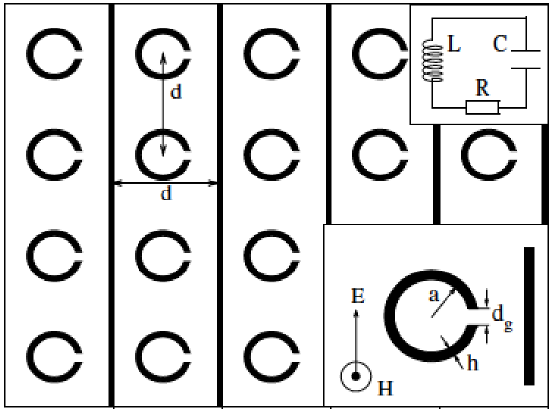


$$n=\sqrt{\varepsilon\mu}=\sqrt{(\varepsilon'+i\varepsilon'')(\mu'+i\mu'')}\cong\sqrt{\varepsilon'\mu'+i(\varepsilon'\mu''+\mu'\varepsilon'')}\cong\pm\sqrt{\varepsilon'\mu'}$$

The wave vector k is in opposite direction wrt the Poynting vector

Nonlinearity

Nonlinearity across the gap of an SRR alters the electric field that in-turn affects self consistently the magnetic field and thus the magnetic permeability



$$\mu_{\rm eff}(\mathbf{H}) = 1 + \frac{F\,\omega^2}{\omega_{0\rm NL}^2(\mathbf{H}) - \omega^2 + i\Gamma\omega},$$

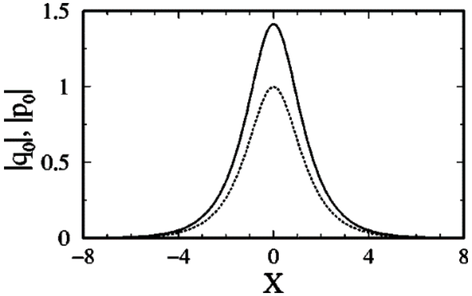
$$\omega_{0\rm NL}^2(\mathbf{H}) = \left(\frac{c}{a}\right)^2 \frac{d_g}{\pi h \epsilon_D(|\mathbf{E}_g(\mathbf{H})|^2)}$$

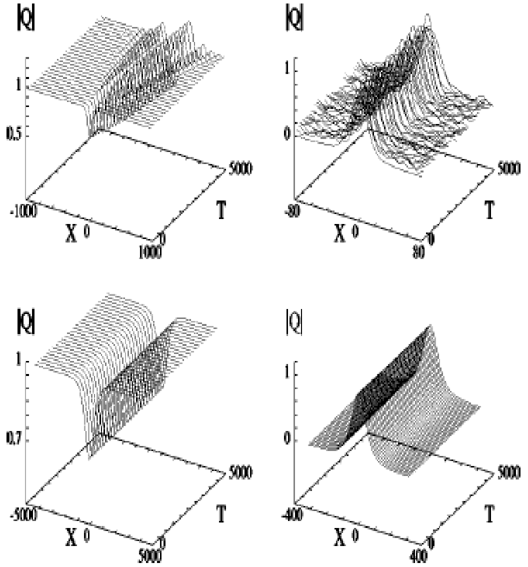
Zharov et al. , PRL 91, 037401 2003

For Kerr nonlinearity and to lowest nonlinear order in the magnetic field compound bright/dark E/M (Manakov) solitons may be generated

$$\varepsilon = \varepsilon_{0D} + \alpha\,|\mathbf{E}|^2$$

$$\mu = \mu_{0D} + \beta\,|\mathbf{H}|^2$$





N. Lazarides and GPT, PRE **71**, 036614 (2005)

Discreteness and nonlinearity

Weak coupling of nonlinear elements leads to modulational instability that generates Intrinsic localized modes (discrete breathers)

$$\frac{dQ_n}{dt} = I_n \quad (2)$$

$$L\frac{dI_n}{dt} + RI_n + f(Q_n) = M\left(\frac{dI_{n-1}}{dt} + \frac{dI_{n+1}}{dt}\right) + \mathcal{E}. \quad (3)$$

RLC model with nonlinear capacitance and weak inductive coupling

$$f(Q_n) = U_n$$

$$Q_n = C_\ell\left(1 + \alpha\frac{U_n^2}{3\epsilon_\ell U_c^2}\right)U_n$$

$$Q_n, I_n, U_n$$

Charge, current, voltage at the n-th SRR

In dimensionless units

$$\frac{d^2 q_n}{d\tau^2} + \gamma\frac{dq_n}{d\tau} + f(q_n) = \lambda\left(\frac{dq_{n+1}}{d\tau} + \frac{dq_{n-1}}{d\tau}\right) + \boldsymbol{\varepsilon}(\boldsymbol{\tau})$$

Linear spectrum

$$f(q_n) \simeq q_n - \frac{\alpha}{3\,\epsilon_\ell}q_n^3 + 3\left(\frac{\alpha}{3\,\epsilon_\ell}\right)^2 q_n^5 + \mathcal{O}(q_n^7)$$

$$\Omega_k = [1 - 2\,\lambda\,\cos(k\,D)]^{-1/2},$$

Localization in translationally invariant systems

Two identical oscillators

$$\ddot{x}_1 + V'(x_1) = kx_2$$
$$\ddot{x}_2 + V'(x_2) = kx_1$$

I. Linear

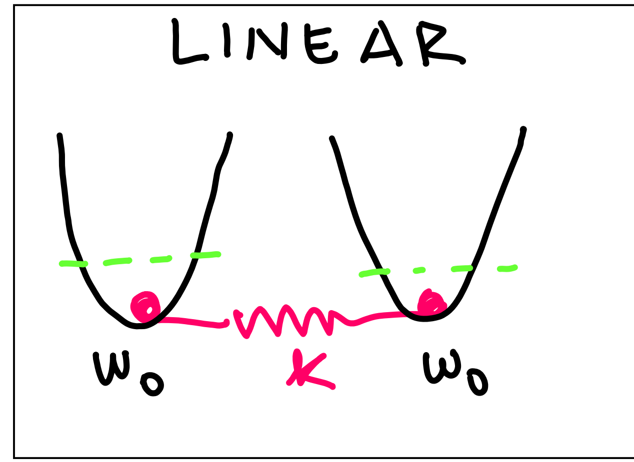
$$V(x) = \frac{1}{2}\omega_0^2 x^2$$

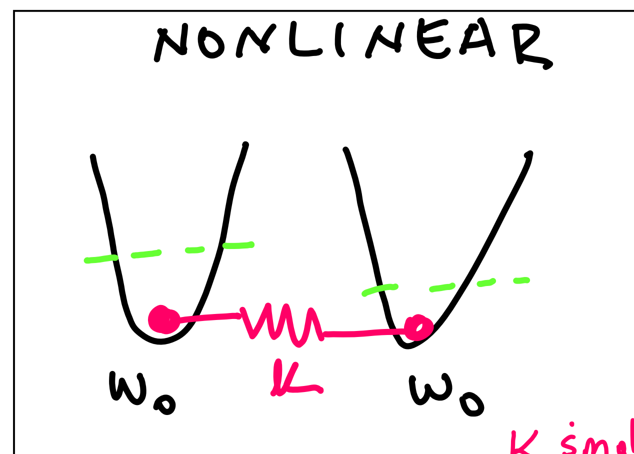
The individual oscillators are always on resonance and thus there is complete energy exchange

