MODULE 2: NEWTON'S LAW OF MOTION

INTRODUCTION TO MECHANICS

MECHANICS

Is the study of motion or states of material bodies under the action of forces. i.e. Mechanics is the branch of physics which deals with the study of force and motion and their relationship. The study of mechanics is divided into three parts:-

1. **STATICS** this deals with the study of objects at rest.
2. **KINEMATICS** this deals with the study of motion of the objects without considering the causes of motion. This word kinematics comes from a Greek word **kinema** which means motion.
3. **DYNAMICS** this deal with the study of motion of the objects taking into consideration the causes of motion. The word '**Dynamics**' comes from a Greek word **dynamics** which means power.

REST AND MOTION

REST an object is said to be at rest if it does not change its position with respect to its surrounding with the passage of time.

MOTION an object is said to be in motion if its changes its position with respect to its surrounding with passage of time.

TYPES OF MOTION

1. Translatory motion
2. Rotational motion
3. Oscillatory motion

- TRANSLATORY MOTION
  Is the type of motion where by every particle of the body have the same displacement in the same time of interval. Example motion of the car on the straight line along the road.

- ROTATIONAL MOTION
  Is the type of motion of the body in which each particle of the body (except those on the axis of rotation) travel in a circle. Example circular motion.

- OCILLATORY MOTION
  Is the type of motion of the body in which the body moves to and fro repeatedly about a mean position example oscillation of the simple pendulum.

TOPIC 1: NEWTON'S LAWS OF MOTION

There are three Newton's laws of motion.

(i) Newton's first law of motion
(ii) Newton's second law of motion
(iii) Newton's third law of motion.

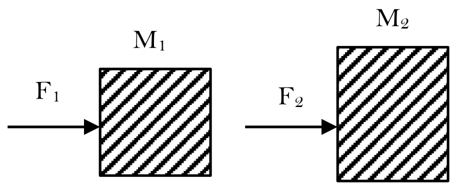
- NEWTON'S FIRST LAW OF MOTION

State that 'everybody in its state of rest or uniform motion in a straight line unless some external force is applied on it to change that state'.
Sometime Newton's first law of motion is known as **law of inertia**.

Reasons

According to the Newton's first law, in the absence of any net force act on a body the body is either at rest or it moves with a constant speed in a straight line in the other words in order to change the motion (i.e. magnitude and direction) of the velocity, net external force is required.
This means that everybody has a tendency to maintain its state of rest or uniform velocity for this reasons, Newton's first law of motion is called law of inertia.

INERTIA is the tendency of a body to maintain its state of rest or uniform motion in the straight line i.e. in the absence of external force, the inability of a body to change its state by itself is called **inertia**. This is the reluctance of the body to be in state of rest or uniform motion. The quantitative measure of inertia is the mass of the body the more the mass of the body, the more its tendency to resist any change in its state of rest or uniform motion in a straight line i.e. a change in its velocity.



If M2 > M1, then F2 > F1 Newton's first law of motion gives a quantitative definition of force. **A FORCE** is a push or pull exerted on a body which produces or tend to produce a change in its velocity.

TYPES OF INERTIA

There are three types of inertia of the body:

1. Inertia of rest
2. Inertia of motion
3. Inertia of direction.

- **INERTIA OF REST**

  It is that property of virtue of which a body in a state of uniform motion tends to maintain its uniform motion. The ability of a body to change its state of uniform motion along a straight line, when no external force acts on it is called **inertia of motion**.

- **INERTIA OF DIRECTION**

  It is that property by virtue of which a body tends to maintain its direction.

APPLICATIONS OF LAW OF INERTIA

1. When a bus suddenly starts, a person sitting in the bus falls backwards.
   Reason
   This is due to inertia of rest as the bus starts , the lower part of this body begins to move but the upper part of his body tends to remain at rest due to inertia.

2. When a man jumps out of a bus he or she falls forward.
   Reason
   This is due to inertia of motion this is because his fact comes to rest suddenly but the upper part of his body continues to move forward.

3. When a blanket (carpet) is given a sudden jerk, the dust particles fall off
   Reason
   It is due to the reason that the blanket is suddenly set in motion but the dust particles tend to inertia of rest.

4. When a bullet is fired into a tightly – fitted glass plane from a reasonably close range , it makes a clear circular hole in the glass pane.
   Reason
   This is due to the fact that particles of glass around the hole tend to remain at rest duet to inertia of rest so they are unable to share the fast motion of the bullet.

5. When the beat a carpet with a stick the carpet is suddenly set into motion but the dust particles tend to remain at the rest due to inertia therefore, dust particles get removed from carpet.

6. A passenger standing in a moving bus falls forward when the bus stops suddenly.
   Reason
   This is due to the fact that the lower part of the body comes to rest along with the bus but the upper part of the body remains in the state of motion on account of inertia of motion the same argument can be applied to the case of a person jumping out of a moving train.

7. An athletes runs for some distance before taking a long jump.
   Reason
   In this way, the athlete gains momentum and due to inertia of motion, he takes a longer jump.

8. During the sharpening of a knife using a grinding wheel sparks fly off tangentially.
   Reason
   This is due to the inertia of direction on.

9. When we shake the branch of a mango tree, the mangoes fall down.
   Reason
   Because due to shaking the branches comes into motion, but due to inertia the mangoes continue to remain at rest and get detached.

10. A ball thrown upwards in a train moving with uniform velocity, returns to the thrower.
    Reason
    Because during the upward and the downward journey; due to inertia, the ball also moves along horizontal with the velocity of the train hence it covers the same horizontal distance as the train does and the ball returns to the thrower.

11. The mud from the wheels of a moving vehicle flies off tangentially.
    Reason
    This is due to the inertia of direction in order that the flying mud does not spoil the clothes of the passer by, the wheels are provided mud guards.

12. If the cloth placed under a book is given a sudden pull, it goes out without disturbing the book, because the book continues to rest at rest due to inertia, when the cloth is suddenly pulled out.

13. Suppose a stone tied to one end of a string, is being rotated in a horizontal circle when the string breaks the stone tends to flies off tangentially along a straight line.
    Reason
    This is due to the inertia of the direction.

<u>**LINEAR MOMENTUM (P)**</u>

Newton introduced the concept of momentum to measure the quantitative effect of a force. The total quantity of motion possessed by a moving body is known as the linear momentum of body.

DEFINITION OF LINEAR MOMENTUM - is defined as the product of the mass and the velocity of the body.

$$P = MV \quad \text{or} \quad \vec{P} = M\vec{V}$$

Linear momentum is the vector quantity S.I unit of linear momentum is **Kgm/s**.

<u>**NEWTON'S SECOND LAW OF MOTION**</u>

State that 'The rate of change of linear momentum of a body is directly proportional to the applied force and takes place in the direction of the force'. Net external force acting on a system or body is equal to the rate of change of linear momentum.

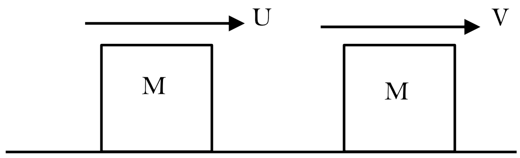
$$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

$$F = M\frac{dv}{dt} = Ma$$

$$F = Ma$$

Derivation

Consider a body of mass, M starts to move with initial velocity, U and finally velocity, V.



The rate of change of linear momentum.

$$\frac{dp}{dt} = M\frac{(v-u)}{t} = ma$$

According to the newton's second law of motion

$$F \,\alpha\, \frac{dp}{dt}, \quad F = K\frac{dp}{dt}$$

$$F = KMa$$

In S.I unit K = 1, F = Ma

<u>**DIFFERENT FORMS OF EXPRESSION OF THE FORCE**</u>

1. $F = M\frac{(u-v)}{t}$
2. $F = Ma$
3. $F = \frac{dp}{dt}$
4. $F = M\frac{dv}{dt} + V\frac{dm}{dt}$
5. $F = M\left[\frac{V^2 - U^2}{2s}\right]$

IMPULSE OF A FORCE

Is defined as the product of the average force and force acts. Impulse of a force is the change in a linear momentum of the body. The total effect of force is called **impulse**. It may also be defined as a measure of the action of force it is vector quantity and is denoted by I.

Expression of impulse of a force

According to the newton's second law of motion.

$$F = \frac{mv - mu}{t}$$

$$Ft = m(v-u) \quad \text{or} \quad dp = Fdt$$

$$I = Fdt = m(v-u)$$

Now

$$dI = Fdt$$

$$I = \int_{t_1}^{t_2} R(t)\,dt$$

S.I unit of impulse of a force is Ns or Kgm/s dimensional formula of impulse as $[MLT^{-1}]$

IMPULSE – MOMENTUM THEOREM

State that 'The given change in momentum can be produced by applying a larger force for a smaller time or by applying smaller force for a longer time.

APPLICATION OF THE CONCEPT OF IMPULSE

1. While catching a ball, a cricket player extends his hand forward so that has plenty of room to let his hands move backward after making contact with the ball.

   Reason

   This extend the time of impact and thus reduces the force of impact.

2. A person is better off falling on a wooden floor than a concrete floor.

   Reason

   The wooden floor allows for a longer time of impact and the before a less force of impact than a concrete floor.

3. A person jumping from an elevated position on a floor below bends his knees upon making contact.

   Reason

   This extends the time of impact there force the force of impact reduced.

4. Automobiles are provided with spring system

   Reason

   When the automobile bumps over an un even road, it receives a jerk. The spring increases the time of jerk, thereby reducing the force this minimizes the damage to the automobiles.
5. It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity.

   Reason

   This is due to the fact that the cricket ball is heavier than tennis ball. The change in momentum is more in the case of a cricket ball than in the case of tennis ball as a results, more force is required to be applied in the case of a cricket ball

- **NEWTON'S THIRD LAW OF MOTION**

  State that 'Action and reaction are equal and opposite' to every action, there is always an equal in (magnitude) and opposite (in direction) reaction. The following points may be noted from the newton's third law of motion:-

  1. Action and reaction are equal in magnitude but are in opposite direction i.e. $|F_1| = |F_2|$ and $-F_1 = F_2$ or $F_1 = -F_2$
  2. This law involves two separate distinct bodies.
  3. Action and reaction forces acts on the different bodies for this reason, they cannot cancel to each other.
  4. Action and reactions forces involved a pair of bodies.
  5. The forces occur in pairs.

APPLICATION OF NEWTON'S THIRD LAW OF MOTION

1. In swimming, the man pushes the water backward and in turns, he is pushed forward due to the reaction of water.

2. When we hit a nail by using a hammer the force exerted by the hammer on nail is equal and opposite to the force exerted by the nail on hammer.
3. Walking on the road, when a person walks on a horizontal road he pushes the ground slantingly with one foot, this is the action force. The ground exerts a reaction force in the opposite direction.
4. The working of rockets is based on Newton's third law of motion. The rocket exerts a strong force on the gases expelling them the gases exerted an equal and opposite force on the rocket and it is this force which moves the rocket forward.
5. A bird flies forward by exerting a force on the air but it is the air pushing back on the birds wing that moves bird forward.
6. When a gun is fired, the force exerted on the bullet is equal to the reaction force exerted on it.
7. When a rubber ball is struck against a wall, the ball bounces back due to the reaction of the wall.
8. When a person jumps out of a boat, the boat is pushed in the backward direction due to reaction.

<u>APPLICATIONS OF THE NEWTON'S LAWS OF MOTION.</u>

1. **NEWTON'S SECOND LAW OF MOTION IS THE REAL LAW OF MOTION.**
   This is because Newton's first law of motion and Newton's third law of motion are derived from Newton's second law of motion.
   (i) Newton's second law of motion is the special case for the Newton's first law of motion. According to the Newton's second law of motion.

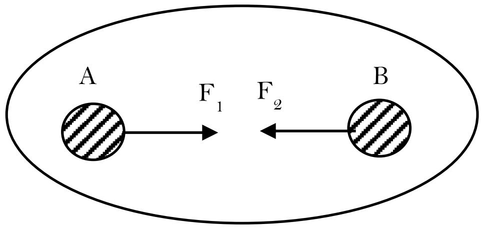
$$F = m\frac{(v-u)}{t} = ma$$

$$a = \frac{F}{M} = \frac{v-u}{t}$$

If $F = 0$, $a = 0$ zero acceleration mean that the body is at the rest or it is moving with constant velocity (i.e. $u = v$). This is the statement of Newton's first law of motion.

∴ Newton's first law of motion contained in the newton's second law of motion

(ii) To prove that Newton's third law follows from the newton's second law of motion. If F_1 is the force exerted on body A by body B (action) and F_2 is the force exerted on the body B by body A (reaction)



According to the Newton's second law of motion.

$$F = \frac{dp}{dt}$$

$$\text{Body A}: F_1 = \frac{dp_1}{dt}$$

$$\text{Body B}: F_2 = \frac{dp_2}{dt}$$

The total rate of change of linear momentum

$$\frac{dp}{dt} = \frac{dp_1}{dt} + \frac{dp_2}{dt}$$

$$\frac{dp}{dt} = F_1 + F_2$$

Since there is net external forces acting on the system

$$dp = 0$$

$$0 = (F_1 + F_2)dt$$

$$0 = F_1 + F_2$$

$$-F_1 = F_2 \text{ OR } F_1 = -F_2$$

This is the statements of the Newton's third law of motion.

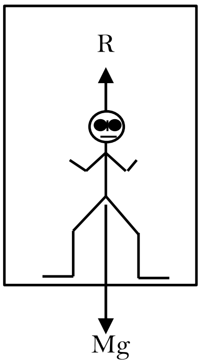
2. APPARENT WEIGHT OF A BODY MOVING IN A LIFT OR ELEVATOR.

Consider a body of mass, M kept in lift. The available forces acting on the body are: -

(i) The weight of the body Mg acting vertically downwards.

(ii) The reaction force R acting vertically upwards different cases for the motion of the lift.

CASE 1: WHEN THE LIFT IS AT REST



In this case a = 0, F = 0

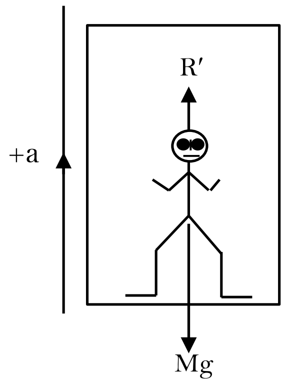
$$R = Mg$$

∴ apparent weight

$$Mg' = Mg$$

$$g' = g$$

CASE 2: WHEN THE LIFT IS MOVING UPWARD WITH AN ACCELERATION, a



Resultant force on a person

$$Ma = R' - Mg$$

$$R' = M(a+g)$$

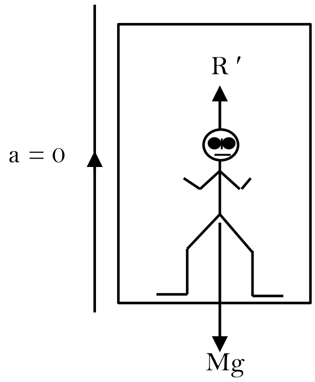
Since the reaction has increased therefore the passenger shall fell that this weight has increased so the apparent weight of the person becomes more than the actual weight of the person.

$$Mg' = M(g+a)$$

$$g' = g+a \quad , \quad g' > g$$

g' = effective value of acceleration due to gravity.

CASE 3: WHEN THE ELEVATOR IS MOVING VERTICALLY UPWARD WITH A UNIFORM VELOCITY, V

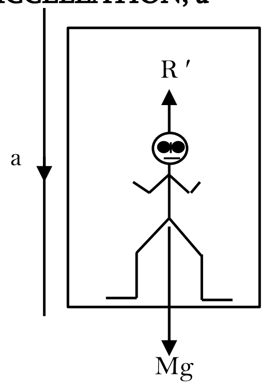


In this case no change in velocity, a = 0

$$R' = Mg$$

$$g' = g$$

CASE 4: WHEN THE LIFT IS MOVING VERTICALLY DOWNWARD WITH ACCELERATION, a



Resultant force

$$F = Mg - Ra'$$

$$Ma = Mg - R'$$

$$R' = M(g-a)$$

Again,

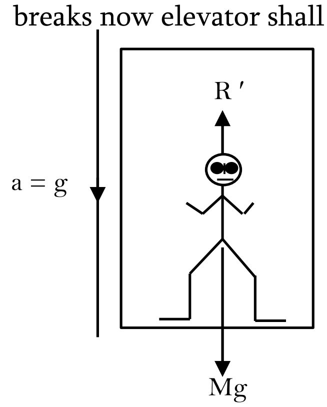
$$Mg' = M(g-a)$$

$$g' = g-a \quad , \quad g' < g$$

g' = effective value of the acceleration due to gravity.

CASE 5: FREELY FALLING ELEVATOR

Suppose the cable supporting of the elevator breaks now elevator shall begin when



$$a = g \quad , \quad R' = 0$$

$$R' = (g-a)$$

$$= (g-g)$$

$$R' = 0$$

Thus, the floor will not exert any reaction force. The apparent weight of the person will be zero. The person will be in a state **weightlessness.**

SOLVED EXAMPLES

Example – 1

(a)

i. Explain why bullet fired from a riffle leave a clean hole in window panes?

ii. The blades of a fan continue to rotate even when the current is switched off why?

(b) A man of 75kg stand in a lift what force does the floor exerts on him when the lift starts moving upwards with an acceleration of $2m/s^2$ ($g = 10m/s^2$).

Solution

(a)

i. This is due to inertia of rest the bullet exerts a large force for a short time. The glass near the hole is not able to share, this quick motion of the bullet and hence remain undisturbed.

ii. This is due to inertia of motion the blades were in motion, when the fan was on but when the switch is off due to inertia it continues to rotate but due to air resistance and friction its becomes to rest.

(b) $R = M(g + a)$

$$R = 75(10 + 2) = 900N$$

<u>Example – 02 NECTA 2020 / P1 /1(b)</u>

Calculate the tension in the cable which delivers the power of 23kw when a pulling a fully loaded elevator at constant speed of 0.75m/s (03 marks)

Solution

Let

T = Tension on the cable

$$P = TV$$

$$T = \frac{P}{V} = \frac{23 \times 1000W}{0.75m/s}$$

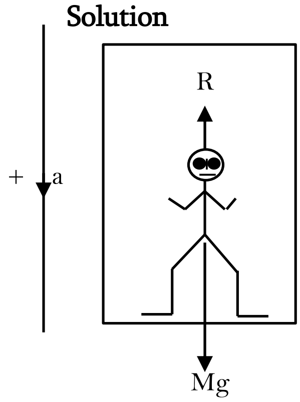
$$T = 3.067 \times 10^4 N$$

Example – 03

(a) A 100kg man stands on a spring balance in an elevator when the elevator starts moving, the scale reads 90kg. What is the magnitude and direction of the acceleration of the elevator? ($g = 10m/s^2$)

(b) What force should be applied on a 10kg body so that it moves down in vacuum with an acceleration of $4m/s^2$ ($g = 9.8m/s^2$)

Solution



Resultant force on a parson

$$R - Mg = Ma$$

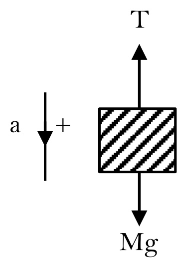
$$\frac{R - Mg}{M} = a$$

$$a = \frac{9 \times 10}{100} - 10$$

$$a = -1m/s^2$$

Negative sign shown that the acceleration of an elevator is downward $a = 1m/s^2$.

(b)



$$Ma = Mg - T$$

$$T = M(g - a)$$

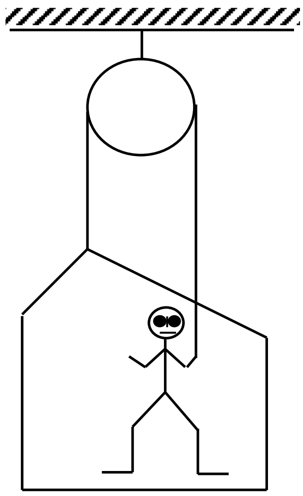
$$= 10(9.8 - 4)$$

$$T = 58N$$

Example – 04

(a) Explain why the load on the back wheels of motor car increases when the vehicle is accelerating.

