Technische Universiteit Delft

Master in Aerospace Engineering

Academic course 2018/19

Group Assignment:
Airline Planning & Optimization (Group 16)

Authors:
4948939 Aguilar Fuertes, Jose Javier
4920341 Lloret Cendales, Kevin
4935918 Muñoz Tejeda, Jesús Manuel

Professor:
Bruno Santos

January 11, 2019

Contents

I Introduction

In this assignment, it is assumed that a new airline called Westpoint Airways, whose hub is in London, has recently started. Throughout this report, optimization methods will be implemented in order to achieve successful results in this new airline for the incoming years. That implies a correct network and frequency plan, together with an adequate fleet plan, which are presented afterwards.

Assumptions and results will be commented and explained, taking into account the references included in the Bibliography section, specially the lecture slides provided in the course [1].

Finally, all necessary data and programs to replicate the results are included attached to the report.

II Parameters modelling

Common parameters that will be used along the various problems to face are explained here, computed as stated in the problem statement appendices:

II.1 Revenue

Yield (or Revenue per RPK -Revenue per Passenger Kilometer-) is divided two different types, depending on if the flight is intra-European or not.

- Yield for Intra-European Passengers:

$$Y_{EUR_{ij}} = 5.9 \cdot d_{ij}^{-0.76} + 0.043 \tag{1}$$

  where $Y_{EUR_{ij}}$ is the yield between origin 'i' and destination 'j', expressed in € and d_{ij} is the distance in km between origin 'i' and the destination 'j'.

- Yield for flight with US origin or destination:

$$Y_{US_{ij}} = € \ 0.05 \tag{2}$$

II.2 Load Factors

Average Load Factors expected are estimated as follows:

- Load Factor for Intra-European Passengers: $LF = 75\%$
- Load Factor for flight with US origin or destination: $LF = 85\%$

II.3 Costs

Costs are subdivided in two different types: Leasing Costs and Operating Costs. Those are explained in detail here:

- **Leasing Costs** (C_{Lea}): All aircraft operated are leased, and their leasing costs depend on the Aircraft type and can be found on assignment data provided [1].

- **Operating Costs** (C_{OP}): those can be divided as well into three different costs:

[1]Henceforth, the nomenclature '*assignment data*' will be included every time data is taken from the assignment guidelines. The reader is therefore redirected there to check that information

- Fixed Operating Costs (C_X^k): costs per flight leg; which includes landing rights, parking fees and fixed fuel costs. They depend on aircraft Type k (assignment data).
- Time-Based Costs (C_T^k): costs defined in € per flight hour, and account for any cost which is time-dependent (cabin crew, flight crew...). Find below how those costs are modelled:
$$C_{T_{ij}}^k = c_T^k \frac{d_{ij}}{sp^k} \tag{3}$$
where $C_{T_{ij}}^k$ is the time cost between two different airports 'i' and 'j', flown by an aircraft of type 'k'; c_T^k is the time cost parameter for an aircraft of type 'k', d_{ij} represents the distance between two airports and sp^k the aircraft speed (assignment data).
- Fuel Costs (C_F^k): they depend on the distance flown (d_{ij}), and are modelled as:
$$C_{F_{ij}}^k = \frac{c_F^k \cdot f}{1.5} d_{ij} \tag{4}$$
where $C_{F_{ij}}^k$ is the fuel cost between two given airports 'i' and 'j' for a specific flight 'k'; c_F^k is the fuel cost parameter that depends on aircraft type (assignment data) and f is the fuel cost ($f = 1.42$ USD/gallon in 2017 and $f = 1.6$ USD/gallon in 2022).

As a result, total cost is computed as:

$$C_{ij}^k = C_{Lea}^k + C_{OP_{ij}}^k \tag{5}$$

where C_{OP} should be computed by flight leg between airports 'i' and 'j' (when is a flight to/from the hub, operating costs are assumed to be 30 per cent less) and per aircraft type 'k' as follows:

$$C_{OP_{ij}}^k = C_X^k + C_{T_{ij}}^k + +C_{F_{ij}}^k \tag{6}$$

II.4 Distance between two points

Distance between two points is necessary to compute, as many factors directly depend on it. To do, it is possible to apply Equation 7 by using the position of two points, given by their respectively latitude and longitude:

$$d_{ij} = 2R_e \cdot \arcsin \sqrt{\sin\left(\frac{\varphi_i - \varphi_j}{2}^2\right) + \cos\varphi_i \cos\varphi_j \sin\left(\frac{\lambda_i - \lambda_j}{2}^2\right)} \tag{7}$$

where d_{ij} represents that distance between points 'i' and 'j', and φ_i, φ_j, λ_i and λ_j are the latitude and longitude respectively of airports 'i' and 'j'.

1 Problem 1

Firstly, it is supposed that Westpoint Airways has leased a certain amount of aircraft (assignment data). Therefore, it is necessary to develop an optimal network and frequency plan, in order to maximise Westpoint Airways profit. As a result, an optimisation problem should be solved.

1.1 Mathematical Model

Before going into the optimisation model itself, it is necessary to specify the notation of the problem. That is: what are the set used, the decision variables, and the different parameters that play an important role in the model.

It is noted that parameters explained in Section II will be used as well. The reader is redirected to that section when any of them appear in the optimisation model.

At the end, optimisation problem is addressed: Network and Frequency plan optimisation model is divided into the objective function (the one that should maximise -in this case- in order to get a numerical solution that allows for the desired plan), and the constraints which this objective function is subjected to.

1.1.1 Problem Notation

Sets

- **N**: set of airports, where 'h' is the hub
- **K**: set of aircraft types

Decision Variables

All decision variables have a lower bound at 0.

- w_{ij}: passengers from airport i to airport j that transfer at the hub
- x_{ij}: direct passengers from airport i to airport j
- z_{ij}^k: number of flights from airport i to airport j with aircraft type k.

Parameters

- q_{ij}: travel demand between airport i to airport j
- g_h: binary parameter: $g_i = 0$ if a hub is located at airport i; $g_i = 1$ otherwise
- d_{ij}: distance between airport i to airport j
- Y: revenue per RPK flown (Yield)
- s^k: number of seats per aircraft type k
- $C_{OP_{ij}}^k$: Cost of operating a flight from i to j
- sp^k: speed of aircraft type k
- $Slot_i$: available slots of each airport type i, (unlimited slots in hub airport are considered according to assignment guidelines).
- BT^k: aircraft type k average utilisation time
- RW^k: runway length needed for each aircraft type k
- RW_i: runway length of an airport i
- RA^k: maximum range of each aircraft type k
- AC^k: number of aircraft of each type k.
- δijk: runway binary parameter: $\delta ijk = 1$ if $RW^k \leq RW_i$ and $RW^k \leq RW_j$; $\delta ijk = 0$ otherwise
- Ψij: range binary parameter: $\Psi ij = 1$ if $d_{ij} \leq RA^k$; $\Psi ij = 0$ otherwise

1.1.2 Optimisation Problem

Objective Function

$$\sum_{i\in N}\sum_{j\in N}\left[Y\cdot d_{ij}(x_{ij}+w_{ij})-\sum_{k\in K}C^k_{OP_{ij}}\cdot z^k_{ij}\right] \tag{8}$$

The objective function represents the profit obtained in a week of operation, given as the revenues from passenger minus the operating costs. As leasing costs are fixed in this case, they do not need to be included in the objective function.

