Dataflow Analysis

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Outline

- what is dataflow analysis? an intuitive understanding
- dataflow problems
- dataflow algorithm
- dataflow algorithm analysis
  - lattice and dataflow analysis
  - dataflow analysis termination
  - dataflow analysis accuracy: monotonicity, distribuitive, pointers
- dataflow algorithm implementation
- dataflow analysis and bug detection

History

1. 1970s, we want to know more about code so we can optimize the code
   - there exists any re-computation in the code?
   - there exists any useless computations?
2. Nowadays, we also want to know if the program will run correctly, what computation the legacy code is performing, ...

What is dataflow analysis?

What are the data usage patterns in a program?

- is this variable always holding a constant value?
- where a defintion of a variable is used?
- does two variables always hold the same value?
- is this expression already "available" (computed) at this program point?

What is dataflow analysis

1. *dataflow analysis*: determining variable and expression relationships throughout a function or a program
2. formulated into a mathematical framework
   - to reason about the termination of dataflow analysis
   - to generalize a set of dataflow problems so we can use one algorithm to address them all

Dataflow problems

Three classical dataflow analysis problems

- Reaching definitions (null-pointer dereference): what definitions can reach a given program point
- Available expressions (performance issue): for each program point, what are the expressions available
- Live variables (memory leak): which variables are live at a program point

See ppt slides for example details

Generalizing dataflow problems

Defining a dataflow problem:

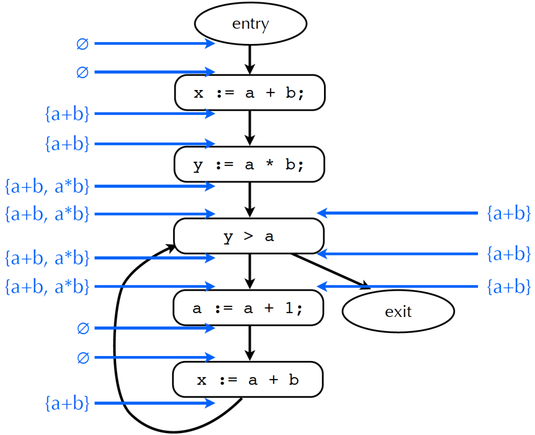
- What is the problem? (what are the dataflow facts to compute at each program point?)
- Applications in compiler optimization and software engineering
- The properties of dataflow problems:
  - backward or forward (determine the direction of the dataflow analysis)
  - may or must (determine how to merge values at the branches)
- Dataflow equations (local information): Gen, Kill, In, Out

Dataflow analysis

Generalizing dataflow analysis

- **Goal**: solving dataflow equations and determining dataflow facts/program information for the program points in the entire program
  - local information can be propagated along program control flow to influence other nodes
  - the dataflow facts will be stablized and the analysis can be terminated?
- **Framework**: a set of dataflow propagation algorithms for a set of dataflow problems
- **Key of the algorithm**:
  - datafow equation computes dataflow facts locally
  - dataflow algorithms: connect dataflow facts globally, especially stabilizing the dataflow facts/solutions in presence of loops via, e.g., fixpoint

Computing available expressions



Forward must data flow algorithm

```
Out(s) = ⊤ for all statements s
W := { all statements }                    (worklist)
repeat {
    Take s from W
    In(s) := ∩_{s' ∈ pred(s)} Out(s')
    temp := Gen(s) ∪ (In(s) - Kill(s))
    if (temp != Out(s)) {
        Out(s) := temp
        W := W ∪ succ(s)
    }
} until W = ∅
```

Forward data flow again

Out(s) = $\top$ for all statements s
W := { all statements }
repeat {
  Take s from W
  temp := $\boxed{f_s(\sqcap_{s' \in pred(s)} Out(s'))}$
  if (temp != Out(s)) {
    Out(s) := temp
    W := W $\cup$ succ(s)
  }
} until W = $\varnothing$