II. Nonlinear

$$V(x) = \frac{1}{2}\omega_0^2 x^2 + \frac{1}{4}\beta x^4$$

The frequency of oscillation depends on the initial energy of each oscillator and thus may be completely out of resonance



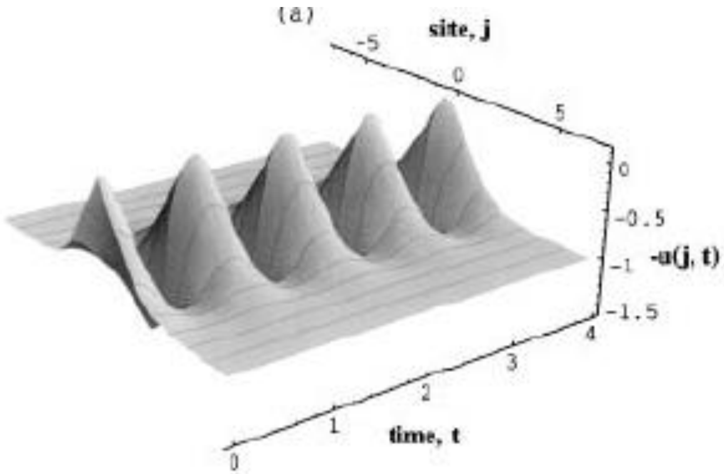


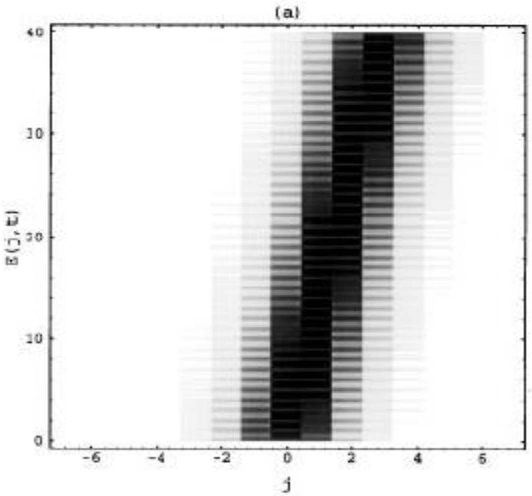
Nonlinearity

Time periodic, localized modes in extended discrete systems
of coupled nonlinear oscillators

$$H = \sum_n \left[\frac{1}{2} \dot{u}_n^2 + \frac{k}{2}(u_{n+1} - u_n)^2 + V(u_n) \right],$$

$$V(u_n) = \frac{1}{4}(1 - u_n^2)^2$$

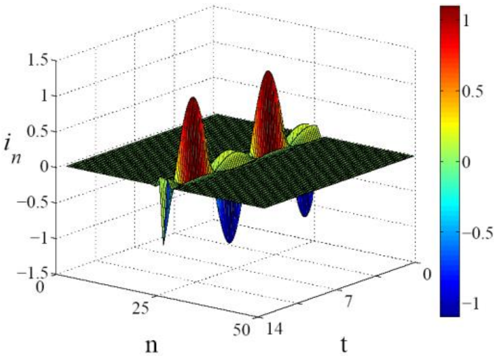




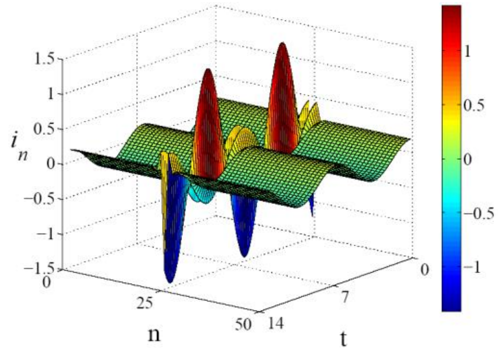
Discrete breathers may be mobile

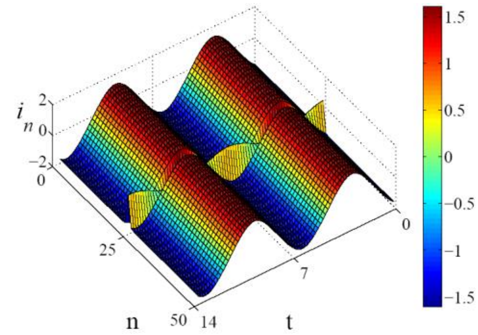
Discrete breathers in nonlinear metamaterials