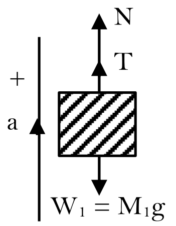
(b) When a man whose weight is 1000N pulls a rope, the force he exerts on the floor of a cage is 450N if the cage weight is 250N, find the acceleration.



Solution

(a) This is due to the inertia, as the car accelerates, the load wants to maintain its velocity as a result its weight is supported mainly with back wheels. This is accordance to the newton's first law of motion.

(b) Consider FBD on the man



$$T + N - W_1 = M_1 a$$

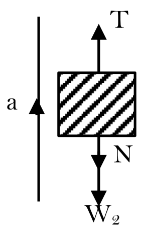
$$M_1 = \frac{W_1}{g} = \frac{1000}{9.8}$$

$$M_1 = 102\text{kg}$$

$$T + 450 - 1000 = 102a$$

$$T - 550 = 102a \;.....(i)$$

FBD on the cage



$$M_2 = \frac{W_2}{g} = \frac{250}{9.8}$$

$$M_2 = 25.5\text{Kg}$$

$$T - 450 - 250 = 25.5a$$

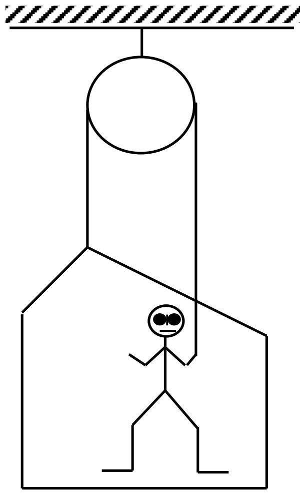
$$T - 700 = 25.5a \;........(ii)$$

On solving simultaneously equation (i) and (ii)

$a = 1.96\text{m/s}^2$

example – 05

a pointer is raising himself and the crate on which he stands with an acceleration of 5m/s^2 by massless rope and pulley arrangement mass of the pointer is 100kg and that of the crate is 50kg. if $g = 10\text{m/s}^2$, find the force of contact between the pointer and the floor.



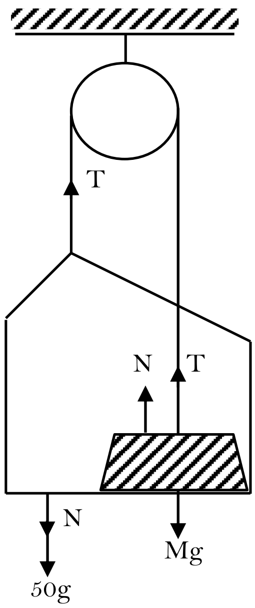
Solution

For the pointer

$$T + N - 1000 = 500 \;..........(i)$$

For the create

$$T - N - 500 = 250 \;.........(ii)$$



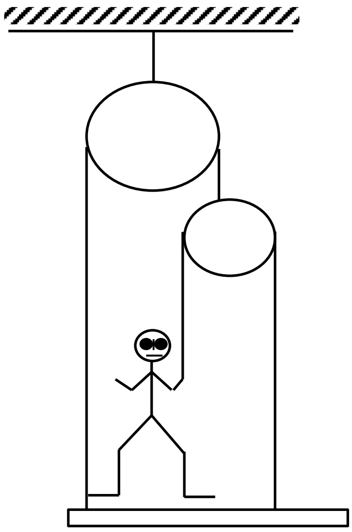
On solving simultaneously equation (i) and (ii)

$$T = 1125\text{N}$$

$$N = 375\text{N}$$

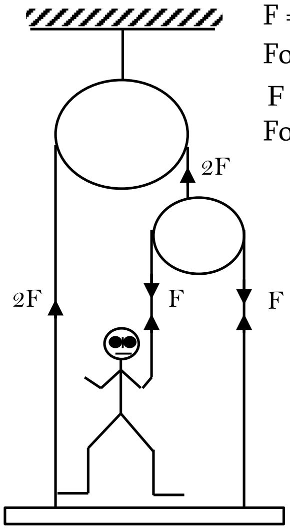
Example – 06

In the figure below, with what force must the man pull the rope to hold the plank in position? Weight of the man is 60kgf neglecting the weight of plank, rope and pulley.



Solution

Consider the FBD as shown below



F = tension on the string

For a man

$$F + N = 60$$

For plank

$$3F = N$$
$$4F = 60$$
$$F = 15Kgf$$

Example – 07

A lift of mass 3,000kg is supported by a thick cable if the tension in the supporting cable is 36,000N.

(i) Calculate the upward acceleration of the lift.

(ii) How far does it rise in 12seconds if it starts from rest?

(iii) If the breaking stress for the rope is 3 × 108N/m², calculate the minimum diameter of the rope.

Solution

M = 3000Kg

Tension in the rope, T = 36,000N

(i) As the lift moves upward with acceleration, a

$$T = M(g + a)$$
$$36,000 = 3000(9.8 + a)$$
$$a = 2.2m/s^2$$

(ii) For upward motion of a lift, u = 0 , a = 2.2m/s²

$$t = 12\,sec$$
$$S = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 2.2(12)^2$$
$$S = 158.4m$$

(iii) $$\text{Breaking stress} = 3 \times 10^8 Nm^{-2}$$

$$3.0 \times 10^8 = \frac{T}{\pi D^2/4} = \frac{4T}{\pi D^2}$$

$$D^2 = \frac{4 \times 36,000 \times 7}{22 \times 3 \times 10^8}$$

Example – 08

A bob of mass 5kg is hung by a spring balance in a lift. Find the reading of the balance when

(i) The lift is moving upwards with an acceleration of 1m/s²

(ii) Moving downwards with an acceleration of 1m/s²

(iii) Moving downwards with a uniform velocity of 3m/s take g = 10m/s².

Solution

Mass of bob, M = 5kg

Let T be the tension on hook of spring balance

(i) When the lift moves upward with acceleration, a resultant force on the bob

$$T - Mg = Ma$$
$$T = M(g + a) = 5(10 + 1)$$
$$T = 55N$$

∴ When the spring balance

$$M' = \frac{T}{g} = \frac{55}{10} = 5.5Kg$$

(ii) When the lift moves downwards with acceleration, a

$$T = M(g-a) = 5(10-1)$$

$$T = 45N$$

Hence reading of spring balance

$$M' = \frac{T}{g} = \frac{45}{10} = 4.5Kg$$

$$M' = 4.5Kg$$

(iii) When the lift is moving downward with acceleration a = 0 i.e. with uniform velocity, V

$$T = Mg = 5 \times 10 = 50N$$

Hence reading of spring balance

$$M' = \frac{T}{g} = \frac{50N}{10m/s^2}$$

$$M' = 5kg$$

Example – 09

A lift moving down with an acceleration of 3m/s². A ball is released 1.7m above the lift floor. Assuming g = 9.8m/s², how long will the ball take to hit the floor?

Solution

Here lift is moving downward with acceleration, the acceleration of the ball relative to the lift.

$$a = g - a = 9.8 - 3 = 6.8m/s^2$$

Now

$$S = ut + \frac{1}{2}a't^2$$

$$1.7 = 0 \times t + \frac{1}{2} \times 6.8t^2$$

$$3.4t^2 = 1.7$$

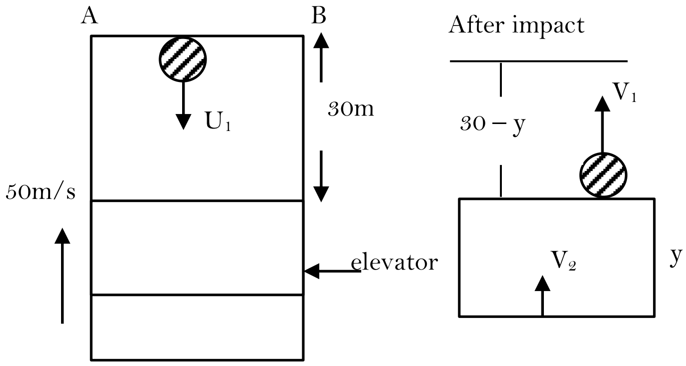
$$t = \sqrt{0.5} = 0.71\,sec$$

$$t = 0.71\,sec$$

Example – 10

An elevator is moving up at 5m/s in a shaft. At the instant, the elevator is 30m from the top, a ball is dropped from the top of the shaft. The ball rebounds elastically from the elevator roof to what height can it rise relative to the top of the shift?

Solution



Let t be the time interval that has elapsed before the ball reaches the elevator.

- Motion of the particle

Since

$$S = ut + \frac{1}{2}at^2$$

$$30 - y = 0 \times t + \frac{1}{2}gt^2 \qquad [a = g]$$

$$30 - y = \frac{1}{2}gt^2$$

- For elevator

$$y = u_2 t = 5t$$

$$t = \frac{y}{5}$$

Now $$30 - y = \frac{1}{2}g\left(y/5\right)^2$$

$$30 - y = 0.196y^2$$

$$0.196y^2 + y - 30 = 0$$

On solving $y = 10m$

The velocity of the ball on hitting the elevator is therefore

$$U_1 = \sqrt{2g(30-y)} = \sqrt{2 \times 9.8(30-10)}$$

$$U_1 = 19.8m/s$$

Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Taking the upward velocities as negative and downward velocities as positive e = 1 for perfect elastic collision.

$$1 = \frac{-5-(-V_1)}{19.8-(-5)} = \frac{-5+V_1}{24.8}$$

$V_1 = 29.8m/s$

The height reached by the rebounding ball is given by

$$H = \frac{V_1^2}{2g} = \frac{29.8^2}{2 \times 9.8} = 45.3m$$

Required height h′ relative to the top of the shift

$h' = H - 20 = 45.3 - 20$

$h' = 25.3m$

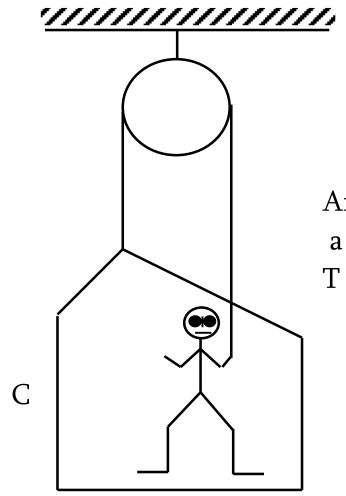
Example – 11

(a) (i) Athletes run some distance before taking a long jump why?

(ii) Why are mud–guards used over the rotating wheel of the vehicles?

(iii) Action and reaction are equal opposite, then why do not cancel each other.

(b) In the figure below, a painter of mass 100kg standing in a crate C of mass 25kg pulls himself up with the crate with an acceleration. If the painter exerts an effective force of 450N on the floor of the crate, find the acceleration of the system and the tension in the string take $g = 10m/s^2$



Answer

$a = 2m/s^2$

$T = 750N$

Solution

(a) (i) Due to inertia of motion the athlete can jump longer the distance covered by the athlete in a long jump depends on the velocity just before his jump. Due to his initial run he acquires a certain velocity so that inertia of motion of his body during jumping may help him in his muscular effect to jump through a greater distance.

(ii) Action and reactions acts on two different bodies so they do not cancel each other

(b) Assignment to the student

Example – 12

(a) In order to swim, a person pushes the water backward, why?

(b) Can a sail boat be propelled by air blown at the sails from a fan attached to the boat?

(c) Why the Newton's First Law of motion is known as Law of Inertia?

Solution

(a) When a man pushes (newton's 3rd law is applied here) water backward with his hands, due to the reaction of water the man is pushed forward.

(b) No, when the air from fan pushes the sail, the air pushes the fan in the opposite direction. The fun is a part of the boat and hence boat cannot be propelled.

(c) Refer to your notes.

ASSIGNMENT NO: 1

1. The mass of a lift is 500kg. Find the tension in the cable of the lift while lift is
   (i) At rest position
   (ii) Moving upwards with the acceleration of $2m/s^2$
   (iii) Coming downwards with the acceleration of $2m/s^2$.
   Answers (a) 4900N (b) 5900N (c) 39000N

2. A person weighing 56kg is standing in a lift. Find the weight as recorded by the weighing machine when
   (a) The lift is stationary
   (b) The lift moves upward with a uniform velocity of $2.1m/s^2$
   (c) The lift moves downwards with a uniform velocity of 2.1m/s
   (d) The lift moves downwards with a uniform acceleration $2.1m/s^2$
   Ans. (a) 59kgwt (b) 56kgwt (c) 56kgwt (d) 68kgwt, (e) 44kgwt

3. An elevator starts form rest and goes in upward direction. What is the acceleration of the elevator and how far does it go upward is 5seconds if its mass is 3,000kg and the upward tension in the supporting cable is 50,000N (Take $g = 10m/s^2$)?

   Ans. $\frac{20}{3}$ m/s^2 , 83.33m

4. A man weighing 55kg on the earth is standing in a lift. Calculate the apparent weight of the man if the lift is: -
   (a) Rising with an acceleration of $1m/s^2$
   (b) Going down with an acceleration of $0.5m/s^2$.
   (c) Falling freely under the action of gravity

   Ans. (a) 594N (b) 511.5N (c) 0

5. A body of mass 100kg stands on a spring weighing machine inside a lift. The lift starts to ascend with acceleration of $2.2m/s^2$. What is the reading of the machine?

   Ans. 1,200N or 122.45kgf.

3. SYSTEM OF VARIABLE MASSES

According to the newton's second law of motion

$$F = \frac{dp}{dt} \quad , \quad P = MV$$

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

If V is regarded as a constant , then

$$F = V\frac{dm}{dt}$$

This is equation for the system of variable masses.

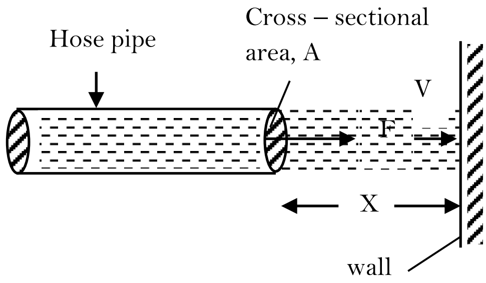
Example of the system of variable masses.

(i) Reaction force on the hose pipe
(ii) Sand falling on a moving conveyor belt.
(iii) Rocket propulsion
(iv) The helicopter
(v) Flying of a bird.

I. REACTION FORCE ON HOSE PIPE (WATER SPRAY)

This is the device which is used to spread water on the tall building or garden. This operates on the based on the Newton's second law and Newton's third law of motion.

Consider the projection of water from the horse pipe as shown on the figure below.



Assumptions made: -

1. A jet water strikes the wall at the right angle
2. Water losses all velocity after impact on the wall when a jet of water strike on the wall, force exerted is due to the rate of change of mass of water strike on the wall.

- **Expression of rate of mass of water strike on the vertical wall.**

  Mass = density × volume

  $M = \rho AX$

  Rate of mass of water strike on the wall

  $$\frac{dm}{dt} = \frac{d}{dt}(\rho AX) = \rho A\frac{dx}{dt}$$

  But $V = \frac{dx}{dt}$

  $$\frac{dm}{dt} = \rho AV$$

  Where

  ρ = density of water

  A = cross – sectional area

  V = velocity of water strike on the wall

- **Expression of the force exerted on the wall.**

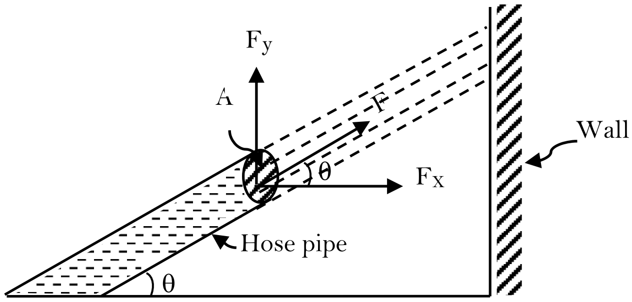
  Since $F = V\frac{dm}{dt} = V(\rho AV)$

$$F = \rho AV^2$$

$$F = V\frac{dm}{dt} = \rho AV^2$$

Special case

Consider when a jet of water strike on the vertical wall at the certain angle as shown on the figure below



Force normal to the wall

$F_x = F\cos\theta$

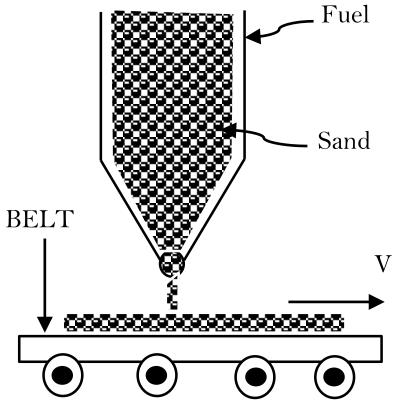
$F_x = \rho AV^2\cos\theta$

II. SAND FALLING ON A MOVING CONVEYOR BELT

Consider the case of sand which is allowed to fall vertically at a steady rate of

$$dm/dt$$

on to a horizontal conveyor belt at steady velocity, V as shown on the figure below



- **Expression of the force exerted on the belt.**

  Since the initial horizontal velocity of the sand is zero. Therefore, the initial horizontal momentum of the sand is zero. According to the Newton's second law of motion.

$$F = V\frac{dm}{dt}$$

$\frac{dm}{dt}$ = rate of mass of sand fall on conveyor belt.

V = speed of the belt

The system is gaining mass with time so the necessary external force must point in the direction in which belt moves.

Note:

The mass of the belt does not enter in our calculation because the belt is supposed to move with a constant velocity.

- **Expression of power supplied by external force so that maintained belt to move with the constant velocity.**

$$P = F\cdot V = V^2\frac{dm}{dt}$$

- **Expression of rate of change of kinetic energy of sand drops on the belt**

$$k.e = k = \frac{1}{2}MV^2$$

$$\frac{dk}{dt} = \frac{v^2}{2}\cdot\frac{dm}{dt}$$

Relationship between P and dk/dt

$$P = V^2\cdot\frac{dm}{dt}, \quad \frac{dk}{dt} = \frac{V^2}{2}\cdot\frac{dm}{dt}$$

$$\frac{dk}{dt} = \frac{1}{2}\cdot P$$

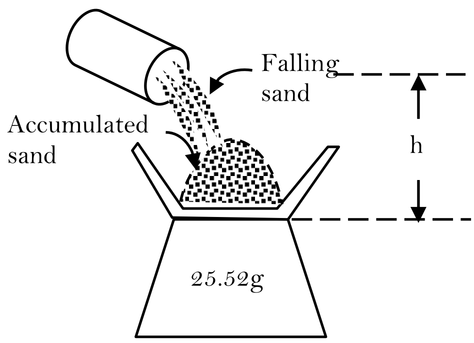
$$P = 2\frac{dk}{dt}$$

Comment

The power supplied is twice a great as the rate of change of kinetic energy. The extra power is due to the fact that the sand does not immediately assume the velocity of the belt; so the belt at first move relative to the sand. The extra power is needed to overcome the friction force between the belt and sand.

III. FALLING OF SAND ON A PAN BALANCE

Consider the figure below which shows falling of the sand on pan balance after time, t.



The effective reading on the pan will be the net weight of the accumulated sand plus the impact force produced by falling sand at a given time

$$F = Mg + V\frac{dm}{dt}$$

F = Total pan reading

Mg = weight of accumulated sand

$V\frac{dm}{dt}$ = impact force due to the of flow of sand.

Velocity of the sand at the impact on the pan.

$$V^2 = U^2 + 2gh \quad [U = 0]$$

$$V = \sqrt{2gh}$$

$$F = Mg + \sqrt{2gh}\cdot\frac{dm}{dt}$$

IV. FORCE DUE TO ROTATING HELICOPTER BLADES

When helicopter blades are rotating, the strike air molecules in a downward direction. The momentum changes per second of the air molecules produces a downward force and by the law of action and reaction, an equal upward force is exerted by the molecules on the helicopter blades. This upward force helps to keep the helicopter hovering in the air because it can balance the downward weight of the machine.