Transfer function for statement s

Further reading after classes

- backward algorithm
- worklist algorithm

Iterative Approach

Example - backward problem (live variables)

```
for i = 1 to N do   IN[B_i] = GEN[B_i]
OUT[B_exit] = ∅
change = true
while change do
    change = false
    for each block B do
        OLDIN = IN[B]
        OUT[B] = ∪_{S ∈ succ(B)} IN[S]
        IN[B] = GEN[B] ∪ (OUT[B] - KILL[B])
        If OLDIN ≠ IN[B] then change = true
    end for
end while
```

Alternative Approach - Worklist Algorithm

Example - backward problem (very busy expressions)

for $i=1$ to N do $IN[B_i] = \{\text{All expressions}\} - KILL[B_i]$

$OUT[B_{exit}] = \emptyset$

Worklist ← All blocks

while Worklist ≠ ∅ do

  get B from Worklist

  OLDIN = IN[B]

  $OUT[B] = \bigcap_{S \in succ(B)} IN[S]$

  $IN[B] = GEN(B) \cup (OUT[B] - KILL[B])$

  If OLDIN ≠ IN[B] then

    Add Pred(B) to Worklist

endwhile

∩ - start with largest solution & keep iterating till it stops shrinking.

Dataflow algorithm analysis: lattice, monotonicity, termination, distributivity, precision

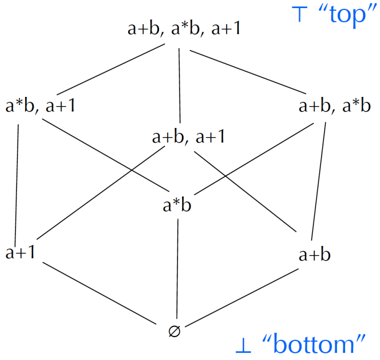
Dataflow facts and lattice [Kildall:1973] [Kam:1976]

Dataflow and lattice: for analyzing termination and for generalizing the aglorithms to dataflow problems

- *Lattice (L)*: the set of elements plus the order of these elements, it has a upper bound and a lower bound
- each dataflow problem has a lattice that describes the domain of the dataflow facts. That is, at each program point, the dataflow fact is an element of a lattice
- "order" among dataflow facts: element a is a conservative approximation of b, a $<$ b
- *transfer (flow) function* for dataflow problem: how each node affects the dataflow fact $F : L \to L$
- *merge function* for dataflow problem: reduce to meet operator $\sqcap$ $(\wedge)$ on lattice

Data flow facts and lattices

- Typically, data flow facts form lattices
- E.g., available expressions

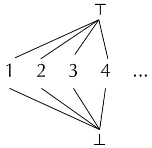


Partial orders and lattices

- A **partial order** is a pair $(P, \leq)$ such that
  - $\leq$ is a relation over P ($\leq \ \subseteq P \times P$)
  - $\leq$ is reflexive, anti-symmetric, and transitive
- A partial order is a **lattice** if every two elements of P have a unique least upper bound and greatest lower bound.
  - $\sqcap$ is the meet operator: $x \sqcap y$ is the greatest lower bound of x and y
    - $x \sqcap y \leq x$ and $x \sqcap y \leq y$
    - if $z \leq x$ and $z \leq y$ then $z \leq x \sqcap y$
  - $\sqcup$ is the join operator: $x \sqcup y$ is the least upper bound of x and y
    - $x \leq x \sqcup y$ and $y \leq x \sqcup y$
    - if $x \leq z$ and $y \leq z$ then $x \sqcup y \leq z$
- A join semi-lattice (meet semi-lattice) has only the join (meet) operator defined

Useful lattices

- $(2^S, \subseteq)$ forms a lattice for any set S
  - 2^S is powerset of S, the set of all subsets of S.
- If $(S, \leq)$ is a lattice, so is $(S, \geq)$
  - i.e., can "flip" the lattice
- Lattice for constant propagation



Which lattice to use?

