Lecture 5

Rank-reduction of Tensors or Multilinaer arrays examples

M D Sacchi
University of Alberta

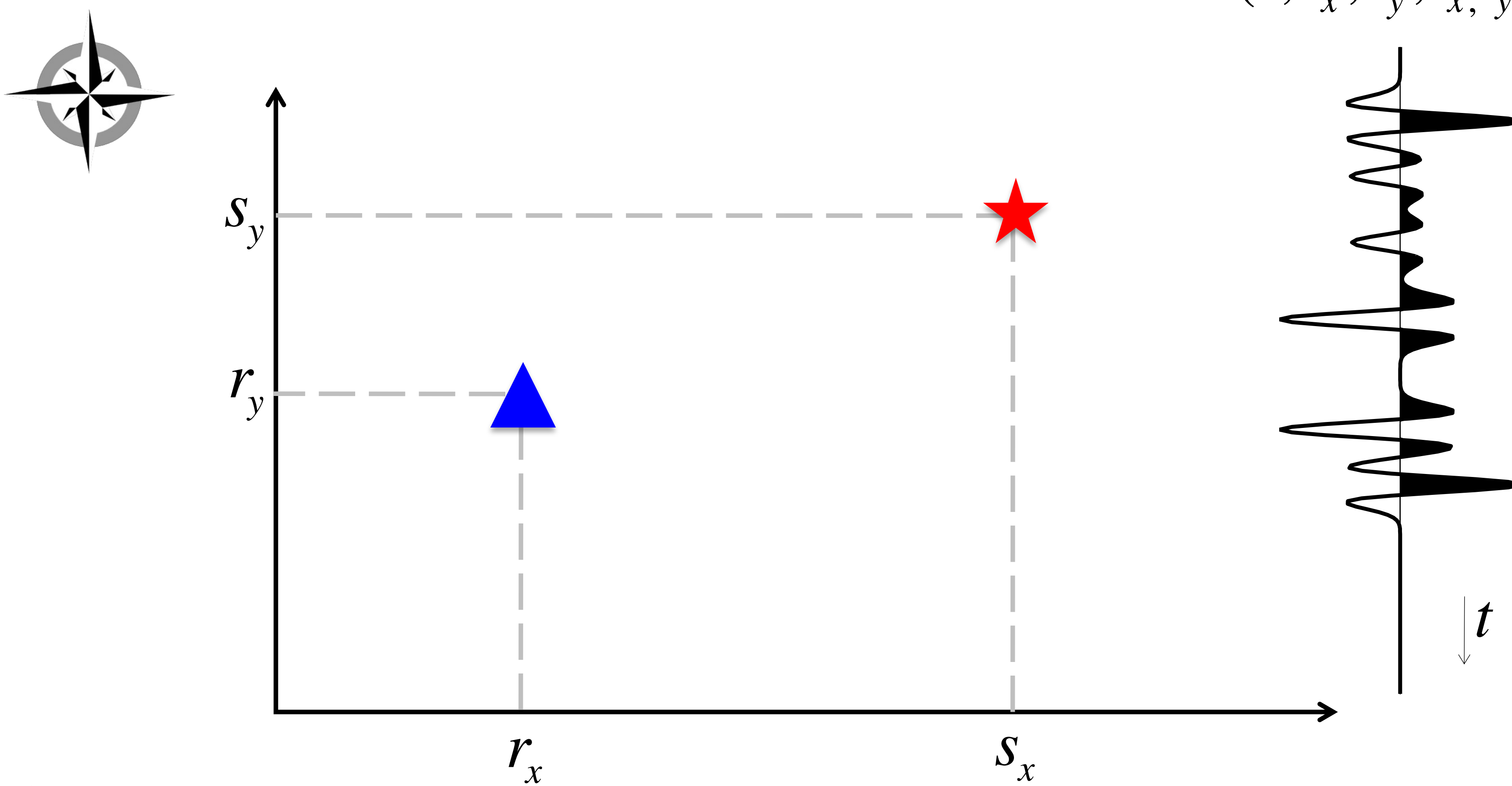
Some Examples of 5D reconstruction

- Moving to Tensors..

5D ?

