

Reaction wheel Inverted Pendulum

(Team 56)

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Design Summary:

This project aims to use the concept of closed loop feedback to control a
reaction wheel based inverted pendulum, it works depending on the
reaction torque produced on the wheel and system from the torque
produced by the motor, this torque is used as an input to our system to
track a predefined reference (angle 0 in our case) to balance the pendulum
and reach equilibrium.

Design Components:

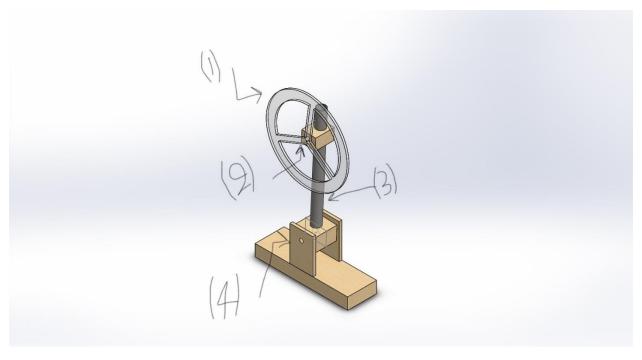


Figure 1 - Design components and mechanism

- 1. 4-mm acrylic reaction wheel
- 2. 12 volt DC motor and the wheel mounted on it
- 3. A 450 mm stainless steel tube used as our pendulum
- 4. A wooden base of a stainless steel rod and bearings used as our hinge of rotation for the motion mechanism

The actual mechanism picture will be placed here

System Details:

The equation of motion of the system was derived in our analysis using two methods to ensure that our system is verified enough and correctly:

1. Using the Euler-Lagrange equation which is basically a differential equation depending on the kinetic and potential energy of the system, then representing the system in State-Space form to further verify its states and initial conditions (if any), the transfer function calculated was a relation between the input voltage to the DC motor and the actual angle of the system

$$=\frac{\frac{-Kt}{(mplp^{2}+4mwlp^{2}+Ip)Ra}+\frac{Kt*Ke}{\frac{(mplp^{2}+4mwlp^{2}+Ip)Ra}{Ra}*\frac{Kt}{Ra}\left(\frac{1}{Iw}+\frac{1}{(mplp^{2}+4mwlp^{2}+Ip)}\right)*S}{S^{2}+\frac{Kt*Ke}{Ra}*\left(\frac{1}{Iw}+\frac{1}{(mplp^{2}+4mwlp^{2}+Ip)}\right)*S}}$$

$$=\frac{S^{2}-\frac{g*lp(mp+2mw)}{mplp^{2}+4mwlp^{2}+Ip}+\frac{Kt*Ke}{\frac{(mplp^{2}+4mwlp^{2}+Ip)Ra}{Ra}*\frac{g*lp(mp+2mw)}{mplp^{2}+4mwlp^{2}+Ip}*S}}{S^{2}+\frac{Kt*Ke}{Ra}*\left(\frac{1}{Iw}+\frac{1}{(mplp^{2}+4mwlp^{2}+Ip)}\right)*S}\right)}$$

2. Using simscape tool in Matlab which helped us derive the system EOM directly from the solidworks design without the need of mathematical analysis like the first method "Simscape is a tool used to import solid parts from solidworks, analyses it and creates the proper block diagram with the proper joints, parts and frames of reference", it resulted in an open loop transfer function where the input is the torque on the system and the output is the actual angle of the system, the motor was not considered as a part of the transfer function as the previous method, so a separate transfer function for the motor was calculated:

$$\frac{T}{V} = \frac{S * Kt * Iw}{S * Ra * Iw + Kt * Ke}$$

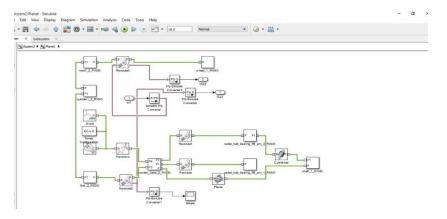


Figure 2 - Block diagram after importing into simscape

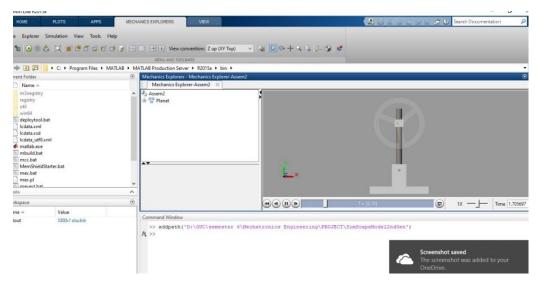


Figure 3 - The system in matlab simulation tool

Simulation and analysis:

