

3.THERMODYNAMICS

The study of heat and its transformation to mechanical energy is called thermodynamic (i.e. movement of heat).

THERMODYNAMICS: Is a branch of physics which deals with the study of transformation of heat energy into other forms of energy and vice-versa. ie. Is a branch of physics with deals with relationship between heat energy and work.

- This deal with study of the exact relationship between heart energy changes and the direction in which these changes take place.
- Its foundation of the law of conservation of energy and the fact that heart flows naturally from a hot body to a cold body and hot other way around.
- Its study helps us to design steam, diesel, jet propulsion engines, refrigerators
- Also, its help us to predict the end of the universe etc.

LAWS OF THERMODYNAMICS

1. ZEROOTH LAW OF THERMODYNAMICS

State that “two bodies or systems that are in thermal equilibrium with a third body are in thermal equilibrium with each other”

The Zeroth law is so called because the need for it was recognized until after the first law has been established.

2. FIRST OF LAW OF THRMODYNAMICS

State that “the quantity of heat energy (ΔQ) supplied to a system is equal to the sum of the

change of internal energy ($\Delta\mu$) and work done (Δw) by expanding gases”

Mathematically

$$\Delta Q = \Delta U + \Delta W \text{ --- (1)}$$

$$\Delta\mu = \Delta Q - \Delta W \text{ --- (2)}$$

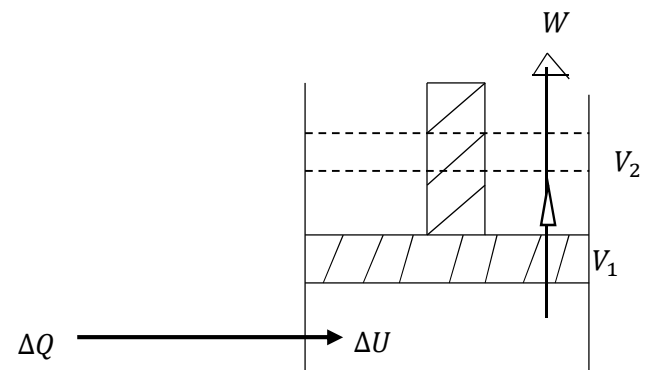
$$\Delta W = \Delta Q - \Delta\mu \text{ --- (3)}$$

Where ΔQ = Change of quantity of heat energy added (supplied)

$\Delta\mu$ = raised of the internal energy

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Δw = change of the work done consider a gas constrained in a cylinder fitted with movable piston a shown in the figure below.



Depending upon the condition imposed on the system, the energy added (supplied) does one or both of two things:

- i) It can raise the temperature of the gas and thus increase its internal energy
- ii) It can cause the gas to expand thus allowing the gas to do work on the piston.

SIGN CONVENTIONS

The first law of the thermodynamics is considered from the point view of gas. Therefore, we can account the following signs;

i) $+\Delta Q$ = Gas gain heat

ii) $-\Delta Q$ = Gas loss heat

iii) $+\Delta \mu$ = increase in internal energy

iv) $-\Delta \mu$ = decrease in internal energy

v) $+\Delta w$ = work done on the gas by the compression

SIGNIFICANCE OF FIRST LAW OF THERMODYNAMICS

This law implies the law of conservation of energy

NOTE THAT

The following point are worth noting regarding the first law of thermodynamic:

1. This law which is basically the law of conservation energy applies to every process in nature

2. The law applies equally to all three phases of matter i.e. solid liquid and gas

3. While applying the first law of thermodynamic are must be takes that all the three quantifies dQ , and dW are expressed either by in joule or in calorie (i.e. some unit)

4. Just as Zeroth law of thermodynamics introduce the concept of temperature, the first law of thermodynamic introduces the concept of internal energy.

5. From the first law of thermodynamic we learn that it is impossible to get work from any machine

without giving an equivalent amount of energy to the machine

6. The first law of thermodynamics establishes the essential equivalence between work and heat as according to this law internal energy (and hence temperature) of a system can be increased either by supplying heat to it or by doing work on the system or both

LIMITATIONS (DRAWBACKS) OF THE FIRST LAW OF THERMODYNAMICS.

1. The law does not tell us the direction in which the change takes place.

2. The law does not give the idea about the extent to which the change can take place.

3. The law does not tell the condition under which a body can convert heat can be converted in to work

SECOND LAW OF THERMODYNAMICS

State that “heat flows naturally from a hot body to a cold body, heat will never of itself to flow from a cold body to a hot body.

Important applications of second law thermodynamics.

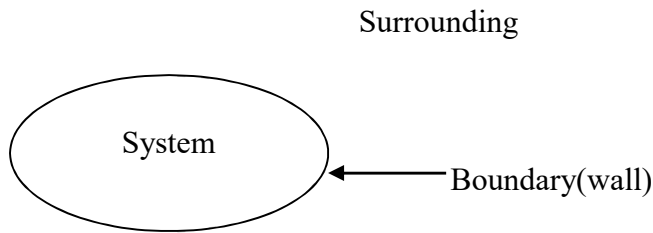
- working of heat engines
- working of heat pump

TERMS USED IN THERMODYNAMICS

1. ASSEMBLE – is the number of N identical particles of the gas molecules.

2. ENSEMBLE -is an identical assemblies of the particles (gas molecules)

3. SYSTEM – is a collection of objects on which attention is being focused.



4. SURROUNDING – is anything outside the system which has got some bearing on the behavior of the system and surrounding are separated by walls of some kind

i) DIATHERMAL WALLS are well separating system and surroundings but permit heat flow through them.

ii) ADIABATIC WALLS: are thick perfectly insulated wall that do not allow heat flow between the system and surrounding

5. THERMODYNAMIC SYSTEM

Is an extremely large number of particles (atoms or molecules) having a certain value of pressure volume and temperature. Example of thermodynamic system in engineering are

- i) A gas
- ii) A vapor
- iii) A mixture of gasoline vapor
- iv) Surface films
- v) Thermocouples
- vi) Stretched strings etc.

6. BOUNDARY – is the surface that separates the system from the surrounding

It may be a real or imaginary surface either ridged or movable.

The relationship between the system and surrounding is the interchanging of the amount of the amount of energy them.

TYPES OF THERMODYNAMICS SYSTEM

- i) Isolated system
- ii) Closed system
- iii) Open system homogeneous system

7. ISOLATED SYSTEM – is the system in which no heat energy leaves or enters to the system and the work done is zero.

Hence internal energy remains constant i.e $\Delta u = 0, \Delta Q = 0$

8. CLOSED SYSTEM – Is the system in which there is heat energy exchange between the system and surrounding.

9. OPEN SYSTEM – Is the one in which the system can exchange both matter and energy with the surrounding

10. HOMOGENEOUS SYSTEM – Is the one which contain the only one molecule.

Example – A system containing pure hydrogen gas a homogeneous system.

11. PROPERTY – is the observable characteristics of the of the system such as pressure, temperature and volume

12. STATE – is the condition of the system specified by the values of temperature, volume and pressure

13. STATE VARIABLE -Is the variable which determine the thermodynamic behavior of a system i. e

State variable of the system are the parameter which describe equilibrium state of the system. For example, if the system is the gas, then pressure volume and temperature (for a given mass) is the state variables.

Further, the thermodynamics state variable is of two type

- i) Extensive state variables
- ii) Intensive state variables

Extensive variables change with size of system, but intensive variable does not for example imagine a gaseous system in thermal equilibrium divided in to two equal part. The state variable, volume V mass M, and internal energy U which get halved in each part extensive variables. the other state temperature T, and density ρ which remain unchangeable for each part are intensive variable.

14. EQUATION OF STATE (STATE FUNCTION)

Is the equation which represent the connection between the state variables of a system is the equation which show the general relationship between pressure, volume and temperature for a given mass of system (gas). Equation of state

. for an ideal gas

$$PV = nRT$$

For real gas

$$\left(p + \frac{an^2}{v^2}\right)(v - nb) = nRT$$

Each system has usual meaning

15. THERMODYNAMIC STATE

Is the system of the body (system) that is completely define the situation of the gas which has specified value of pressure P, volume V, and temperature T

16. THE THERMODYNAMIC COORDINATE

Is the quantities which are used to specify the state of the system. i.e.

Is the coordinate which completely defined the particular situation of the gas which has specified value of pressure P, volume V and temperature, T

17. P- V CURVE OR INDICATOR DIAGRAM

Is the graph which show the variation of the pressure and volume of a gas exerted at particular temperature

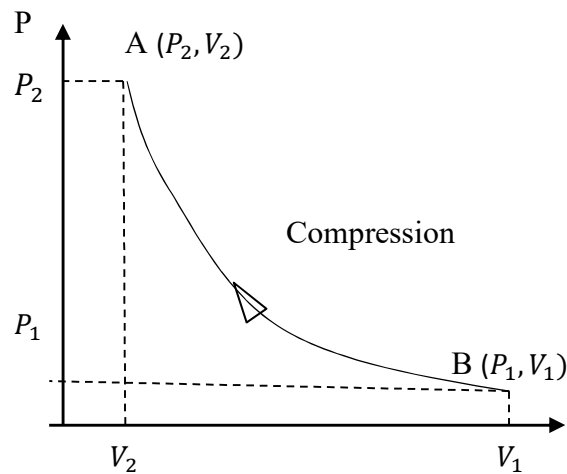
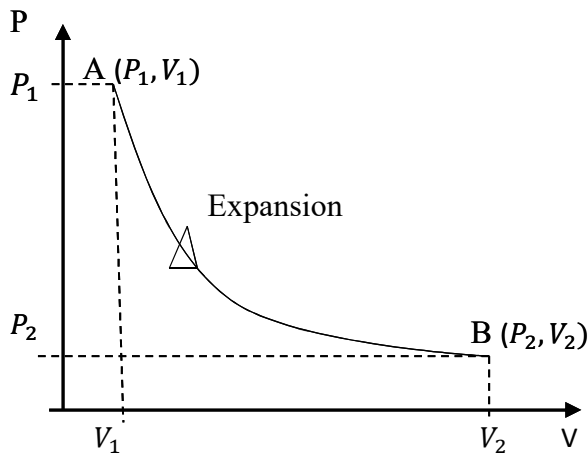
$$p \propto \frac{1}{v} [PV = nRT]$$

A graph representation of the state of a system with the help of two thermodynamic variable is called indicator diagram of the system

IMPORTANCE OF P-V DIAGRAM

It can be mathematically proved that work done by a system or on the system on the numerically equal the area under the PV diagram

P-V Curve



18. QUASI-STATE /EQUILIBRUM PROCESS

Is the thermodynamic process which proceeds extremely slow such that every instant of time the temperature and pressure are the same in all part of the system.

A process or a change in the variable of thermodynamically which is infinitely slow is called quasi- static process.

19.THERMODYNAMIC PROCESS

Is the any continuous change of a gas (system)

TYPES OF THERMODYNAMIC PROCESS

There are four types of thermodynamic process

- i) Isobaric process.
- ii) Isovolumetric (Isochoric) process.
- iii) Isothermal process.
- iv) Adiabatic process.

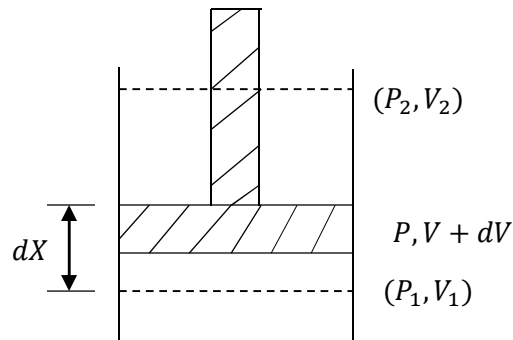
WORK DONE DURING EXPANSION OR COMPRESSION OF GAS

1. Analytical method

Suppose the initial state of the gas is A (p_1, v_2)

You require to obtain expression of work done by the gas

Let A = Cross -sentinel area



Suppose a piston move through a small P be considered to be constant

$$dw = p dx \text{ but } F = PA = PADX$$

$$dw = p dv$$

$$[Adx = dv]$$

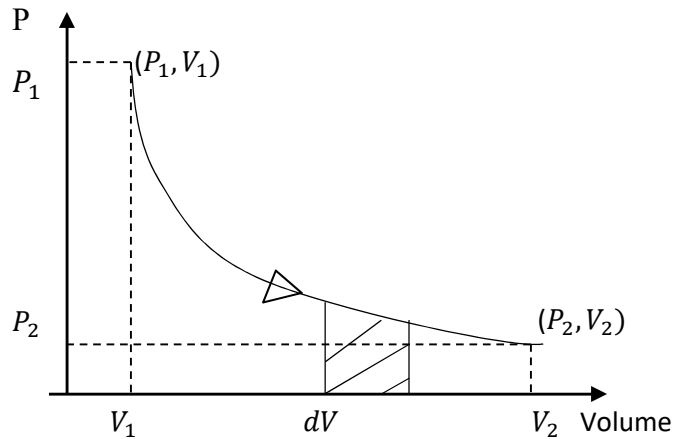
Total work done by expanding gas from the volume V_2

$$w = \int_{V_1}^{V_2} p dv$$

2.WORK DONE FROM P-V DIAGRAM

Consider a gas vs cylinder expanding from a volume

V_1 to V_2 are shown in figure below



Dividing the area under the curve in to small strips each of volume dv work done for volume change, dv

$$dw = p dv$$

Total work done is equal to the are under the curve

$$w = \sum p dv$$

$$w = \int_{v_1}^{v_2} p dv$$

Work done = are under P-V graph and volume axis

w = Area under the curve from volume v_1 to v_2 in the

P – V diagram

POSITIVE AND NEGATIVE WORK DONE FROM P – V CURVE

1.WORK DONE BY EXPANSION OF GAS FROM P -V CURVE (POSITIVE WORK DONE)

The positive work done represent the work done by expansion of gas

When the piston fitted in the cylinder is moving upwards i.e. Gas expands when $v_2 > v_1$

V_1 = initial volume

V_2 = final volume

If $dv > 0, w > 0$ This represent positive work done.

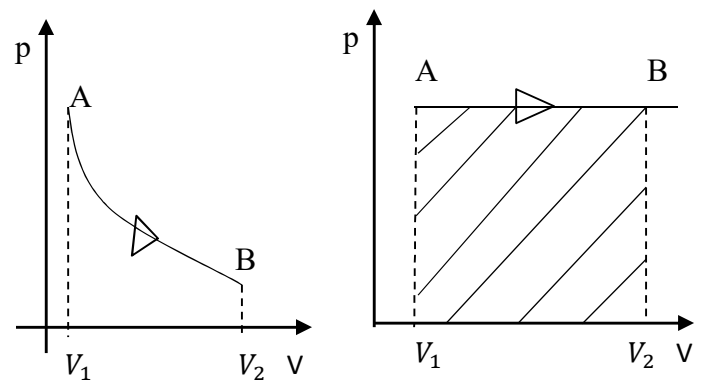
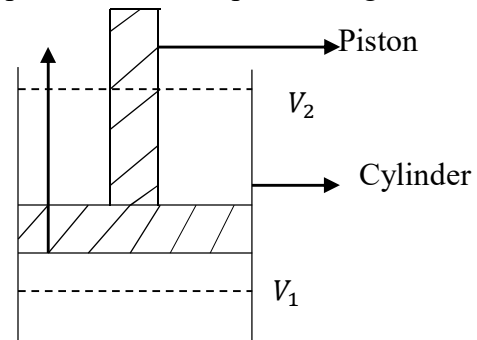
Work done = area shaded under the curve ABCD

If the curve traced in clockwise direction represent the work done

2.WORK DONE BY COMPRESSION OF GAS (NEGATIVE WORK DONE)

When the position is moving downward the gas compressed $V_2 < V_1$ Then $dv < 0, w < 0$

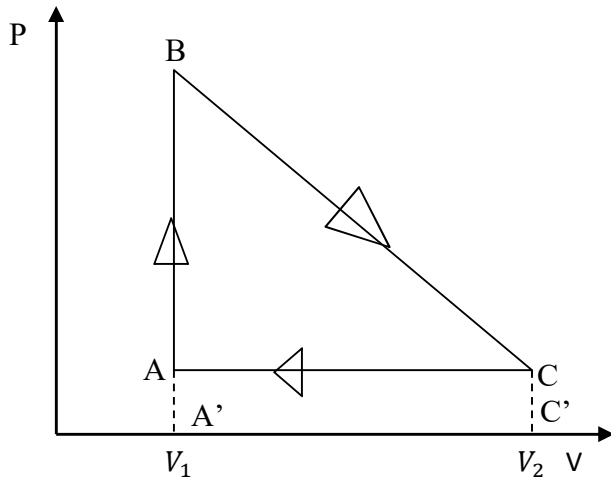
The force exerted by the gas is opposite direction to the displacement, this represent negative work done.



If the curve traced out in anticlockwise direction represent negative work done

WORK DONE FROM P -V CURVE

Consider a P-V curve as shown on the figure below



Area under curve ABC is equal to the net work done by gas in the following process

$$A \longrightarrow B: w_1 = 0, dv = 0$$

$$B \rightarrow C: w_2 = \text{area of trapezium } AA'BC'$$

$$w_2 = \frac{1}{2} \overline{A'C'} (\overline{A'B} + \overline{CC'})$$

$$C \rightarrow A: W_3 = -\text{Area of } ACC'A'$$

$$W_3 = -\overline{AC} \times \overline{AA'}$$

Network done for the complete cycle

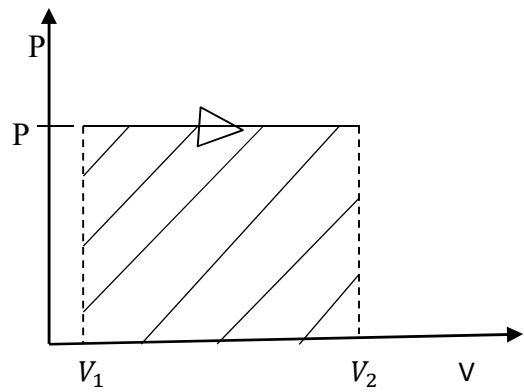
$$w = w_1 + w_2 + w_3$$

Note that:

Thermodynamic system may be

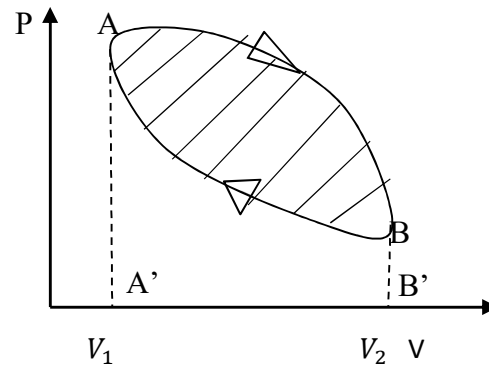
- (i) Non – cyclic process
- (ii) Cyclic process

NON -CYCLIC PROCESS – Is the process in which the system does not return back to its initial state i. e. the final state of the system is different from initial state.



CYCLIC PROCESS

Is the process in which the system returns to its initial state.



Work done a cyclic process by expansion of the gas

$$A \rightarrow B: W_1 = \int_{V_1}^{V_2} P dv$$

$$= \text{area of } ABB'A' (+ve)$$

Now gas compressed and return back to the original state A

$$B \rightarrow C: W_2 = \int_{V_2}^{V_1} P dv = \text{Area of } BAA'B' (-ve)$$

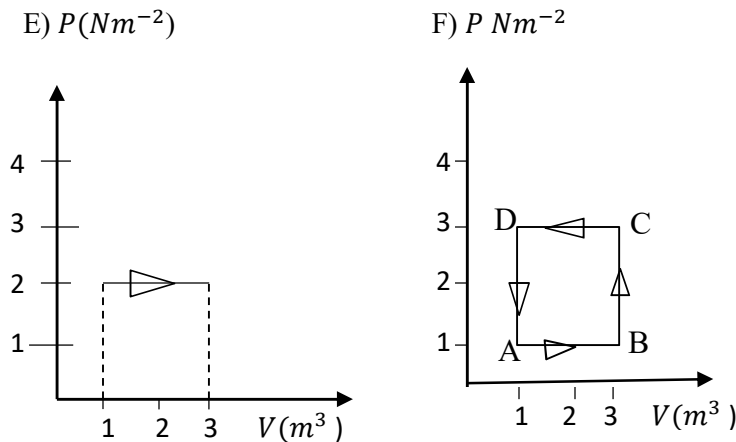
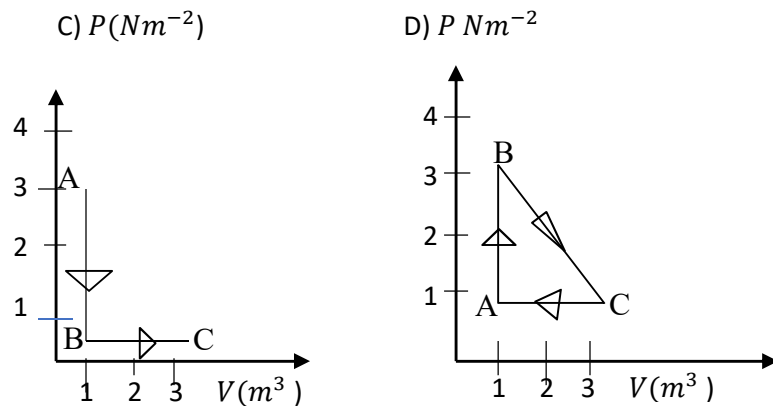
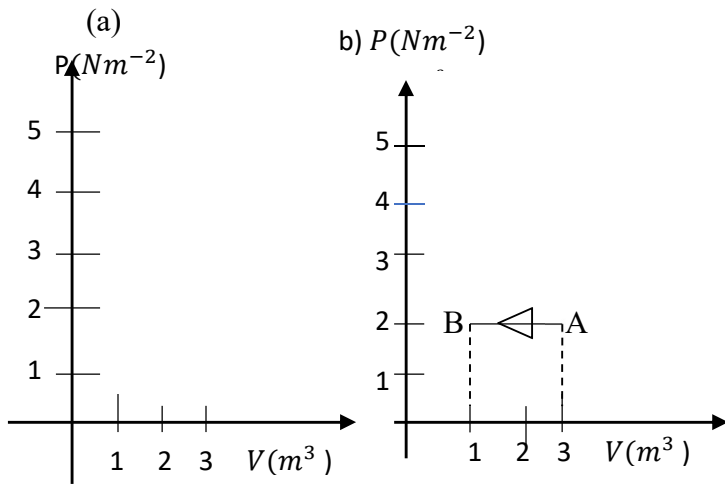
Total work done is equal to the area of shaded enclosed loop.