Source receiver coordinates

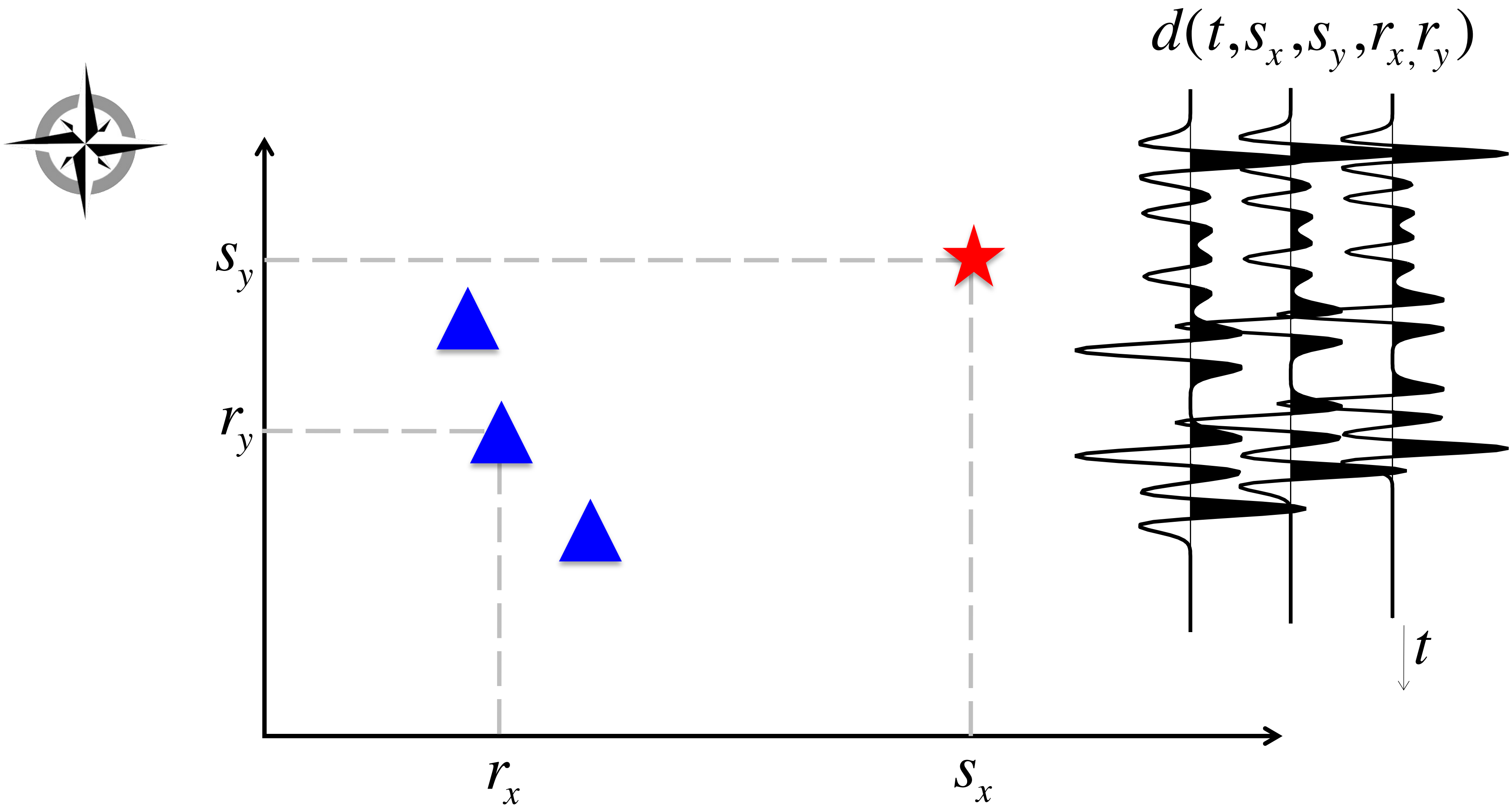
$$d(t, s_x, s_y, r_x, r_y)$$



: Source

: Receiver

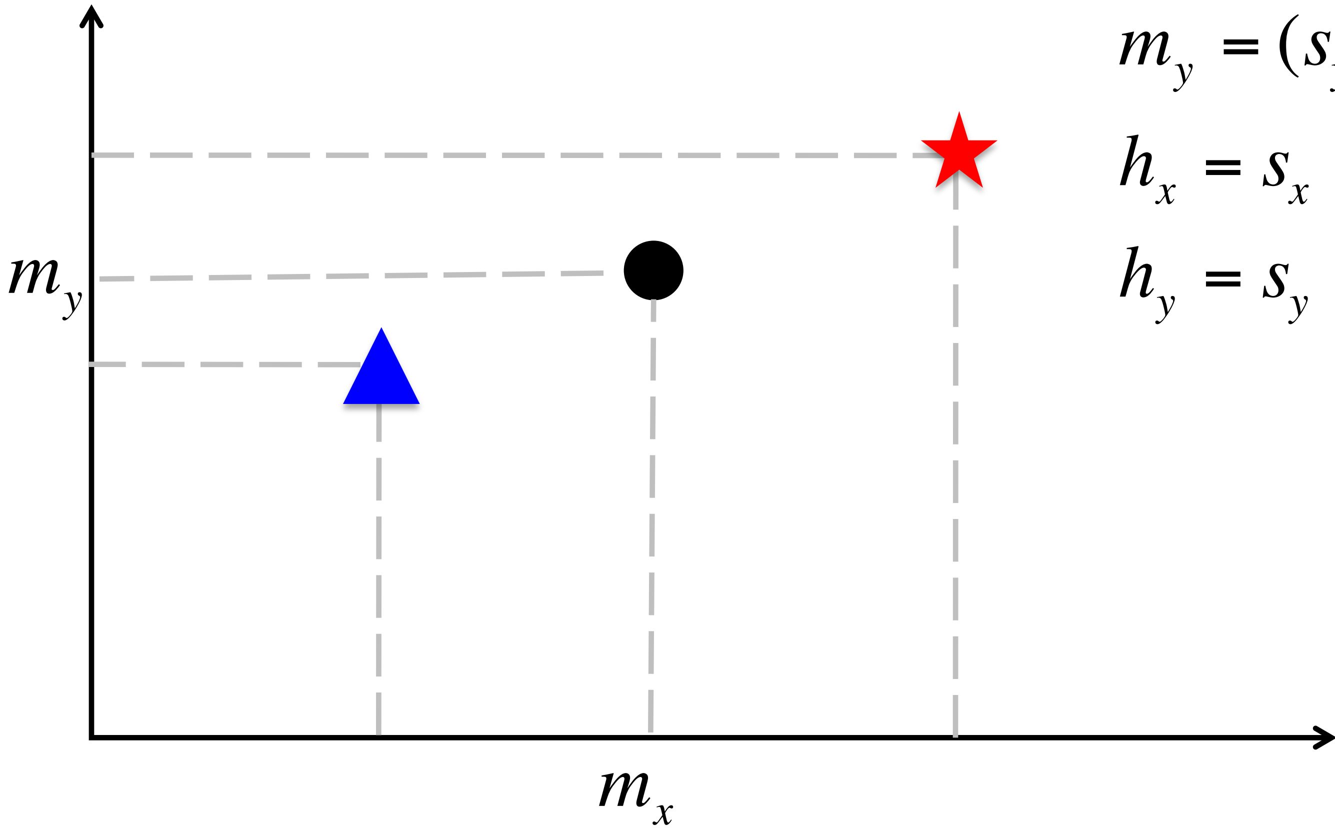
Source receiver coordinates



 : Source

 : Receiver

Midpoint-offset coordinates



$$m_x = (s_x + r_x)/2$$

$$m_y = (s_y + r_y)/2$$

$$h_x = s_x - r_x$$

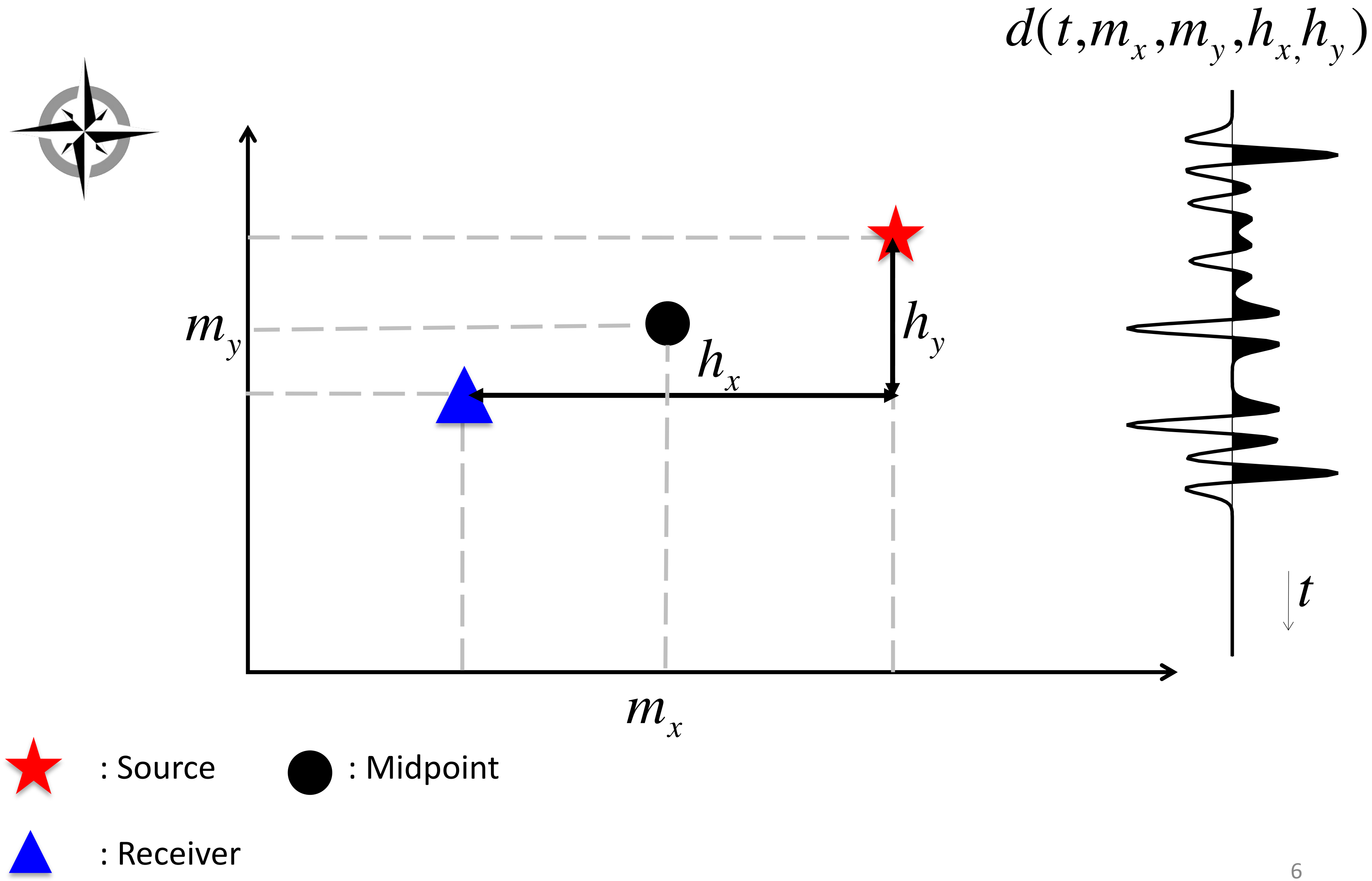
$$h_y = s_y - r_y$$

: Source

: Midpoint

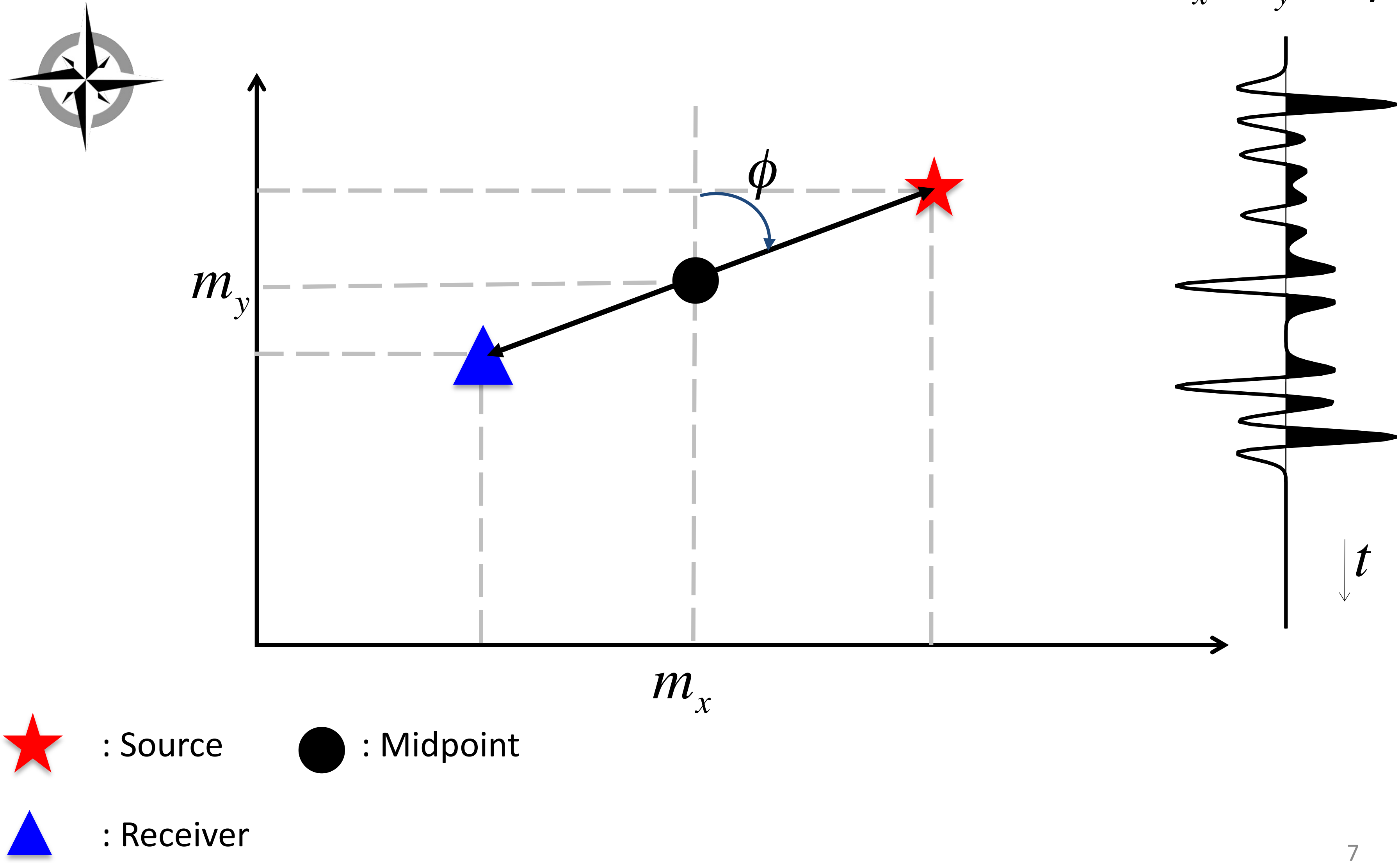
: Receiver

Midpoint-offset coordinates



Midpoint-offset coordinates

$$d(t, m_x, m_y, h, \phi)$$



5D data (4 spatial coordinates + time)

- Source Receiver coordinates $d(t, s_x, s_y, r_{x,} r_y)$
- Midpoint, inline and cross-line offsets $d(t, m_x, m_y, h_{x,} h_y)$
- Midpoint, offset and azimuth $d(t, m_x, m_y, h, \phi)$

5D data (4 spatial coordinates + frequency)

$$f(t) \leftrightarrow F(\omega)$$

- Source Receiver coordinates $\quad D(\omega, s_x, s_y, r_x, r_y)$
- Midpoint, inline and cross-line offsets $\quad D(\omega, m_x, m_y, h_x, h_y)$
- Midpoint, offset and azimuth $\quad D(\omega, m_x, m_y, h, \phi)$

We often work with one 4D cube per frequency

Fourier Industrial modules for ND reconstruction

- MWNI (Minimum Weighted Norm Interpolation)
- ALFT (Anti leakage Fourier Transform)
- POCS (Projection onto Convex Sets)
- Sparse Fourier Reconstruction
- Matching Pursuit Reconstruction
- and 10^{10} versions of the aforementioned algorithms

  - All of the above assume a sparse (simple) distribution of Fourier Coefficients.
  - Different solvers but similar concepts/assumptions are used
  - Tricks make them robust (as always)
  - I/O and patching strategies are extremely important
  - Operators
    - NUDFT to honor true coordinates
    - FFT after Binning
    - Could replace Fourier kernels by any localized transform

Rank reduction techniques

- So far I have described methods that are based on Fourier synthesis with sparsity/simplicity constraints. These methods are extensively used by industry and the core of the so called 5D Interpolation methods

- New classes of methods have started to develop in recent years (based on some old and interesting ideas). They assume that seismic data are low rank tensor structures
  - Interesting area of research because connects to Data Analytics, Collaborative Filtering, Personalized Medicine etc.

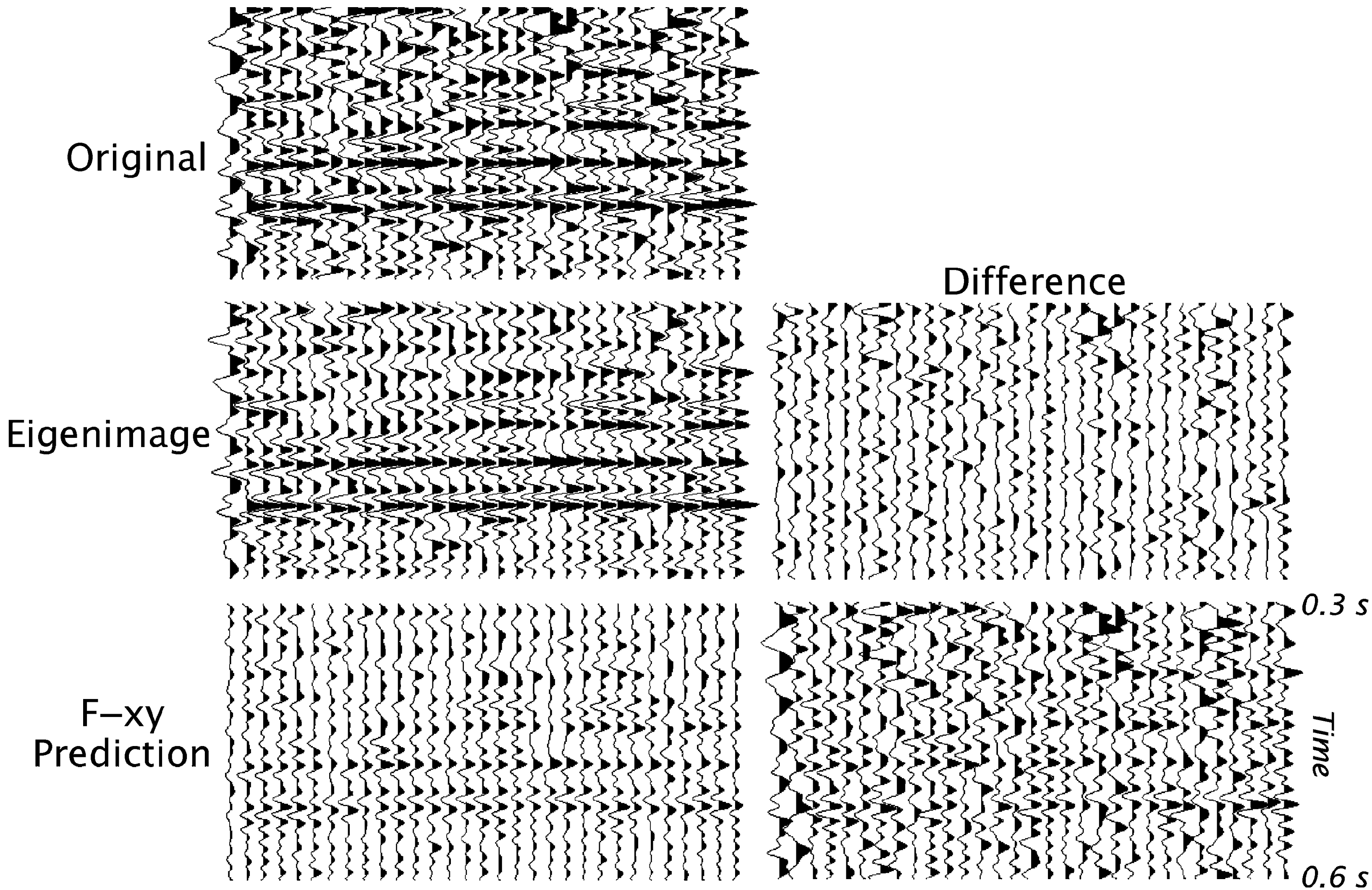
Denoising via rank-reduction

Hemon , C. H., and Mace, D., 1978, Use of the Karhtmen-Lolve transformation in seismic data processing: Geophys. Prosp., 26, 600-606.

Jones, I. F., 1985, Applications of the Karhunen-Loeve transform in reflection seismology: Ph.D. thesis, Univ. of British Columbia.

Al-Yahya, K. M., 1991, Application of the partial Karhunen-Loeve transform to suppress random noise in seismic sections: Geophys. Prosp., 39, 77–93.

Trickett (2003). ”F-xy eigenimage noise suppression.” GEOPHYSICS, 68(2), 751-759.



From Trickett GEO-2003

Recommender System

- A recommendation system (or recommender system) is an algorithm that attempts to predict the rating that a user or customer will give to an item. Recommendation systems have become quite popular in the field of e-commerce for predicting ratings of movies, books, news, research articles etc.

Netflix Prize

- From http://www.netflixprize.com/
- *“The Netflix Prize sought to substantially improve the accuracy of predictions about how much someone is going to enjoy a movie based on their movie preferences"*
- Netflix provided a training data set of 100,480,507 ratings that 480,189 users gave to 17,770 movies (only 1.17% of the elements of the data table/matrix are known)
- On September 21, 2009 Netflix awarded the $1M Grand Prize to team BellKor’s Pragmatic Chaos.

Matrix/Tensor Completion and the famous NETFLIX problem

Movie →

User ↓

	Taxi Driver	**Sense and Sensibility**	**Battleship Potemkin**	**Raging Bull**	**Titanic**	**Alexander Nevsky**
John	5	1	5	4	1	3
Mary	1	4	?	1	4	?
Pepe	4	2	2	3	4	?
Adrian	3	1	?	3	3	?
Tony	?	?	?	?	4	?
Kevin	3	3	?	3	2	?
Jianjung	2	1	?	2	4	?
Natasha	?	?	3	?	5	3

Hypothetical portion of the Netflix matrix

Matrix completion with minimal math

For Matrix completion

$$\text{Find } M_{ij}, \quad \text{such that } S_{ij} M_{ij} = M_{ij}^{obs} \text{ and } \operatorname{rank}(M) = K$$

For Tensor completion

$$\text{Find } M_{ijkl}, \quad \text{such that } S_{ijkl} M_{ijkl} = M_{ijkl}^{obs} \text{ and } \operatorname{multirank}(M) = K$$

Or, using min Nuclear Norm

$$\text{Find } M_{ijkl}, \quad \text{such that } S_{ijkl} M_{ijkl} = M_{ijkl}^{obs} \text{ and } \| \mathrm{M} \|_* = \min$$

N. Kreimer, A. Stanton and M. D. Sacchi, 2013, Tensor completion based on nuclear norm minimization for 5D seismic data reconstruction, Geophysics, 78 (6), V273-V284.

Rank (Review)

- The number of linearly independent columns of the matrix **A** is called the **rank** of **A**.
- A matrix is said to have full **rank** if all columns are linearly independent

Full Rank, rank = 4

1	0	1	-1
1	0	-3	-1
0	2	2	0
0	-1	-1	-1

Rank deficient, rank = 1

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{matrix}$$

Rank deficient, rank = 1

0	**0**	**0**	**0**
0	**0**	**0**	**0**
0	**0**	**0**	**0**
1	**1**	**1**	**1**
2	**2**	**2**	**2**
1	**1**	**1**	**1**
0	**0**	**0**	**0**
0	**0**	**0**	**0**
0	**0**	**0**	**0**

Low-rank Approximation and SVD

- **Assumption:** Data can be approximated by a matrix of rank r

$$M \approx M_r = \sum_{k=1}^{r} \lambda_k u_k v_k^T$$

$$= \sum_{k=1}^{r} E_k, \qquad E_k = \lambda_k u_k v_k^T$$

A matrix of rank r has r non-zero singular values

Algorithm

Simple matrix completion algorithm

$$M^{obs} = SM$$

$$M^{k} = M^{obs} + (1 - S)R[M^{k-1}]$$

$$R[\ ] = \text{rank reduction}$$

The algorithm can be derived by minimizing a cost function constrained by the assumption that the unknown image can be approximated by a low-rank matrix

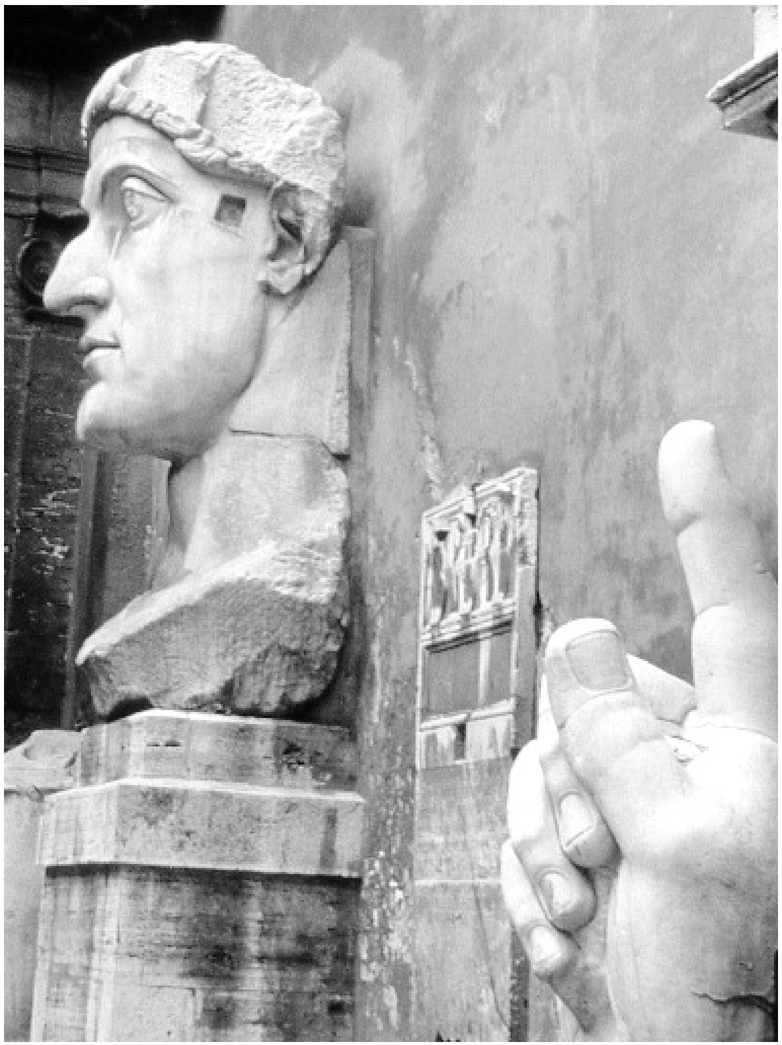
Algorithm

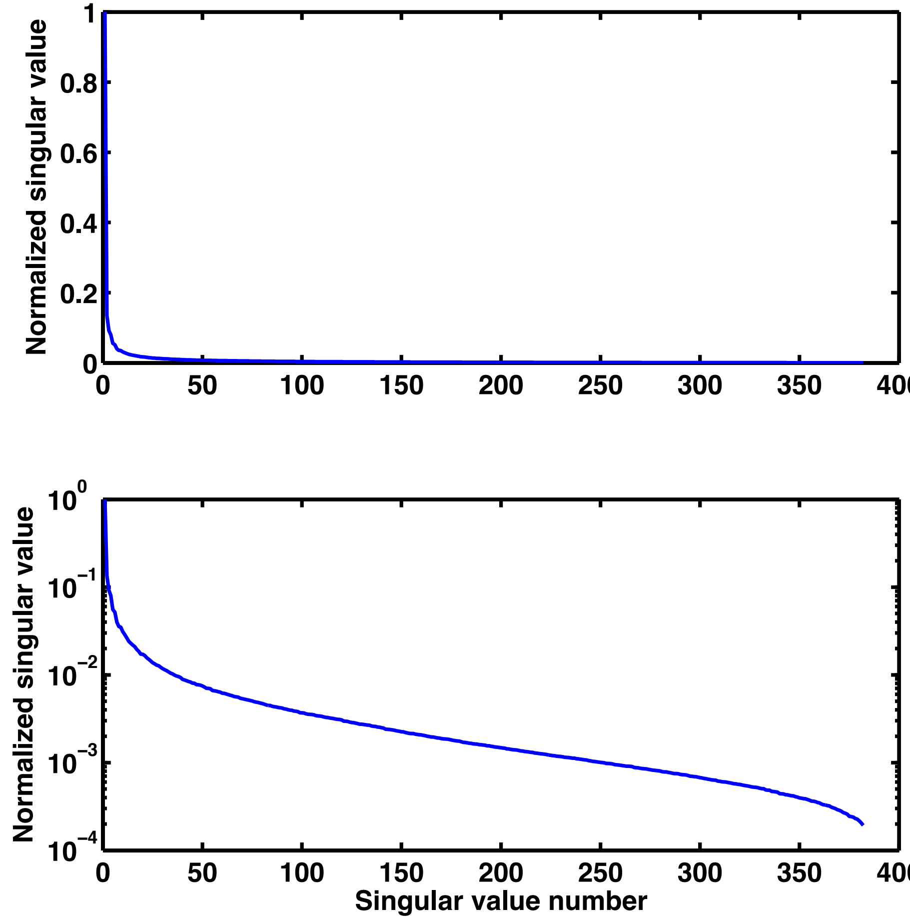
Simple matrix completion algorithm

$$M^k = \overbrace{M^{obs}}^{\text{Insert existing data}} + \underbrace{(1-S)R[M^{k-1}]}_{\text{Replace low rank approximation in pixels with missing data}}$$