Constraints

The different constraints that apply in Problem 1 are:

Demand Constraint

$$x_{ij}+w_{ij}\le q_{ij},\ \forall i,j\in N \tag{9}$$

This constraint means that the sum of direct and connecting passengers between two airports i and j should not be greater than the demand.

Transfer Passengers Constraint

$$w_{ij}\le q_{ij}\cdot g_i\cdot g_j,\ \forall i,j\in N \tag{10}$$

This constraint means it is not possible to have connecting passengers if the origin or destination airport is the hub (so either g_i or g_j will be zero in those cases).

Capacity, Range and Runway constraints

$$x_{ij}+\sum_{m\in N}w_{im}\cdot(1-g_j)+\sum_{m\in N}w_{mj}\cdot(1-g_i)\le\sum_{k\in K}z^k_{ij}\cdot s^k\cdot LF\cdot\delta ijk\cdot\Psi jk,\ \forall i,j\in N \tag{11}$$

This constraint means that the flow of passengers in each leg ij (composed by direct and indirect passengers if the origin or destination is the hub) cannot be higher than the actual capacity, defined as the frequency times the number of seats multiply by the aircraft load factor.

Inside this capacity constraint, there are two conditions that the aircraft should comply. Otherwise, the capacity of this aircraft in this leg is zero.

First, the Range Constraint: $d_{ij}\le RA^k,\forall i,j\in N$ expressed with the binary variable Ψ_{jk}. The distance in that route should be lower than the aircraft range to be able to add capacity to that route.

Second, the Runway Constraint: $RL_i\ge RL^k\ and\ RL_j\ge RL^k,\forall i,j\in N$, expressed with the binary variable δ_{ijk}. The Runway length of the origin and destination airports should be greater or equal than the Runway length needed for that aircraft type.

Balance constraint

$$\sum_{j\in N}z^k_{ij}=\sum_{j\in N}z^k_{ji},\quad\forall i\in N,k\in K \tag{12}$$

This constraint means that the number of aircraft going out of a node should be equal than the number of aircraft coming into that one.

Utilisation constraint

$$\sum_{i \in N} \sum_{j \in N} \left(\frac{d_{ij}}{sp^k} + LTO \right) \cdot z_{ij}^k \leq BT^k \cdot AC^k, \quad \forall k \in K \tag{13}$$

This constraint means that the time spent by an aircraft, either on ground or flying, should be lower than the block time of this aircraft.

1.2 Analysis of results

After implementing in *MATLAB* the mathematical model explained previously, it is optimised with *CPLEX* looking for the maximum profit of Westpoint Airways, considering all constraints mentioned above and a typical week of operations with 10 operating hours per day on average (70 hours on a week). Key performance indicators of the airline (KPI's), network routes, number of flights per route, expected passengers and use of different types of aircraft can be obtained from the solution obtained.

In Table 1 the values of KPI's of this airline and its operative results are listed. ASK and RPK are according to a new company operating in Europe with 14 different destinations, mainly short flights. Its CASK is similar to the one of another ULCC (Ultra Low Cost Carrier) as WizzAir, 0.05 € represents low costs per ASK [2], and its marginal profit around 13% is also in similar values of this type of airlines. A load factor of 74.90 is quite low in comparison with them but more usual in new companies, and a BELF of 64.68 represents a good business model. Finally, this affirmation is verified with a weekly profit of 70,127 €.

Key Performance Indicators	
Available Seat Kilometer (ASK)	8,565,155 ASK
Revenue Passenger Kilometer (RPK)	6,414,930 RPK
Cost per ASK (CASK)	0.0518 €/ASK
Revenue per ASK (RASK)	0.0600 €/ASK
Yield per RPK	0.0802 €/RPK
Average Network Load Factor (ANLF)	74.90%
Break-Even Load Factor (BELF)	64.68%

Operative Results	
Costs	444,068.06 €
Revenue	514,195.12 €
Profit	**70,127.06 €**

Table 1: KPI's and Operative Results Exercise 1

As it is shown on Figure 1, there are 14 European destinations from London, without operating any direct flight between other cities but with some transfer passengers. They are just few people (15-30 people each transfer route) for whom is profitable travelling at the longest possible routes of the network, as revenue is based on distance. Also, it is worth mention that the hub lies close to the line joining the origin and destination of connecting passengers, e.g. Dublin-Munich or Madrid-Helsinki.