$$w = w_1 + w_2$$

SOLVED EXAMPLES

Example 01

Find the work done corresponding to the P -V curve



d) $A \rightarrow B: W_1 = 0$ Since $\Delta v = 0$

$$B \rightarrow C: W_2 = \frac{1}{2} (3 - 1)(3 + 1) W_2 = 4J$$

$$C \rightarrow A: W_3 = P(V_2 - V_1) = 1 \times (1 - 3)$$

$$W_3 = -2J$$

NET WORK DONE

$$W = W_1 + W_2 + W_3$$

$$= 0 + 4 + -2$$

$$W = 2J$$

$$e) w = P(V_2 - V_1) = 2 \times (3 - 1)$$

$$W = 4J$$

$$f) A \rightarrow B: W_1 = 1 \times (3 - 1) = 2J$$

$$B \rightarrow C: W_2 = 0, \text{ Since } \Delta V = 0$$

$$C \rightarrow D: W_3 = 3(1 - 3) = -6J \quad D \rightarrow A: W_4 = 0, \Delta V = 0$$

Net work done

$$w = W_1 + W_2 + W_3 + W_4$$

$$= 0 + 2 + -6 + 0$$

$$W = -4J$$

Negative sign shows that the work done by the compression of the gas

Solution

a) $w = 0$ since $\Delta V = 0$

b) $W = P(V_2 - V_1) = 2 \times (1 + 3) W = -4J$

c) $A \rightarrow B: W_1 = 0$ Since $\Delta V = 0$

$$B \rightarrow C: W_2 = 1 \times (3 - 1) = +2J$$

$$\text{Net work done } W = W_1 + W_2 = 0 + 2$$

$$W = 2J$$

EXAMPLE 02

a) (i) Can you cool a room by keeping the fridge open?

ii) What are the uses of the indicator diagram or P-V diagram

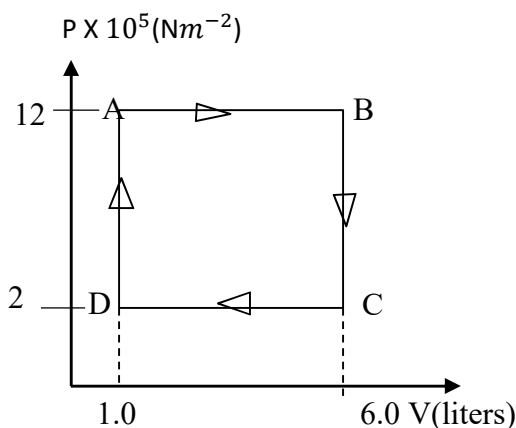
b) The figure below shows the P-V graph of the thermodynamic behavior of an ideal gas find out from the graph:

(i) Work done in the processes

$A \rightarrow B, B \rightarrow C, C \rightarrow D$, and

$D \rightarrow A$

ii) Work done in the complete cycle



solution

A)(i) From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

No. when work is done on compressor the working substance absorbs heat from a low temperature body and rejects the heat absorbed + work done by compressor to the room. Therefore, the room will become warmer

ii) It is used to find the work done by the system or on the system. The area enclosed by P-V graph with the X-axis gives the work done

a)(i) $A \rightarrow B : W_1 = P(V_1 - V_2)$

$$W_1 = (6 - 1) \times 10^{-3} \times 12 \times 10^5 W_1 = 6000 J$$

$B \rightarrow C : W_2 = 0, \Delta V = 0$

$$C \rightarrow D : W_3 = (1 - 6)10^3 \times 2 \times 10^5 W_3 = 1000 J$$

$$D \rightarrow A : W_4 = 0, \Delta V = 0$$

(ii) Net work done

$$w = w_1 + w_2 + w_3 + w_4 = 6000 + 0 + -1000 + 0$$

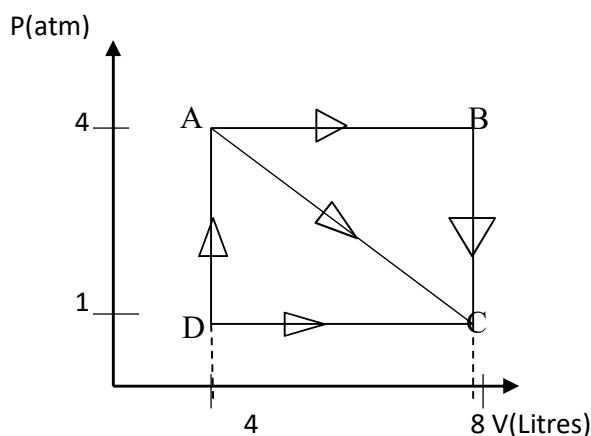
$$w = 5000 J$$

EXAMPLE 03

a) (i) Explain first law of thermodynamics. what is its physical significance?

ii) What do you mean by saying that the work done by a system is a path function?

b) A gas expands from A to C along three different paths as shown in the work done by gas along the path ABC, AC, ADC?



Solution

a) (i) refer to your notes

(ii) work done depends not only on the initial and final states but also on the path followed by the system

$$b) W_{ABC} = W_{AB} + W_{BC}$$

$$W_{AB} = P(V_2 - V_1) = 4 \times 10^3 (8 - 4) \times 10^{-3}$$

$$W_{AB} = 1620 \text{ J}$$

$$W_{BC} = 0 \text{ Since } \Delta V = 0$$

$$W_{ABC} = 1620 + 0$$

$$W_{ABC} = 1620 \text{ J}$$

Also

$$W_{AC} = \frac{1}{2} (4 + 1) \times 103125 \times 4 \times 10^{-3}$$

$$W_{AC} = 1013.2 \text{ J}$$

$$\text{also } W_{ADC} = W_{AD} + W_{DC}$$

$$W_{AD} = 0 \text{ Since } \Delta V = 0$$

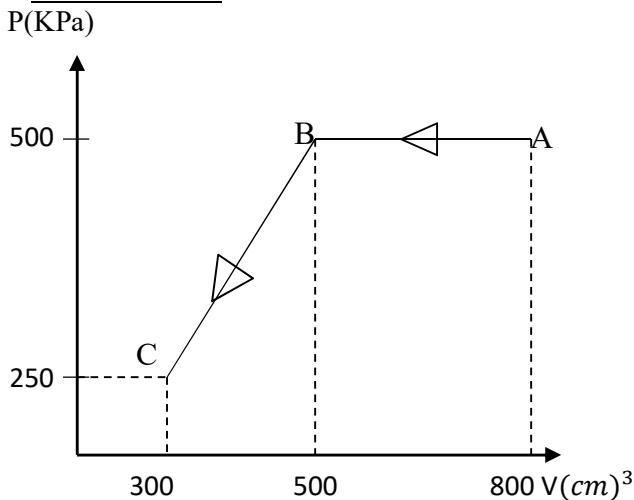
$$W_{DC} = 1 \times 103135 \times (8 - 4) \times 10^{-3}$$

$$W_{DC} = 405.3 \text{ J}$$

$$W_{ADC} = 0 + 405.3$$

$$W_{ADC} = 405.3 \text{ J}$$

EXAMPLE 04



The figure above shows P-V graph of thermodynamics of a gas. Find the work done by gas in going the situation represented by point A through point B to point C.

Solution

Work done by gas in process

$$A \rightarrow B : W_1 = -[\text{area under curve AB}]$$

$$= 500 \times 10^5 (800 - 500) \times 10^{-6}$$

$$W_1 = -150 \text{ J}$$

$$B \rightarrow C : W_2 = -[\text{Area of curve BC}]$$

$$W_2 = -\frac{1}{2} [500 - 300] \times 10^{-6} \times [250 + 500] \times 10^5$$

$$W =$$

$$\text{Net work done } w = w_1 + w_2$$

EXAMPLE 05

A certain amount of water of mass M_w at a temperature T_2 cools to temperature T_1 the heat given by the water is absorbed by n moles of an ideal gas. The gas expands at constant temperature and change its volume from v_1 to v_2 . What is the initial volume v_1 ?

Solution

Heat given out by the water

$$H = M_w C_w (T_2 - T_1) \dots \dots \dots (1)$$

Work done by the gas

$$W = \int_{v_1}^{v_2} p dv \text{ but } p = \frac{nRT}{V}$$

$$= \int_{v_1}^{v_2} \frac{nRT}{V} dv$$

$$= nRT [\log_e V]_{v_1}^{v_2}$$

$$W = nRT \log_e \left(\frac{v_2}{v_1} \right)$$

$$\log_e \left(\frac{V_2}{V_1} \right) = \frac{w}{nRT}$$

In the exponential from

$$\frac{V_2}{V_1} = e^{\frac{w}{nRT}}$$

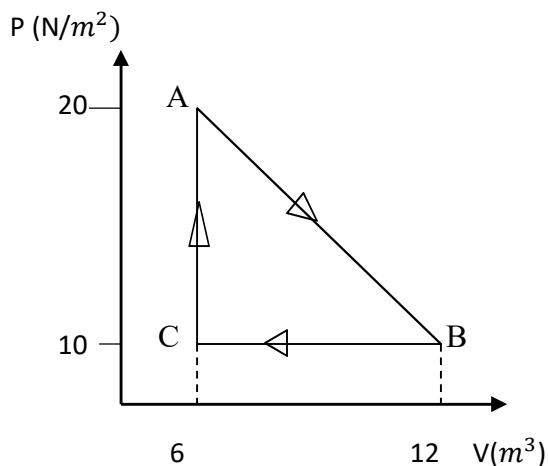
$$V_1 = V_2 e^{\frac{-w_1}{nRT}} = V_2 e^{\frac{-MwCm(T_2 - T_1)}{nRT}}$$

Where $W = Mw Cw (T_2 - T_1)$ because the expansion of the gas take place at the constant temperature T, the heat absorbed is used to do external work done

Example 06

In the figure below show the PV graph for the thermodynamic behavior of a system determine from the graph

- i) Work done on the system in the process
 - (a) $A \rightarrow B$
 - (b) $B \rightarrow C$
 - (c) $C \rightarrow A$
- ii) Net work done by the system in the complete cycle ABC



Solution

$$\begin{aligned} i) A \rightarrow B \quad W_1 &= +[\text{area under the curve } AB] \\ &= \frac{1}{2} (12 - 6) (20 + 10) \end{aligned}$$

$$W_1 = 90 \text{ J}$$

$$b) B \rightarrow C; \quad W_2 = P(V_2 - V_1)$$

$$W_2 = 10(6 - 12) = 60 \text{ J}$$

$$c) C \rightarrow A; \quad W_3 = 0, \Delta V = 0$$

$$ii) W = W_1 + W_2 + W_3 = 90 + -60 + 0$$

$$W = 30 \text{ J}$$

Example 07

a) (i) Distribution between a closed and open system.

(ii) A match stick can be lighted by rubbing it against a rough surface, why?

iii) 'water is slightly warmer at the base of a water fall than at the top' why?

iv) 'As hammered n iron nail become hot when it'. Why?

b) A gas at a pressure of 1aim is compressed at a constant pressure from 10 liter to 2 liters if 500 J of heat energy leaves the gas. Calculate

- i) The work done by the gas
- ii) The change in its internal energy

Solution

a)(i) refer to your notes

(ii) Work done against friction is converted to heat and is used to light the match stick

(iii) The potential energy of the water at the top is converted in to heat energy at the bottom, which rises temperature of water

iv) The kinetic energy of the hammer is converted into heat energy which in turn raises the temperature

$$a) 1 \text{ atm} = 101325 \text{ N/m}^2$$

$$\Delta V = 2 - 10 = -8 \text{ L} = -8 \times 10^{-3} \text{ m}^3$$

$$(i) \quad \Delta W = P \Delta V = 101325 \times -8 \times 10^{-3}$$

$$\Delta W = -826 \text{ J}$$

ii) Heat leaving the gas

$$\Delta Q = -500 \text{ J} \text{ Apply 1st law of thermodynamic}$$

$$\Delta \mu = \Delta Q - \Delta W$$

$$= -500 - (-826)$$

$$\Delta \mu = 326 \text{ J}$$

EXAMPLE 09

A system gains 1500 J of heat while the internal energy of the system increases by 4500 J and volume and volume by 0.01 m^3 . Assume the pressure is constant and find its value.

Solution

Work done by system

$$\Delta W = \Delta Q - \Delta \mu$$

$$= 1500 - 4500 \quad \Delta W = -3000 \text{ J}$$

$$\text{Now } P = \frac{\Delta W}{\Delta V} = \frac{-3000}{-0.01}$$

$$P = 3 \times 10^5 \text{ N/m}^2$$

EXAMPLE 10

(a) A thermos bottle containing tea is vigorously shaken. What will be the effect on the temperature of tea?

b) An electric fan is switched on in a closed room. Will the air in the room be heated?

c) Why is a spark produced when two substances are struck hard against each other?

d) Why do the brake drums of an automobile get heated up when the automobile moves down a hill at constant speed.

Solution

a) There will be a slight increase in temperature of tea. This is because work done will be converted into heat. It should be however noted that the whole of the work will be converted into heat.

b) No, in fact the temperature of air shall rise due to increase in the speed of air molecules.

c) The work is converted into heat which produces a spark.

d) Since the speed is constant therefore, there is no change of kinetic energy. The loss in gravitational potential energy is partially the gain in the heat of the brake drum.

EXAMPLE 11

A) A thermos bottle containing coffee is shaken. Coffee is considered as a system.

- (i) Does its temperature rise?
- (ii) Has heat been added to it?
- (iii) Has work been done on it?

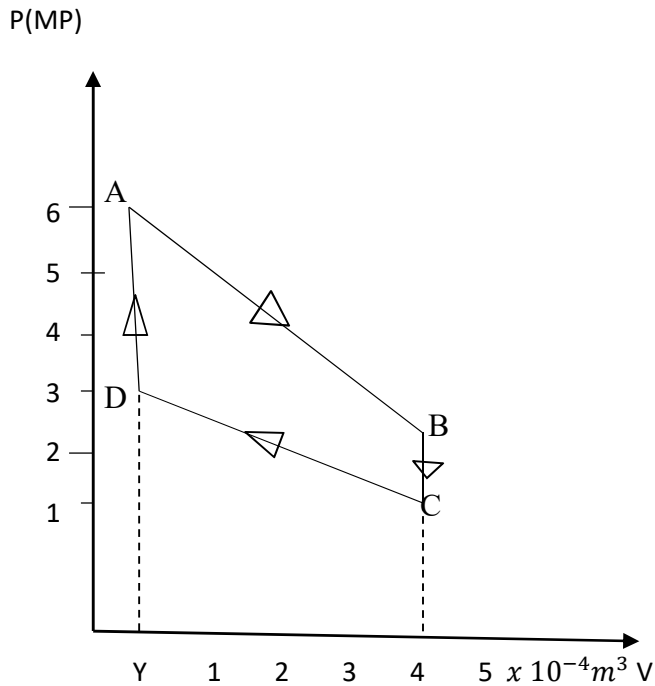


Figure above shows a simplified indicator diagram (pressure volume cycle) for one cylinder of an engine. Calculate

- The work done by the gas on expansion of A to B.
- The work done by the gas on construction C to D
- The network by the gas during one cycle ABCD

If the engine rotates at 50 cycles per second and it has four cylinders

iv) Calculate power generated by the engine.

SOLUTION

- (i) yes, there is the rise of temperature
- (ii) No, heat has been added because thermos bottle is an insulator

iii) Yes, the work has been done against the viscous force

a)(i) $A \rightarrow B$, work done by expanding gas

$$W_1 = \frac{1}{2} XY \cdot [AY + BX]$$

$$= \frac{1}{2} (4 - 0.5) \times 10^{-4} (6 + 3) \times 10^6$$

$$W_1 = 1575 \text{ J}$$

(ii) $C \rightarrow D$, Work done by construction

$$W_2 = [\text{Area of } CDYX]$$

$$W_2 = -\frac{1}{2} [DY + CX] \cdot XY$$

$$= -\frac{1}{2} [3 + 2] \times 10^6 \times 3.5 \times 10^{-4}$$

$$W_2 = -875 \text{ J}$$

$$(iii) W = W_1 + W_2 = 1575 + -875$$

$$W = 700 \text{ J}$$

iii) In one second each cylinder is taken through 50 cycles and there are 4 cylinders. Thus, the power generated is $50 \times 4 = 200$ times the work done by the gas in one cycle

Power generated

$$P = 200 \times 700 = 140 \times 10^3 \text{ W}$$

The power generated is partly used to overcome friction within the engine or car system and partly to provide a driving force.

THERMAL AGITATION AND INTERNAL ENERGY OF A GAS

THERMAL AGITATION -is define as the random motion of the gas molecules whose kinetic energy depends upon the temperature

INTERNAL ENERGY of a system is the system due to molecule motion and molecular configuration

Internal energy is the kinetic energy of the thermal agitation. The sum of the kinetic energy and potential energy of the gas molecules is called internal energy

$$\mu = p.e + k.e$$

REAL GASES

Real gases have intermolecular force of attraction. this the internal energy of real gas is the sum of internal kinetic energy and internal potential energy of molecules

$$\mu = p.e + k.e$$

It would obviously depend on both the temperature and the volume of gas

Under the limit of low-pressure P.e will be negligible since the average separation of any two gas molecules is very grate. Then, the internal energy is totally kinetic energy and depends on only temperature

Ideal gas

We assume that intermolecular forces are zero. therefore, the internal energy of ideal gas is wholly kinetic energy in nature and is the function of the temperature

$$\mu = k.e$$

EXPRESSION FOR INTERNAL ENERGY OF ANY GAS

From the basic of the kinetic theory of gases

$$PV = nRT = \frac{1}{3} N m \overline{c^2}$$

$$nRT = \frac{2}{3} N \left(\frac{1}{2} \right) M \overline{C^2} \quad \frac{N}{N_A} RT = \frac{2}{3} N \left(\frac{1}{2} \right) M \overline{C^2}$$

$$\frac{1}{2} M \overline{C^2} = \frac{3}{2} \left(\frac{R}{N_A} \right) T$$

$$K.e = \frac{1}{2} m \overline{C^2} = \frac{3}{2} \left(\frac{R}{N_A} \right) T = \frac{3}{2} KT$$

Internal energy of gas molecules

$$u = N - K \text{ for } n = 1 \text{ mole}, N = N_A$$

$$u = N_A \cdot K \cdot \bar{e} = N_A \cdot \frac{3}{2} \left(\frac{R}{N_A} \right) T$$

$$u = \frac{3}{2} RT$$

Internal energy of n-number of moles of the gas

$$u = \frac{3}{2} nRT$$

TYPE OF GAS MOLECULES

There are three types of gas molecules

1. Monatomic gas
2. Diatomic gas
3. Polyatomic gas

DEGREE OF FREEDOM, f

Vs the number of independent ways in which the gas can possessing energies.

$$f = 3N - K$$

$$F = 5$$

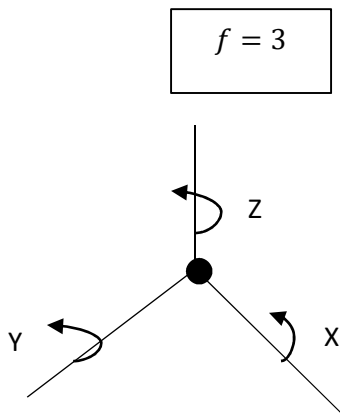
Each symbol has usual meaning

1. MONATOMIC GAS

Is the gas molecules consisting of a single atom

Example: mercury, sodium, helium, neon gas etc.

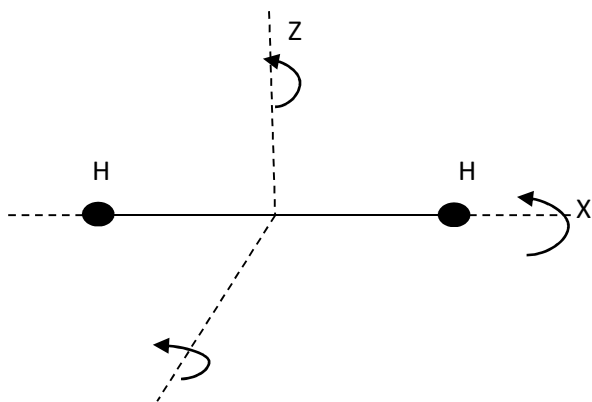
Monatomic gas has negligible moment of inertia and have three degree of freedom



2. DIATOMIC GAS

Is the gas molecules which consists of two atoms of the same or different element.

EXAMPLE: Oxygen, hydrogen gas, chlorine gas (CL_2). Co etc.



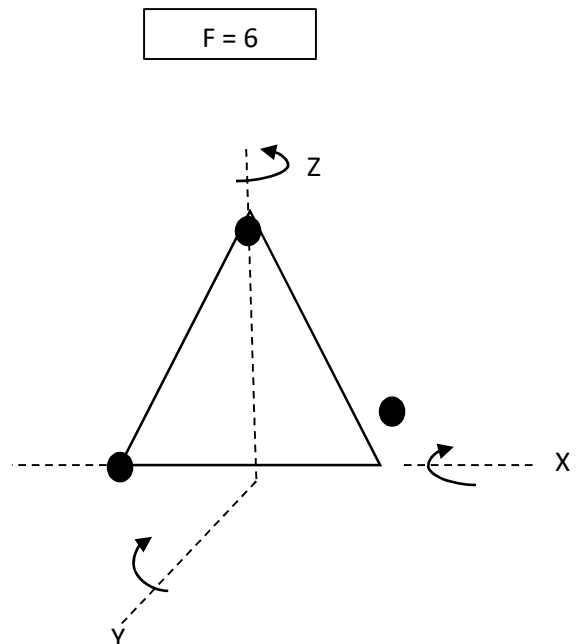
have five (5) degree of freedom

3. POLYATOMIC GAS

Is the gas molecules consist more than two atomics.