After using the previously mentioned methods to get the transfer function, we imported our transfer function into Simulink and created the following block diagram:

• Using the Lagrange equation to get the EOM, we were able to construct the following closed loop system "treating our sensor as a unity feedback":



Figure 4 - Closed loop system

 Investigating the system's response to the loop after inserting an initial condition to our system to respond to (an angle of 15 degrees) and using Matlab auto tune tool for the PID block to get the proper gains for a proper performance, we got the following response which satisfied our intentions for the project:

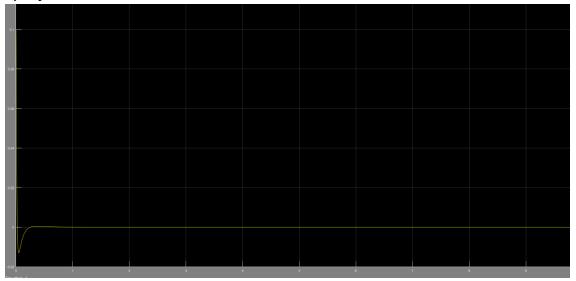


Figure 5 - Closed loop response

Using simscape to create our transfer function from the solidworks design
was no different in the response we got, the loop was created after
suppressing our system's block diagram into a subsystem and using it as a
black box in our simulation, we used the calculated transfer function of the

DC motor as the transfer function produced by the simscape tool provided the input to be the torque of the motor and the output is the actual angle of the system:

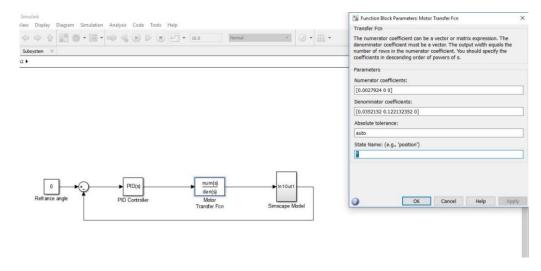


Figure 6 - Closed loop system2

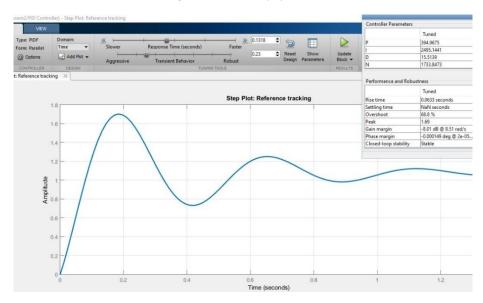


Figure 7 - auto tuning for gains and the closed loop response

A brief description of mechanism:

The system is a simple inverted pendulum pivoted on a stainless steel rod and two ball bearings to support rotational motion due to reaction torque

attained from the motor when it rotates the wheel, the DC motor provides a torque via the reaction wheel mounted on it mechanically and the systems exhibits a reaction from newton's third law (for every action there's a reaction equal in magnitude and opposite in direction) and that reaction is the source of actuation of the system.

Components:

1. A 12 volts DC motor with 250 rated RPM and 8.8 Kgf.cm rated torque: it's our project's actuator and the source of the torque that balances our project.



Figure 8 - DC motor

2. <u>IMU sensor (Accelerometer + Gyroscope + temperature sensor) MPU 6050:</u> It's the feedback of our project, it calculates the angle of tilting of the pendulum and sends it to the arduino in order to process the error with respect to our reference angle (0 in our case)

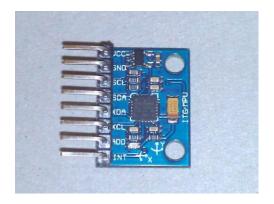


Figure 9 - MPU 6050 sensor

3. <u>H-Bridge L298N module to drive the DC motor</u>: It controls our motor via receiving the PWM signal from the arduino board and produces the proper voltage to rotate the motor with the desired speed.