Constantine the Great (c. 280-337)

M

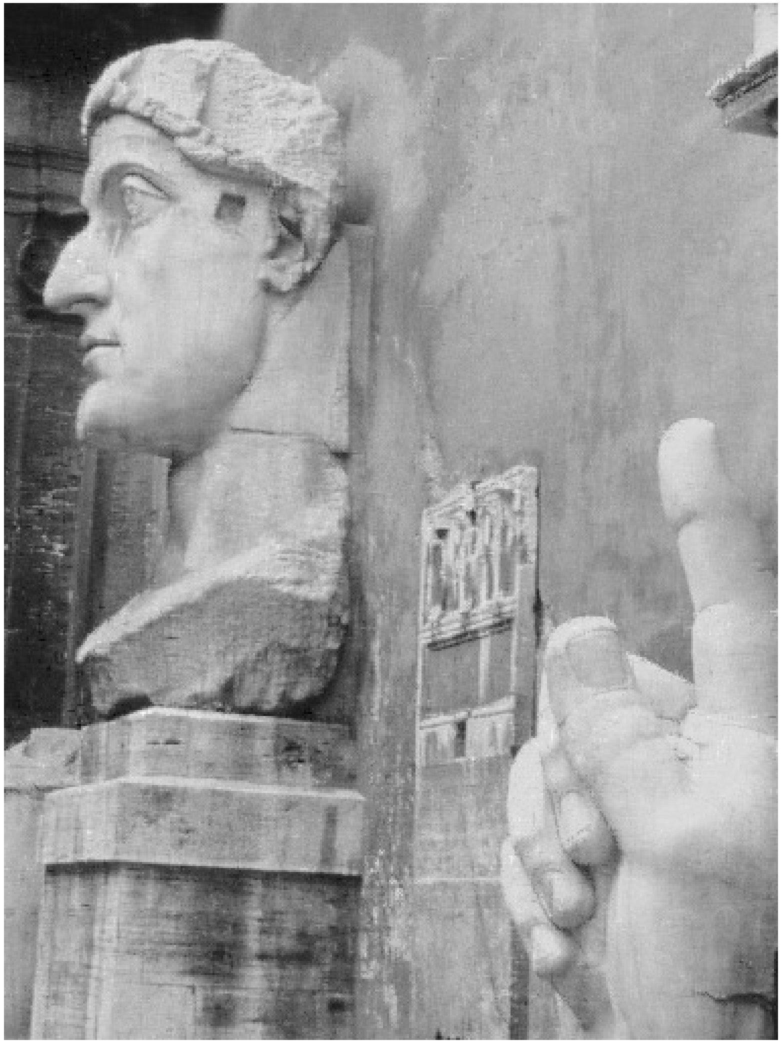




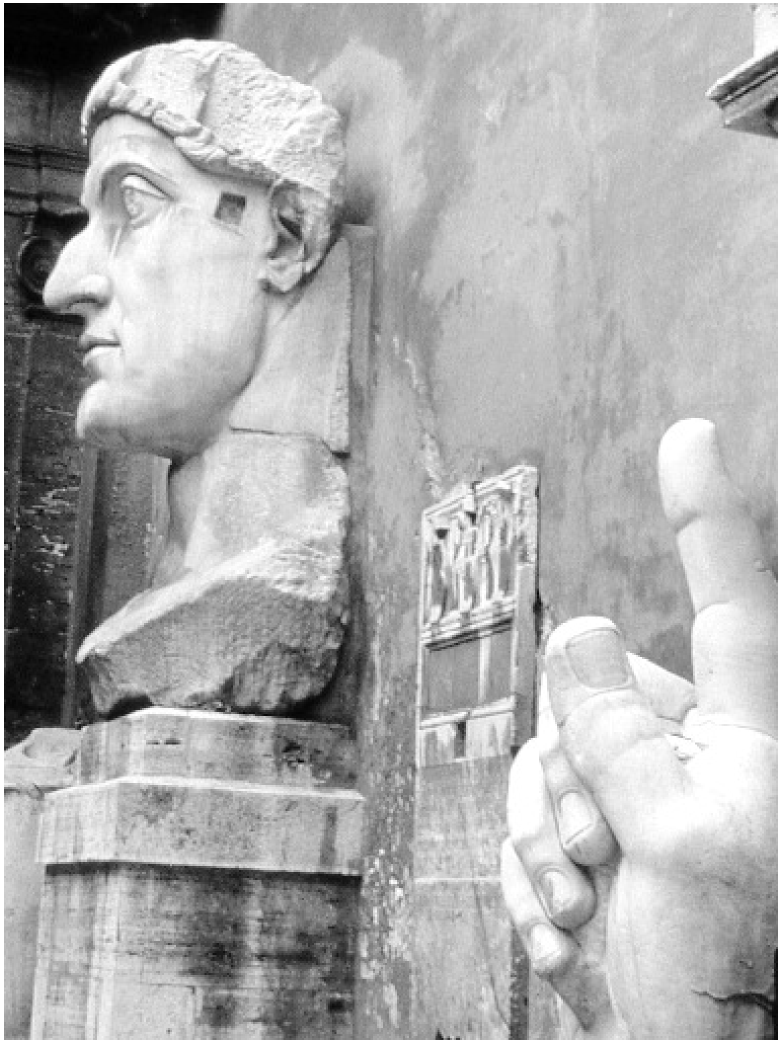
Constantine the Great after decimation

$$M^{obs} = SM$$

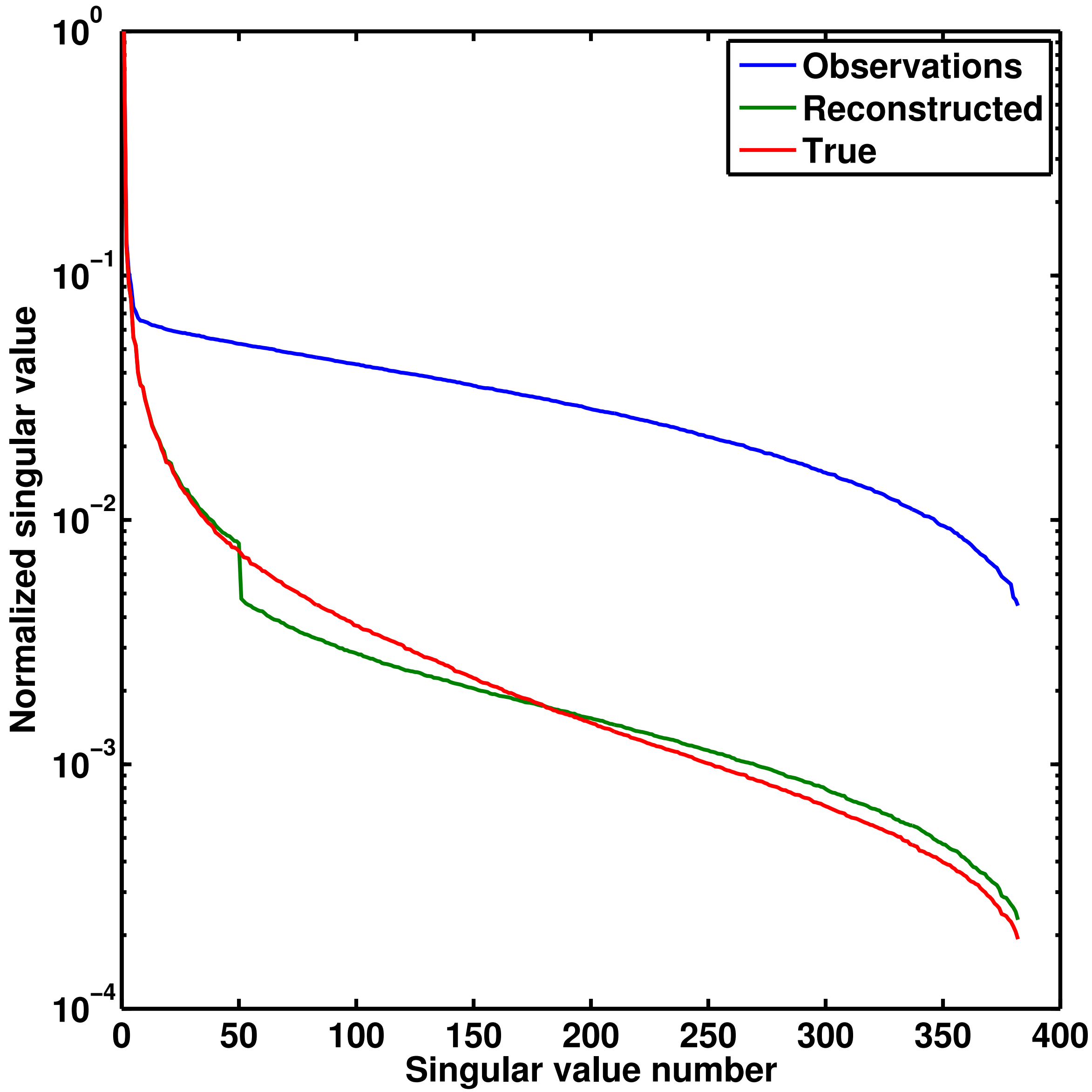
Constantine the Great after reconstruction



Constantine the Great – original image



Constantine the Great – Singular values



Why tensors ?

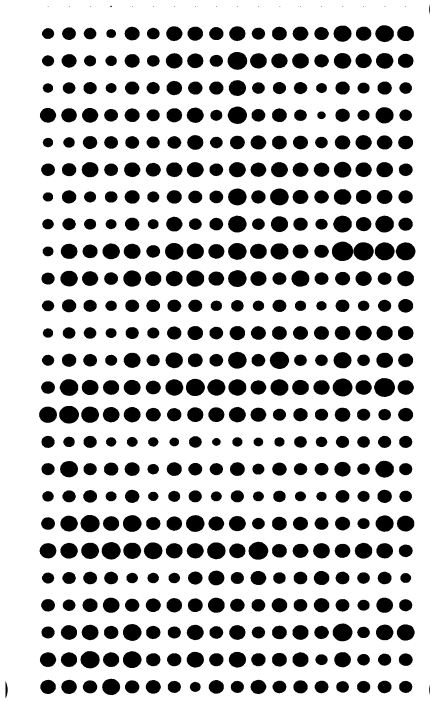
$$D(\omega, m_x, m_y, h, \phi) \rightarrow D(\omega, i, j, k, l) \rightarrow \mathbf{D}$$

$\mathbf{D}$: 4th order tensor

Sampling

- 5D volumes are irregularly sampled in space due to
  - Logistic constraints
  - Insufficient equipment
  - Acquisition costs
  - Provincial/Municipal regulations
  - Environmental constraints

$$\mathbf{D}^{obs} = \mathbf{T}\mathbf{D}$$



Fold Map

PMF (Xu et al., 2013)

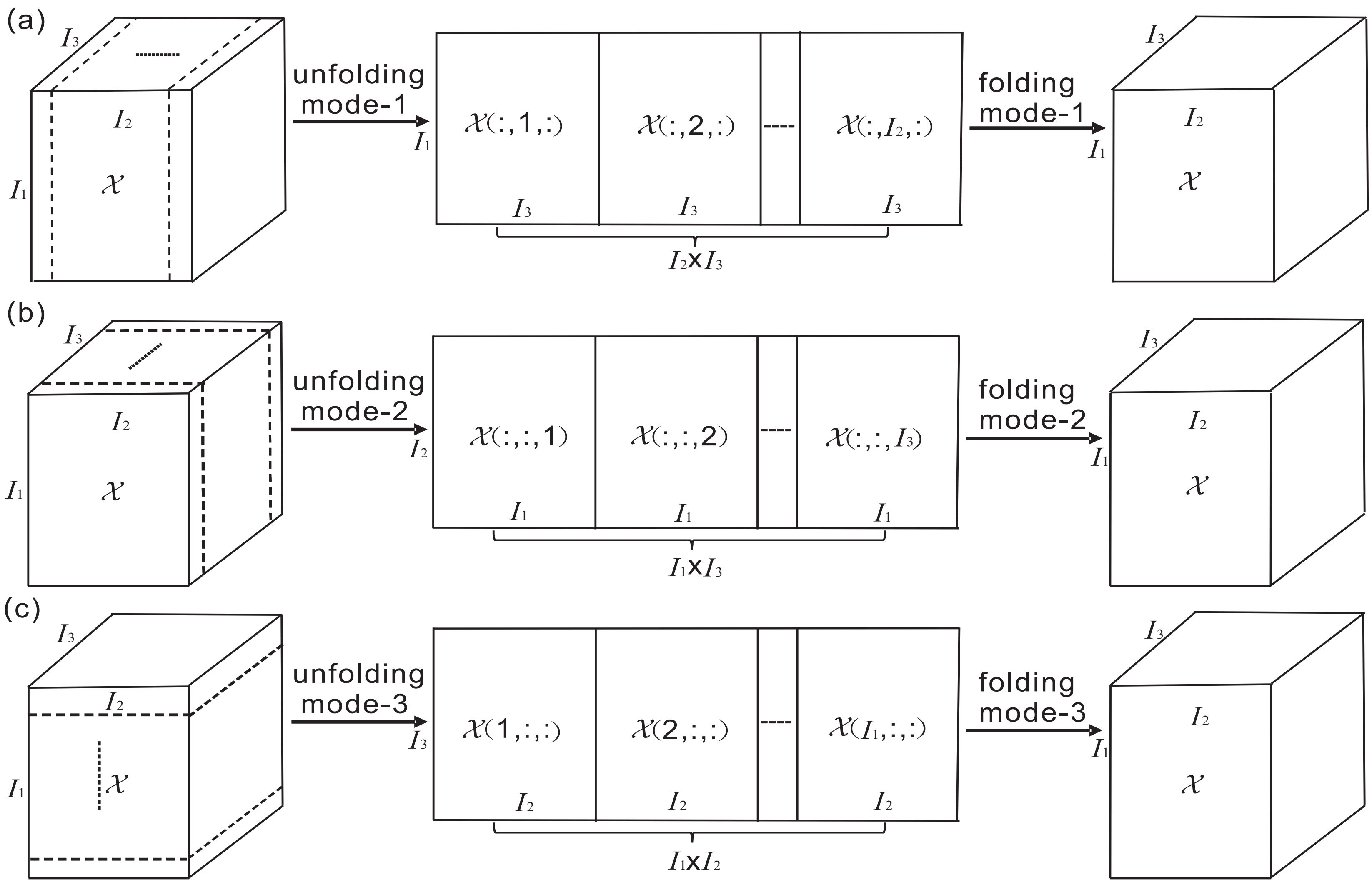
- In the Parallel Matrix Factorization method we minimize

$$\| \mathbf{D}^{obs} - \mathbf{T}\mathbf{D} \|_F$$

- Subject to

$$\mathrm{Rank}\,[\mathrm{Unfold}_i(\mathbf{D})] = k_i \quad i = 1,2,3,4$$

Unfolding and folding



Reconstruction algorithm

- The math leads to a simple algorithm

$$\mathbf{D}^{n} = \alpha \mathbf{D}^{obs} + (1 - \alpha \mathbf{T}) R[\mathbf{D}^{k-1}]$$