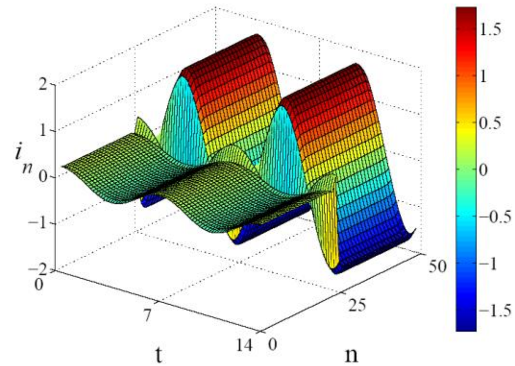
Hamiltonian breather



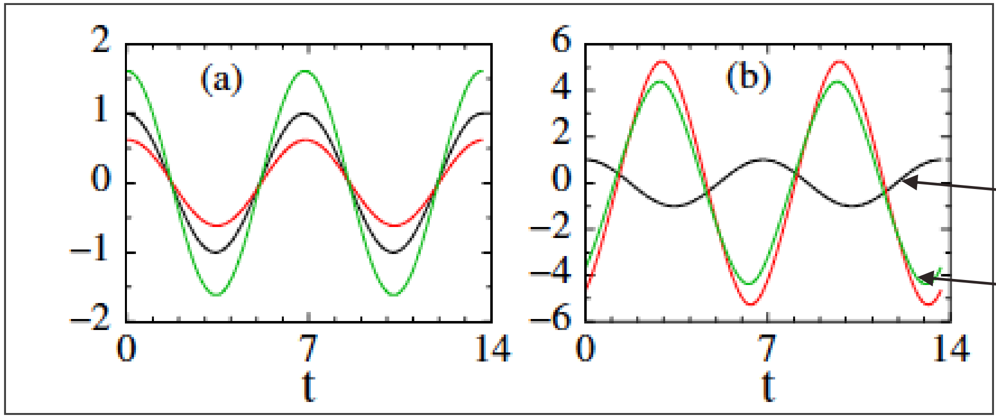
Dissipative breahers







Left-handedness below RLC resonance



Driver

Induced local field

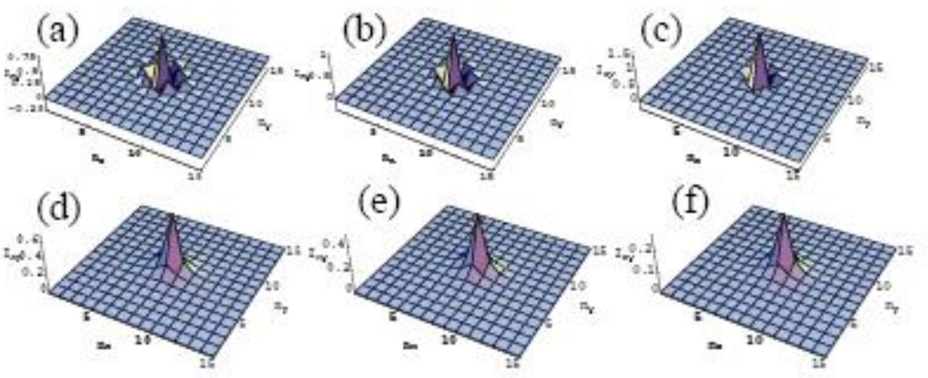
in-phase response

Out-of-phase response

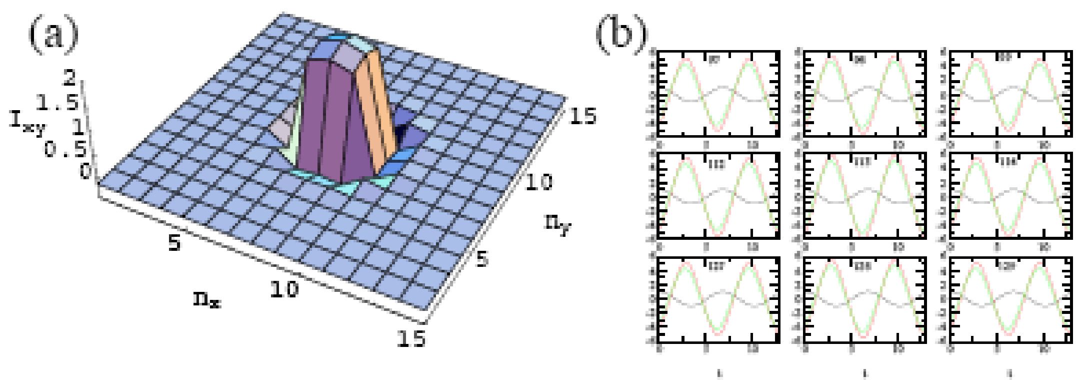
Lazarides, et al. PRL **97**, 157406 (2006)

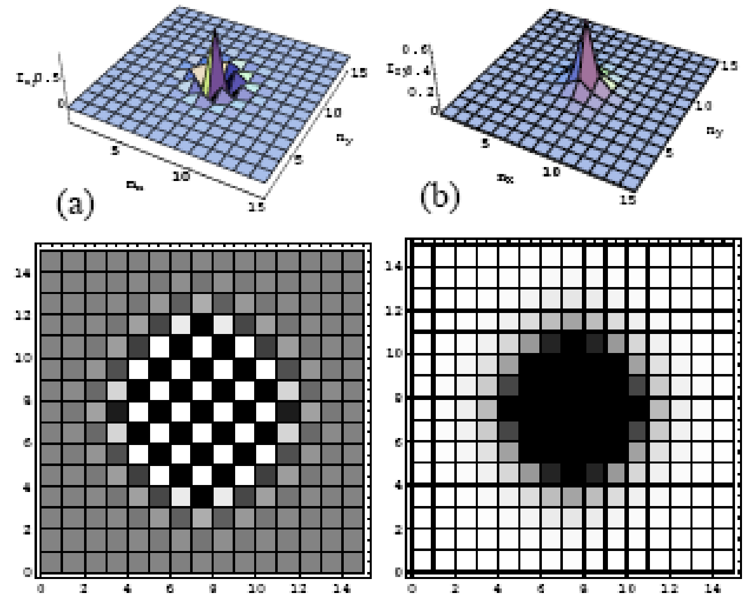
Nonlinear localization in 2D

Hamiltonian breathers



α=+1





α=-1

Left-handed
dissipative
multibreather

M. Eleftheriou et al. PRE 80, 017601 (2009)

$\mathcal{PT}$ – symmetric Quantum Mechanics

Should a Hamiltonian be Hermitian in order to have real eigenvalues?

$$\hat{P} \equiv \begin{cases} \hat{p} \to -\hat{p} \\ \hat{x} \to -\hat{x} \end{cases} \qquad \hat{T} \equiv \begin{cases} \hat{p} \to -\hat{p} \\ \hat{x} \to \hat{x} \\ i \to -i \end{cases}$$

$$PT-Hamiltonian \quad \Leftrightarrow \quad V^*(x) = V(-x)$$

Parity and time operators

$\mathcal{PT}$ symmetric Hamiltonian share common eigenfunctions with the $\mathcal{PT}$ operator. As a result they can exhibit entirely real spectra.

Pseudo-Hermitian quantum mechanics?

$$\boxed{PT - Potential \Leftrightarrow V^*(x) = V(-x)}$$

Real part: even
Imaginary part: odd

A complex $\mathcal{PT}$-potential, below threshold, has real eigenvalues

*C. M.Bender et al, Phys. Rev. Lett., 80, 5243 (1998); C. M.Bender et al, Phys. Rev. Lett., 89, 270401 (2002)
C. M.Bender et al, Phys. Rev. Lett., 98, 040403 (2007); C. M.Bender, Contemporary Physics, 46, 277 (2005)

$\mathcal{PT}$ symmetric two level system

$$H = \begin{pmatrix} i\gamma & V \\ V & -i\gamma \end{pmatrix}$$

$$\gamma = 0 \qquad \lambda_{\pm} = \pm V$$

$$\gamma \neq 0 \qquad \lambda_{\pm} = \pm\sqrt{V^2 - \gamma^2}$$

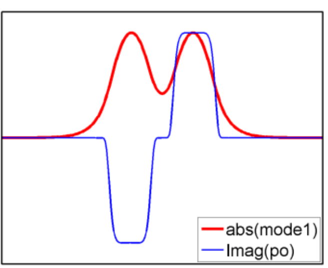
Solution

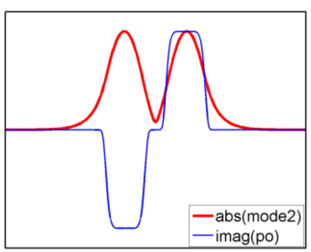
$$\psi_{1,2}(t) = A_{1,2}e^{\lambda_+ t} + B_{1,2}e^{\lambda_- t}$$

Eigenvalues

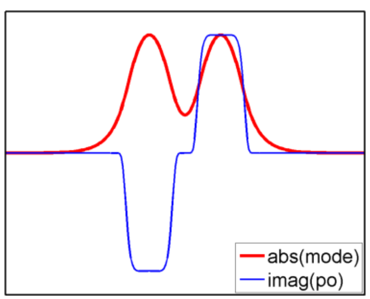
When the eigenvalues become imaginary, the solutions explode/decay respectively