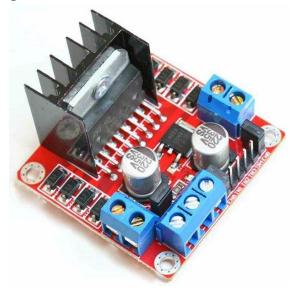


Figure 10 - DC motor driver

4. <u>Arduino uno R3 board</u>: The controller and brain of the project, it receives input from the sensor, evaluates the error and produces the output control signal to our motor.



Figure 11 - Arduino uno

Circuit schematic:

The circuit schematic was created with "Fritzing", a software used for creating circuit schematics and PCB layouts.

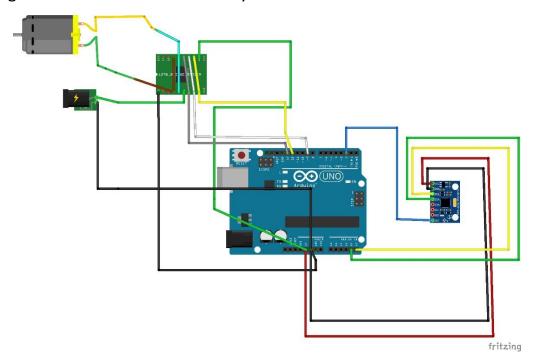


Figure 12 - Circuit Schematic

A brief flowchart describing our control logic:

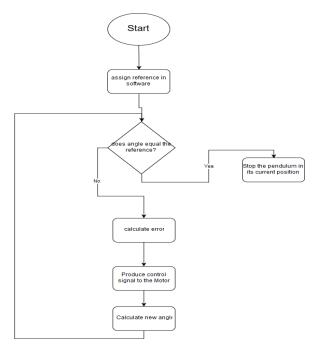


Figure 13 – Flowchart

Design evaluation:

After tuning the PID parameters and due to the fluctuations in the reading
of the sensor, we had to go with the trial and error to adjust our gains a bit
more or less of what we got from modelling and simulation on matlab, the
system responded aggressively but in an accepted manner, it balanced
itself for a while reaching the zero position and suffering from fluctuations
about the zero position as the performance produced by the matlab
simulations showed, so the performance was a success in overall

Partial parts list:

Part	Image	Price	Brief
Name			description
Ardunio UNO R3	AROUTHO TO THE PARTY OF THE PAR	150 L.E.	The arduino board is the brain and controller of our project, the software is uploaded on it, it processes the input signal, calculates he error and produces the proper control signal to control the system

<u>DC Motor</u>		250 L.E.	It's the actuator that moves our project, it's a 12V rating DC motor with a 34:1 gear box and produces a rated torque of 8.8 kgff.cm and 250 RPM at its rated operating point, it receives varying voltage depending on the error and moves with a specific speed to produce the intended torque via the reaction wheel attached to it.
Reaction wheel	8	100 L.E.	It's the source of the reaction torque that will move the system when the motor exerts a specific torque on it.
<u>H-Bridge</u> <u>driver</u>		35 L.E.	It receives the control signal from the arduino and produce the proper voltage via the output port that the motor is

MPU-6050		70 L.E.	connected to and cause it to rotate, it also receives the direction commands from the arduino via the input port to control the direction of rotation of the motor It's the sensor
<u>sensor</u>	SOLUTION OF THE PARTY OF THE PA		responsible for the feedback in our project and is the one calculating the current tilt angle and inputs it to the arduino to process as previously mentioned

Appendix:

