PSDM and LS-PSDM

UNLP Inverse Problems

Tamas Nemeth, Chengjun Wu, and Gerard T. Schuster, (1999), “Least-squares migration of incomplete reflection data," *GEOPHYSICS* 64: 208-221.

Prestack depth migration (PSDM)

- Workhorse of seismic exploration
- PSTM is Time-migration
- Easy derivation via Born and Ray-based Green functions

Modeling

$$d(t, \mathbf{r}, \mathbf{s}) = \int_{\mathbf{x}} W \,.\, m(\mathbf{x}) w(t - \tau(\mathbf{s}, \mathbf{x}) - \tau(\mathbf{x}, \mathbf{r}))\mathbf{dx}$$

Simplify $W = 1$

$$d(t, \mathbf{r}, \mathbf{s}) = \int_{\mathbf{x}} m(\mathbf{x}) w(t - \tau(\mathbf{s}, \mathbf{x}) - \tau(\mathbf{x}, \mathbf{r}))\mathbf{dx}$$

Let $m(\mathbf{x}) = \eta\delta(\mathbf{x} - \mathbf{x}_0)$

Response of scattering point of strength η

$$d(t, \mathbf{r}, \mathbf{s}) = \eta w(t - \tau(\mathbf{s}, \mathbf{x}_0) - \tau(\mathbf{x}_0, \mathbf{r}))$$

Modeling

$$d(t, \mathbf{r}, \mathbf{s}) = \int_{\mathbf{x}} m(\mathbf{x}) w(t - \tau(\mathbf{s}, \mathbf{x}) - \tau(\mathbf{x}, \mathbf{r}))\mathbf{dx}$$

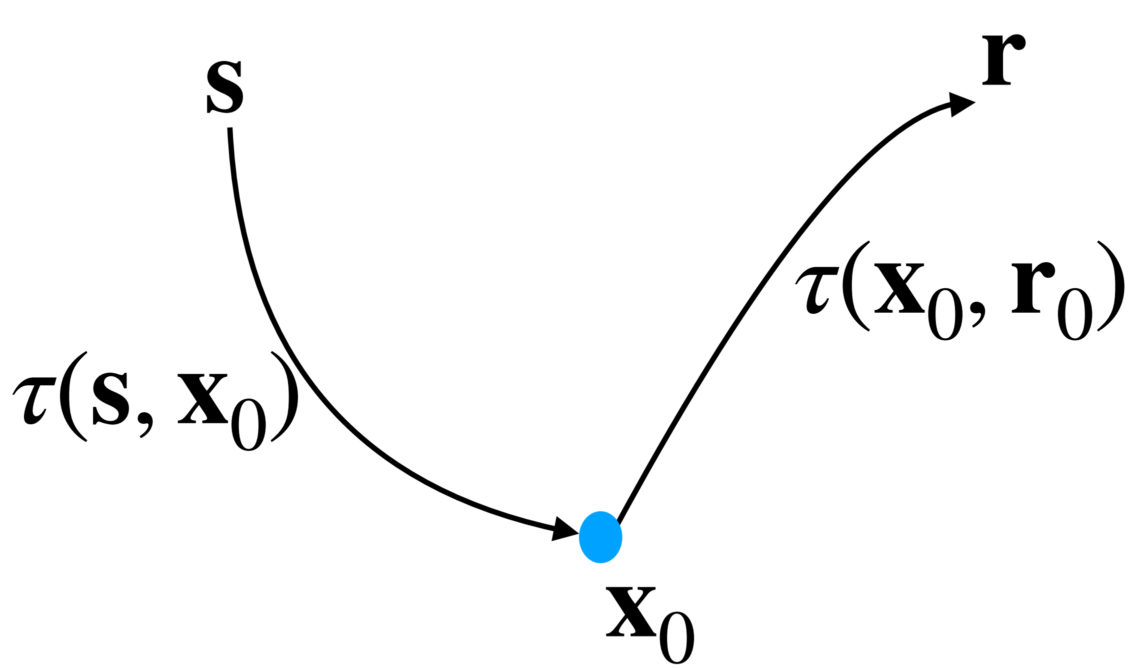
We interpret data as the sum of the response of scatterers of strength m

$$d(t, \mathbf{r}, \mathbf{s}) = \sum_i m(\mathbf{x}_i) w(t - \tau(\mathbf{s}, \mathbf{x}_i) - \tau(\mathbf{x}_i, \mathbf{r}))$$