EXAMPLE: water vapor (H_2O) ozone (O_3) , H_2S, CO_2, CH_4 , etc

Polyatomic gases have six degree of freedom



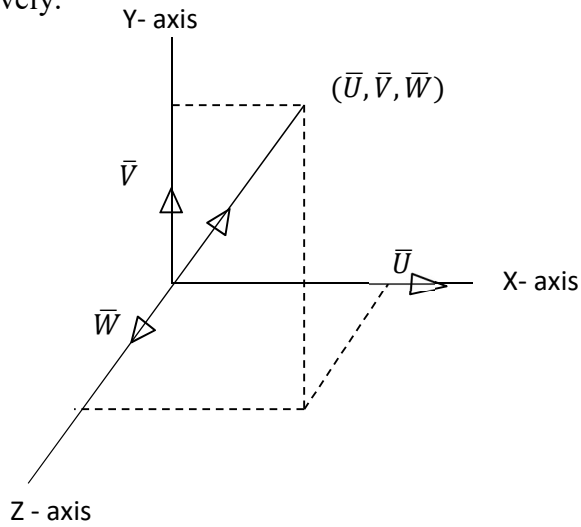
PRINCIPLE OF EQUIPARTITION OF ENERGY

State that for a dynamical system in thermal equilibrium the energy of the system is equally distributed among the various degree of freedom and the energy associated with each degree of freedom per molecule is $\frac{1}{2} KT$, where K is the Boltzmann constant

PROOF

Consider the figure below which show on mole of a monatomic gas in thermal equilibrium at temperature

T and whose velocity C have been resolved In to its components u, V and W a long X, Y and Z-axes respectively.



By using Pythagoras theorems

$$\overline{C^2} = \overline{u^2} + \overline{V^2} + \overline{W^2} \quad [\overline{u^2} = \overline{V^2} = \overline{W^2}]$$

$$\overline{C^2} = 3\overline{u^2} = 3\overline{V^2} = 3\overline{W^2}$$

Average kinetic energy of a gas molecule of a gas

$$\frac{1}{2} m \overline{C^2} = \frac{3}{2} KT$$

In x- direction

$$\frac{1}{2} m (3\overline{u^2}) = \frac{3}{2} KT$$

$$\frac{1}{2} M \overline{u^2} = \frac{1}{2} KT$$

NOW

$$\frac{1}{2} m \overline{u^2} = \frac{1}{2} m \overline{v^2} = \frac{1}{2} M \overline{w^2} = \frac{1}{2} KT$$

This the molecule has kinetic energy $\frac{1}{2} KT$

Per degree of freedom

Generally, the total energy of a molecule of a gas at temperature, T of F degree of freedom.

$$K.E = \frac{f}{2} KT \text{ Per molecule.}$$

EXPRESSION OF INTERNAL ENERGY

1.MONATOMIC GAS

$$f = 3 \text{ since } K.E = \frac{f}{2} KT = \frac{3}{2} KT$$

$$u = N_A KT = \frac{3}{2} (KT) N_A$$

$$= \frac{3}{2} (N_A K) T = \frac{3}{2} RT$$

$$u = \frac{3}{2} RT \text{ OR } du = \frac{3}{2} R dT$$

2.DIATOMIC GAS

$$f = 5 \quad K.E = \frac{5}{2} KT$$

$$u = N_A K \bar{E} = \frac{5}{2} (N_A K) T$$

$$u = \frac{5}{2} RT \text{ or } du = \frac{5}{2} R dT$$

3.POLTATOMIC GAS

$$f = 6 \quad K.E = \frac{6}{2} KT = 3KT$$

$$u = N_A K \bar{E} = \frac{6}{2} RT = 3RT$$

$$u = \frac{6}{2} RT = 3RT$$

Note that

$$K.E = \frac{f}{2} KT$$

$$u = N_A K \cdot E = \frac{fRT}{2}$$

SPECIFIC HEAT CAPACITIES OF GAS

HEAT CAPACITY (THERMAL CAPACITY)

Is defined as the amount of heat energy required to raise its temperature by $1^\circ\text{C}(K)$

$$\text{Heat capacity} = \frac{\text{Quantity of heat energy}}{\text{Temperature rise}}$$

$$C_1 = \frac{dQ}{dT}, dQ = C_1 dT$$

S-I unit of heat capacity is JK^{-1}

SPECIFIC HEAT CAPACITY OF A GAS

Is the amount of heat energy required to rise the temperature of unit mass (1Kg) of the substance through $1K (1^\circ\text{C})$

$$C = \frac{\text{Thermal capacity}}{\text{mass}} = \frac{dQ}{MdT}$$

$$dQ = MCdT$$

S-I unit of the specific heat capacity is

$$JKg^{-1}K^{-1}$$

LIMITS OF SPECIFIC HEAT CAPACITY

If dQ is a small amount of heat required to raise the temperature of a gas

$$C = \frac{dQ}{MdT}$$

1. Let the gas be suddenly compressed. In this case no heat supplied to the gas i.e. $dQ = 0$, even the temperature of gas raises

$$C = 0$$

2. If heat is applied to the gas and at the same time the gas is allowed to expand such that there is no rise in temperature i.e.

$$dT = 0 \quad C = \frac{dQ}{MdT} \rightarrow \infty$$

3. If dT is positive, then C is positive and dT is negative

Conclusion

Therefore, specific heat capacity of gas varies from zero to the infinite. The specific heat capacity of a gas depends on manner in which the gas is being heated.

TYPES OF SPECIFIC HEAT CAPACITY OF A GAS

The gas has two principal heat capacities

1. Specific heat capacity at constant volume C_V
2. Specific heat capacity at constant pressure C_P

SPECIFIC HEAT CAPACITY AT CONSTANT VOLUME (C_V)

Is defined as the amount of heat required to raise the temperature of 1 Kg (unit mass) of the gas through 1°C or $1K$ at constant volume

Suppose a gas of mass M , is heated in such a way that its temperature raised by dT at constant volume, $dv = 0$, $dQ = MC_V dT$

Apply the 1st law of thermodynamics

$$dQ = du + dw [dw = pdv]$$

$$MC_V dT = du + 0$$

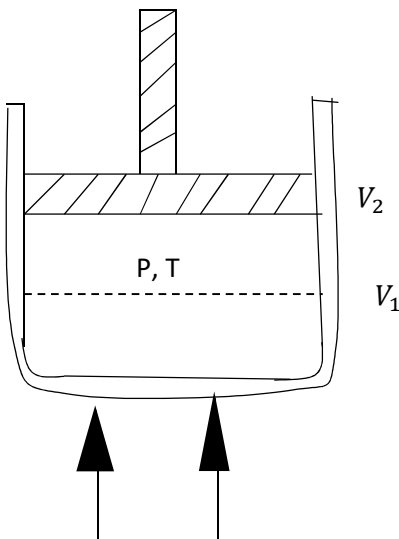
$$du = MC_V dT$$

There is no work done by heating gas due to the fact that $dv = 0$. according to the law of conservation of energy the entire energy and hence the temperature of the gas

2.SPECIFIC HEAT CAPACITY AT CONSTANT PRESSURE

Is define as the amount of heat require to rise the temperature of unit mass at constant pressure

Suppose the gas of mass, M is heated and piston moves inside of cylinder as shown below



Heat supplied

$$dQ = dU + PdV$$

According to the 1 st law of the thermodynamic

$$dQ = du + pdV$$

$$mCpdT = mCVDt + PdV$$

WHEN THE GAS Is heated at constant pressure, it expands and does work, so that only part of heat is used up to increasing the internal energy and hence rise the temperature of the gas. Therefore, at constant pressure more heat is needed to achieve a given

temperature change than that of constant volume process.

$$mCpdT = mCvdT + PdV$$

Assume that the gas is an ideal

$$PV = nRT = \frac{M}{Mr} \cdot RT$$

$$PdV = \frac{M}{Mr} RdT$$

Now

$$MCpdT = MCvdT + \frac{M}{Mr}$$

$$RdT \quad Cp = Cv + \frac{R}{Mr} \text{ or}$$

$$Cp = Cv + r$$

$$r = \frac{R}{Mr}$$

MOLAR HEAT CAPACITY OF A GAS

Define

Molar specific heat required to raise the temperature of one mole of a gas 1°C

$$C' = \frac{dQ}{ndT}$$

S. I unit of c' is $J K^{-1} \text{ mol}$

TYPE OF MOLAR SPECIFIC HEAT CAPACITY

1. Molar specific heat capacity at constant volume, C_v
2. Molar specific heat capacity at constant volume C_p

MOLAR SPSCIFIC HEAT CAPACITY AT A CONSTANT VOLUME (C_v)

Is the amount of heat required to raise the temperature of 1 mole of gas through 1K at a constant volume.

$$Cv' = \frac{dQ}{ndT} mr Cv$$

$$dQ = nCv' dT$$

MOLAR SPECIFIC HEAT CAPACITY AT CONSTANT PRESSURE (Cv')

Is the amount of heat required to raise the temperature of 1 mole of the gas through 1K (1°C) at constant pressure

$$Cp' = \frac{dQ}{ndT} = Mr Cp$$

Also

$$dQ = nCp' dT$$

Addition concepts

1. Since

$$Cp = Cv + \frac{R}{Mr} Mr Cp = rCv + R Cp'$$

$$= Cv' + R$$

2. TO SHOW THAT $c_p > c_v$

When a gas is heated at constant volume supplied to the gas is wholly used up to raise its temperature i.e to increase the internal energy of a gas only

On the other hand, when a gas is heated at a constant pressure, a part of heat is used to increase the internal energy and a part used to do external work done to keep the pressure constant.

Therefore, heat required to raise the temperature of a unit mass of a gas through 1k 1°C at a constant

pressure is greater than that at constant volume ($Cp > Cv$)

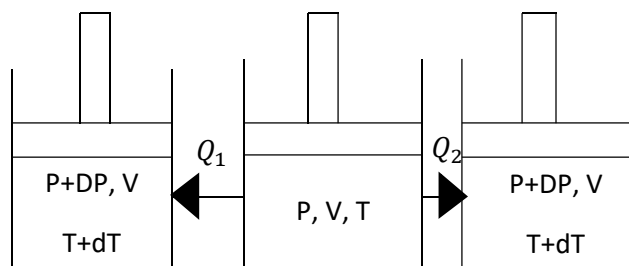
3. Gases have two principal specific heat capacities due to the following reason

This is due to the greater change of pressure and volume of the gases when temperature rises or falls on the gas

4. Solid and liquid substances have only one specific heat capacity i.e. $Cp \cong Cv$ because there is no change of pressure and volume when temperature rises or falls i.e. the expansion of solid and liquid is minimum when heat is supplied to them at constant volume

MAYER'S RELATION CONNECTING Cp AND Cv

Consider one mole of an ideal gas enclosed in a cylinder fitted with a frictionless piston



When the gas is heated at constant volume

$$Q_1 = Cv dT$$

Cv = molar specific heat capacity at constant volume when the gas is heated at constant pressure

Change from V to $V + dV$

$$Q_2 = Cp dT$$

The rise in internal energy is the same for both cases according to 1st law of thermodynamics

$$Q_2 = Q_1 + PdV$$

$$Cp dT = Cv dT + p dV$$

$$Cp dT = Cv dT + PdV \quad Cp dT = R dT$$

$$Cp = Cv + R$$

$$Cp - Cv = R$$

R is Positive, $Cp > Cv$

1. For n moles of a gas

$$nCp = nCv + nR$$

2. Also $Cp = Cv + \frac{R}{Mr}$

For ideal gas

$$PV = \frac{M}{Mr} \cdot RT$$

$$P = \left(\frac{M}{N}\right) \frac{RT}{Mr} \quad \frac{pRT}{Mr}$$

$$\frac{P}{\delta T} = \frac{R}{Mr}$$

$$Cp - Cv = \frac{p}{\phi T}$$

P = pressure of a gas

δ = density of a gas

ATOMICITY (γ)

Is defined as the ratio of principal molar specific heat capacity at constant pressure to the molar specific heat capacity at constant volume.

$$\gamma = Cp / Cv$$

CALCULATE OF Cp / Cv FOR A GAS

Since

$$\gamma = \frac{Cp}{Cv}$$

$$Cp = \gamma Cv$$

$$\text{now } Cp = Cv + R$$

$$\gamma = \frac{R}{\gamma - 1}$$

1. MONOATOMIC GAS

For $n = 1 \text{ mole of gas}$

$$du = \frac{3}{2} R dT$$

$$Cv = \frac{3}{2} R$$

$$\text{Also } Cp = Cv + R = \frac{3}{2} R + R$$

$$Cp = \frac{5}{2} R$$

$$\text{Now } \gamma = \frac{Cp}{Cv} = \frac{5}{2} R / \frac{3R}{2}$$

$$\gamma = \frac{5}{3} = 1.67$$

2. DIATOMIC GAS

For $n = 1 \text{ mole of a gas}$

$$du = \frac{5}{2} R dT$$

$$Cv dT = \frac{5}{2} R dT$$

$$C_v = \frac{5}{2} R$$

$$\text{Also } C_p = C_v + R = \frac{5}{2}R + R$$

$$C_p = \frac{7}{2} R$$

now

$$\gamma = \frac{C_p}{C_v} = \frac{7R}{2} / \frac{5R}{2}$$

$$\gamma = \frac{7}{5} = 1.4$$

3. POLYATOMIC GAS

$$du = 3RdT$$

$$C_v dT = 3RdT$$

$$C_v = 3R$$

also

$$C_p = 3R + R = 4R$$

$$C_p = 4R$$

now

$$\gamma = \frac{C_p}{C_v} = \frac{4R}{3R} = \frac{4}{3}$$

$$\gamma = \frac{4}{3} = 1.33$$

RELATION BETWEEN γ AND f

Internal energy with 1 mole of a gas for f -degree of freedom

$$u = \frac{f}{2} RT, du = \frac{f}{2} R dT$$

but

$$du = C_v dT$$

$$C_v dT = \frac{f}{2} R dT$$

$$C_v = \frac{fR}{2}$$

Also

$$C_p = C_v + R = \frac{fR}{2} + R$$

$$C_p = R \left(\frac{f}{2} + 1 \right)$$

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

IMPORTANCE OF ATOMICITY (γ) OF A GAS

1. It is used to show the variation between pressure and volume of a gas in adiabatic process.

$$PV^\gamma = \text{Constant}$$

2. It is used to determine the speed of sound in the gas

$$V = \sqrt{\frac{\gamma RT}{Mr}}$$

3. It is used to identify the kind of gas molecules or atom which the gas consist

Example

$$\gamma = 1.67 \text{ for monatomic}$$

$\gamma = 1.4$ for diatomic gas

$\gamma = 1.33$ for polyatomic gas

4. The knowledge of γ is used to determine the value of principal heat capacity at constant pressure and constant volume without carrying any experiment.

$$C_p = C_v + \frac{R}{Mr}, \gamma = \frac{C_p}{C_v}$$

$$C_v = \frac{R}{(\gamma - 1)Mr}$$

$$C_p = \frac{\gamma R}{(\gamma - 1)Mr}$$

SOLVED EXAMPLE

Example 01

For a constant mass of the gas, explain how r.m.s speed of molecules change if at all when

- The gas expands at constant temperature
- The gas expands by pushing back the piston so that work is done without entering or leaving the system

Solution

- The root mean square speed of the gas is the function of the temperature so when the gas expands at constant temperature the r.m.s remain constant
- According to the 1st law of thermodynamics

$$dQ = du + dw$$

$$\text{if } dQ = 0, 0 = du + dw$$

Negative sign of change of the internal energy means the decrease of internal energy which is a decrease of temperature therefore r.m.s speed of the gas molecules decreases

Example 02

a) (i) Define the atomicity of a gas

(ii) if f is the number of degrees of freedom, of a gas show that

$$\gamma = 1 + \frac{2}{f}$$

b). The density of a gas is 1.775 Kg m^{-3} at 27°C and 10^5 Nm^{-2} pressure and specific heat capacity at constant pressure is $846 \text{ J Kg}^{-1} \text{ K}^{-1}$ find atomicity of a gas.

Solution

a) Refer to your notes

$$b) C_p - C_v = \frac{P}{\rho T} \gamma = \frac{C_p}{C_v} \rightarrow C_p = \gamma C_v$$

$$\gamma C_v - C_v = \frac{p}{\rho T}$$

$$\gamma = \frac{p}{\rho T C_v} + 1 = \frac{10^5}{1.775 \times 300 \times 846} + 1$$

$$\gamma = 1.22$$

Example 03

Two cylinders A and B, fitted with pistons contain equal amounts of an ideal diatomic gas at 300K . The piston A and B is fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K , find the rise in temperature

Solution

Cylinder A > since the piston of the cylinder A is free to move, gas is heated at constant pressure

$$dQ = nC_p dT \text{ but } C_p = \frac{7}{2}R$$

$$dQ = \frac{7}{2}nRdT, \text{ --- (i)}$$

Cylinder B: the piston of a cylinder B is fixed, the gas is heated at constant volume

$$dQ = nC_v dT_2, C_v = \frac{5}{2}R$$

$$dQ = \frac{5}{2}nRdT_2 \text{ --- (ii)}$$

$$(i) = (ii)$$

$$\frac{5}{2}nRdT_2 = \frac{7}{2}nRdT_1$$

$$dT_2 = \frac{7}{5}$$

$$dT_1 = \frac{7}{5} \times 30 dT_2 = 42K$$

Example 04

a). (i) What are the two principal specific heat capacities of a gas and why are they different?

(ii) show that the difference between these specific heat capacities is given by

$$C_p - C_v = \frac{p}{\rho T}$$

T are the pressure, density and temperature respectively

b) A steel pressure vessel of volume

$2.2 \times 10^{-2} m^3$ contain $4.0 \times 10^{-2} Kg$ of gas at a pressure of 1.0×10^5 Pa and temperature 300K. An explosion suddenly releases $6.48 \times 10^4 J$ of energy which rises the pressure instantaneously to 1.0×10^6 Pa. Assuming no loss of heat to the vessel and ideal gas behavior calculate

- The maximum temperature attained
- Two principal specific heat capacities of the gas

What is the velocity of sound in this gas at a temperature of 300k?

a) Refer to your notes

$$b)(i) T_2 = \left(\frac{P_2}{P_1}\right)$$

$$T_1 = \frac{10^6 \times 300}{10^5}$$

$$(ii) C_p - C_v = \frac{p}{\rho T}$$

$$= \frac{10^5 \times 2.2 \times 10^{-2}}{300 \times 0.04}$$

$$C_p - C_v = 183.33 JKg^{-1}K^{-1}$$

$$\text{Since } C_v = \frac{Q}{M \Delta T} = \frac{6.48 \times 10^4}{0.04(3000-300)}$$

$$C_v = 600 JKg^{-1}K^{-1}$$

$$C_p = 183.33 + 600$$

$$C_p = 783.33 JKg^{-1}K^{-1}$$

Velocity of sound at 300k

$$V = \sqrt{\frac{\gamma P}{\rho}}, \gamma = \frac{C_p}{C_v}, \int = \frac{M}{V} V = \sqrt{\left(\frac{C_p}{C_v}\right) \left(\frac{PV}{M}\right)}$$
$$= \left(\frac{783.33}{600}\right) \frac{10^5 \times 2.2 \times 10^{-2}}{0.04}$$

$$V = 267.97 M/S \approx 268 M/S$$

Example 05 (NECTA 1997/P1/17)

- a) Define the principle molar heat capacity of a gas
- b) Why is the energy needed to raise the temperature of a fixed mass of a gas by a specific amount is greater if pressure is kept constant than if volume is kept constant $C_p > C_v$?
- c) Find the two principal of heat capacities for oxygen molecules where ratio of $\frac{C_p}{C_v} = 1.4$ at *S.T.P* Given that

$$R = 8.3 \text{ J mole}^{-1} \text{ K}^{-1}$$

Solution

(a) And (b) refer to your notes

$$(c) \frac{C_p}{C_v} = 1.4, C_p = 1.4 C_v$$

$$C_p = C_v + R \quad 1.4 C_v = C_v + R \quad .04 C_v C_p = C_v + R \quad 1.4$$

$$C_v = C_v + R \quad .04 C_v = R$$

$$C_v = \frac{R}{0.4} = \frac{8.3}{0.4} \quad C_v = 20.75 \text{ J mol}^{-1} \text{ K}^{-1} \quad C_p = 1.4 C_v$$

$$C_p = 29.5 \text{ J mol}^{-1} \text{ K}^{-1}$$

Example 06

The heat capacity C_v at constant volume of 8 moles of oxygen gas is $116.2 \text{ J mole}^{-1}$

Solution

$$nC_p = nC_v + nR \quad 8C_p = 8C_v + 8$$

$$R = 1.66.2 + 8 \times 8.38$$

$$C_p = 232.6 \text{ J K}^{-1}$$

Example 07

- a) Show that $C_v = \frac{R}{\gamma - 1}$
- b) At 27°C and pressure of $1.0 \times 10^5 \text{ Pa}$ of an ideal gas has a volume of 0.04 m^3 it is heated at constant pressure until its volume increases to 0.05 m^3 . Find
- The external work done
 - The new temperature of gas
 - The change in internal energy of a gas if its mass is 45 g its molar mass is 28 g and its molar heat capacity at constant volume is $0.6 \text{ J Mol}^{-1} \text{ K}^{-1}$
 - The total gain by the gas

Solution

a) Refer to your notes

$$b) (i) w = p(v_2 - v_1) = 1 \times 10^5 (0.05 - 0.04) = 1000 \text{ J}$$

ii) By Charles's law

$$T_2 = \left(\frac{V_2}{V_1} \right) T_1 = \left(\frac{0.05}{0.04} \right) \times 300 = 375 \text{ K}$$

$$iii) n = \frac{M}{M_r} = \frac{45}{28}$$

$$du = nC_v dT = \frac{45}{28} \times 0.6 (375 - 300)$$

$$du = 72.32 \text{ J}$$

iv) Apply the 1st law of thermodynamics

$$dQ = du + dw$$

$$= 72.32 + 1000$$

$$dQ = 1072.32J$$

EXAMPLE 07

a) State first law of thermodynamic

b) At a temperature of 100°C and pressure of $1.01 \times 10^5 \text{ Nm}^{-2}$, 1.00 Kg of steam occupies 1.67 m^3 but the same mass of water occupies only $1.04 \times 10^{-3} \text{ m}^3$. The specific heat of vaporization of water at 100°C is $2.26 \times 10^6 \text{ J Kg}^{-1}$ for a stream of 5.00 Kg of water changing to steam at 100°C at a pressure of 1.01

- i) Heat supplied to the system
- ii) Work done by the system
- iii) The increase in internal energy of the system

Solution

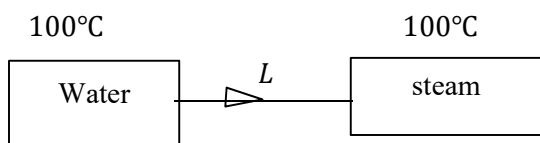
a) Refer to your notes

b) (i) $dQ = \text{heat energy supplied}$ $Dq =$

$$Ml = 5 \times 2.26 \times 10^6$$

$$dQ = 11.25 \times 10^6 J$$

(ii) situation



$$V_L = 5 \times 1.04 \times 10^{-3} \text{ m}^3$$

$$V_L = 5 \times 1.67 \text{ m}^3$$

$$\Delta w = p(V_V - V_L)$$

$$\Delta w = 1.01 \times 10^5 \times 5[1.67 - 1.04 \times 10^{-3}]$$

$$\Delta w = 8.42 \times 10^5 J$$

Example 08

a). The specific heat capacity of helium at constant pressure $5240 \text{ J Kg}^{-1} \text{ K}^{-1}$. Explain why they are different

b)

Gas	He	Ne	Ar	N_2	CO	CO_2
$C_p (\text{J mol}^{-1} \text{ K}^{-1})$	21.0	20.8	20.8	29.1	29.1	29.1
$C_v (\text{J mol}^{-1} \text{ K}^{-1})$	12.6	12.7	12.4	20.7	20.7	20.8

The table above show the molar heat capacity at constant pressure at constant volume for number of gases

i) Show that the molar capacity of substance can be obtained from its specific heat capacity. Find the heat capacity of oxygen at constant pressure

ii) The molar heat capacity of He, Ne and Ar are similar. what conclusion can be drawn from this?

iii) The molar heat capacities of N_2 , C_0 and O_2 have some volume of atomicity gas molecule since have only two atoms

Example 09

When 1.50 kg of water is converted to steam (at 100°C) at atmospheric pressure

$1.01 \times 10^5 \text{ Pa}$) 3.39 MJ . heat vs required. During the transformation from liquid to the volume of the water is 2.50 m^3 . Calculate Work done against the external pressure during the process of vaporization to the rest of the energy.

