An Example of Tensor-based reconstruction

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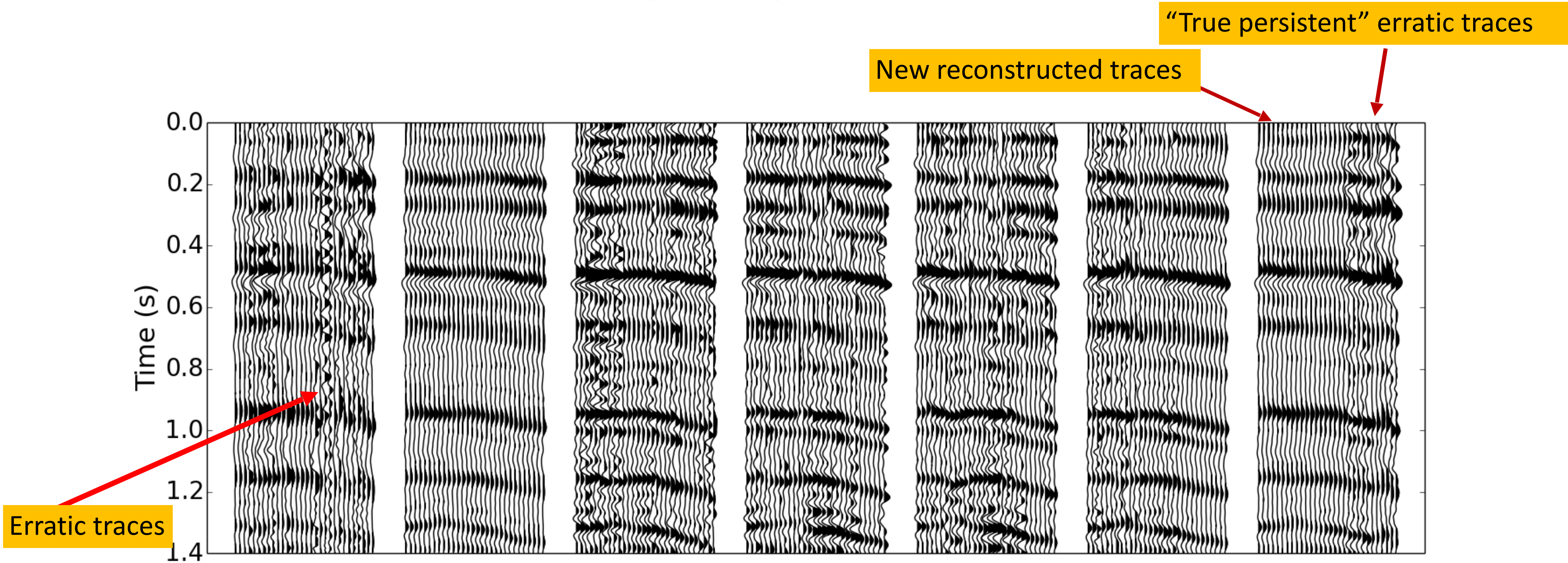
Outline

- Motivation
- Introduction
- Theory
- Examples
- Conclusions

Motivation
5D reconstruction going kind of wrong...



5D reconstruction, patch of data showing CMP-X vs CMP-y for fixed Offset-x and Offset-y. The algorithm does not forget erratic traces and results show traces of different character...

Motivation

- Erratic noise can have a negative impact on reduced-rank denoising and reconstruction methods.
- In particular, land data volumes often contain many erratic traces that negatively affect 5D seismic data reconstruction.
- We have two options:
  - We could kill <u>bad traces</u> and then reconstruct the seismic volume (a bad idea because data scarcity increases)
  - We can automatically deemphasize <u>bad traces</u> during the reconstruction process
- This presentation investigates <u>Robust Tensor Completion</u> to attenuate the influence of erratic/bad traces on 5D reconstruction.

Introduction

- Robust rank-reduction methods
  - Trickett and Burroughs and Milton, 2012, Robust rank-reduction filters for erratic noise, GeoConvention
  - Chen and Sacchi, 2014, Robust reduced-rank filtering for erratic seismic noise attenuation, Geophysics
  - Cheng, Chen and Sacchi, 2015, Robust reduced-rank filtering for erratic seismic noise attenuation, SEG
- Tensor completion
  - Kreimer and Sacchi, 2012, A tensor higher-order singular value decomposition for prestack seismic data noise reduction and interpolation, Geophysics
  - Gao, Stanton and Sacchi, 2015, Parallel matrix factorization algorithm and its application to 5D seismic reconstruction and denoising, Geophysics
  - Carozzi and Sacchi, 2018, Robust tensor-completion algorithm for 5D seismic-data reconstruction, Geophysics
  - Carozzi, Sacchi, 2020, Making seismic reconstruction more robust via a generalized loss function. SEG 2020
- Robust FX filters
  - Chen and Sacchi, 2017, Robust f-x projection filtering for simultaneous random and erratic seismic noise attenuation, Geophysical Prospecting

Theory

Tensor Completion

Prestack seismic data (after binning) can be organized into a **tensor** that depends on 5 coordinates:

Time or Frequency, Midpoint-x, Midpoint-y, Offset-x and Offset-y

$$\text{Data Tensor} : \mathcal{D} \equiv D(i_\omega, i_1, i_2, i_3, i_4)$$

For fixed frequency we have a 4th order tensor in space

Tensor Completion

Minimize cost function

$$J = \|\mathcal{T}\mathcal{Z} - \mathcal{D}\|_2^2 + \mu\|\mathcal{Z} - \mathcal{Z}_0\|_2^2$$

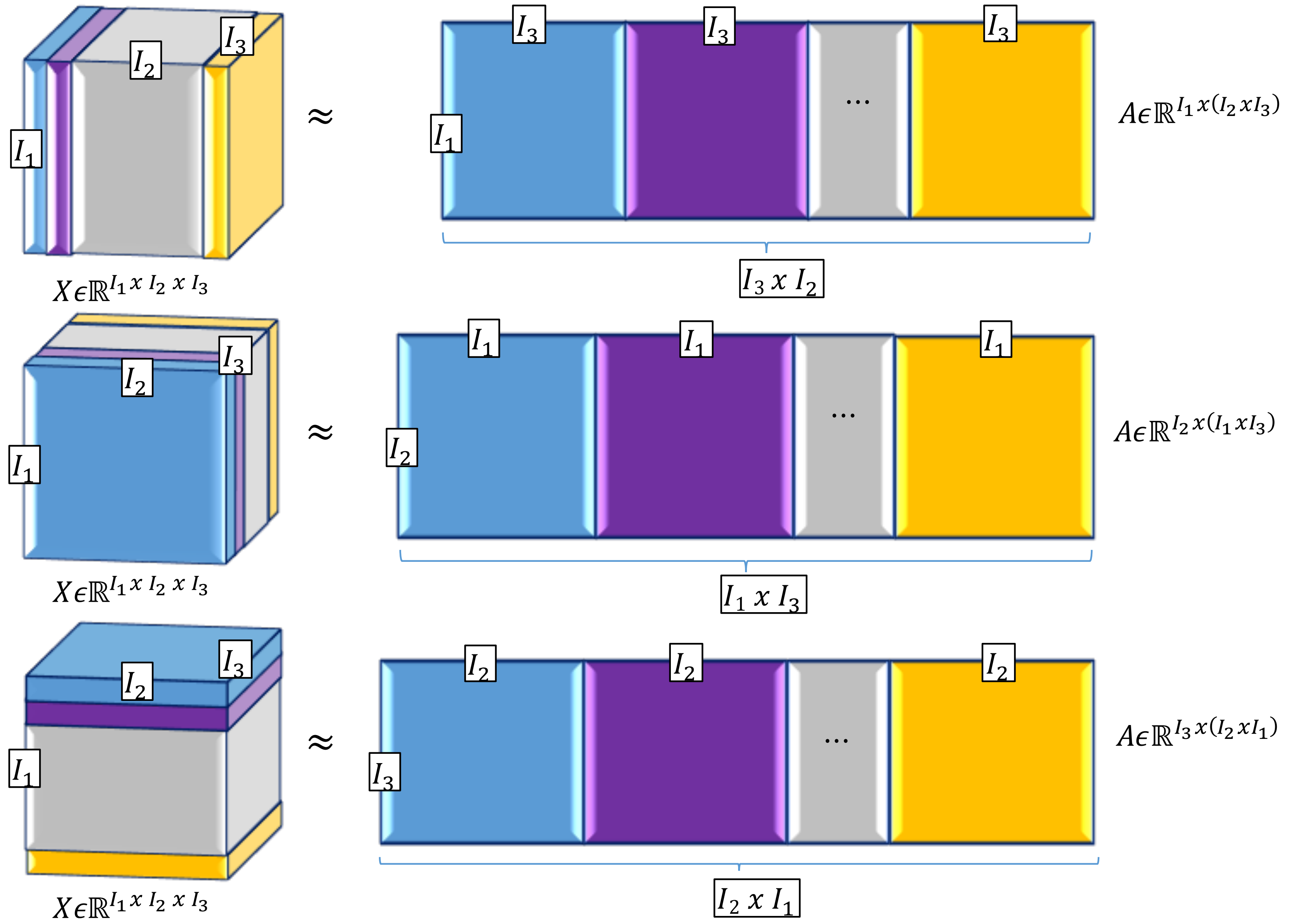
$\mathcal{D}$: Observed and incomplete data

$\mathcal{Z}$: Desired data

$\mathcal{T}$: Sampling Operator

$\mathcal{Z}_0$: Low-rank Tensor approximation

Unfold →



← fold

3rd order tensor can be unfolded into into 3 matrices.

PMF (Parallel Matrix Factorization)

- Alternating minimization with respect to all variables leads to iterative solver

$$
\begin{cases}
\mathcal{Z}^k = \alpha\mathcal{D} + (1 \underset{4}{-} \alpha\mathcal{T})\mathcal{Z}_0^{k-1} \\
\mathcal{Z}_0^{k-1} = (1/4)\sum_{k=1} \texttt{Fold}\left[\mathcal{R}[\texttt{Unfold}[\mathcal{Z}^{k-1}]]\right]
\end{cases}
$$

Rank Reduction

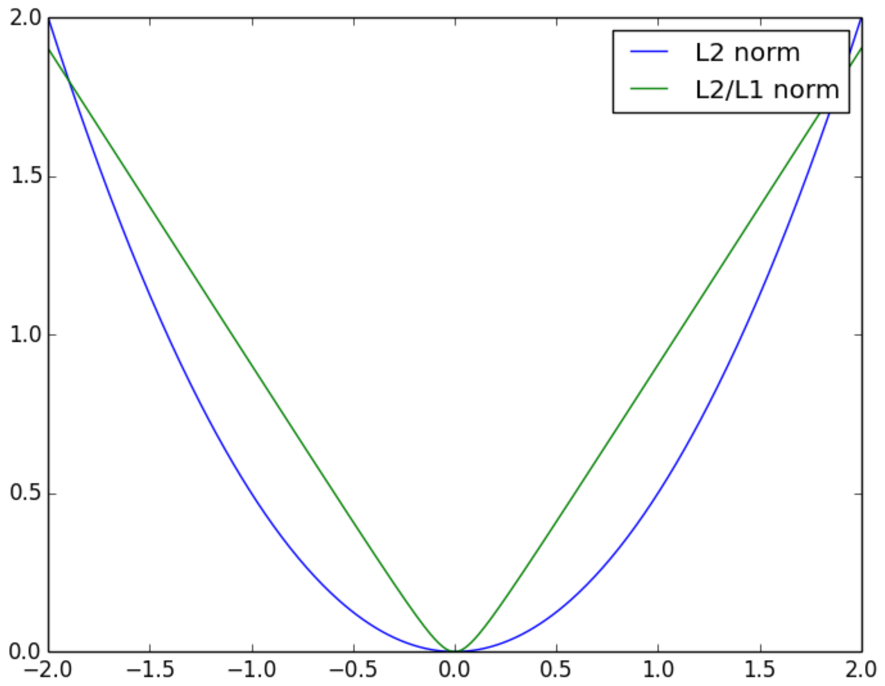
Robust PMF

$$J = \|\mathcal{T}\mathcal{Z} - \mathcal{D}\|_r + \mu\|\mathcal{Z} - \mathcal{Z}_0\|_2^2$$

