Delft University of Technology

SUBJECT

AE4423-19 - Airline Planning and Optimization

Assignment 1

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December 20, 2019

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1 Introduction

The aim of this assignment is to find the optimal solution for a new airline to successfully grow in the coming years. For each problem, an optimization on a different sector will be done. This new airline will operate flights between 19 different European destinations and its hub, which is the Paris Charles de Gaulle Airport (LFPG) in France.

To achieve this objective, the assignment is divided into two parts. In the first one, it is developed a network and fleet plan for one standard week on operations based on demand forecast for the year 2019 following a gravity model. The model generated is aimed to maximize the profit of the airline. In the second part, given a daily flight schedule, a passenger mix flow solution is computed that seeks maximizing the revenue of the company. Both models have been implemented in a Matlab code, which can be found in a separate attached file.

This paper is divided into three sections: Methodology, Results and Conclusion. In the Methodology, the different steps followed to obtain the results for each of the problems are explained, as well as the developed mathematical models and the considered assumptions. In the Results section, different figures and tables are portrayed to adequately present the solution obtained with both models. Finally, in the Conclusion section, a brief description of the results is given together with final comments regarding what has been found conducting this study.

2 Methodology

2.1 Problem 1

In this first problem, the objective is to generate the weekly flight schedule for the new airline based on accurate demand data for the year 2019. The demand is calculated calibrating a gravity model for a given demand of the year 2014, of which GDP and Population data are also known. The expression for the gravity model employed is presented in Equation 1, where Dij and dij are respectively the demand and the distance between the different airports, pop refers to the population per city and f represents the fuel cost, which is assumed to remain constant between years 2014 and 2019.

$$D_{ij} = k\frac{(pop_i pop_j)^{b_1}(GDP_i GDP_j)^{b_2}}{(f \cdot d_{ij})^{b_3}} \tag{1}$$

The Population and GDP for 2019 are forecast assuming a linear variation based on given data from years 2014 and 2017. Once they are calculated, the demand for the 2019 year can be computed following the next steps:

- First, the gravity model presented in Equation 1 is linearized applying logarithms to both sides of the previous expression:

$$log(D_{ij}) = log(k) + b_1 log(pop_i pop_j) + b_2 log(GDP_i GDP_j) - b_3 log(f \cdot d_{ij}) \tag{2}$$

- Then, the scaling factor, k, as well as the coefficients b_1, b_2 and b_3 have been determined from the given demand data of 2014 using the expression that defines the Ordinary Least Squares method in Matrix/vector formulation:

$$\beta = (X^T X)^{-1} X^T y \tag{3}$$

where

$$X = \begin{pmatrix} 1 & log(pop_1 pop_1) & log(GDP_1 GDP_1) & -log(f \cdot d_{1,1}) \\ 1 & log(pop_1 pop_2) & log(GDP_1 GDP_2) & -log(f \cdot d_{1,2}) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & log(pop_N pop_N) & log(GDP_N GDP_N) & -log(f \cdot d_{N,N}) \end{pmatrix} \tag{4}$$

$$\beta = \begin{pmatrix} log(k) \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} y = \begin{pmatrix} log(D_{1,1}) \\ log(D_{1,2}) \\ \vdots \\ log(D_{N,N}) \end{pmatrix} \tag{5}$$

- Solving Equation 3 for those linear equations where the demand is different from 0 (for i $\neq$ j) the different coefficients gathered in vector β are obtained.

- Finally, having those coefficients, the population and the GDP in 2019, the demand is computed using again Equation 1.

The following coefficient values were obtained in the calculation of the demand.

Coefficient	**k**	**b1**	**b2**	**b3**
Value	7.6480e-08	0.5864	0.6317	1.2442

Once the demand is obtained, a fleet & network model is developed which seeks to maximize the profit for the company determining the number of aircraft that it should lease depending on given data for 4 different kinds of aircraft. For each of the aircraft, information about their number of seats s^k, speed V^k, average TAT, maximum range and runway required is known. Regarding the costs of each aircraft, there is data about their weekly leasing costs, fixed operating costs C_x^k, a time cost parameter C_T^k, and a fuel cost parameter C_F^k.

2.1.1 Objective Function

The function to maximize, i.e. the profit of the airline, is obtained by subtracting the costs from the company revenues during the week of operation.

The revenues are directly obtained multiplying the yield, which is expressed in €/RPK and depends on the distance following Equation 6, by the flow, $x_{i,j}+w_{i,j}$, and the distance, $d_{i,j}$, between airports i and j.

$$Yield_{i,j} = 5.9 \cdot d_{i,j}^{-0.76} + 0.043 \tag{6}$$

Two different costs are taken into account in this problem. The first one is the cost associated with leasing a certain number of a specific aircraft for the week of operations, which is denoted by C_L^k. The second kind of costs are the operating costs, which depend on the number of flights of each aircraft from airport i to airport j, $z_{i,j}^k$. These costs are subdivided into three components: fixed operating costs per flight leg, C_x^k; time based costs, defined in € per flight hour and dependent on the time cost parameter, C_T^k; and fuel costs, which depend on the distance flown and the fuel cost parameter, C_F^k. A conditional statement is employed in the Matlab code to take into account that the fixed operating costs for flights departing or arriving at the hub are assumed to be 30% lower.

