AE4-205 MDO for Aerospace Applications 2019/2020

Homework Assignment

Multi-disciplinary optimization of a wing to minimize aircraft take off mass

The objective of this assignment is to optimize the wing (planform and airfoil shape) of an assigned reference aircraft, in order to **minimize the aircraft take off mass.** To this purpose a multidisciplinary optimization problem will be set up, using the **MDF (Multi Discipline Feasible) architecture, with the Gauss-Seidel coordination scheme**, including the following distinct disciplines:

1. **Loads**
2. **Structures**
3. **Aerodynamics**
4. **Performance**

The level of detail will be that typical of the conceptual design phase.
No other MDO architecture and coordination schema than what stated above is allowed.
To support the set-up of your MDO framework, an aerodynamic analysis tool - to be used also for loads estimation - called *Q3DSolver*, and a structural sizing tool, called *EMWET*, are provided via Brightspace. Concerning the performance, you will take care of implementing the fuel fraction methods based on Brequet equation, as detailed later in this document. These four discipline blocks work as black boxes and their output does not include any gradient information.

You will obtain your reference aircraft (the one for which you have to optimize the wing) by enrolling to one of the MDO groups made available on Brightspace. In the moment you enroll to an MDO group, a matrix of numbered aircraft data sets (stored as Excel file) will become visible to you, in the same Brightspace section “Assessment policy- Deadlines- Assignment” where you found this assignment. **Your reference aircraft is the one with the same number as your MDO group number**. You are <u>not</u> allowed to choose any other aircraft.

<u>Requirements on wing parameterization</u>

You will parameterize the **wing planform** as a composition of **two trapezoidal components**, in order to model **the typical kink in the wing trailing edge (TE)**. Generally, the trailing edge segment spanning from the wing root to the kink is unswept (or barely swept) to guarantee the maximum efficiency of the inboard high lift devices (HLD) and to provide space for a landing gear support spar. Since both HLDs sizing and landing gear positioning are <u>outside the scope of this MDO study</u>, you will make sure this kink is maintained (if originally present) during the planform optimization, thus: BOTH the sweep angle of the trailing edge of the inboard wing segment AND the spanwise distance of the kink with respect to the fuselage centerline will remain constant.
In case your reference aircraft wing has no kink, you will still parameterize the wing by means of **two trapezoidal elements**, where the span of the inboard trapezium will be assumed equal to 40% of the semi span the original reference aircraft. Also in this case, you will keep BOTH the TE

sweep of the inboard wing segment AND the spanwise distance between the introduced kink and the fuselage centerline constant.

Each one of the two trapezoidal components will have to be parameterized in order to allow the modification of their lifting areas, sweep and twist angles (the wing incidence angle, i.e. the angle between the wing root chord and the longitudinal fuselage axis can be kept constant), chord lengths and taper ratios. Only the span of the outboard wing will be allowed to change. You can assume constant dihedral angle(s).

Concerning the **wing outer shape**, you will define **two** different airfoils, one at the root and one at the wing tip. The shape of the airfoil at the kink will be defined as **linear interpolation** of the root and tip airfoils. Note that the provided aerodynamic analysis tool requires the airfoils to be defined using the 2D CST parameterization method. You shall use **CST-curves of order not lower than 5**, both for the upper and the lower part of each airfoil.

<u>Loads and Aerodynamic analysis</u>

A MATLAB tool, called Q3DSolver is available on Brightspace to analyze the aerodynamic performance of a given wing. The implemented aerodynamics models and the functionalities of this tool, its required input and generated output files are discussed in the tutorial material and tool documentation. Q3D can be operated with or without viscous calculations. Wave and profile drag are computed only when the viscous calculation mode is activated. The inviscid calculation mode is faster and suitable for wing loads estimation, but does not provide the required drag components necessary to predict the wing aerodynamic efficiency. Thus the same tool (operated with different input, of course) can be used both for the loads[1] and the aerodynamics disciplines.

<u>Initial point and design point</u>

Concerning the **initial design point** for your wing optimization, you will use the planform of the reference aircraft assigned to your team and suitable airfoils, i.e. the actual airfoils of the reference aircraft, if you can find them in literature, or airfoils of your choice, such as, for example, the Withcomb airfoil presented during the MDO tutorials, one scaled to a thickness ratio of about 14% for the wing root section and one scaled to 8% for the tip section).