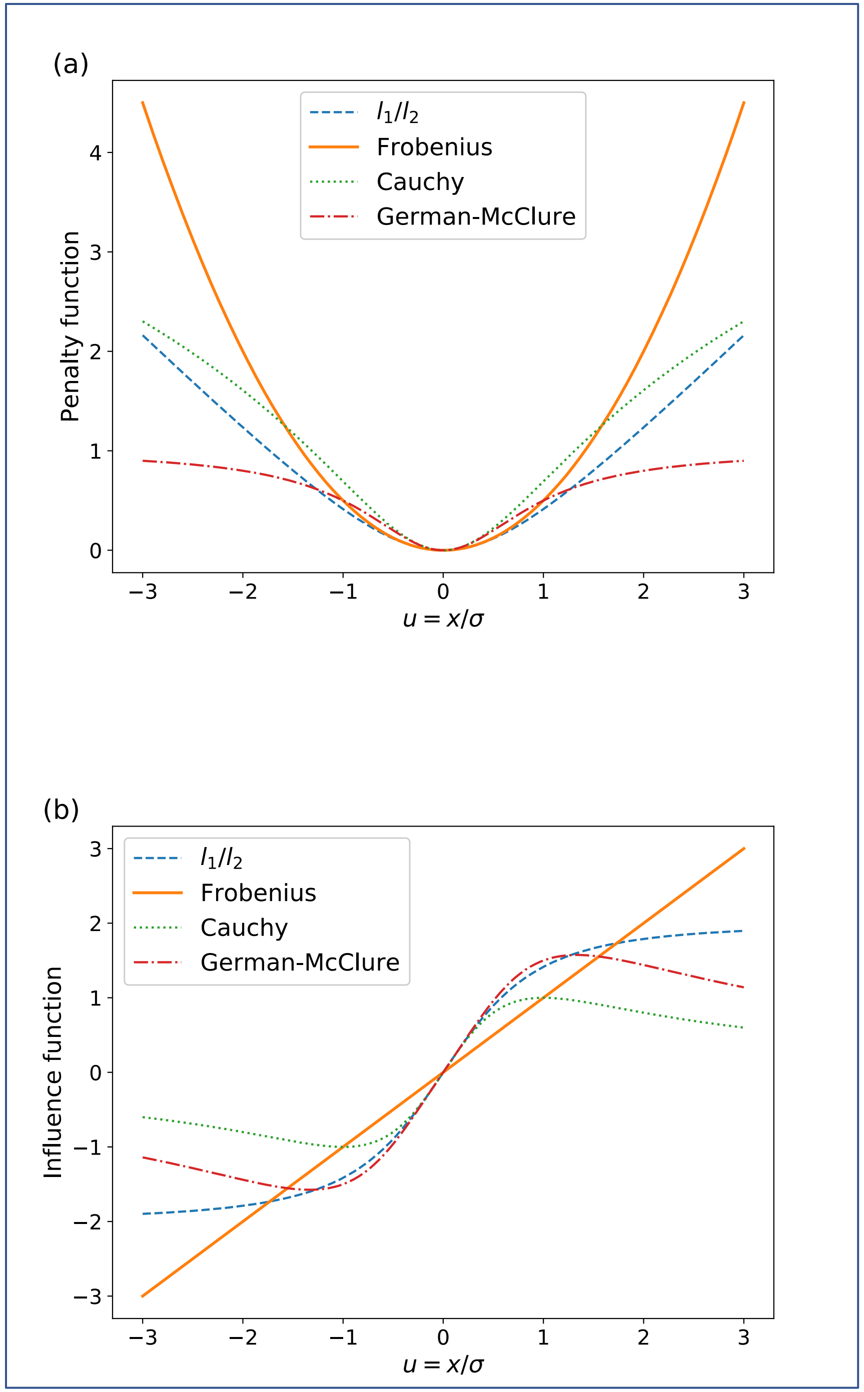
PMF can be modified via a Robust *l1/l2* norm to decrease influence of erratic noise

$$\|x\|_r = \sum_i \sqrt{|x_i|^2 + \epsilon^2}$$

The main idea is to deform the *l2* norm to obtain a new norm that attenuates outliers (Maronna et al. Robust Statistics: Theory and Methods)



Many options for robust norms…



Robust PMF

- Minimization with respect to all variables leads to an iterative solver with weighs

$$\begin{cases} \mathcal{Z}^k = \mathcal{A}^{k-1}\mathcal{D} + (1 - \mathcal{A}^{k-1}\mathcal{T})\mathcal{Z}_0^{k-1} \\ \mathcal{Z}_0^{k-1} = (1/4)\sum_{k=1}^{4} \texttt{Fold}\left[\mathcal{R}[\texttt{Unfold}[\mathcal{Z}^{k-1}]]\right] \end{cases}$$

Robust PMF

- Non-robust (*L2*) (Scalar weight)

$$\mathcal{Z}^k = \alpha\mathcal{D} + (1 - \alpha\mathcal{T})\mathcal{Z}_0^{k-1}$$

- Robust (*L2/L1*)

$$\mathcal{Z}^k = \mathcal{A}^{k-1}\mathcal{D} + (1 - \mathcal{A}^{k-1}\mathcal{T})\mathcal{Z}_0^{k-1}$$

Estimator	Misfit Criterion	$\rho(u)$	Weights $\mathcal{A}(u)$
Non-robust	l_2	$\|u\|^2$	$\frac{1}{1+N\mu\sigma^2}$
Robust	l_1/l_2	$\sqrt{1+\|u\|^2}$	$\frac{1}{1+N\mu\sigma^2\sqrt{1+\|u\|^2}}$
	Cauchy	$ln(1+\|u\|^2)$	$\frac{1}{1+N\mu\sigma^2(1+\|u\|^2)}$
	Geman-McClure	$\frac{\|u\|^2}{1+\|u\|^2}$	$\frac{1}{1+N\mu\sigma^2(1+\|u\|^2)^2}$

Weights de-emphasize large residuals

Examples

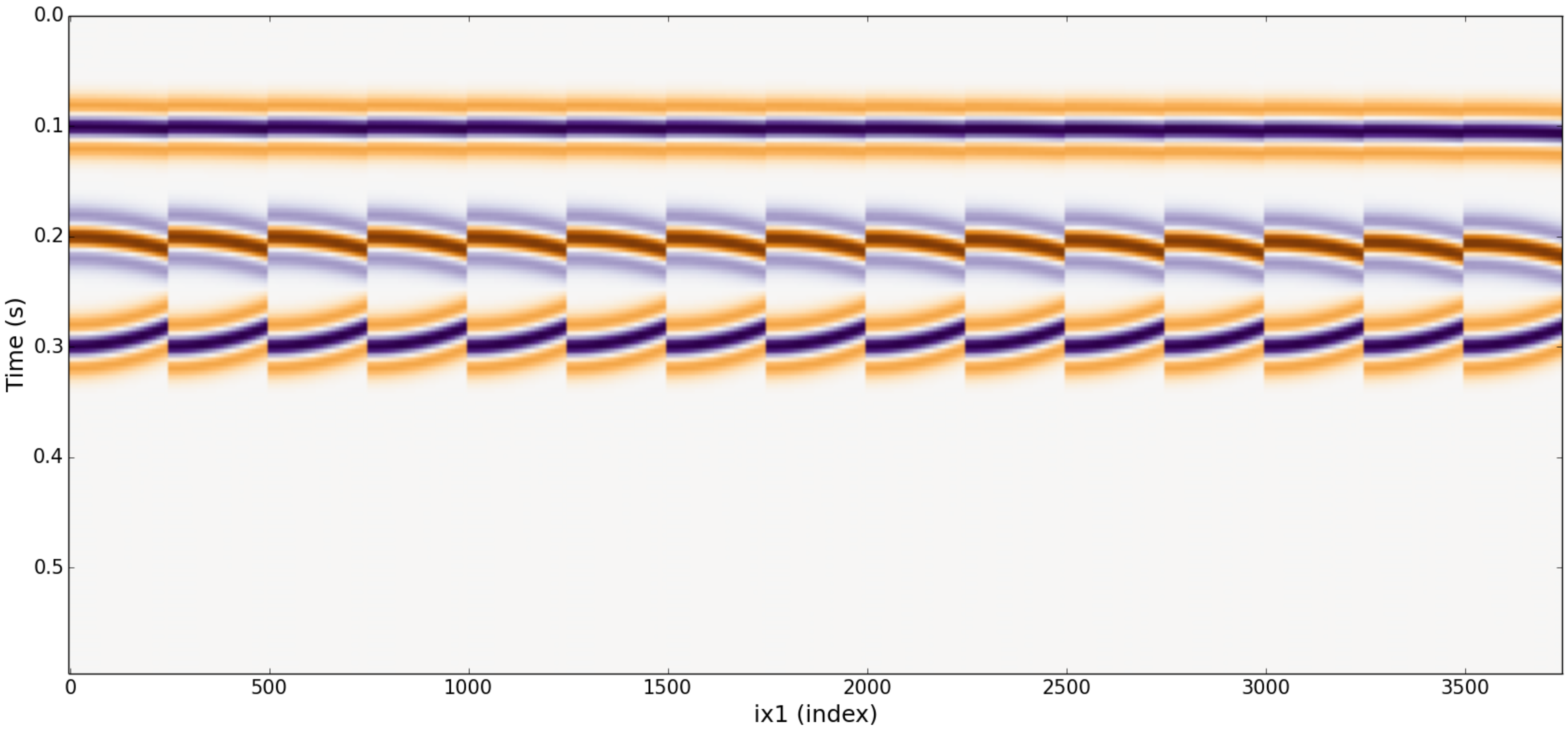
Synthetic experiment 1

- D(1:125, 1:25, 1:15, 1:8, 1:8)
- N_CMP_x = 25
- N_CMP_y = 15
- N_offset_x = 8
- N_offset_y = 8
- f_min=1Hz
- f_max=80Hz
- Decimation = 50% (Random)
- Noise = Gaussian + Erratic