The objective function to maximize is therefore given by the following expression:

$$\begin{aligned} MaxProfit = \sum_{i\in N}\sum_{j\in N}(5.9 \cdot d_{i,j}^{-0.76} + 0.043) \times d_{ij}(x_{ij}+w_{ij}) \\ -\sum_{k\in K} AC^k \times C_L^k - \sum_{i\in N}\sum_{j\in N}\sum_{k\in K}(C_T^k \frac{d_{i,j}}{V^k} + C_F^k \frac{f}{1.5} d_{i,j} + C_x^k) \times z_{ij}^k \end{aligned} \tag{7}$$

Considering that there are 20, N, airports (with only one hub) and 4, K, types of aircraft, the definition and number of decision variables for this problem are: $x_{i,j}$, 400, direct flow from airport i to airport j; $w_{i,j}$, 400, flow from airport i to airport j that transfers at the hub; $z_{i,j}^k$, 1600, number of flights from airport i to airport j with aircraft type k; and AC^k, 4, number of aircraft type k.

2.1.2 Constraints

Constraint 1. Demand Verification: The number of passengers following a specific flight leg between airports i and j must be equal to or less than the demand, $q_{i,j}$, between those airports.

$$x_{ij} + w_{ij} \leq q_{ij}, \forall i,j \in N \tag{8}$$

Constraint 2. Hub consistency: Taking into account that $w_{i,j}$ represents the flow of passengers from airport i to j that transfers at the hub, it must be ensured that if the hub is the origin or the destination of the flight leg no passengers are transferred, since it is direct flow between the hub and another airport. This constraint requires introducing the new variable g_h, which presents a null value if h corresponds to the hub and a value of 1 otherwise.

$$w_{ij} \leq q_{ij} \times g_i \times g_j, \forall i, j \in N \tag{9}$$

Constraint 3. Capacity: Assuming a load factor, LF, of 75%, this constraint verifies that the capacity for a certain type of aircraft is not exceeded for each of the flight legs.

$$x_{ij} + \sum_{m \in N} w_{im} \times (1 - g_j) + \sum_{m \in N} w_{mj} \times (1 - g_i) \leq \sum_{k \in K} z_{ij}^k \times s^k \times LF, \forall i, j \in N \tag{10}$$

Constraint 4: Continuity: There must be a balance between the incoming and outgoing flights in each node.

$$\sum_{j \in N} z_{ij}^k = \sum_{j \in N} z_{ji}^k, \forall i \in N, k \in K \tag{11}$$

Constraint 5: Aircraft productivity: The use of each specific type of aircraft is limited to the number of aircraft leased and the block hours associated. It is assumed that aircraft are only available for operations for an average time of 10 hours per day, resulting in 70 block hours for the whole week. The Turn-Around-Time, TAT^k, which includes landing and take-off times, must be corrected taking into consideration that this time is doubled for flights to the hub.

$$\sum_{i \in N} \sum_{j \in N} \left(\frac{d_{ij}}{sp^k} + TAT^k \left(1 + (1 - g_j)\right) \right) \times z_{ij}^k \leq BT^k \times AC^k, \forall k \in K \tag{12}$$

Constraint 6: Range: An aircraft can only fly between airports i and j if its range is greater than or equal to the distance between those airports. To generate this constraint, it is necessary to employ a new variable, $a_{i,j}$, that sets its value to 0 when the range of the aircraft is not enough, forcing the number of flights in that flight leg to be 0, and takes a really high value otherwise so as not to limit the number of flights of the specific aircraft.

$$z_{ij}^k \leq a_{ij}^k \rightarrow a_{ij}^k = \begin{cases} 10000 & \text{if } d_{ij} \leq R^k \\ 0 & \text{otherwise} \end{cases}, \forall i, j \in N, k \in K \tag{13}$$

Constraint 7: Runway required: Similar to the previous constraint, an aircraft can only perform a flight leg if the runway of both the airport of origin and the airport of destination are greater than or equal to the runway required for that specific aircraft. Again, a new variable need to be added, $b_{i,j}$.

$$z_{ij}^k \leq b_{ij}^k \rightarrow b_{ij}^k = \begin{cases} 10000 & \text{if (Runway)}^k \leq \text{(Runway)}_i \text{ and (Runway)}_j \\ 0 & \text{otherwise} \end{cases}, \forall i, j \in N, k \in K \tag{14}$$