Solution

When the liquid is converted in to steam the molecules have to push atmosphere dirty they

accompanying an increase to the work done against the work pressure

$$dw = PdV$$

$$= 1.01 \times 10^5 \times 2.5$$

$$\underline{dw = 0.253 \times 10^6 J \text{ } 0.253 MJ}$$

the rest of energy goes to the increases of the internal energy of water

Apply 1st law of thermodynamic

$$du = dQ - dw$$

$$= 3.39 - 0.253$$

$$\underline{du = 3.14 MJ}$$

This is need to work in separating of water molecule during the liquid – vapor transmission. It thus becomes potential energy No kinetic energy change occur because there is no rise in temperature

Example 10

$56 \times 10^{-3} Kg$ of nitrogen is to be heated from

270K to 310 k when this occur in an insulation 2.33Kj of heat is required when contained in an insulated rigid container 1.60KJ of heat is required calculate the principal molar heat capacities of nitrogen (molar mass of nitrogen = $28 \times 10^{-3} Kg$)

Solution

The number of moles of nitrogen

$$n = \frac{56 \times 10^{-3}}{28 \times 10^{-3}} = 2 \text{ mole}$$

Heat require, $dQ = nC_p dT$

$$C_p = \frac{dQ}{ndT} = \frac{2.33 \times 10^3}{2 \times (310 - 270)}$$

$$\underline{C_p = 29.125 JMO^{-1} K^{-1}}$$

Since the container is insulated no change /rise in volume

$$C_v = \frac{dQ}{ndT} = \frac{1.66 \times 10^3}{2(310 - 270)}$$

$$\underline{C_v = 20.75 JMO L^{-1} K^{-1}}$$

Example 11.Necta 2007/p1/5c

When a metal cylinder of mass $2.0 \times 10^{-2} kg$ and special heat capacity $500 JKg^{-1} k^{-1}$ vs heated at constant power ,the initial rase of rase of temperature is $3 Kmin^{-1}$. After a time of heater is switched off and initial rate of fall of temperature is $0.3 Kmin^{-1}$. What vs the rate at which the cylinder gains het energy immediately before the heater is within switched off?

Solution

Rate of heat energy gain immediately by the cylinder before the heater is switched off is equivalent to the net power

$$P = P_1 - P_2 = MC \left[\frac{dQ_1}{dt} - \frac{dQ_2}{dt} \right]$$

$$= 2 \times 10^{-2} \times 500 \left[\frac{3}{60} - \frac{0.3}{60} \right]$$

$$\underline{P = 0.45 W}$$

Example 12

When a quantity of monatomic ideal gas expands at constant pressure of $4 \times 10^4 Pa$ the volume of the gas increases from $2 \times 10^{-3} m^3$ to $8 \times 10^{-3} m^3$,

what vs the change in the internal energy of the gas and the total heat flow to the system ?

Solution

Case 1: For monatomic gas

$$C_V = \frac{3}{2}R$$

The change of internal energy

$$du = nC_V dT = \frac{3}{2}nRdT \text{ --- (i)}$$

For an ideal gas

$$PV = nRT \quad PdV = nRdT = nRdT \text{ --- (2)}$$

Putting equation (2) into (1)

$$\begin{aligned} du &= \frac{3}{2}PdV = \frac{3}{2}P(V_2 - V_1) \\ &= \frac{3}{2} \times 4 \times 10^4 (8 - 2) \times 10^{-3} \\ \underline{du} &= 360J \end{aligned}$$

Case 2 $ddw = pdv = p(V_2 - V_1)$ According to the 1st law of thermodynamics

$$\begin{aligned} dQ &= dW + du \\ &= 4 \times 10^4 \times (8 - 2) \times 10^{-3} + 360 \\ dQ &= 600J \end{aligned}$$

EXAMPLE 13 (roger C72)

a). A gas has volume of $0.02m^3$ at a pressure of $2.0 \times 10^5 Pa$ and temperature of $27^\circ C$ it is heated at constant pressure until its volume increases to $0.03m^3$. calculate

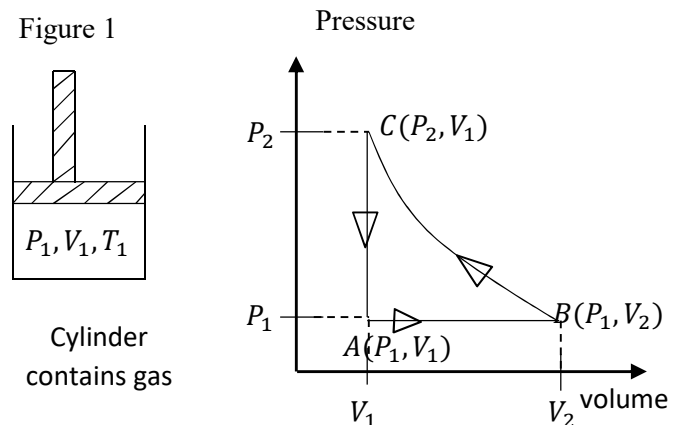
- (i) The external work done
- (ii) The new temperature of the gas

- (iii) The increase in internal energy of a gas if its mass is 16g, its molar heat capacity at constant volume is $0.8Jmol^{-1}K^{-1}$ AND Its molar mass is 32 gm.

b). The cylinder in figure 1 below holds a volume $V_1 = 1000cm^3$ of air at an initial pressure

$P_1 = 1.1 \times 10^5 Pa$ and temperature $T_1 = 300K$.

Assume air behaves as an ideal gas figure below shows a sequence of change imposed on the air in the cylinder



- (i) AB – The air is heated to $375K$ at constant pressure calculate V_2
- (ii) BC – The air is compressed isothermally to V_1 . Calculate the new pressure P_2
- (iii) CA – The air cools at constant volume to pressure P_1 state the value of the work done on the air during the full sequence of changing may be found from the graph.

Solution

$$\begin{aligned} i). w &= (p(V_2 - V_1)) \\ &= 2 \times 10^5 [0.03 - 0.02] \end{aligned}$$

$$W = 2000J$$

ii). According to the Charles's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_2 = T_1 \left[\frac{V_2}{V_1} \right] = 300 \left[\frac{0.03}{0.02} \right]$$

$$T_2 = 450 K$$

iii). du = change of internal energy

$$du = nC_v dT = \frac{m}{Mr} C_v [T_2 - T_1]$$

$$= \frac{16}{32} \times 0.8 \times [450 - 300]$$

$$du = 60 J$$

a)(i) Apply Charles's law

$$V_2 = V_1 \left[\frac{T_2}{T_1} \right] = 1000 \left[\frac{375}{300} \right]$$

$$V_2 = 1250 cm^3$$

ii) For an isothermal condition, temperature remain constant

apply Boyles law

$$P_1 V_2 = P_2 V_1 \quad P_2 = P_1 \left[\frac{V_2}{V_1} \right]$$

$$= 1.1 \times 10^5 \left[\frac{1250}{1000} \right]$$

$$P_2 = 1.375 \times 10^5 P_a$$

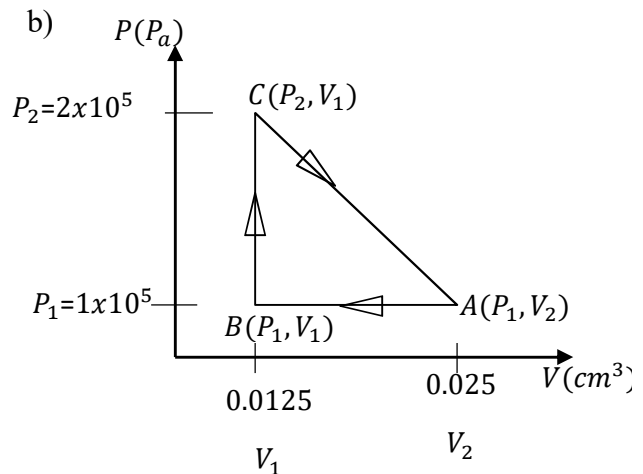
iii) The corresponding work done is equal to the total area under the curve ABC

Example 14 [roger c74]

$$[R = 8.314 J mol^{-1} K^{-1}, C_v = \frac{5}{2} R]$$

a) (i) Prove that work done by a gas in expanding through δV at constant pressure, P is $P\delta V$ and show that for large change in volume, the work done is equal to the area under the curve

ii). Deduce the relationship $C_p - C_v = R$ for the difference between principle molar heat capacity C_p and C_v) for an ideal gas.



The diagram above represent an energy cycle where by a mole of an ideal gas is firstly cooled at constant pressure ($A \rightarrow B$) . the heated at constant volume ($B \rightarrow C$); and then returned to its original state ($C \rightarrow A$)

- (i) Calculate the temperature of the gas at A, B, and C
- (ii) Calculate the heat given out by the gas in the process $A \rightarrow B$
- (iii) Calculate the heat given absorbed process $B \rightarrow C$
- (iv) Calculate the net amount of work transferred in the cycle

- (v) Calculate the net amount of heat transferred in the cycle

Solution

a) Refer to your notes

b) (i) assume that the gas is an ideal of 1 mole [$PV = RT$]

$$\begin{aligned} \text{at } A: P_1 V_2 &= RT_A T_A = \frac{P_1 V_2}{R} \\ &= \frac{1 \times 10^5 \times 0.025}{8.314} T_A = 300.69K \\ &\cong 301K \end{aligned}$$

$$\begin{aligned} \text{At } B: T_B &= \frac{P_1 V_1}{R} \\ &= \frac{1 \times 10^5 \times 0.0125}{8.314} \end{aligned}$$

$$T_B = 150.6K \cong 151K$$

$$\begin{aligned} \text{At } C: T_c &= \frac{p_2 v_1}{R} \\ &= \frac{2 \times 10^5 \times 0.00125}{8.314} \\ \underline{T_c = 301K} \end{aligned}$$

ii) $A \rightarrow B$: Gas cooled at constant pressure

$$dQ = nC_p dT \text{ BUT } C_p = \frac{7}{2} R \text{ } n = 1$$

$$\begin{aligned} dQ &= \frac{7}{2} R [T_A - T_B] \\ &= \frac{7}{2} \times 8.314 [301 - 151] \end{aligned}$$

$$\underline{dQ = 4375J}$$

iii) $B \rightarrow C$: Gas heated at constant volume, $dV = 0$

According to 1st law of thermodynamic

$$dQ = du + PdV [dV = 0]$$

$$dQ = du = nC_v dT$$

$$= \frac{5}{2} Rn [T_C - T_B]$$

$$= \frac{5}{2} \times 8.314 [301 - 151]$$

$$\underline{dQ = 3125 J}$$

iv) Net work done = Area under the graph

$$\Delta W = \frac{1}{2} \Delta P \times \Delta V$$

$$= \frac{1}{2} \times [2 - 1] \times 10^5 \times [0.025 - 0.0125]$$

$$\underline{\Delta W = 625 J}$$

v) Heat transferred = Net work done to the cycle

$$\underline{Q = 625 J}$$

EXAMPLE 15

Three moles of an ideal monatomic gas expand at constant pressure of 2.50 atmosphere, the volume of gas changes from $3.2 \times 10^{-2} m^3$ to $4.5 \times 10^{-2} m^3$.

- Calculate the initial and final temperatures of the gas.
- Calculate the amount of work done in expanding gas.
- Calculate the amount of heat added to the gas.
- Calculate the change in internal energy of the gas.

Solution

a) For an ideal gas of n -moles

$$T_1 = \frac{PV_1}{nR}$$

$$= \frac{2.5 \times 1.01 \times 10^5 \times 4.5 \times 10^{-2}}{3 \times 8.314}$$

$$T_2 = 455.6K$$

$$b). \Delta w = P\Delta V = P(V_2 - V_1)$$

$$= 2.5 \times 1.01 \times 10^5 [4.5 - 3.2] \times 10^{-2}$$

$$\underline{\Delta W = 3282.5 J}$$

c) The amount of heat added

$$dQ = nC_p dT = nC_p(T_2 - T_1) \text{ but } C_p = \frac{5}{2} R$$

$$dQ = \frac{5}{2} nR[T_2 - T_1]$$

$$= \frac{5}{2} \times 3 \times 8.314 [455.6 - 324]$$

$$\underline{dQ = 8206 J}$$

$$d). du = nC_v[T_2 - T_1] = \frac{3}{2} nR[T_2 - T_1]$$

$$= \frac{3}{2} \times 3 \times 8.314 [455.6 - 324]$$

$$\underline{du = 4923 J}$$

Example 16 (roger C70)

A vessel of volume $1.0 \times 10^{-2} m^3$ contains an ideal gas at a temperature of $300K$ and pressure $1.5 \times 10^5 Pa$. calculate the mass of the gas, given that the density of the gas at temperature $285K$ and pressure $1.0 \times 10^5 Pa$ is $1.2 kg m^{-3}$. 750J of heat energy is suddenly released in the gas, causing an instantaneous rise of pressure to $1.8 \times 10^5 Pa$. Assuming ideal gas behavior and no loss of heat to the constraining vessel, rise and hence the specific heat capacity at constant volume of a gas

Solution

Case 1

$$V = 1.0 \times 10^{-2} m^3, T_1 = 300K$$

$$P_1 = 1.5 \times 10^5 Pa, T_2 = 285K, P_2 = 1.0 \times 10^5 Pa$$

$$\text{since } \frac{P}{\delta} = \frac{RT}{Mr} \frac{P_1}{\delta_1} = \frac{RT}{Mr} \frac{P_2}{\delta_2} = \frac{RT_2}{Mr}$$

$$P_1 = \frac{RT_1 \delta_1}{Mr}, \frac{RT_2 \delta_2}{Mr}$$

$$\frac{P_1}{P_2} = \left(\frac{\delta_1}{\delta_2} \right) \left(\frac{T_1}{T_2} \right)$$

$$\delta_1 = \delta_2 \left(\frac{P_1}{P_2} \right) \left(\frac{T_2}{T_1} \right)$$

$$= 1.2 \times \left[\frac{1.5 \times 10^5}{1.0 \times 10^5} \right] \left[\frac{285}{300} \right]$$

$$\delta_1 = 1.71 kg m^{-3}$$

let M – mass of the gas

$$M = \delta_1 V = 1.71 \times 10 \times 10^{-2}$$

$$\underline{M = 0.0171 Kg}$$

Case 2

Since

$$\frac{p}{T} = \text{constant}$$

$$T = \frac{PT_2}{P_1} = \frac{1.8 \times 10^5 \times 300}{1.5 \times 10^5}$$

$$\underline{T = 360K}$$

$$\Delta T = \text{temperature rise}$$

$$\underline{\Delta T = 360 - 300 = 60 K}$$

Since the amount of heat energy added

$$dQ = MC_v dT$$

$$C_v = \frac{dQ}{MdT} = \frac{750}{0.0171 \times 60}$$

$$C_v = 730 \text{ J K}^{-1} \text{ g}^{-1} \text{ (approx)}$$

APPLICATION OF THE FIRST LAW OF THERMODYNAMICS

1. Derive relation between C_p and C_v
2. Describe the boiling process of the liquid
3. Describe the melting process of solid
4. Describe thermodynamic process
5. Derivation of equation cyclic adiabatic process cyclic process.

1.DERIVATION OF MAYER'S EQUATION

$$[C_p - C_v = R]$$

Apply first law of thermodynamic

$$dQ = du + dw = du + PdV$$

$$nC_p dT = nC_v dT + nRdT$$

$$C_p = C_v + R$$

2.BOILING PROCESS

When a liquid is heated, it changes in to vapor at constant temperature (called boiling point) and pressure when water is heated at normal atmospheric pressure its boils at 100°C . The temperature remains unchanged during the boiling process.

Let

M = mass of liquid at its boiling point

V_1 = Volume of liquid

V_2 = Volume of vapor

p = constant pressure at which boiling process occur

L = latent heat of vaporization of liquid

\therefore work done in expansion

$$dW = PdV = P(V_2 - V_1)$$

The amount of heat energy required to convert the whole liquid in to vapor i.e heat absorbed by the liquid in boiling process

$$dQ = ML$$

According to the first law of thermodynamic

$$dQ = du + dw$$

$$du = dQ - dw = ML - P(V_2 - V_1)$$

$$du = u_f - u_i = ML - P(V_2 - V_1)$$

u_f = internal energy of vapour

u_i = internal energy of the liquid

3.MELTING PROCESS

When a solid substance change in to liquid state (melting), its internal energy increases. This can be calculated from the first law of thermodynamics

Let M = mass of the solid

L = specific latent heat of fusion

Heat absorbed during the melting process

$$dQ = ML$$

Since during the melting process, the change in volume is very small ($dV = 0$)

$$dw = 0$$

$$\text{now } dQ = du$$

$$du = dQ = ML$$

Therefore, during the melting process internal energy increases by ML , since temperature remain constant during melting the kinetic energy remain the same. Thus, the increase in internal energy is due to the increases in potential energy

Note that

The latent -heat of substance is related to the increase in internal energy per unit mass

4.CYCLIC PROCESS

In such processes, the system remains to its initial state after any number of changes, in that case, no intralipid property of the system would change \therefore

$$dQ = du + dw$$

$$dQ = dw$$

\therefore net work done during a cyclic process exactly equal to the amount of heat energy transferred

SOLVED EXAMPLE

Example 01

1gm of water (1cm^3) become 1671 cm^3 of steam at a pressure of $1\text{atm} = 1.03 \times 10^5\text{Pa}$

The latent heat of evaporations at at this pressure is 2254 Jgm^{-1} Calculate the external work done and the increase in internal energy.

Solution

External work done

$$w = p(V_2 - V_1) = 1.013 \times 10^5$$

$$[1671 - 1.0] \times 10^6$$

$$\underline{W = 169J}$$

The heat required to convert 1gm of water in to vapor

$$Q = ML = 1 \times 2256$$

$$\underline{Q = 2256J}$$

According to the first law of the thermodynamic

$$u = Q - W$$

$$= 2256 - 169$$

$$\underline{u = 2087J}$$

$$\therefore \text{increase in internal energy} = 2087\text{ J}$$

Example 2.

1.0m^3 of water is converted in to 1671 m^3 of steam at (atmosphere) atmospheric pressure and 100°C temperature. The latent heat of vaporization of water $2.3 \times 10^6\text{Jkg}^{-1}$ if 2.0kg of water is converted in to steam at atmospheric pressure and 100°C temperature , then how much will be the increase in its internal energy?

$$\text{density of water} = 1000\text{kgm}^{-3}$$

$$\text{Atmospheric pressure} = 1.01 \times 10^5\text{N/m}^2.$$

Solution

Heat supplied to 2.0 kg of water to convert in to steam at 100°C

$$dQ = ML$$

$$= 2 \times 2.3 \times 10^6$$

$$dQ = 4.6 \times 10^6\text{J}$$

Volume of water of 2 kg

$$V_1 = \frac{M}{\delta} = \frac{2}{1000} = 2 \times 10^{-3}\text{m}^3$$

Volume of stream

$$1m^3 \rightarrow 1671m^3$$

$$2 \times 10^{-3}m^3 \rightarrow V_2$$

$$V_2 = 1671 \times 2 \times 10^{-3} = 3342 \times 10^{-3}m^3$$

Work done

$$\begin{aligned} dW &= P(V_2 - V_1) \\ &= 1.01 \times 10^5 [3342 - 2] \times 10^{-3} \end{aligned}$$

$$dW = 0.337 \times 10^6 J$$

According to the first law of the thermodynamic

$$dU = dQ - dW = (4.6 - 0.337) \times 10^6$$

$$\underline{du = 4.263 \times 10^6 J}$$

EXAMPLE 3

At 0°C and normal atmospheric pressure the volume of 1 gm of water increases from $1cm^3$ to $1.091cm^3$ on freezing. What will be the change in its internal energy normal atmospheric pressure is $1.013 \times 10^5 N/m^2$ and latent heat of melting ice

$$L = 80 \text{ Cal/gm}$$

Solution

Heat given out by 1 gm of water in freezing

$$dQ = -ML = -1 \times 80$$

$$dQ = -80 \text{ Cal}$$

Negative sign shows that dQ is given out by water during the freezing, the water expands against atmospheric pressure

External work done

$$dW = Pdv = 1.013 \times 10^5 [1.091 - 1] \times 10^{-6}$$

$$dw = 0.0092J$$

$$\text{Since } 1 \text{ cal} \rightarrow 4.2 \text{ J}$$

$$dw \rightarrow 0.0092 \text{ J}$$

$$dw = \frac{0.0092}{4.2} = 0.0022 \text{ cal}$$

$$\underline{dW = 0.0022 \text{ Cal}}$$

Since the work has been done by ice, it will be taken as positive

According to the first law of thermodynamic

$$du = dQ - dw$$

$$= -80 - 0.0022 \text{ du} = -80.0022 \text{ cal}$$

Negative sign shows decrease in internal energy on freezing this decrease is more than the heat is given up by the water

THERMODYNAMIC PROCESS

1. ISOBARIC POCCESS

Is the thermodynamic process that take place at constant pressure.