• First trial code: #include <Wire.h> #include "Kalman.h" // Source: https://github.com/TKJElectronics/KalmanFilter //#include "MPU6050 6Axis MotionApps20.h" #define RESTRICT PITCH // Comment out to restrict roll to ±90deg instead please read: http://www.freescale.com/files/sensors/doc/app_note/AN3461.pdf //MPU6050 mpu; Kalman kalmanX; // Create the Kalman instances Kalman kalmanY; /* IMU Data */ double accX, accY, accZ; double gyroX, gyroY, gyroZ; int16_t tempRaw; double gyroXangle, gyroYangle; // Angle calculate using the gyro only double compAngleX, compAngleY; // Calculated angle using a complementary filter double kalAngleX, kalAngleY; // Calculated angle using a Kalman filter uint32 t timer; uint8 ti2cData[14]; // Buffer for I2C data /* PID data */ //The motor parameters int Pwm = 9; int int1 = 12;int int2 = 13; //Define Variables we'll be connecting to

```
double Setpoint;
//Specify the links and initial tuning parameters
double Kp=-5795.6724490643, Ki=-22671.6412364561, Kd=-
363.798837982102;
float delta t = 0;
                                                //initialize sampling time
float time old = micros();
float error old = 0;
float diff term = 0;
                                                 //derivartive term
                                                //inegrator term
float int term = 0;
float agressive;
                                               //set agressive input limit
int guard;
void setup() {
 Serial.begin(115200);
 Wire.begin();
 TWBR = ((F CPU / 400000L) - 16) / 2; // Set I2C frequency to 400kHz
 i2cData[0] = 7; // Set the sample rate to 1000Hz - 8kHz/(7+1) = 1000Hz
 i2cData[1] = 0x00; // Disable FSYNC and set 260 Hz Acc filtering, 256 Hz
Gyro filtering, 8 KHz sampling
 i2cData[2] = 0x00; // Set Gyro Full Scale Range to ±250deg/s
 i2cData[3] = 0x00; // Set Accelerometer Full Scale Range to ±2g
 while (i2cWrite(0x19, i2cData, 4, false)); // Write to all four registers at
once
 while (i2cWrite(0x6B, 0x01, true)); // PLL with X axis gyroscope reference
and disable sleep mode
 while (i2cRead(0x75, i2cData, 1));
 if (i2cData[0] != 0x68) { // Read "WHO AM I" register
  Serial.print(F("Error reading sensor"));
  while (1);
```

```
delay(100); // Wait for sensor to stabilize
 /* Set kalman and gyro starting angle */
 while (i2cRead(0x3B, i2cData, 6));
 accX = (i2cData[0] << 8) | i2cData[1];
 accY = (i2cData[2] << 8) | i2cData[3];
 accZ = (i2cData[4] << 8) | i2cData[5];
accX=-3671;
accY=1518;
accZ=4130;
gyroX=145;
gyroY=2;
gyroZ=36;
 // Source:
http://www.freescale.com/files/sensors/doc/app_note/AN3461.pdf eq. 25
and eq. 26
// atan2 outputs the value of -\pi to \pi (radians) - see
http://en.wikipedia.org/wiki/Atan2
 // It is then converted from radians to degrees
#ifdef RESTRICT PITCH // Eq. 25 and 26
 double roll = atan2(accY, accZ); //the output of this function is radians
 double pitch = atan(-accX / sqrt(accY * accY + accZ * accZ)); //the output
of this function is radians
#else // Eq. 28 and 29
 double roll = atan(accY / sqrt(accX * accX + accZ * accZ)); //the output of
this function is radians
 double pitch = atan2(-accX, accZ) * RAD_TO_DEG;
#endif
 kalmanX.setAngle(roll); // Set starting angle
 kalmanY.setAngle(pitch);
 gyroXangle = roll;
```

```
gyroYangle = pitch;
 compAngleX = roll;
 compAngleY = pitch;
 //Turn PID on and ser your reference angle in radians
 Setpoint = 0;
 timer = micros();
}
void loop() {
/* Update all the values */
 while (i2cRead(0x3B, i2cData, 14));
 accX = ((i2cData[0] << 8) | i2cData[1]);
 accY = ((i2cData[2] << 8) | i2cData[3]);
 accZ = ((i2cData[4] << 8) | i2cData[5]);
 tempRaw = (i2cData[6] << 8) | i2cData[7];
 gyroX = (i2cData[8] << 8) | i2cData[9];
 gyroY = (i2cData[10] << 8) | i2cData[11];
 gyroZ = (i2cData[12] << 8) | i2cData[13];
 double dt = (double)(micros() - timer) / 1000000; // Calculate delta time
 timer = micros();