2.2 Problem 2

For the second problem, a passenger mix flow solution needs to be obtained that maximizes the revenue of the company for a given daily flight schedule. There is data about the different flights in the schedule, the path, demand and price of the itineraries and the recapture rate, b, for each combination of itineraries. Following a keypath formulation, the objective is to identify the best mix of passengers from each itinerary to carry on each flight leg for a single day of operations. The solution to this problem is achieved by means of a column generator algorithm. This methodology, which is based on the observation that in most problems not all decision variables (columns) are used in the final solution, works as follows. First, the problem is analysed with a reduced number of decision variables that makes the problem feasible: this is named the initial Restricted Master Problem (RMP). Then, more columns are continuously added depending on a pricing problem that determines if adding any specific column improves the objective function value. The column generation algorithm continues running until there are no extra columns which are beneficial.

2.2.1 Objective Function

The decision variables for this problem are t_p^r: the number of passengers that would like to travel on itinerary p and are reallocated by the airline to itinerary r. In order to maximize the revenue of the company, the objective function to minimize is the difference between the fare that passengers would pay following their desired itinerary p and the fare they would pay if they are reallocated by the airline to itinerary r. Reallocation by the airline is necessary when there is lack of capacity in the aircraft, which would result spilled pax. For this problem, the are 431, P, itineraries and 132, L, number of flights.

$$min \sum_{p \in P} \sum_{r \in P} (fare_p - b_p^r \times fare_r) \times t_p^r \tag{15}$$

2.2.2 Constraints

Constraint 1. Capacity verification: So as to generate this constraint, three new variables need to be taken into account. The parameter CAP_i, that represents the capacity of a specific flight leg i; Q_i, that determines the daily unconstrained demand on flight leg i; and δ_i^p, which is 1 if flight i belongs to itinerary p and 0 otherwise. The parameter Q_i is calculated with Equation 16, where D_p represents the daily unconstrained demand for itinerary p. On the one hand. this constraint detects if the capacity of a flight leg i is exceeded and consequently forces the airline to reallocate passengers to different itineraries. On the other hand, if there is capacity surplus, spilled passengers can be recaptured to paths that use the flight leg i.

$$Q_i = \delta_i^p \times D_p, \forall i \in L \tag{16}$$

$$\sum_{p\in P}\sum_{r\in P} \delta_i^p \times t_p^r - \sum_{r\in P}\sum_{p\in P} \delta_i^p \times b_r^p \times t_r^p \geq Q_i - Cap_i, \forall i \in L \tag{17}$$

Constraint 2. Limited demand: The number of passengers reallocated from their desired itinerary p to itinerary r must be less than or equal to the daily unconstrained demand of the considered itinerary p, D_p.

$$\sum_{r\in P} t_p^r \leq D_p \forall p \in P \tag{18}$$

Constraint 3. Consistency of decision variables: The value of each specific decision variable t_p^r must be greater than or equal to 0 in order to guarantee the applicability of the problem in reality. This constraint is satisfied simply by selecting appropriate bounds for the decision variables.

2.2.3 Restricted Master Problem

The initial set of columns must be such that an initial feasible solution is obtained. In order to fulfil this objective, an extra fictitious' itinerary 0 is created as the only option to reallocate from other itineraries in the first iteration. Those passengers spilled from other itineraries due to the flight capacity are reallocated in this itinerary. The "fictitious" itinerary has a null fare and it is assumed that b_p^0 and b_p^p are equal to 1.

Once computed the first iteration, the pricing problem is conducted in order to see if there are columns that should be added.

2.2.4 Pricing Problem

The dual variables need to be calculated in order to compute the pricing problem. Cplex allows to obtain these variables once it has reached the solution of a continuous problem. There are two sets of dual variables for this problem: π_i, associated with Constraint 1; and σ_p, associated with Constraint 2.

The pricing problem is solved by means of Equation 19. For every possible combination of itineraries the number of moved passengers is calculated, resulting in a matrix of dimensions $p \times p$. The negative coefficients of this matrix determine which possible reallocations "price out" the current solution: adding the columns t_p^r related to those coefficients would result in reduced spilling costs. The columns added in each iteration are those associated with a negative coefficient with the highest absolute value.

$$t_p^{r'} = (fare_p - \sum_{i \in P} \pi_i) - b_p^r \times (fare_r - \sum_{j \in P} \pi_j) - \sigma_p \tag{19}$$

The optimal solution is reached when there are no columns that "price out" the existing solution. Bearing in mind that there are $(P+1)$ itineraries, the total number of decision variables t_p^r for this problem is $(P+1) \times (P+1) = 186624$, this being the reason for using the column generation algorithm.

2.2.5 Verification

Since the column generator algorithm was implemented in a while loop, it was important to check that there are no values creating problems from loop to loop. For example, a value not updating. To prove that this was not the case the last iteration with the new number of columns and its respective positions was run in the optimizator alone. The same objective function value should be outputted.