R : Do Something Operator

R: Amplitude Thresholding (POCS)

R: Tensor Rank-reduction (HOSVD, PMF, etc)

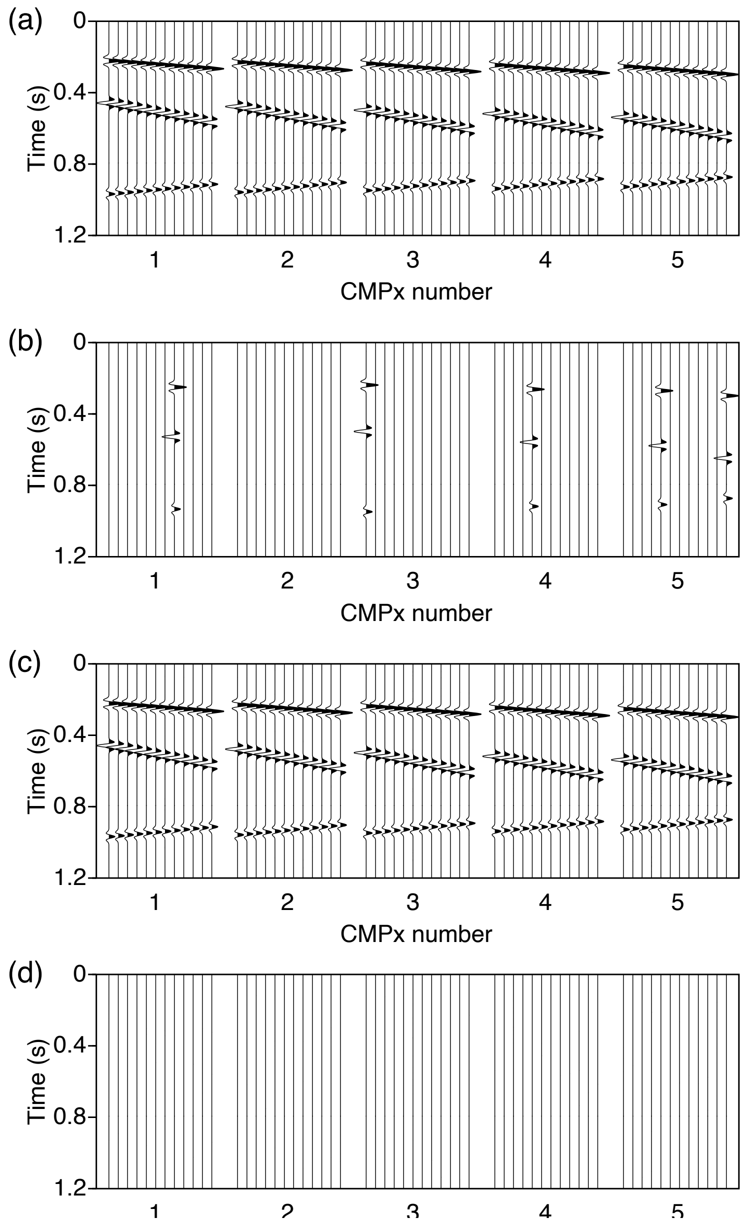
Reconstruction algorithm

- In PMF

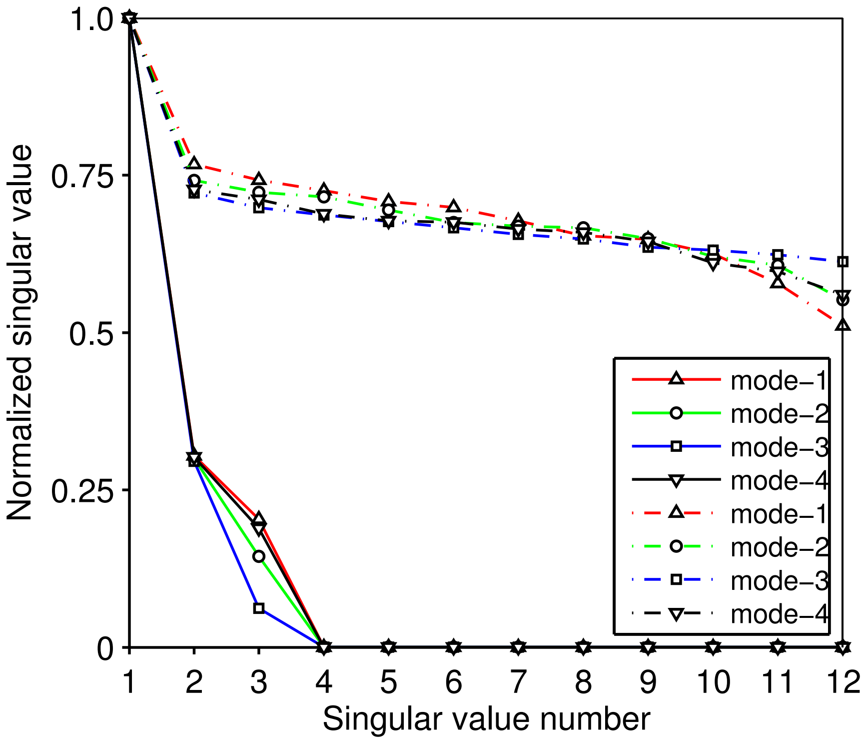
$$\mathbf{D}^{n} = \alpha \mathbf{D}^{obs} + (1 - \alpha \mathbf{T}) R[\mathbf{D}^{k-1}]$$

$R[\ ]$ = average low rank-approximation over all modes

Synthetic

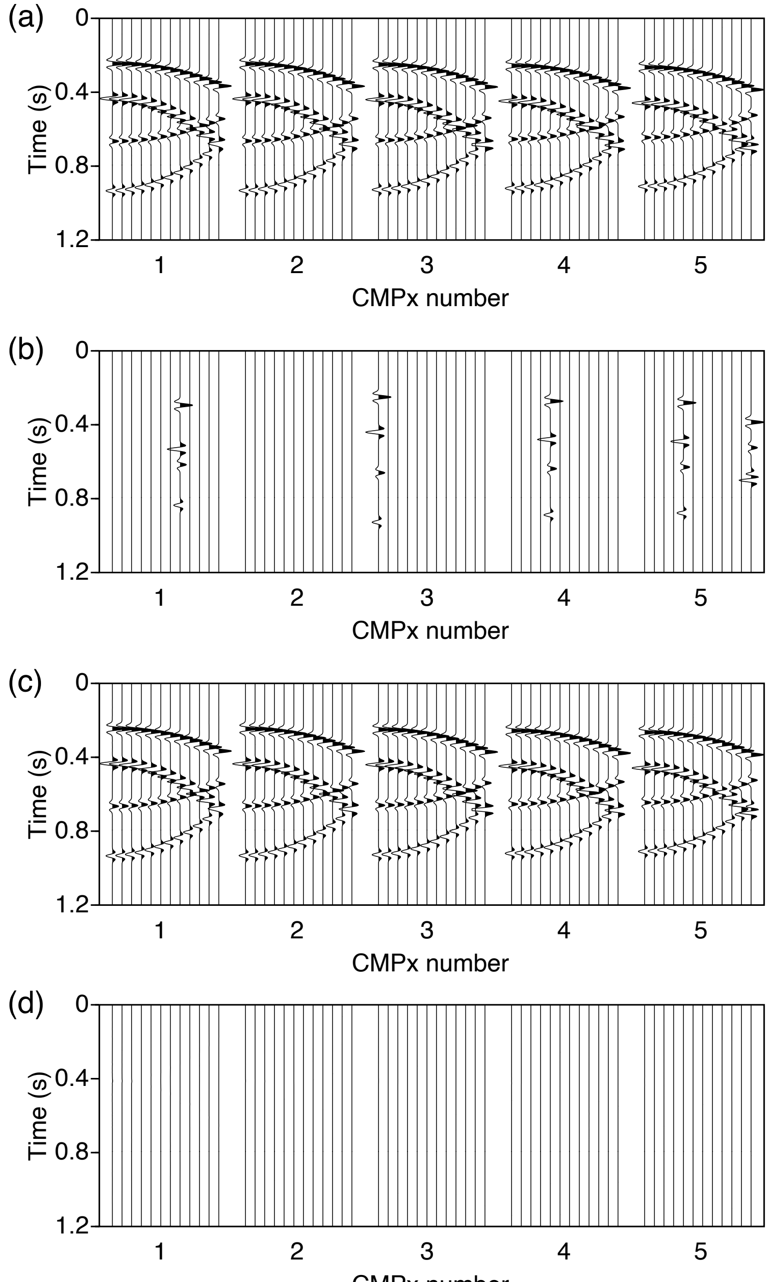


(a)
(b)
(c)
(d)
Time (s)
0
0.4
0.8
1.2
1
2
3
4
5
CMPx number

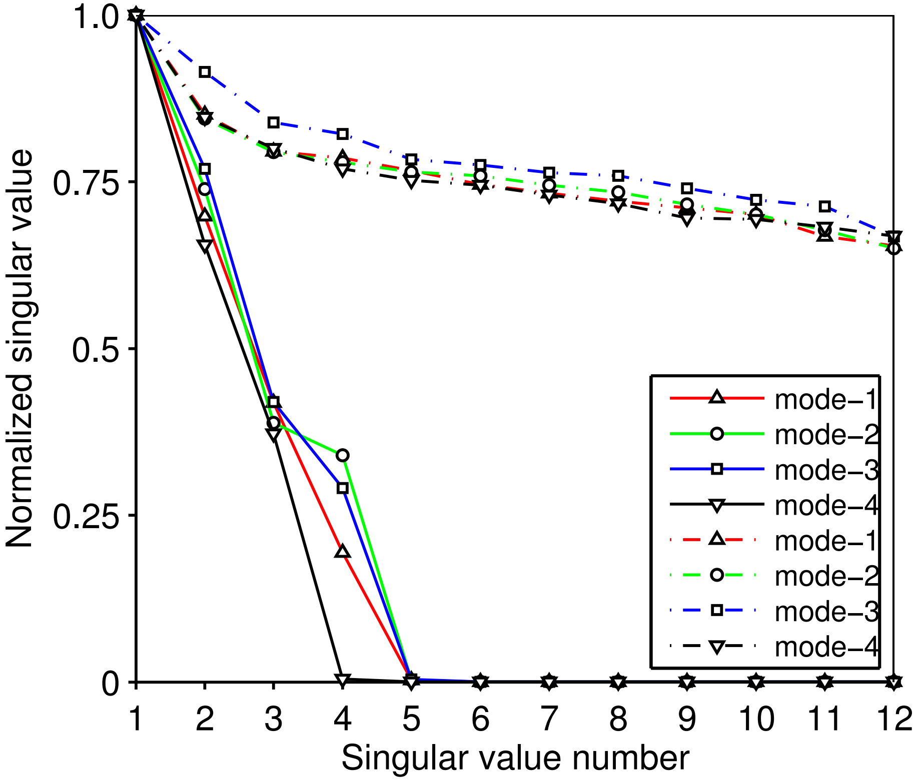


Normalized singular value
1.0
0.75
0.5
0.25
0
1
2
3
4
5
6
7
8
9
10
11
12
Singular value number
mode−1
mode−2
mode−3
mode−4
mode−1
mode−2
mode−3
mode−4

Synthetic

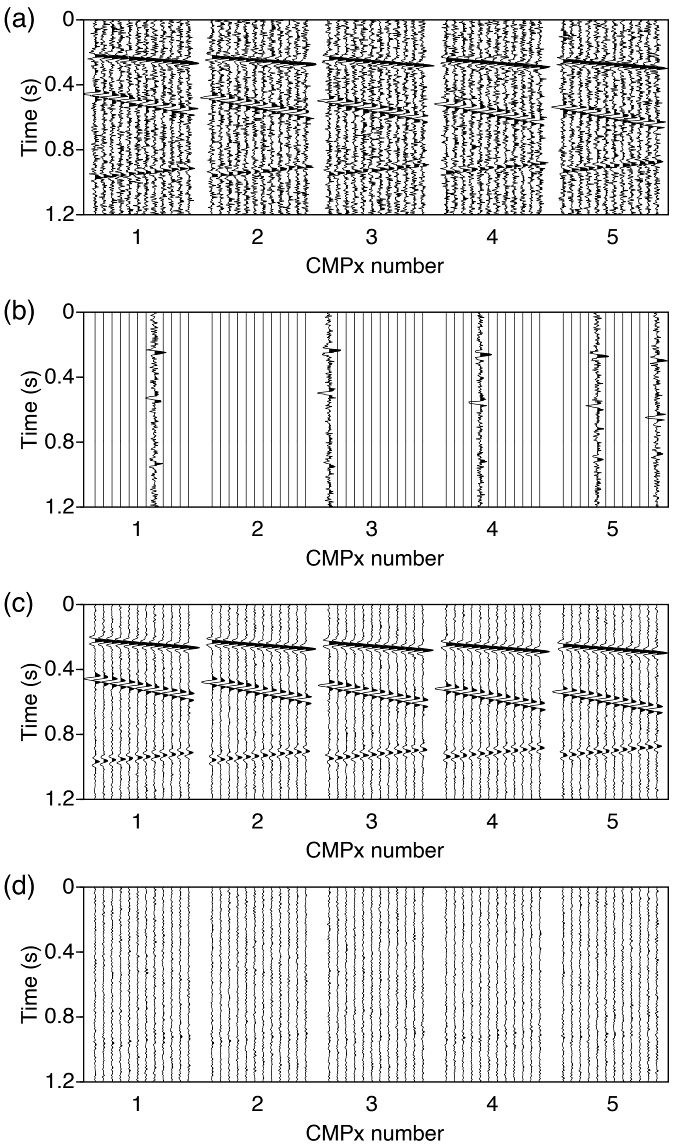


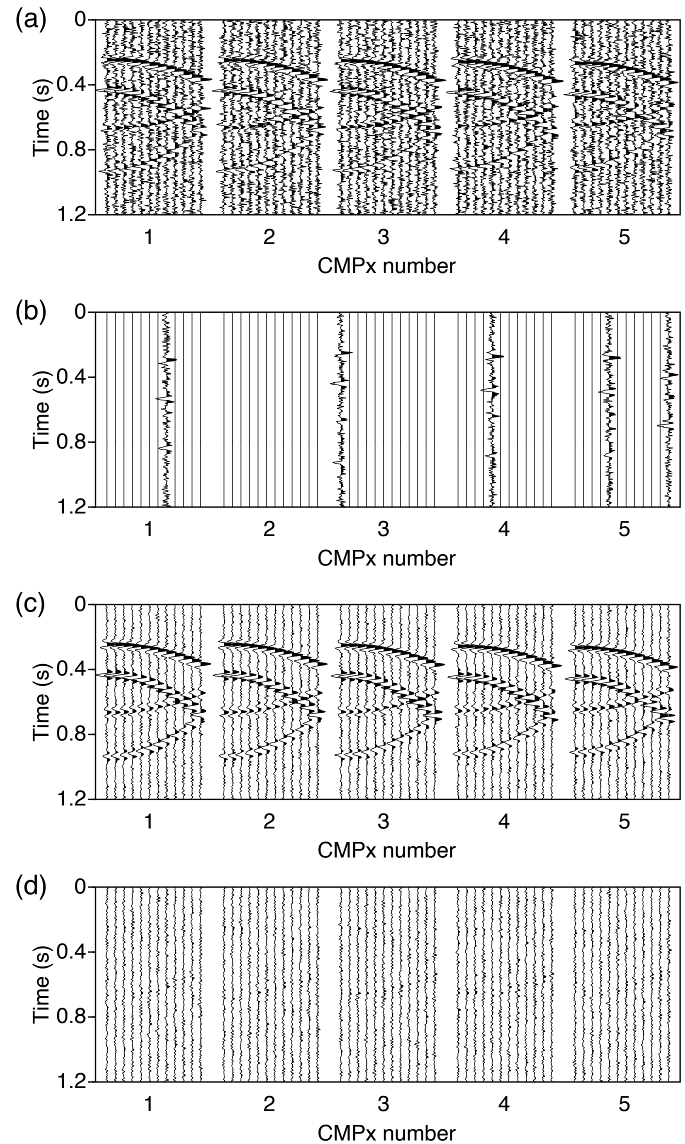
(a)
(b)
(c)
(d)
Time (s)
0
0.4
0.8
1.2
1
2
3
4
5
CMPx number