- Available expressions
  - P = sets of expressions
  - Meet operation $\sqcap$ is set intersection $\cap$
  - $\top$ is set of all expressions
- Reaching definitions
  - P = sets of definitions (assignment statements)
  - Meet operation $\sqcap$ is set union $\cup$
  - $\top$ is empty set
- Monotonic **transfer function** f_s is defined based on gen and kill sets.

Monotonicity

- A function f on a partial order is **monotonic** if
  - if $x \leq y$ then $f(x) \leq f(y)$

- Functions for computing In(s) and Out(s) are monotonic
  - $In(s) := \bigcap_{s' \in pred(s)} Out(s')$
  - $temp := Gen(s) \cup (In(s) - Kill(s))$ A function f_s of In(s)

- Putting them together: $temp := f_s(\bigcap_{s' \in pred(s)} Out(s'))$

Termination

- We know the algorithm terminates
- In each iteration, either W gets smaller, or Out(s) decreases for some s
  - Since function is monotonic
- Lattice has only finite height, so for each s, Out(s) can decrease only finitely often

```
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} until W = ∅
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Termination

- A **descending chain** in a lattice is a sequence $x_0 < x_1 < \ldots$
- The **height of a lattice** is the length of the longest descending chain in the lattice
- Then, dataflow must terminate in O(nk) time
  - n = # of statements in program
  - k = height of lattice
  - assumes meet operation and transfer function takes O(1) time

Distributive data flow problems

- If f is monotonic, then we have

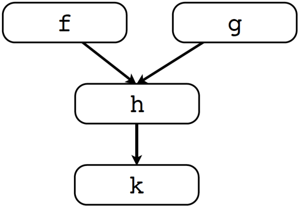
$$f(x \sqcap y) \leq f(x) \sqcap f(y)$$

- If f is **distributive** then we have

$$f(x \sqcap y) = f(x) \sqcap f(y)$$

Benefit of distributivity

- Joins lose no information



- $k(h(f(\top) \sqcap g(\top)))$
$= k(h(f(\top)) \sqcap h(g(\top)))$
$= k(h(f(\top))) \sqcap k(h(g(\top)))$

Accuracy of data flow analysis

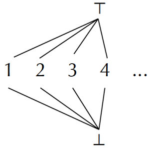
- Ideally we would like to compute the **meet over all paths** (MOP) solution:
  - Let f_s be the transfer function for statement s
  - If p is a path $s_1,\ldots,s_n$, let $f_p = f_{sn};\ldots f_{s1}$
  - Let paths(s) be the set of paths from the entry to s
  - $MOP(s) = \sqcap_{p \in paths(s)} f_p(\top)$
- If the transfer functions are distributive, then solving using the data flow equations in the standard way produces the MOP solution

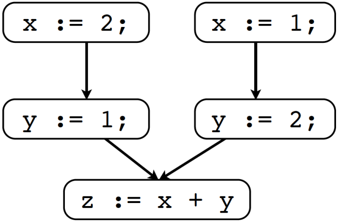
What problems are distributive?

- Analyses of *how* the program computes
  - E.g.,
    - Live variables
    - Available expressions
    - Reaching definitions
    - Very busy expressions
- All Gen/Kill problems are distributive

Non-distributive example

- Constant propagation





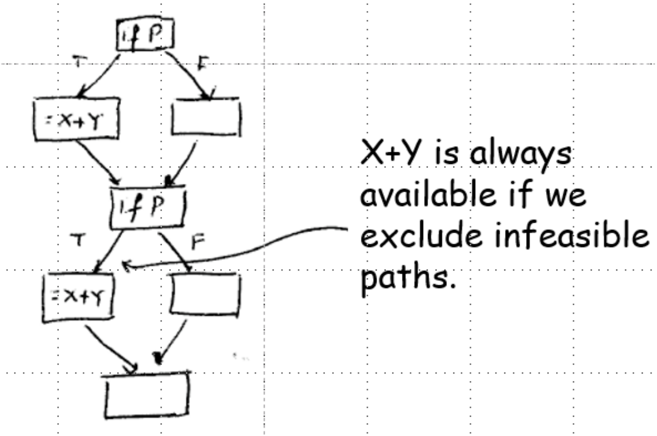
- In general, analysis of *what* the program computes is not distributive
- Thm: MOP for In(s) will always be $\sqsubseteq$ iterative dataflow solution

Dataflow analysis precision - conservative analysis