Other verification techniques carried out during the coding of the problem were:

- Manually check that the objective function and the constraints are correctly implemented in the .lp file that is generated.
- Running the program for the column generator problem described in class.
- Check whether the values make sense or not, regarding the number of passenger spilled.
- Check whether the columns were added correctly, manually checking some random entries to see if the solution was correct.

3 Results

3.1 Problem 1

The following section describes the main results obtained from the Matlab code for the fleet & network model. The value of the optimal objective function obtained giving to the solver a time limit of 5 minutes is **53595.09 €** being the best objective value **54592.77 €** whose gap is **1.86**%.

Key Performance Indicators (KPI)

Not all airlines operate in the same way. Some airlines have a higher budget, some have more planes or better locations. Some airlines operate in Hub and Spoke while others on direct link. Therefore we can not simply compare the total revenues of the business to compare them all. KPIs are introduced to comprehend and integrate the performance of the airline, and be able to compare the different airlines.

The KPIs used in this study are: ASK, RPK, CASK, RASK and BELF. The first two are traffic based indicators, the second two are financial based indicators and the fifth one is a load factor indicator. **ASK** calculates the number of seats available per flown kilometres. It relates the distance travelled with the capacity offered. It is calculated multiplying the distance flown by the number of seats of the aircraft. **RPK** Calculates the used seats on that same aircraft. It relates the number of passengers and the distance they have been transported. It is calculated by multiplying the flying passengers times the distance they travelled.

CASK is the amount of the operational costs incurred for each ASK supplied. It can be explained as the operational costs of flying an aircraft of "x" seats over a distance of "d". It is calculated by dividing the costs of flying an airplane divided by the number of seats times the distance travelled. **RASK** is the operational revenue per ASK supplied. This can be seen as the revenue that the airline has obtained per kilometre flight per number of seats available. To calculate it, the operational revenue is divided by the ASK.

BELF is the break even load factor, when the revenues cover exactly the operational cost. In order to calculate the BELF, the CASK is divided by the total yield, which is obtained dividing the total revenues by the RPK. The following table displays the KPI results of our airline:

KPI	**ASK [seats/km]**	**RPK [pax/km]**	**CASK [€]**	**RASK [€]**	**BELF [-]**
Value	9.2390e+06	6.9174e+06	0.0607	0.0665	0.6834

Table 1: Airline KPI Results

In Figure 1 a comparison of current CASK and RASK values for different airlines is outlined. It can be observed that the values obtained in problem 1 do not differ much from the global average.