Below transition

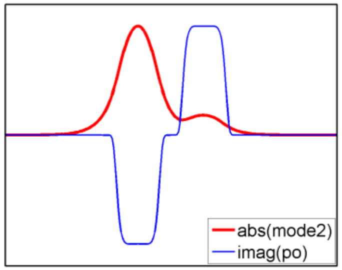


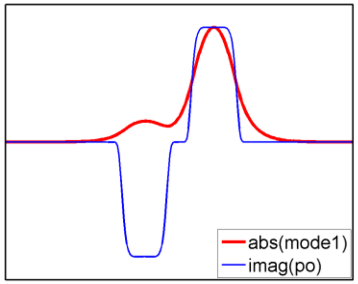


Transition



Above transition





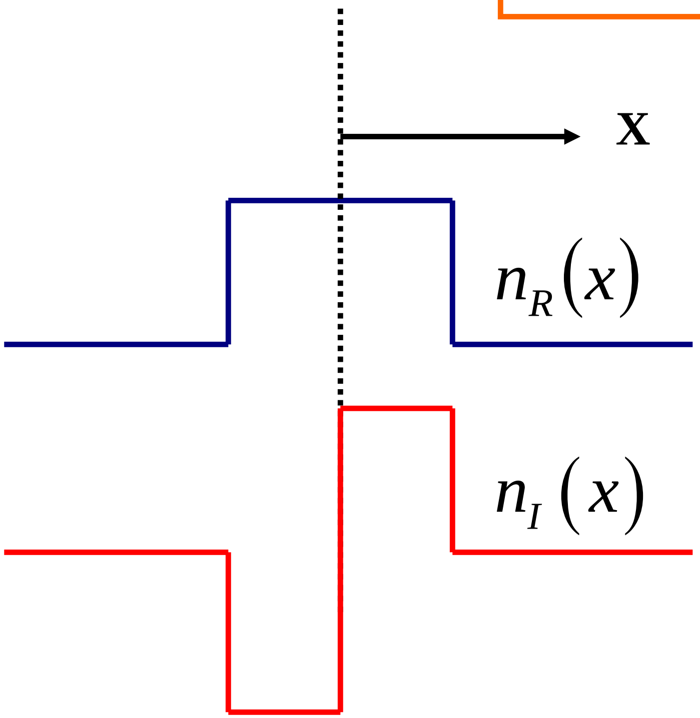
Bender C.M. et al, Phys. Rev. Lett. 98 040403 (2007).

$\mathcal{PT}$ symmetry in Optics

Normalized Paraxial Wave Equation

$$i\frac{\partial \Psi}{\partial z}+\frac{\partial^2 \Psi}{\partial x^2}+V(x)\Psi=0$$

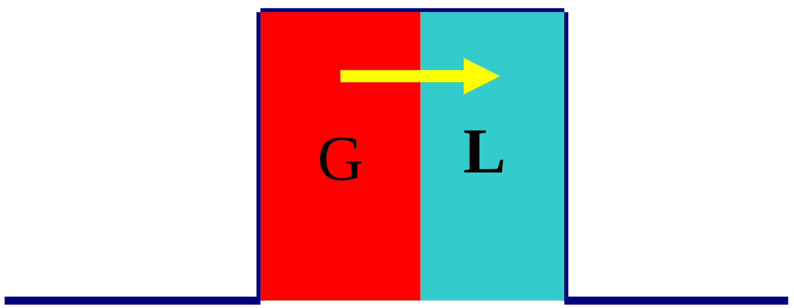
$$V(x)=V^{*}(-x)$$



Typical parameters

$0.5\mu m<\lambda_0<1.6\mu m$ $\quad \Delta n_R^{\max}\approx 10^{-3}$

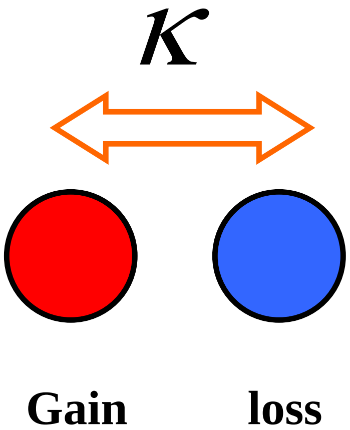
$g=-a\approx 30cm^{-1}$ $\quad \Delta n_I^{\max}\approx 5\times 10^{-4}$



R. El-Ganainy, K.G.Makris, D. N. Christodoulides, and Z. H. Musslimani, Opt. Lett. 32, 2632 (2007).

K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Phys. Rev. Lett. 100, 103904 (2008).

Two-level $\mathcal{PT}$ systems-supermodes

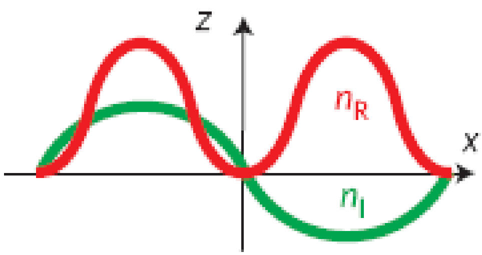


$$\boxed{i\frac{da}{dz}-i\frac{g}{2}a+\kappa\ \ b=0, \qquad i\frac{db}{dz}+i\frac{g}{2}b+\kappa\ \ a=0}$$

$$Z=\kappa z$$

$$i\frac{d}{dz}\begin{pmatrix} a \\ b \end{pmatrix}+\begin{pmatrix} -i\,g/2 & \kappa \\ \kappa & i\,g/2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}=0$$

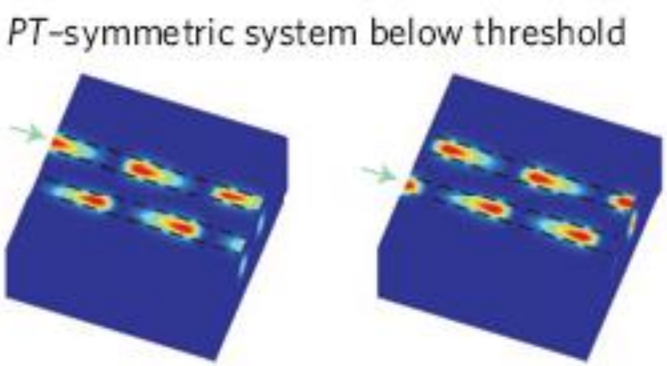
$\mathcal{PT}$-symmetric Hamiltonian

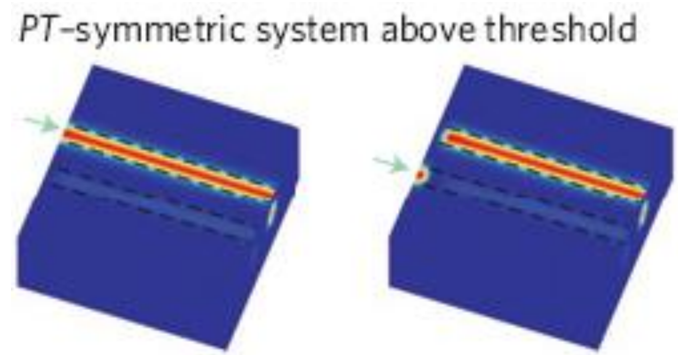


$$\lambda=\pm\sqrt{\kappa^2-(g/2)^2}\Rightarrow\lambda=\begin{cases}\pm\sqrt{\kappa^2-(g/2)^2}, & g<2\kappa \\ 0, & g=2\kappa \\ \pm i\sqrt{(g/2)^2-\kappa^2}, & g>2\kappa\end{cases}$$

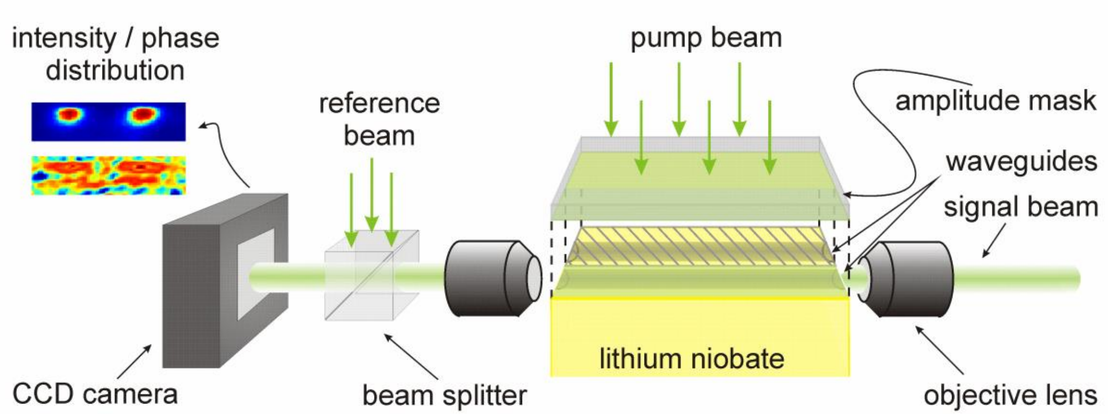
*Bender C.M., Brody D.C., Jones H.F., Meister B.K., Faster than Hermitian quantum mechanics, Phys. Rev. Lett. 98 (2007).