Example

- (i) Heat of water at atmospheric process
- (ii) Melting process

EQUATION FOR AN IDEAL GAS FOR ISOBARIC PROCESS

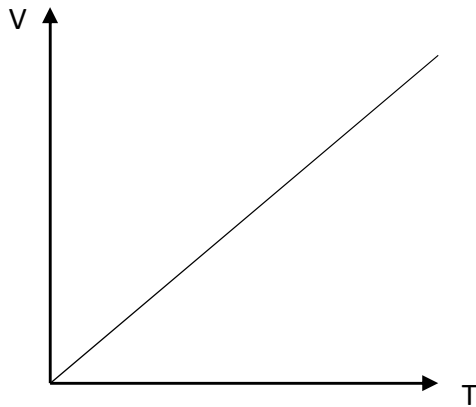
$$\text{Since } Pv = nRT$$

$$\frac{V}{T} = \frac{nR}{P} = \text{Constant}$$

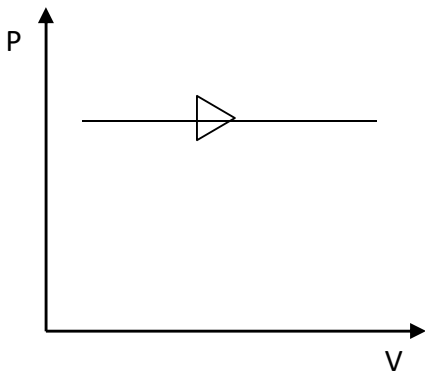
$$\frac{V}{T} = \text{Constant}$$

$V \propto T$. Charle's law

GRAPH OF V AGAINST T

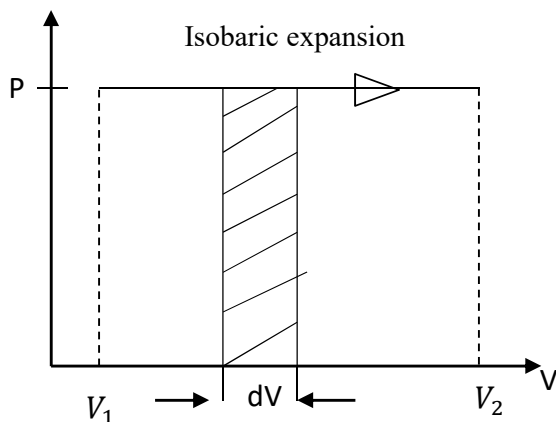


GRAPH OF P AGAINST V



WORK DONE BY EXPANSION OF GAS UNDER ISOBARIC PROCESS

When heat is given to the gas the volume and temperature change without any change in pressure.
When gas expands isobarically its temperature rises



Total work done to expand gas from volume V_1 to V_2

$$W = \int_{V_1}^{V_2} P dV = P(V_2 - V_1)$$

$$W = P(V_2 - V_1)$$

In this case heat added increases the internal energy of the gas as well as gas does external work done since

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}, V_2 = \frac{T_2 V_1}{T_1}$$

$$W = P(V_2 - V_1)$$

$$W = P \left[\frac{T_2 V_1}{T_1} - V_1 \right] = P V_1 \left[\frac{T_2}{T_1} - 1 \right]$$

$$W = P V_1 \left[\frac{T_2}{T_1} - 1 \right] = n R T_1 \left[\frac{T_2}{T_1} - 1 \right]$$

OR

$$W = n R (T_2 - T_1)$$

According to the first law of thermodynamic

$$dQ = du + dw$$

$$dQ = n C_v (T_2 - T_1) + P(V_2 - V_1)$$

T_1 = initial temperature

T_2 = Final temperature

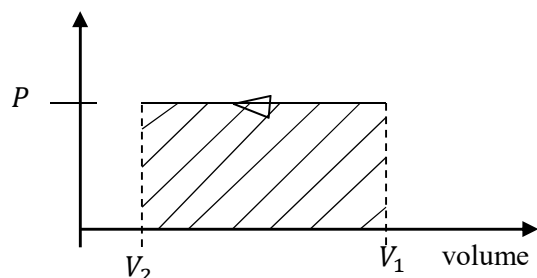
Note that

1. When a gas expands isobarically, that the Heat supplied dQ Cannot be negative

Proof. for an expansion of gas $V_2 > V_1$, ΔW is positive

- Since $V \propto T$ It follows that $T_2 > T_1$ and $du = n C_v (T_2 - T_1)$ is positive.
- Since dW and du is positive

2. When the gas is compressed isobarically, heat must be removed from the system and dQ is negative



2.ISOCHORIC (ISOVOLUMETRIC) PROCESS

Is the process of the thermodynamic in which the volume of the gas remains constant

Example

- i) A gas enclosed in a cylinder having rigid walls and fixed piston. When heat is added to the gas there would be no change in the volume of the gas
- ii) When the substance melts the change in volume is negligible small. So, this may be regarded as nearly isochoric process.

EQUATION FOR AN IDEAL GAS

UNDERGOING ISOCHORIC PRCESS

For one mole of an ideal gas

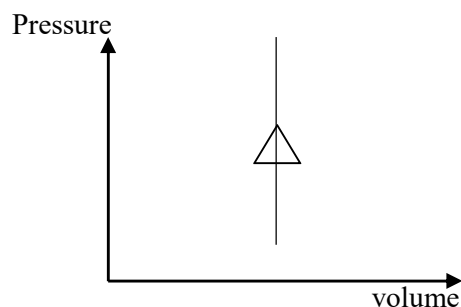
$$PV = RT \quad P = \frac{R}{V} = \text{CONSTANT}$$

$$\frac{P}{T} = \text{CONSTANT}$$

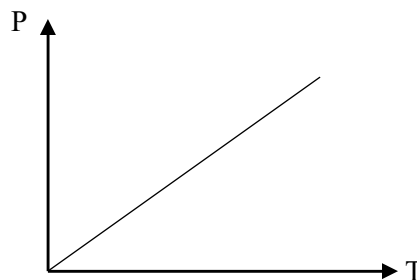
$$(P \propto T)$$

This represent equation of isochoric process. It is obeying pressure law

THE GRAPH OF P AGAINST V



THE GRAPH OF P AGAINST T



WORK DONE BY EXPANDING GAS UNDER ISOCHORIC PROCESS

For an isochoric process, there is no work done by expanding gas since the process taken place at the constant volume $dV = 0$

Apply first law of thermodynamic

$$dQ = du + pdV \text{ but } dV = 0$$

$$dQ = du$$

The heat energy supplied on the system is only due to the internal energy of the gas

$$\text{Since } dU = nC_v dT$$

$$\text{now } dQ = DU = nC_v dT \quad \int dQ = nC_v \int_{T_1}^{T_2} dT \quad Q$$

$$Q = nC_v(T_2 - T_1)$$

Also

$$PV = nRT, T = \frac{PV}{nR}$$

$$Q = nC_v \left[\frac{P_2 V}{nR} - \frac{P_1 V}{nR} \right]$$

$$Q = \frac{VC_v}{R} [P_2 - P_1]$$

NOTE THAT

1. When heat energy is supplied to the gas under isochoric internal energy increases so that the temperature of the gas rises
2. If heat is taken from a system process, isochoric of the gas decreases so that temperature of the gas falls.
3. **ISOTHERMAL PROCESS**

Is the process of the thermodynamic which takes place when temperature of the gas is kept constant.

It is physical change in which pressure and volume of a system change at constant temperature.

CONDITION NECESSARY FOR ISOTHERMAL PROCESS TO TAKE PLACE

1. This process takes place very slowly.
This allow enough time for heat change
2. The gas container must be surrounded by constant temperature reservoir.
3. The gas should be kept in a container made of diathermal (very thin) walls.
This is to allow heat exchange between system and surrounding.

Example

i) Melting process is an isothermal change because temperature of a substance remains constant during melting

ii) Similarly boiling process is also an isothermal operation

iii) Slow expansion or compression of an ideal gas at constant temperature

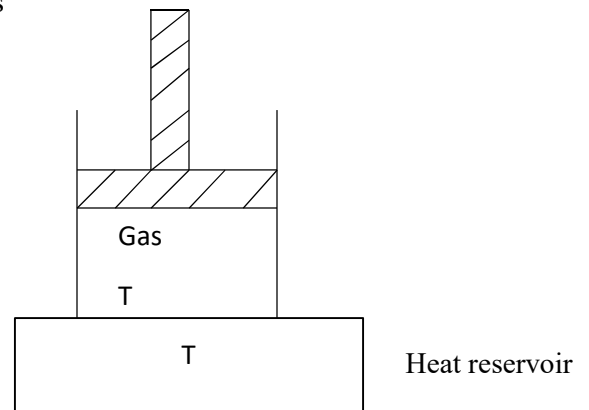
Note that

Consider an ideal gas contained in a cylinder of heat-conducting bottom and fitted with a piston. To carry out isothermal process, we place the heat conducting both bottom of the cylinder in good thermal

Conduct with second system of very large heat capacity and same temperature as the gas

This second system is known 'HEAT RESERVOIR'

since the heat capacity of a heat reservoir is very large and volume of the gas is changed very slowly, the temperature remains constant throughout the process



EQUATION OF ISOTHERMAL PROCESS

Assume the gas is ideal gas of 1 mole of a gas

$$PV = RT$$

For an isothermal process temperature is kept constant

$$PV = \text{Constant}$$

The equation of an isothermal process obeys

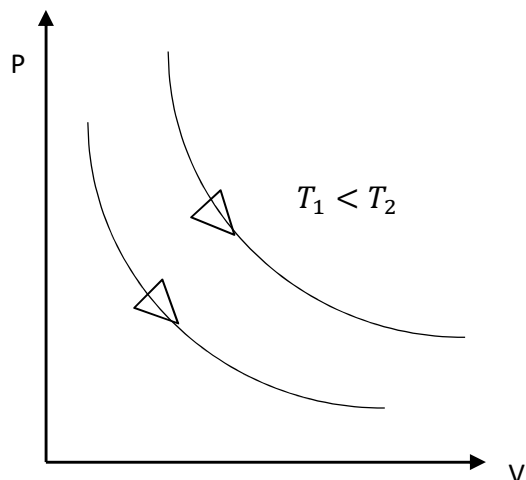
Boyle's law

ISOTHERM – Is the path of isothermal process on PV curve for ideal gas

ISOTHERMAL CURVE – is the graph or curve between volume (V) and pressure (P) kept temperature constant

Since $PV = \text{Constant}$, $P \propto \frac{1}{V}$

GRAPH OF P AGAINST VOLUME, V

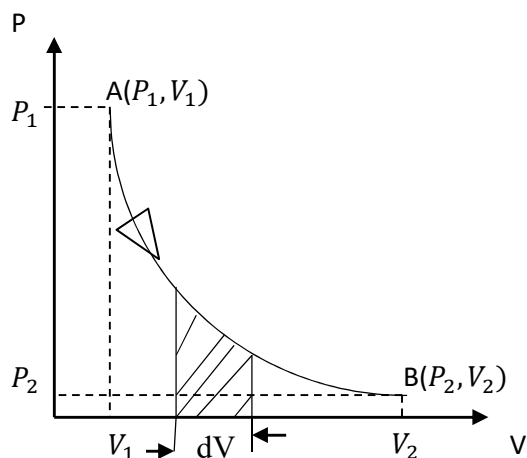


EXPRESSION DURING WORK DONE DURING ISOTHERMAL PROCESS

1. for n ideal gas of one mole

$$PV = RT, P = \frac{RT}{V}$$

Total work done by the gas to expands from volume V_1



$$W = \int_{V_1}^{V_2} P dv = \int_{V_1}^{V_2} \frac{RT}{V} dv$$

$$= RT \left[\log_e V \right]_{V_1}^{V_2}$$

$$W = RT \log_e \frac{V_2}{V_1}$$

For n – number of moles

$$W = nRT \log_e \left(\frac{V_2}{V_1} \right)$$

Let p_1 and P_2 be the pressure of gas corresponding to the volume V_1 and V_2 respectively.

$$P_1 V_1 = P_2 V_2 = nRT$$

$$w = P_1 V_1 \log_e \left(\frac{V_2}{V_1} \right) = P_2 V_2 \log_e \left(\frac{V_2}{V_1} \right)$$

Also

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$w = RT \log_e \left(\frac{P_1}{P_2} \right) n = \text{mole}$$

For n- number of moles

$$w = nRT \log_e \left(\frac{P_1}{P_2} \right)$$

- If we can consider one gram of an ideal gas.

$$w = \frac{RT}{Mr} \cdot \log_e \left(\frac{V_2}{V_1} \right) = rT \log_e \left(\frac{V_2}{V_1} \right)$$

- Work done by expanding real gas of one mole from volume V_1 to V_2

- For real gas of one mole

$$\left(P + \frac{a}{V^2} \right) (V - b) = RT$$

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \left(\frac{RT}{V - b} - \frac{a}{V^2} \right) dV$$

$$dV = RT \int_{V_1}^{V_2} \frac{dV}{V - b} - a \int_{V_1}^{V_2} V^{-2} dV$$

$$= RT [\log_e (V - b)] \Big|_{V_1}^{V_2} - a \left[\frac{-1}{V} \right] \Big|_{V_1}^{V_2}$$

$$w = RT \log_e \left(\frac{V_2 - b}{V_1 - b} \right) + a \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

- For n moles of real gas

$$\left(P + \frac{an^2}{V^2} \right) (V - nb) = nRT$$

$$P = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}$$

$$W = \int_{V_1}^{V_2} \left(\frac{nRT}{V - nb} - \frac{n^2 a}{V^2} \right) dV$$

$$W = nRT \log_e \left(\frac{V_2 - nb}{V_1 - nb} \right) + an^2 \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$$

APPLICATION OF FIRST LAW OF THERMODYNAMICS TO ISOTHERMAL PROCESS

According to the first law of thermodynamics

$$dQ = du + dW$$

The internal energy of an ideal depend on temperature. Since temperature does not change for an isothermal process.

$$dT = 0 \quad du = nC_v dT = 0$$

$$dQ = dW = nRT \log_e \left(\frac{V_2}{V_1} \right)$$

Therefore, the system is known as isolated system

Definition

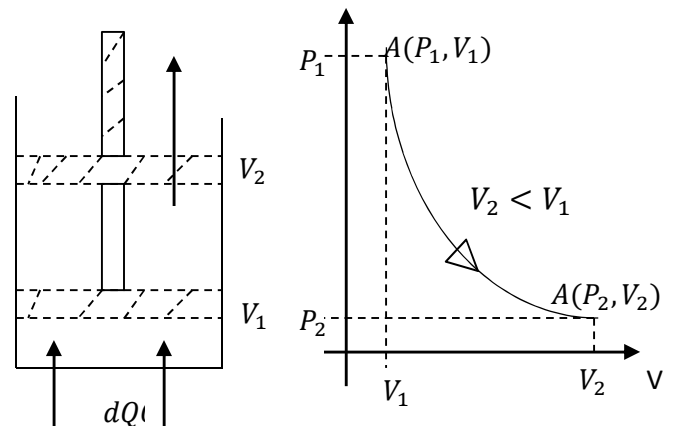
ISOLATED SYSTEM – is the system in which the amount of internal energy of the gas always remain constant i.e. $du = 0$

Since the energy supplied is equal to the external work done by the expanding gas. This implies the principle of conservation of energy

Its applied in mechanical engineering and industrial

(i) ISOTHERMAL EXPANSION -

When a gas expands isothermally then dV Is positive and so dW and dQ both are positive.



Expansion of Heat

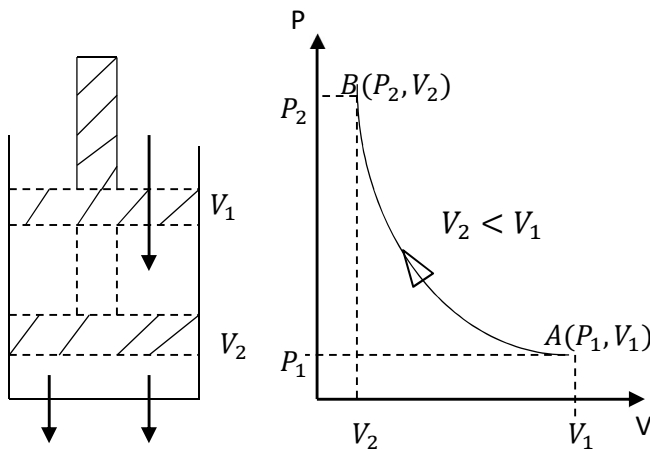
This means that for an isothermal expansion of the gas an amount of heat vs equal to work done the gas

will have to be supplied from the external source in order to maintain constant temperature of the gas. This heat comes from the heat reservoir. ($w > 0$)

$$w = nRT \log_e \left(\frac{V_2}{V_1} \right) = nRT \log_e \left(\frac{P_1}{P_2} \right)$$

(ii) ISOTHERMAL COMPRESSION

When a gas is compressed isothermally, dV is negative, and so dW and dQ will be also negative. This means that for isothermal compression of the gas an amount of heat energy is equal to the work done on the gas if its temperature is not change. This heat is given by the heat reservoir



$$W = nRT \log_e \left(\frac{V_2}{V_1} \right) = nRT \log_e \left(\frac{V_2}{V_1} \right) \quad w < 0$$

Additional concepts

1. In an isothermal change $T = \text{constant}$,

$$dT = 0 \text{ As}$$

$$C = \frac{dQ}{MdT} \quad dT = 0$$

$$C = \infty$$

During an isothermal change, specific heat capacity of a gas is infinite.

2. $du = 0$ for an isothermal expansion or compression

3. For isothermal expansion wholly heat supplied dQ does external work done.

4. For isothermal compression all work done on the gas is lost as heat to surrounding

5. Isothermal bulk modulus of a gas

For reversible isothermal change of a gas.

$$PV = \text{constant}, C$$

$$\frac{d}{dv} [PV] = \frac{d}{dV} [C]$$

$$P + \frac{vdp}{dv} = 0 \quad \frac{dp}{dv} = \frac{-p}{v}$$

$$\text{Gradient of isothermal} \left(\frac{dV}{dV} \right) = \frac{-P}{V}$$

$$P = \frac{-VdP}{dV} \text{ Kiso}$$

$$\boxed{\text{kiso} = E_i = P}$$

Pressure of the gas equal to the bulk modulus of a gas under isothermal process

04. ADIABATIC PROCESS

Is the process of thermodynamic in which no heat energy leaves or enters the system i.e.

$$dQ = 0$$

It is that thermodynamic process in which pressure, volume, and temperature of the system change but

there is no exchange of heat energy between the system and the surrounding

Example of adiabatic process

1. If a gas is suddenly expanded by moving the piston outward there will be the decrease of temperature of a gas.
2. A sudden explosion
3. Busting of bus type (i.e rapid escape of air from burst type tube)
4. Propagation of sound in air
5. Expansion of hot gases in internal combustion engine.
6. Expansion of a stream in the cylinder of a steam engine
7. Quickly inflating of bicycle type
8. All heat engine like automobile engine, jet engines etc.

CONDITION NECESSARY FOR ADIABATIC CHANGE (PROCESS)

1. The process of compression or expansion should be sudden so that there is no time for the exchange of heat.
i.e $dQ = 0$
2. The gas must be contained in a thick walled poorly conducting cylinder and fitted with a frictionless movable piston
3. The process should take place very rapidly

APPLICATION OF FIRST LAW OF THERMODYNAMICS IN AN ADIABATIC PROCESS

According to the first law of thermodynamic

$$dQ = du + dW$$

1. ADIABATIC EXPANSION

When the gas expand adiabatically, dV is positive and dW is positive

$$dW = -du = -nC_vdT$$

negative sign In an internal energy means the decreases of internal energy and temperature of the gas falls.

Therefore, when a gas expand adiabatically, decreases in internal energy is equal to the work done by the gas and temperature of the gas falls. This adiabatic expansion produces “cooling process”

2. ADIABATIC COMPRESSION

When a gas is compressed adiabatically, dv is negative, so that dW is negative

$$-dw = du = nC_vdT$$

Positive sign of du is means the increases of internal energy is equal to the work done on the gas and temperature of the gases rises.