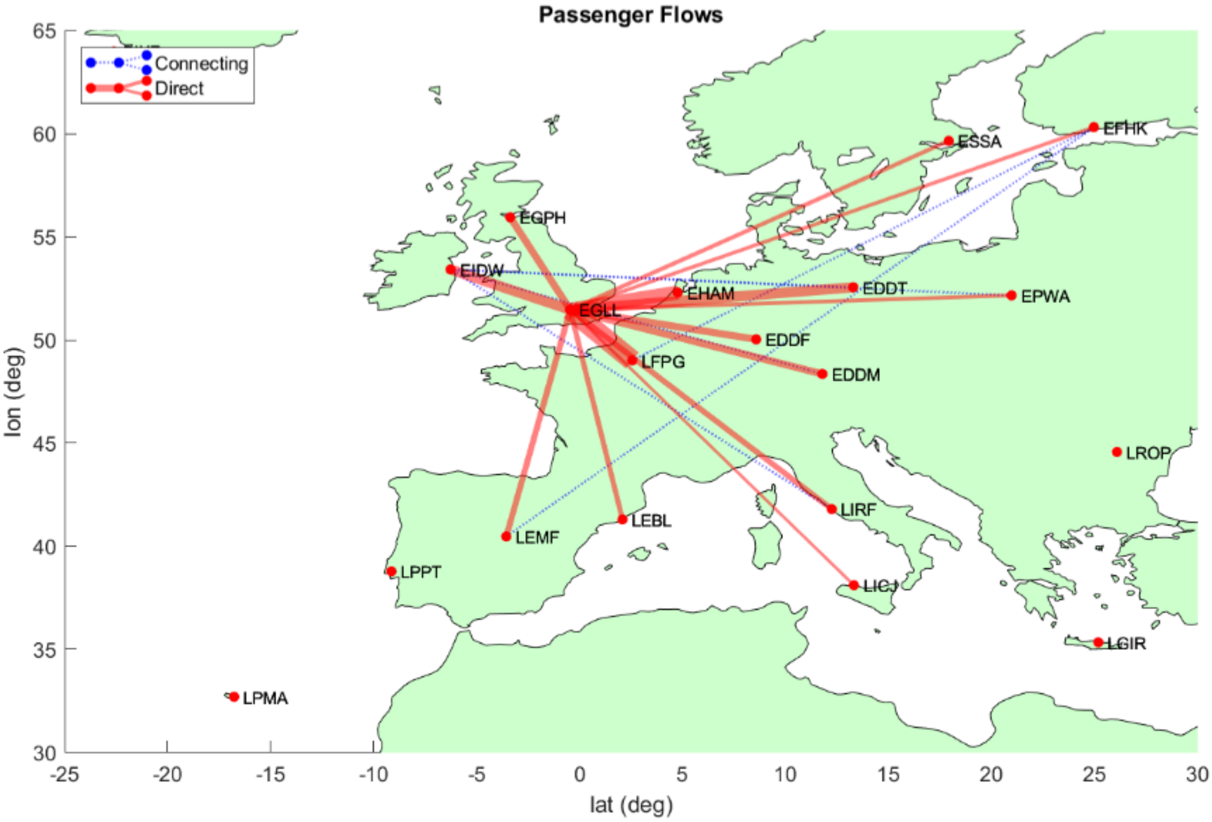

Figure 1: Passengers flows on Westpoint Airways

All flights available from London are represented in Figure 2[2] , with their respective passengers and number of flights according to the type of aircraft. Main destinations are Paris, Amsterdam and Dublin, with estimated passengers of 968, 626 and 433, respectively. These routes are also the closest from London, having lower operational costs with the smallest aircraft (regional turboprop, i.e. ATR-42). In that way, it is possible to offer a good number of daily connections, up to 4 in the case of Paris with just regional turboprop.

Another type of regional aircraft (regional jet, i.e. Bombardier CRJ-700) is used for destinations with considerable demand and short-mid range, as German cities or Edinburgh. In these airports usually two types of aircraft are used, a combination of regional jet with turboprop or with single-aisle jet (i.e. Boeing 737-700). This last type of aircraft is used for furthest routes of the network, as Warsaw or Rome, with just 1 or 2 weekly services. Finally, the use of aircraft is divided among 2 turboprops, 1 regional jet and 1 single-aisle jet and presented together with its productivity in Figure 3. A remarkable aspect is how precise the optimisation is, having a utilisation of 139.92 out of a maximum of 140 on turboprops or 69.98 out of 70 on regional jet, which is a 99.97% of use. Also, it shows how is the importance of Single-aisle jet on productivity with more than 50% of this indicator, meanwhile, in hours used, it is the less used with 24.95%.

[2]Note that Figure 2 upper values of Paris and Amsterdam are cut off to account for a better visual representation of data. Actually, their value is much higher, as it can be seen on the tab.

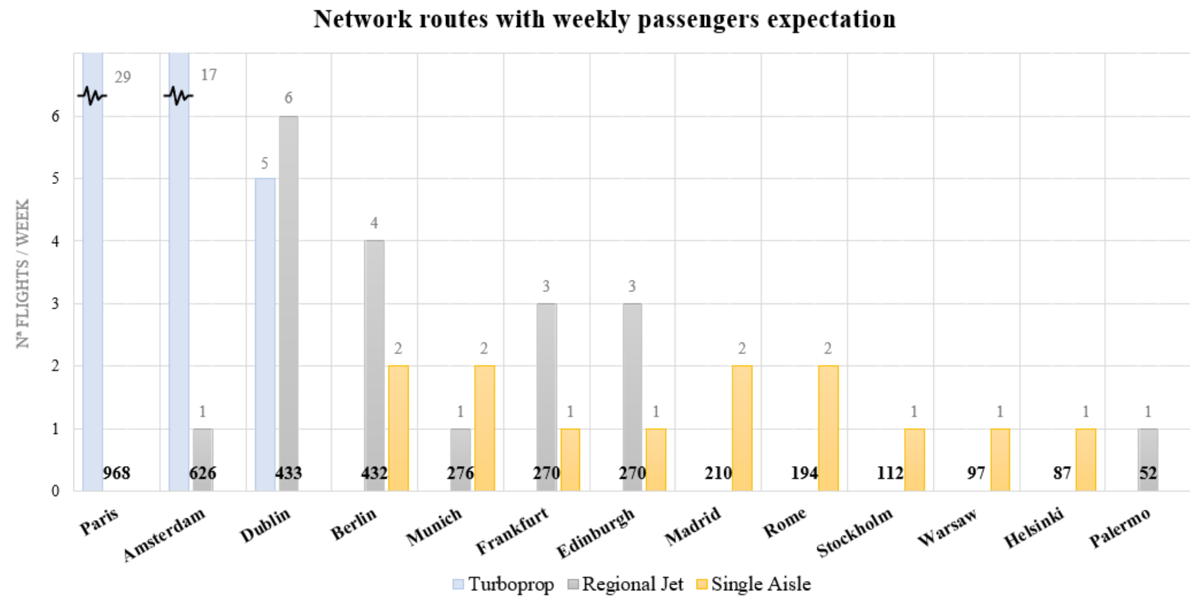

Figure 2: Routes from Westpoint Airways' hub (London)

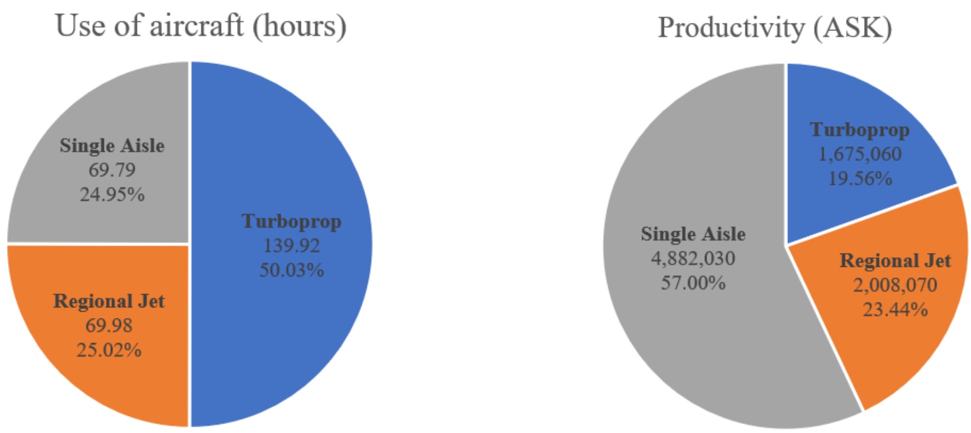

Figure 3: Utilization of aircraft

2 Problem 2.1

In this problem, a demand forecast will be assessed. The results obtained here will be used in next problems to evaluate new results using a fleet expansion; which will be needed to fly to US destinations.