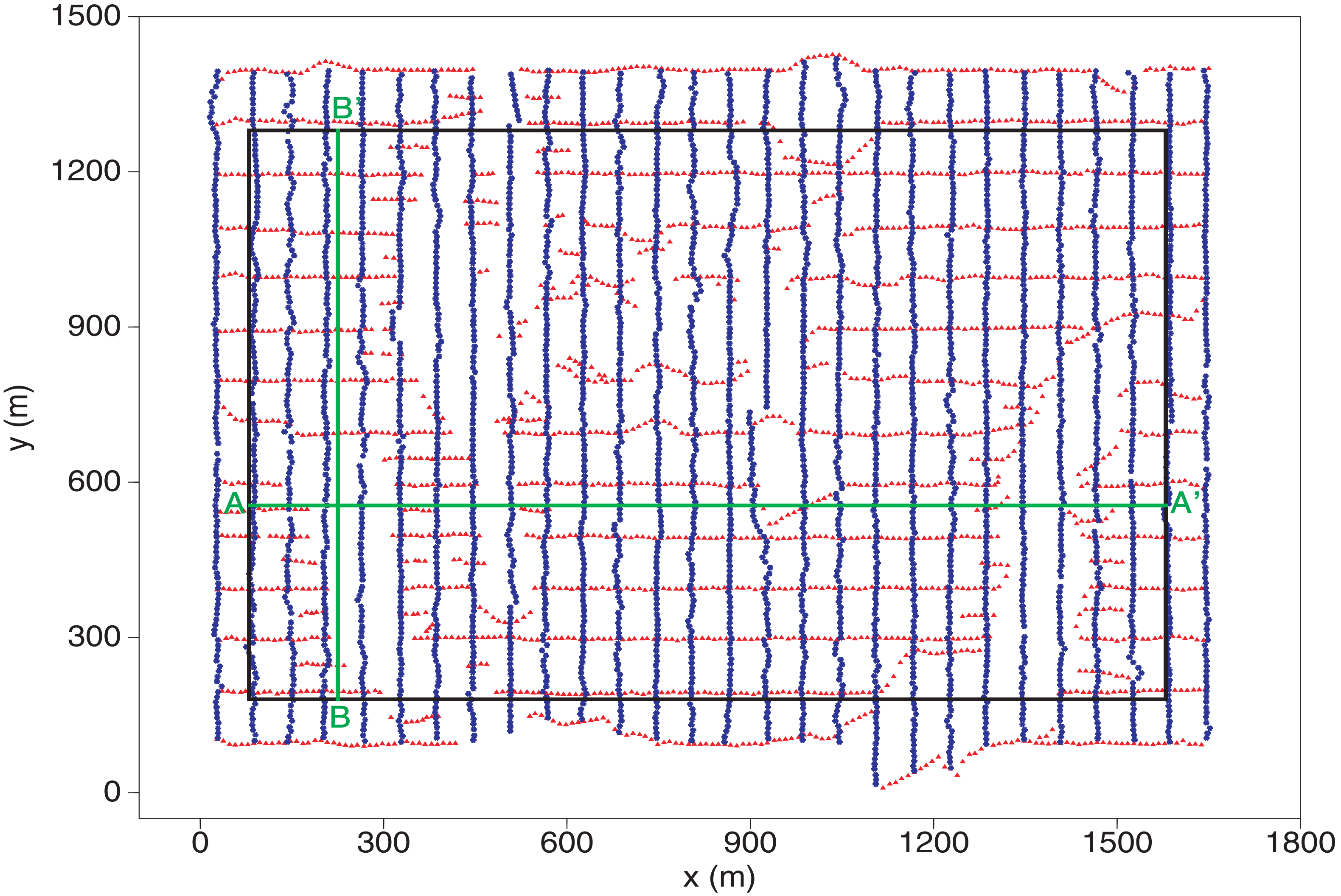
Normalized singular value
Singular value number
1.0
0.75
0.5
0.25
0
1
2
3
4
5
6
7
8
9
10
11
12
mode−1
mode−2
mode−3
mode−4
mode−1
mode−2
mode−3
mode−4

Linear and curved events SNR=1

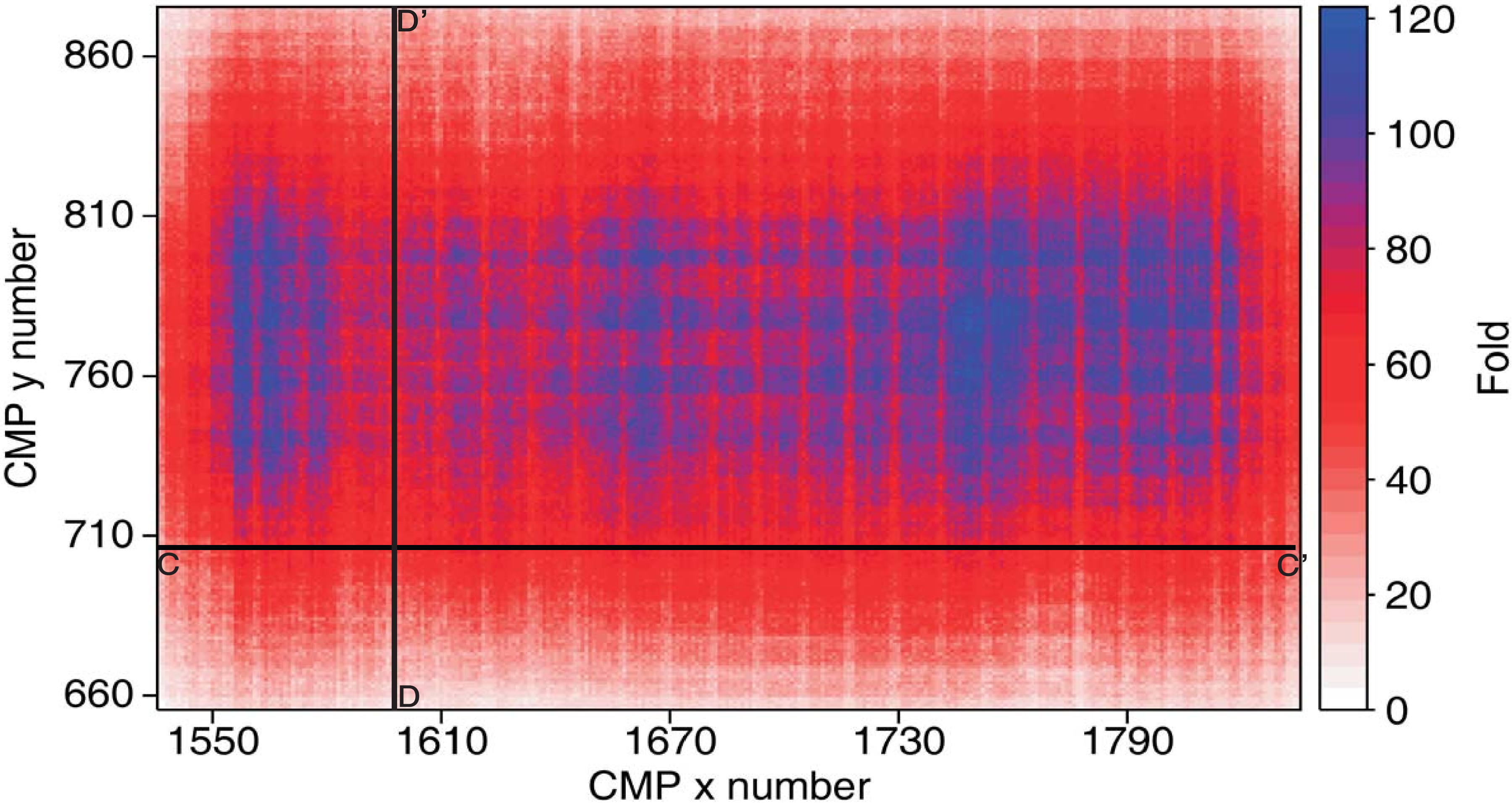




Field data example (WCB)



Fold Map

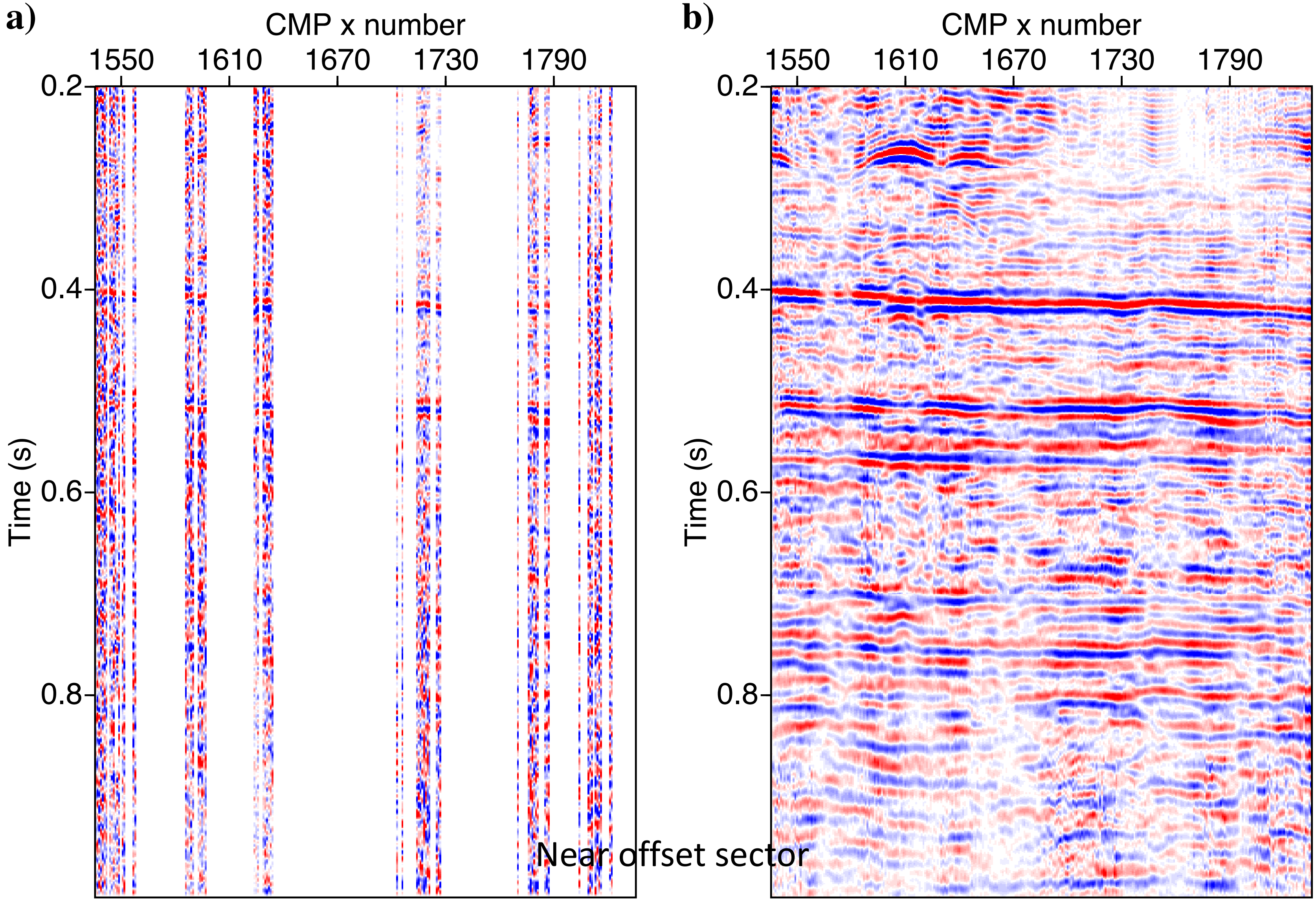


D’
D
C
C’
860
810
760
710
660
CMP y number
1550
1610
1670
1730
1790
CMP x number
120
100
80
60
40
20
0
Fold

Processing Parameters

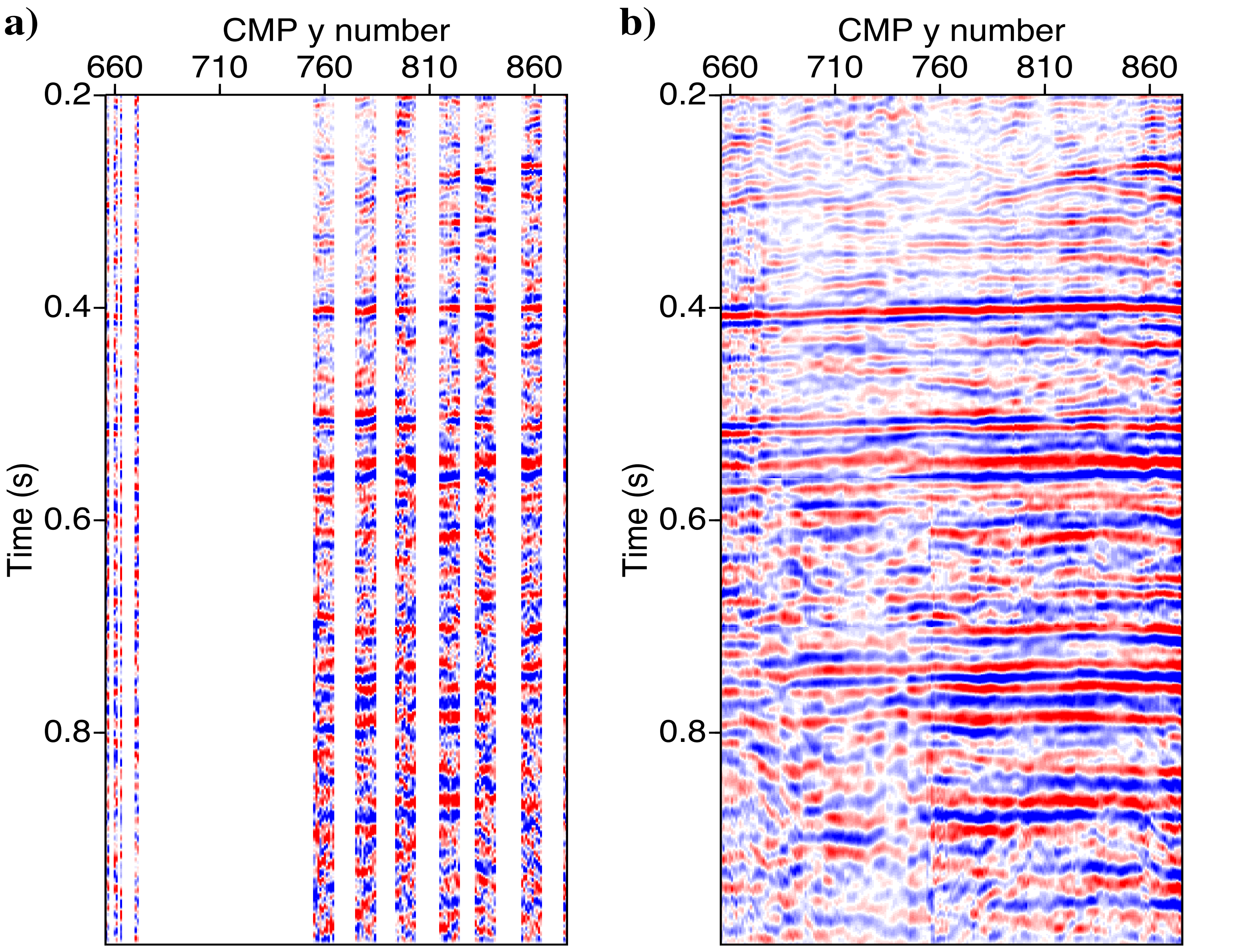
- 5m X 5m CMP Bins
- 100m offset sectors (x and y)
- 300 CMPx and 220 CMPy bins
- All survey was divided in 2640 overlapping blocks
- Each block has about 85% of missing traces (15% alive)
- First part of analysis is with reconstruction in offset-midpoint