Reaction force upward = weight of the helicopter

$$F = \rho AV^2 = Mg$$

ρ = density of compressed air

A = Area of rotating blades

V = average speed of air

M = mass of the helicopter

Note that:

(i) A vertical rotor on the tail provides a counter thrust and control the direction of motion of the helicopter

(ii) Helicopter and air plane cannot operate outside of the earth atmosphere where there is no air because the working thrust is given by the concept of propelling the air.

V. FLYING MECHANISM OF A BIRD

All birds fly by biting their wings up and down. The physics law being applied here is the forced air like an air jet. When the wings bits down, the air is forced downward producing the thrust

At the equilibrium of a bird.

$$\rho AV^2 = Mg$$

$$V = \sqrt{\frac{Mg}{\rho A}}$$

SOLVED EXAMPLES

Example – 13

(a) (i) Define the term inertia. (01 mark)

(ii) Why is Newton's first law of motion is called law of inertia? (01 mark)

(b) A jet of water from a fire hose is capable of reaching a height of 20M if the cross -sectional area of the hose outlet is $4 \times 10^{-4}m^2$, calculate the :-

(i) Minimum speed of water from the hose. (01 mark)

(ii) Mass of water leaving the hose each second. (02 marks)

(iii) Force on the hose due to the water jet. (02 marks)

Takes $g = 9.8m/s^2$

Density of water = $1000kgm^{-3}$

Solution

(a) Refer to your notes

(b) (i) since

$$V^2 = U^2 - 2gh$$

$$V = 0$$

Figure pg 57

$$U = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 20}$$

$$U = 19.798m/s$$

(ii) Since $\frac{dm}{dt} = \rho AV$

$$\frac{dm}{dt} = 1000 \times 4 \times 10^{-4} \times 19.798$$

$$\frac{dm}{dt} = 7.92kg/sec$$

∴ mass of water leaving a hose each second = 7.92kg

(iii) $F = V\frac{dm}{dt}$

$$= 19.798 \times 7.92$$

$$F = 156.784N$$

Example – 14

A jet of water emerges from a hose pipe of cross – sectional area $5 \times 10^{-3}m^2$ with a velocity of 3m/s strikes a wall at the right angle. Assume that water to be brought to rest by the wall and does not rebound. Calculate the force on the water. Give that $\rho = 1000kgm^{-3}$.

Solution

$$F = \rho AV^2$$

$$= 1000 \times 5 \times 10^{-3} \times 3^2$$

$$F = 45N$$

Example – 15

A hose ejects water at a speed of 20cm/s through a hole of area $100cm^2$. If the water strikes a wall normally, calculate the force on the wall in Newton, assuming the velocity of the water normal to the wall is zero after collision.

Solution

Rate of volume of water

Strike on the wall

$$AV = 100 \times 20 = 2000cm^3/s$$

Rate of mass of water

Strike on the wall

$$\frac{dm}{dt} = \rho AV = 1 \times 2000$$

$$= 2000g/s = 2kg/s$$

Force on the wall

$$F = V\frac{dm}{dt} = 0.2 \times 2$$

$$F = 0.4N$$

Example – 16

Sand drops vertically at the rate of 2kg/s on the conveyor belt moving horizontally with a velocity of 0.1m/s calculate.

(i) The extra power needed to keep the belt moving

(ii) The rate of change of kinetic energy of sand. Why is the power twice as great as the rate of kinetic energy?

Solution

(i) Extra power need to keep the belt on moving

$$P = V^2 \cdot \frac{dm}{dt}$$

$$P = (0.1)^2 \times 2 = 0.02W$$

(ii) $\frac{dk}{dt} = \frac{V^2}{2} \cdot \frac{dm}{dt} = \frac{0.02W}{2}$

$$\frac{dk}{dt} = 0.01W$$

Comment: refer to your notes.

Example – 17

(a) State the newton's laws of motion and reduce from them relation between the distance travelled and time for the case of a body acted upon by a constant force.

(b) A fire engine pumps water at such a rate that the velocity of water leaving the nozzle is 15m/s. if the jet be directed perpendicular onto a wall and rebound of the water be negligible. Calculate the pressure on the wall [1cm³ of water has a mass of 1000kg]

Solution

(b) $F = \rho A V^2$

$$P = \frac{F}{A} = \rho V^2$$

$$= 1000 \times 15^2$$

$$P = 225,000 Nm^{-2}$$

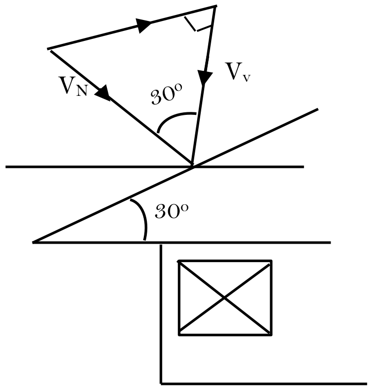
Example – 18

Rain falls vertically on a plane roof 1.5m square is inclined to the horizontal at an angle of 30°. the rain drops strike the roof with a vertical velocity of 3.0m/s and a volume of $2.5 \times 10^{-3} m^3$ of water is collected from the roof in one minute. Assuming that the conditions are steady and that the velocity of the rain drops after the impact is zero. Calculate;

(a) The vertical force exerted on the roof by the impact of the rain falling.

(b) The pressure normal to the roof due to the impact of the rain. If in steady, the roof were subject to a rain of hard spheres which collided elastically with the same mass per unit time and the same vertical velocity, what would the normal pressure on the roof than be? Density of water = $1000kgm^{-3}$.

Solution



(a) Area of roof, $A = (1.5m)^2 = 2.25m^2$

$$\frac{dV_o}{dt} = \frac{2.5 \times 10^{-3} m^3}{60se}$$

$$\frac{dV_o}{dt} = 4.16 \times 10^{-5} m^3$$

The vertical component of the force

$$F_v = V_v \frac{dm}{dt} = \rho V_v \frac{dV_o}{dt}$$

$$= 1000 \times 3 \times 4.16 \times 10^{-5}$$

$$F_V = 0.125N$$

(b) CASE 1: P_N = pressure normal to the roof of the house

$$P_N = \frac{F_N}{A} = \frac{1}{A} V_N \rho \frac{dV_o}{dt}$$

$$\cos 30^\circ = \frac{V_V}{V_N}, \quad V_N = \frac{V_V}{\cos 30^\circ}$$

$$P_N = \frac{1}{A} \cdot \frac{V_V}{\cos 30^\circ} \rho \cdot \frac{dV_o}{dt}$$

$$= \frac{1000}{2.25} \times \frac{3}{\cos 30^\circ} \times 4.16 \times 10^{-5}$$

$$P_N = 0.064 Nm^{-2}$$

CASE 2: when water collides elastically
Rate of change in momentum

$$\frac{dp}{dt} = V_N \frac{dm}{dt} - \left(-V_N \frac{dm}{dt}\right)$$

$$F_N = 2V_N \frac{dm}{dt} = 2V_N \rho \frac{dv_o}{dt}$$

Now

$$P_N = \left(\frac{2\rho}{A}\right) \cdot \frac{V_V}{\cos 30^\circ} \cdot \frac{dv}{dt}$$

$$= \frac{2 \times 1000}{2.25} \times \frac{3}{\cos 30^\circ} \times 4.16 \times 10^{-5}$$

$$P_N = 0.12894 Nm^{-2}$$

<u>Example – 19 NECTA 2008/P1/2</u>

(a)

i. can a body have energy without momentum? Explain.

ii. Two masses M_1 = 10kg and M_2 = 250gm are acted upon by a force of 10N and 5N respectively in opposite direction after a certain instant the two masses collide and coalesce. If the force remains the same both before and after collision. Calculate the relative acceleration before collision and their acceleration after collision.

(b) Rain fall vertically on a plane roof 1.5m square which inclined to the horizontal at angle of 30o. the rain drops strike the roof with a velocity of 3m/s and a volume of $2.5 \times 10^{-2} m^3$ of water is collected from the roof in one minute. Assuming that the velocity of the rain drops after impact is zero. Calculate the

(i) Vertical force exerted on the roof by the impact of the falling rain.

(ii) Pressure exerted normal to the roof due to the impact of the rain.

Solution

(a)

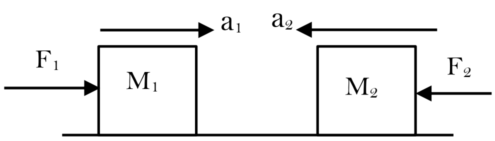
i. Yes, the body have energy without momentum because when the body is at rest, its possessing only the potential energy due to its virtual position while momentum acquired by the body is equal to zero

ii. <u>Data</u>

M_1 = 10kg M_2 = 0.25kg

F_1 = 10N F_2 = 5N

- Before collision of the bodies



$$a_1 = \frac{F_1}{M_1} = \frac{10}{10} = 1m/s^2$$

$$a_2 = \frac{F_2}{M_2} = \frac{5}{0.25} = 20m/s^2$$

a_{21} = Acceleration of M_2 relative to M_1 before collision

$$a_{21} = a_2 - a_1$$

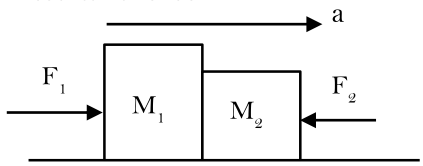
$$= 20 - (-1)$$

$$a_{21} = 12m/s^2$$

- a = acceleration of two bodies after collision.

Figure

Resultant force



$$F = F_1 - F_2$$

$$(M_1 + M_2)a = F_1 - F_2$$

$$a = \frac{F_1 - F_2}{M_1 + M_2} = \frac{(10-5)}{10 + 0.25}$$

$$a = 0.4878m/s^2$$

(b) Refer to your notes

Example – 20

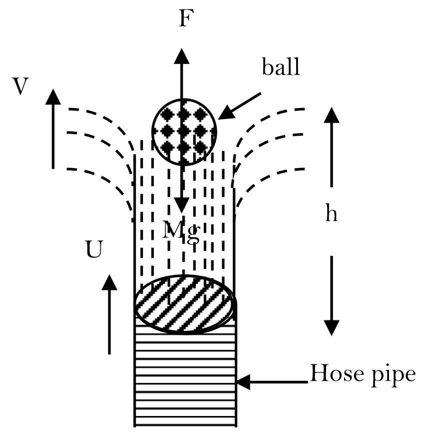
(a) Newton's third law of motion state that 'Action and reaction are equal and opposite yet when you kicking the ball will move away from you explain.

(b) A jet of water issues vertically at a velocity of 14m/s from a nozzle $0.4cm^2$ in area of cross – section. A ball of mass 400gm is maintained in equilibrium by the impact of water on its underside. How high is the ball above the nozzle? ($g = 9.8m/s^2$)

Solution

(a) When you kicking the ball will move away from you because the ball will exert the force from the legs i.e. reaction force on the leg and this according to the newton's third law of motion.

(b)



At the equilibrium of the ball

$$F = Mg$$

$$U\frac{dm}{dt} = mg$$

$$u\frac{d}{dt}(\rho Ah) = mg$$

$$u\rho A\frac{dh}{dt} = mg$$

$$u\rho AV = mg$$

$$V = \frac{mg}{\rho Au}$$

Since

$$V^2 = u^2 - 2gh$$

$$h = \frac{1}{2g}\left[u^2 - v^2\right]$$

$$= \frac{1}{2g}\left[u^2 - \left(\frac{mg}{\rho Au}\right)^2\right]$$

$$= \frac{1}{2\times 9.8}\left[14^2 - \left(\frac{9.8\times 0.4}{1000\times 0.4\times 10^{-4}\times 14}\right)^2\right]$$

$$h = 7.5\text{m above the nozzle.}$$

Example – 21

Sand is poured at a steady rate of 5kg/s onto the pan of a direct reading balance calibrated in grams. If the sand falls from a height of 0.2m on to the pan and it doesn't bounce off the pan then, neglecting any motion of the pan. Calculate reading on the balance 10sec after he sand first hits the pan ($g = 10m/s^2$)

Solution

Force due to the falling of sand at any time, t.

$$F = Mg + V\frac{dm}{dt}$$

But

$$M = t\cdot\frac{dm}{dt}$$

$$V^2 = U^2 + 2gh \quad , \quad U = 0$$

$$V = \sqrt{2gh}$$

$$F = t\cdot\frac{dm}{dt} + \sqrt{2gh}\cdot\frac{dm}{dt}$$

$$= 10\times 5\times 10^{-3} + \sqrt{2\times 10\times 0.2}\times 5\times 10^{-3}$$

$$F = 0.51N$$

Reading

$$M = \frac{F}{g} = \frac{0.51N}{10m/s^2}$$

$$M = 0.051kg$$

Example – 22

(a)

i. A person falling from a certain height receives more severe injuries if he falls on a cemented floor than on a heap of sand? Why?

(b) A bird of wing of mass 0.5kg moves upwards by beating its wings of effective area $0.3m^2$, estimate the velocity of imparted air by the beating of the wings. Assume the air to be at S.T.P and the density of air at S.T.P = $1.3kgm^{-3}$, $g = 9.8m/s^2$.

Solution

(a) (i) Impulse = force × time larger time , smaller the force , if change in momentum is constant the change in momentum in the two cases is the same. When he falls on a cemented floor he is stopped suddenly (time is small) so the force applied against him is very large and this cause injury. But when he falls on san it gets depressed under his weight and thus the time in which is stopped is more, force is less.

(ii) When he lowers his hands along with the ball he increases the time to stop the ball. Then he can stop the ball by applying a small force thus the force of reaction of the ball on his hand is very small and he hand are hurt.

(b)

At the equilibrium of the wing of a bird

$$\rho AV^2 = Mg$$

$$V = \sqrt{\frac{mg}{\rho A}}$$

$$= \sqrt{\frac{0.5\times 9.8}{1.3\times 0.3}}$$

$$V = 3.5445m/s$$

Example – 23

(a) Based on newton's laws of motion explain how a helicopter gets its lifting force.

(b) A helicopter of mass 5000kg hovers when its rotating blades move through an area of $30m^2$ and gives an average speed of V to the air. Estimate V assuming the density of air is $1.3kgm^{-3}$ and $g = 10m/s^2$.

Solution

(a) Refer to your notes

(b) Since $\rho AV^2 = Mg$

$$V = \sqrt{\frac{Mg}{\rho A}}$$

$$= \sqrt{\frac{500\times 10}{1.3\times 30}}$$

$$V = 11m/s$$

Example – 24 NECTA 2018/P1/2

(a) (i) Under what condition a passenger in a lift feels weight less?

(ii) Calculate the tension in the supporting cable of an elevator of mass 500kg which was originally moving down – wards with a velocity of 4m/s and brought to rest with a constant acceleration at a distance of 20m.

(b) (i) The rotating blades of a hovering helicopter swept out an area of radius 2m imparting a downward velocity of 8m/s of the air displaced. Find the mass of helicopter.

(ii) Compute the mass of water striking the wall per second when the jet water with velocity 5m/s and cross – sectional area of $3 \times 10^{-2}m^2$ strikes the wall at right angle losing its velocity to zero ($g = 10m/s^2$)

Density of water = $1000kgm^{-3}$.

Solution

(a) (i) This occurs when the lift is moving vertically downward in such a way that acceleration of the lift a = g i.e. R = 0

(ii) Since $V^2 = U^2 - 2as$

$$0 = u^2 - 2as$$

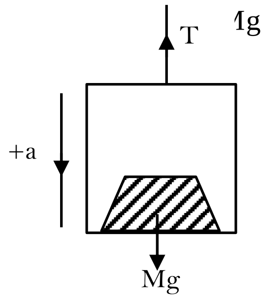
$$2as = u^2$$

$$a = \frac{u^2}{2s}$$

$$a = \frac{4^2}{2\times 20} = \frac{16}{40}$$

$$a = 0.4m/s^2$$

Resultant force



$$Mg - T = Ma$$

$$T = Mg - Ma$$

$$= M(g-a)$$

$$= 500(10-0.4)$$

$$T = 4800N$$

Example – 25

(a) What is the connection between force and momentum?

(b) A helicopter of total mass 1000kg is able to remain in a stationary position by imparting a uniform velocity to a cylinder of air below it of effective diameter 6m. assuming the density of air to be 1.2kgm^{-3}. Calculate the downward velocity given to the air.

Solution

(a) $F = \dfrac{dp}{dt}$

F = Force

$\dfrac{dp}{dt}$ = rate of change of momentum

(b) $\rho AV^2 = Mg$

$$V = \sqrt{\frac{4Mg}{\rho \pi d^2}}$$

$$= \sqrt{\frac{4 \times 1000 \times 10}{1.2 \times 3.14 \times 6^2}}$$

$$V = 17.172\text{m} / \text{s}$$

Example – 26

An elevator can carry a maximum load of 1,6000kg (elevator + passengers) is moving up with a constant speed of 2m/s. the frictional force opposing the motion is 3, 320N. Determine the maximum power delivered by the motor to the elevator.

Solution

Weight of elevator + passengers

$W = Mg = 1600 \times 9.8$

$W = 15,680\text{N}$

The force applied by motor

$F + W + f = 15,680 + 3320$

$F = 20,000\text{N}$

Power delivered by the motor to the elevator

$P = FV$

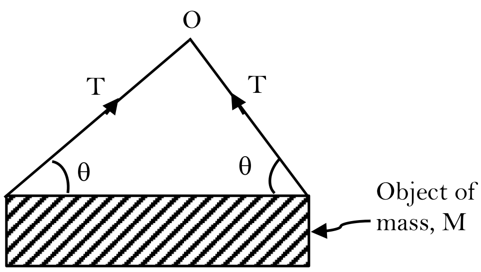
$= 20,000 \times 2$

$P = 40,000\text{W}$

Example – 27 NECTA 1998/P2/3

(a) Explain why a length of hope pipe which lying in a curve on a smooth horizontal surface straightness out when a fast flowing stream of water passes through it.

(b)

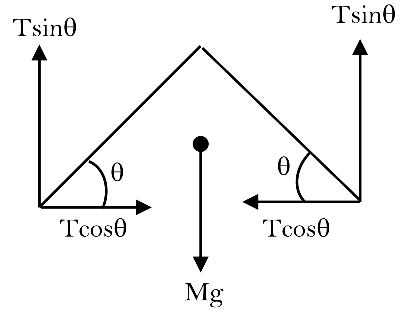


The diagram above shows a cord supporting a heavy object of mass , M form a fixed point , O if the total length of the cord is varied , sketch a graph of the tension in the cord against the angle , θ.

Solution

(a) This is due to the fact that the fast flowing water wants to continue in its state of motion in the straight line within the pipe and surface is smooth to make sure that no centripetal force acts on it to prevent circular motion. Hence the pipe straightness out according t the newton's first law of motion.

(b)

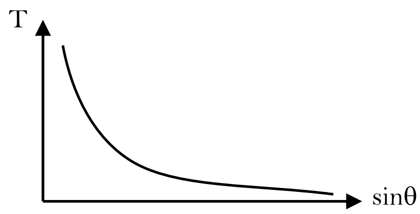


Horizontal components of tensions cancelled to each other at the equilibrium

$T \sin\theta + T \sin\theta = Mg$

$$T = \frac{Mg}{2\sin\theta} \quad , \quad T \; \alpha \; \frac{1}{\sin\theta}$$

Graph of T against sin θ



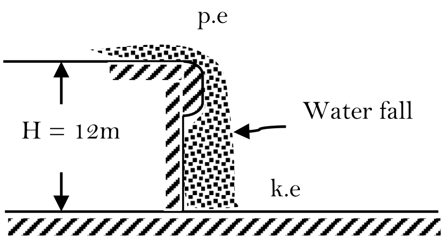
Example – 28

(a) Some 10kg of water poured to the floor from a height of 12m. If the downward motion of the water is uniformly stopped by the floor in 5.0sec. estimate the maximum force exerted onto the floor by the water.