The **design point** at which you will optimize the **is the mid-cruise condition**. The following semi-empirical relationship can be used to estimate the weight W_{des} of the aircraft (hence the design lift L_{des}) at the design point:

$$L_{des}\left(\bar{x}\right) = W_{des}\left(\bar{x}\right) = \sqrt{W_{TO\_max}\left(\bar{x}\right)\cdot\left[W_{TO\_max}\left(\bar{x}\right) - W_{fuel}\left(\bar{x}\right)\right]}$$

<u>Wing structural sizing & weight prediction</u>

A dedicated computational tool, called EMWET is provided on Brightspace, to perform the preliminary sizing of the wing structure and estimate the overall wing weight (including the weight of fixed LE and TE edges, control surfaces, High-Lift devices, etc.). An aluminium structure is considered.

[1] Thus, we assume here that the wing sizing loads will only be of aerodynamic nature

EMWET expects as input both the wing geometry (outer shape and internal layout) and the sizing aerodynamic loads, expressed as spanwise distribution of lift and moment, computed at the critical conditions specified below.

Concerning the wing structural layout, you will assume a simple two spars configuration, consisting of one front spar and one back spar, which will delimi the wing fuel tank. **Do not** consider the support spar that is often located in the inboard part of the wing, from root to kink, and used to support the main gear. It is **your choice** to keep the positions of the spars fixed or to include them as design variables. Typical positions of front and back spar are around 15-20% and 55-60% of the local chord, respectively. These positions changes according to the particular airfoil shape and the amount of space (as percentage of the chord) reserved for high lift devices. The rib pitch and the type of stringers (to be selected via EMWET input file among a number of possible alternatives) will be kept constant.

Note that, during the wing structural sizing process, EMWET automatically takes care of satisfying all relevant structural constraints (e.g., buckling) and delivers a minimum weight design, for the given sizing loads and structure configuration.

In order to compute the **sizing aerodynamic loads** for the wing structure, you **cannot** use the same flight conditions used to estimate the aerodynamic performance of the wing in cruise, neither the same aircraft weight! A reasonable estimation of the critical (thus sizing) aerodynamic loads can be obtained using the Q3DSolver at this flight condition: $\mathbf{n = n_{max}, W= W_{TO\_max}, h=h_{cruise}, V=V_{MO}}$ (where V_{MO} is the maximum operative speed of the aircraft, which is specified in the data set of your reference aircraft).

The necessary set of constants and reference values required for the wing structural sizing and weight estimation can be found in Table 1, or retrieved from the data set of the assigned reference aircraft.

Table 1: Constants and reference values to be used in the optimization.

Constant	Value
Aluminum elasticity modulus, E_{al}	70.10^3 N/mm^2
Aluminum tension yield stress, $\sigma_{yield,tens}$	295 N/mm^2
Aluminum compression yield stress, $\sigma_{yield,comp}$	295 N/mm^2
Aluminum density, ρ_{al}	2800 kg/m^3
Cruise speed, V_{cr}	To be taken from reference aircraft Excel table
Cruise altitude, h_{cr}	To be taken from reference aircraft Excel table
Maximum load factor, n_{max}	n_{max} to be selected according to **Table 2**
Rib Pitch	0.5 m
Maximum wing-loading, W/S	**Not higher*** than the maximum wing-loading of the reference aircraft (W_{TO_max}/S)
Reference aircraft design range	To be taken from reference aircraft Excel table
Type of stringer in wing-structure	Select one type from the manual of EMWET
*to allow fulfilling at least the same takeoff and landing requirements, with same HLDs	

Table 2: Maximum positive limit load factor n_{max} for different aircraft categories

CS	Aircraft type		n_{max}
CS-23	Normal + Commuter		$2.1+\left(24{,}000/\left(W_{TO}+10{,}000\right)\right)$
	Utility		4.4
	Aerobatic		6.0
CS-25	Transport	$\leq$ 4100 [lbs]	3.8
		4100 < $W_{TO} \leq$ 50,000 [lbs]	$2.1+\left(24{,}000/\left(W_{TO}+10{,}000\right)\right)$
		> 50,000 [lbs]	2.5

The total weight of the aircraft, W_{TO_max}, is determined by the following equation:

$$W_{TO\_\max}\left(\bar{x}\right)=W_{A-W}+W_{fuel}\left(\bar{x}\right)+W_{str,wing}\left(\bar{x}\right)$$

Where:

- W_{TO_max} is the maximum take-off weight of the aircraft (**which must be minimized in this assignment**).
- W_{fuel} is the weight of the fuel required to fulfill the design range,.
- $W_{A\text{-}W}$ is the weight of the entire aircraft (design payload included), excluding the weight contributions of fuel and wing.
- $W_{str,wing}$ is the structural weight of the wing. This is the weight of the complete wing (with empty fuel tanks), which can be determined using EMWET.