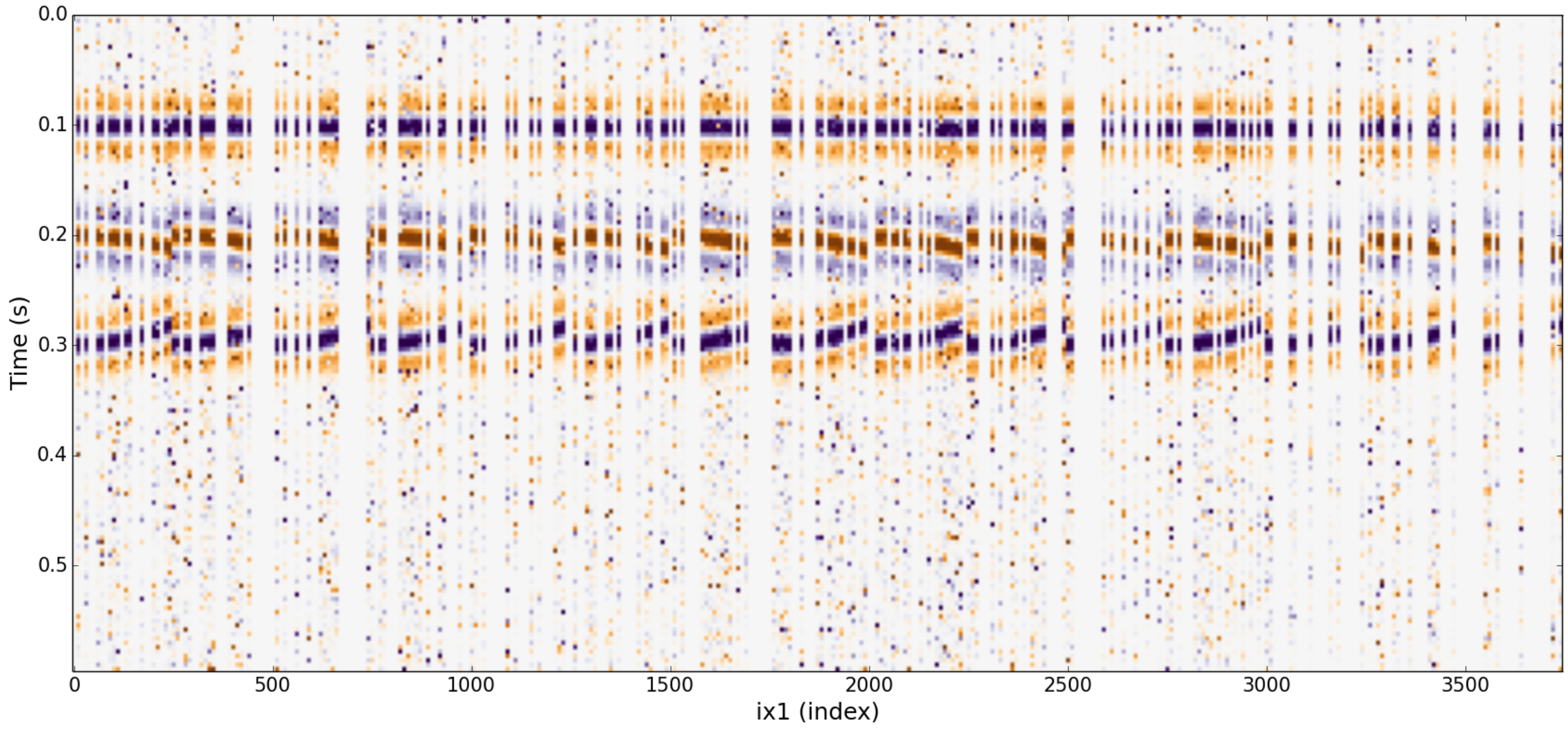
Total number of traces in Patch: **24000**

Traces alive: **12047**

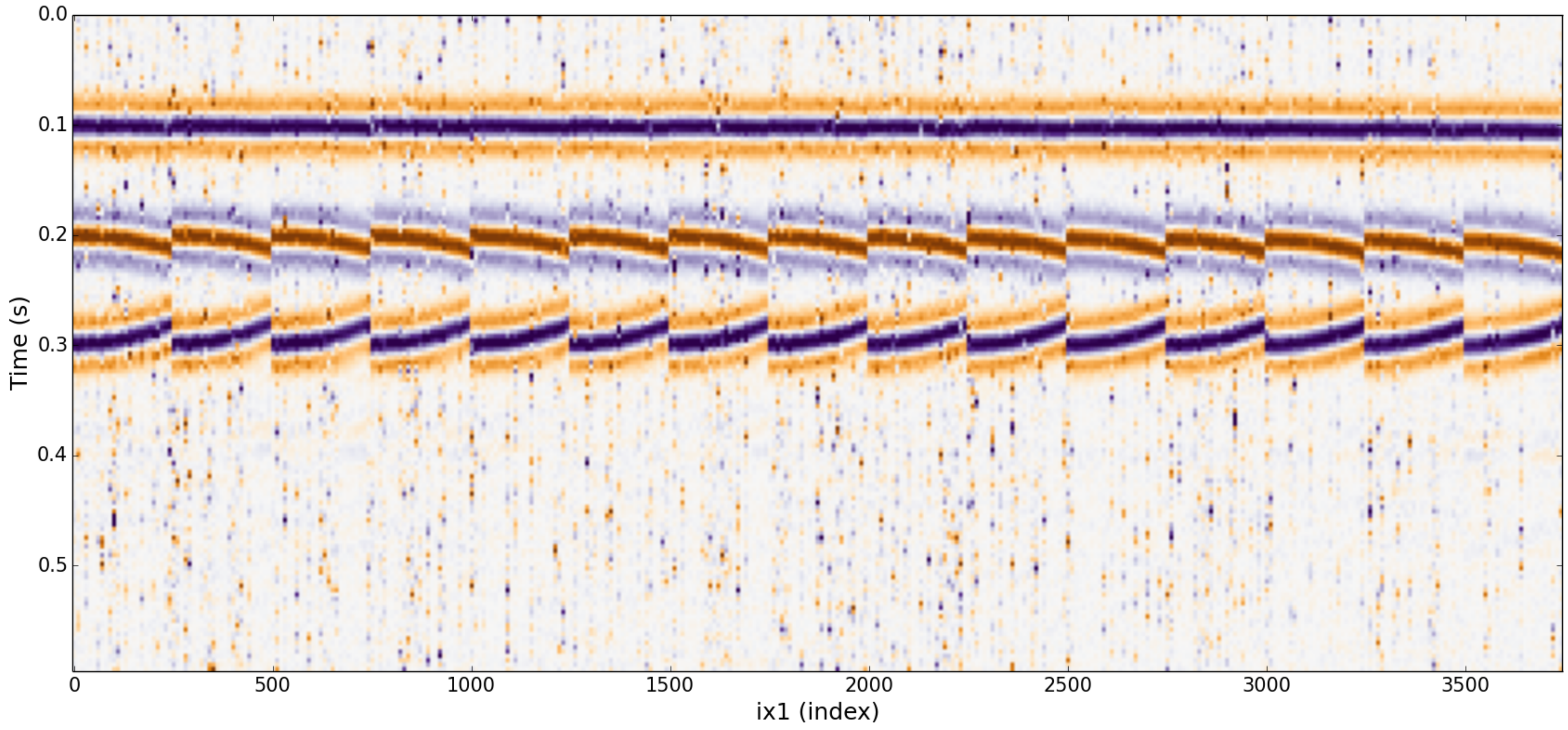
Patch of data D(1:125, 1:25, 1:15, 2, 2)



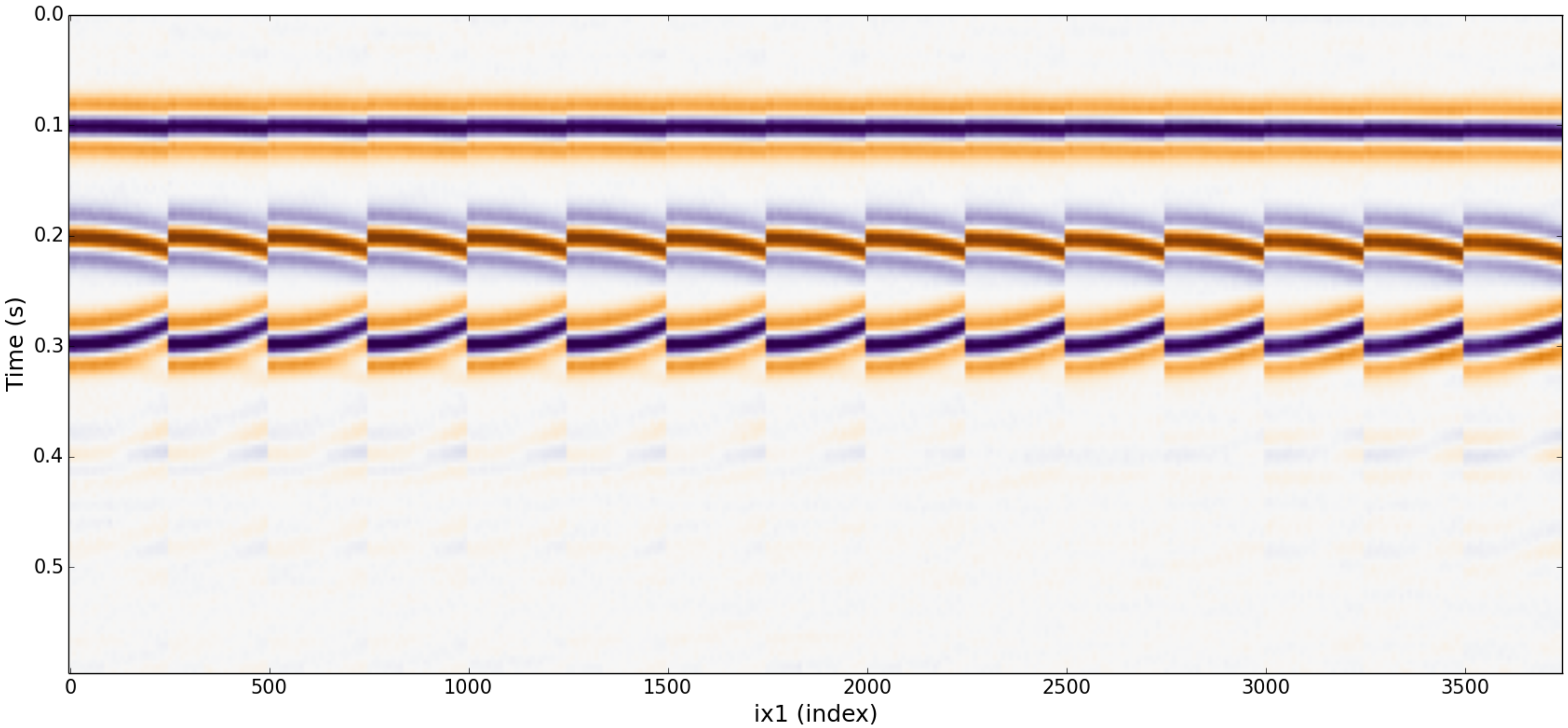
Data decimated & contaminated with erratic and Gaussian noise

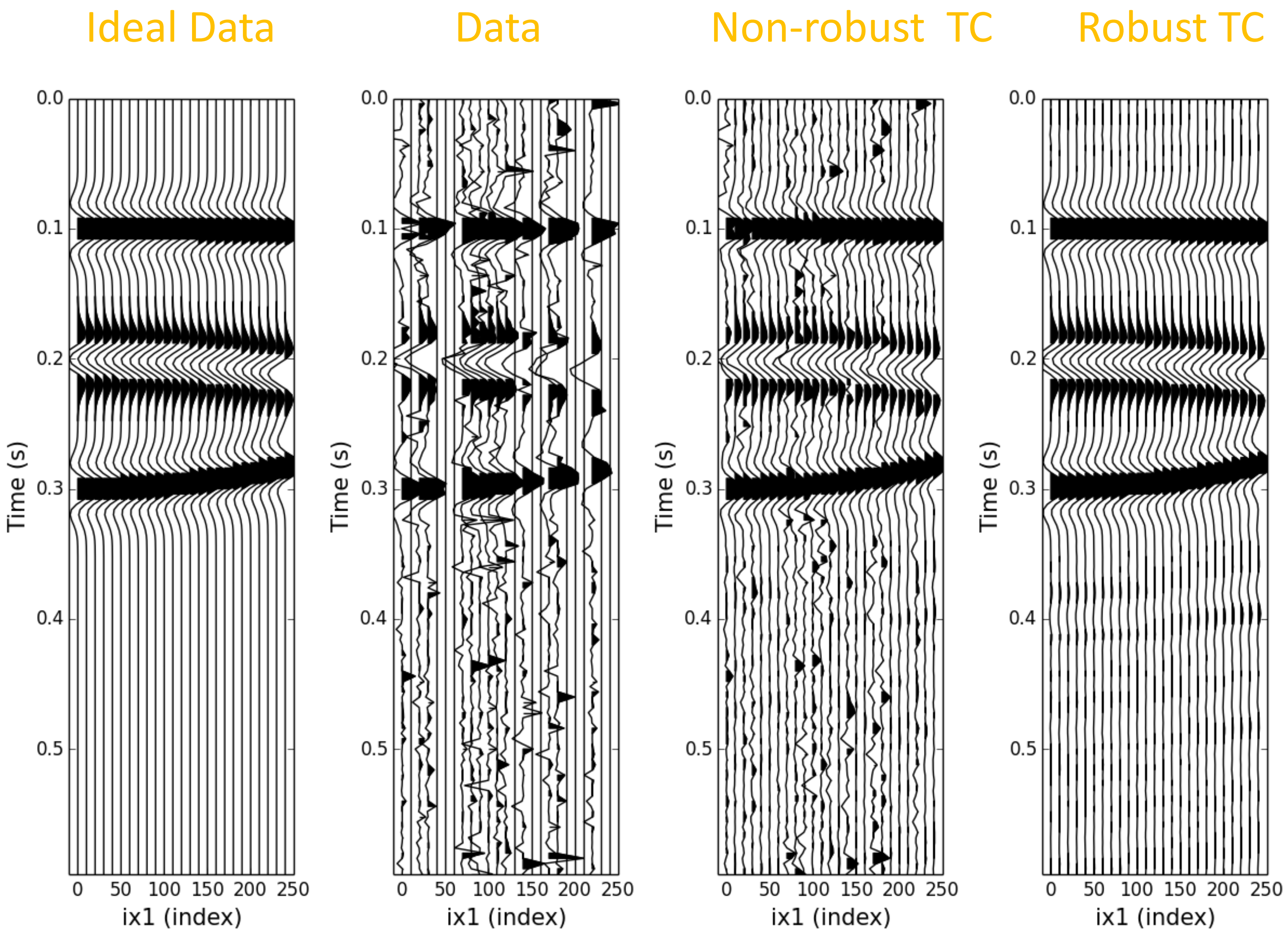


Reconstructed via L_2 PMF (non-robust completion)