Fix offsets and CMPy



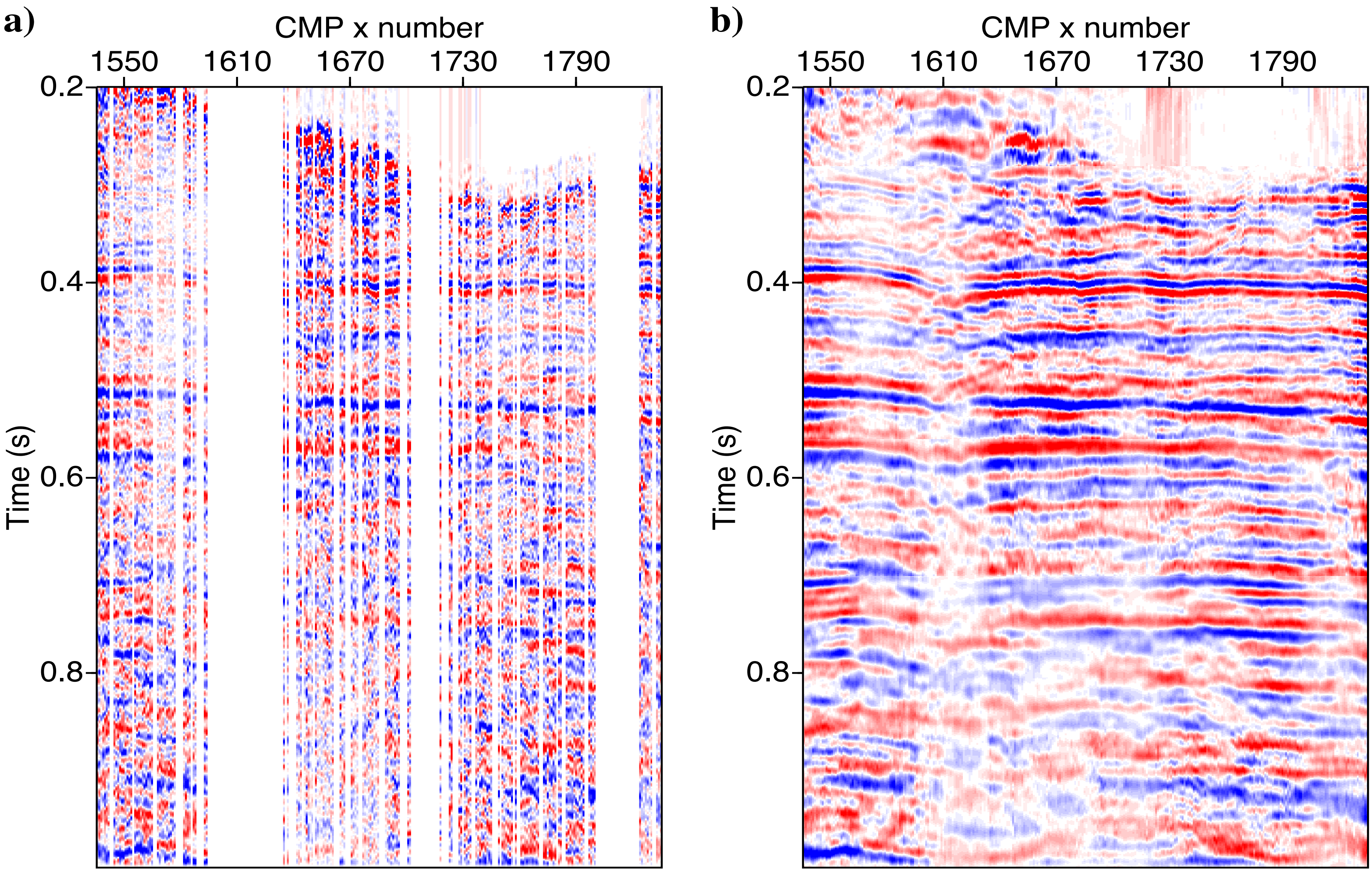
Fix offsets and CMPx

Near offset sector

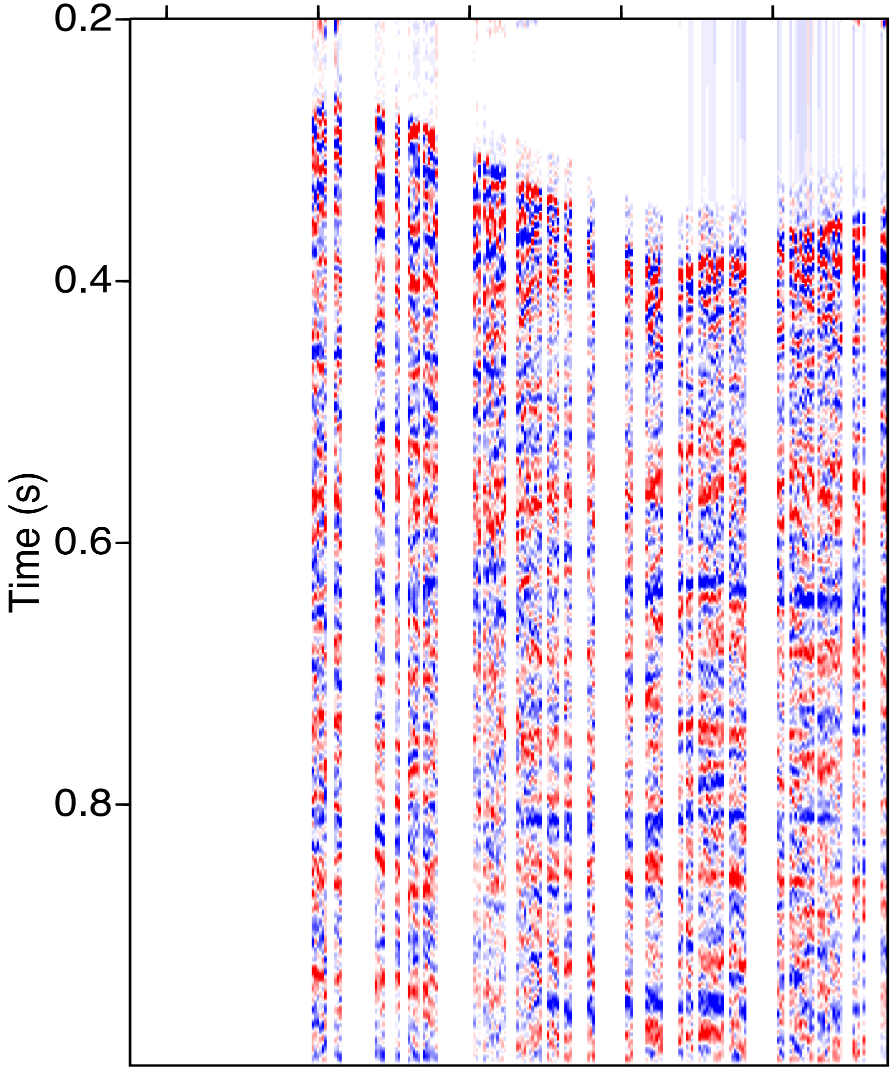


Fix offsets and CMPy

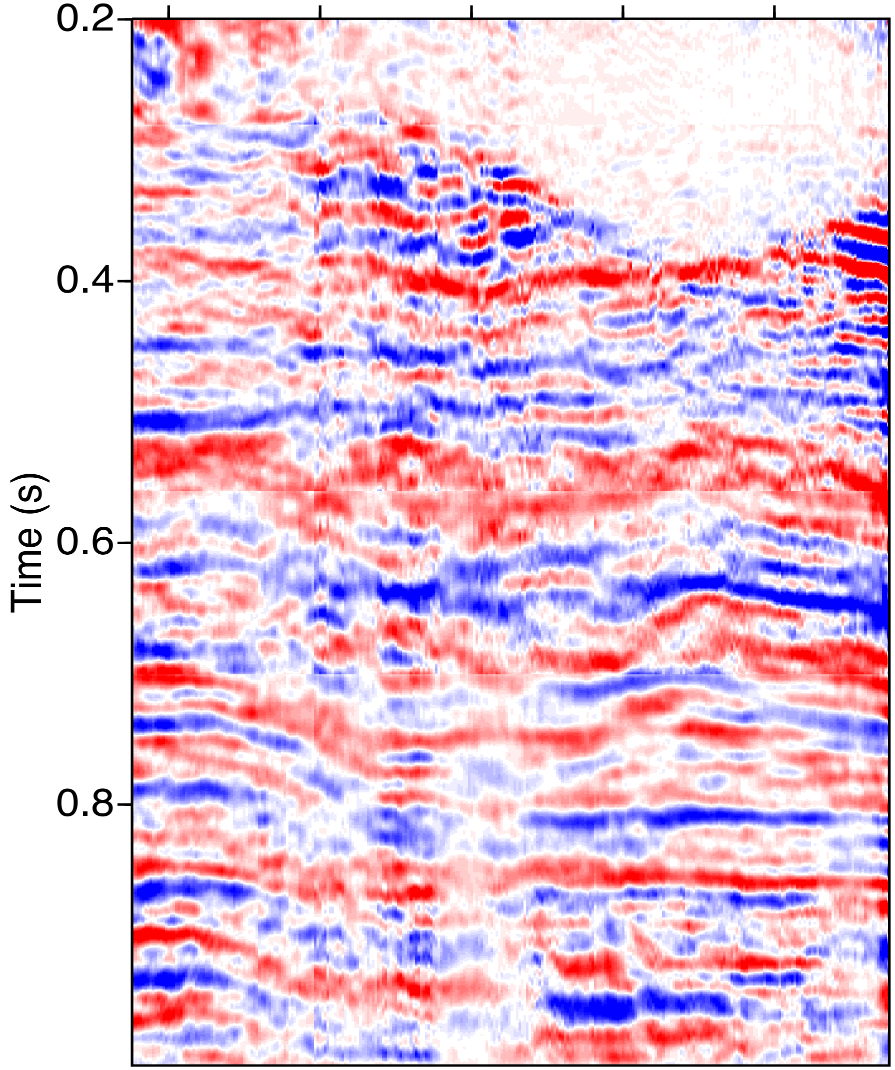
Mid range offset sector



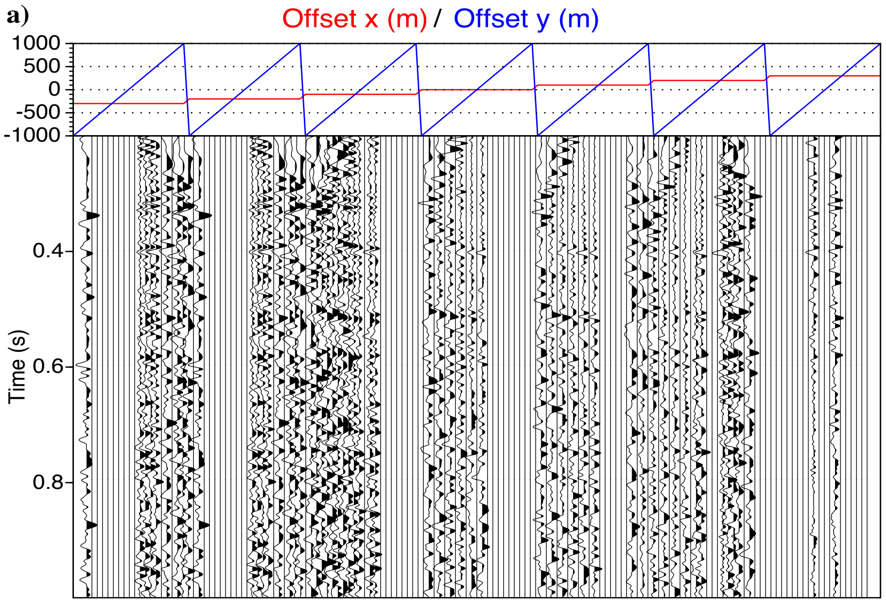
Fix offsets and CMPy

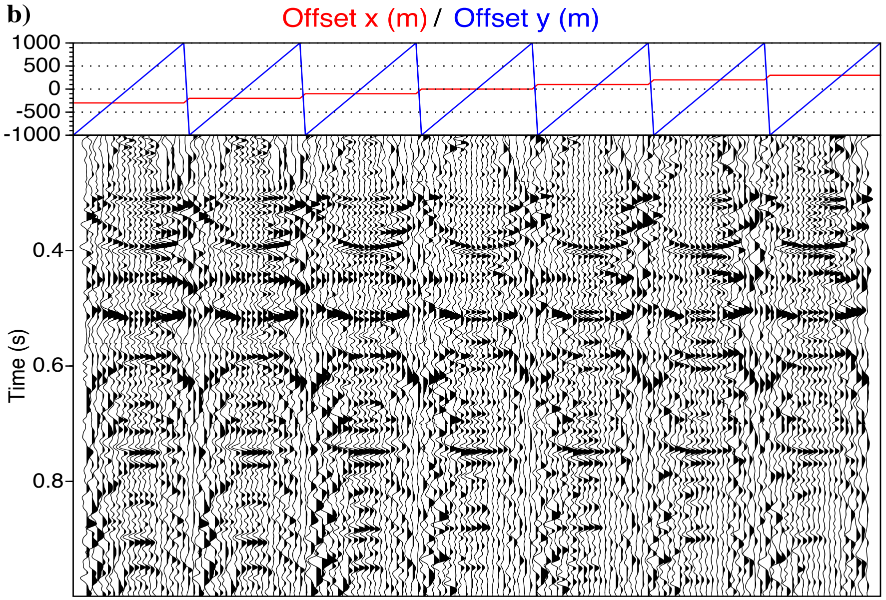


Mid range offset sector

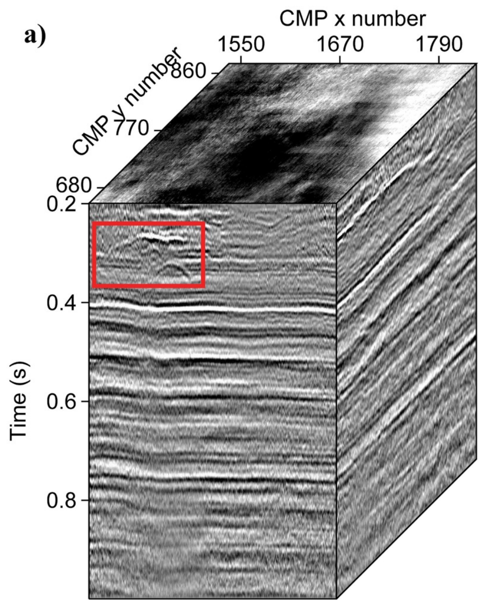


Fix CMPx and CMPy

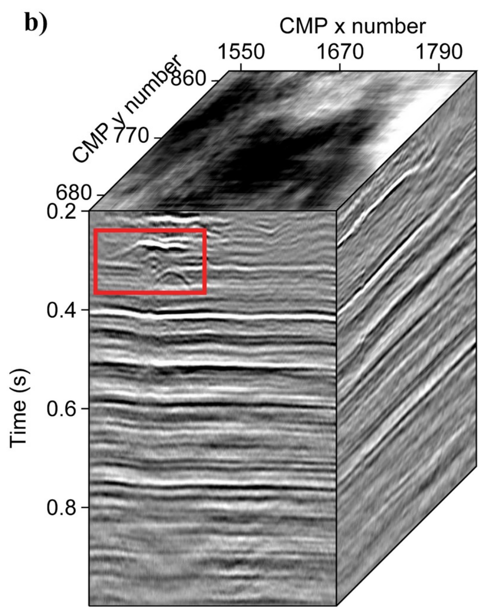