(b) Rain is falling to the ground. The rain drops makes tracks on the side window of a car at an angle of 30° below the horizontal. Calculate the speed of the car.

Solution

(a)



Apply the law of conservation of energy

Loss in p.e = gain in k.e

$$Mgh = \frac{1}{2}MV^2$$

$$V = \sqrt{2gh}$$

Since

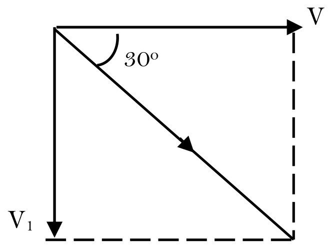
$$F = Ma = \frac{MV}{t}$$

$$= M\frac{\sqrt{2gh}}{t}$$

$$= \frac{10 \times \sqrt{2 \times 9.8 \times 12}}{5}$$

$$F = 30.672N$$

(b)



V_1 = vertical component of the velocity

$$\tan 30^\circ = \frac{V_1}{V}$$

$$V = \frac{V_1}{\tan 30^\circ} = \frac{8}{\tan 30^\circ}$$

$$V = 13.856m/s$$

Example – 29

(i) An empty train starts move quickly than a loaded train why?

(ii) If we jerk a piece of paper from under a book quick enough, the book will not move why?

(iii) When the branch of a tree is shaken, the fruits fall why?

Solution

(i) The momentum of the empty train is less so force needed is less because force is the rate of change of momentum.

(ii) This is due to inertia of rest.

(iii) This is due to inertia of rest. The force is exerted on the tree and it moves but the fruits try to remain in the initial position due to inertia of rest so the fruits fall.

Example – 30

A helicopter of mass 1,000kg rises with a vertical acceleration of $15m/s^2$. The crew and the passengers weigh 300kg. Given the magnitude and direction of the

(a) Force on the floor by the crew and the passengers.

(b) Action of the rotor of the helicopter on the surround air.

(c) Force on the helicopter due to the surrounding air ($g = 10m/s^2$)

Solution

Mass of crew and the passenger, $M_1 = 300kg$

Mass of helicopter, $M_2 = 1000kg$, $a = 15m/s^2$

(a) Let R = Force on floor by the crew and the passengers

According to the newton's third law of motion

$$R - M_1g = M_1a$$

$$R = M_1(g+a)$$

$$= 300(10+15)$$

$$R = 7,500N \text{ vertically}$$

downwards.

(b) Let M be the total mass of the helicopter, crew and the passengers.

$$M = M_1 + M_2 = 1000 + 300$$

$$M = 1,300kg$$

R1 be the force on the helicopter by air in upward direction.

$$R_1 - Mg = Ma$$

$$R_1 = M(g+a)$$

$$= 1300(10+15)$$

$$R_1 = 32,500N = 3.25\times10^4 N$$

(Vertically upwards)

By newton's third law the motor of the helicopter will exerts 3.25×10^4N vertically downwards.

(c) Force on the helicopter due to surrounding air = 3.25×10^4N vertically upwards.

ASSIGNMENT NO 2.

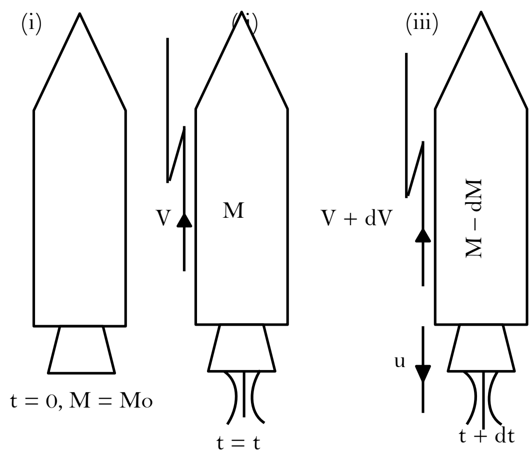
1. Rain drops of mass $5\times10^{-7}kg$ fall vertically in still air with uniform speed of 3m/s. if such drops are falling when a wind is blowing with a speed of 2m/s , what is the angle which the paths of the drops makes with the vertical? What is the kinetic energy of a drop?
   Answer: 33.7^o , $3.25 \times 10^{-6}J$

2. A hose with a nozzle 80mm in diameter ejects a horizontal stream of water at a rate of $0.044m^3/s$ with what velocity will the water leave the nozzle? What will be the force exerted on a vertical wall situated close to the nozzle and at a right – angles to the stream of water, if after hitting the wall.
   (a) The water falls vertically to the ground
   (b) The water rebounds horizontally
   Ans. 8.75m/s (a) 385 N (b) 770N

3. (a) State the Newton's law of motion
   (b) A bird of mass 0.50kg hovers by beating its wings of effectively area $0.30m^2$.
      (i) What is the upward force of the air on the bird?
      (ii) What is the downward force of the bird on the air as it beats its wings?
      (iii) Estimate the velocity imparted to the air, which has a density of $1.3kgm-3$, by the beating of the wings. Which of Newton's laws is applied in each of (i), (ii) and (iii) in above.
   Ans. (i) 5.0N (ii) 5.0N (iii) 3.6m/s

VI. ROCKET PROPULSION AND JET PROPULSION

- **ROCKET PROPULSION**
  This is the one of examples of the system of variable masses.
  Principle of rocket propulsion
  This based on the principle of conservation of linear momentum and Newton's third law of motion. Before the rocket is fired the total momentum of the rocket plus fuel is zero since the system, the total momentum remains the same. When the rocket is fired fuel is burnt and very hot gases are formed. These gases are expelled from the nozzle on the back size of the rocket. Since the momentum acquired toward the rear, the rocket must acquire an equal momentum in the opposite direction (forward) in order to conserve momentum this is how a rocket propelled the lift of the rocket can be achieved when R > Mg.
  Different cases
  Case 1:
  If the rocket is moving out – of space, the effect of acceleration due to gravity is neglected.



- **Expression of the thrust or reaction force experienced on the rocket.**
  Apply the principle of conservation of the linear momentum
  Total initial Momentum = total final momentum

$$MV = (M - dM)(V + dV) + UdM$$

$$MV = MV + MdV - VdM - dVdM + UdM$$

$$0 = MdV + (U - V)dM$$

$$-MdV = (U - V)dM$$

$$-M\frac{dV}{dt} = V_r\frac{dm}{dt}$$

$$F = -V_r\frac{dm}{dt}$$

Negative sign shows that increases of speed rate of mass of rocket decreases

- **Expression of velocity of rocket at any time, t in out of space.**

  Since

$$MdV = -V_r dV$$

$$dV = -V_r\frac{dm}{m}$$

$$\int_{V_o}^{V} dv = -V_r\int_{M_o}^{U}\frac{dm}{m}$$

$$\left[V\right]_{V_O}^{V} = -V_r\left[\log_e m\right]_{Mo}^{M}$$

$$V = V_O + V_r\log_e\left(M_O/M\right)$$

Special case

If the initial velocity of the rocket, Vo = 0

$$V = V_r\log_e\left(M_O/M\right)$$

This shows that

(a) $V\alpha V_r$

(b) $V\alpha\log_e\left(M_O/M\right)$

- **Expression of the acceleration of the rocket**

  In magnitude

$$MdV = -V_r dM$$

$$MdV = V_r dM$$

$$\frac{dV}{dt} = \frac{V_r}{M}\frac{dm}{dt}$$

$$a = \frac{V_r}{M}\cdot\frac{dm}{dt}$$

- **Burnt out speed of the rocket.**

  Is the speed acquired by the rocket when the whole of the fuel gets burnt it is maximum speed that can be acquired by the rocket? Let Mc be the mass of the empty container.

$$V_b = V_r\log_e\left(M_o/M_C\right) \ ; \ V_O = 0$$

- **Expression of mass of the rocket at any time , t**

  From the equation

$$V = V_o + V_r\log_e\left(M_o/M\right)$$

$$\frac{V - V_o}{V_r} = \log_e\left(M_o/M\right)$$

$$\frac{M_o}{M} = e^{\frac{V - V_o}{V_r}}$$

$$\frac{M}{M_o} = e^{-\left(\frac{V-V_o}{V_r}\right)}$$

$$M = Moe^{-\left(\frac{V-V_O}{V_r}\right)}$$

Case 2

When the rocket is under the influence of earth's gravity, the equation of the reaction force

$$F = M\frac{dv}{dt} = V_r\frac{dm}{dt} - mg$$

Instantaneous acceleration

$$a = \frac{V_r}{m}\cdot\frac{dm}{dt} - g \qquad \text{or}$$

$$a = \frac{V_r}{M_O - \left(\frac{dm}{dt}\right)t}\cdot\frac{dm}{dt}$$

- **Expression of the velocity at any time, t**

$$-M\frac{dv}{dt} = V_r\frac{dm}{dt} - g$$

$$dv = -V_r\cdot\frac{dm}{m} - gdt$$

$$\int_{V_o}^{v} dv = -V_r\int_{m_o}^{m}\frac{dm}{m} - g\int_0^t dt$$

$$V = V_o + V_r\log_e\left(M_o/M\right) - gt$$

If

$$V_O = 0$$

$$V = V_r\log_e\left(\frac{M_o}{M}\right) - gt$$

Acceleration at this time

$$a = \frac{V_r}{t}\log_e\left(\frac{M_o}{M}\right) - g$$

This applies where the force of gravitation is experienced

DELAY TIME

Is the extra time taken by the rocket to move (stay) along the ground in order to reduce the weight of the rocket to be less than the reaction force (thrust) when the estimated time of the rocket to lift off is finished.

Now

$$R > W - W'$$

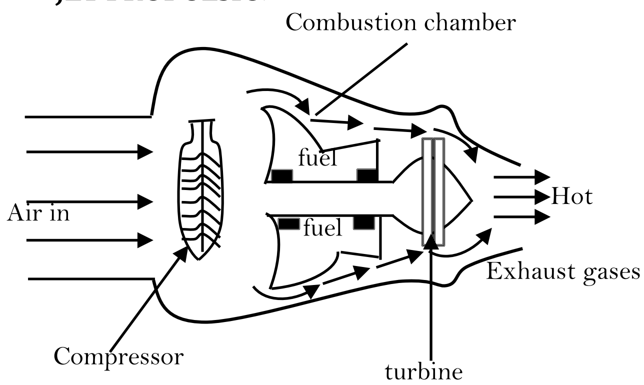
$$R > Mg - gt\frac{dm}{dt}$$

$$gt\frac{dm}{dt} > mg - R$$

$$t > (mg - R)\frac{1}{g\frac{dm}{dt}}$$

$$t > \frac{1}{g\frac{dm}{dt}} \cdot [mg - F]$$

- **JET PROPULSION**



A jet engine uses the surrounding air for its oxygen supply and so is un suitable for space travel. The compressor draws in air at the front, compresses it, fuel is injected and the mixture burns to produce hot exhaust gases which escape at high speed from the rear end of the engine. This cause forward propulsion and drive the turbine which in turn rotates the compressor and hence jet takes off.

Mathematically

Suppose air of mass Ma enters the front end of the jet with incoming velocity V_i in the combustion chamber the mixture of air and fuel burn and the exhaust (burnt) gases will be exerted with velocity V_o through the rear end of the jet. Initial linear momentum of incoming air.

$P_i = M_aV_i$

Final linear momentum P_f of outgoing burnt gases is given by

$$P_f = (M_a + M_f)V_o$$

The change of linear momentum

$$dp = (M_a + M_f)V_o - M_aV_i$$

According to the newton's second law of motion

$$F = \frac{dp}{dt}$$

$$F = \left(\frac{M_a}{t} + \frac{M_f}{t}\right)V_o - \left(\frac{M_a}{t}\right)V_i$$

SOLVED EXAMPLE

Example – 01

A rocket moving in free space has a speed of 3.0×10^3m/s relative to the earth its engines are turned on , and fuel is ejected in a direction opposite the rocket's motion at a speed of 5.0×10^3m/s relative to the rocket.

(a) What is the speed of the rocket relative to the earth once the rockets mass is reduced to one – half its mass before ignition?

(b) What is the thrust on the rocket if it burns fuel at 0.77kgs^{-1}?

Solution

(a) $V - V_o = V_r \log_e\left(Mo/M\right)$

$$V = V_o + V_r \log_e\left(M_o/M\right)$$

$$= 3\times10^3 + 5\times10^3 \log_e\left[\frac{M_o}{0.5M_o}\right]$$

$$V = 6465.7\text{m/s}$$

(b) $F = V_r\frac{dm}{dt}$

$$= 5.0\times10^3 \times 0.77$$

$$F = 3850\text{N}$$

Example – 02

(a) What do understand by the term 'Thrust on the rocket'?

(b) In the first second of its flight a rocket ejects 1/60 of its mass with a relative velocity of 2073m/s. what is the acceleration of the rocket?

Solution

(a) Thrust on the rocket is the reaction force exerted on the rocket by the escaping burnt gases.

(b) $$a = \frac{V_r}{M}\cdot\frac{dm}{dt} - g$$

$$= \frac{2073}{M}\cdot\frac{M}{60\times 1} - 9.8$$

$$a = 24.75m/s^2$$

Example – 03

A rocket of initial mass 6000kg ejects mass at a constant rate of $16kgs^{-1}$ with constant relative speed of 11km/s. what is the acceleration of the rocket a minute after the blast? Neglect gravity.

Solution

Acceleration of the rocket

$$a = \frac{V_r}{M_o - t\frac{dm}{dt}}\cdot\frac{dm}{dt}$$

$$= \frac{11000}{6000 - 16\times 60}\cdot 16$$

$$a = 34.9m/s^2$$

Example – 04

A rocket burns 50g of fuel per second ejecting it as a gas with a velocity of 5 × 105cm/s. what is the force exerted by the gas on the rocket?

Solution

$$F = V_r\cdot\frac{dm}{dt}$$

$$F = 5\times 10^3\times 50\times 10^{-3}$$

$$F = 250N$$

Example – 05

A fully fueled rocket has a mass of 21,000kg of which 15,000kg is fuel. The burnt fuel is exhausted at a rate of 190kg/sec at a speed of 2800m/s relative to the rocket. If the rocket is fired vertically upward calculate: -

(i) The thrust of the rocket.

(ii) The net force on the rocket at blast off

Solution

(i) $$F = V_r\frac{dm}{dt}$$

$$= 2800\times 190$$

$$F = 5.3\times 10^5 N$$

(ii) Net force on rocket

$$F_{net} = F - Mg$$

$$= 5.3\times 10^5 - 21{,}000\times 9.8$$

$$F_{net} = 3.2\times 10^5 N$$

Example – 06

A fully fueled rocket of mass 5000kg is set to be fired vertically if the rocket ejects its gases at speed of 3×10^3m/s w.r.t the rocket and burns fuel at the rate of $50kgs^{-1}$, what is the rockets initial upward acceleration? Include the effect of gravity.

Solution

Thrust on the rocket

$$F = V_r\frac{dm}{dt}$$

Net upward force initially (i.e. at blast off)

$$Ma = F - Mg$$

$$Ma = V_r\frac{dm}{dt} - Mg$$

$$a = \frac{V_r}{M}\cdot\frac{dm}{dt} - g$$

$$= \frac{3\times 10^3\times 50}{5000} - 9.8$$

$$a = 20m/s^2 \text{ (upward)}$$

Example – 07

A rocket of mass 20kg has 180kg of fuel. The exhaust velocity of the fuel is 1.6km/s. calculate the minimum rate of consumption on of fuel so that the rocket may rise from the ground also calculate the ultimate vertical speed gained by the rocket when the rate of consumption of fuel is 2kg/sec.

Solution

The rate of consumption of fuel is minimum when the upward acceleration of the rocket is zero i.e. the rocket moves with uniform velocity under this condition, up thrust of rocket has to overcome the weight of the rocket

$$V\frac{dm}{dt} = Mg$$

$$\frac{dm}{dt} = \frac{Mg}{V} = \frac{200 \times 9.8}{1600}$$

$$\frac{dm}{dt} = 1.225\text{kg/sec}$$

If the rate of consumption of fuel is 2kg/sec, the time taken for the consumption of 180kg fuel.

$$2\text{kg} \longrightarrow 1\text{sec}$$

$$180\text{kg} \longrightarrow t$$

$$t = \frac{180 \times 1\text{sec}}{2} = 90\text{sec}$$

The ultimate vertical velocity attained by the rocket.

$$V = V_r \log_e\left(M_o/M\right) - gt$$

$$= 1600\log_e\left(\frac{200}{20}\right) - 9.8 \times 90$$

$$V = 2798\text{m/s}$$

Example – 08

An 8000kg rocket is set for a vertical firing. If the exhaust speed is 800m/s, how much gas must be ejected per second to supply thrust needed (i) to overcome the weight of the rocket. (ii) to give the rocket an initial upward acceleration of 19.6m/s²?

Solution

(i) Since $V_r\frac{dm}{dt} = Mg$

$$\frac{dm}{dt} = \frac{Mg}{V_r} = \frac{8000 \times 9.8}{800}$$

$$\frac{dm}{dt} = 98\text{kg/sec}$$

(ii) Net force on the rocket

$$Ma = V_r\frac{dm}{dt} - Mg$$

$$\frac{dm}{dt} = \frac{M(a+g)}{V_r}$$

$$= \frac{8000(19.6+9.8)}{800}$$

$$\frac{dm}{dt} = 294\text{kg/sec}$$

$$\frac{dm}{dt} = 294\text{kg/sec}$$

Example – 09

(b)

i. State the principles on which the rocket propulsion is based (01 mark)

ii. A jet engine on a test bed takes in 40kg of air per second at a velocity of 100m/s and burns 0.8kg of fuel per second after compression and heating the exhaust gases are ejected at 600m/s relative to the air craft. Calculate the thrust of the engine.

(c) An object of mass 2kg is attached to the hook of a spring balance which is suspended vertically to the roof of a lift. What is the reading on the spring balance when the lift is?

(i) Going up with the rate of 0.2m/s².

(ii) Going down with an acceleration of 0.1m/s².

(iii) Ascending with uniform velocity of 0.15m/s.

Solution

(b) (i) • Principle of conservation of linear momentum third law of motion

(ii) $$F = \left(\frac{Ma}{t} + \frac{M_f}{t}\right)V_o - \left(\frac{Ma}{t}\right)V_i$$

$$= (0.8 + 40) \times 600 - 40 \times 100$$

$$F = 20,480\text{N}$$

(c) (i) $$R = M(g+a)$$

$$= 2(9.8 + 0.2)$$

$$R = 20\text{N}$$

(ii) $$R = M(g-a)$$

$$= 2(9.8 - 0.1)$$

$$R = 19.4\text{N}$$

(iii) $a = 0$

$R = Mg = 2\times 9.8$

$R = 19.6N$

Example – 10

An aero plane flying at 250m/s takes air into its jet engines at the rate of 500kg/s fuel burnt at the rate of 20kg/s and engine expel the gases at speed of 600m/s relative to the jet. Calculate the driving force of the engine on the aero plane

Solution

$$F = \left(\frac{M_a}{t}+\frac{M_f}{t}\right)V_O - \left(\frac{M_a}{t}\right)V_i$$

$$= (500+20)\times 600 - 500\times 250$$

$$F = 1.9\times 10^5 N$$

Example – 11

A rocket, set for vertical firing weighs 500kg and contains 900kg of fuel. It can have a maximum exhaust velocity of 3km/s.

(a) What should be its minimum rate of fuel consumption.
   (i) To just lift the launching pad.
   (ii) To give it an acceleration of 16m/s^2.

(b) What will be the speed of the rocket when the rate of consumption of fuel is 60kg/sec?