Reconstructed data via Robust PMF





Ideal Data
Data
Non-robust TC
Robust TC
Time (s)
0.0
0.1
0.2
0.3
0.4
0.5
0 50 100 150 200 250
ix1 (index)

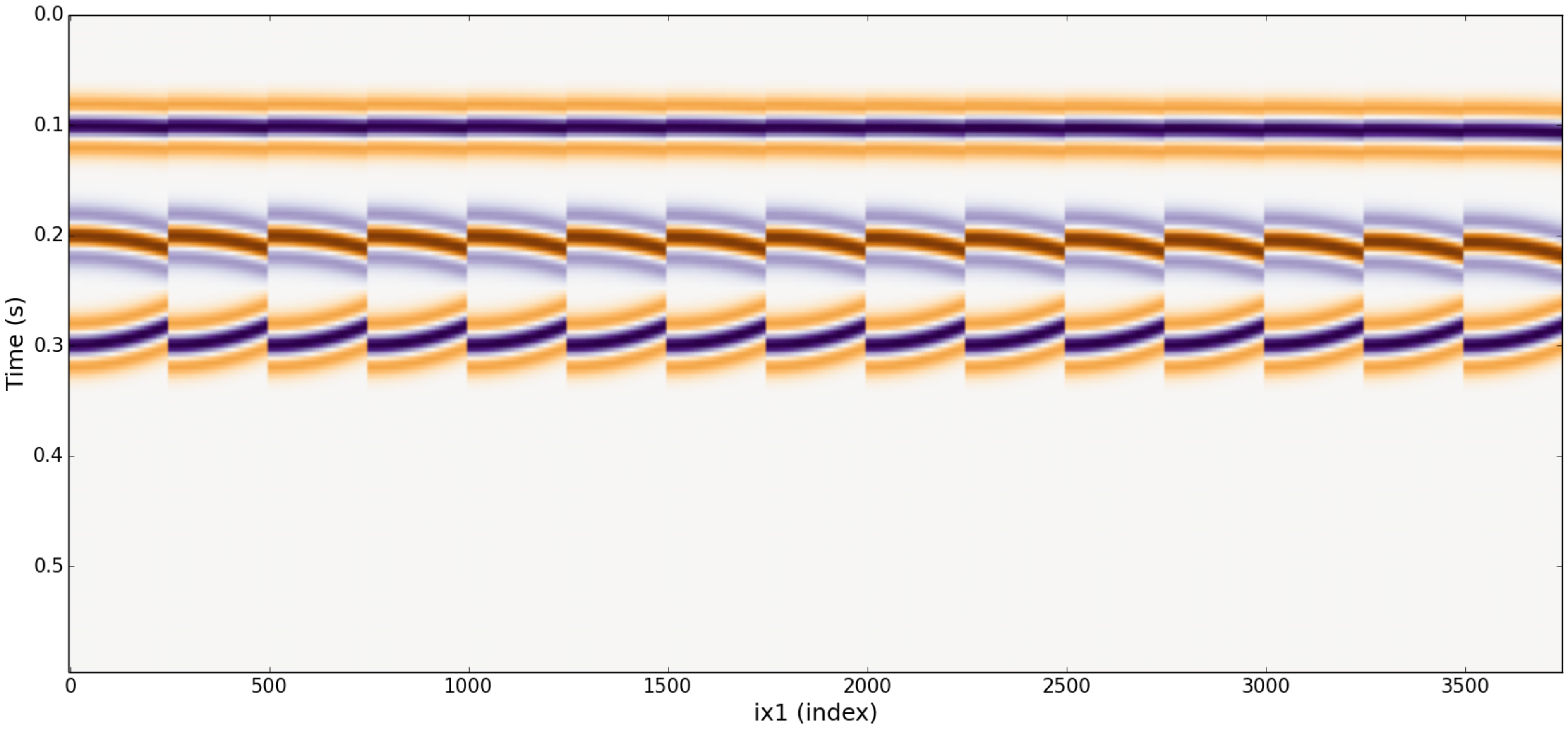
Synthetic experiment 2

- D(1:125, 1:25, 1:15, 1:8, 1:8)
- N_CMP_x = 25
- N_CMP_y = 15
- N_offset_x = 8
- N_offset_y = 8
- f_min=1Hz
- f_max=80Hz
- Decimation = 90% (Random)
- Noise = Gaussian + Erratic

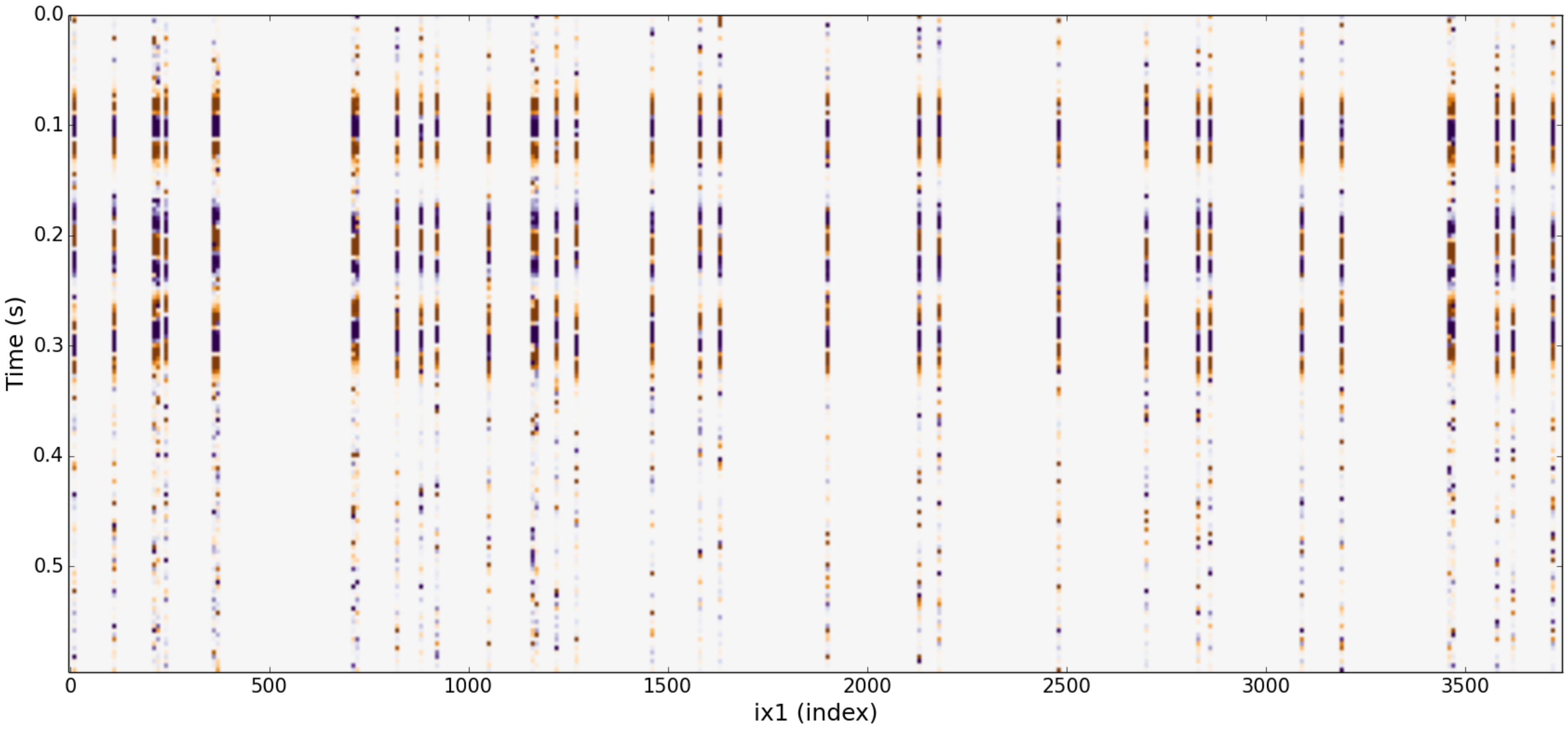
Total number of traces in Patch**: 24000**

Traces alive**: 2398**

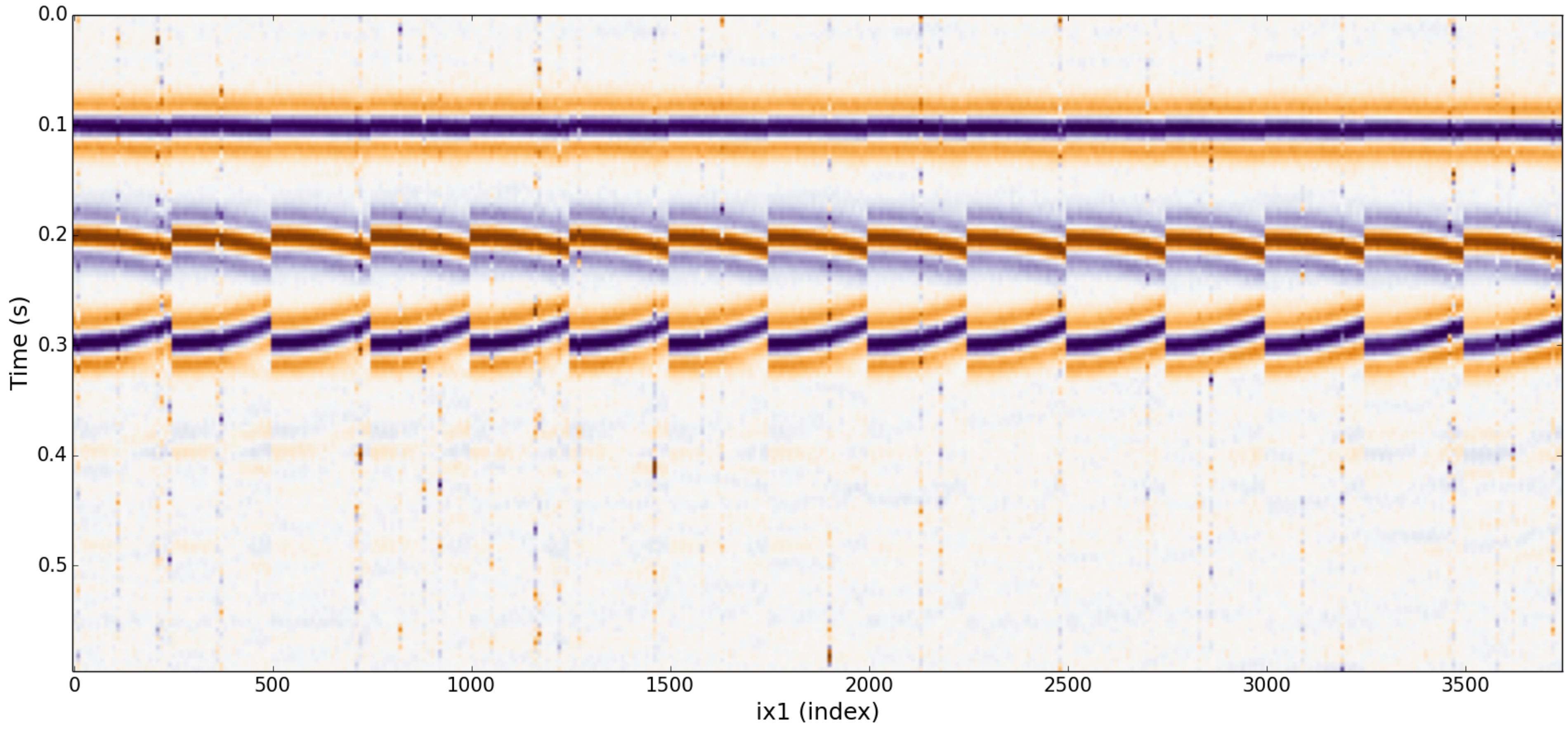
Patch of data D(1:125, 1:25, 1:15, 2, 2)