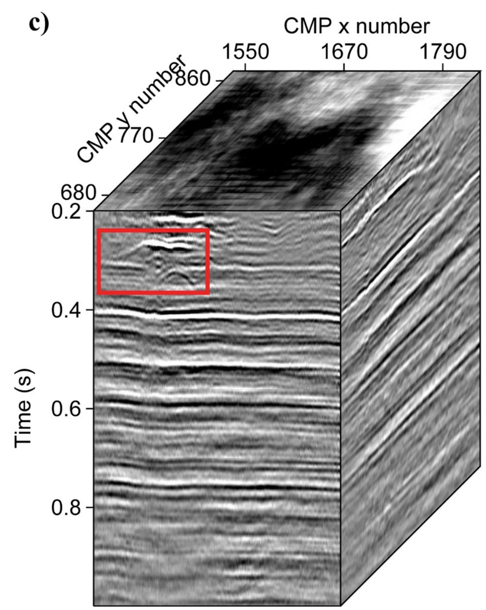


Before and after reconstruction of one CMP

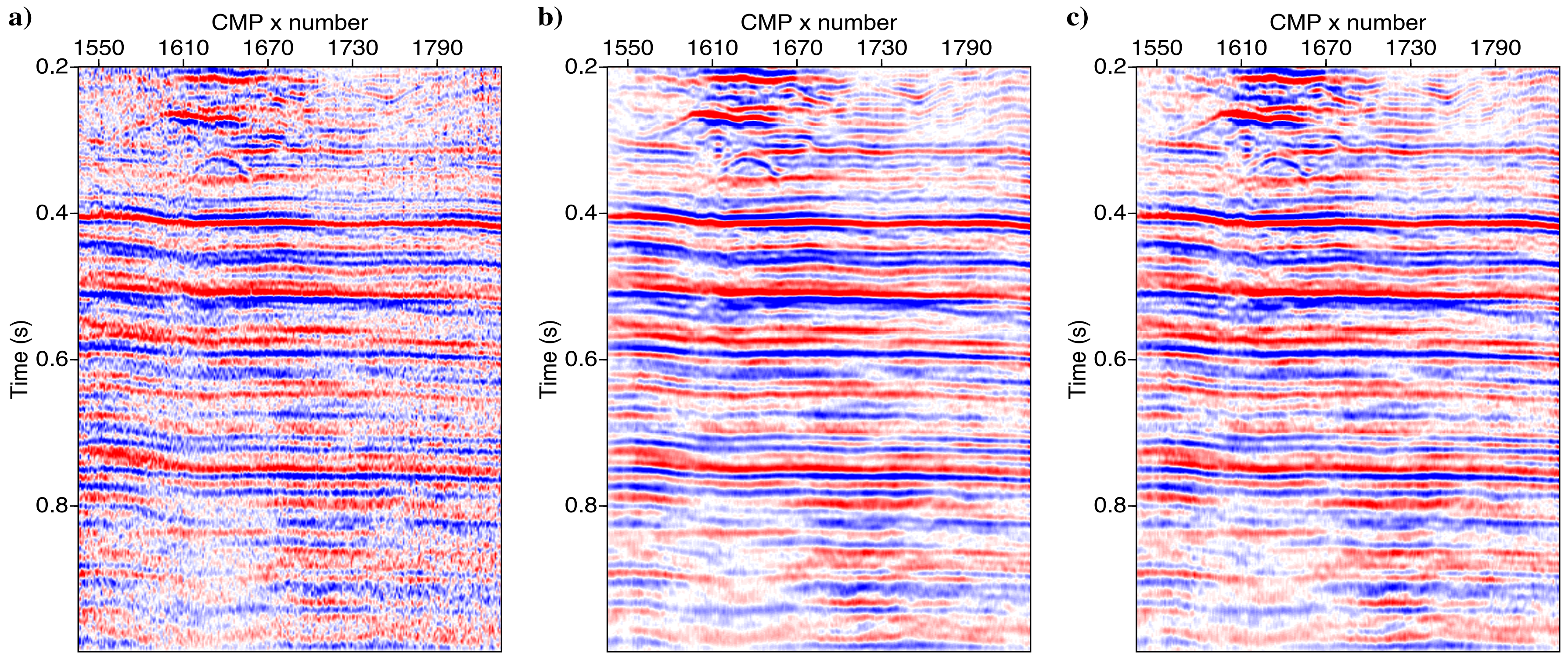


a) Observations
b) All traces (Observed + Reconstructed)
c) Only new traces



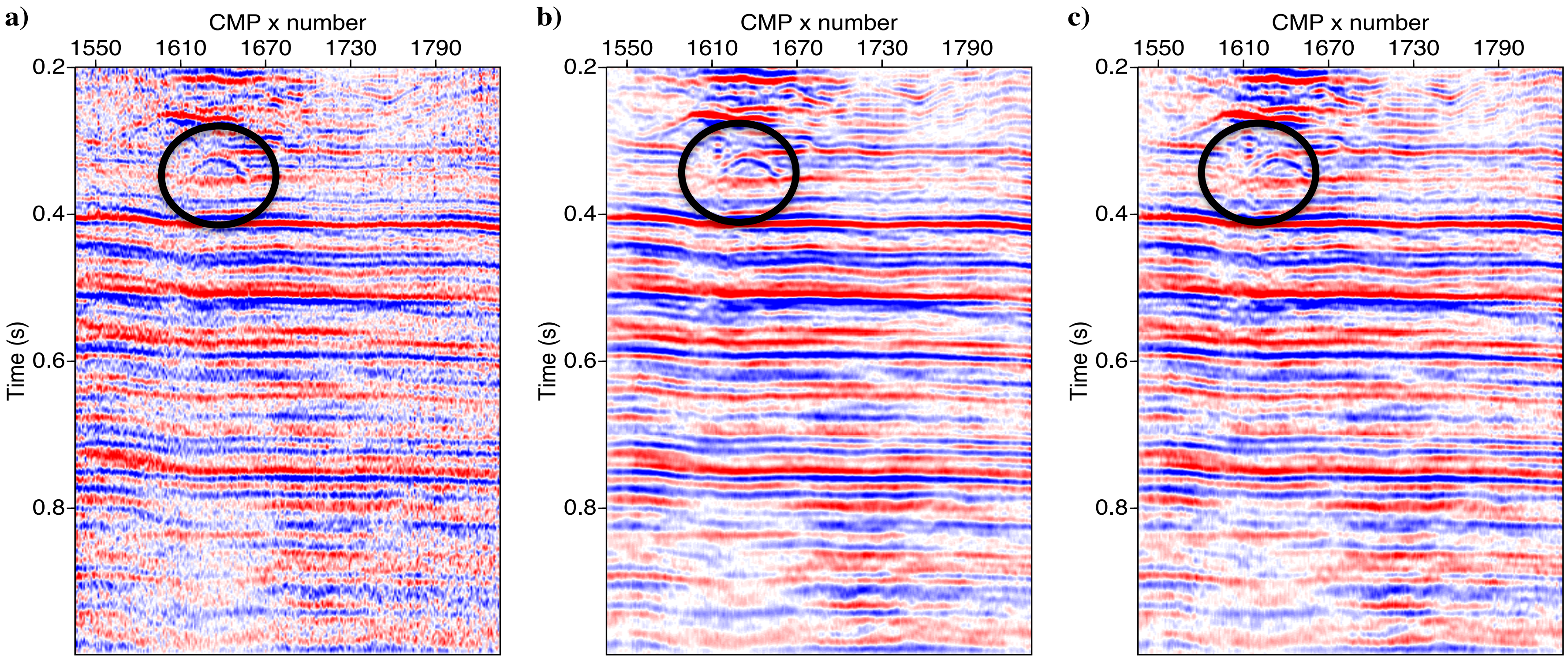


Stacks



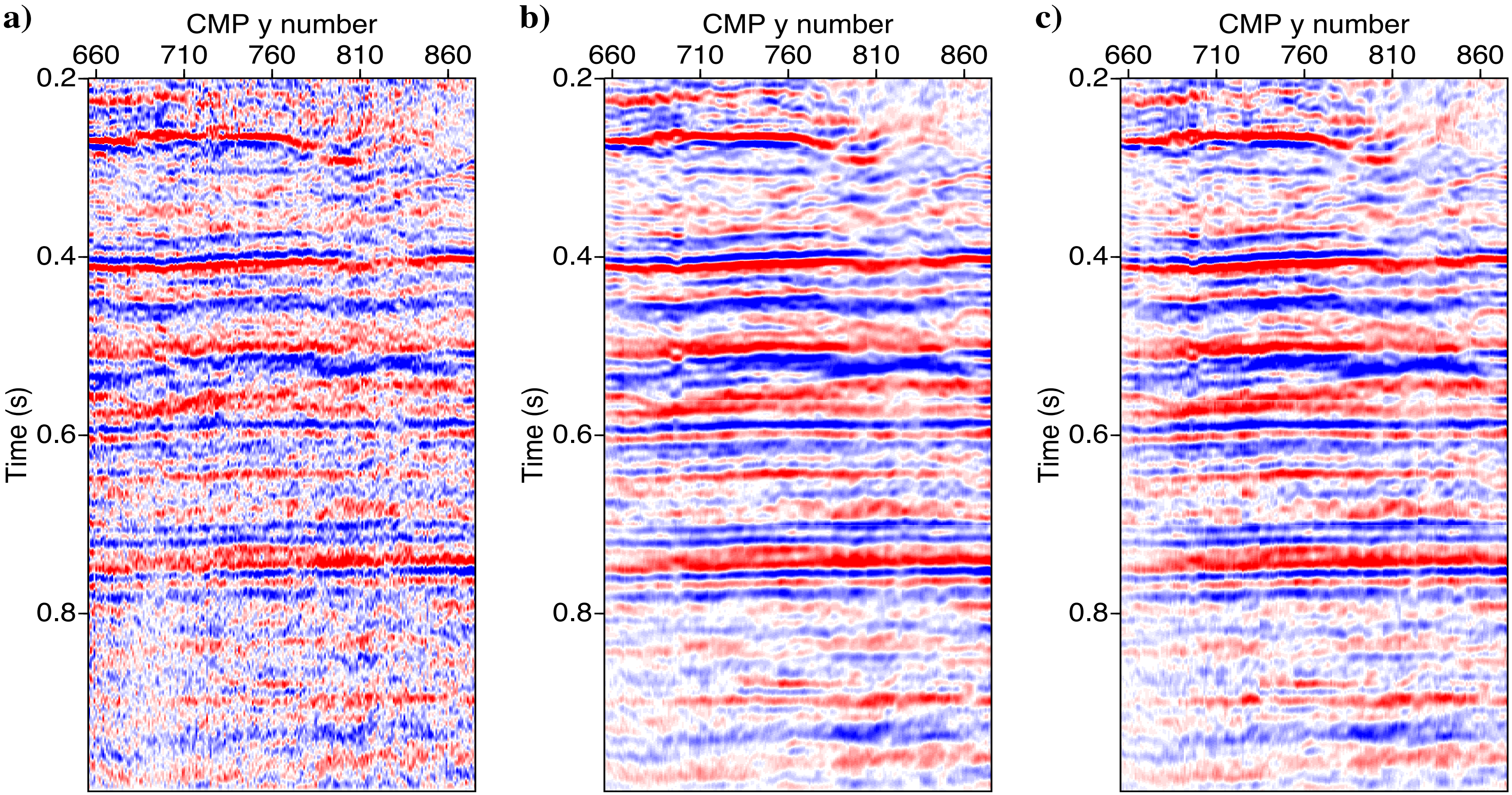
a) Observations b) All traces (Obs +Reconstructed) c) Only new traces

Stacks



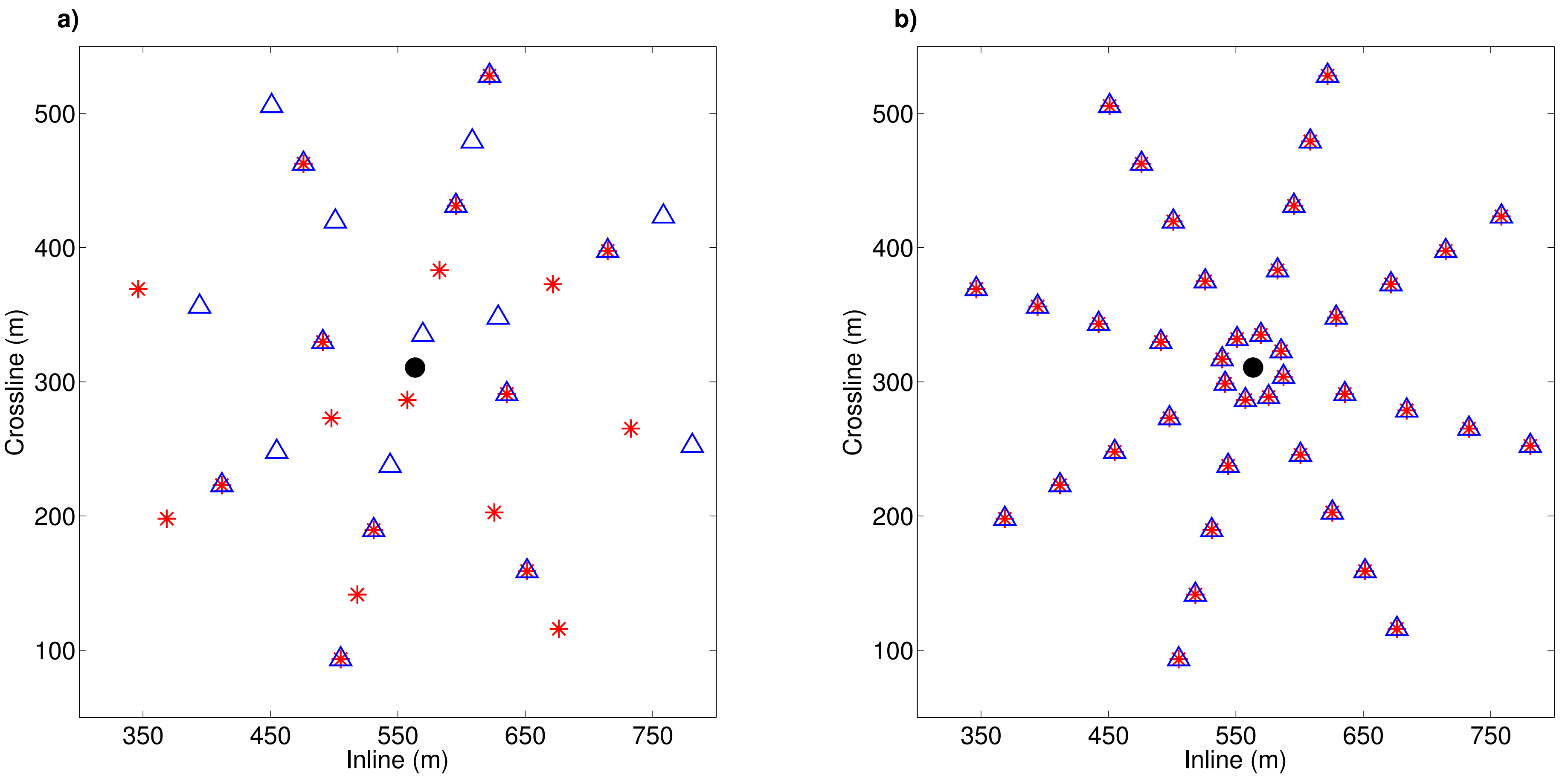
a) Observations b) All traces (Obs +Reconstructed) c) Only new traces