Solution

(a) Since $F = V_r\cdot\frac{dm}{dt} - Mg$

(i) Just at lift off, M = Mo

$Mo = 500+900 = 1400kg$

$F = 0$

$$\frac{dM_o}{dt} = \frac{M_o g}{V_r} = \frac{1400\times 9.8}{3000}$$

$$\frac{dM_O}{dt} = 4.573kg/\ sec$$

(ii) $a = 16m/\ s^2$

$$M_o a = V_r\frac{dM_o}{dt} - M_o g$$

$$\frac{dM_o}{dt} = \frac{M_o}{V_r}(a+g)$$

$$= \frac{1400}{3000}[16+9.8]$$

$$\frac{dM_o}{dt} = 12.04kg/\ sec$$

(b) $V = V_r\log_e\left(\frac{M_o}{M}\right) - gt$

Rate of consumption

$$\frac{dm}{dt} = 16kg/\ sec$$

Time taken to consume 900kg

$$t = \frac{900}{60} = 15\ sec$$

$$V = 3000\log_e\left(\frac{1400}{500}\right) - 9.8\times 15$$

$$V = 2.94km/\ s$$

Example – 12

A jet aircraft is travelling at 225m/s in a horizontal flight. The engine takes in air at a rate of 85kg/sec and burns fuel at a rate of 3kgs^{-1}. If the exhaust gases are ejected at 650m/s relative to the aircraft, find the thrust of the jet engine

Solution

Since

$$F = \left(\frac{M_a}{t}+\frac{M_f}{t}\right)V_o - \left(\frac{M_a}{t}\right)V_f$$

$$= 88\times 650 - 85\times 225$$

$$F = 38075N$$

$\therefore$ Thrust of the jet engine is F = 38075N

Example – 13

A rocket moving in free space has a speed of 3.0×10^3m/s relative to the earth its engines are turned on and fuel is ejected in a direction opposite to the rocket motion at a speed of 5×10^3m/s relative to the rocket.

(a) What is the speed of the rocket relative to the earth once the rocket's mass is reduced to one half its mass before ignition?

(b) What is the thrust on the rocket if it burns fuel at 0.77kg/s^{-1}?

Solution

(a) $V_f - V_i = V_r\log_e\left(M_o/M\right)$

$$V_f = V_i + V_r \log_e \left({M_o}/{M} \right)$$

$$= 3\times10^3 + 5\times10^3 \log_e\left[\frac{M_o}{0.5M_o}\right]$$

$$V_f = 6465.7m/s$$

(b) $F = V_r \frac{dm}{dt}$

$$= 5\times10^3 \times 0.77$$

$$F = 3850N$$

Example – 14

Two fire fighters must apply a total force of 600N to a steady hose that is discharging water at 3600ligres/min. estimate the speed of the water as it exists the nozzle.

Solution

$$F = V_r \frac{dm}{dt} \quad , \quad V_r = \frac{F}{dm/dt}$$

$$= \frac{F}{\rho\, dv/dt} = \frac{600}{1000\times0.06}$$

$$V_r = 10m/s$$

∴ The speed of water relative to nozzle,

Vr = 10m/s

Example – 15

A rocket of mass 1000kg containing 1500kg of fuel. When it is launched its burns fuel at the rate of 50kg/s and the product of combustion are expelled at constant rate of 400m/s.

(i) Calculate the thrust produced by the rocket.

(ii) Draw the diagram of force acting on the rocket, why there is delay time (time lapse) before the rocket takes off?

(iii) Calculate the time lapse between start of combustion and taking off.

Solution

(i) Thrust on the rocket.

$$F = V_r \cdot \frac{dm}{dt}$$

$$= 400\times50$$

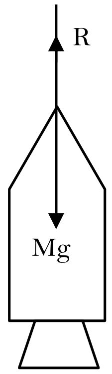
$$F = 20,000N$$

(ii) Diagram of force on the rocket. Weight of rocket before taking off

$$W = (M + M_f)g$$

$$= (1000 + 500)\times10$$

$$W = 25,000N$$



F = R = Thrust on the rocket

W = Mg = weight of the rocket

There is delay in time since the initial weight of the rocket is greater than that of the thrust on the rocket

(iii) For lift off of the rocket

R > Weight of the rocket

$$R > W - V_r \cdot \frac{dm}{dt}$$

$$2\times104 > 2.5\times10^4 - V_r\frac{dm}{dt}$$

$$2\times104 > 2.5\times10^4 - 50\times10t$$

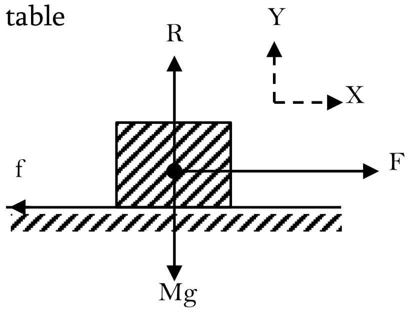
$$2\times104 > 2.5\times10^4 - 500t$$

$$t > 10sec$$

4. EQUILIBRIANT FORCES ON THE BODY AND SYSTEM OF CONNECTED BODIES.

DEFINITION FREE BODY DIAGRAM (FBD)

Is the diagram which shows all components of the forces acting on the body. Consider the block of mass M which lies on the horizontal rough table



EQUILIBRIUM AND RESOLUTION OF THE FORCES

RESULTANT FORCE

Is a single force that has the same effect of number of forces acting together equilibrant of a number of forces acting on a body is the single force which acting along with a number of forces keep the body in equilibrium.

EQUILIBRIUM

Is the condition where by the resultant forces acting on the body is equal to zero. Forces in the same plane are called coplanar forces. The condition of equilibrium of a rigid body under the action of coplanar forces:

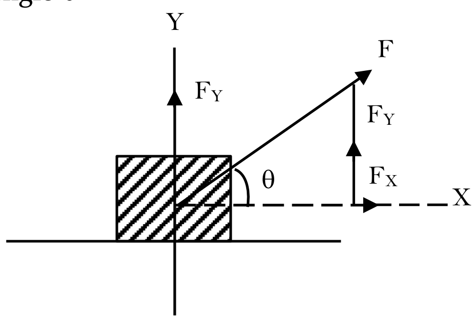
1. The algebraic sum of the resolved component of the forces in any fixed direction must be zero.
   i.e. $\sum Fx = 0, \sum Fy = 0, \sum Fz = 0$
2. The algebraic sum of the resolved components of the forces in a perpendicular direction must be zero.
3. The algebraic sum of torques or moments of the forces about any point in their plane must be zero.

COMPONENTS OF THE FORCES

There are two components of the forces acting on the given point if the force acting at an angle θ with the horizontal:-

1. Horizontal components of the force, Fx
2. Vertical component of the force, Fy

Consider figure which a force F acting on the block at an angle θ



$$\cos\theta = \frac{Fx}{F}, \quad Fx = F\cos\theta$$

$$\sin\theta = \frac{Fy}{F}, \quad Fy = F\sin\theta$$

By using Pythagoras theorem

$$F^2 = F_X^2 + F_y^2$$

Resultant force

$$F = \sqrt{F_x^2 + F_y^2}$$

Direction of the force, F

$$\tan\theta = \frac{Fy}{Fx}, \quad \theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$$

$$\theta = \tan^{-1}\left[\frac{Fy}{Fx}\right]$$

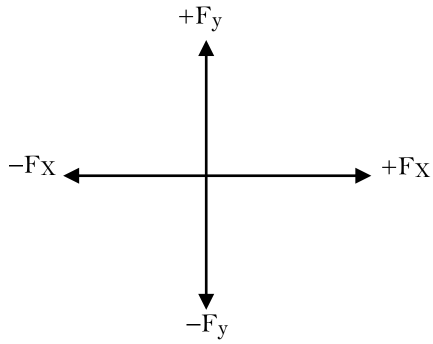
Additional conformations

1. Generally, for the given number of forces acting on the given point

$$F = \sqrt{\left(\sum Fx\right)^2 + \left(\sum Fy\right)^2}$$

$$\tan\theta = \frac{\sum Fy}{\sum Fx}$$

Reference diagram

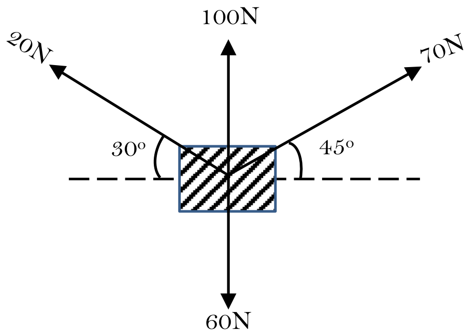


2. Reasoning
   It is possible for the body to be in equilibrium when is in motion?
   Yes. This is happening when the body is moving with constant velocity, then acceleration of the body is equal to zero. As a result, the resultant force acting on the body is equal to zero. This is the condition for the body to be in equilibrium.

SOLVED EXAMPLE

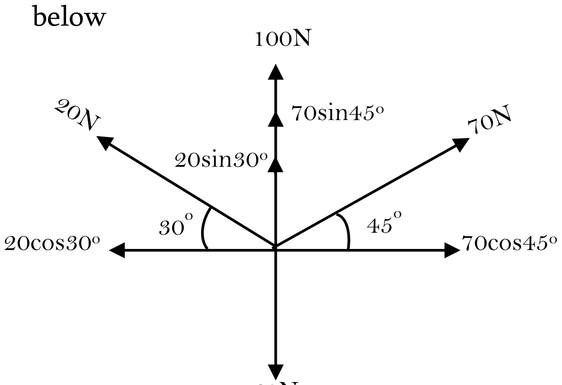
Example – 01

Find the resultant force acting on the point, P as shown on the figure below



Solution

Consider the FBD as shown on the figure below



Force in X – dire

$\sum Fx \;=\; 70\cos 45^\circ - 20\cos 30^\circ$

$\sum Fx \;=\; 32.18N$ (Towards to the right)

Forces in Y – direction

$\sum Fy \;=\; 100 + 70\sin 45^\circ + 20\sin 30^\circ - 60^\circ$

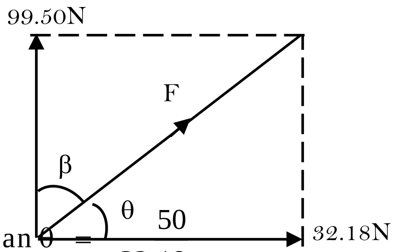
$\sum Fy \;=\; 99.50N$ (Vertically upward)

Resultant force

$$F \;=\; \sqrt{(32.18)^2 + (99.50)^2}$$

$$F \;=\; 104.57N$$

Direction of force

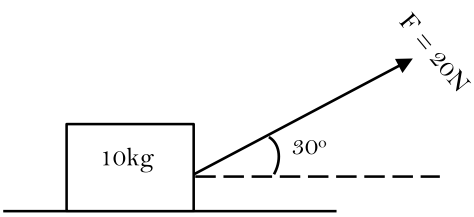


$\tan\theta = \dfrac{50}{32.18}$

$\theta \;=\; 72^\circ$ (approx.)

Example – 02

The figure given below shows a block of mass 10kg acted upon by a force F = 20N, inclined to the horizontal at an angle 30°. find the acceleration of the block if it moves in the horizontal direction



Solution

The horizontal component of the force

$Fx \;=\; F\cos 30^\circ \;=\; 20\cos 30^\circ$

$Fx \;=\; 8.66N$

Acceleration of the block

$$a \;=\; \frac{Fx}{M} \;=\; \frac{8.66}{10}$$

$$a \;=\; 0.866m/s^2$$

Example – 03

Two forces of magnitude $3\sqrt{2}$ and $5\sqrt{2}N$ acting on a block of mass 1, 500kg at an angle of 60° to each other. Calculate the acceleration, distance covered and velocity of the block after 20seconds.

Solution

Resultant force on the block

By using parallelogram law

$$F \;=\; \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(3\sqrt{2})^2 + (5\sqrt{2})^2 + 2\times 3\sqrt{2}\times 5\sqrt{2}\cos 60^\circ}$$

$$F \;=\; 9.89N$$

Acceleration

$$a \;=\; \frac{F}{M} \;=\; \frac{9.89}{1500}$$

$$a \;=\; 0.0066m/s^2$$

Velocity after 20s

$V \;=\; u + at \;=\; 0 + 0.0066\times 20$

$V \;=\; 0.132m/s$

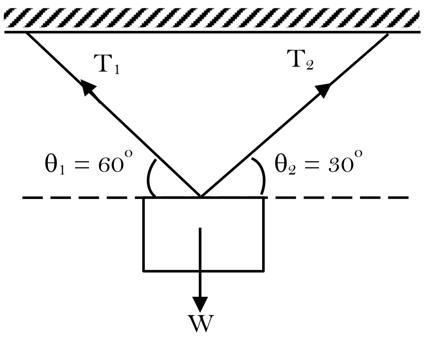
Distance covered

$$S \;=\; ut + \frac{1}{2}at^2$$

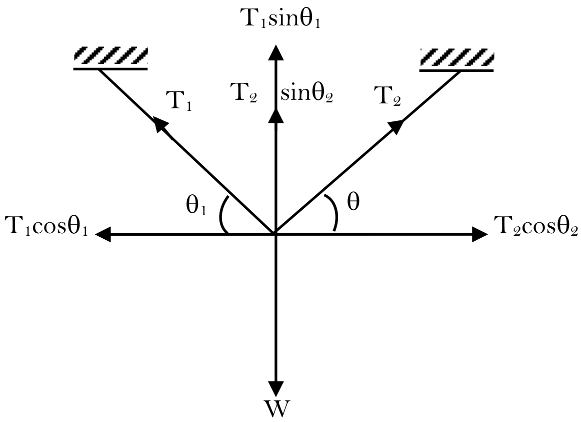
$$= 0 + \frac{1}{2}\times 0.0066\times (20)^2$$

$$S \;=\; 1.32m$$

Example – 04
A box weighing 8.0N is supported by two wires with tension T_1 and T_2. Find the tension on each wire



Solution
Consider the FBD as shows below.



Net force along the x – axis

$$\sum Fx = T_2 \cos\theta_2 - T_1 \cos\theta_1$$

Since the box does not accelerate, $a_x = 0$

$$T_1 \cos\theta_1 = T_2 \cos\theta_2$$

$$T_1 = T_2 \frac{\cos\theta_2}{\cos\theta_1}$$

$$T_1 = T_2 \frac{\cos 30^\circ}{\cos 60^\circ}$$

$$T_1 = 1.732 T_2 \ldots\ldots\ldots\ldots \text{(i)}$$

Net force along y – axis

$$\sum Fy = T_2 \sin\theta_2 + T_1 \sin\theta_1 - W$$

But $a_y = 0$

$$W = T_2 \sin\theta_2 + T_1 \sin\theta_1$$

$$8 = T_2 \sin 30^\circ + T_1 \sin 60^\circ$$

$$8 = 0.5T_2 + 0.866T_1 \ldots\ldots (2)$$

On solving simultaneously equation (1) and (2)

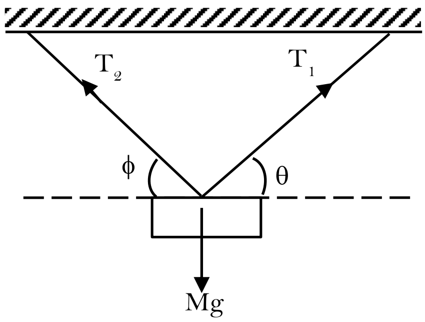
$$T_1 = 6.9\text{N}$$

$$T_2 = 4.0\text{N}$$

Example – 05
A body of mass M is suspended by two strings making angle θ and ϕ with the horizontal as shown in the figure below show that

$$T_1 = \frac{Mg\cos\phi}{\sin(\theta+\phi)}$$

$$T_2 = \frac{Mg\cos\theta}{\sin(\theta+\phi)}$$



Solution
At the equilibrium
Forces in X – direction

$$T_1\cos\theta - T_2\cos\phi = 0$$

$$T_1\cos\theta = T_2\cos\phi$$

Forces in Y – direction

$$T_1\sin\theta + T_2\sin\phi = Mg$$

But $T_2 = T_1 \dfrac{\cos\theta}{\cos\phi}$

Now

$$T_1\sin\theta + T_1\frac{\cos\theta\sin\phi}{\cos\phi} = Mg$$

$$T_1\left[\sin\theta\cos\phi + \cos\theta\sin\phi\right] = Mg\cos\phi$$

$$T_1\sin(\theta+\phi) = Mg\cos\phi$$

$$T_1 = \frac{Mg\cos\phi}{\sin(\theta+\phi)}$$

Also

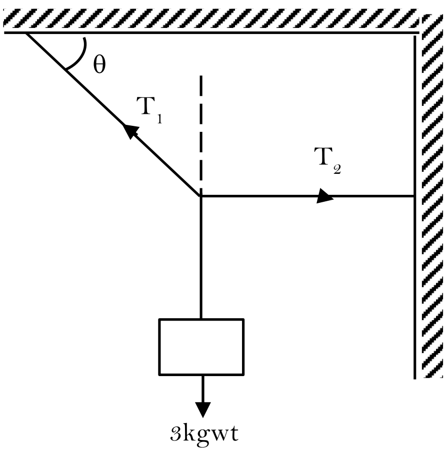
$$T_2 = T_1 \frac{\cos\theta}{\cos\phi}$$

$$T_2 = \frac{\cos\theta}{\cos\phi}\left[\frac{Mg\cos\phi}{\sin(\theta+\phi)}\right]$$

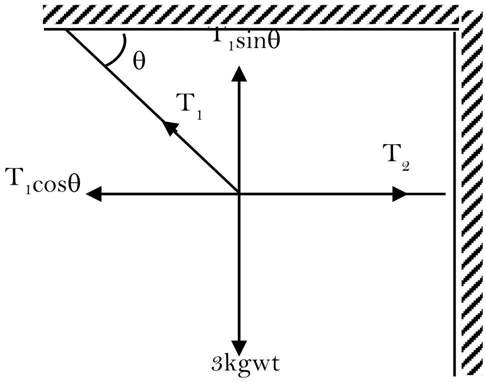
$$T_2 = \frac{Mg\cos\theta}{\sin(\theta+\phi)}$$

Example – 06

In the figure given below , tension T_1 = 45N. Determine the angle θ and magnitude of tension T_2.



Solution



As the body is in equilibrium

$$T_1 \sin\theta = 3kgwt = 3\times 9.8N$$

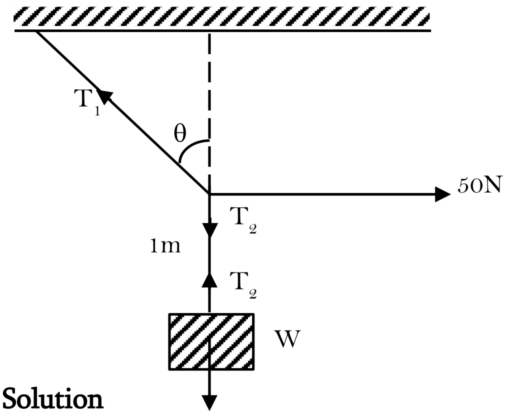
$$\sin\theta = \frac{3\times 9.8}{T_1} = \frac{3\times 9.8}{45}$$

$$\sin\theta = 0.653$$

$$\theta = 40.8^{\circ}$$

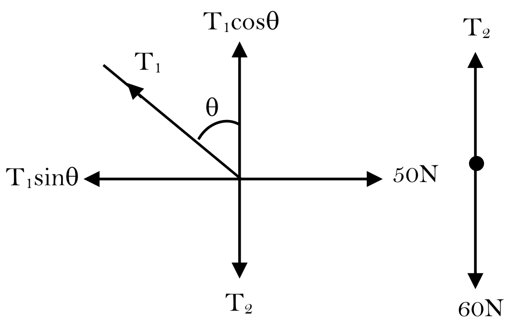
Example – 07

A mass of 6kg is suspended by a rope of length 2m from a ceiling. A force of 50N in horizontal direction is applied to the mid – point of the rope as shown. What is the angle the rope makes with vertical in equilibrium.