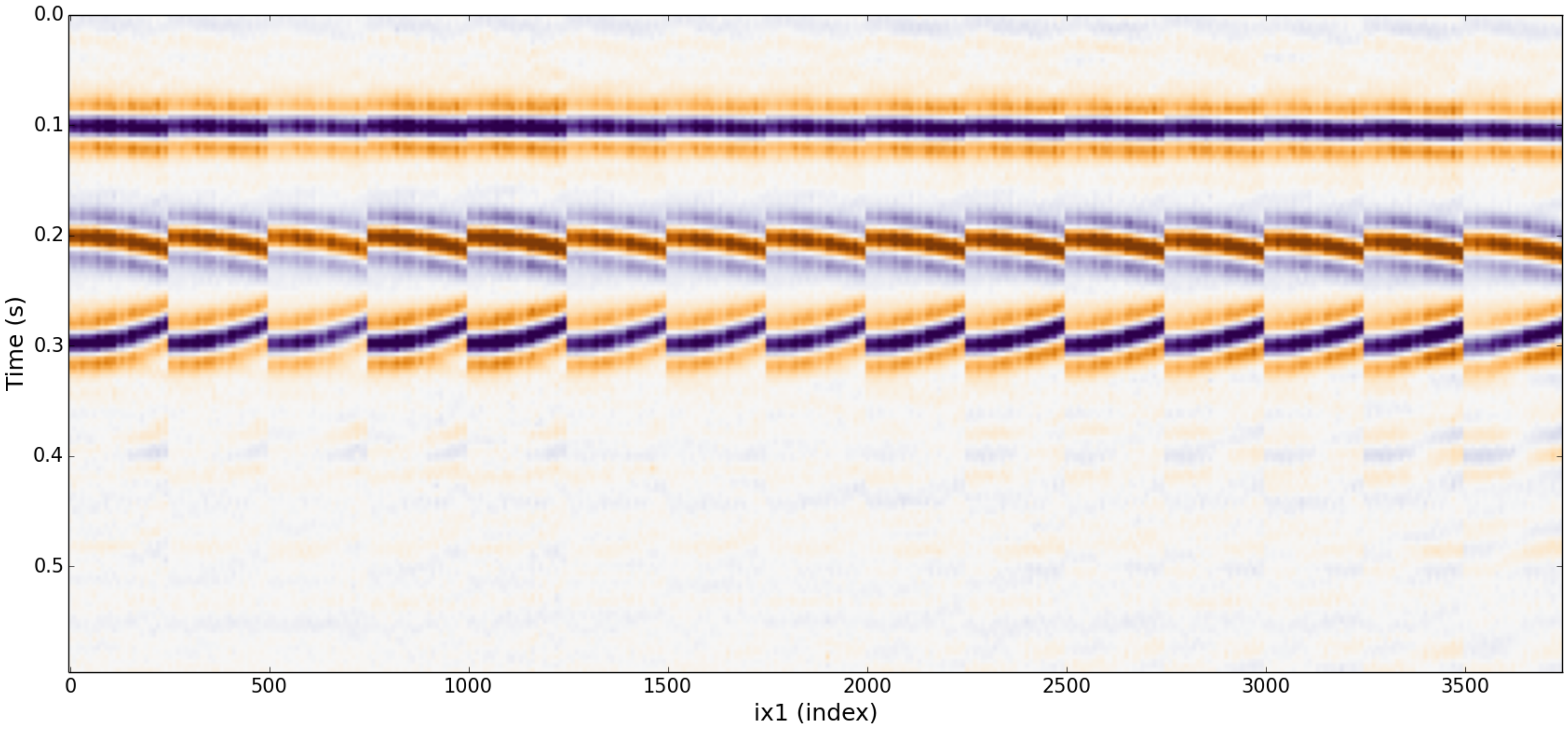
Data decimated contaminated with erratic and Gaussian noise

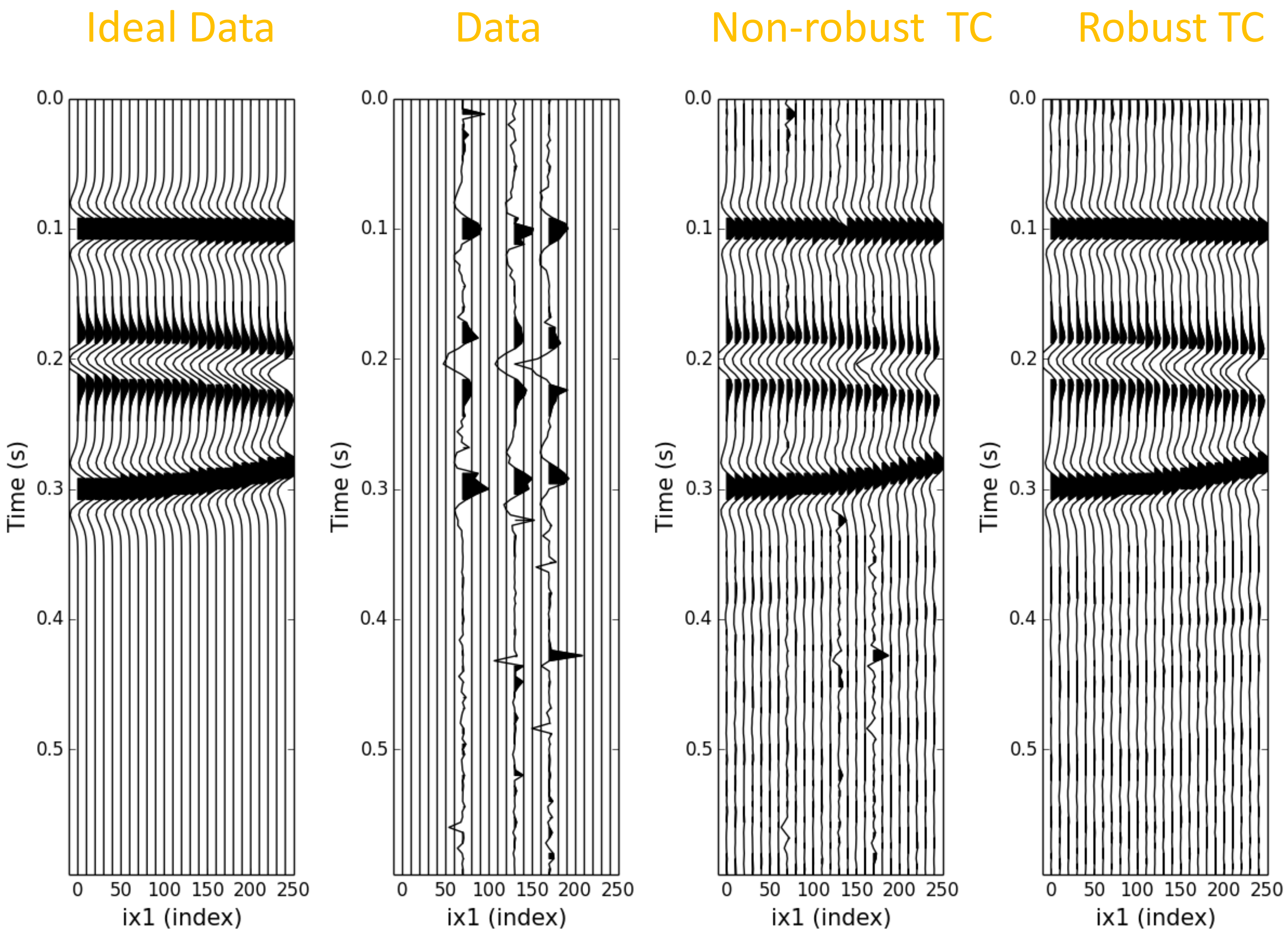


Reconstructed via L_2 PMF (non-robust completion)



Reconstructed data with Robust PMF

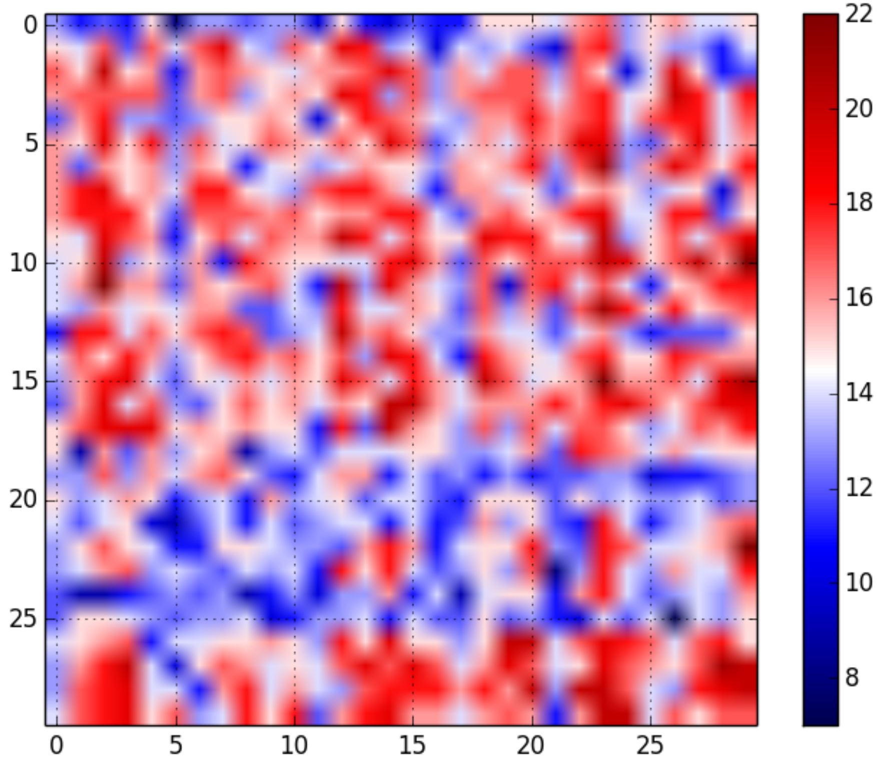



Ideal Data
Data
Non-robust TC
Robust TC
0.0
0.1
0.2
0.3
0.4
0.5
Time (s)
0 50 100 150 200 250
ix1 (index)

Field data example

Patch of data for 5D reconstruction

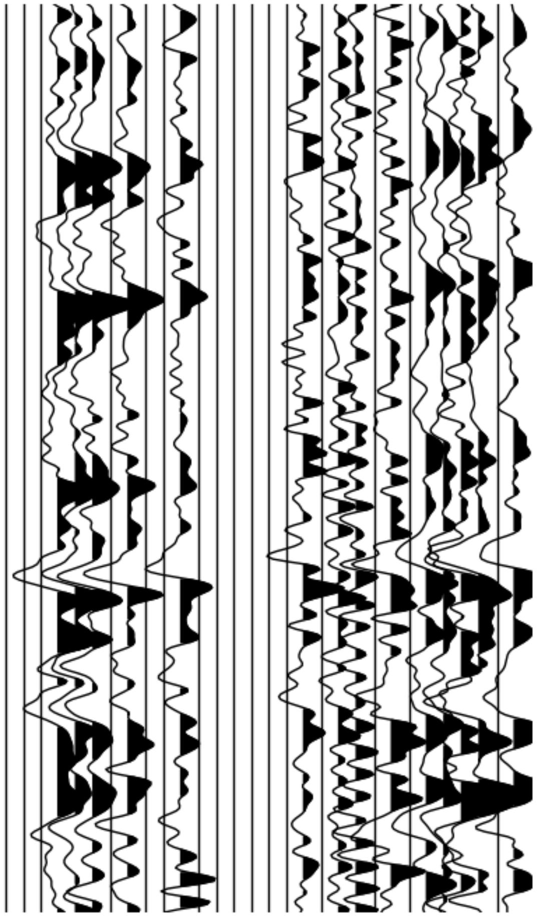
- Total number of traces in Patch: **37800 (1 of 1000s of patches)**
- Traces alive: **13554**
    - N_CMP_x = 30
    - N_CMP_y = 30
    - N_offset_x = 7
    - N_offset_y = 6
    - f_min=1Hz
    - f_max=180Hz
    - Decimation = 70%
    - Noise = ??