$$\longrightarrow \quad \mathbf{d} = \mathbf{L}\mathbf{m}$$

Modeling

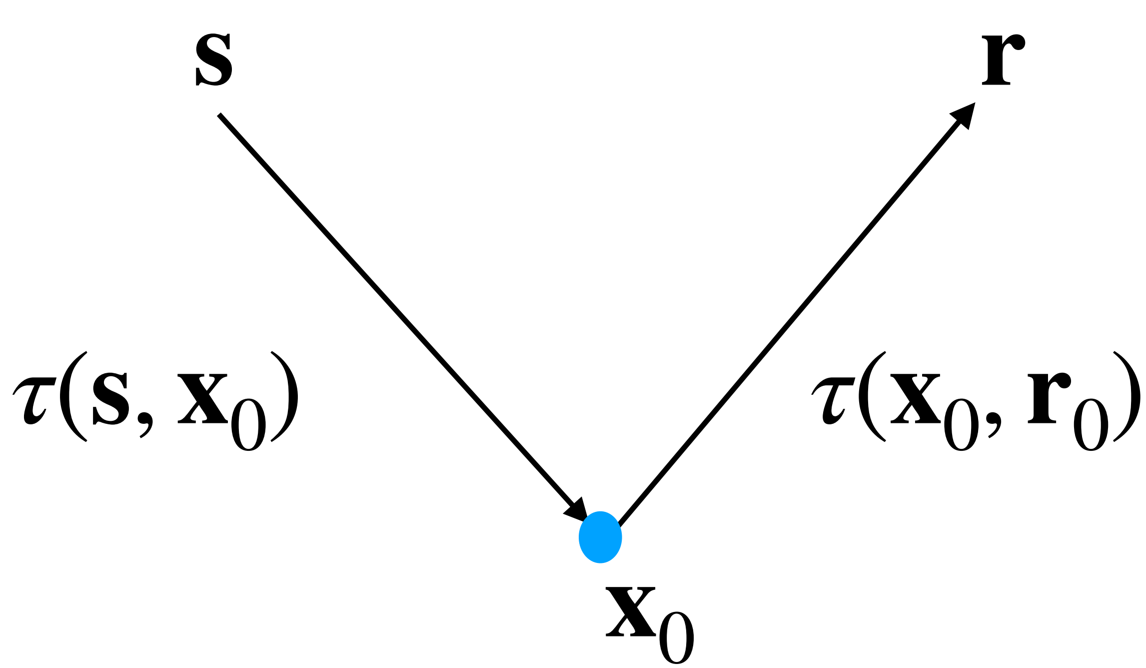
$$d(t, \mathbf{r}, \mathbf{s}) = \eta w(t - \tau(\mathbf{s}, \mathbf{x}_0) - \tau(\mathbf{x}_0, \mathbf{r}))$$



$$m(\mathbf{x}) = \eta \delta(\mathbf{x} - \mathbf{x}_0)$$

$v(\mathbf{x})$ Media velocity is variable, ray tracing is needed

Constat velocity



$$\tau(\mathbf{x}_0, \mathbf{r}_0) = \frac{|\mathbf{x}_0 - \mathbf{r}|}{v}$$

$$\tau(\mathbf{s}, \mathbf{x}_0) = \frac{|\mathbf{s} - \mathbf{x}_0|}{v}$$

$v(\mathbf{x}) = v$ Media velocity constant, rays are straight lines

Modeling

$$d(t, \mathbf{r}, \mathbf{s}) = \sum_i m(\mathbf{x}_i) w(t - \tau(\mathbf{s}, \mathbf{x}_i) - \tau(\mathbf{x}_i, \mathbf{r}))$$

If $\quad \mathbf{r} = (r, 0) \quad \mathbf{s} = (s, 0) \quad \mathbf{x} = (x, z)$

$$d(t, r, s) = \sum_{x,z} m(x, z) w(t - \frac{1}{v}\sqrt{(x-s)^2 + z^2} - \frac{1}{v}\sqrt{(x-r)^2 + z^2})$$

For time imaging $\quad t_0 = z/v$

$$d(t, r, s) = \sum_{x,t_0} m(x, t_0) w(t - \sqrt{(x-s)^2/v^2 + t_0^2} - \sqrt{(x-r)^2/v^2 + t_0^2})$$

Modeling: Post stack case

$$d(t, r, s) = \sum_{x,z} m(x, z)\, w(t - \frac{1}{v}\sqrt{(x - s)^2 + z^2} - \frac{1}{v}\sqrt{(x - r)^2 + z^2})$$

$$s = r$$

$$d(t, r) = \sum_{x,z} m(x, z)\, w(t - \sqrt{4(x - r)^2/v^2 + (2z/v)^2})$$

For time imaging $t_0 = 2z/v$

$$d(t, r) = \sum_{x,t_0} m(x, t_0)\, w(t - \sqrt{4(x - r)^2/v^2 + t_0^2})$$

Question: What is the adjoint of the modeling operator?

- Visualizing importance of loops - First code (Input driven) data is in the outer loop.

```
For trace (get r,s positions)
 For x
  For z
      time = 1/v | x-r| + 1/v |s - x|
          If forward = true,     d(time, r,s) = d(time,r,s) + m(x,z) —> Forward
          If forward = false,    ma(x,z) = ma(x,z) + d(time, r,s)  —>  Adjoint
   End
  End
End
```