Solution

Consider the shown below



At the equilibrium

$$T_1 \sin\theta = 50$$

$$T_1 \cos\theta = T_2 = 60$$

Takes

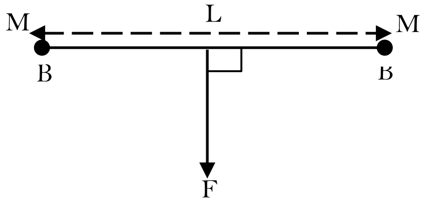
$$\frac{T_1 \sin\theta}{T_1\cos\theta} = \frac{50}{60}$$

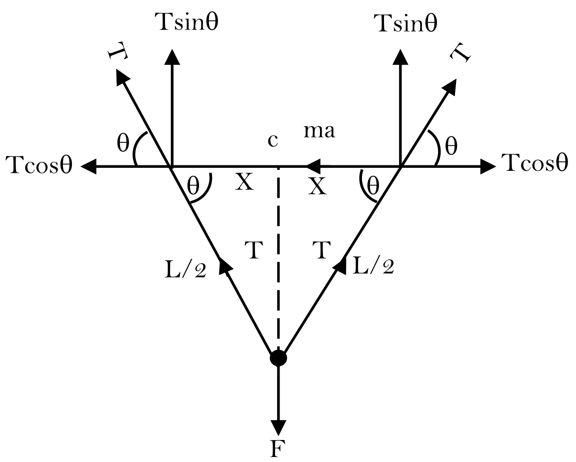
$$\tan\theta = \frac{5}{6} \text{ or } \theta = 39.8^{\circ}$$

Example – 08

To the two ends of the light string of length L two equal masses in each are attached as shown in the figure below. At the mid – point of the string, a constant force F is applied at right angles to the initial position of the string. Prove that the acceleration of mass in the direction at right to F is

$$a_x = \frac{Fx}{2M \cdot \sqrt{\frac{L^2}{4} - X^2}}$$





At the equilibrium

$$T\sin\theta + T\sin\theta = F$$

$$F = 2T\sin\theta \ldots\ldots \text{(i)}$$

For the motion of either of two particles.

$$Ma_x = T\cos\theta$$

[Dividing equation (1) by (2)]

$$\frac{F}{Ma_x} = \frac{2T\sin\theta}{T\cos\theta}$$

$$\frac{F}{Ma_x} = 2\tan\theta$$

$$\tan\theta = \frac{\overline{OC}}{\overline{AC}} = \frac{\sqrt{L^2/4 - X^2}}{X}$$

$$\frac{F}{Ma_x} = 2\sqrt{L^2/4 - X^2}\Big/X$$

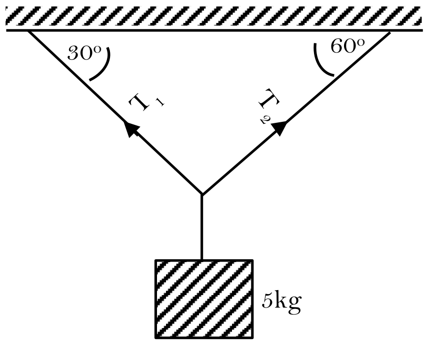
$$a_x = \frac{Fx}{2M \cdot \sqrt{\frac{L^2}{4} - X^2}}$$

Example – 09

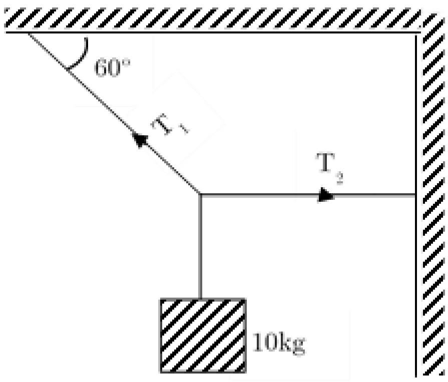
Find the tension in each cord of for the system described in the figure below.

(neglecting the mass of the cord)

(a)



(b)



Ans. (a) $T_1 = 25N$, $T_2 = 43.30N$

(b) $T_1 = 115N.49N$, $T_2 = 57.735N$

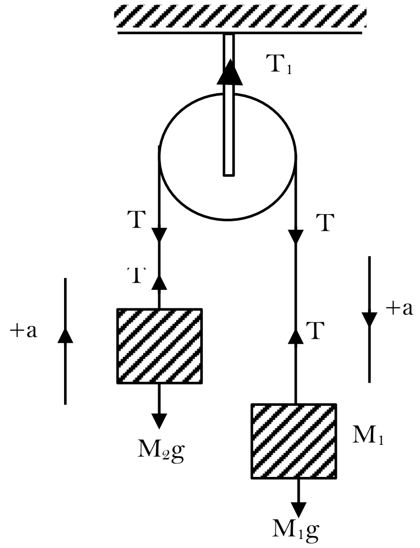
SYSTEM OF CONNECTED BODIES

I. SMOOTH SURFACE

This is the surface which have no frictional force. Different cases for the system of the connected bodies.

CASE 1: PULLEY SYSTEM

Two blocks of masses M_1 and M_2 are connected to the two ends of a string and the string is passing over light pulley. This system is known as Atwood machine



We have to find the accelerations of blocks and tensions developed in the string. Assumptions made: -

(i) The string is light inextensible string and massless.

(ii) The pulley is the frictionless and massless.

(iii) Assume that $M_1 > M_2$ resultant forces on each block.

M_1: $M_1g - T = M_1a$.............. (i)

M_2: $T - M_2g = M_2a$............(ii)

Add equation (i) and (ii)

$$(M_1 + M_2)a = g(M_1 - M_2)$$

$$a = g\frac{(M_1 - M_2)}{M_1 + M_2}$$

Expression of the tension on the string

$$T - M_2g = M_2a$$

$$T = M_2g + M_2a$$

$$= M_2g + M_2g\frac{(M_1 - M_2)}{M_1 + M_2}$$

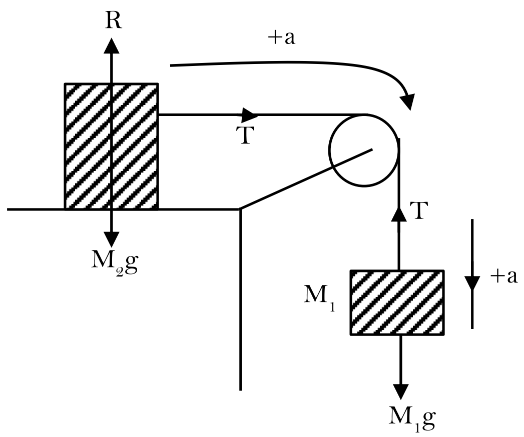
$$T = \frac{2M_1M_2g}{M_1 + M_2}$$

Expression of T_1

$$T_1 = 2T = \frac{4M_1M_2g}{M_1 + M_2}$$

CASE 2:

Consider the system of connected bodies as shown on the figure below



Resultant forces on each block.

M_1: $M_1g - T = M_2a$ ....(i)

M_2: $T = M_2a$ ....(ii)

Adding equation (i) and (ii)

$$M_1g = (M_1 + M_2)a$$

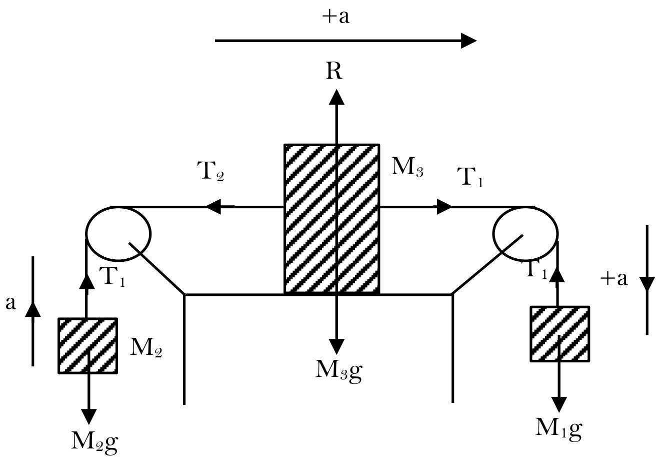
$$a = \frac{M_1g}{M_1 + M_2}$$

Expression of the tension, T

$$T = M_2a = \frac{M_1M_2g}{M_1 + M_2}$$

CASE 3:

Consider the system of the connected bodies as shown on the figure below



Assume that $M_1 > M_2$, then the resultant force on each block

M_1: $\quad M_1g - T_1 = M_1a$ ....(i)

M_2: $\quad T_2 - M_2g = M_2a$ ....(ii)

M_3: $\quad T_1 - T_2 = M_3a$ ....(iii)

Adding equation (i), (ii) and (iii)

$$(M_1 + M_2 + M_3)a = (M_1 - M_2)g$$

$$a = g\frac{(M_1 - M_2)}{M_1 + M_2 + M_3}$$

Expression of T_1

$$M_1g - T_1 = M_1a$$

$$T_1 = M_1g - \frac{M_1g(M_1 - M_2)}{M_1 + M_2 + M_3}$$

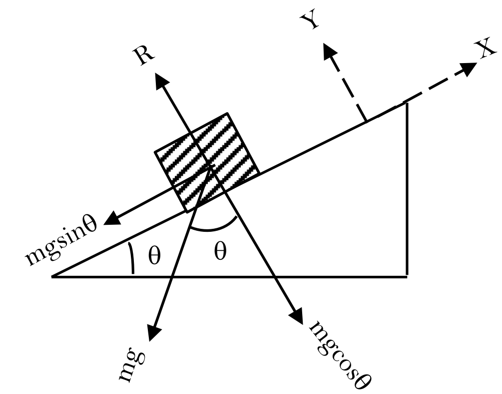
$$T_1 = \frac{M_1g(2M_2 + M_3)}{M_1 + M_2 + M_3}$$

Expression of T_2

$$T_2 - M_2g = M_2a$$

$$T_2 = M_2g + \frac{M_2g(M_1 - M_2)}{M_1 + M_2 + M_3}$$

FORCES ACTING ON AN OBJECT LIES ON SMOOTH INCLINED PLANE



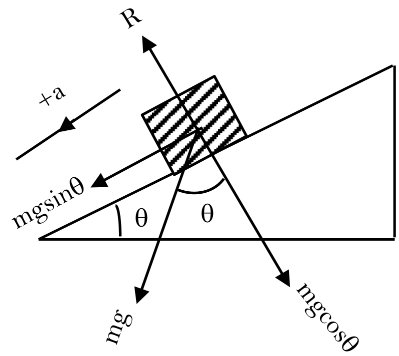
The available forces acting on an object lies on the smooth inclined plane:

(i) Normal reaction force, R

(ii) Weight of an object

CASE 4:

Expression of an acceleration for body moving along the smooth inclined plane.



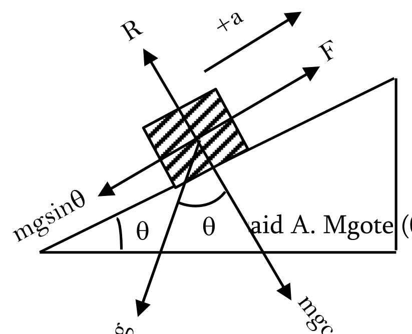
Resultant force on the block

$$Ma = Mg\sin\theta$$

$$a = g\sin\theta$$

CASE 5:

If the block moves up to the top of smooth inclined plane under the action of applied force, F



Resultant force on the block

$$F - Mg\sin\theta = Ma$$

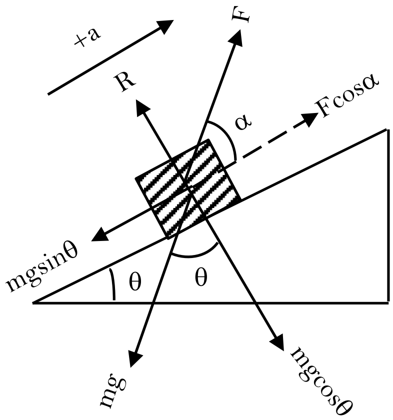
$$a = \frac{F - Mg\sin\theta}{M}$$

Magnitude of force

$$F = M\left(a + g\sin\theta\right)$$

CASE 6:

Consider the figure below



Resultant force on the block

$$Ma = F\cos\alpha - Mg\sin\theta$$

$$a = \frac{F\cos\alpha - Mg\sin\theta}{M}$$

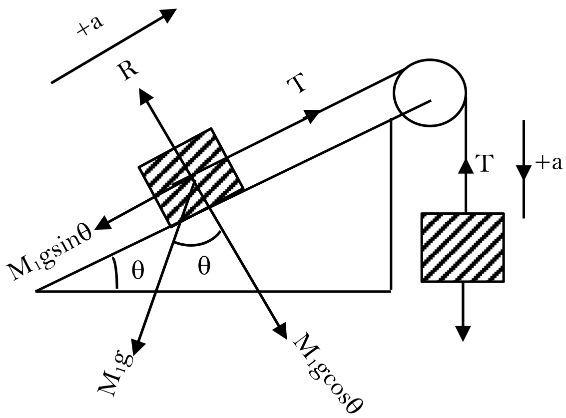
$$a = \frac{F\cos\alpha - Mg\sin\theta}{M}$$

Expression of applied force, F

$$F = \frac{M\left(a + g\sin\theta\right)}{\cos\alpha}$$

CASE 7:

Consider the system of connected bodies as shown on the figure



Resultant force on each block

$$M_1: \; T - M_1 g\sin\theta = M_1 a \;\ldots\ldots.(i)$$

$$M_2: \; M_2 g - T = M_2 a \;\ldots\ldots(ii)$$

Expression of the acceleration, a

Adding equation (i) and (ii)

$$\left(M_1 + M_2\right)a = g\left(M_2 - M_1\sin\theta\right)$$

$$a = g\frac{\left(M_2 - M_1\sin\theta\right)}{M_1 + M_2}$$

Expression of the tension on the string

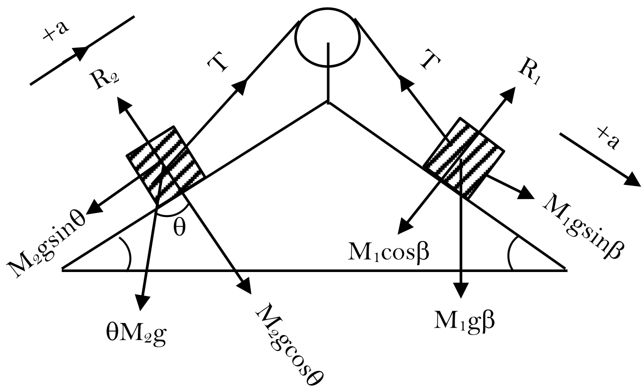
$$T = M_1 g\sin\theta + M_1 a$$

$$= M_1 g\sin\theta + \frac{M_1 g\left(M_2 - M_1\sin\theta\right)}{M_1 + M_2}$$

$$T = \frac{M_1 M_2 g\left(1 + \sin\theta\right)}{M_1 + M_2}$$

Case 8:

Consider the system of connected bodies as shown on the figure below



Resultant forces on each block

$$M_1: \; M_1 g\sin\beta - T = M_1 a \;\ldots..(i)$$

$$M_2: \; T - M_2 g\sin\theta = M_2 a \;\ldots..(ii)$$

Adding equation (i) and (ii)

$$\left(M_1 + M_2\right)a = g\left(M_1\sin\beta - M_2\sin\theta\right)$$

$$a = g\frac{\left(M_1\sin\beta - M_2\sin\theta\right)}{M_1 + M_2}$$

Expression of the tension on the string

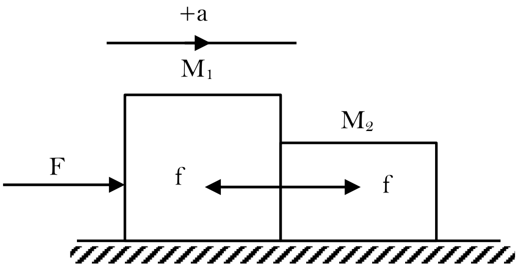
$$T - M_2 g\sin\theta = M_2 a$$

$$T = M_2 g \sin\theta + M_2 a$$

$$T = \frac{M_1 M_2 g(\sin\theta + \sin\beta)}{M_1 + M_2}$$

CASE 9: TWO BODIES INCONTACT

(a) Consider two blocks of masses M_1 and M_2 are in contact and applied force F acting on the bock of mass M_1.



Let F be force in contact between M_1 and M_2.

Resultant forces on the block of mass.

M_1: $F - F = M_1 a$ ....(i)

M_2: $F = M_2 a$ ....(ii)

Adding equation (i) and (ii)

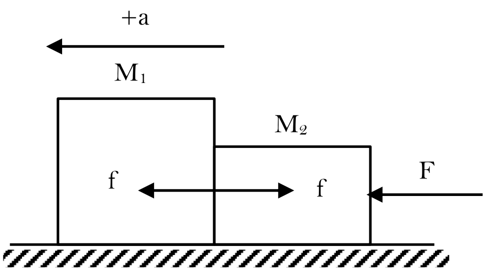
$$F = (M_1 + M_2)a$$

$$a = \frac{F}{M_1 + M_2}$$

Expression of force, F

$$F = M_2 a = \frac{M_2 F}{M_1 + M_2}$$

(b) If the applied force, F acting on the block of mass M_2.



Acceleration of the system

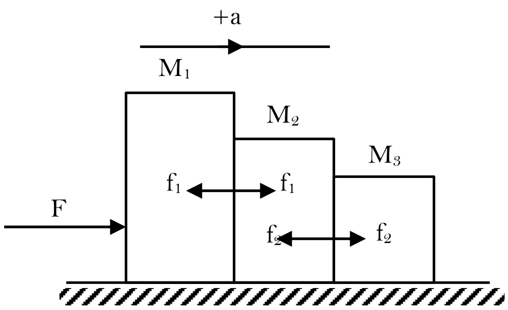
$$a = \frac{F}{M_1 + M_2}$$

Expression of force in contact

$$F = \frac{M_1 F}{M_1 + M_2}$$

CASE 10: THREE BODIES IN CONTACT

Consider three blocks of masses M_1, M_2 and M_3 placed on the contact as shown on the figure below.



Resultant forces on each block

M_1: $F - F_1 = M_1 a$ ......(i)

M_2: $F_1 - F_2 = M_2 a$ ......(ii)

M_3: $F_2 = M_3 a$ ......(iii)

Adding equation i), (ii) and (iii)

$$F = (M_1 + M_2 + M_3)a$$

$$a = \frac{F}{M_1 + M_2 + M_3}$$

Expression of the force, F_1

$$F - F_1 = M_1 a = \frac{FM_1}{M_1 + M_2 + M_3}$$

$$F_1 = F\left[\frac{M_2 + M_3}{M_1 + M_2 + M_3}\right]$$

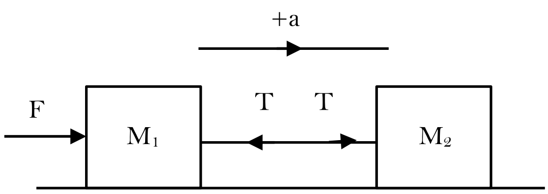
Expression of the force, F_2

$$F_2 = M_2 = \frac{M_3 F}{M_1 + M_2 + M_3}$$

$$F_2 = \frac{M_3 F}{M_1 + M_2 + M_3}$$

CASE 11: A TYPICAL PROBLEM OF TENSION IN THE STRING CONNECTING TWO BODIES.

(a) Consider two bodies of masses M_1 and M_2 connected by a string and placed over a smooth horizontal surface.



Resultant forces on the blocks

M_1: $F - T = M_1 a$

M_2: $T = M_2 a$

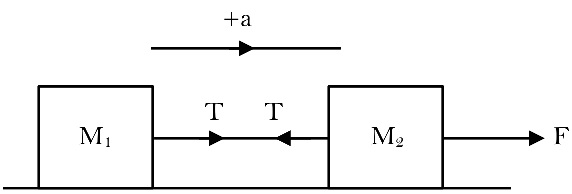
$$F = (M_1 + M_2)a$$

$$a = \frac{F}{M_1 + M_2}$$

Expression of the tension on the tow bar

$$T = M_2 a = \left(\frac{M_2}{M_1 + M_2}\right)F$$

(b) If the force F is applied on M_2



Acceleration of the system

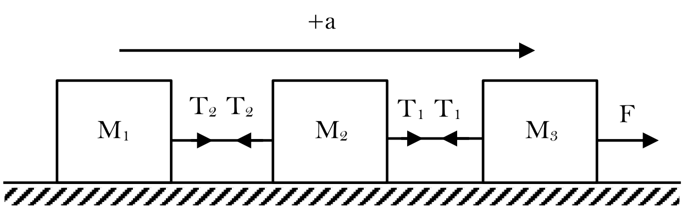
$$a = \frac{F}{M_1 + M_2}$$

Tension on the string

$$T = M_1 a$$

$$T = \left(\frac{M_1}{M_1 + M_2}\right)F$$

THREE BODIES CONNECTED BY TWO STRINGS.