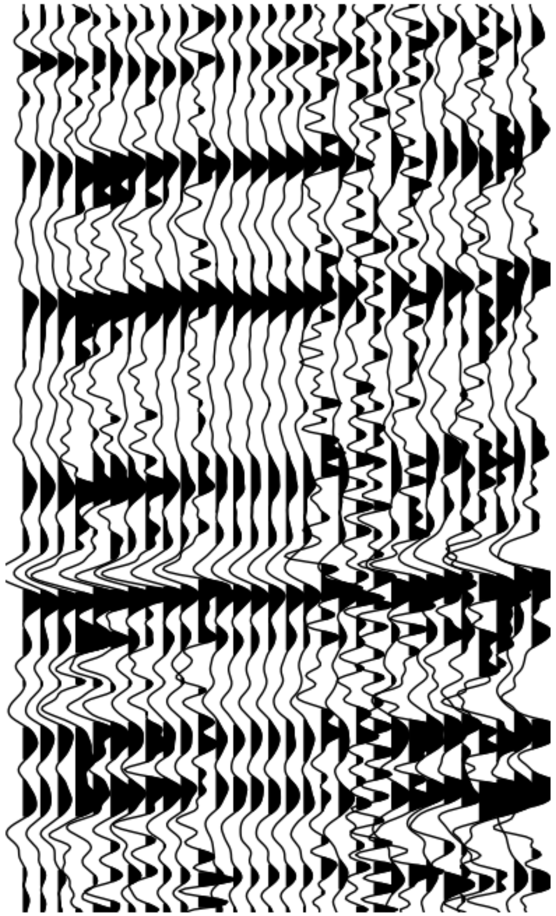
Fold of the Patch

Inline D[:,4,:,2,2]

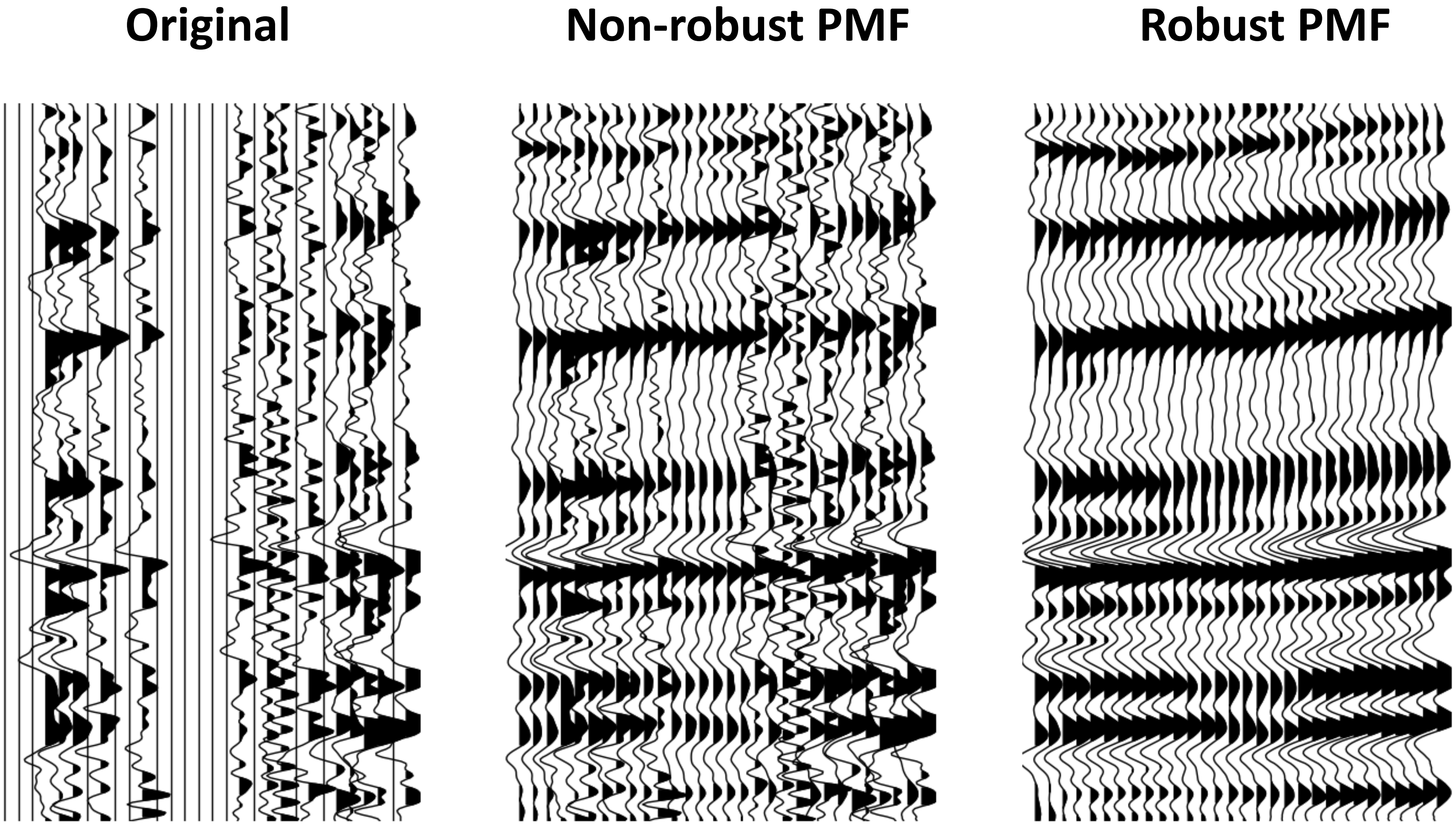
Original



Non-robust PMF

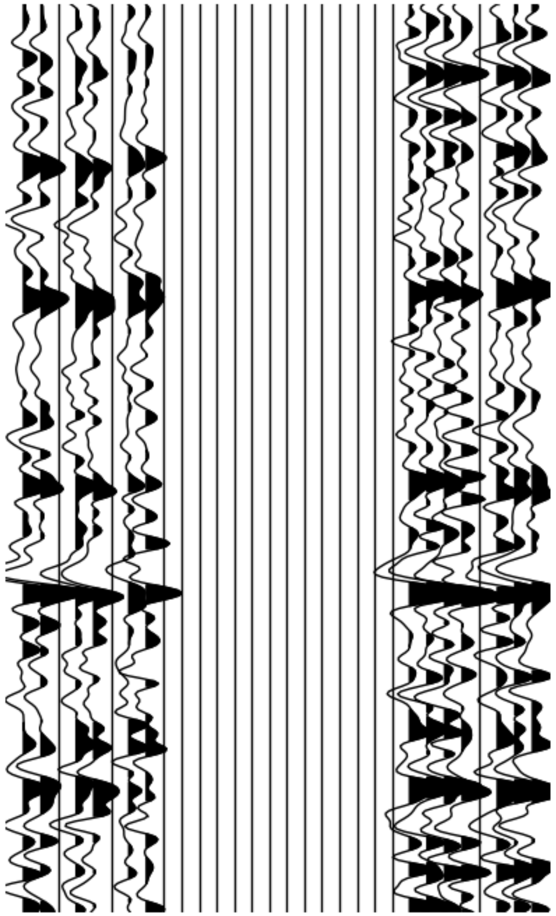


Inline D[:,4,:,2,2]

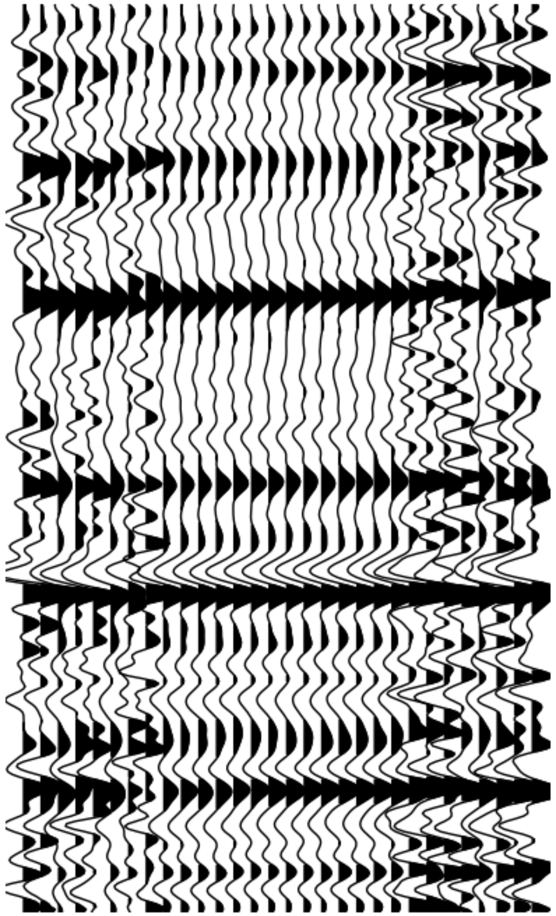


X-line D[:,:,8,2,2]