A-W group (aircraft less wing) contributions

The weight contribution $W_{A\text{-}W}$, can be considered constant during the optimization, hence independent of the wing weight. Although this is generally not true (can you explain why?), it is a suitable assumption at this stage of the design. This contribution can be evaluated **once**, using the data of your reference aircraft and running EMWET for the reference wing design.

Concerning the aerodynamic contribution of the A-W group (i.e., fuselage, tail and nacelles), it shall be assumed that the wing is the only component that generates lift, so the A-W group does not produce any lift. Besides that, the **drag** generated by the A-W group can be assumed constant under the flight-condition of interest and irrespective of the wing design. Please note that while the value of the A-W **drag** contribution can be assumed constant, the associated **drag-coefficient** will vary in relation with the reference area of the wing!

Hint: you can derive the A-W group drag contribution using the reference aircraft data provided for this assignment and assuming the following relation for the overall aircraft C_L/C_D-ratio:

$$\frac{C_L}{C_D}=\frac{C_{L,wing}}{C_{D,wing}+C_{D,A-W}}$$

Note that, in order to separate $C_{D,wing}$ and $C_{D,A\text{-}W}$, you will have to run the Q3DSolver aerodynamic analysis tool **once** for the reference wing-design. For the reference aircraft you may assume an overall C_L/C_D-ratio at cruise equal to 16 (**unless you have a more reliable value at hand, from literature**).

Performance

The amount of fuel necessary to perform the mission (design range), can be estimated using the well-known fuel fractions method, where the Breguet-range equation is used to estimate the cruise fuel fraction (see equations below).

$$R = \frac{V}{C_T} \cdot \frac{L}{D} \cdot \ln\left(\frac{W_{start-cr}}{W_{end-cr}}\right)$$

$$W_{fuel} = \left[1 - 0.938 \cdot \frac{W_{end-cr}}{W_{start-cr}}\right] \cdot W_{TO\_\max}$$

In the equations above, $W_{start-cr}$ and W_{end-cr} are the aircraft's weights at the start and at the end of the cruise-phase, respectively.

The factor 0.938 in the equation above accounts for the fuel fractions used outside of the cruise-stage of the flight (i.e., taxi, take-off, climb, descent, etc).

For this problem, the engines' **specific fuel consumption** C_T may be taken as 1.8639 10^{-4} N/Ns (**unless you get a more reliable value from literature**).

Fuel tank

The amount of fuel required to fly the mission must fit inside the wing integral fuel tanks. The fuel tanks are assumed to be placed between the front and rear spars. The following constraint applies:

$$V_{fuel}\left(\bar{x}\right) \leq V_{\tan k}\left(\bar{x}\right) \cdot f_{\tan k}$$

$$V_{fuel}\left(\bar{x}\right) = \frac{W_{fuel}\left(\bar{x}\right)}{\rho_{fuel}}$$

In which f_{tank} is a factor to account for the wing-tank volume occupied by structural elements, fuel systems, unusable fuel, gas, etc. You may consider **f_{tank} as 0.93**. For aviation **fuel a density** ρ_{fuel} of $0.81715 \cdot 10^3$ kg/m^3 may be assumed.

Note, that fuel tanks generally extend from the aircraft center line or from the fuselage/wing intersection span station up to (about) **85% of the wing span**, being the tip area at higher risk of (lightning) strikes.

The volume of the integral fuel tanks (i.e., the part of the wing-box used to accommodate fuel) will have to be determined through adequate geometric evaluations and must be consistent with the definition of the wing shape and structural layout (e.g. thickness and curvature of the airfoil and chordwise position of spars).

Assignment deliverables

What

The deliverables for this course consist of the Matlab implementation of the requested MDO system and some reporting, split in two parts as follows:

- **Part 1:** report including the wing parameterization, the formal statement of the optimization problem according to required MDO architecture and a detailed XDSM
- **Part 2:** Matlab code + report containing generated results and their discussion

The specific requirements and scores for Part 1 and 2 are provided below in this document.

When

All deliverable material will have to be **uploaded** on Brightspace within the deadlines stated in the section “Assessment policy - Delivery deadlines – Assignment”.
You can decide to deliver Part 1 and Part 2 together, not later than the set deadlines , or – as warmly suggested – to deliver first Part 1 (well before the deadline) and ask for its quick assessment. This will prevent you to waste time implementing in Matlab a wrongly defined MDO system.
Please note that **Part 2 will be checked <u>only if</u> the score for Part 1 is >50%** (20/40 points).
For the overall Assessment policy, please refer to the section “Assessment policy - Delivery deadlines – Assignment”.