Resultant forces on each block

M_1: $T_2 = M_1 a$ ……(i)

M_2: $T_1 - T_2 = M_2 a$

M_3: $F - T_1 = M_3 a$ …….(iii)

$$F = (M_1 + M_2 + M_3)a$$

$$a = \frac{F}{M_1 + M_2 + M_3}$$

Expression of the tension T_1

$$F - T_1 = M_3 a$$

$$T_1 = F - M_3 a$$

$$= F - \frac{M_3 F}{M_1 + M_2 + M_3}$$

$$T_1 = F\left[\frac{M_2 + M_3}{M_1 + M_2 + M_3}\right]$$

Expression of the tension, T_2

$$T_2 = M_1 a$$

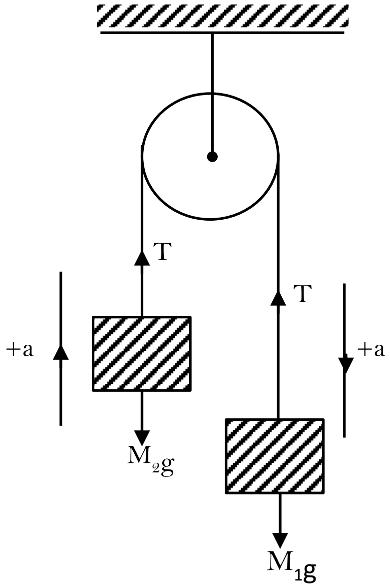
$$T_2 = \left(\frac{M_1}{M_1 + M_2 + M_3}\right)F$$

SOLVED EXAMPLES

Example – 10

Two masses 340g and 300gm are connected at the two ends of a light inextensible string that passes over frictionless pulley. What is the distance travelled by the masses in 6seconds, starting from the rest?

Solution



Resultant forces on the block

M1: $M_1g - T = M_1a$

M2: $(M_1 + M_2)a = (M_1 - M_2)g$

$$a = \frac{g(M_1 - M_2)}{M_1 + M_2} = \frac{9.8(350 - 300)}{350 + 300}$$

$$a = 0.6125\text{m}/\text{s}^2$$

Since $S = ut + \frac{1}{2}at^2 \quad , \quad u = 0$

$$S = \frac{1}{2}at^2 = \frac{1}{2} \times 0.6125(6)^2$$

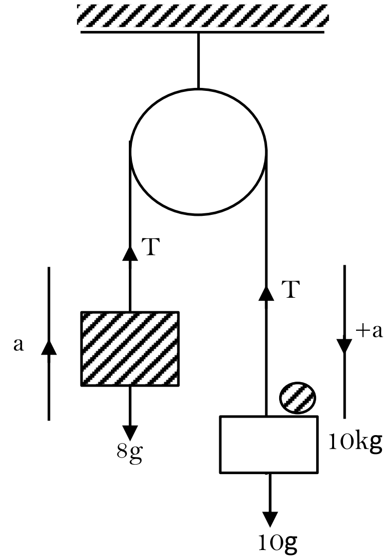
$$S = 11.025\text{m}$$

Example – 11

The two blocks of each of mass 8.0kg are suspended at opposite ends of a soft non – elastic cord passing over a frictionless and stationary pulley. A body of mass 2.0kg is then carefully placed on one of the blocks. Calculate the distance moved by each of the block after 2.0seconds.

Solution

After adding 2kg of one of block.



Resultant force on the

8kg: $T - 8g = 8a$ ....(i)

10kg: $10g - T = 10a$ ....(ii)

Adding equation (i) and (ii)

Distance moved by each block

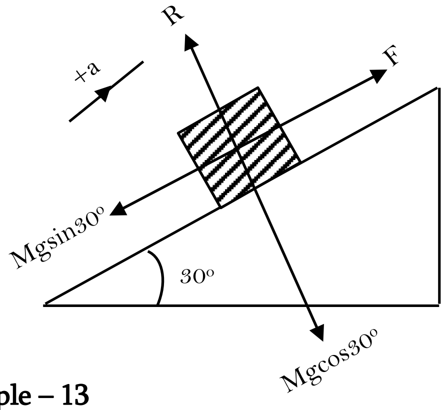
$$S = ut + \frac{1}{2}at^2 \quad , u = 0$$

$$S = \frac{1}{2} \times 1.089 \times 2^2$$

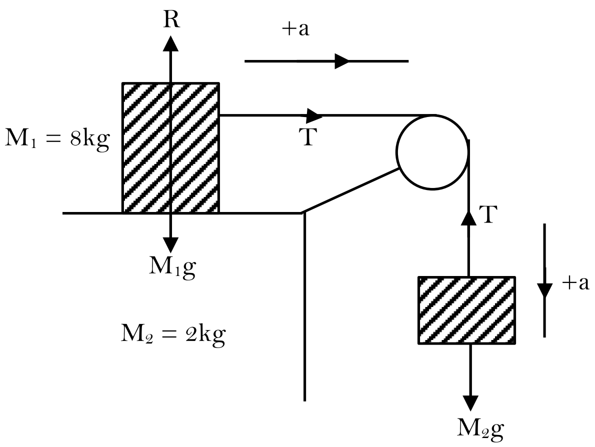
$$S = 2.18\text{m}$$

Example – 12
What force is required to push a 200N body up a 30^0 smooth inclined plane with an acceleration of $2m/s^2$. The force is to applied along the plane ($g = 10m/s^2$)
Solution
Resultant force on the block

$$F - Mg\sin\theta = Ma$$
$$F = M(a + g\sin\theta)$$
$$F = 20(2 + 10\sin 30^\circ)$$
$$F = 140N$$



Example – 13
A body of mass 8kg placed on a smooth horizontal table is connected by a light string passing over a pulley to hanging body of mass 2kg. find the acceleration of the masses and the tension in the string ($g = 10m/s^2$)
Solution



Resultant forces on

M1: $T = M_1 a$ ....(1)

M2: $M_2 g - T = M_2 a$ ....(2)

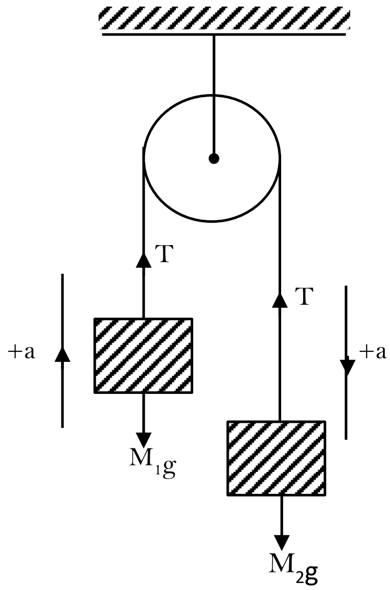
Adding equation (1) and (2)

$$M_2 g = (M_1 + M_2)a$$
$$a = \frac{M_2 g}{M_1 + M_2} = \frac{2\times 10}{2+8}$$
$$a = 2m/s^2$$

Tension on the string

$$T = M_1 a = 8\times 2$$
$$T = 16N$$

Example – 14
Masses of 50kg and 40g are connected by a string passing over a smooth pulley. If the system travels 2.18m in the first two seconds, find the value of acceleration due to gravity
Solution
Let $M_1 = 50g$
$M_2 = 40g$
$S = 2.18m$, $t = 2sec$
$g = ?$



resultant forces

M1: $T - M_1 g = M_1 a$

M2: $M_2 g - T = M_2 a$

$$(M_2 - M_1)g = (M_1 + M_2)a$$

Since $S = ut + \frac{1}{2}at^2$, $u = 0$

$$S = \frac{1}{2}gt^2\frac{(M_2 - M_1)}{M_1 + M_2}$$

$$g = \frac{2s(M_1 + M_2)}{t^2(M_2 - M_1)}$$

$$= \frac{2 \times 2.18(50 + 40)}{2^2(50 - 40)}$$

$$g = 9.81 m/s^2$$

Example – 15 NECTA 1984/P1/3

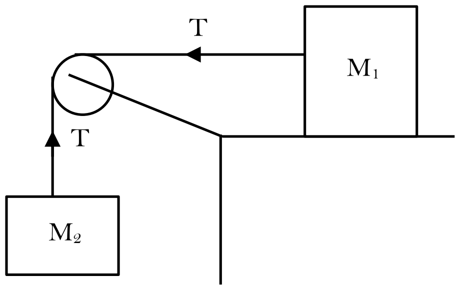
(a) State the Newton's second law of motion and show that it contains Newton's first law of motion.

(b) A block of mass M_1 rest on a smooth horizontal surface another block of mass M_2 is attached to the first by a string such that its hangs over pulley as shown on the figure below. Find an expression for:

(i) The acceleration of the system

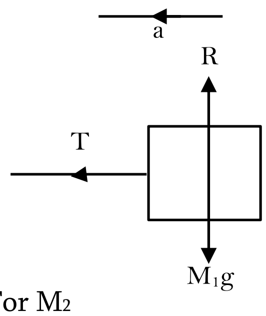
(ii) The tension in the string assuming the pulley to be massless and frictionless and that $M_2 > M_1$ when the acceleration of the system is equal to that of gravity?

Solution



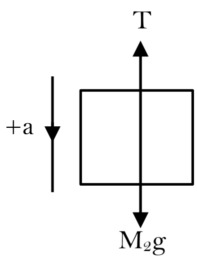
(a) Refer to your notes

(b) (i) consider FBD for M_1



$T = M_1a$……. (1)

For M_2



$M_2 - T = M_2a$…. (2)

$$(M_1 + M_2)a = M_2g$$

$$a = \frac{M_2g}{M_1 + M_2}$$

(ii) Expression of tension, T

$$T = M_1a$$

$$T = \frac{M_1M_2g}{M_1 + M_2}$$

For the case, $a = g$

$$g = \frac{M_2g}{M_1 + M_2}$$

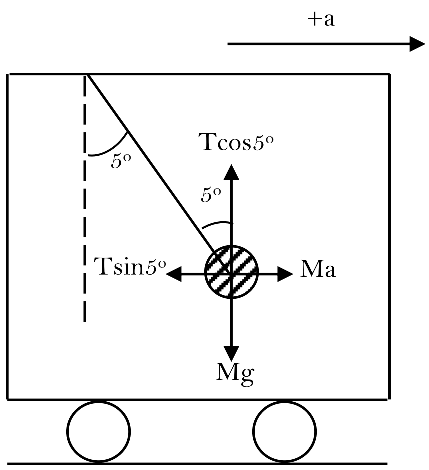
$$1 = \frac{M_2}{M_1 + M_2}$$

In the absence of M_1 (i.e. $M_1 = 0$) then $a = g$.

Example – 16

A train is moving along a straight horizontal track. A pendulum suspended from the roof makes an angle of 5° with the vertical. Find the acceleration of the train (take $g = 10m/s^2$).

Solution



At the equilibrium

$T \sin 5^\circ = Ma$ ………. (1)

$T \cos 5^\circ = Mg$………. (2)

Dividing equation (1) and (2)

$$\frac{T \sin 5^\circ}{T \cos 5^\circ} = \frac{Ma}{Mg} = \frac{a}{g}$$

$$\tan 5^\circ = \frac{a}{g}$$

$$a = g\tan 5^\circ = 10\tan 5^\circ$$

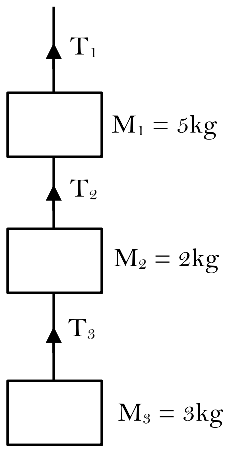
$$a = 0.875 m/s^2$$

Example – 17

The masses M_1, M_2 and M_3 of the three bodies shown in the figure below are 5kg, 2kg and 3kg respectively. Calculate the values of tension T_1, T_2 and T_3 when.

(a) The whole system is going upward with an acceleration of $2m/s^2$

(b) The wholes system is stationary ($g = 9.8m/s^2$)



Solution

(a) In this case, all the bodies are moving upward together.

$$T_1 - (M_1 + M_2 + M_3)g = (M_1 + M_2 + M_3)a$$

$$T_1 = (M_1 + M_2 + M_3)(a + g)$$

$$= (5 + 2 + 3)(2 + 9.8)$$

$$T_1 = 118N$$

Again

$$T_2 - (M_2 + M_3)g = (M_2 + M_3)a$$

$$T_2 = (M_2 + M_3)(a + g)$$

$$= (2 + 3)(2 + 9.8)$$

$$T_2 = 59N$$

The net upward force on M_3

$$T_3 - M_3 g = M_3 a$$

$$T_3 = M_3(a + g)$$

$$= 3(2 + 9.8)$$

$$T_3 = 35.4N$$

(b) When the whole system is stationary, a = 0

$$T_1 = (M_1 + M_2 + M_3)g$$

$$= (5 + 2 + 3) \times (9.8)$$

$$T_1 = 98N$$

Also

$$T_2 = (M_2 + M_3)g$$

$$= (2 + 3) \times 9.8$$

$$T_2 = 49N$$

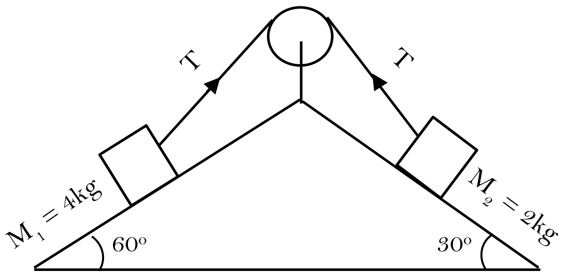
Again

$$T_3 = M_3 g = 3 \times 9.8$$

$$T_3 = 29.4N$$

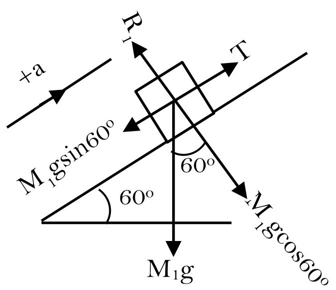
Example – 18

What is the magnitude of the acceleration of the system and tension on the string as shown in the figure below?

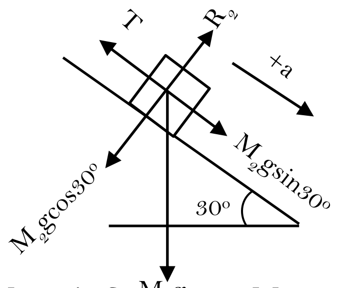


Solution

Consider the FBD on M_1



Resultant force on M_1



$M_1 g \sin 6 \quad = M_1 a \ldots\ldots (i)$

Resultant force on M_2

$T - M_2 g \sin 30^\circ \quad = \quad M_2 a$

Adding equation (i) and (ii)

$$g\left(M_1 \sin 60^\circ - M_2 \sin 30^\circ\right) \quad = \quad \left(M_1 + M_2\right)a$$

$$a \quad = \quad \frac{g\left(M_1 \sin 60^\circ - M_2 \sin 30^\circ\right)}{M_1 + M_2}$$

$$= \quad \frac{9.8\left(4 \sin 60^\circ - 2 \sin 30^\circ\right)}{4+2}$$

$$a \quad = \quad 4.02 m / s^2$$

Tension on the string

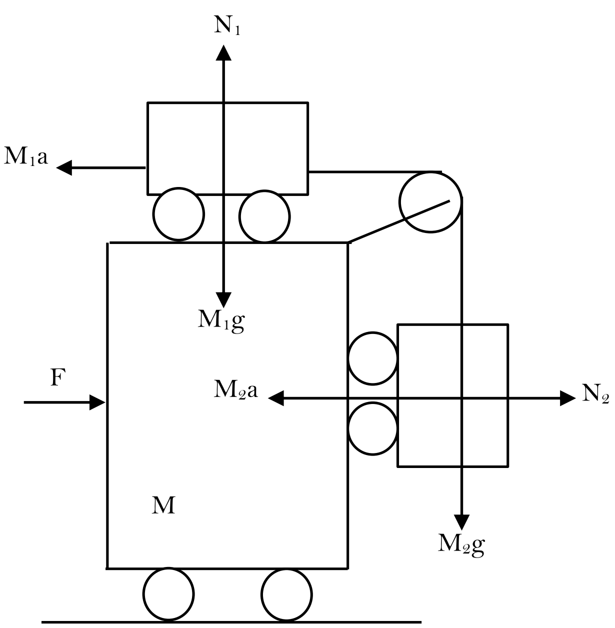
$$T \quad = \quad M_2\left(a + g \sin 30^\circ\right)$$

$$= \quad 2\left(4.02 + 9.8 \sin 30^\circ\right)$$

$$T \quad = \quad 17.8N$$

Example – 19

A frictionless cart of mass M carries two other frictionless carts having masses M_1 and M_2 connected by a string passing over a pulley as shown in the figure below. What is the horizontal force, F must be applied on M so that M_1 and M_2 do not move relative to its?



Solution

Since M_1 and M_2 are in accelerating frame we can assume that inertial force M_1a and M_2a acts on the system respectively. Let a be acceleration of the system.

$$F \quad = \quad \left(M_1 + M_2 + M\right)a$$

$$a \quad = \quad \frac{F}{M + M_1 + M_2} \ldots . (1)$$

Resultant forces on

M_1: $T \quad = \quad M_2 g$

M_2: $T \quad = \quad M_2 g$

$$M_1 a \quad = \quad M_2 g$$

$$a \quad = \quad \frac{M_2}{M_1} g \ldots\ldots (2)$$

$(1) = (2)$

$$\frac{F}{M + M_1 + M_2} \quad = \quad \frac{M_2 g}{M_1}$$

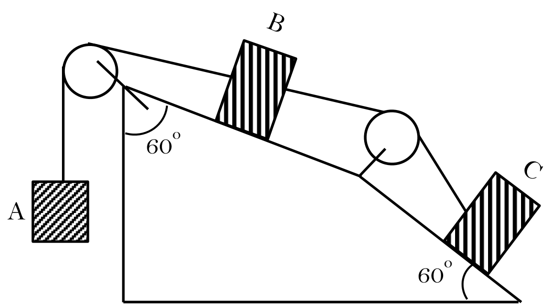
$$F \quad = \quad \left(M + M_1 + M_2\right)\frac{M_2 g}{M_1}$$

Example – 20

Three blocks A, B and C of masses 10kg, 6kg and 4kg respectively are connected by the light inextensible string over a frictionless pulley as shown in the figure below

Find

(i) The direction of the system
(ii) The acceleration of the system
(iii) The tension in the string



Solution

(i) Let M_1, M_2 and M_3 be the masses of blocks A, B and C respectively.

$$M_1 g \quad = \quad 10 \times 9.8 \quad = \quad 98N$$

$M_2 g\sin 30^\circ + M_3 g\sin 60^\circ$

$= 6\times 9.8\sin 30^\circ + 4\times 9.8\sin 60^\circ$

$= 63.3N$

Since $M_1 g > M_2 g\sin 30^\circ + M_3 g\sin 60^\circ$

The motion of the system in anticlockwise.