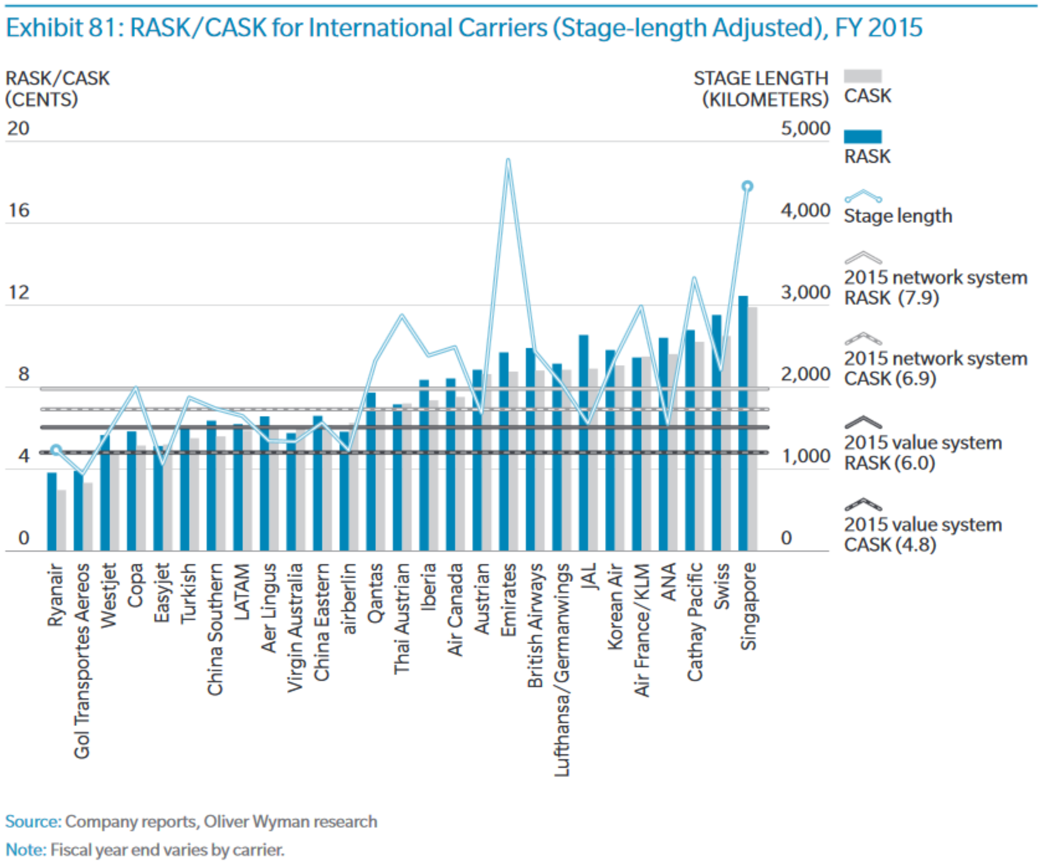


Figure 1: Multiple Airlines CASK and RASK comparison. Data obtained from [1].

Graphs

The figures displayed in this section show the flight connections resulting from the solution of problem 1. It can be observed that some airports are not connected with flights. When the demand of a flight leg is very low, the airplane will not be filled enough with passengers and it might happen that the costs of generating that connection are greater than the revenues. Therefore, the optimized solution does not take into account these flight legs in the weekly flight schedule.

Figure 2 shows the type of airplane chosen per flight leg. It can be observed that the shortest flights are connected with the regional turboprop, while the longest with the regional jet. Moreover, the twin aisle twin engine jet is not used. The reason for this is that the hub is located close to the centre of all other airports, reducing considerably the length of all flight legs. Consequently, big aircraft are not required since they will only add significant operational cost. The distribution of aircraft leased by the airline is shown in the following table:

Type	Regional		Twin engine jet	
	Turboprop	**Jet**	**Single aisle**	**Twin aisle**
Leased	3	1	1	0

Table 2: Distribution of aircraft leased by the airline

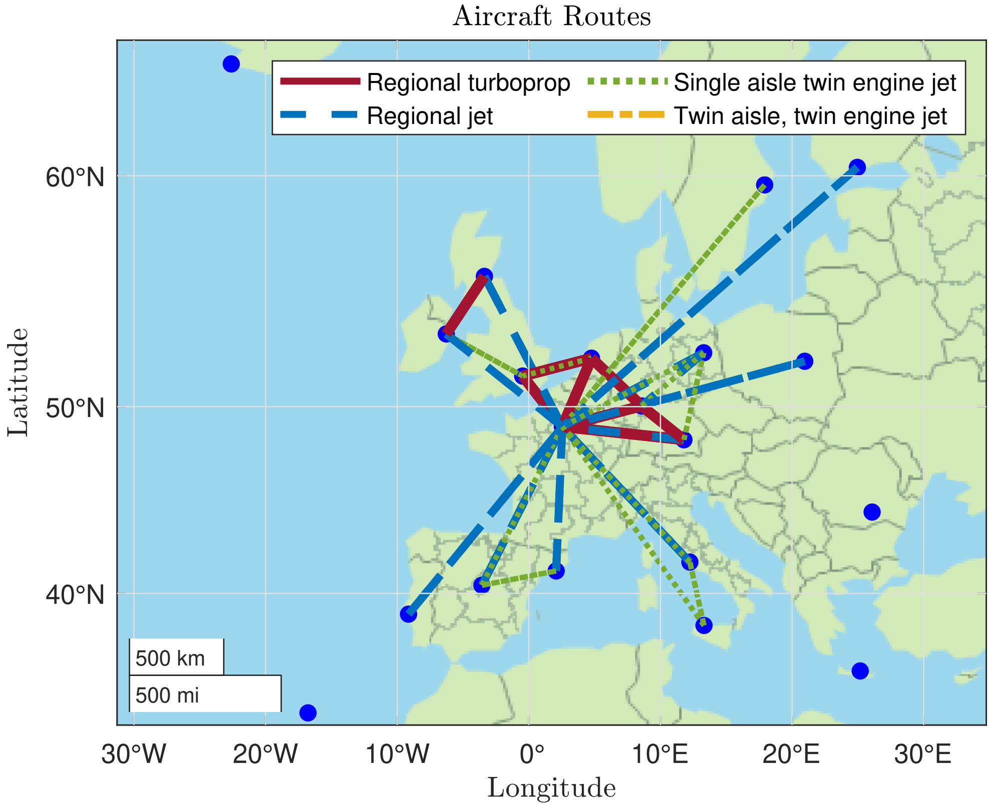


Figure 2: Map displaying the type of aircraft per flight leg.

In Figure 3 it is represented the flow of passengers per flight leg. The thicker the line, the more passengers travel within that flight. It can quickly be recognized in the graph that the flights with the highest flow are London - Paris and London - Amsterdam. This is consistent with the 2014 data, in which the two aforementioned flights had the highest demand, 967 and 615 respectively.

Finally, in Figure 4 the type of passengers flying in each route is displayed. Direct passengers represent the ones that do not pass through the hub to arrive at the destination, while no direct passengers need to stop-over in the hub.