PART 1 Deliverables:

Part 1 yields 40% of the total score for the assignment (i.e. 40/100 points).
A minimum of 20 points is necessary to “pass” Part 1 and have Part 2 assessed

Part 1.1 Parametrization and Optimization problem specification (15 points)

Formal specification (i.e., in mathematical terms, not only in words!) of the MDO optimization problem, as implemented using the **requested MDO architecture**.
This will include:

- The full design vector
- The bounds on the design variables
- The objective function (equation)
- The inequality constraints, if present (equations)
- The equality and consistency constraints, if present (equations)

Make sure to provide a **nomenclature table**, where you shall indicate **all** the used symbols, their description and units.
Make use of **simple drawings** to clearly illustrate the **adopted parameterization** and clarify, for example, how you make use of the selected design variables to evaluate the constraints.
Very briefly justify your choices (e.g. why that parameterization, why those bounds, etc..).

Part 1.2: XDSM (25 points)

Provide a detailed formalization of the (to be) implemented MDO architecture by means of an Extended Design Structure Matrix (XDSM).
Make sure to have one block per discipline and to use one separate block for the objective function and **one block for each** single inequality and equality constraint. In case of consistency constraints, you can use one single block for the whole set.

Indicate **<u>in detail</u> and explicitly** (thus, not just “wing geometry”) the input and output to/from each block in the XDSM. Don’t just provide generic symbols but those specified in the nomenclature table assembled in Part 1. Write explicitly also the objective function and constraints (thus not just f, h_i, g_i, etc.).
Include in your XDSM **both the sequential numbers** of the MDO process steps and the **process lines**.

For the generation of the XDSM feel free to use dedicated editing tools (for example those available here[2]) or generic diagramming tools like Visio, as far as you **fully** respect the XDSM style presented in literature and in the lecture notes.

A proper XDSM allows an exhaustive description of any MDO system; therefore extra text and diagrams are not necessary to illustrate your MDO implementation. Any other diagram different than the XDSM will not be considered to the purpose of the evaluation.

<u>PART 2 Deliverables:</u>

Part 2 yields 60% of the total score for the assignment (i.e. 60/100 points).
It is assessed only when a minimum of 20 points has been scored for Part 1.
You will upload on Brightspace the Matlab **code** and a **report** organized according to the subparts described below.
Do **not** include in your report any printout of the matlab code, not even in Appendix.

Part 2.1: MDO system set up and results (50 points)

The following items shall be included in the report:

- Your selected **termination criteria** and **tolerances** for the optimization process
- **Criteria responsible for the optimization process termination** (include a printout of MATLAB termination message)
- Table comparing the **initial <u>and</u> the optimized design**. **For both** you will include the following values and figures of merit:
    - objective function value
    - design vector
    - value of all constraints
    - W_{fuel} fuel weight
    - W_{TO_max}
    - W_{str_wing} wing structure weight
    - *Fuel volume* V_{fuel} and available tank capacity V_{tank}
    - C_L, $C_{D,wing}$ and C_L/C_D at design point
    - Plot of the spanwise lift distribution ($C \cdot C_l$) at the design point
    - Plot of the spanwise drag coefficients at the design point. Show in the same plot <u>two</u> separate curves, one for the induced drag and one for the profile and wave drag contributions combined
    - Plot of the spanwise lift distribution ($C \cdot C_l$) at the critical conditions used to size the wing. Specify the critical conditions (load factor, weight, flight conditions) in the plot caption

[2] http://mdolab.engin.umich.edu/content/xdsm-overview

- $D_{A\text{-}W}$ (constant during the optimization)
- $C_{D,A\text{-}W}$
- $W_{A\text{-}W}$ (constant during the optimization)
- Wing area, wing loading, sweep angles (for each wing trapezoidal element), chords, span (for the whole wing and for the two wing trapezoidal elements), aspect ratio and twist angles
- Plot of the convergence history of the objective
- Convergence history of **each single** constraint. You can use a single plot, as far as you provide a proper legend.
- Overlapped plots of the initial and optimized airfoils (only root and tip). Plot them for a chord of length 1.
- Overlapped plot of the initial and optimized wing planform
- plot in **isometric** view of the final wing shape (possibly overlapped with initial wing shape)

* Table including the following values:
    - Time needed to converge to optimum (or to reach termination) (use *tic..toc* in Matlab)
    - number of **iterations** and **objective** function evaluations required
    - average time required per iteration

Part 2.2 Conclusions and Critical reflection (10 points)

Briefly discuss the obtained results. Make sure to answer the following questions:

1. Did the optimization converged to an optimum? How did you verify that?
2. How did the optimization modified the initial design? Do these changes match your expectations and why (not)?
3. Are there any active constraints?
4. Is your optimum point dictated by the bounds of your design variables?