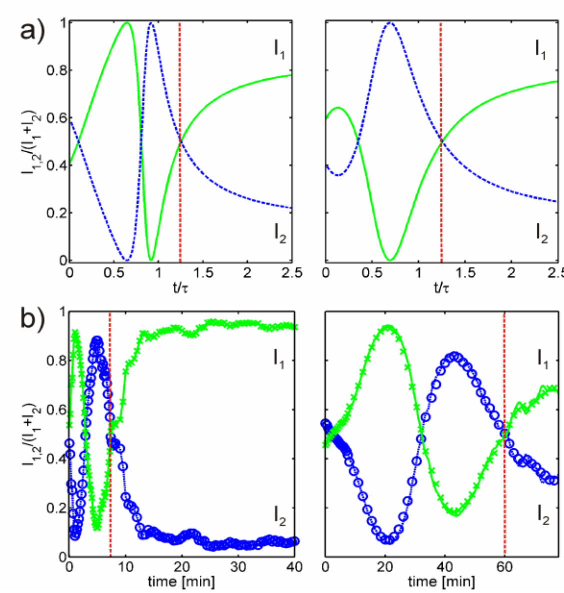
Theory





Experiment



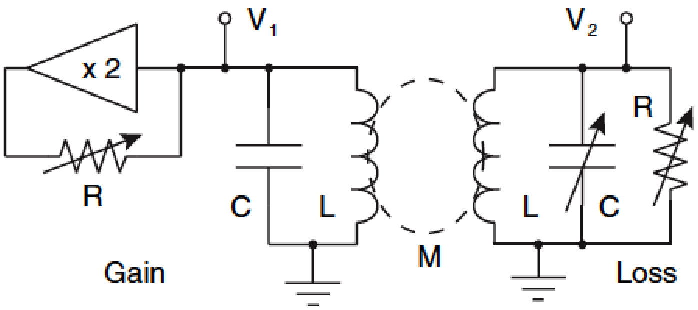


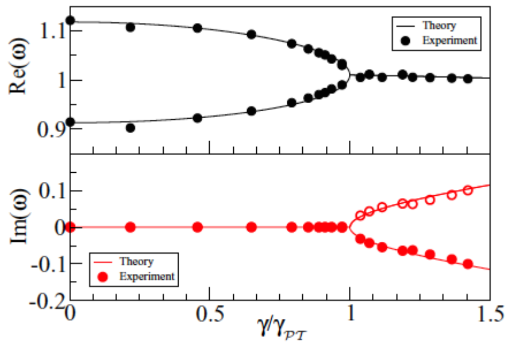
C.E. Ruter, K.G. Makris, R. El-Ganainy, D.N. Christodoulides, M. Segev, D. Kip, Nature Physics 6, 192 (2010).

$\mathcal{PT}$-symmetric metamaterials

Metamaterials are lossy and to overcome losses we can intruduce gain

Active RLC electronic circuit with PT-symmetries





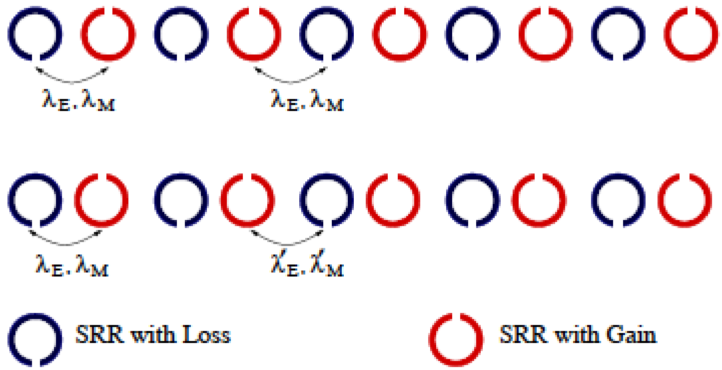
Ω_0=200KHZ

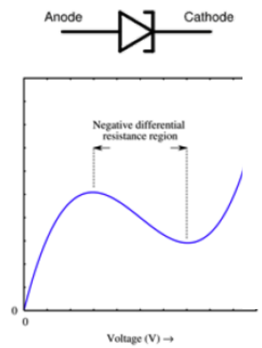
Schindler et al., PRA 84 040101 (2011)

Coupled split ring resonators with gain and loss

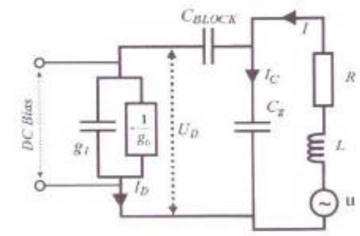
equidistant

dimerized





Esaki diode



Boardman et al. (2007)

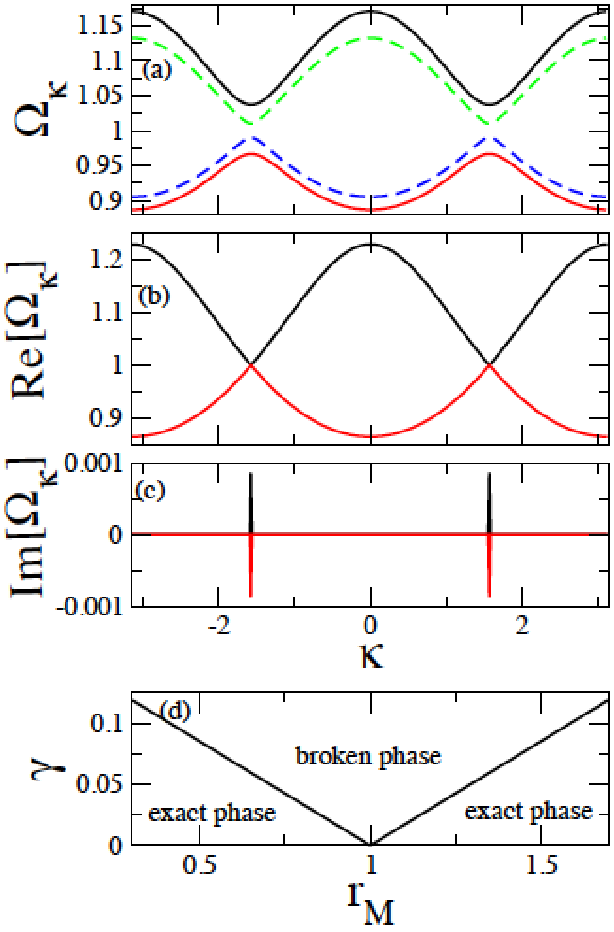
$$\lambda'_M \ddot{q}_{2n} + \ddot{q}_{2n+1} + \lambda_M \ddot{q}_{2n+2} + \lambda'_E q_{2n} + q_{2n+1} + \lambda_E q_{2n+2} = \varepsilon_0 \sin(\Omega\tau) - \alpha q_{2n+1}^2 - \beta q_{2n+1}^3 - \gamma \dot{q}_{2n+1} \quad (1)$$

$$\lambda_M \ddot{q}_{2n-1} + \ddot{q}_{2n} + \lambda'_M \ddot{q}_{2n+1} + \lambda_E q_{2n-1} + q_{2n} + \lambda'_E q_{2n+1} = \varepsilon_0 \sin(\Omega\tau) - \alpha q_{2n}^2 - \beta q_{2n}^3 + \gamma \dot{q}_{2n} \quad (2)$$