Demand can be modelled using the following gravity model:

$$D_{ij} = k\frac{(pop_i pop_j)^{b1} \cdot (GDP_i GDP_j)^{b2}}{(f \cdot d_{ij})^{b3}} \tag{14}$$

where D_{ij} is the demand between airports i and j; pop_i and pop_j are the population in thousands of cities i and j; GDP_i and GDP_j is GDP of airports i and j; f is the fuel cost (see Parameters Modelling section, Costs), and d_{ij} is the distance between two airports in km. The main goal here will be to obtained the demand for each leg in 2022.

Making use of the data about the population (in the correct units), distance and GDP between all set of legs between two points *ij* in the year 2017, it is possible to compute the parameters *k*, *b1*, *b2* and *b3* using Non-Linear Regression of multiple variables. Notice that it is supposed that this model only differs from European to US models into a factor of 10 of the coefficient *k*. As a result, those coefficients can be taken from the European data and then apply this scaling factor when computing the demand in US flights.

To do so, it is necessary to express the gravity model in a linear system of equations of $y_i = a_0 + a_1 \cdot x_i + a_2 \cdot u_i + a_3 \cdot w_i$, where i is the number of European airports. By taking logarithms at both sides of the Equation 14:

$$\log D_{ij} = \log k + b1 \cdot \log(pop_i pop_j) + b2 \cdot \log(GDP_i GDP_j) - b3 \cdot \log(f \cdot d_{ij})$$

Then, just by calling $y_i = \log D_{ij}$, $b_0 = \log k$, $x_i = \log(pop_i pop_j)$, $u_i = \log(GDP_i GDP_j)$ and $w_i = -\log(f \cdot d_{ij})$, it is possible to obtain the required coefficients by implementing the following problem:

$$\underbrace{\begin{bmatrix} 1 & x_1 & u_1 & w_1 \\ 1 & x_2 & u_2 & w_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_N & u_N & w_N \end{bmatrix}}_{\mathbf{X}} \underbrace{\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{\mathbf{\Phi}} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_n \end{bmatrix}}_{\mathbf{Y}}$$

where N = Number of European Airports, and that can be solved in this way:
$\Phi = \left(X^T X\right)^{-1} X^T Y$. By doing so, it is obtained:

- k = 0.00376136
- b1 = 0.539187
- b2 = 0.560755
- b3 = 1.32012

Now that this coefficients are known, it is necessary to forecast both, population and GDP to obtained the demand in 2022 by using Equation 14. A linear variation is assumed to do so.

As population and GDP values are known in both 2010 and 2017, it is now a Linear single variable regression problem, in order to obtained the fitting line between those two values. Afterwards, this trend line will give us the new values at the following years.

To calibrate Linear single variable regression line, a similar approach as done before can be performed:

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where a_0 and a_1 are the coefficients of the linear fit line $y = a_0 + a_1 x$, x_1 and x_2 are the values of the two years 2010 ($x_1 = 0$) and 2017 ($x_2 = 7$), and finally y_1 and y_2 are the population/GDP values in the years 2010 and 2017.

Once the fitting line is obtained, just by evaluating this equation with the new year 2022 ($x_3 = 12$), the new forecast values of GDP and Population are obtained.

Finally, by introducing those new values in 2022 of population and GDP in each airport, together with the gravity coefficients, it is possible to forecast the new demand in 2022 by using Equation 14.

2.1 Analysis of results

Due to differences in GDP and population expectation in each airport, there are positive and also negative growing on demand, mainly around + or - 20%. Focusing on our hub, London, these higher growth are on the routes with more demand (Paris, Amsterdam and Dublin), and a significant decrement is concentrated on Spain (Madrid and Barcelona) and Italy (Rome) because of lower expectations on GDP and population for 2022.

Top 3 Positive Growth		**Top 3 Negative Growth**	
Paris	126 (+13%)	Madrid	36 (-15%)
Dublin	106 (+20%)	Barcelona	31 (-16%)
Amsterdam	71 (+11%)	Rome	24 (-12%)

Nevertheless, the principal change on demand respect before is the opening of flights to US, even more, due to a scaling factor of 10, which gives a considerable demand from the whole European network to US destinations, especially New York. In these cases, demand goes up to 830 potential passengers. All US cities have a demand between them higher than 1000, but the airline cannot operate those as it lacks the required Air Services Agreement with the US.

2 Problem 2.2

By making use of the demand forecast in 2022, here new results obtained by flying to US destinations with a fleet expansion are studied.

2.2 Steps to adapt this new model

The steps made in order to adapt code from Problem 1 to the new requirements in Problem 2.2. The reader will be redirected to Problem 2.2 Notation, Objective Function and Constraints (explained afterwards) when needed.

As step zero, new data from Excel sheet should be included as step zero, considering, for example, all nodes (European and US) and slots in each airport [3].

First of all, flights from US should be considered. To do so, it is necessary to take into account that some factors (such as Yield, Load Factor) change from European to US flights. Then, those changes should be implemented (see new Notation from Problem 2.2 and its Constraints that affect that notation parameters). The new constraints needed because of US flights will be explained afterwards.

Secondly, it should be clear what are our new decision variables. We start considering the already ones x_{ijs}, w_{ijs} and z_{ijs}, but now the total number of aicraft changes, because now it is possible to lease a new aircraft and cancel an already existing contract. Those decision variables will be denoted as a^K, and b^K (cancelled leasing contracts and new

[3] Slots from Hub are unlimited according the statement guidelines

leasing contracts, see Notation). Now that the program is ready to include US flights and the decision variables have been analysed, it is time to included all changes related to this new variables.

To start with, lease matrix should be created, containing the weekly lease cost of each aircraft type k (C^k_{Lea}). Notice that this amount should be also given back if a lease contract is cancelled. Those operations have different fees associated which can be seen on Problem 2.2 Notation and Objective Function. [4]

Then, objective function is redefined (Equation 15). Notice that cancelled aircraft cannot be higher than the ones our airline already has, so upper bound of this decision variable is included.Demand and Transfer Passengers constraints (Equations 9 and 10) are the same than in Problem 1, so no changes apply there.

Nevertheless, Capacity Constraint needs to be updated (16). Specially, new constraints affecting US flights should be considered. On one hand, it is only possible to fly to US from the Hub (and viceversa). On the other hand, Aircraft types 4 and 5 cannot fly inside EU. Those changes are expressed with the binary variables σ_{ijk} and Φ_{ij} (see Problem 2.2 Notation).

Balance constraint (Equation 12) remains the same as in Problem 1. Utilisation constraint (Equation 13) is updated, as the number of aircraft is no longer constant. Therefore, the utilisation time of the initial number of aircraft (AC^k, see notation) should be updated with the new fleet because of those new leasing or cancelled contracts.