// Source:
http://www.freescale.com/files/sensors/doc/app_note/AN3461.pdf eq. 25
and eq. 26
// atan2 outputs the value of -\pi to \pi (radians) - see
http://en.wikipedia.org/wiki/Atan2
// It is then converted from radians to degrees
#ifdef RESTRICT PITCH // Eq. 25 and 26
 double roll = atan2(accY, accZ) * RAD_TO_DEG;
```

```
double pitch = atan(-accX / sqrt(accY * accY + accZ * accZ)) *
RAD TO DEG;
#else // Eq. 28 and 29
 double roll = atan(accY / sqrt(accX * accX + accZ * accZ)) * RAD_TO_DEG;
 double pitch = atan2(-accX, accZ) * RAD TO DEG;
#endif
 double gyroXrate = gyroX / 131.0; // Convert to deg/s
 double gyroYrate = gyroY / 131.0; // Convert to deg/s
#ifdef RESTRICT PITCH
// This fixes the transition problem when the accelerometer angle jumps
between -180 and 180 degrees
 if ((roll < -90 && kalAngleX > 90) | | (roll > 90 && kalAngleX < -90)) {
  kalmanX.setAngle(roll);
  compAngleX = roll;
  kalAngleX = roll;
  gyroXangle = roll;
 } else
  kalAngleX = kalmanX.getAngle(roll, gyroXrate, dt); // Calculate the angle
using a Kalman filter
 if (abs(kalAngleX) > 90)
  gyroYrate = -gyroYrate; // Invert rate, so it fits the restriced
accelerometer reading
 kalAngleY = kalmanY.getAngle(pitch, gyroYrate, dt);
#else
// This fixes the transition problem when the accelerometer angle jumps
between -180 and 180 degrees
 if ((pitch < -90 && kalAngleY > 90) | | (pitch > 90 && kalAngleY < -90)) {
  kalmanY.setAngle(pitch);
  compAngleY = pitch;
  kalAngleY = pitch;
  gyroYangle = pitch;
 } else
```

```
kalAngleY = kalmanY.getAngle(pitch, gyroYrate, dt); // Calculate the angle
using a Kalman filter
 if (abs(kalAngleY) > 90)
  gyroXrate = -gyroXrate; // Invert rate, so it fits the restriced
accelerometer reading
 kalAngleX = kalmanX.getAngle(roll, gyroXrate, dt); // Calculate the angle
using a Kalman filter
#endif
 gyroXangle += gyroXrate * dt; // Calculate gyro angle without any filter
 gyroYangle += gyroYrate * dt;
 gyroXangle += kalmanX.getRate() * dt; // Calculate gyro angle using the
unbiased rate
 gyroYangle += kalmanY.getRate() * dt;
 compAngleX = 0.93 * (compAngleX + gyroXrate * dt) + 0.07 * roll; //
Calculate the angle using a Complimentary filter
 compAngleY = 0.93 * (compAngleY + gyroYrate * dt) + 0.07 * pitch;
// Reset the gyro angle when it has drifted too much
 if (gyroXangle < -180 | gyroXangle > 180)
  gyroXangle = kalAngleX;
 if (gyroYangle < -180 | gyroYangle > 180)
  gyroYangle = kalAngleY;
//Arduino PID without the Library
 delta t = micros() - time old;
                                                    //calculate sampling
time
 sensor read = kalAngleX*DEG TO RAD;
//The PID code without the library
```

```
//calc. error "e"
float error = reference - sensor read;
float E = E + (error * delta t);
                                                    //calc. error integration
float e_dot = error - error_old;
                                                     //calc. error
diffrentiation
// -----Two tricks-----
if (delta t != 0){
 diff_term = e_dot / delta_t;
                                                    //calc. derivative term
}else { diff term = 0;}
if (e_dot < agressive){</pre>
 guard = 1;
}else { guard = 0 ;}
// -----
int_term = E;
                                          //calc. itegrator term
float u = kp* error + guard * ki * int_term + kd * diff_term;
 if(sensor_read > 0){
   sensor_read = -1 * sensor_read;
   digitalWrite(int1, LOW);
   digitalWrite(int2, HIGH);
 else if (sensor_read == 0){
   digitalWrite(int1, LOW);
   digitalWrite(int2, LOW);
 }
 else{
   digitalWrite(int1, HIGH);
   digitalWrite(int2, LOW);
 }
analogWrite(Pwm, u);
```

```
Serial.print(" | Angle = ");
Serial.print(Input);
Serial.print(" ");
Serial.println(Output);

delay(2);
}
```

```
Second trial code:
  /*****************
   * PID Basic Example for the RWIP
   #include <PID v1.h>