λ_E, λ_E’ electric coupling

λ_M,λ_M’ inductive coupling

Linear Band picture



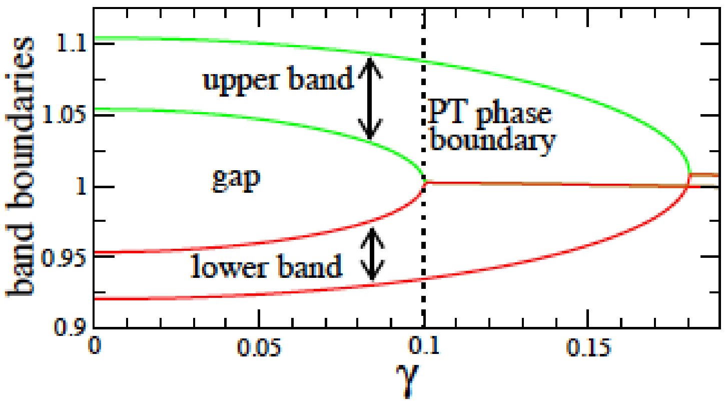
$$r_M = \lambda'_M/\lambda_M\,,$$

For $\lambda_E=\lambda_E'=0$

$$\Omega_\kappa^2 = \frac{2-\gamma^2 \pm \sqrt{\gamma^4 - 2\gamma^2 + (\lambda_M - \lambda'_M)^2 + \mu_\kappa \mu'_\kappa}}{2(1-(\lambda_M - \lambda'_M)^2 - \mu_\kappa \mu'_\kappa)}.$$

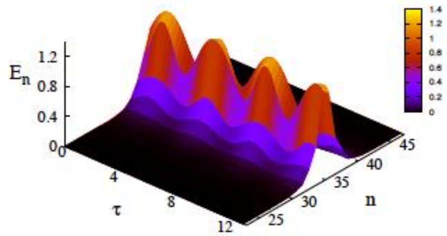
$$\mu_\kappa = 2\lambda_M \cos(\kappa),\ \mu'_\kappa = 2\lambda'_M \cos(\kappa)$$

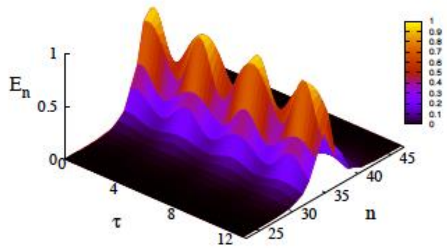
For $\lambda_M=\lambda_M$' the lattice is always in the broken phase: only the dimerized PT-metamaterial has propagating modes