Original

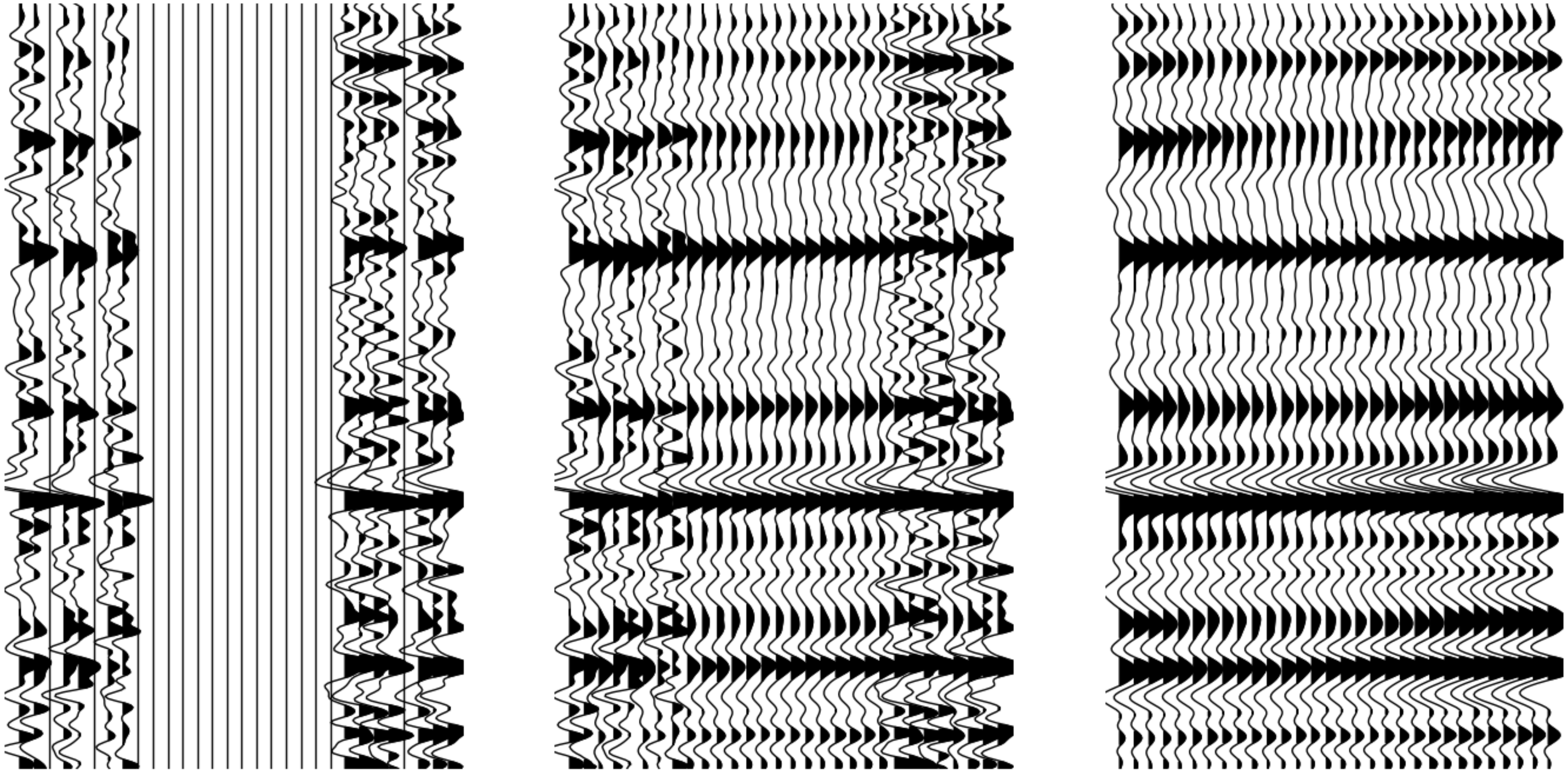


Non-robust PMF

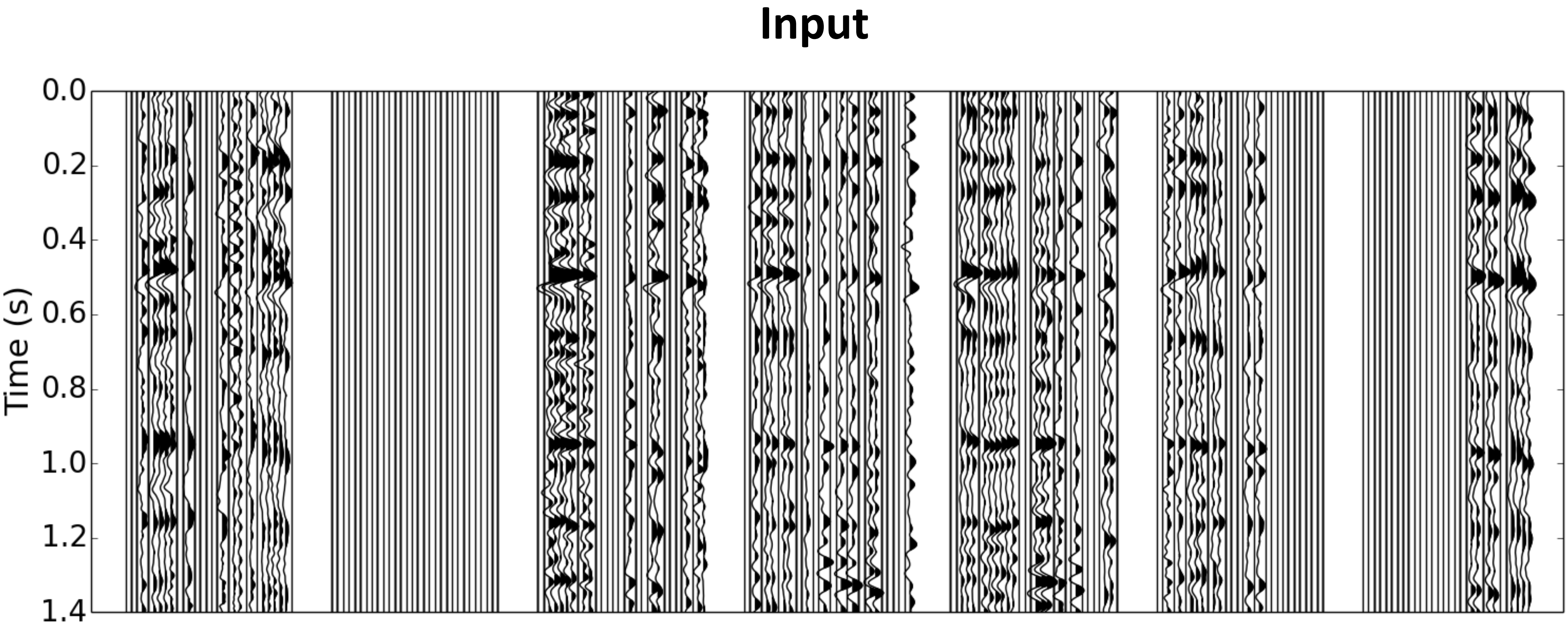


X-line D[:,:,8,2,2]

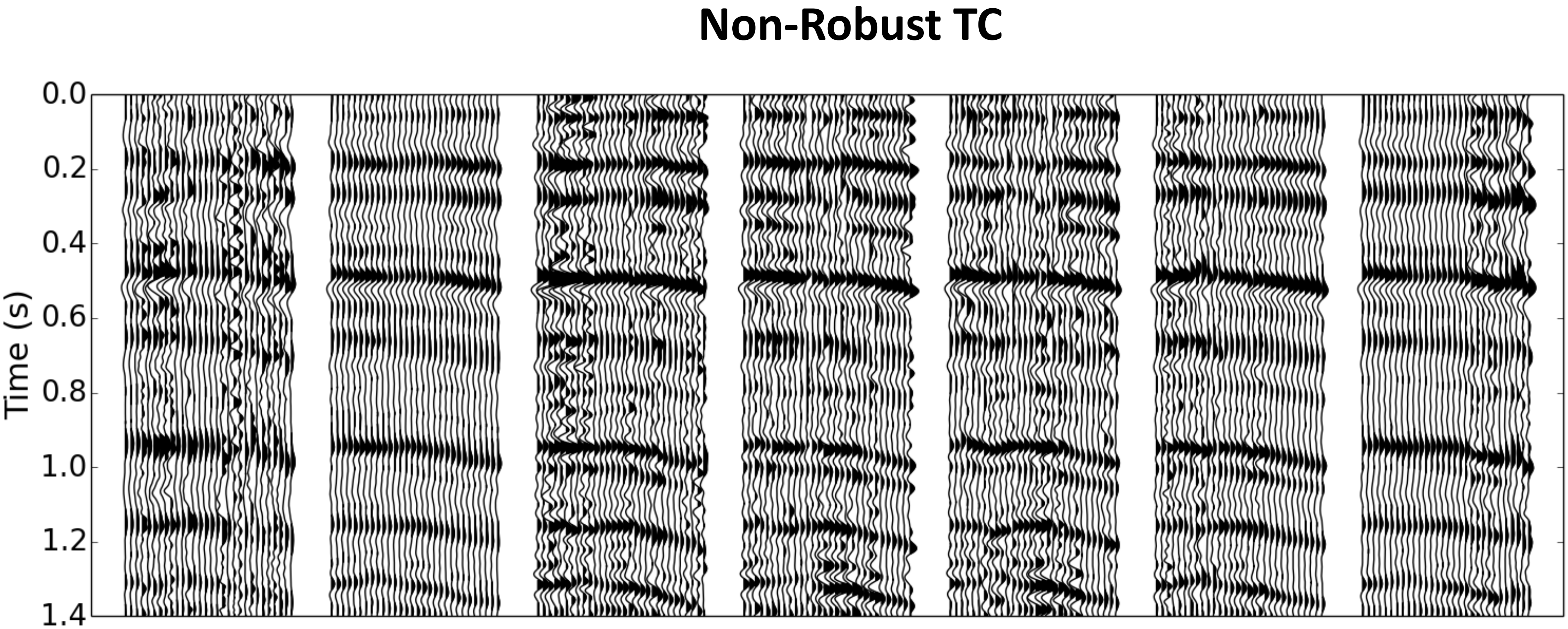
Original **Non-robust PMF** **Robust PMF**



In-line D[:,14,:,:,2]

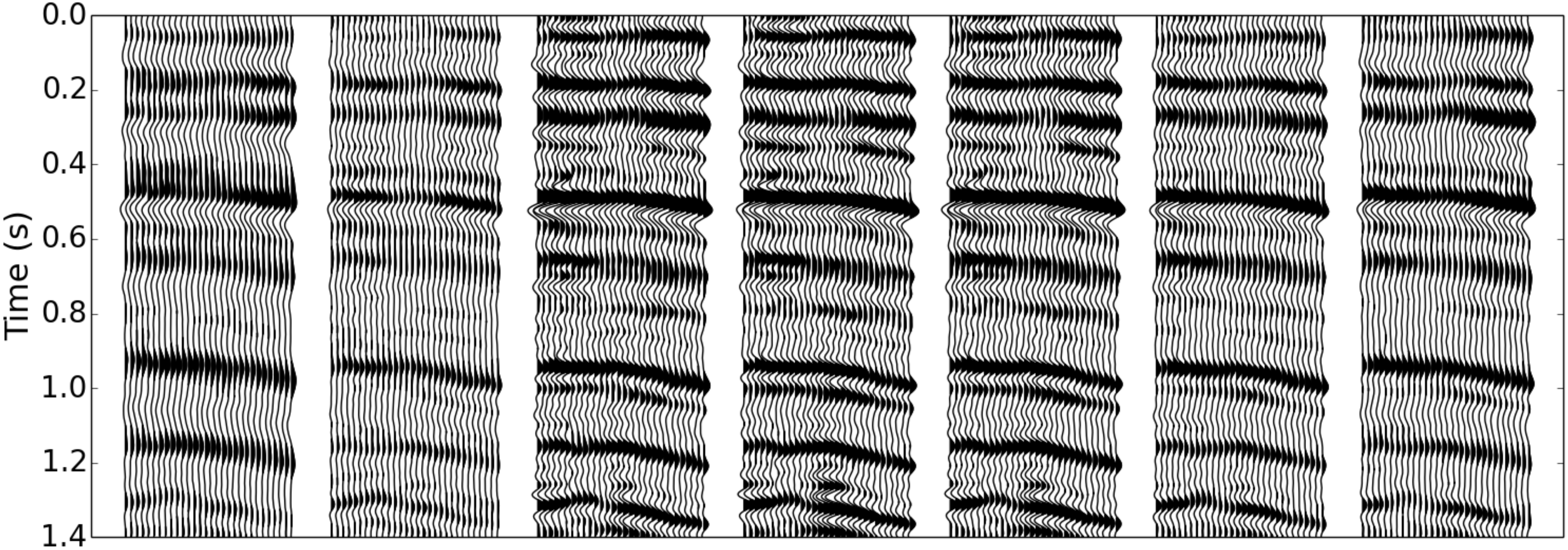


Inline D[:,14,:,:,2]