- For compiler optimizations, the dataflow facts we compute should definitely be true (not simply possibly true).
- However, we may miss optimization opportunities (not able to compute a complete and correct solution): two main reasons
  - Control Flow
  - Pointers & Aliasing

Dataflow analysis precision- control flow

We assume that all paths are executable; however, some may be infeasible.



Dataflow analysis precision- pointer analysis

we may not know what a pointer points to:

1. X = 5
2. *p = ...   // p may or may not point to X
3. ... = X

Constant propagation: assume p does point to X (i.e., in statement 3, X cannot be replaced by 5).

Dead Code Elimination: assume p does not point to X (i.e., statement 1 cannot be deleted).

Dataflow analysis implementation

Dataflow analysis implmentation

- Does a definition reach a point ? T or F, each variable definition is a bit, each program point has a bitvector
- Is an expression available/very busy ? T or F, each expression is a bit, each program point has a bitvector
- Is a variable live ? T or F, each variable is a bit, each program point has a bitvector

Intersection and union operations in dataflow equations can be implemented using bitwise *and* & *or* operations.

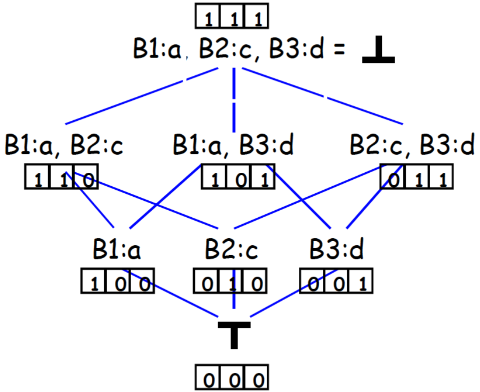
Practical implementation

- Data flow facts are assertions that are true or false at a program point
- Can represent set of facts as bit vector
  - Fact i represented by bit i
  - Intersection=bitwise and, union=bitwise or, etc
- “Only” a constant factor speedup
  - But very useful in practice

Implementation Issues: Bit-vector Problems

The set of facts (universe of facts) can often be expressed as finite subsets of a finite base set. Such sets can be represented as bit-vectors.

Example: RD - { B1: a:=2, B2: c := d+2, B3: d := a-5 }



Implementation Issues: Bit-vector Problems

Meet operation is either bit-wise logical AND or bit-wise logical OR .

GEN and KILL sets can be expressed as single bit-vectors.

Bit-wise logical - is bit-wise negation followed by bit-wise AND .

Implementation steps:

1. - bit-vector construction/interpretation
2. - bit-vector CFG initialization (RD, GEN, and KILL vectors)
3. - bit-vector CFG propagation
4. - information post-processing (e.g.: DU/UD chains)

Dataflow analysis for bug detection

basic dataflow algorithms: what are the dataflow facts?

- How to detect uninitialized variables?
- How to detect memory leaks?

make the analysis more precise and less false positives

- How to detect infeasible paths
- How to make it more precise by considering pointer aliasing information
- How to track inter-procedural bugs?
- ...

Static analysis for bug detection

- construct a cfg
- map to a dataflow problem
- dataflow analysis
- extend interprocedurally
- further improve precision: add pointer analysis and infeasible paths detection

Further Reading

Lattice Theory by Patrick Cousot

Data flow analysis in Principles of Program Analysis