EQUATION OF ADIABATIC PROCESS

1. Equation of adiabatic relation between the pressure P and volume V of an ideal gas

$$PV^\gamma = \text{Constant}$$

$$\gamma = \frac{C_p}{C_v} = \text{Atomicity}$$

$$\text{Derivation} :: PV^\gamma = \text{Constant}$$

According to the first law of thermodynamics

$$dQ = du + pdV$$

$$\text{Since } dQ = 0$$

$$0 = du + PdV$$

$$0 = nC_v dT + PdV \text{ --- (i) for ideal gas}$$

$$\frac{d}{dT}(PV) = \frac{d}{dT}(nRT)$$

$$\frac{PdV}{dT} + \frac{VdP}{dT} = nR$$

$$PdV + VdP = nRdT$$

$$dT = \frac{PdV + VdP}{nR} \text{ --- (ii)}$$

Putting equation (ii) in to (i)

$$0 = nC_v \left[\frac{PdV + VdP}{nR} \right] + PdV$$

$$0 = C_v PdV + VC_v dP + RPdV$$

$$\text{but } R = C_p - C_v$$

$$0 = C_v PdV + VC_v dP + (C_p - C_v)PdV$$

$$0 = PC_p dV + VC_v dP$$

$$0 = \frac{PC_p dV}{C_v PV} + \frac{VC_v dP}{C_v PV}$$

$$0 = \gamma \frac{dV}{V} + \frac{dP}{P} \int 0 = \gamma \int \frac{dV}{V} + \int \frac{dP}{P}$$

$$C = \gamma \log_e V + \log_e P = \log_e PV^\gamma$$

In exponential form

$$PV^\gamma = e^C$$

$$PV^\gamma = \text{Constant}$$

Let P_1 and P_2 be pressure of gas

at the volume V_1 and V_2 respectively

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

2. Equation of adiabatic relation volume, V and temperature T

$$TV^{\gamma-1} = \text{Constant}$$

Derivation $TV^{\gamma-1} = \text{constant}$

According to the law of thermodynamics

$$dQ = du + PdV [dQ = 0]$$

$$0 = nC_v dT + PdV$$

$$\text{since } PV = nRT, P = \frac{nRT}{V}$$

$$0 = nC_v dT + \frac{nRT}{V} dV$$

$$0 = \frac{C_v dT}{T} + (C_p - C_v) \frac{dV}{V}$$

$$0 = \frac{dT}{T} + \left(\frac{C_p}{C_v} - 1 \right) \frac{dV}{V}$$

$$0 = \frac{dT}{T} + (\gamma - 1) \frac{dV}{V}$$

$$\int 0 = \int \frac{dT}{T} + (\gamma - 1) \int \frac{dV}{V}$$

$$\text{constant} = \log_e T + (\gamma - 2) \log_e V$$

$$C = \log_e T + \log_e V^{\gamma-1}$$

$$C = \log_e [TV^{\gamma-1}]$$

In exponential form

$$TV^{\gamma-1} = \text{Constant}$$

Alternative

From the equation: $PV^\gamma = \text{constant}$

$$P = \frac{nRT}{V} \frac{nRT}{V} \cdot V^\gamma = C$$

$$TV^{\gamma-1} = \text{constant}$$

Let V_1 and V_2 be volume of the gas at the temperature T_1 and T_2 respectively

3. Equation of an adiabatic relation pressure, p temperature, T

$$\frac{T^\gamma}{P^{1-\gamma}} = \text{constant}$$

proof : $T^\gamma P^\gamma = \text{constant}$

$$PV^\gamma = \text{constant}$$

$$V = \frac{RT}{P}$$

$$P \left[\frac{RT}{P} \right]^\gamma = \text{Constant}$$

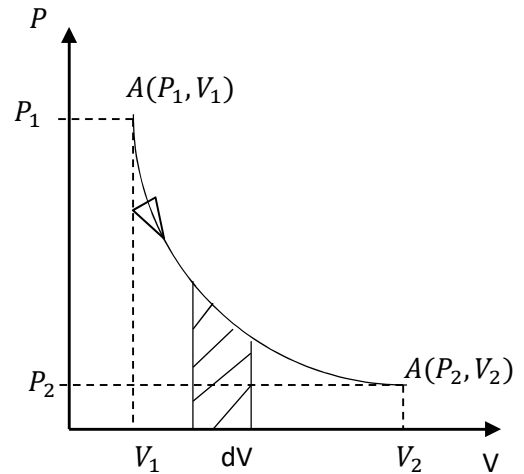
$$T^\gamma P^{1-\gamma} = \text{Constant}$$

$$T_1^\gamma P_1^{1-\gamma} = T_2^\gamma P_2^{1-\gamma}$$

WORK DONE DURING ADIABATIC PROCESS

Adiabatic curve – is the curve which represent relative variations of pressure and volume of a give mass of a gas

$$PV^\gamma = k, P = \frac{K}{V^\gamma}, P \propto \frac{1}{V^\gamma}$$



1. Suppose the gas expands adiabatically the initial state $A(P_1, V_1)$ TO Final state $B(P_2, V_2)$

$$[PV^\gamma = K, P = KV^{-\gamma}]$$

$$W_{adi} = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} KV^{-\gamma} dV$$

$$W_{adi} = \frac{K}{1-\gamma} [V_2^{1-\gamma} - V_1^{1-\gamma}]$$

$$W_{adi} = \frac{1}{1-\gamma} [KV_2^{1-\gamma} - KV_1^{1-\gamma}]$$

$$\text{But } P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$W_{ad} = \frac{1}{1-\gamma} [KV_2^{1-\gamma} - KV_1^{1-\gamma}]$$

$$\text{but } P_1 V_1^\gamma = P_2 V_2^\gamma = K$$

$$W_{adi} = \frac{1}{1-\gamma} (P_2 V_2 - P_1 V_1) = \frac{1}{\gamma-1} (P_1 V_1 - P_2 V_2)$$

Special case: from ideal gas

$$\text{Equation: } PV = nRT.$$

$$P_1 V_1 = nRT_1, \quad P_2 V_2 = nRT_2$$

$$W_{adi} = \frac{nR}{1-\gamma} [T_1 - T_2]$$

For $n = 1$ mole of an ideal gas

$$W_{adi} = \frac{R}{\gamma-1} (T_1 - T_2)$$

If we consider one gram of an ideal gas i.e $M = 1$ gm

$$PV = nRT = \frac{m}{Mr} RT$$

$$PV = \frac{RT}{Mr} = rT, P_1 V_1 = rT_1, P_2 V_2 = rT$$

$$W_{adi} = \frac{1}{\gamma-1} (P_1 V_1 - P_2 V_2) = \frac{1}{\gamma-1} [rT_1 - rT_2]$$

$$W_{adi} = \frac{r}{\gamma-1} [T_1 - T_2]$$

$$W_{adi} = \frac{R}{Mr(\gamma-1)} (T_1 - T_2)$$

2. According to the first law of thermodynamic

$$dQ = du + dw \quad [dQ = 0]$$

$$0 = du + dw$$

$$dw = -du = -C_v dT$$

Work done when gas expand adiabatically from temperature T_1 to T_2

$$W_{adi} = - \int_{T_1}^{T_2} C_v dT$$

$$W_{adi} = -C_v [T_2 - T_1] = C_v [T_1 - T_2]$$

Special case: for n - moles of the gas

$$W_{adi} = nC_v [T_1 - T_2]$$

3. From the basic of the kinetic theory of gases

$$u = \frac{f}{2} KT \text{ per molecule}$$

For N molecules of gas

$$u = \frac{f}{2} NKT. \text{ but } k = \frac{R}{N_A} = \frac{f}{2} \frac{N}{N_A} RT$$

$$u = \frac{fnRT}{2}$$

$$du = \frac{nfR}{2} dt. [dT = T_2 - T_1]$$

$$du = \frac{nfR}{2} [T_2 - T_1] \text{ but}$$

$$du = -dw - dw = \frac{nfR}{2} (T_2 - T_1)$$

$$W_{adi} = \frac{nfR}{2} (T_2 - T_1)$$

f =degree of freedom

n = number of mole

R =universal gas constant

Since

$$T_1 = \frac{P_1 V_1}{nR}, T_2 = \frac{P_2 V_2}{nR}$$

$$W_{adi} = \frac{f}{2} [P_1 V_1 - P_2 V_2]$$

relation between γ and f

$$\gamma = 1 + \frac{2}{f}$$

ADIABATIC BULK MODULUS OF A GAS

Since $PV^\gamma = \text{Constant}$

$$\frac{d}{dv} (PV^\gamma) = \frac{d}{dv} (c)$$

$$P \frac{d}{dv} V^\gamma + V^\gamma \frac{dP}{dv} = 0$$

$$\gamma PV^{\gamma-1} + V^\gamma \frac{dP}{dV} = 0$$

$$V^\gamma \frac{dP}{dV} = -\gamma PV^{\gamma-1}$$

$$K_{ad} = \frac{dP}{\frac{dV}{V}} = \gamma P$$

$$K_{adi} = \gamma P$$

NOTE THAT

In an adiabatic change,

$$dQ = 0$$

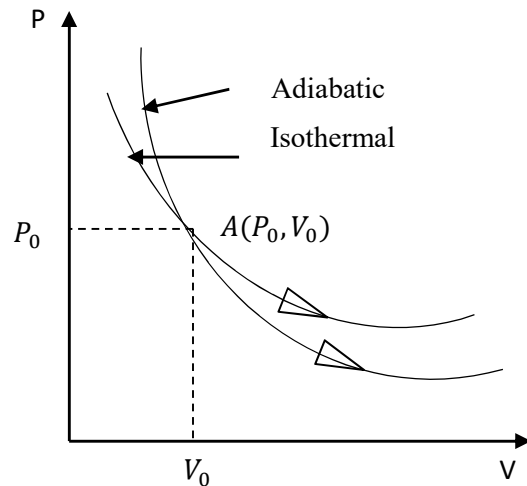
$$C = \frac{dQ}{MdT} = \frac{0}{MdT} = 0$$

Hence in an adiabatic change specific heat capacity of a gas is equal to zero.

COMPARISON OF THE SLOPE OF AN ISOTHERMAL AND ADIABATIC CURVE

Slope of line

$$\tan \theta = \frac{dY}{dX} = \frac{\text{Change in } y - \text{Coordinate}}{\text{Change in } x - \text{Coordinate}}$$



- For an isothermal process

$$PV = \text{Constant}$$

$$PdV + VdP = 0 \quad \frac{dP}{dV} = \frac{-P}{V}$$

$$\text{At point A} \left(\frac{dP}{dV} \right)_{adi} = \frac{-\gamma P_0}{V_0} = \gamma \left| \frac{P_0}{V_0} \right|$$

$$\left(\frac{dP}{dV} \right)_{adi} > \left(\frac{dP}{dV} \right)_{iso}$$

Slope of adiabatic is greater than the slope of isothermal. An adiabatic is steeper than isothermal curve.

$$\left(\frac{dP}{dV} \right)_{ad} / \left(\frac{dP}{dV} \right)_{iso} = \gamma$$

Ratio of bulk modulus

$$\frac{E_a}{E_i} = \frac{\gamma P}{P}$$

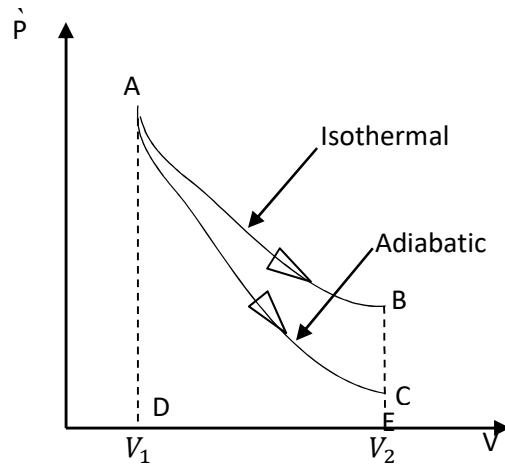
$$\frac{E_a}{E_i} = \frac{B_{adi}}{B_{iso}} = \gamma$$

COMPARISON BETWEEN ISOTHERMAL AND ADIABATIC CHANGE

ISOTHERMAL CHANGE	ADIABATIC CHANGE
1.Occur at constant temperature	1.No heat enter or leave from the system ($dQ = 0$)
2.The change occur slowly	2.The change occur suddenly
3.System is thermally conducting to the surrounding	3.System is thermal insulated from the surrounding
4. Internal energy (u) remain constant $du=0$	4. Internal energy change $u \neq$ constant i.e $du \neq 0$
5.Specific heat become infinite.	5.Specific heat become zero
6.Equation of state $PV = \text{Constant}$	6.Equation of the state $PV^\gamma = \text{Constant}$
7.Slope of isothermal curve $\frac{dP}{dv} = \frac{-p}{v}$	7.Slope of adiabatic curve $\frac{dp}{dv} = \frac{-\gamma P}{v}$
8.Coeff of isothermal elastic $E_i = P$	8.Coeff of adiabatic elasticity, $E_a = \gamma P$

COMPARISONS OF ISOTHERMAL AND ADIABATIC PROCESS CURVE

1. Expansion. Isothermal and adiabatic expansion from volume V_1 to V_2 are represented by isothermal curve AC as show below



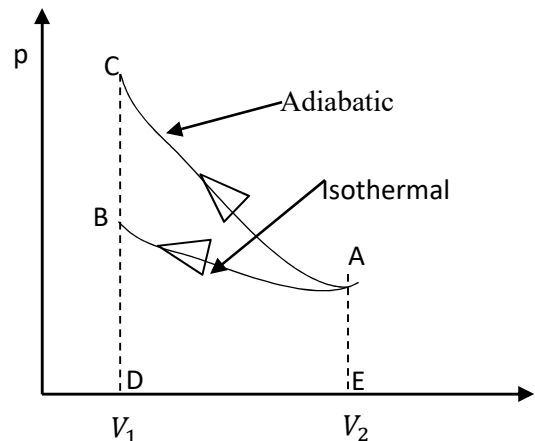
$$W_{\text{iso}} = \int_{V_1}^{V_2} P dV = \text{area of ABEDA}$$

$$W_{\text{adi}} = \int_{V_1}^{V_2} P dV = \text{Area of ACEDA}$$

As area ABEDA > area of ACEDA

$$\therefore W_{\text{iso}} > W_{\text{ad}}$$

2. Compression. Isothermal compression and adiabatic compression from volume V_1 to V_2



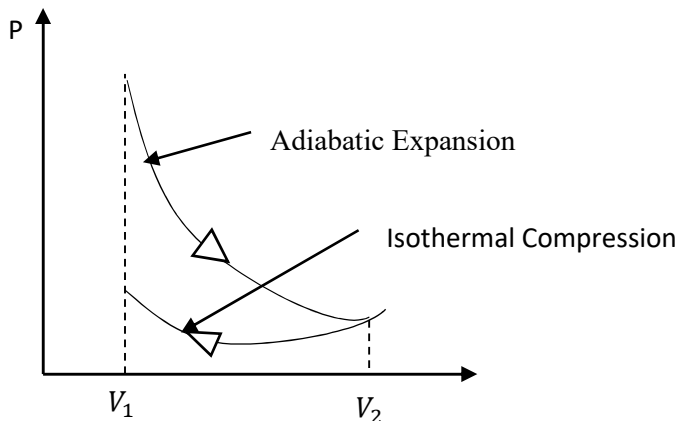
$$W_{\text{iso}} = \int_{V_1}^{V_2} P dV = \text{Area ABDEA}$$

$$W_{\text{ad}} = \int_{V_1}^{V_2} P dV = \text{Area of ACDEA}$$

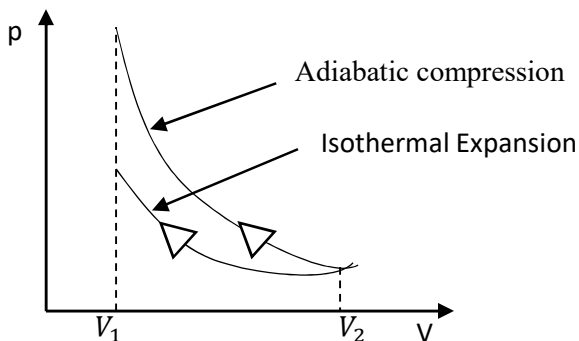
Since Area ACDEA > Area ABDEA

$$W_{\text{adi}} > W_{\text{iso}}$$

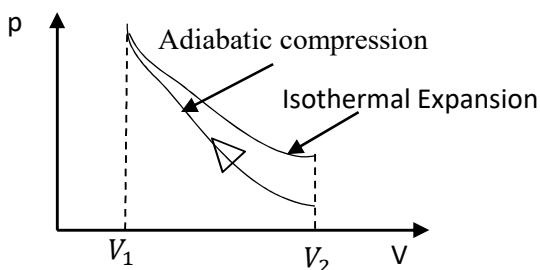
3. The gas expands adiabatic and compress isothermal



4. A gas expands isothermal and compressed adiabatically.



5. A gas compressed adiabatically and then extend isothermally

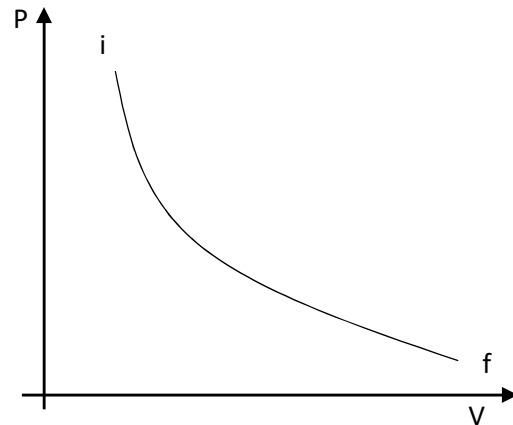


FREE EXPANSION – is the expansion of the gas against external pressure.

REVERSIBLE PROCESS- is the one which can be retraced in the reverse order and in the opposite sense, so that it passes exactly through some state as in the direct process.

Reversible process is the process that can be turn back such that both the system and the surrounding return to their original state, with no other change anywhere else in the universe.

P-V Diagram for Reversible process



Thus during the direct process if heat is absorbed, then during the reverse process and equal amount of heat is given out. If the work is done by the system during the direct process then work is done on the system. At the end of this process system return to the initial state.

Example of reversible process

1. Slow compression and expansion of a spring can also be treated as a reversible process
2. An ideal gas allowed to expand slowly and then compressed slowly in cylinder with frictionless movable piston

3. Electrolysis can be taken as a reversible process provided resistance offered by electrolyte is zero

Condition for the reversible process to take place

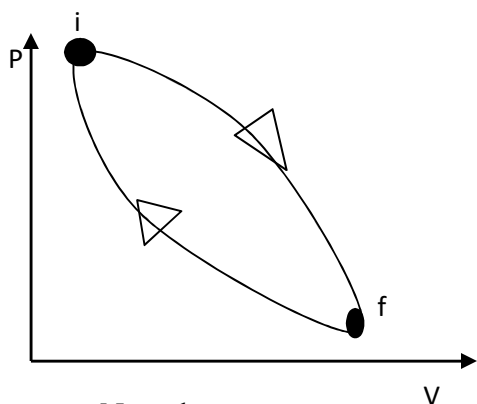
1. The process should take place very slowly so that the system is in thermodynamic equilibrium
2. There should be no loss of heat due to conduction, convection, radiation, friction, viscosity etc.

Their condition cannot be satisfied in practice so all real processes are irreversible processes.

Irreversible process

Is the process that cannot be retraced in the reverse order and in the opposite sense

Graph of P against V



Note that

Irreversibility arises mainly from two causes

- i). Many processes like free expansion or an explosive chemical reaction take the system to a non-equilibrium state.
- ii). Most processes involve friction, viscosity and other dissipative effects.

Example of irreversible process

1. Almost all processes in nature are irreversible processes.
2. Rusting of iron, dissolving of soap in water
3. Heat energy exchange by any process
4. Decay of matter.
5. Flowing of current through a conductor
6. Diffusion of gas
7. Stopping of moving gases
8. Adiabatic (sudden) compression and expansion of gas are irreversible processes.
9. Most of chemical reactions.

SOLVED EXAMPLE

Example 1 [Roger C77]

- a) Explain what is meant by a reversible change.
- b) State the first law of thermodynamics and discuss the experimental observation on which it is based.
- c) A mass of 0.35 kg of ethanol is vaporized at its boiling point of 78°C and a pressure of $1.0 \times 10^5 \text{ Pa}$. At this temperature the specific latent heat of vaporization of ethanol is $0.95 \times 10^5 \text{ J kg}^{-1}$ and the densities of the liquid and vapor are 790 kg m^{-3} and 1.6 kg m^{-3} respectively. Calculate

- i) The work done by the system.
- ii) The change in internal energy of the system.

Explain in terms of molecular terms what happens to the heat supplied to the system.

Solution

a), (b) refer to your notes

$$(c)(i) dW = PdV = [V_1 - V_L]$$

$$dW = P(V_2 - V_1)$$

$$= P \left[\frac{M}{\delta_2} - \frac{M}{\delta_1} \right]$$

$$= 1 \times 10^5 \times 0.35 \left[\frac{1}{1.60} - \frac{1}{790} \right]$$

$$dW = 2.2 \times 10^4 J (\text{approx})$$

ii) du = change in internal energy

The change of amount of heat energy supplied

$$dQ = ML = 0.35 \times 0.95 \times 10^6$$

$$dQ = 332.5 \times 10^3 J$$

According to the first law of thermodynamics

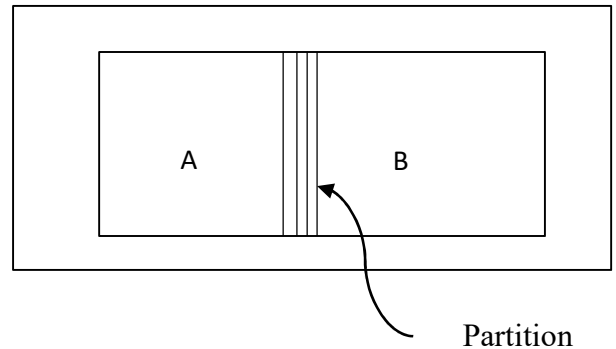
$$du = dQ - dW = 332.5 \times 10^3 - 2.2 \times 10^4$$

$$du = 310.5 \times 10^3 J$$

In molecular term: The heat supplied causes increase in thermal agitation of the molecules hence apart of this thermal agitation to the expansion of the molecules

Example 2

a)(i) in the figure below the container is very poor conductor of heat and is divided by a removable partition into two compartments A and B. compartment A contains a gas at a temperature 0°C and B is evacuated. Briefly explain why the temperature of the gas remains constant when the partition is suddenly removed.



b) A gas of volume of 500cm^3 and pressure $1.0 \times 10^5 \text{ N/m}^2$ expands adiabatically to 600 cm^3 compute

(i) The final pressure of the gas

(ii) The work done by the gas [$\gamma = 1.40$]

Solution

a). This is because of the free expansion.

-No work is performed on the system.

-Initial internal energy is equal to the final internal energy of the gas.

b)

(i) For an adiabatic change

$$PV^\gamma = \text{Constant } P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 1 \times 10^5 \left[\frac{500}{600} \right]^{1.4}$$

$$P_2 = 0.774 \times 10^5 \text{ Nm}^{-2}$$

$$(ii) W_{adi} = \frac{1}{\gamma - 1} [P_1 V_1 - P_2 V_2]$$

$$= \frac{1}{1.4 - 1} [1 \times 10^5 \times 500 \times 10^{-6} - 0.774 \times 10^5 \times 600 \times 10^{-6}]$$

$$W_{adi} = 8.8 J$$

Example 03

a) Distinguish between isothermal and adiabatic change. Show that for an ideal gas the curve relating pressure and volume for an adiabatic change have a greater slope than those for isothermal change at the same pressure

b) A quantity of oxygen is compressed in to isothermally until its pressure doubled. It is then allowed to expand adiabatically until its original volume is restored. Find the final pressure in term of initial pressure.

$$\gamma = 1.4 \text{ for oxygen gas}$$

Solution

a) Isothermal change – is the physical change which takes place at constant temperature
 adiabatic change – Is the physical change that energy which will enter or leave from the system.