Finally, two more constraints are needed to implement: first, slots constraints are included (Equation 19) to ensure the maximum slot capacity at each airport is not surpassed. Second, the maximum weekly capacity to the US is also included (Equation 20), to limit the maximum seats between the hub and the US destinations and viceversa.

2.3 Problem Notation

In this problem, notation from Problem 1 is still valid. Nevertheless, there are more Sets, Decision Variables and Parameters that should be defined. The reader is therefore redirected to Problem 1 to take any notation not explained here and used in the problem formulation.

Sets

- **E**: set of load factors, either intra-European or to and from the US flight legs

Decision Variables

All decision variables have a lower bound at 0, and a^k has an upper bound at the initial number of aircraft.

- a^k: cancelled leasing contracts from each aircraft type k
- b^k: new leasing contracts from each aircraft type k

Parameters

- n_{EUR}: number of European airports.

[4] Although the assignment is not focused on code implementation, it is worth mention that before defining them, it is mandatory to update the indexing function and incorporate this new decision variables.

- n_{TOT}: total number of airports (Europe + US).
- C^k_{Lea}: weekly amount to pay for a new aircraft (also, the amount to receive weekly if the contract is cancelled), depends on aircraft type k.
- f_N: weekly contract fee to pay for a new aircraft contract (2,000 €).
- f_C: weekly contract fee to pay for a cancelled aircraft contract (8,000 €).
- MC: maximum weekly capacity between hub and US destinations (7,500 seats).
- AC^k: initial number of aircraft of each type k
- Y_{ij}: yield for a flight between i and j.
- LF_{ij}: load factorfor a flight between i and j.
- σ_{ijk} binary parameter: $\sigma_{ijk} = 0$ if $k \in [4,5]$ and if both $i \in [1, n_{EUR}]$ and $j \in [1, n_{EUR}]$; $\sigma_{ijk} = 1$ otherwise.
- Φ_{ij} binary parameter: $\Phi_{ij} = 1$ if both $i \in [n_{EUR}+1, n_{TOT}]$ and $j = 1$ or if both $j \in [n_{EUR}+1, n_{TOT}]$ and $i = 1$; $\Phi_{ij} = 0$ otherwise.
- ρ_i binary parameter: $\rho_i = 1$ if $i \in [n_{EUR}+1, n_{TOT}]$; $\rho_i = 0$ otherwise.
- ρ_j binary parameter: $\rho_j = 1$ if $j \in [n_{EUR}+1, n_{TOT}]$; $\rho_j = 0$ otherwise.

2.3.1 Optimisation Problem

Objective Function

$$\sum_{i \in N} \sum_{j \in N} \left[Y_{ij} \cdot d_{ij}(x_{ij} + w_{ij}) - \sum_{k \in K} C^k_{OP_{ij}} \cdot z^k_{ij} \right] + \sum_{k \in K} \left[a^k \left(C^k_{Lea} - f_C \right) + b^k \left(-C^k_{Lea} - f_N \right) \right] \tag{15}$$

This objective function represents the profit of the airline, to be maximised, expressed at the revenue minus the operating costs minus the leasing and fees costs of new aircraft plus the savings obtained by cancelling leases.

Constraints

Find below the different constraints that apply into Problem 2. Some constraints are shared with Problem 1. When this is the case, the reader is redirected to that section.

Demand Constraint
(See Problem 1)

Transfer Passengers Constraint
(See Problem 1)

Balance Constraint
(See Problem 1)

Restricted Capacity Constraint

$$x_{ij} + \sum_{m \in N} w_{im} \cdot (1 - g_j) + \sum_{m \in N} w_{mj} \cdot (1 - g_i) \leq$$

$$\leq \sum_{k \in K} z^k_{ij} \cdot s^k \cdot LF_{ij} \cdot \delta ijk \cdot \Psi jk \cdot \sigma_{ijk} \cdot \Phi_{ij}, \ \forall i, j \in N, \ \forall i, j \in N \tag{16}$$

Although this equation exists on Problem 1, some modifications need to be done here.

First, Load Factor is not a constant anymore, as it is different depending on if we consider European or US flights. Then, binary variables should be added such that capacity is only added if a certain number of constraints are fulfilled. Here, not only Range and Runway constraints are included (see Problem 1) but also Freedoms of Air and Aircraft Type restrictions. Regarding Freedoms of Air, it is only possible to reach US destinations from the hub. This is expressed by Φ_{ij}. Last, Aircraft types 4 and 5 cannot be used in intra-European flight legs. Therefore, capacity is only added when this situation does not happen. It is expressed by σ_{ijk}

Utilisation Constraint

$$\sum_{i \in N} \sum_{j \in N} \left(\frac{d_{ij}}{sp^k} + LTO \right) \cdot z_{ij}^k \leq BT^k \cdot AC^k - a^k \cdot BT^k + b^k \cdot BT^k, \quad \forall k \in K \tag{17}$$

This constraint, also used in Problem 1, is modified by introducing the utilization time because of new contracts (more time), or the cancelled contracts (less time).

Slots Constraints

$$\sum_{k \in K} \sum_{j \in N} Z_{ij}^k \leq Slot_i \quad \forall i \ \in N \tag{18}$$

$$\sum_{k \in K} \sum_{i \in N} Z_{ij}^k \leq Slot_j \quad \forall j \ \in N \tag{19}$$

Which are equivalent by means of the Balance Constraint. In this constraints, the number of flights is restricted to the available slot quantity, both for the origin airport i and the destination airport j. Maximum Weekly capacity Hub-US

$$\sum_{k \in K} \sum_{i \in N} \sum_{j \in N} \rho_i \cdot Z_{ij}^k \cdot Seats^k \leq MC$$

$$\sum_{k \in K} \sum_{i \in N} \sum_{j \in N} \rho_j \cdot Z_{ij}^k \cdot Seats^k \leq MC \tag{20}$$

Those two constraints means that the maximum number of seats between the hub and any US destinations should not exceed the maximum capacity (MC). One constraint is for flights between Hub and US, and the other one for flights between US and the Hub.

2.4 Analysis of results

All these new constraints build a new model, specially due to incorporation of US flights, which give results quite different than Problem 1. ASK and RPK are 8 times higher because of these transatlantic flights and new units of aircraft. But Cost per ASK, Revenue per ASK and Yield per RPK are a little bit smaller for the same reason, more distance travelled but with similar unit costs (preference of single-aisle aircraft even for US flights). The average network load factor goes up considering that flights to the States have a load factor of 85%, also the Break-Even Load Factor increases, having a narrow margin profit (around 7%) depending more on economies of scale, this is a typical characteristic of transatlantic airlines. Finally, as the airline is moving more passengers with lower costs, total profit is almost 3 times than previous case, almost 200,000 € for one week.