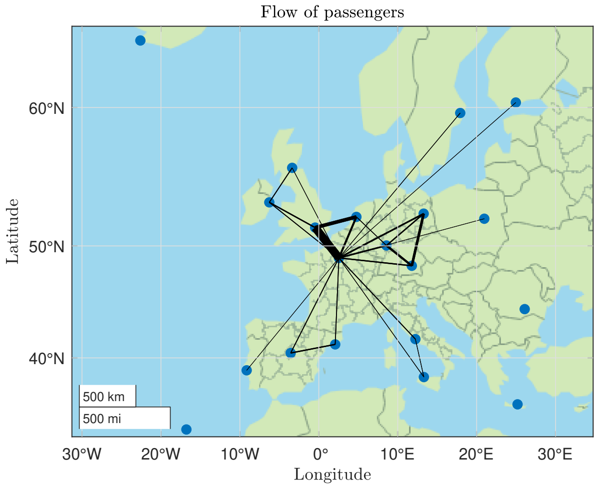


Figure 3: Map displaying the flow of passengers per flight.

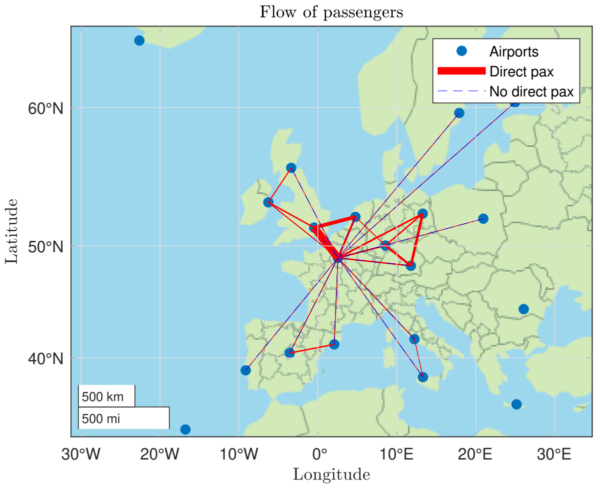


Figure 4: Map displaying the passenger type fer flight.

3.2 Problem 2

In this section, the results of the passenger mix flow problem are displayed. The solution obtained is shown for both the initial Restricted Master Problem (RMP) and the global problem. In addition, the already mentioned verification of the solution has been carried out to make sure that the results obtained are correct.

3.2.1 Initial RMP

As commented in the section "Methodology", the initial RMP of a column generation algorithm is solved for an initial set of columns which leads to a feasible solution. The set of columns selected starts with the "fictitious" itinerary (with a null fare) as the only option to reallocate from other itineraries. This itinerary works as a buffer for spilled passengers. The number of columns employed for the first iteration of the column generation algorithm is therefore $(P + 1) = 432$, where P represents the number of itineraries provided at the data sheet.

The optimal objective value for the initial RMP is **1.7562E6 €**. As a sample, the optimal decision variables associated with the first 5 passengers itineraries, i.e., the number of passengers reallocated (spilled) to the fictitious itinerary, are: **0,12,0,10,50**. The total number of passengers spilled is **11765**, it must have a high value since there is no reallocation of passengers to other actual itineraries offered by the airline. The optimal dual variables associated with the first 5 flights for the first iteration are $\pi =$ **62,41,47,69,69**, and the ones associated with the first 5 passengers itineraries are $\sigma =$ **0,0,0,0,0**.

3.2.2 Global Problem

The passenger mix flow problem has been solved to the end. The number of final columns is 496, so 64 more columns have been added through the process of column generation. This means that the majority of the decision variables are not used in the final solution, and therefore present a null value in the solution obtained. The number of iterations required to converge is 49. This number is lower than the number of columns added because when the pricing problem finds out that there are more than one columns which share the highest (negative) value of “slackness”, all the mentioned columns are added as decision variables.

The optimal objective value for the global problem is **1.7192E6 €**. The decision variables associated with the first 5 passenger itineraries are displayed in Table 3 where the third row indicates first the desired itinerary and the itinerary in which the passengers have been reallocated. It has to be mentioned that since in the data the itineraries started with a 0 value, in the matlab code, one extra unit has been added to all the itineraries. So, in fact those values would be substracting 1 (example: 7-5 for the first case). The table shows non-null values of passengers reallocated to other itineraries from the first 5 and non-null values of passengers reallocated from other itineraries to the first 5. The number of spilled passengers for the global solution is 10321. The optimisation run time is 2974.9 s. Furthermore, the value of the dual variables for the first 5 flights is displayed in Table 4.

Table 3: Table displaying the values of the non zero objective values of the first 5 passenger itineraries.

Passenger Itinerary	**1**	**2**	**3**	**4**	**5**
Pax reallocated:	24	85	41	36	13
Reallocated to:	8-6	16-6	19-17	135-23	195-23

Table 4: Table displaying the values of the dual variables for the first 5 flights.

