Part 1 Preliminaries, Notation and Linear Algebra

M D Sacchi Problemas Inversos - UNLP

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Preliminaries: Notation

 $f(\mathbf{x})$: scalar function of a vector

x: a vector

x: a scalar

 α : a scalar

M: a matrix

 $M_{i,j}$: element of the matrix

J: scalar cost function

In general, we will upper case bold fonts for matrices, and lower case bold fonts for vectors.

Lower case non-bold are scalars. Operators are denoted by calligraphic fonts. All the above is true unless we indicate there is an exception.

Preliminaries: Notation

•
$$J = f(\mathbf{x})$$

•
$$\mathbf{u} = f(\mathbf{x})$$

•
$$\alpha = \mathbf{M}$$

•
$$AB = C$$

•
$$\alpha + \mathbf{M} = \mathbf{v}$$

$$\|\mathbf{v}\|_2^2 = \alpha$$

•
$$\|\mathbf{v}\|_2^2 = \mathbf{u}$$

Preliminaries: Notation

•
$$J = f(\mathbf{x})$$

ok

•
$$\mathbf{u} = f(\mathbf{x})$$

X

•
$$\alpha = \mathbf{M}$$

X

•
$$AB = C$$

ok

•
$$\alpha + \mathbf{M} = \mathbf{v}$$

X

•
$$\|\mathbf{v}\|_2^2 = \alpha$$

ok

•
$$\|\mathbf{v}\|_2^2 = \mathbf{u}$$

X

Preliminaries: ell-2 norm

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_N \end{pmatrix}$$

Real vector

$$\mathbf{u}: N \times 1 \longrightarrow \|\mathbf{u}\|_{2}^{2} = \mathbf{u}^{T}\mathbf{u} = \sum_{i=1}^{N} u_{i}^{2}$$

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_N \end{pmatrix} \qquad \mathbf{u} : N \times 1 \longrightarrow \|\mathbf{u}\|_2^2 = \mathbf{u}^T \mathbf{u} = \sum_{i=1}^N u_i^2$$

$$\bullet \quad \text{Complex vector}$$

$$\mathbf{u} : N \times 1 \longrightarrow \|\mathbf{u}\|_2^2 = \mathbf{u}^H \mathbf{u} = \sum_{i=1}^N u_i u_i^* = \sum_{i=1}^N |u_i|^2$$

The ell-2 norm

$$l_2 = \|\mathbf{u}\|_2 = \sqrt{\|\mathbf{u}\|_2^2}$$

Preliminaries: ell-2 norm

$$\|\mathbf{u}\|_{2}^{2} = \mathbf{u}^{T}\mathbf{u} = (u_{1} \quad u_{2} \quad u_{3} \quad u_{4}) \begin{pmatrix} u_{1} \\ u_{2} \\ u_{3} \\ u_{4} \end{pmatrix} = \sum_{k=1}^{4} u_{k}^{2}$$

$$\|\mathbf{u}\|_{2}^{2} = \mathbf{u}^{H}\mathbf{u} = \begin{pmatrix} u_{1}^{*} & u_{2}^{*} & u_{3}^{*} & u_{4}^{*} \end{pmatrix} \begin{pmatrix} u_{1} \\ u_{2} \\ u_{3} \\ u_{4} \end{pmatrix} = \sum_{k=1}^{4} |u_{k}|^{2}$$

A: Matrix

x : Vector

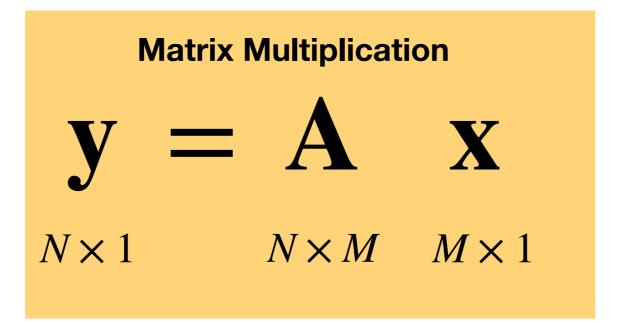
y: Vector

 α : scalar

 A_{ij} : Element of the Matrix **A**

 x_i : Element of Vector **x**

 y_i : Element of Vector **y**



$$y_i = \sum_{j=1}^{M} A_{ij} x_j \quad i = 1...N$$
Output index
Output index

Notation and Review of Linear Algebra

$$y = A x$$

Matrix-times-vector multiplication

My own code...

$$\mathbf{y} = \mathbf{A} \quad \mathbf{x}$$

$$N \times 1 \qquad N \times M \quad M \times 1$$

$$y_i = \sum_{j=1}^{M} A_{ij} x_j$$
 $i = 1...N$ $x'_j = \sum_{i=1}^{N} A_{ij} y_i$ $j = 1...M$

Forward mode

$$\mathbf{x}' = \mathbf{A}^T \quad \mathbf{y}$$

$$M \times 1 \qquad M \times N \qquad N \times 1$$

$$x'_{j} = \sum_{i=1}^{N} A_{ij} y_{i} \quad j = 1...M$$

Transpose mode

$$\mathbf{y} = \mathbf{A} \quad \mathbf{x}$$

$$N \times 1 \qquad N \times M \quad M \times 1$$

$$y_i = \sum_{j=1}^{M} A_{ij} x_j$$
 $i = 1...N$ $x'_j = \sum_{i=1}^{N} A_{ij}^* y_i$ $j = 1...M$

Forward mode

$$\mathbf{x}' = \mathbf{A}^H \mathbf{y}$$

$$M \times 1 \qquad M \times N \qquad N \times 1$$

$$x'_{j} = \sum_{i=1}^{N} A_{ij}^{*} y_{i} \quad j = 1...M$$

Conjugate mode