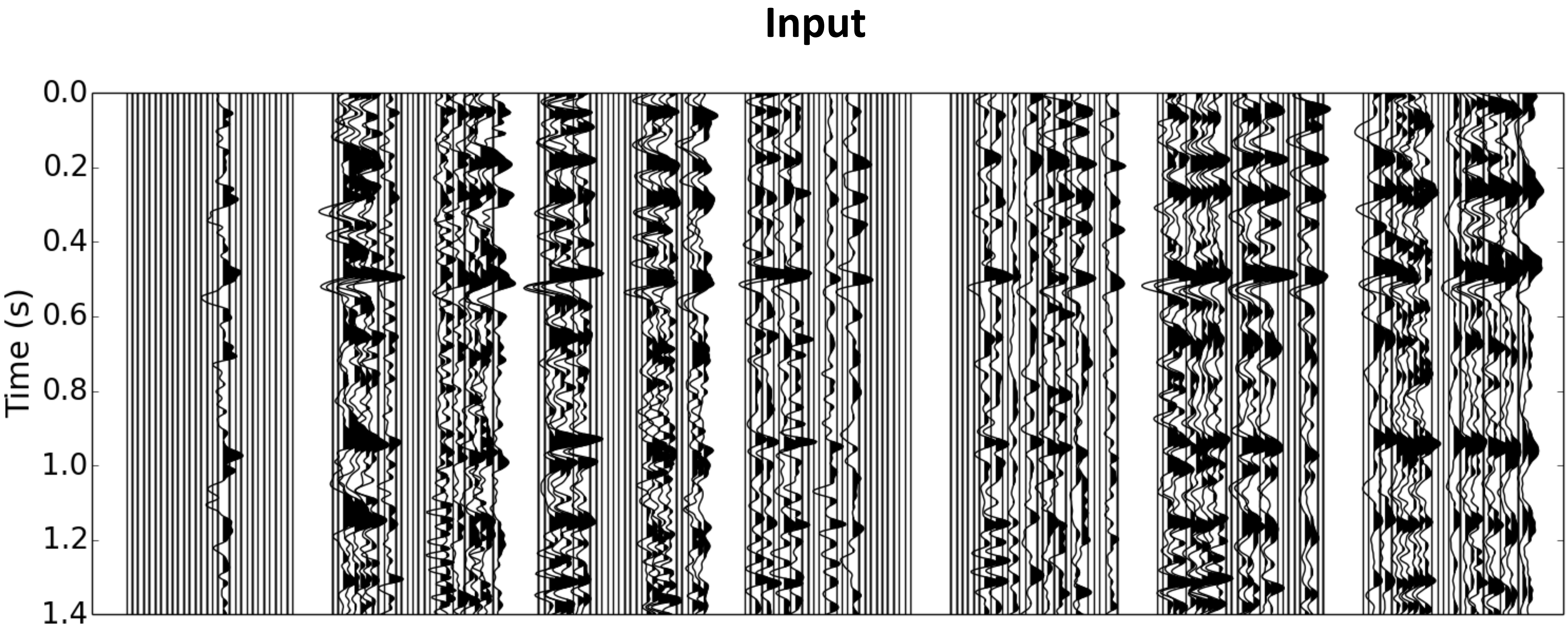


In-line D[:,14,:,:,2]

Robust TC

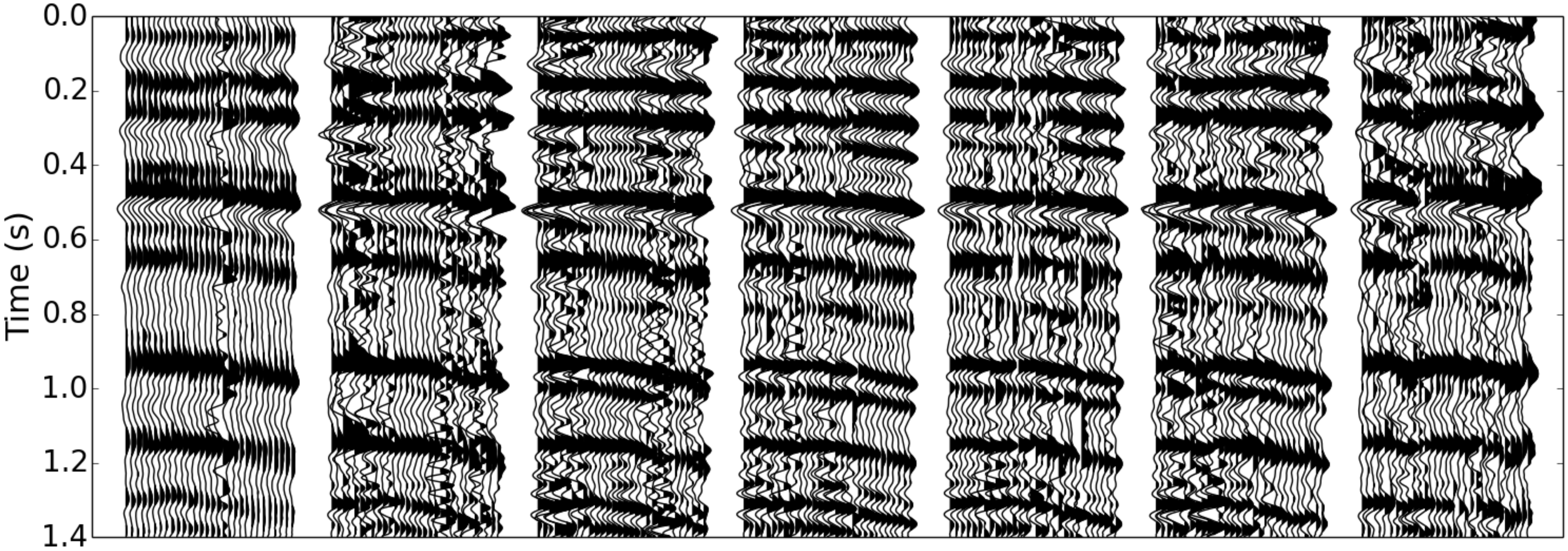


In-line D[:,2,:,:,2]



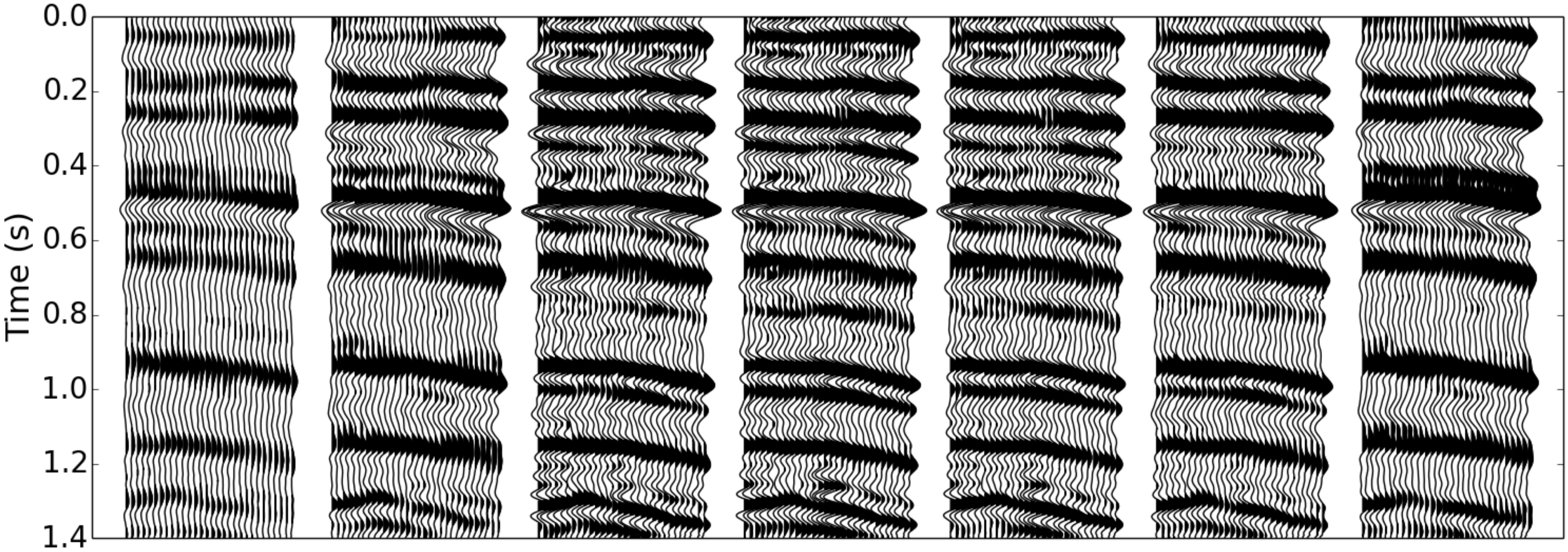
in-line D[:,2,:,:,2]

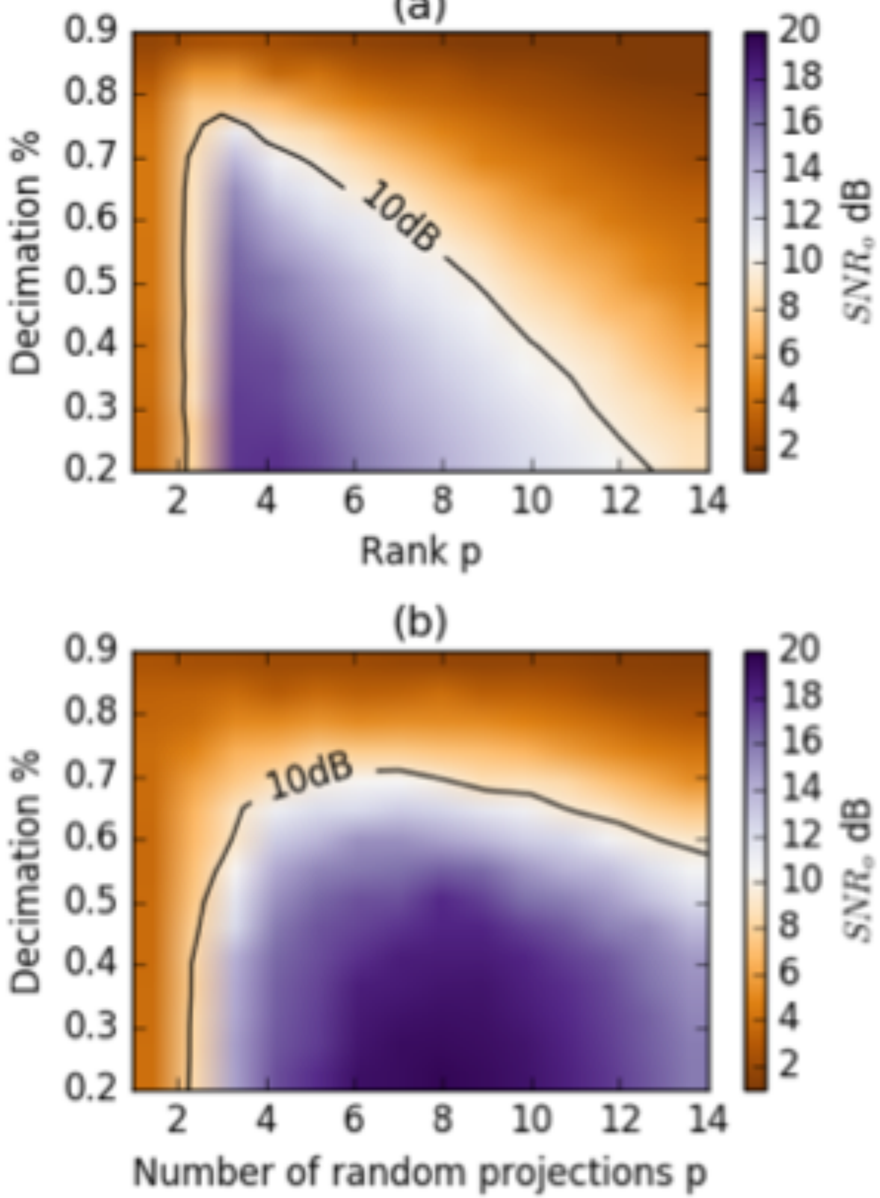
Non-robust TC



In-line D[:,2,:,:,2]

Robust TC





Rank reduction via SVD

Rank reduction via Randomized QR Decomposition

Probability of success for tensor completion SVD vs R-QR decomposition

Chiron et al., 2014, Efficient denoising algorithms for large experimental datasets and their applications in Fourier transform ion cyclotron resonance mass spectrometry: *PNAS*