Key Performance Indicators	
ASK	67,208,243 ASK
RPK	55,828,326 RPK
CASK	0.0408 €/ASK
RASK	0.0437 €/ASK
Yield	0.0527 €/RPK
ANLF	83.07%
BELF	77.43%
Operative Results	
Costs	2,740,017.67 €
Yield	2,939,646.18 €
Balance	**199,628.52 €**

Leasing table				
Type aircraft	Current	Initial	New	Cancel
Turboprop	2	2	0	0
Regional Jet	1	1	0	0
SA Jet	7	1	6	0
TA Jet	1	0	1	0
LR Jet	0	0	0	0

Table 2: KPI's and Operative Results Exercise 2

Table 2 shows number of leased aircraft, keeping same aircraft as previously (2 turboprop, 1 regional jet and 1 single-aisle jet), but adding 6 more single-aisle jet (mainly for operations to New York) and 1 two-aisle jet (used for long range destinations: LA and Houston).

The route map of the company have changed adding 3 US destinations and also more European destinations, as Bucharest or Heraklion, attracted for these new routes to North America. More than the half of European cities would have transfers passengers to US, specially to New York. This route would be the one with the highest capacity of the network, 3450 seats would be offered per week to New York, operated with 3 daily flights each direction. From this number, 830 would be direct passengers from London but the rest (75%) would be connecting passengers. Some of them from cities with more connecting than direct passengers because the more profitable routes are the ones to the US, e.g. Berlin would have 456 and 219 respectively or Frankfurt with 200 connecting passengers to New York and just 25 direct passengers to London.

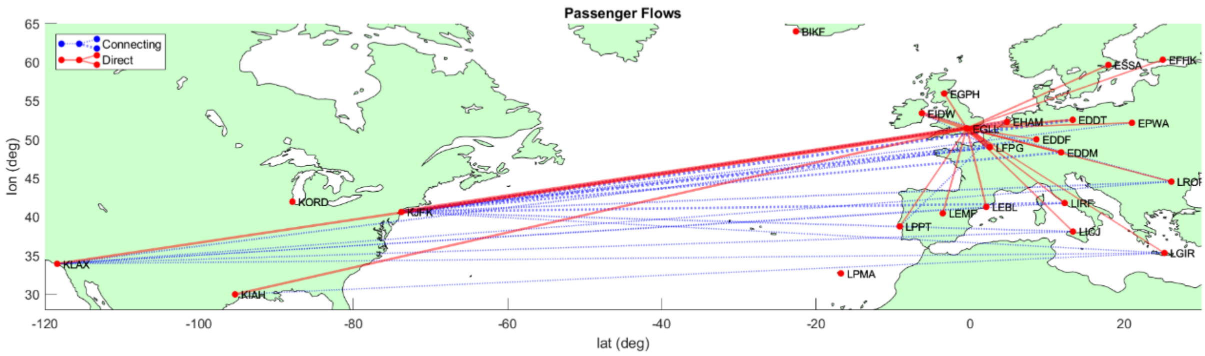

Figure 4: Route map of Westpoint Airways with US flights

Use of aircraft has changed adding a new type a model, a two-aisle jet (i.e. Airbus A330-300), which provides more capacity and range than previous ones. This type would have the smallest amount of operation hours but would be the second on ASK because of long routes to the US. Anyway, the dominant type of aircraft in this case, both hours and ASK, would be single-aisle. Being the model with more units (6), 2-3 units are exclusively used for NY flights every day, and the rest of the units operate to almost all European destinations. The ones which are closer and high demand (Paris, Amsterdam and Dublin) continue being operated by smaller aircraft, turboprop and regional jet.

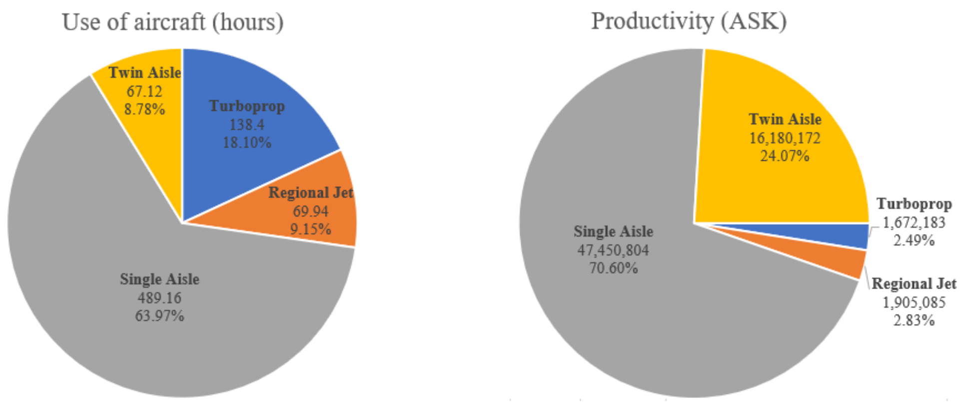

Figure 5: Use of aircraft with US flights

3 Problem 3

3.1 Problem Methodology

This problem is the continuation of Problem 2.2, applying a more refined model taking into account a variable demand between high and low season, which depends on market share. Two weeks will be planned (high demand season and low demand season) with a constant fleet to maximise the overall profit of the airline. As market share depends on frequency; which depends on demand as well, an iterative process should be performed to achieve the most precise values.

To do so, first of all, all problem is computed (objective function, constraints) according to Problem 2.2 (but considering two weeks instead of one -new index 's' will be added-). Demand, changing parameter in each iteration, is affected by two constraints (Demand Constraint and Transfer Passengers Constraint). As a result, those will be included last in the matrix. Then, new frequency is calculated according to previous iteration results. Thanks to that frequency, market share can be recomputed, and therefore, the new demand. With this new demand value, previously computed Demand and Transfer Passengers Constraint are deleted from previous model; and then computed again with the new refreshed demand value.

Notice that Market Share can be obtained from Equation 3.1,where FS_1 is the frequency share of the airline which is needed to compute the market share, consisting of $FS_{1,d}$ (frequency share of direct flight), and $FS_{1,i}$ (frequency share of indirect flight). FS_2 is the market share of the other airlines. Finally, coefficients a and b, accounts fro competition parameters, whose values will be assumed to be $a = 1$ and $b = 1.7$ in this model.

$$MS_1 = \frac{FS_{1,d}^a + FS_{1,i}^b}{FS_{1,d}^a + FS_{1,i}^b + FS_2^a} \tag{21}$$

It is important to highlight how $FS_{1,i}^b$ is computed. According to statement guidelines, when a flight is consisting on multiple legs, the minimum leg is leading. As a result, the minimum value between $FS_{1,i}^b$ and $FS_{j,1}^b$ is used. Nevertheless, if there is no frequency, (minimum is zero), the market share goes to 0 or numerical errors if the competition does not operate flights either. To deal with that, a hypothetical flight is introduced if the current offer is 0, which produces converged results and no numerical error.