For reversible isothermal change

$$PV = C$$

$$\frac{d}{dV}[PV] = \frac{d}{dV}[C]$$

$$P + \frac{VdP}{dV} = 0$$

$$\left(\frac{dP}{dV}\right)_{\text{iso}} = \frac{-P}{V} \text{ --- (1)}$$

For reversible adiabatic change

$$PV^\gamma = C$$

$$\frac{d}{dV}[PV^\gamma] = \frac{d}{dV}[C]$$

$$\gamma PV^{\gamma-1} + V^\gamma \frac{dP}{dV} = 0$$

$$\left(\frac{dP}{dV}\right)_{\text{ad}} = \frac{\gamma P}{V} \text{ --- (2)}$$

$$\left(\frac{dP}{dV}\right)_{\text{adi}} = \gamma \left(\frac{dP}{dV}\right)_{\text{iso}}$$

Since $\gamma > 1$ then adiabatic slope is greater than slope of isothermal

b) $P_1 = P_0 = \text{original pressure}$

$$V_1 = V_0 = \text{Original volume}$$

$$P_2 = 2P_0 = \text{New pressure}$$

. for isothermal compression

$$P_1 V_1 = P_2 V_2, V_2 = V_1 \left[\frac{P_1}{P_2} \right]$$

$$V_2 = V_0 \left[\frac{P_0}{2P_0} \right] = \frac{V_0}{2}$$

Suppose the gas expand adiabatically to its volume V_0 when pressure is P

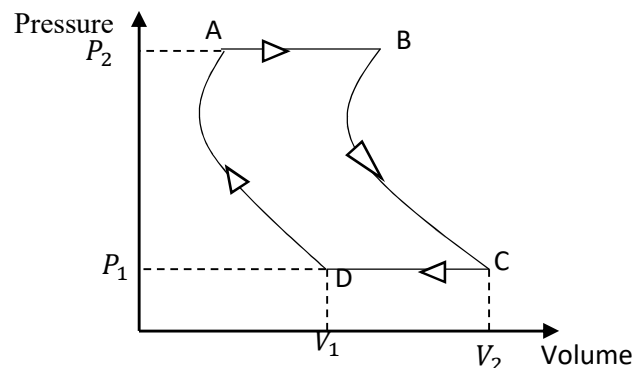
$$PV^\gamma = \text{Constant}$$

$$PV_0^{1.4} = 2P_0 \left[\frac{V_0}{2} \right]^{1.4}$$

$$P = 0.8P_0$$

Example 4

a) An ideal gas undergoes a thermodynamic process as shown in the figure below



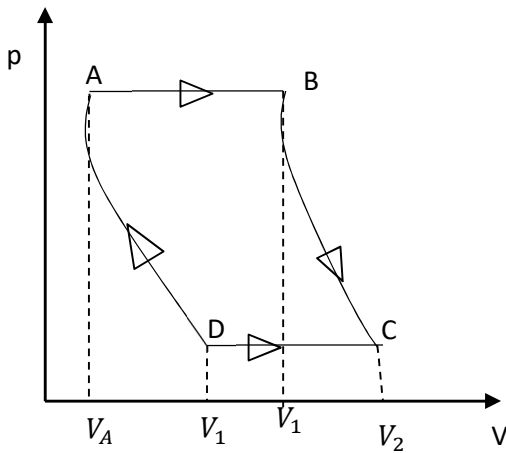
The process consists of two isobaric and two isothermal steps. Show that the network done during the whole process is given by

$$W = P_1(V_2 - V_1) \log_e \left(\frac{P_2}{P_1} \right)$$

a) An ideal gas at 75.0mmHg pressure is compressed isothermally until its volume is reduced to three quarter (3/4) of its original volume. It is allowed to expand adiabatically to a volume 20% greater than its original volume. If the initial temperature of a gas is 17°C, calculate its final pressure and temperature of gas. Given that

$$C_p = 3.6 \text{ KJ Kg}^{-1} \text{ K}^{-1}, C_v = 2.4 \text{ KJ Kg}^{-1} \text{ K}^{-1}$$

Solution



A → B: Isobaric expansion

$$W_{AB} = P_2(V_B - V_A)$$

B → C: Isothermal expansion

$$W_{BC} = P_1 V_2 \log_e \left(\frac{V_2}{V_B} \right)$$

C → D: Isobaric compression

$$W_{CD} = P_1(V_1 - V_2)$$

D → A: Isothermal compression

$$W_{DA} = P_1 V_1 \log_e \left(\frac{V_A}{V_1} \right)$$

Net work done

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$\begin{aligned} &= P_2(V_B - V_A) + P_1 V_2 \log_e \left(\frac{V_2}{V_B} \right) + P_1(V_1 - V_2) \\ &\quad + P_1 V_1 \log_e \left(\frac{V_A}{V_1} \right) \\ &= P_2 V_B - P_2 V_A + P_1 V_2 \log_e \left(\frac{P_2}{P_1} \right) + P_1 V_1 - P_1 V_2 \\ &\quad - P_1 V_1 \log_e \left(\frac{P_2}{P_1} \right) \end{aligned}$$

Since $P_2 V_B = P_1 V_2$, $P_2 V_A = P_1 V_1$

$$\frac{P_2}{P_1} = \frac{V_2}{V_B} \frac{V_A}{V_1} = \frac{P_2}{P_1}$$

$$\begin{aligned} W &= P_1 V_2 \log_e \left(\frac{P_2}{P_1} \right) - P_1 V_2 \log_e \left(\frac{P_2}{P_1} \right) \\ &\quad - P_1 V_1 \log_e \left(\frac{P_2}{P_1} \right) \end{aligned}$$

$$W = P_1(V_2 - V_1) \log_e \left(\frac{P_2}{P_1} \right)$$

Hence shown

$$(b) . P_1 = 75 \text{ mmHg}, V_1 = V_0, V_2 = \frac{3}{4} V_0, P_2 = ?$$

For isothermal change

$$P_1 V_1 = P_2 V_2, P_2 = P_1 \left[\frac{V_1}{V_2} \right]$$

$$P_2 = 75 \left[\frac{V_0}{\frac{3}{4} V_0} \right]$$

$$\underline{P_2 = 100 \text{ mmHg}}$$

Adiabatically

$$V_3 = V_0 + \frac{20V_0}{100} \frac{6}{5} V_0$$

$$T_1 = 17^\circ\text{C}(290\text{K}), P_3 = ? \quad T_2 = ?$$

$$\gamma \frac{CP}{Cv} = \frac{3.6}{2.4} = 1.5 \quad PV^\gamma = \text{Constant}$$

$$P_2 V_2^\gamma = P_3 V_3^\gamma \quad P_3 = P_2 \left[\frac{V_2}{V_3} \right]^\gamma$$

$$= 100 \left[\frac{\frac{3}{4} V_0}{\frac{6}{5} V_0} \right]^\gamma$$

$$\underline{P_3 = 49.41 \text{ mmHg}}$$

Also

$$TV^{\gamma-1} = \text{Constant} \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left[\frac{V_1}{V_2} \right]^{\gamma-1} = 290 \left[\frac{V_0}{\frac{6}{5} V_0} \right]^{1.5-1}$$

$$\underline{T_2 = 241.66\text{K}}$$

Example 05

a). An ideal gas at 17°C has a pressure of 760mmHg and is compressed (i) isothermally and (ii) adiabatically until its volume is halved. Calculate in each case the final pressure and temperature of the gas ($\gamma = 1.4$)

b). A motor car tyre has a pressure of four atmospheres at a room temperature of 27°C

If the tyre suddenly bursts. Calculate the temperature of escaping air [volume of $\gamma = 1.4$ for air]

Solution

(a) (i) isothermal compression

$$PV = \text{Constant}$$

$$P_1 V_1 = P_2 V_2, P_2 = P_1 \left[\frac{V_1}{V_2} \right]$$

$$P_2 = \frac{760V}{\frac{V}{2}}$$

$$P_2 = 1520\text{mmHg}$$

The final temperature will remain 17°C because will isothermal change temperature remain constant.

ii) for an adiabatic compression

$$PV^\gamma = \text{Constant} \quad P_1 V_1^\gamma = P_2 V_2^\gamma \quad P_2 = P_1 \left[\frac{V_1}{V_2} \right]^\gamma$$

$$= 760 \left[\frac{V}{\frac{V}{2}} \right]^{1.4}$$

$$P_2 = 2006\text{mmHg}$$

Let $T_2 = \text{New temperature}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad T_2 = T_1 \left[\frac{V_1}{V_2} \right]^{\gamma-1}$$

$$= 290 \left[\frac{V}{V/2} \right]^{1.4-1}$$

$$T_2 = 383\text{K} = 110^\circ\text{C}$$

b) The suddenly bursts of type is an adiabatic process is an adiabatic process

$$T^\gamma P^{1-\gamma} = \text{Constant}$$

$$T_1^\gamma P_1^{1-\gamma} = T_2^\gamma P_2^{1-\gamma}$$

$$T_2^\gamma = T_1^\gamma \left[\frac{P_1}{P_2} \right]^{1-\gamma} \quad \text{But } \frac{P_1}{P_2} = 4$$

$$[300]^{1.4} = T_1^\gamma [4]^{1-1.4} \quad [\gamma = 1.4]$$

$$[300]^{1.4} = T_1^{1.4} [4]^{1-1.4}$$

$$T_1 = 201.8K = -71.2^\circ C$$

EXAMPLE 6

Three moles of an ideal gas kept at a constant temperature of 300k are compressed from a volume of 4 liters to 1 liter calculate work done in the process

$$R = 8.31J \text{ mole}^{-1}k^{-1}$$

Solution

The process is an isothermal process

$$w = nRT \log_e \left(\frac{V_2}{V_1} \right)$$

$$W = 3 \times 8.31 \times 300 \log_e \left(\frac{1}{4} \right)$$

$$w = -1.037 \times 10^4 J$$

Negative sign shows the work done by the compression of the gas.

Example 07

Air in the cylinder of a diesel engine is compressed to $\frac{1}{15}$ of its initial volume if initial temperature is 300k and initial pressure is $10^5 Pa$, find and final temperature and final pressure. Take $\gamma = 1.4$

Solution

$$\frac{V_2}{V_1} = \frac{1}{15}, P_1 = 10^5 Pa$$

$$T_1 = 300K, T_2?$$

$$P_2 = ?, P_2 V_2^\gamma = P_1 V_1^\gamma, P_2 = P_1 \left[\frac{V_1}{V_2} \right]^\gamma$$

$$P_2 = 10^5 (15)^{1.4} = 44.3 \times 10^5 Pa$$

$$P_2 = 44.3 \times 10^5 Pa$$

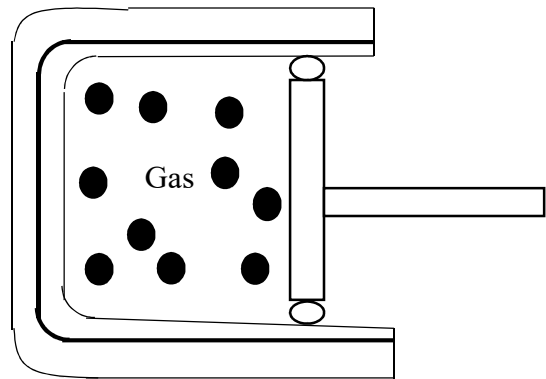
$$\text{Also } T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$T_2 = T_1 \left[\frac{V_1}{V_2} \right]^{\gamma-2} = 300 [5]^{1.4-1}$$

$$T_2 = 886K$$

Example 08[Roger C54]

Some gas assumed to behave ideal is constrained within a cylinder which is surrounded by insulation to prevent loss of heat as shown below



Initially the volume of gas is $2.9 \times 10^{-4} m^3$ its pressure is $1.04 \times 10^5 Pa$ and its temperature is 314.

- Use the equation of state for an ideal gas to find the amount in moles of gas in cylinder
- The gas is then compressed to volume $2.9 \times 10^{-5} m^3$ and its temperature rises to 790K. Calculate the pressure of gas after this compression.
- The work done on the gas during the compression is 91J. Use the first law of thermodynamic to find the increase in the internal energy to the gas during the compression.
- Explain the meaning the of internal energy as supplied to this system and use your result in to

explain arise in the temperature of the gas taken place during compressing (Molar gas constrain,

Solution

$$a). n = \frac{PV}{RT} = \frac{1.04 \times 10^5 \times 2.9 \times 10^{-4}}{8.3 \times 314}$$

b) From ideal gas equation

$$P_2 V_2 = nRT_2$$

$$P_2 = \frac{nRT_2}{V_2}$$

$$= \frac{1.2 \times 10^{-2} \times 8.3 \times 790}{2.9 \times 10^{-5}}$$

$$P_a = 2.6 \times 10^6 \text{ Pa (approx)}$$

c). Since the system is insulated no heat leaves or enters to the system. Therefore, the system increases in internal energy

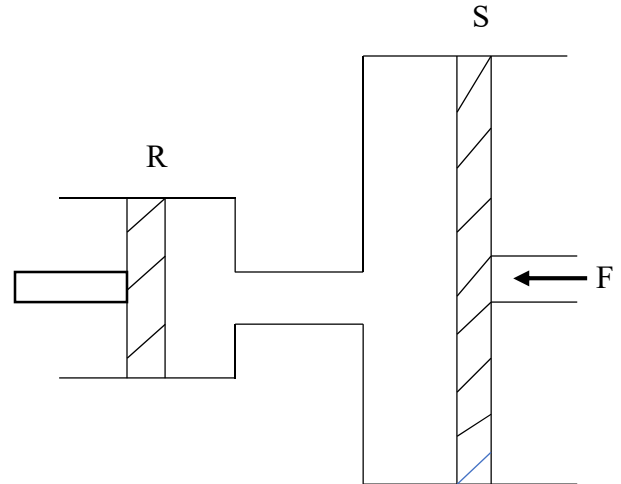
$$du = 91 \text{ J}$$

d). The internal energy of the system can be depending on the temperature. The rise in temperature is due to the fact that the work done on the gas appears to be the increase in internal energy of the gas hence its temperature.

Example 9 (Roger C64)

a). Does a gas do any work when expanding adiabatically? if so what is the source of energy needed to do this work.

b). In a particular automated process, an action by a piston R is transferred by means of moving air to a second piston which is maintained in equilibrium by a variable force F as shown below



Area of piston R = 10^{-3} m^2 area of piston S = $2 \times 10^{-3} \text{ m}^2$ after being at rest for a long time, the volume and pressure of the air between R and S are 10^{-4} m^3 and $2 \times 10^5 \text{ Pa}$ respectively. Piston R is then moved distance of 50mm towards S and this because S move a distance of 10mm.

- (i) Calculate the force exerted on S, the gas before R, is moved
- (ii) Calculate the value of F after R is moved, assuming that the change has taken place isothermally.
- (iii) Calculate the value of F after R is moved assumed that the change taken place adiabatically take the value of γ for air is 1.4]

Solution

a). Yes, the gas does work when it expands adiabatically with reference to thermodynamics, work is a means of converting one form of energy to another form so, the work done under adiabatically with reference to thermodynamics work is a means of converting of

form of energy to another form so , the work done under adiabatic process is the result of conversion of the gas expands but also if the gas in is supplied but the no heat is supplied but the internal energy of the gas increases ,this increases ,this increases in that energy of the work done by surrounding on the system being converted to internal energy.

According to the first law of thermodynamics

$$dQ = du + dw$$

For adiabatic process $dQ = 0$

$$0 = du + dw$$

$$du = -dw \text{ or } dw = -du$$

$$b) A_R = 10^{-3} m^2, A_S = 2 \times 10^{-3} m^2$$

$$V_1 = 10^{-4} m^3, P_1 = 2 \times 10^5 Pa$$

$$\Delta X_R = 50 mm \Delta X_S = 10 mm$$

$$i) F = P_1 A_S = 2 \times 10^5 \times 2 \times 10^{-3}$$

$$F = 400 N$$

ii). 1st obtain the value of volume of air

$$V_2 = V_1 - \Delta V$$

$$V_1 = \text{Volume of air at R}$$

$$V_2 = \text{Volume of air at S}$$

$$\text{but } \Delta V = V_1 - V_2$$

$$\begin{aligned} \text{but } A_R \Delta X_R - A_S \Delta X_S \\ &= 10^{-3} \times 50 \times 10^{-3} - 2 \times 10^{-3} \\ &\times 10 \times 10^{-3} \Delta V = 30 \times 10^{-6} m^3 V_2 \\ &= V_1 - \Delta = 10^{-4} - 30 \times 10^{-6} \end{aligned}$$

Example 10 NECTA 1989/P2

a) With the help of sketch diagram distinguish between isothermal and adiabatic change illustrate your answer with an example of a gas changing from state A to state B

b). Argon gas (specific heat capacity ratio is 1.67) is contained in $250 cm^3$ vessel at a pressure of 750mmHg and temperature of $0^\circ C$. The gas is expanded isothermally to the final volume of $400 cm^3$

i) Calculate the final pressure of the gas

ii). By how much work would the pressure be lowered if the change is made adiabatically instead of isothermally.

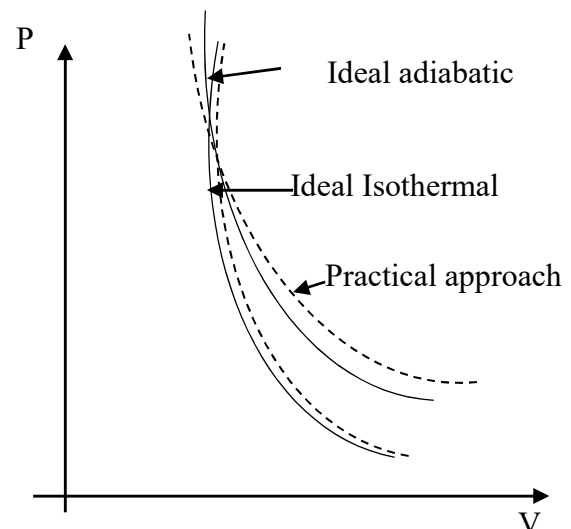
Solution

a). Hints: define and give the condition necessary for isothermal and adiabatic change

Adiabatic change

Isothermal change: $PV = \text{CONSTANT}$

Adiabatic change : PV^γ



$$b)(i) P_1 V_1 = P_2 V_2, P_2 = P_1 \left[\frac{V_1}{V_2} \right]$$

$$P_2 = 750 \left[\frac{250}{400} \right]$$

$$P_2 = 468.75 \text{ mmHg}$$

ii) For adiabatic change

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad P_2 = P_1 \left[\frac{V_1}{V_2} \right]^\gamma = 750 \left[\frac{250}{400} \right]^{1.67}$$

$$P_2 = 342.12 \text{ mmHg}$$

$$W_{adi} = \frac{1}{\gamma - 1} [P_1 V_1 - P_2 V_2]$$

$$= \frac{1}{1.67 - 1} [750 \times 250 \times 10^{-6} - 342.12 \times 400 \times 10^{-6} \times \frac{1.01 \times 10^5}{760}]$$

$$W = 6.731 \text{ J}$$

EXAMPLE 11 NECTA 2000/P2/3

a). Derive an expression for the work done per mole in an isothermal expansion of van der Waals gas from volume v_1 to v_2

b). A number 16 mole of an ideal gas which is kept at constant temperature of 320K is compressed isothermally from its initial volume of 18 liter to the final volume of 4 liter

i) Calculate the total work done in the whole process

ii). Comment on the sign of numerical answer you have obtained.

c). A cylinder fitted contains 1.0 g of oxygen at a pressure of 760 mmHg and at temperature 27°C

The following operations are performed in stage

1. The gas of oxygen is heated at the constant pressure to 127°C and then

2. It is compressed isothermally to its original volume and final stage

3. It is cooled at a constant volume to its original temperature

i) Illustrate the stage in a sketch P-V diagram,

ii) What is the input of heat to the cylinder in the stage (1) above?

iii) How much work does the oxygen in the stage (2)?

iv) How much heat is extracted from the oxygen in stage (3)? [for oxygen, density = 1.43 kg m⁻³, at S.T.P $C_v = 670 \text{ J K g}^{-1} \text{ K}^{-1}$ molecular mass = 32]

Solution

$$a). W = \int_{V_1}^{V_2} P dv$$

$$\text{since } \left(p + \frac{a}{v^2} \right) (v - b) = RT$$

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

$$W = \int_{V_1}^{V_2} \left(\frac{RT}{V - b} - \frac{a}{V^2} \right) dV$$

$$W = RT \log_e \left(\frac{V_2 - b}{V_1 - b} \right) + A \left[\frac{1}{V_2} - \frac{1}{V_1} \right]$$

$$b).(i) W = nRT \log_e \left(\frac{V_2}{V_1} \right)$$

$$= 16 \times 8.314 \times 320 \log_e \left(\frac{4}{18} \right)$$

$$W = -63994.3 \text{ J}$$

ii). Minus sign show that the work is done on the gas system. i.e. work done by compression of gas

b) i)

$$\frac{V}{T} = \text{Constant}$$

$$\frac{V_1}{T_1} = \frac{V}{T} \rightarrow V_1 = V \left[\frac{T_1}{T} \right]$$

$$V_1 = 22.4 \times 10^{-3} \left[\frac{300}{273} \right]$$

$$V_1 = 7.683 \times 10^{-4} m^3$$

$$T_1 = 300K$$

Stage 1 : heating gas at constant pressure

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \rightarrow V_2 = V_1 \left[\frac{T_2}{T_1} \right]$$

$$V_2 = 7.683 \times 10^{-4} \times \left[\frac{400}{300} \right]$$

$$V_2 = 10.24 \times 10^{-4} m^3$$

$$P_2 = 760 mmHg, V_2 = 10.24 \times 10^{-4} m^3$$

$$T_2 = 400K$$

Stage 2 : isothermal change

$$PV = \text{Constant } P_3 V_3 = P_2 V_2 \rightarrow P_3 = P_2 \left[\frac{V_2}{V_3} \right]$$

$$P_2 = 760 \left[\frac{10.24 \times 10^{-4}}{7.683 \times 10^{-4}} \right]$$

$$P_3 = 1013.3 mmHg$$

$$P_3 = 1013.3 mmHg, V_3 = 7.683 \times 10^{-4} m^3$$

$$T_3 = 400K$$

Stage 3 : cooling gas at constant volume

$$\frac{P}{T} = \text{Constant}, T_1 = T_4 = 300K$$

$$\frac{P_4}{T_4} = \frac{P_3}{T_3} \rightarrow P_4 = P_3 \left[\frac{T_1}{T_3} \right]$$

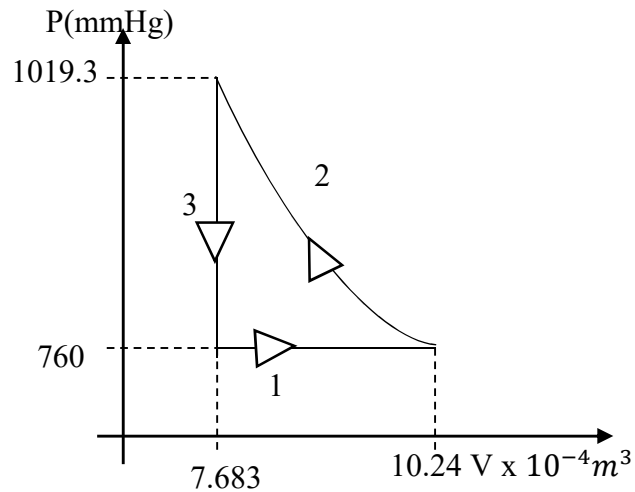
$$P_4 = 1013.3 \left[\frac{300}{400} \right]$$

$$P_4 = 760 mmHg$$

$$P_4 = P_1 = 760 mmHg$$

$$V_3 = V_1 = 7.683 \times 10^{-4} m^3$$

The sketch of P- V Diagram



(ii) According to the first law of thermodynamic

$$dQ = du + Pdv$$

$$dQ = McV (T_1 - T_2) + P(V_2 - V_1)$$

$$= 10^{-3} \times 670(400 - 300) + 10^5 \times [10.24 - 7.683] \times 10^{-4}$$

$$dQ = 92.6 J$$

Heat input in stage 1 ,

$$\underline{dQ = 926 J}$$

(iii) work done in stage (2) isothermal change at 400k

$$W = nRT \log_e \left[\frac{V_3}{V_2} \right] \frac{m}{mr} RT \log_e \left[\frac{V_3}{V_2} \right]$$

$$= \frac{1}{32} \times 8.3 \times 400 \log_e \left[\frac{7.683}{10.24} \right]$$

$$\underline{W = -29.8J}$$

$$(iv) dQ = du = MC_v T [dv = 0]$$

$$= 1 \times 10^{-3} \times 670 \times 100$$

$$dQ = 67J$$

Example 12. NECTA 2003 /P2 14

(a) State and explain the term in the first law of thermodynamic what principle does the law represent?