Flight	**1**	**2**	**3**	**4**	**5**
Dual variable π**:**	62	41	47	69	66
Dual variable σ**:**	0	0	0	0	0

In Figure 5 the passengers spilled per column added is displayed. Each iteration corresponds to a point in the graph. In the graph it can be observed that the number of columns added per iteration varies. It must be said that the line itself does not provide any information since the approach is discrete.

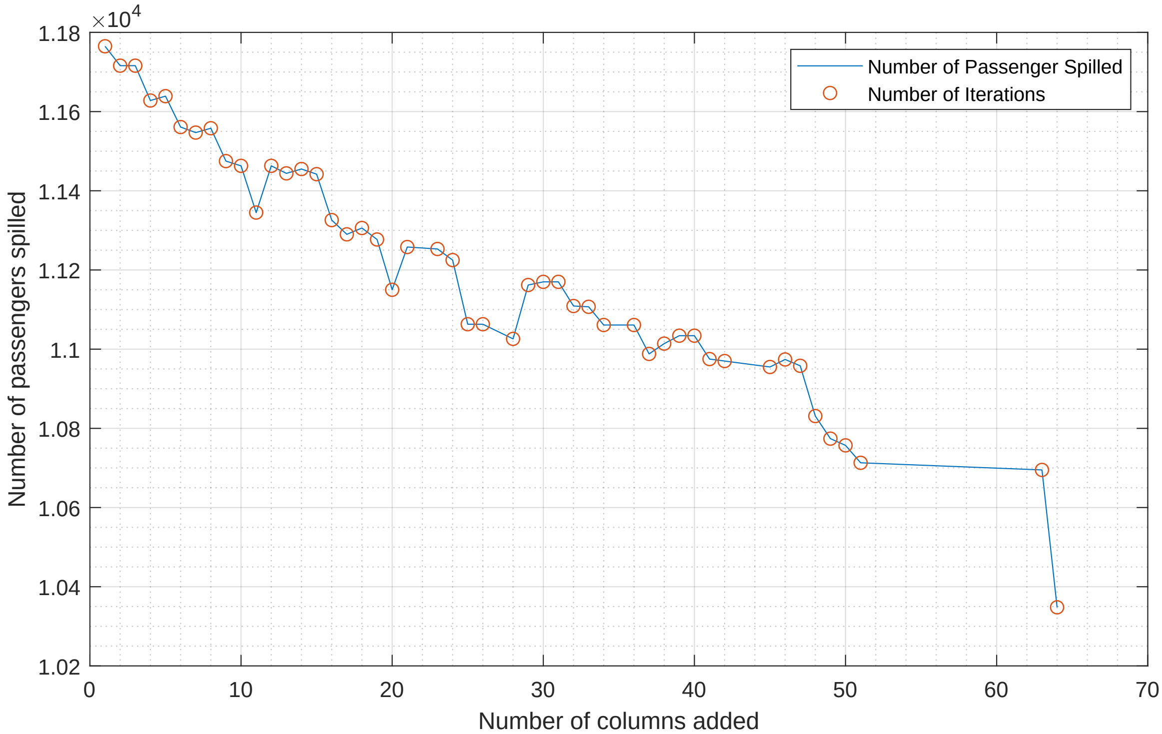


Figure 5: Number of passengers spilled per column added.

3.2.3 Verification

The results obtained when using the optimization for the found number and position of columns match the ones obtained by the column generator algorithm. This means that the while loop is well implemented, and there is no variables wrongly updating the solution. When the program described in class was ran, the results were satisfactory. The number of passengers spilled make sense as well. There is a total demand of 18143 and the total capacity of the flights is 11300. This means that a total of 6843 passengers will be spilled for sure. Furthermore, accounting for the recapture ratio, if the average recapture ratio (0.295) is multiplied by the flight capacity, the average spilled passengers should be 3336, which added to 6843 gives a total of 10179 spilled passengers, quite close to the solution given by the program of 10321. This one is higher because we are not accounting for the specific flights, but gives a good overview if the recapture rate is random. It is important to note that, although this value does not have a clear sense (since we are not taking into account that some passengers fly in the desired schedule, thus the recapture ratio is 1), the order of magnitude should be similar, and it is.