  #include "I2Cdev.h"
  #include "MPU6050 6Axis MotionApps20.h"
  #if I2CDEV_IMPLEMENTATION == I2CDEV_ARDUINO WIRE
    #include "Wire.h"
  #endif
  MPU6050 mpu;
  //The motor parameters
  #define pwm 3
  const int in1 = 12;
  const int in 2 = 13;
  //the sensor parameters
  #define OUTPUT READABLE YAWPITCHROLL
  bool dmpReady = false; // set true if DMP init was successful
  uint8 t mpuIntStatus; // holds actual interrupt status byte from MPU
                    // return status after each device operation (0 =
  uint8_t devStatus;
  success, !0 = error)
  uint16 t packetSize; // expected DMP packet size (default is 42 bytes)
  uint16_t fifoCount; // count of all bytes currently in FIFO
  uint8 t fifoBuffer[64]; // FIFO storage buffer
  double time1,time2;
  // orientation/motion vars
  Quaternion q; //[w, x, y, z]
                                  quaternion container
  VectorInt16 aa;
                  // [x, y, z]
                                 accel sensor measurements
```

```
VectorInt16 aaReal; // [x, y, z]
                                  gravity-free accel sensor
measurements
VectorInt16 aaWorld; // [x, y, z] world-frame accel sensor
measurements
VectorFloat gravity; // [x, y, z]
                                  gravity vector
float euler[3]; // [psi, theta, phi] Euler angle container
               // [yaw, pitch, roll] yaw/pitch/roll container and gravity
float ypr[3];
vector
// packet structure for InvenSense teapot demo
uint8_t teapotPacket[14] = { '$', 0x02, 0,0, 0,0, 0,0, 0,0, 0x00, 0x00, '\r', '\n'
};
//
===
// ===
             INTERRUPT DETECTION ROUTINE
______
===
volatile bool mpuinterrupt = false; // indicates whether MPU interrupt
pin has gone high
void dmpDataReady() {
  mpuInterrupt = true;
}
//Define Variables we'll be connecting to
double Setpoint, Input, Output;
//Specify the links and initial tuning parameters
double Kp=1612.62764385817, Ki=1760.60255185303,
Kd=241.395867072866;
PID myPID(&Input, &Output, &Setpoint, Kp, Ki, Kd, DIRECT);
```

```
void setup()
 //initialize the variables we're linked to
 pinMode(pwm,OUTPUT);
 pinMode(in1,OUTPUT);
 pinMode(in2,OUTPUT);
//MPU configuration
// join I2C bus (I2Cdev library doesn't do this automatically)
  #if I2CDEV IMPLEMENTATION == I2CDEV ARDUINO WIRE
    Wire.begin();
    TWBR = 24; // 400kHz I2C clock (200kHz if CPU is 8MHz)
  #elif I2CDEV IMPLEMENTATION == I2CDEV BUILTIN FASTWIRE
    Fastwire::setup(400, true);
  #endif
  // initialize serial communication
  // (115200 chosen because it is required for Teapot Demo output, but it's
  // really up to you depending on your project)
  Serial.begin(115200);
  while (!Serial); // wait for Leonardo enumeration, others continue
immediately
  // NOTE: 8MHz or slower host processors, like the Teensy @ 3.3v or
Ardunio
  // Pro Mini running at 3.3v, cannot handle this baud rate reliably due to
  // the baud timing being too misaligned with processor ticks. You must
use
  // 38400 or slower in these cases, or use some kind of external separate
  // crystal solution for the UART timer.
```

```
// initialize device
  Serial.println(F("Initializing I2C devices..."));
  mpu.initialize();
  // verify connection
  Serial.println(F("Testing device connections..."));
  Serial.println(mpu.testConnection()? F("MPU6050 connection
successful"): F("MPU6050 connection failed"));
  // wait for ready
  Serial.println(F("\nSend any character to begin DMP programming and
demo: "));
  while (Serial.available() && Serial.read()); // empty buffer
                                     // wait for data
  while (!Serial.available());
  while (Serial.available() && Serial.read()); // empty buffer again
  // load and configure the DMP
  Serial.println(F("Initializing DMP..."));
  devStatus = mpu.dmpInitialize();
  // supply your own gyro offsets here, scaled for min sensitivity
  mpu.setXGyroOffset(220);