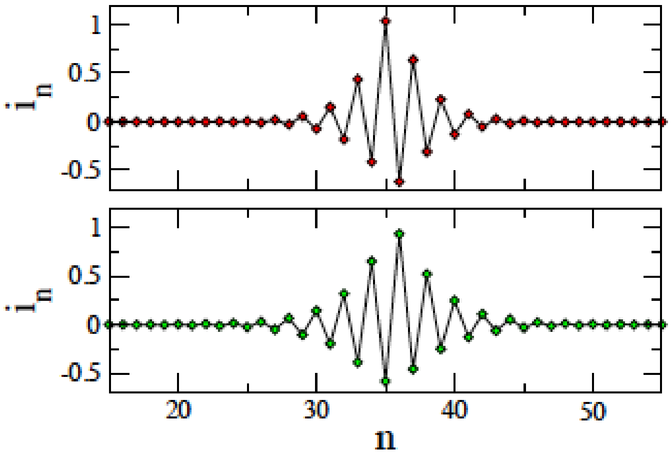


$$\gamma_c \approx \left|\lambda_M - \lambda'_M\right|$$

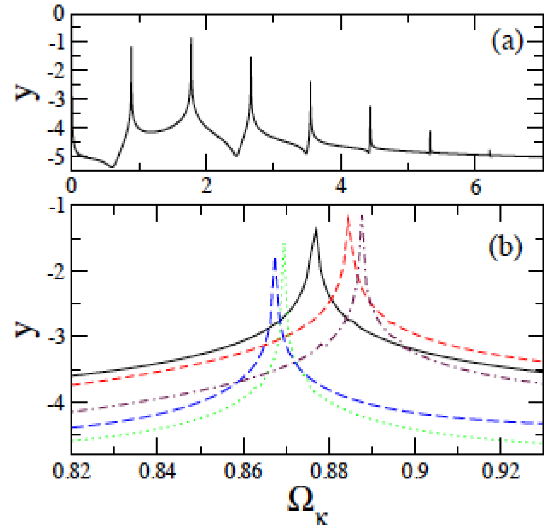
Localized nonlinear modes



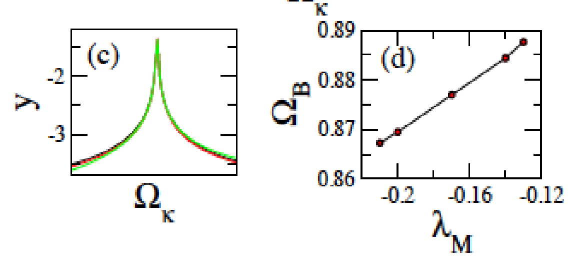




Breather power spectrum

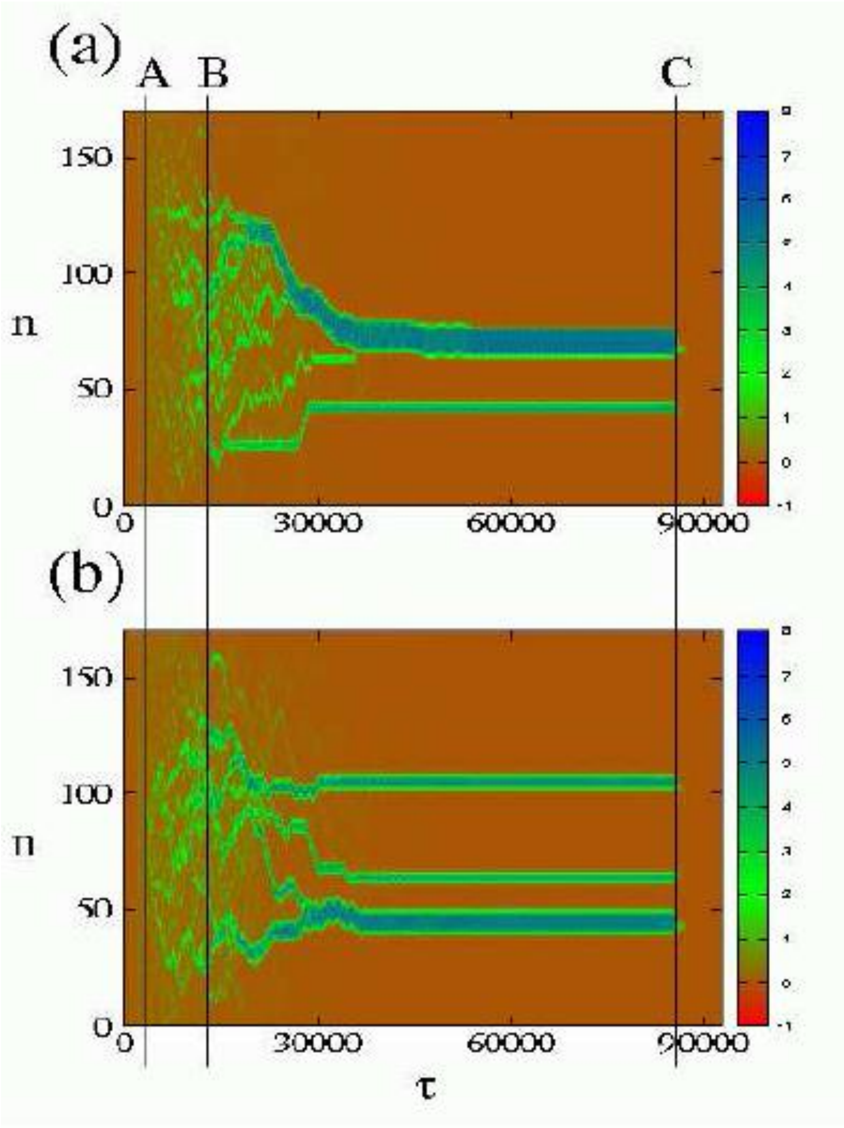


Frequency tuning with λ_M



Weak dependence on γ

Breather generation through frequency chirping



Spatiotemporal local energy density landscape

Time=0 to point A

The lattice is not driven

Point A to point B

Frequency chirping from $1.01\ \Omega_0$ to $0.997\ \Omega_0$

Point B to C

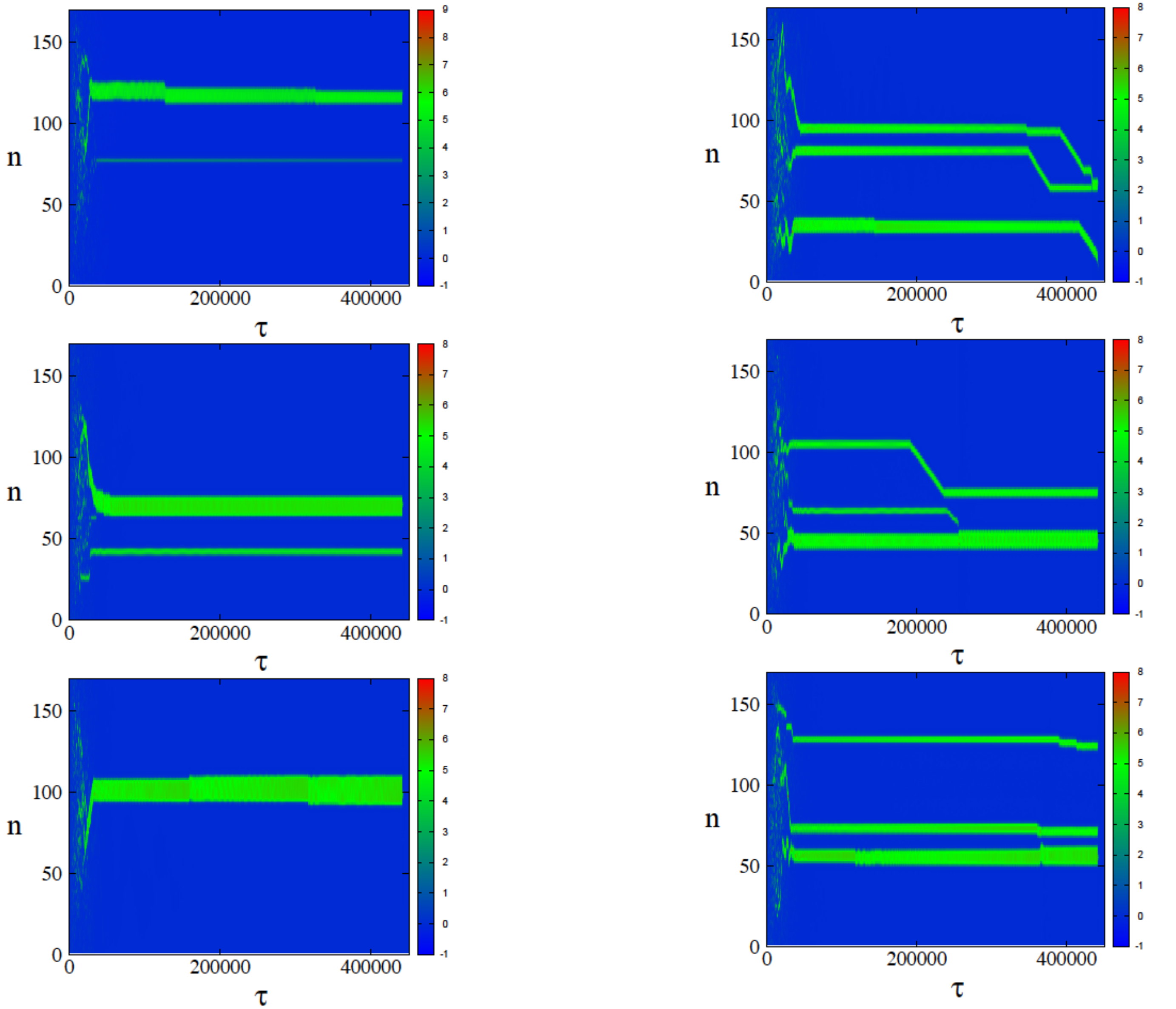
No external driving

Point C to end

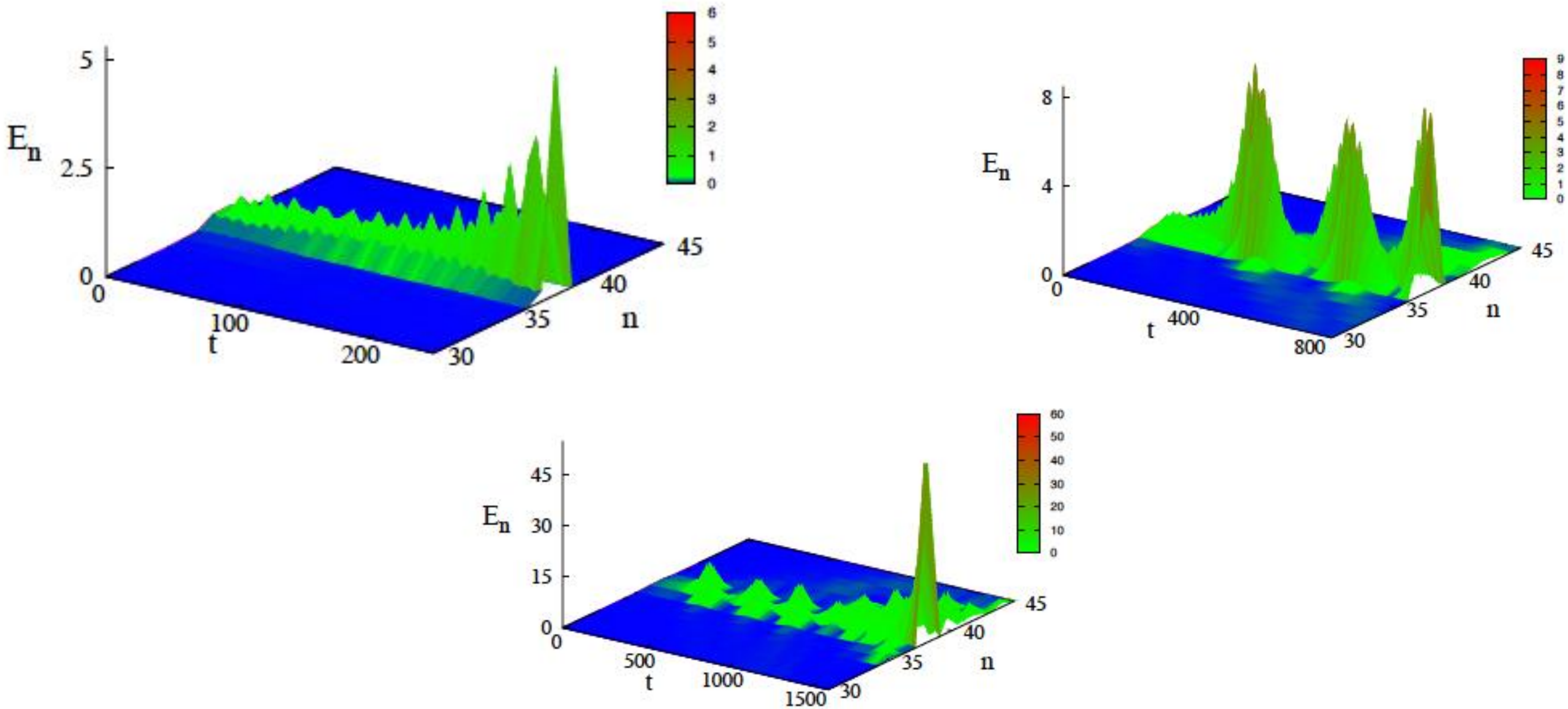
Turn off gain

Ω_0 freq. at the bottom of lower band

With Gain/loss mismatch 0.1%



Breather instability



The breather becomes unstable for gamma values smaller than the critical one

Classical PT symmetry in zero dimensions

$$\ddot{x} + 2\theta(t)\dot{x} + \omega_0^2 x = 0,$$

$$\theta(t) = \begin{cases} +\gamma & \text{if } 0 \le t < \tau_1; \\ -\gamma & \text{if } \tau_1 \le t < \tau_2. \end{cases}$$

$$\begin{pmatrix} x \\ \dot{x} \end{pmatrix} = M_{G/L}(\tau) \begin{pmatrix} x_0 \\ \dot{x}_0 \end{pmatrix}$$