(b) Distinguish between isothermal and adiabatic process 0.15 mol of an ideal monatomic gas is enclosed in a cylinder at a pressure of 250 k Pa and temperature of 320K. the gas is allowed to expand adiabatically and reversibly until its pressure is 100k Pa

- Sketch the P – V curve for the process
- Calculate final temperature and amount of work done by the gas $[\gamma = \frac{5}{3}]$

Solution

(a) Refer to your notes

(b) For an ideal gas

$$PV = nRT$$

$$V_1 = \frac{nRT_1}{P_1}$$

$$= \frac{0.15 \times 8.31 \times 320}{250 \times 10^3}$$

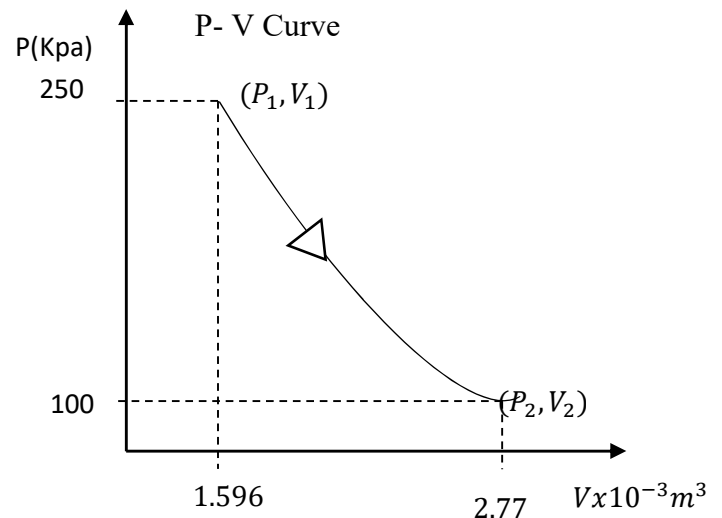
$$\text{INITIALLY : } V_1 = 1.596 \times 10^{-3} m^3 P_1 = 250 KPa T_1 = 320K$$

For adiabatic change

$$P_1 V_1^\gamma = P_2 V_2^\gamma V_2^\gamma = V_1^\gamma \left[\frac{P_1}{P_2} \right] V_2^{\frac{5}{3}}$$

$$= \left[1.596 \times 10^{-3} \right]^{\frac{5}{3}} \left[\frac{250}{100} \right]$$

$$= V_2 = 2.77 \times 10^{-3} m^3$$



ii) Let $T_2 = \text{final temperature}$

$$T_2 = \frac{P_2 V_2}{nR}$$

$$= \frac{100 \times 10^{-3} \times 2.77 \times 10^{-3}}{0.15 \times 8.31}$$

$$\underline{T_2 = 222.2K}$$

Work done by adiabatic

$$W = \frac{1}{\gamma - 1} [P_1 V_1 - P_2 V_2]$$

$$W = \frac{1}{\frac{5}{3} - 1} [250 \times 1.596 - 100 \times 2.77] \times 10^{-3} m^3$$

$$\underline{W = 183J}$$

Example 13 (Roger e 59)

- (a) Explain what meant by the term internal energy and molar heat capacity.
- (b) Explain why the molar heat capacity of a gas at constant pressure is different from the molar heat capacity at constant volume, which from these is larger? explain your answer
- (c) What additional fact may be stated about the internal energy of a gas if the gas is ideal?
- (d) (i) A quantity of 0.2 mole of air enters a diesel engine at a pressure of $1.04 \times 10^5 \text{ Pa}$ and at a temperature of 297K. assuming that air behaves as an ideal gas . find the volume of this quantity of air
- (ii) the air is then compressed to one twentieth of this volume ,the pressure having raised to $6.89 \times 10^6 \text{ Pa}$ find the new temperature
- (iii) Heating of the air then taken place by burning a small quantity of a fuel in it to supply 6150J. this is done at a constant pressure of $6.89 \times 10^{-3} \text{ m}^3 \text{ Pa}$ as the volume of air increases and temperature rises 2040K find
- (1) The molar heat capacity of air at constant pressure
- (2) The volume of air after burning the fuel
- (3) The work done by the air during this expansion
- (4) The change in the air energy of the internal this expansion

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

Solution

(a), (b), (c) refer to our notes

(d)(i) $PV = nRT$

$$V = \frac{nRT}{P} = \frac{0.2 \times 8.31 \times 297}{1.04 \times 10^5}$$

$$V = 4.75 \times 10^{-3} \text{ m}^3$$

(ii) $T_2 = \text{new temperature after comprssion}$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad T_2 = \left(\frac{P_2}{P_1} \right) \left(\frac{V_2}{V_1} \right) T_1$$

$$= \left(\frac{6.89 \times 10^6}{1.04 \times 10^5} \right) \left(\frac{1}{20} \right) \times 297$$

$$T_2 = 983.8 \text{ K} \approx 984 \text{ K}$$

(iii)(1)

$$C_p = \frac{dQ}{ndT}$$

$$= \frac{6150}{0.2(2040 - 983.8)}$$

$$C_p = 29.11 \text{ J mol}^{-1} \text{ K}^{-1}$$

(2) volume of air after burning fuel

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad V_3 = \frac{2040}{983.8} \times \frac{1}{20} \times 4.75 \times 10^{-3}$$

$$V_3 = 4.92 \times 10^{-4} \text{ m}^3$$

(3) $W = P\Delta V$

$$6.89 \times 10^6 \left[4.92 \times 10^{-4} - \frac{1}{20} \times 4.75 \times 10^{-3} \right]$$

$$W = 1757 \text{ J}$$

(4) According to first law of thermodynamic

$$du = dQ - dW$$

$$= 6150 - 1757$$

$$du = 4393J$$

Example 14 (Roger C61)

What are the two principal specific heat capacities of a gas and why are they different?

Show that the difference between these specific heat capacities is given by

$C_p - C_v = \frac{P}{\delta T}$ where P , δ and T are the pressure, density and temperature respectively.

A steel pressure vessel of volume $2.2 \times 10^{-2} m^3$ contains $4.0 \times 10^{-2} kg$ of a gas at a pressure $1.0 \times 10^5 Pa$ and temperature $300K$. An explosion suddenly releases $6.48 \times 10^4 J$ of energy which raises the pressure instantaneously to $1.0 \times 10^6 Pa$. Assume no loss of heat to the vessel and ideal gas behavior. Calculate

- The maximum temperature attained
- The two principal specific heat capacities of the gas. What is the velocity of the sound in this gas at a temperature $300K$?

Solution

- The maximum temperature attained when the volume of gas is kept constant. Apply pressure law

$$\frac{P_2}{T_2} = \frac{P_1}{T_1} \rightarrow T_2 = \left(\frac{P_2}{P_1}\right) T_1$$

$$T_2 = \frac{10^6 \times 300}{10^5}$$

$$T_2 = 3000K$$

- $C_p - C_v = \frac{p}{\delta T_1}$ but $\delta = \frac{M}{V}$

$$C_p - C_v = \frac{P_1}{MT_1}$$

$$C_p - C_v = \frac{10^5 \times 2.2 \times 10^{-2}}{4 \times 10^{-2} \times 300} C_p - C_v = 183.33 J kg^{-1} K^{-1} \text{ Now}$$

$$C_v = \frac{dQ}{M dT} = \frac{6.48 \times 10^4}{0.04(3000 - 300)}$$

$$C_v = 600 J kg^{-1} K^{-1}$$

$$C_p = C_v + 183.33$$

$$C_p = 783.33 J kg^{-1} K^{-1}$$

Velocity of sound in gas at $300K$

$$V = \sqrt{\frac{\gamma P}{\delta}} = \sqrt{\left(\frac{C_p}{C_v}\right) \left(\frac{PV}{M}\right)}$$

$$= \sqrt{\frac{783.33 \times 10^5 \times 2.2 \times 10^{-2}}{600 \times 0.04}}$$

$$V = 267.97 m/s \approx 268 m/s$$

Example 15

(a) The density of a gas is $1.775 kg m^{-3}$ at $27^\circ C$ and $10^5 N m^{-2}$ pressure and its specific heat capacity at constant pressure is $0.846 KJ kg^{-1} K^{-1}$. Find the ratio of its specific heat capacity at constant pressure to the constant volume

(b) An ideal gas at $17^\circ C$ has a pressure of $760 mmHg$ and it is compressed

(i) isothermally

(ii) adiabatically until its volume is halved in each case reversible. Calculate in each case the final pressure and temperature of the gas. Assuming that $C_p = 2.1$, $C_v = 1.5 KJ kg^{-1} K^{-1}$.

Solution

$$a) C_p - C_v = \frac{p}{\delta T_1} = \frac{10^5}{1.775 \times 300} C_p - C_v = \frac{p}{\delta T_1} = 187.793 \text{ KJ Kg}^{-1} \text{ K}^{-1}.$$

$$C_v = 0.846 \times 10^3 - 187.793$$

$$C_v = 658.207 \text{ KJ Kg}^{-1} \text{ K}^{-1}$$

$$\gamma = \frac{c_p}{c_v} = \frac{846}{658.207}$$

$$\gamma = 1.285 = 1.3(\text{approx}).$$

b) Isothermal, T= constant

$$p_2 = P_1 \left[\frac{V_1}{V_2} \right] = 760 \left[\frac{V_0}{\frac{V_0}{2}} \right]$$

$$P_2 = 1520 \text{ mmHg}$$

(ii) Adiabatically

$$\gamma = \frac{c_p}{c_v} = \frac{2.1}{1.5} = 1.4$$

now

$$P_1 V_1^\gamma = P_2 V_2^\gamma P_2 = P_1 \left[\frac{V_1}{V_2} \right]^\gamma$$

$$= 760 \left[\frac{V_0}{\frac{V_0}{2}} \right]^{1.4}$$

$$P_2 = 2005.65 \text{ mmHg}$$

Also

$$TV^{\gamma-1} = \text{constant}$$

$$T_2 = T_1 \left[\frac{V_1}{V_2} \right]^{\gamma-1} = 290 \left[\frac{V_0}{\frac{V_0}{2}} \right]^{1.4-1}$$

$$T_2 = 382.657 \text{ K}$$

Example 16

(a) (i) Find the work done by an ideal gas is expanding adiabatically from a state $A(P_1, V_1)$ to a state $A(P_2, V_2)$

(ii) A fixed mass of a gas is taken through the closed cycle ABCD as shown in figure below calculate the work done by the gas during this cycle of event.

(b) Show that if an ideal gas is compressed isothermally its compressibility is $1/p$ whereas if it compressed adiabatically its compressibility is $\frac{1}{\gamma p}$

Solution

(a) (i) see your notes.

(ii) Work done = area under the curve

$$W = AB \times \overline{BC}$$

$$= (8 - 2) \times 10^{-2} \times (4 - 2) \times 10^5$$

$$W = 8000 \text{ J}$$

(b) for isothermal compression

$$PV = \text{constant} \quad \frac{d}{dv}(PV) = \frac{d}{dv}(c)P + \frac{Vdp}{dv} = 0$$

$$B = -\frac{Vdp}{dv} = P$$

$K = \text{Compressibility}$

$$K = \frac{1}{B} = \frac{1}{P} \text{ shown}$$

• For adiabatic compression

$$PV^\gamma = \text{CONSTANT}$$

$$\frac{d}{dv}(PV^\gamma) = \frac{d}{dv}(C)$$

$$\gamma PV^{\gamma-1} + V^{\gamma} \frac{dp}{dv} = 0$$

$$B = \frac{-Vdp}{dv} = \gamma P$$

$$\text{Since } K = \frac{1}{B} = \frac{1}{\gamma P}$$

Hence shown

Example 17. (roger c 52)

At a temperature of 100°C and pressure of $1.01 \times 10^5 \text{ Pa}$ 1.0 Kg of steam occupies 1.67 m^3 but the same mass of water occupies only $1.04 \times 10^{-3} \text{ m}^3$. The specific latent heat of evaporation of water at 100°C is $2.26 \times 10^6 \text{ J kg}^{-1}$ for a system consisting of 1.0kg of water changing to steam at 100°C and 1.01×10^5 find

- (a) The heat supplied to the system
- (b) The work done by the system
- (c) The increase in internal energy of the system

Solution

$$(a) Q = ML$$

$$= 1 \times 2.26 \times 10^6$$

$$Q = 1.686 \times 10^6 \text{ J}$$

$$(b) W = P\Delta V = P(V_2 - V_1)$$

$$= 1.01 \times 10^5 [1.67 - 1.04 \times 10^{-3}]$$

$$W = 1.686 \times 10^5 \text{ J}$$

$$(c) du = dQ - DW$$

$$= 2.26 \times 10^6 - 1.686 \times 10^5$$

$$dU = 2.09 \times 10^6 \text{ J}$$

Example 18

A cylinder with a movable piston contains 3.00 mol of gas (assumed to behave like an ideal gas)

- (a) The nitrogen gas is heated at constant volume until 1557 J of heat have been added. Calculate the change in temperature.
- (b) Suppose the same amount of heat is added to the Nitrogen gas but this time the gas is allowed to expand while remaining at constant pressure.
- (c) In which case (a) or (b) is the final internal energy of the higher? how do you know? what account for the difference between the two case

Solution

(a) Let

$dQ = \text{Heat added at constant volume}$

$$dQ = nC_v dT \left[C_v = \frac{5}{2} R \right]$$

$$dT = \frac{dQ}{nC_v} = \frac{1557 \times 2}{5 \times 8.314 \times 3}$$

$$dT = 25 \text{ K}$$

(b) Let $dQ =$

$\text{Heat added at constant pressure}$

$$dQ = nC_p dT \quad dT = \frac{dQ}{nC_p}$$

$$C_p = C_v + R = \frac{5}{2} R + R = \frac{7}{2} R$$

$$dT = \frac{1557 \times 2}{7 \times 3 \times 8.134}$$

c) in case (a) the final internal energy is higher than that of case (b) and the internal energy of the gas depend on the temperature ($du \propto dT$) the difference between the two is due to the fact that when heat is supplied at constant pressure some of it used as the

work done and the other will cause the increase in internal energy

Example 19 NECTA 2010/P2/4

- (a) State four assumption of kinetic theory of gases
- (b) Calculate the work done per mole in an isothermal expansion of van der Waals gas from initial volume v_1 to final volume v_2
- (c) A cylinder fitted with a frictionless piston contains 1.0 gm of oxygen at a temperature of 27°C and pressure of 760 mmHg. The following operation were performed. In the first stage oxygen is heated at constant pressure to 127°C . In second stage it is compressed isothermally to its original volume and in the third stage it is cooled at a constant volume to its original temperature
- Illustrate these change in sketch p - v diagram (2marks)
 - What is the heat input to the cylinder in the first stage?
 - How much work is done on the oxygen in the third stage (3marks)
 - What amount of heat is extracted from oxygen in the third stage (2marks)?
(density of oxygen at S.T.P is 1.43 kg m^{-3} , $C_v = 670\text{ J K}^{-1}\text{ mol}^{-1}$)

And its molecular mass is (32)

Hints: see solution of example 11

Example 20 NECTA 2007/P2/4

- (a) State the expression for the first law of thermodynamics
- (b) What do you understand by the terms?
- Critical temperature
 - Adiabatic change

(c) Air bubble is observed in pipe of braking system of a car. The pipe is filled with a compressible liquid (figure below). When the brake are applied the increased pressure in the pipe causes the bubble to become smaller before the brake are applied the pressure is $110 \times 10^3 \text{ N m}^{-2}$, the temperature is 290 K and the length of the bubble is 15 mm. when the air bubble is compressed adiabatically and if the change in its exceeds 12 mm the breaks fails. If the internal cross-sectional area of the pipe is $2 \times 10^{-5} \text{ m}^2$

- Explain briefly why the compression of the bubble is considered to be adiabatic?
- What is the maximum safe pressure in the system during rapid braking if the bubble change in length does not exceed 12 mm? (take $\gamma_{\text{air}} = 1.4$)
- Determine the temperature of the air in the bubble at the end of the adiabatic compression.

d)(i) Find the number of molecules and their mean kinetic energy for a cylinder of volume $5 \times 10^{-4} \text{ m}^3$ containing oxygen at pressure of $2.0 \times 10^5 \text{ Pa}$ and a temperature of 300 K

(ii) When the gas is compressed adiabatically to a volume of $2 \times 10^{-4} \text{ m}^3$ the temperature rises to 43 K. determine atomicity, γ the ratio of the principal heat capacities (molar gas constant $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ Avogadro's number $N_A = 6.0 \times 10^{23} \text{ mol}^{-1}$)

Solution

$$(a) \Delta Q = \Delta U + \Delta W$$

Each symbol have usual meaning

(b) (i) Critical temperature is the temperature below which the gas can be liquified by the application of the pressure alone

(ii) Adiabatic change is the process of thermodynamics which takes place where there is no amount of heat energy which will enter or leave from the system [$dQ = 0$]

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The process takes place very rapidly (quickly) so that there is not enough time for energy to be supplied or released

The bubble can be surrounded by the liquid and is well insulated by which is considered as an insulation material. So no heat enters or leaves from the system

ii) for adiabatic change

$$PV^\gamma = \text{constant}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_2 = P_1 \left[\frac{V_1}{V_2} \right]^\gamma$$

$$V_1 = AL_1 = 2 \times 10^{-5} \times 15 \times 10^{-3} \\ = 3.0 \times 10^{-7} \text{ m}^3$$

$$V_2 = AL_2 = 2 \times 10^{-5} \times 12 \times 10^{-3} \\ = 2.4 \times 10^{-7} \text{ m}^3$$

$$P_2 = 110 \times 10^3 \left[\frac{3.0 \times 10^{-7}}{2.4 \times 10^{-7}} \right]^{1.4}$$

$$P_2 = 150.3373 \times 10^3 \text{ Nm}^{-2}$$

$$\text{iii) } TV^{\gamma-1} = \text{Constant}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left[\frac{V_1}{V_2} \right]^{\gamma-1}$$

$$= 290 \left[\frac{3 \times 10^{-7}}{2.4 \times 10^{-7}} \right]^{1.4-1}$$

$$T_2 = 317.1 \text{ K (approx)}$$

$$\text{d) (i) } PV = \frac{NRT}{NA} \Rightarrow N = \frac{PVNA}{RT} = \frac{2 \times 10^5 \times 5 \times 10^{-4} \times 6 \times 10^{23}}{8.31 \times 300}$$

$$N = 2.4 \times 10^{22} \text{ Molecules}$$

Example 21

(a) Explain the following observation

- (i) The barrel of the bicycle pump becomes hot when the tyre is being inflated quickly
- (ii) When a gas at high pressure in a container is suddenly released the container cools

(b) (i) adiabatic expansion produces cooling, why?

(ii) It is not advisable to put on wet clothes why?

(c) For the hydrogen, the molar heat capacities at a constant volume and constant pressure are respectively $20.5 \text{ J mol}^{-1} \text{ K}^{-1}$ and $28.8 \text{ J mol}^{-1} \text{ K}^{-1}$. What does this mean?

(d) An ideal gas having initial pressure P , volume V and temperature T is allowed to expand adiabatically until its volume becomes $5.66V$ while its temperature falls to $\frac{T}{2}$

(i) How many degrees of freedom do gas molecules have?

(ii) Obtain the work done by the gas during the expansion as a function of the initial pressure and volume

Example 22

(a) (i) Is it possible to increase the temperature of the gas without adding heat to it? If so, how?

(ii) Milk is poured in to a cup of tea and is mixed with a spoon. Is this an example of a reversible process? Give reason in support of your answer.

(b) The work done by one mole of a monatomic ideal gas ($\gamma = \frac{5}{3}$) in expanding adiabatically is 825J. The initial temperature and volume of the gas are 393K and 0.1 m^3 respectively calculate

- (i) The final temperature
- (ii) The final volume of the gas