This iteration process will last up to a certain number of iterations, which depends on the accuracy needed and the computation time needed to do so.

3.2 Problem Notation

Notation from Problem 1 and Problem 2 is still valid, so just the new notation is explained here.

Sets

- **S**: set of seasons, either high demand season or low demand season.

3.2.1 Optimisation Problem

Objective Function

$$\sum_{s\in S}\sum_{i\in N}\sum_{j\in N}\left[Y_{ij}\cdot d_{ij}\cdot(x_{ijs}+w_{ijs})-\sum_{k\in K}C^k_{OP_{ij}}\cdot z^k_{ijs}\right]+\sum_{k\in K}\left[a^k\left(C^k_{Lea}-f_C\right)+b^k\left(-C^k_{Lea}-f_N\right)\right] \tag{22}$$

The same objective function as Problem 2.2 applies, but now the two weeks period should be summed. Notice that decision variables x_{ijs}, w_{ijs} and z_{ijs} now depend on the season considered, whereas A^k, and B^k are fixed (so the number of aircraft should remain the same for the two seasons).

Constraints

Same constraints as in Problem 2.2 apply here. Nevertheless, there are two main differences between those approaches: now, as said before, decision variables x_{ijs}, w_{ijs} and z_{ijs} depend on the season considered; and demand is computed according to market share. The reader is redirected to Problem 2.2 to know the constraints that apply, considering those new changes.

3.3 Analysis of results

This last scenario will divide all results among two different weeks, one high-demand and other low-demand, but the leasing table will be constant for both situations. In comparison with the previous case, ASK and RPK have almost doubled their values, basically due to more flights and capacity to the US. But the CASK, RASK, yield and ANLF have maintained similar numbers, in concordance of a low-cost airline. Also the break-even load factor has increased to 80% on high-demand and overtakes a little bit the ANLF on low-demand, what will lead to a negative profit of -44,982 €, although in high-demand would make a profit of 143,012 €. Total costs and revenues have also doubled, new routes with 3 units of the biggest aircraft (long-range jet, 400 seats, i.e. Boeing 777-300ER). In the leasing table, besides these long-range jets, one unit of turboprop has been cancelled in exchange for more use of larger aircraft, and 7 more units of single-aisle jet have been leased again mainly for NY flights and the rest of European destinations.

Key Performance Indicators		
	High Dem.	Low Dem.
ASK	111,723,454	106,652,557
RPK	93,123,103	88,947,099
CASK	0.0411	0.0416
RASK	0.0424	0.0412
Yield	0.0509	0.0494
ANLF	83.35	83.40%
BELF	80.84	84.25%
Operative Results		
Costs	4,595,758 €	4,439,757 €
Yield	4,738,770 €	4,394,775 €
Balance	**143,012 €**	**-44,981.8 €**

Leasing Table				
Aircraft	Current	Initial	New	Cancel
Turbop	1	2	0	1
Reg Jet	1	1	0	0
SA Jet	8	1	7	0
TA Jet	0	0	0	0
LR Jet	3	0	3	0

Table 3: KPI's and Operative Results Exercise 3

Main differences between the two seasons will be on demand, which will transform some routes and passengers flows, having more connections passengers due to less demand for direct passengers. This affirmation is possible to appreciate on Figure 6, where all European airports have connection passengers to the US in low season (except Reykjavik that only has connections to Europe), even some cities as Frankfurt, Munich or Rome do not have direct passengers to London but they have flights to provide for transfer

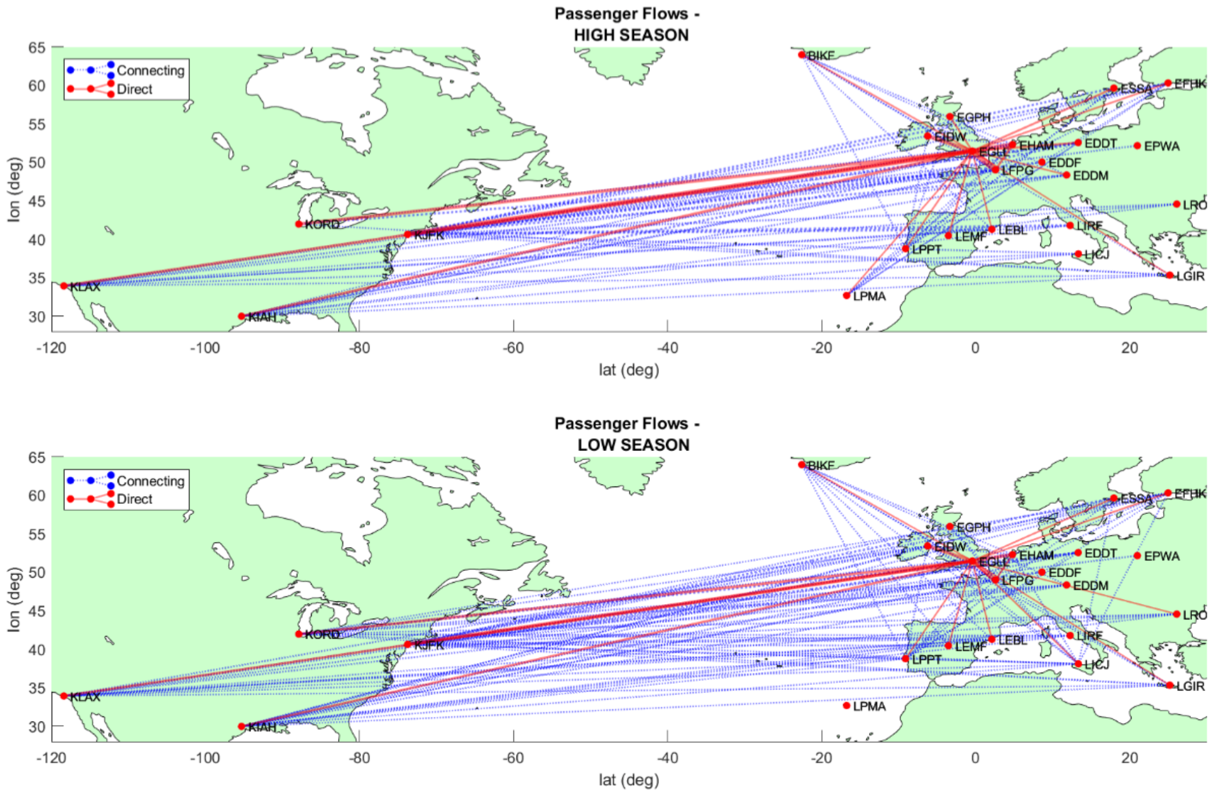

Figure 6: Passenger Flows (Connecting and Direct) - High/Low Demand

passengers. Another component of this variety of flights is competition, which changes some demand on important routes like Paris or Amsterdam, or European connections to New York. Although frequency of this competition is low, around one daily flight, meanwhile frequency of this airline to Paris (more frequent route) is 5 or 6 daily flights. Considering both weeks, annual profit would be positive. Assuming an operational year with 26 weeks of high season and 26 of low season, the total annual profit would be 2.548.785 €