4 Conclusion

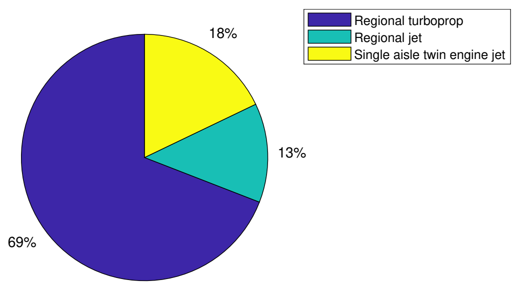
The following conclusions are drawn from the obtained results:

Fleet & Network problem:

- Given that the airports considered in this problem are all located in Europe, there are no long distance connections (transatlantic flights, connections with Asia...). Therefore, it makes sense that the solver has decided not to lease any twin aisle twin engine jet. Its lease, fixed, time and fuel costs are significantly higher than for the other aircraft. With a load factor of 75%, the spoilage of passengers in the flight legs performed by the twin aisle jet would be too high. Moreover, there are no profitable flights which require a range greater than 6300 km. Thus, it is not necessary to use the twin aisle aircraft to perform flight legs that the single aisle twin engine jet could not.

- The employed load factor for this problem, 75%, does not differ a lot from the average value that airlines usually have in reality. We wanted to study the influence of this factor in the results. Of course, the different flight legs and aircraft used would present different values of load factor in reality, making some combination of flight schedules more profitable than others. For the employed load factor, only 5 aircraft are leased: 3 regional turboprop, 1 regional jet and 1 twin engine single aisle jet. If we increase the load factor to a value of 90% and run the solver again, suddenly it is much more profitable to perform more flight during the week, the value of the objective function is increased by 500%. Consequently, the number of aircraft leased increases a lot, a total of 9 aircraft are leased: 3 regional turboprop, 2 regional jet, 4 twin engine single aisle jet and again no twin aisle jet. We even notice that the distribution changes, it is now more profitable to lease more twin engine aircraft, which present a much higher number of seats. In Figure 6, it is represented the percentage of flights flown by each type of aircraft for the two different load factors analysed.

(a) Load factor: 75%



(b) Load factor: 90%

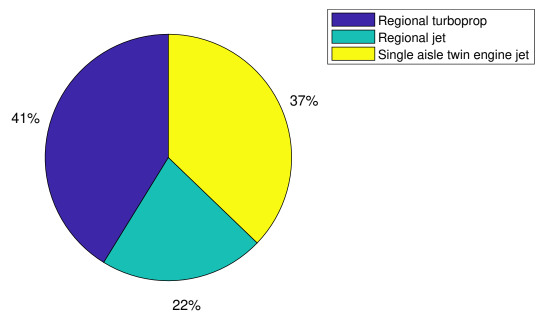


Figure 6: Percentage of flights flown by each type of aircraft.

- Another parameter of which we wanted to study its influence on the results was the price of fuel, which has been set at 1.42 USD/gallon for this problem. If we reduce the fuel cost by 15% (1.2 USD/gallon), the objective function value is incremented by approximately a 91%. It is interesting to study what happens with the distribution of leased aircraft in this case. The number of leased aircraft is increased to 8: 3 regional turboprop, 1 regional jet and 4 twin engine single aisle jet. The benefits of using the twin engine aircraft, which can carry many more passengers, are triggered taking into account that it presents a really high fuel cost parameters when comparing with the other 2 aircraft: 2 times the value of the regional jet and 4 times the value of the regional turboprop. Once again, the twin aisle aircraft is not used by the solver, making it clear that it makes no sense to use this aircraft for the routes considered.

Passenger mix flow problem:

- The recapture ratios and the flight prices do not reflect reality. This allows for the optimizer to redirect the passengers to a multi stop flight since the fare charged to the passenger is higher. That is why multi stop flights are more filled than direct flights. It is difficult to believe that passengers present similar recapture ratios when they are forced to stop over at an airport in their itinerary.
- Since the demand is very low for the first itineraries, the reallocation of the passengers is as well negligible.
- The spillage of passengers for the final solution is at first glance intolerable high: 10321 passengers. Taking into consideration that the total unconstrained demand for all itineraries is 18143, approximately 57% of passengers are spilled. In spite of the outrageous results, is has to be observed that the total capacity of the weekly flights (summing the capacity of each flight without considering anything related to timetables or passenger itineraries) is 11300, meaning that ideally only 62% of passengers could actually fly with the airline.
- The total optimization run time for the second problem is too high: 2974.9 s. Once we got all the results of the problem, we believe we found the source of error in the code that was causing this high computational time. We had included all the information related to all the possible columns of the global problem in each iteration. By means of another variable obtained through the pricing problem in each iteration, the columns added as decision variables from the total set of decision variables were selected. The reason for this way of working is that cplex eliminates all columns from the vector of decision variables which are not considered, computing the same number of columns and rows for the simplex method as if the columns of the vector of decision variables were added in each iteration. Nevertheless, this does not hold true for the other functions which were implemented in the code, which are too time consuming due to the fact that they have to take into account information regarding all other itineraries provided in the data sheet as for example, the time that it takes to complete constraint 1 is 50s each time a new column is added.

References

T. Stalnaker, K. Usman, and A. Taylor. *AIRLINE ECONOMIC ANALYSIS*. Marsh & McLennan comapnies, 2016-2017.