Stacks



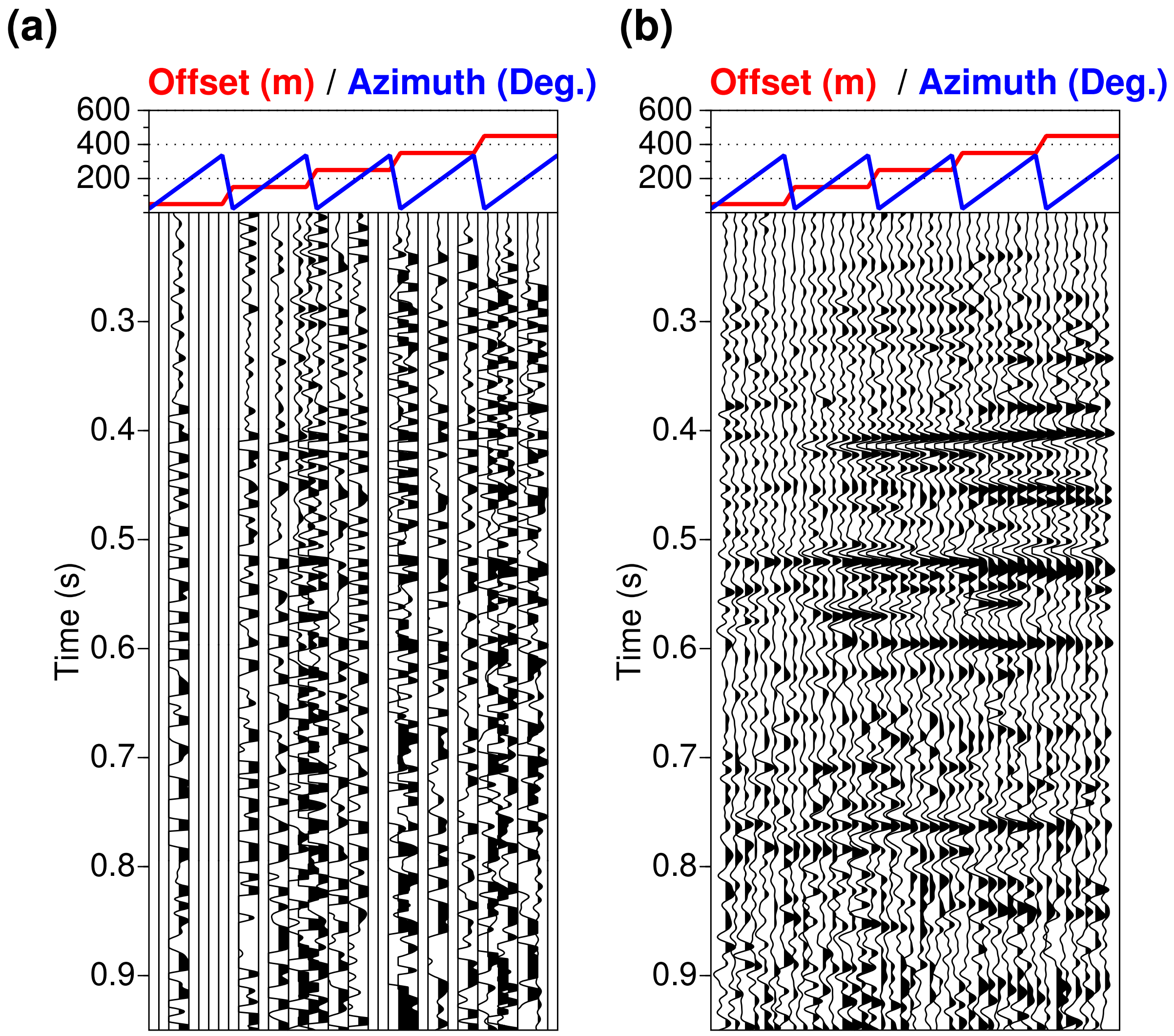
a) Observations b) All traces (Obs +Reconstructed) c) Only new traces

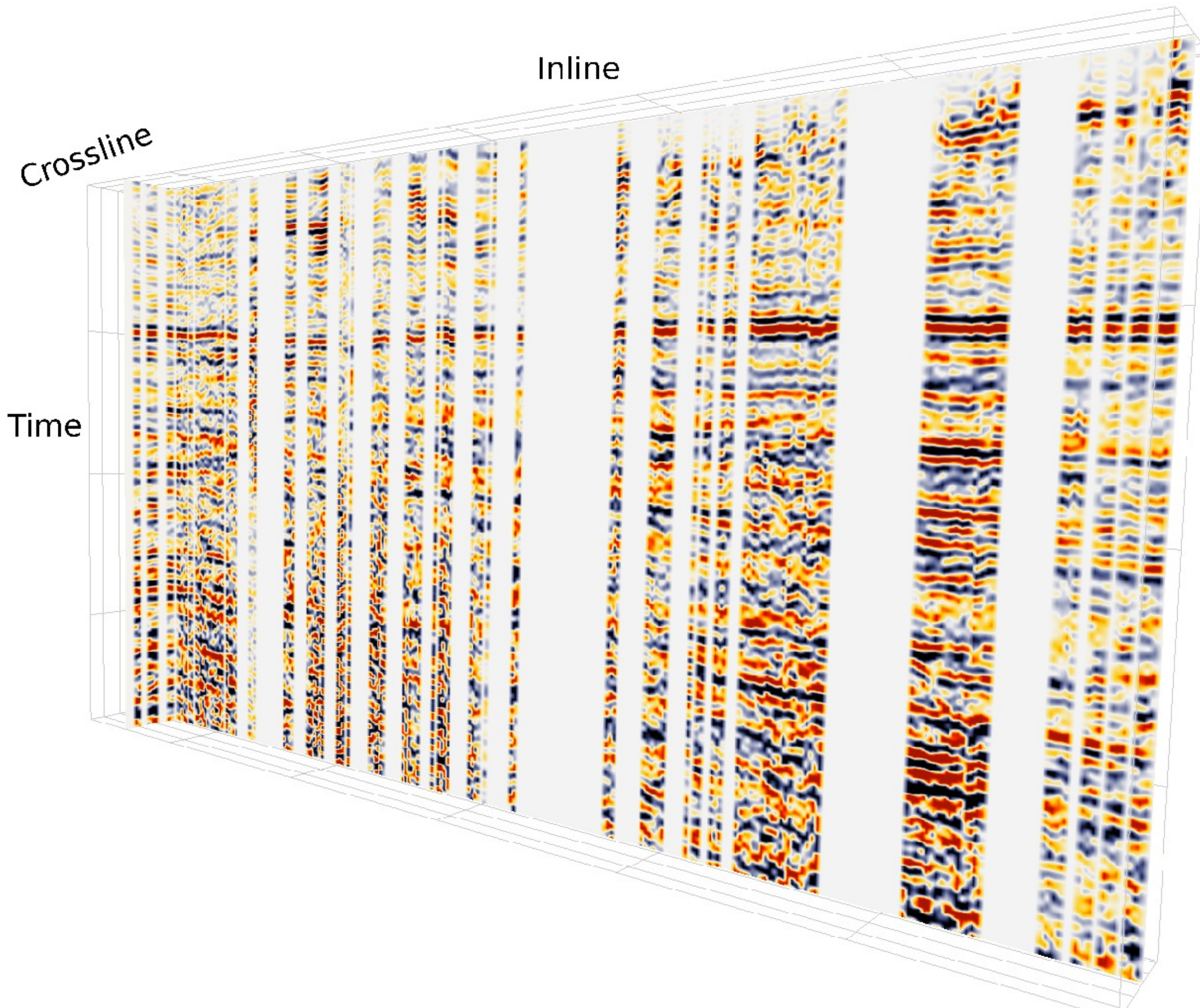
Tensor completion in azimuth offset midpoint

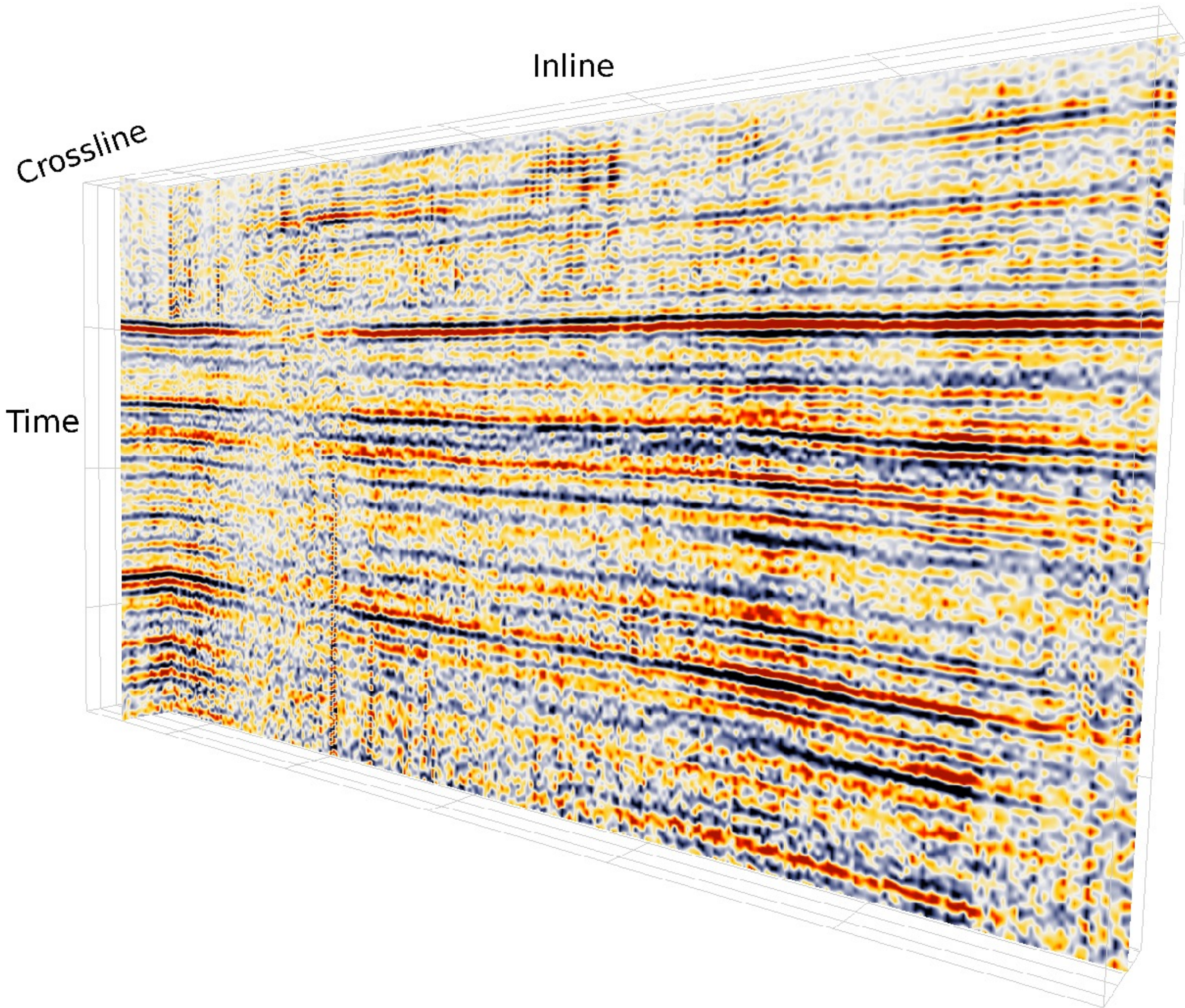


Kreimer et al., 2013

Tensor completion in azimuth offset midpoint

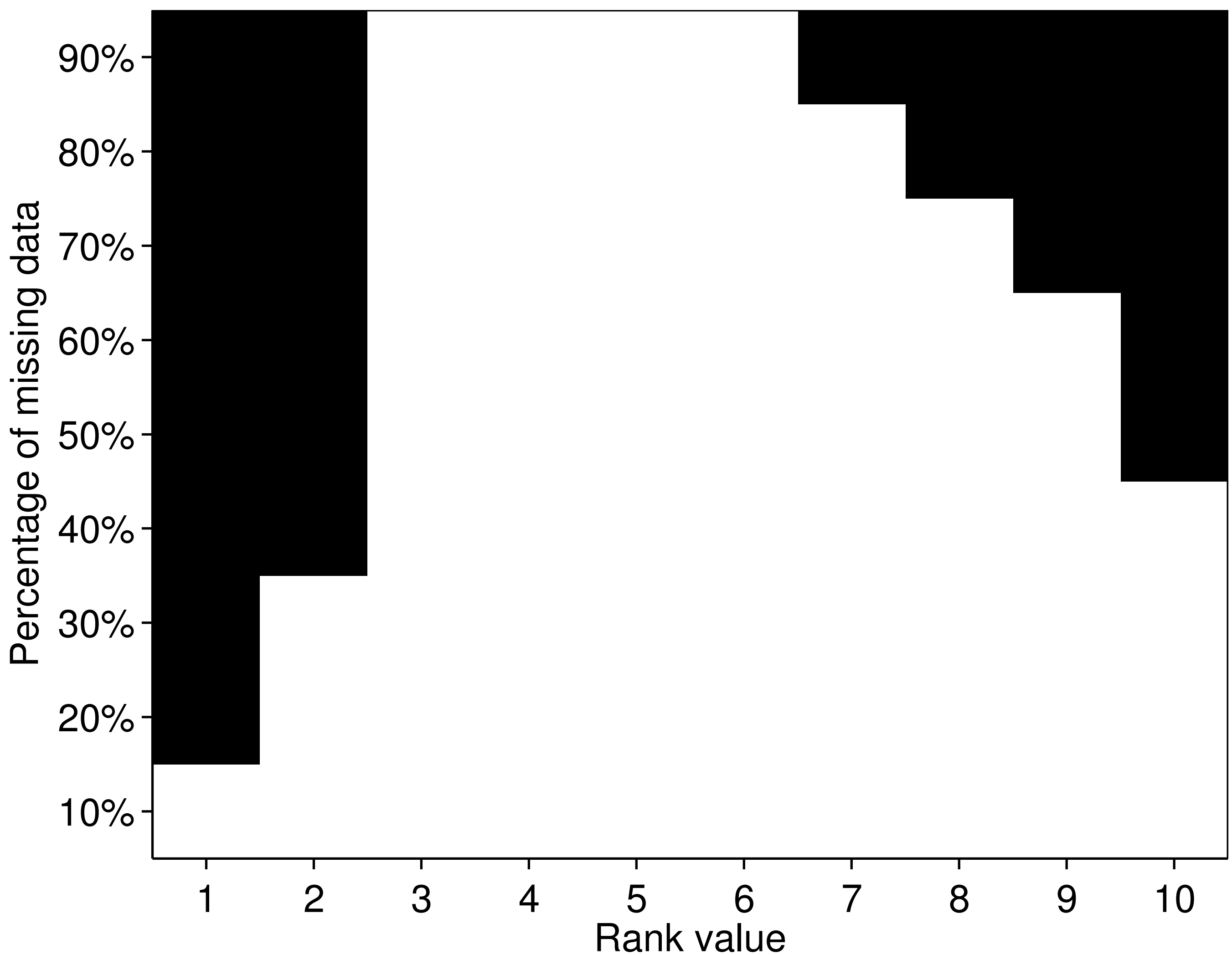



Inline
Crossline
Time


Inline
Crossline
Time

Probability of Success (PMF)

Black: recovery was unsuccessful
White: recovery was successful



References

Abma, R. and N. Kabir, 2005, Comparison of interpolation algorithms: The Leading Edge, **24**, no. 10, 984–989.

Ely, G., S. Aeron, N. Hao, and M. Kilmer, 2013, 5D and 4D pre-stack seismic data completion using tensor nuclear norm (TNN): SEG Technical Program Expanded Abstracts, 3639–3644.

Gao, J., M. Sacchi, and X. Chen, 2013, A fast reduced-rank interpolation method for prestack seismic volumes that depend on four spatial dimensions: Geophysics, **78**, no. 1, V21–V30.

Herrmann, F. J. and C. D. Silva, 2013, Structured tensor missing-trace interpolation in the hierarchical Tucker format: SEG Technical Program Expanded Abstracts, **702**, 3623–3627.

Kreimer, N. and M. D. Sacchi, 2011, Evaluation of a new 5d seismic volume reconstruction method - Tensor Completion versus Fourier reconstruction: CSPG CSEG CWLS Convention, 5 pages.

——— 2012, A tensor higher-order singular value (HO-SVD) decomposition for prestack seismic data noise reduction and interpolation: Geophysics, **77**, V113–V122.

Kreimer, N., A. Stanton, and M. D. Sacchi, 2013, Tensor completion based on nuclear norm minimization for 5D seismic data reconstruction: Geophysics, **78**, V273–V284.

Oropeza, V. and M. D. Sacchi, 2011, Simultaneous seismic data denoising and reconstruction via multichannel singular spectrum analysis: Geophysics, **76**, no. 3, V25–V32.

Trickett, S., L. Burroughs, A. Milton, L. Walton, and R. Dack, 2010, Rank-reduction-based trace interpolation: SEG, Expanded Abstracts, **29**, no. 1, 3829–3833.

Xu, Y. Y., R. R. Hao, W. T. Yin, and Z. X. Su, 2013, Parallel matrix factorization for low rank tensor completion: UCLA Computational and Applied Mathematics, 13–77.