Finally, Figure 7 represents differences of use of aircraft in both situations. As the previous case, single-aisle jet has high utilisation due to European and New York Flights, nevertheless twin-aisle jet has been changed for long range jet with larger capacity which make this type of aircraft as predominant in ASK and operation hours. This jet is only used to all 4 US destinations. Now 3 units of this aircraft are leased in comparison of just one two-aisle jet previously. Also, a remarkable thing is the non-use of turboprop aircraft in low-season. The demand of turboprop destinations (Paris, Amsterdam and Dublin) goes down, e.g. in Paris with high demand there are 1100 direct passengers and 800 connecting passengers to just 650 direct and 500 connecting at low demand week. Frequency in that case is affected severely, from 5-6 daily flights to 2-3, using regional and single-aisle jets.

That lack of use of turboprop aircraft during low season and maybe even the cancellation of one lease can be due to the problem implementation: as the relation between market share and frequency is non-linear, the linear optimiser only receives a constant demand for

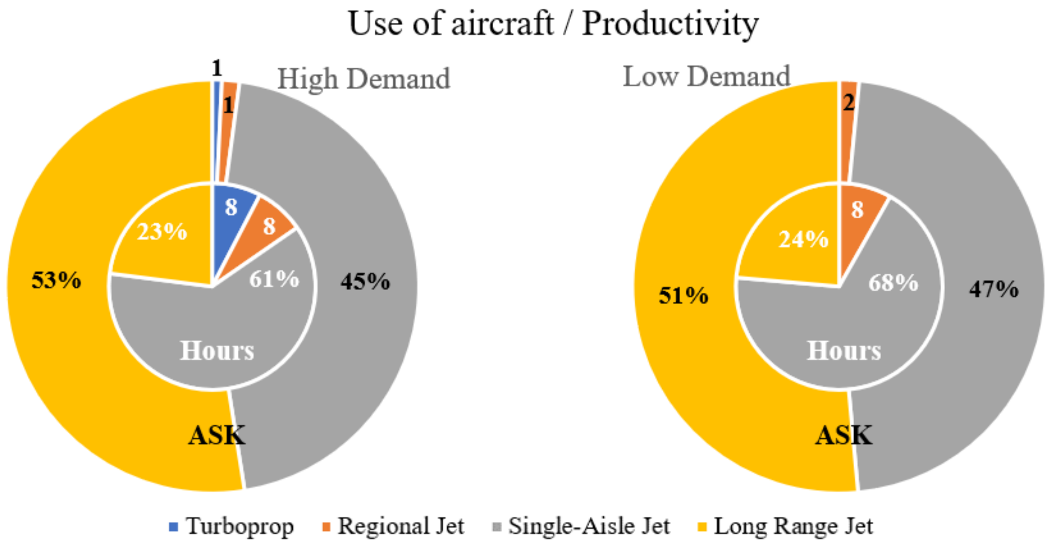

Figure 7: Use of aircraft and aircraft Productivity - High/Low Demand

which it produces a maximum-revenue solution, from which the market share is calculated and the process repeated until convergence. In that structure, the optimiser does not *know* that the frequency solution changes the demand, and therefore produces a frequency plan that does not take into account the fact that a high frequency produces better demand. To overcome that problem, a better model can be developed in future studies: demand can be considered (for both high and low season) as a decision variable, adding a new constraint linking the demand to the frequency via a first order Taylor expansion of the market share, so that the problem would still be linear. Then, iteration would still be necessary to adjust the constants in the Taylor expansion until convergence is reached, probably to a solution with better profits.

Conclusions

To summarize this report, first a mathematical model to calculate maximum profit for an airline has been introduced, in this case it has been applied for Westpoint Airways with its hub in London (U.K.). First results are just for the European network, having 14 different destinations and using 3 small and mid-size aircraft (Regional turboprop, regional jet and single-aisle jet).

In the next step, it has been introduced US destinations (New York, Chicago, Houston and Los Angeles) in 2022, for that a new demand has been calculated using a forecast based on population and GDP. The same mathematical model as before has been used to calculate maximum profit, but adding new constraints about capacity in US, slots, range and runway length, and new/cancel leasing. These new results show a larger network, with more European routes especially interested on connecting flights to US. In this country, New York is the most demand route with transfers to all European destinations operating 3 daily flights with single-aisle jets, having enough range from London to New York to operate this kind of jet. Although LA and Houston also have their own connecting passengers but using a larger aircraft, a twin-aisle jet.

In the last scenario, it has been applied two different weeks of demand situations (high and low) while also considering market share of Westpoint Airways. Based on the same mathematical model used previously (with small variations to accommodate new constraints), results are displayed. For high season, there is a positive profit flying to all 4 US destinations from almost all possible European destinations via London. Besides using several single-aisle jets, 3 long range aircraft are used for US airports. However, in low season, there is a negative profit due to high costs of aircraft fleet and a lower demand. Number of connection passengers to the US increases because demand from London to American destinations decreases; but those seats can be replaced with transfer passengers from Europe network, so demand is served trying to minimise losses.

As possible improvements for this study, a more precise model on demand and revenue would be necessary, considering more variations based on reality and using technologies like machine learning for better forecasts. Also, it has been studied how basic parameters change the total results. In the case of petrol price, with an increasing of 10%, it would alter its weekly average positive profit of 50.000 €, to loses of 125.000 €. This represents how volatile is this industry, a small change on something that airlines can not control would be decisive on their final economical results. It is worth mentioning that computational time increases significantly when performing this change.

To conclude this report, a small group analysis of this situation has been done. Although this is an academic report which does not represent a total reality (e.g. no consideration of flight timetables or days of operation), economic inputs and outputs make the idea that Westpoint Airways is a LCC with its hub on a strategic location (London) between US and continental Europe. As it has been said before, their costs are similar to the ones of a Low-Cost Carrier, and prices are also in the same line, a ticket London-Paris with Westpoint Airways would cost 39 € and London-New York 275 €. This airline would be the style of Norwegian (which has hubs at London-Gatwick and Scandinavian capitals) or WOWair (hub in Iceland), low-cost airlines with a hub between mainland Europe and America, operating mainly with a fleet of single-aisle jets when range allows it, or long-range aircraft when is necessary.

References

[1] Bruno Santos. Aircraft planning & optimisation slides, 2018.

[2] Center for aviation. `https://centreforaviation.com/analysis/reports`. Accessed: 2018-12-15.

[3] Peter Belobaba, Amedeo Odoni, and Cynthia Barnhart. *The Global Airline Industry, Second Edition*. John Wiley Sons, 2015.