  mpu.setYGyroOffset(76);
  mpu.setZGyroOffset(-85);
  mpu.setZAccelOffset(1788); // 1688 factory default for my test chip
  // make sure it worked (returns 0 if so)
  if (devStatus == 0) {
    // turn on the DMP, now that it's ready
    Serial.println(F("Enabling DMP..."));
    mpu.setDMPEnabled(true);
    // enable Arduino interrupt detection
```

```
Serial.println(F("Enabling interrupt detection (Arduino external
interrupt 0)..."));
    attachInterrupt(0, dmpDataReady, RISING);
    mpuIntStatus = mpu.getIntStatus();
    // set our DMP Ready flag so the main loop() function knows it's okay
to use it
    Serial.println(F("DMP ready! Waiting for first interrupt..."));
    dmpReady = true;
    // get expected DMP packet size for later comparison
    packetSize = mpu.dmpGetFIFOPacketSize();
  } else {
    // ERROR!
    // 1 = initial memory load failed
    // 2 = DMP configuration updates failed
    // (if it's going to break, usually the code will be 1)
    Serial.print(F("DMP Initialization failed (code "));
    Serial.print(devStatus);
    Serial.println(F(")"));
  }
 Setpoint = 0.00;
 //turn the PID on
 myPID.SetMode(AUTOMATIC);
 myPID.SetSampleTime(20);
}
//-----
void loop()
```

```
// if programming failed, don't try to do anything
  //time1= millis();
  if (!dmpReady) return;
  // wait for MPU interrupt or extra packet(s) available
  while (!mpuInterrupt && fifoCount < packetSize) {
    // other program behavior stuff here
    // .
    // .
    // .
    // if you are really paranoid you can frequently test in between other
    // stuff to see if mpuInterrupt is true, and if so, "break;" from the
    // while() loop to immediately process the MPU data
    // .
    // .
    // .
  }
  // reset interrupt flag and get INT STATUS byte
  mpuInterrupt = false;
  mpuIntStatus = mpu.getIntStatus();
  // get current FIFO count
  fifoCount = mpu.getFIFOCount();
  // check for overflow (this should never happen unless our code is too
inefficient)
  if ((mpuIntStatus & 0x10) || fifoCount == 1024) {
    // reset so we can continue cleanly
    mpu.resetFIFO();
    Serial.println(F("FIFO overflow!"));
  // otherwise, check for DMP data ready interrupt (this should happen
frequently)
```

```
} else if (mpuIntStatus & 0x02) {
   // wait for correct available data length, should be a VERY short wait
   while (fifoCount < packetSize) fifoCount = mpu.getFIFOCount();</pre>
   // read a packet from FIFO
   mpu.getFIFOBytes(fifoBuffer, packetSize);
   // track FIFO count here in case there is > 1 packet available
   // (this lets us immediately read more without waiting for an interrupt)
   fifoCount -= packetSize;
   #ifdef OUTPUT READABLE YAWPITCHROLL
     // display Euler angles in degrees
     mpu.dmpGetQuaternion(&q, fifoBuffer);
     mpu.dmpGetGravity(&gravity, &q);
     mpu.dmpGetYawPitchRoll(ypr, &q, &gravity);
    // time2 = millis();
    // Serial.print(time2-time1);
     //Serial.print("ypr\t");
     //Serial.print(ypr[0] * 180/M_PI);
     //Serial.print("\t");
     //Serial.print(ypr[1] * 180/M_PI);
    // Serial.print("\t");
     Serial.println(ypr[2]);
    // delay(10000);
   #endif
 }
Input = ypr[2];
myPID.Compute();
if(Output < 0){
 digitalWrite(in1,HIGH);
```

```
digitalWrite(in2,LOW);
}else if(Output > 0){
    digitalWrite(in1,LOW);
    digitalWrite(in2,HIGH);
}else{
    digitalWrite(in1,LOW);
    digitalWrite(in2,LOW);
}
Serial.println(Output);
analogWrite(pwm, abs(Output));
delay(20);
}
```

Kt : motor torque constant

Ke: motor back emf constant

Iw: moment of inertia of the wheel about its center of mass

Ip: moment of inertia of the pendulum about the pivot

Ra: motor armature resistance

Mw: mass of the wheel

Mp: mass of the pendulum

T : reaction torque actuating the system

 $\ensuremath{\mathsf{V}}$: armature voltage of the motor

Lp : half the length of the pendulum