$$\theta(t+T) = \theta(t)$$

$$M(T) = \frac{1}{\delta^2} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

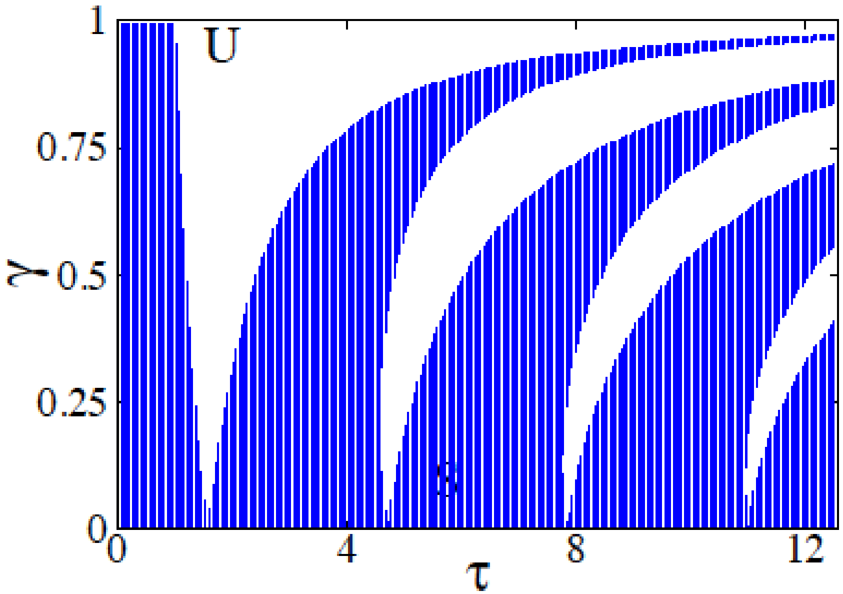
$$\begin{aligned} M_{11} &= -\gamma^2 + \omega_0^2 \cos(2\phi), \\ M_{12} &= +2\sin\phi(\delta\cos\phi + \gamma\sin\phi) \\ M_{21} &= -2\omega_0^2 \sin\phi(\delta\cos\phi - \gamma\sin\phi) \\ M_{22} &= -\gamma^2 + \omega_0^2 \cos(2\phi) \end{aligned}$$

$$\delta = \sqrt{\omega_0^2 - \gamma^2}. \qquad \phi = \delta\tau \equiv \delta T/2.$$

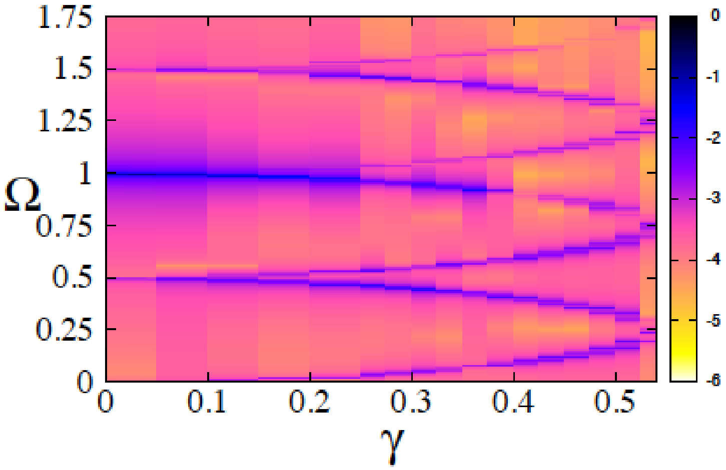
Stability equation

$$\left| \cos\left[\frac{\pi\sqrt{1-\gamma^2}}{\Omega} \right] \right| = \gamma,$$

$$\Omega = \omega/\omega_0$$



Blue: stable region



GPT and N. Lazarides, arXiv:1304.0556

Conclusions

- Localized nonlinear modes may appear in metamaterials
- PT-symmetric metamaterials balance loss with gain and experience a transition in propagation properties
- Nonlinear modes may appear in PT-metamaterials
- May operate in microwave frequencies with tunnel diodes or higher

N. Lazarides and GPT, PRL **110**, 053901 (2013)

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