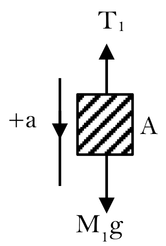
Consider FBD on M1 (Block, A)

Resultant force

$$M_1 a = M_1 g - T$$
$$10a = 98 - T \ldots\ldots (1)$$

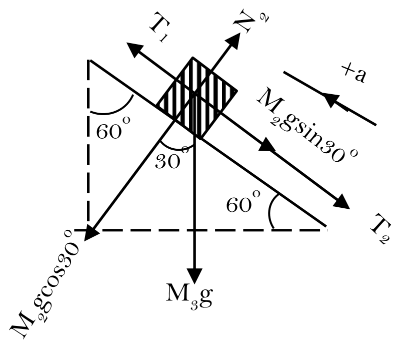
(ii) Consider FBD on M_1 (Block, A)

Resultant force



$$M_1 a = M_1 g - T$$
$$10a = 98 - T \ldots\ldots (1)$$

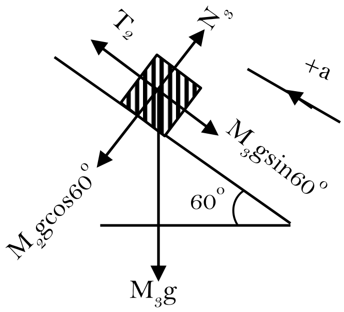
FBD for M_2 i.e. block B



Resultant force on M_2

$$T_1 - T_2 - M_2 g\sin 30^\circ = M_2 a$$
$$T_1 - T_2 - 29.4 = 6a \ldots\ldots\ldots\ldots (2)$$

For M_3 i.e. block C



Resultant force on M_3

$$T_2 - M_3 g\sin 60^\circ = M_3 a$$
$$T_2 - 33.95 = 4a \ldots\ldots\ldots\ldots (3)$$

Adding equation (1), (2) and (3)

$$10a + 6a + 4a = 98 - 29.4 - 33.95$$
$$20a = 34.65$$
$$a = 1.73 m/s^2$$

(iii) $T_1 = M_1\left(g - a\right)$

$= 10\left(9.8 - 1.73\right)$

$T_1 = 80.7N$

Also

$T_2 = M_3\left(a + g\sin 60^\circ\right)$

$= 4\left(1.73 + 9.8\sin 60^\circ\right)$

$T_2 = 40.9N$

Example – 21

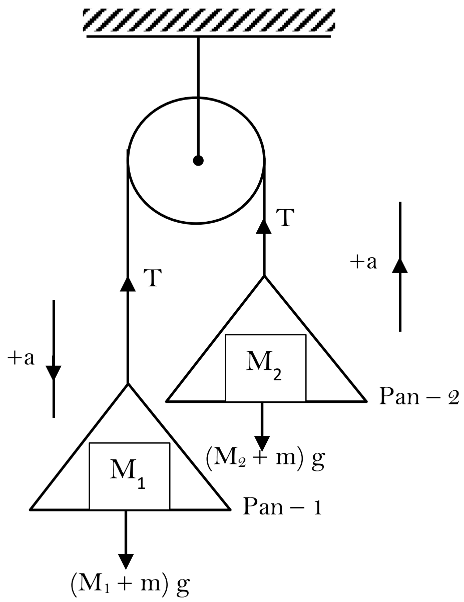
Two scale pans, each of mass M and connected by a light string passing over a small pulleys and in them are placed masses M_1 and M_2. Show that the reactions of the scale pans during the motions are

$$R_1 = \frac{2M_1\left(M + M_2\right)g}{M_1 + M_2 + 2M}$$

$$R_2 = \frac{2M_2\left(M + M_1\right)g}{M_1 + M_2 2M}$$

Solution

Assume that $M_1 > M_2$



Resultant forces on the

$$\text{Pan}-1:\ \left(M+M_1\right)g-T = \left(M+M_1\right)a$$

$$\text{Pan}-2:\ T-\left(M+M_2\right)g = \left(M+M_2\right)a$$

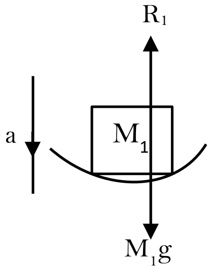
Adding equation (i) and (ii)

$$\left(M_1-M_2\right)g = \left(2M+M_1+M_2\right)a$$

$$a = \frac{\left(M_1-M_2\right)g}{2M+M_1+M_2}$$

Let R_1 and R_2 be the reaction forces on the scale pan – 1 and pan – 2 respectively

- For the scale pan – 1



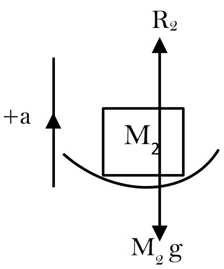
$$M_1g-R_1 = M_1a$$

$$R_1 = M_1g-M_1a$$

$$= M_1g-\frac{M_1g\left(M_1-M_2\right)}{2M+M_1+M_2}$$

$$R_1 = \frac{2M_1\left(M+M_2\right)g}{2M+M_1+M_2}$$

- For the scale pan – 2



$$R_2-M_2g = M_2a$$

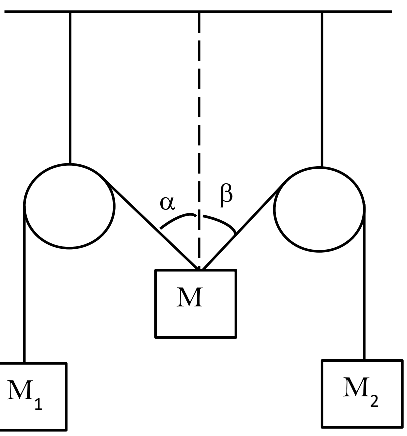
$$R_2 = M_2g+M_2a$$

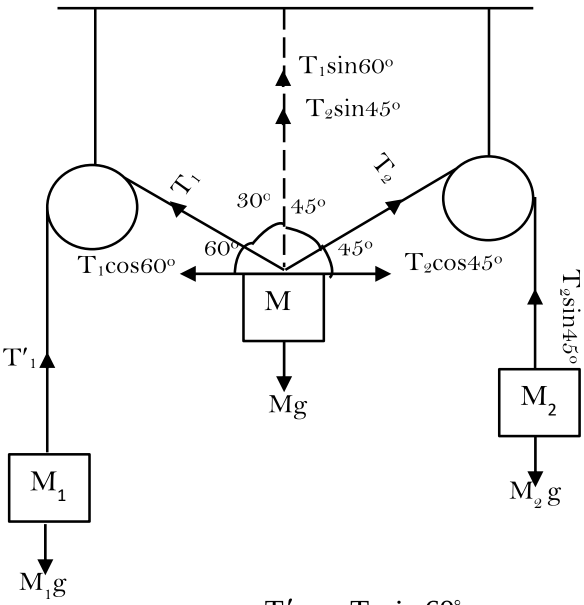
$$R_2 = M_2g+\frac{M_2g\left(M_1-M_2\right)}{2M+M_1+M_2}$$

$$R_2 = \frac{2M_2\left(M+M_1\right)g}{2M+M_1+M_2}$$

Example – 22

A mass M = 6kg hangs by two strings making angles α and β to the vertical where α = 30° and β = 45°. The strings are connected through the pulleys to two masses M_1 and M_2 as shown on the figure below. Calculate the masses M_1 and M_2 such that the mass M hangs in equilibrium.





$$T_1' = T_1\sin60^\circ$$

Solution

At the equilibrium for M

$$T_1\cos30^\circ+T_2\cos45^\circ = Mg\ \ldots\ldots\ (1)$$

Also

$$T_1\cos60^\circ = T_2\cos45^\circ$$

$$T_1 = 1.414T_2\ \ldots\ldots\ (2)$$

Insert equation (2) into (1)

$$1.414T_2\cos30^\circ+T_2\cos45^\circ = Mg$$

$$T_2 = \frac{Mg}{1.414\cos3^\circ+\cos45^\circ}$$

$$T_2 = \frac{6\times9.8}{1.414\cos30^\circ+\cos45^\circ}$$

$$T_2 = 30.4N$$

Also

$$T_1 = 1.414\times30.4$$

$$T_1 = 43N$$

Again

$$T_2\sin45^\circ = M_2g$$

$$M_2 = 2.2kg$$

Also

$$T_1\sin60^\circ = M_1g$$

$$M_1 = 3.8kg$$

Example – 23

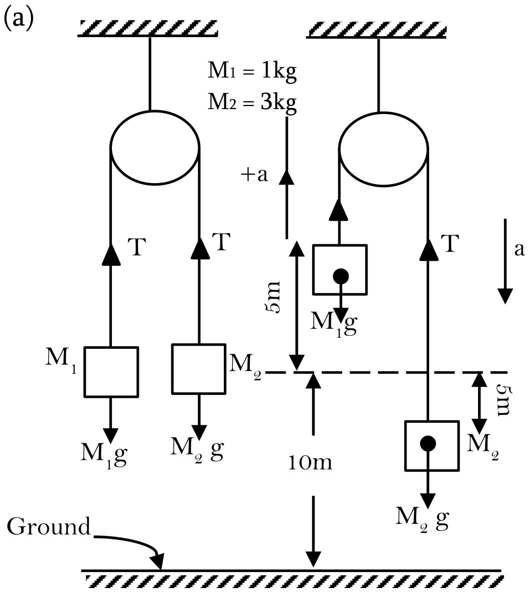
Two masses $M_1 = 1.0kg$ and $M_2 = 3.0kg$ are connected by a massless string which passes over a frictionless pulley. The masses are initially 10m above the ground.

(a) Find the velocity of the masses at the instant, the higher mass moves up a distance of 50m

(b) The string is suddenly cut at that instant, calculate the velocity with which the two masses hit the ground.

Solution

(a)



Resultant force on the block

M_1: $T - M_1g = M_1a$ ....(i)

M_2: $M_2g - T = M_2a$ ....(ii)

Adding equation (i) and (ii)

$$(M_1 + M_2)a = (M_2 - M_1)g$$

$$a = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g$$

$$= \left(\frac{3-1}{3+1}\right) \times 9.8$$

$$a = 4.9m/s^2$$

Let t be the time taken by 1.0kg mass to cover 5.0m.

$$S = ut + \frac{1}{2}at^2 \quad , \quad u = 0$$

$$t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 5}{9.8}}$$

$$t = 1.0\,sec$$

Let V be the velocity acquired by M_1

$$V = u + at\,,\, u = 0$$

$$V = at = 4.9 \times 1$$

$$V = 4.9m/s$$

(b) At this instant, the string is cut and M_1 moves freely under the effect of gravity the displacement of M_1 from the ground

$$h = 10 + 5 = 15m$$

and its initial velocity at that instant is V. Let V_1 be its final velocity when strike the ground.

$$V_1^2 = V^2 + 2gh$$

$$= (4.9)^2 + 2 \times 9.8 \times 15$$

$$V_1 = 17.83m/s$$

The displacement of M_2 from the ground, when the string is cut.

$$H = 10 - 5 = 5m$$

Velocity of mass M_2 at that instant

$$Vo = u + at = 0 + 4.9 \times 1$$

$$Vo = 4.9m/s$$

Let V_2 be its final velocity on reaching on the ground

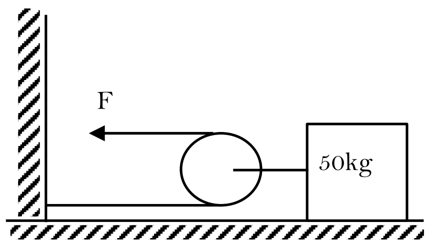
$$V_2^2 = V_o^2 + 2gh$$

$$= 4.9^2 + 2 \times 9.8 \times 5$$

$$V_2 = 1.1m/s$$

Example – 24

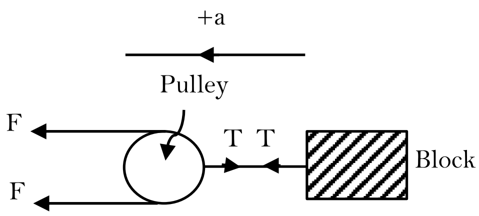
A block of mass 50kg is attached to a frictionless pulley and a rope system as shown below. A horizontal force F must be applied to produce an acceleration of $0.2m/s^2$ in the block. Calculate the value of F.



Assume all the surfaces to be frictionless and rope massless

Solution

Consider the FBD as shown below



Let T be tension in the rope connecting block and pulley.

Then $2T = F$

For the block only

$T = Ma$

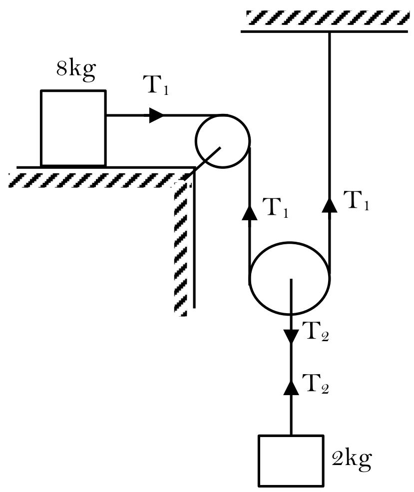
$T = 50 \times 0.2 = 10N$

Now $2F = T = 10N$

$F = 5N$

Example – 25

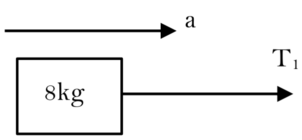
In the figure shown below, calculate the acceleration of 8kg block and 2kg block



Solution

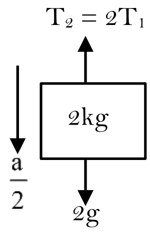
In the above situations, when the block of 8kg moves towards right by a distance X, block of 2kg moves down by a distance X/2. This shows that if acceleration of 8kg block is a, then that of 2kg block is a/2

FBD for 8kg block



$T_1 = 8a$

FBD of 2kg block



$$2g - T_2 = 2 \times \frac{a}{2}$$
$$2g - 2T_1 = a$$
$$2g - 16a = a$$
$$2g = 17a$$
$$a = \frac{2g}{17} = \frac{2 \times 9.8}{17}$$

$a = 1.153m / s$

$\therefore$ acceleration of 8kg block

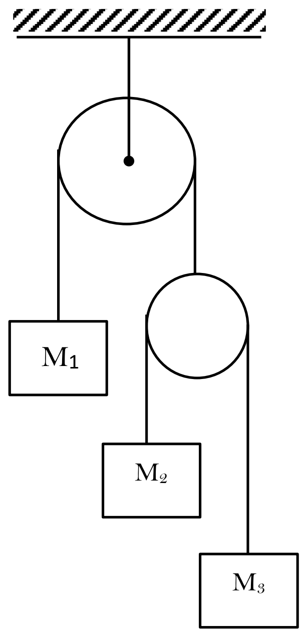
$a = 1.153m/s^2$

acceleration of 2kg block

$$\frac{a}{2} = 0.576m / s^2$$

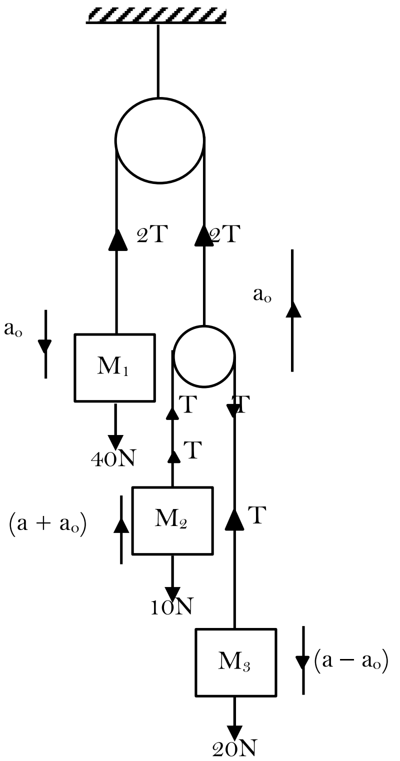
Example – 26

Find the accelerations of blocks as shown in the figure below. Given that the masses of blocks are $M_1 = 4kg$, $M_2 = 1kg$ and $M_3 = 2kg$, $g = 10m/s^2$.



Solution

In this question block of mass M_1 of mass 4kg is assumed to be moving with acceleration a_o in the downward direction and thus movable pulley, carrying other two blocks, must move in the upward direction with the same acceleration, ao in this case movable pulley is fixed at its location, then let us assume that the block M_3 of mass 2kg is moving downward and so block M_2 of mass 1kg must move upward with the same acceleration, a. this acceleration a is relative acceleration of blocks with respect to movable pulley.



So we can now add accelerations of pulley a_o with the acceleration of blocks to get net accelerations of M_2 and M_3. Then acceleration of M_3 and M_3 can be shown on the figure above. The pulley is massless, then net force is equal to zero.

Resultant forces on the

Block M_1: $40 - 2T = 4a_o$ ....(1)

Block M_2: $T - 10 = a + a_o$ ....(2)

Block M_3: $20 - T = 2(a - a_o)$ ....(3)

For equation (1)

$$2T = 40 - 4a_o$$

$$T = 20 - 2a_o$$

Putting T in equation (2) and (3)

$$20 - 2a_o - 10 = a + a_o$$

$$a + 3a_o = 10 \text{ ....} (4)$$

Also

$$20 - 20 + 2a_o = 2a - 2a_o$$

$$a = 2a_o$$

Now

$$2a_o + 3a_o = 10$$

$$5a_o = 10$$

$$a_o = 2m/s^2$$

$$a = 2 \times 2 = 4m/s^2$$

$$T = 20 - 2 \times 2 = 16N$$

∴ Block M_1 moves down with acceleration $a_o = 2m/s^2$ block M_2 moves up with acceleration $(a + a_o) = 6m/s^2$ block M_3 move down with acceleration $(a - a_o) = 2m/s^2$.

Example 27

A lift of mass 3,000kg is supported by a thick cable. If the tension in the supporting cable is 36,000N

(i) Calculate the upward acceleration of the lift.

(ii) How far does it rise in 12seconds if it starts from the rest?

(iii) If the breaking stress for the rope is 3.0 × 108N/m² ; calculate the minimum diameter of the rope.

Solution

(i) As the lift move upward with acceleration

$$T = M(g + a)$$

$$36,000 = 3000(9.8 + a)$$

$$a = 2.2m/s^2$$

(ii) For upward motion of lift

U = 0, a = 2.2m/s², t = 12s

$$S = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 2.2 \times 12^2$$

$$S = 158.4m$$

(iii) Breaking stress = 3 × 108N/m².

$$3.0 \times 10^8 = \frac{\text{Force}}{\text{Area}} = \frac{T}{\pi D^2 / 4}$$

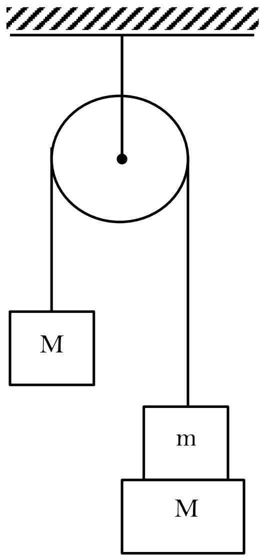
$$3\times10^{8} = \frac{4T}{\pi D^2}$$

$$D^2 = \frac{4\times36,000\times7}{22\times10^{8}}$$

$$D = 1.236\times10^{-2}\text{m} = 1.24\text{cm}$$

Example – 28 (TIE)

An experiment is performed to determine the value of gravitational acceleration g on earth. Two equal masses M hang at rest from the ends of a string on each side of frictionless pulley shown below. A mass m = 0.01M is placed on the right hand side after the heavier side has moved down by h = 1m, the small mass M is removed. The system continues to move for next 1sec, covering distance of H = 0.312M. Find the value of g from these data.



Solution

H = 0.312m, t = 1 sec

First obtain constant

Velocity when the system covers H = 0.312m in 1sec.

$$V = \frac{H}{t} = \frac{0.312M}{1\,\text{sec}}$$

$$V = 0.312\text{m / s}$$

From the equation

$$V^2 = U^2 + 2as \qquad (u = 0)$$

$$V^2 = 2as$$

$$a = \frac{V^2}{2H}$$

$$= \frac{(0.312)^2}{2\times1}$$

$$a = 0.048672\text{m / s}^2$$

Acceleration of system

Resultant forces

$$M: \quad T - Mg = Ma$$

$$M + m: \quad (M+m)g - T = (M+m)a$$

$$Mg + mg - Mg = (2M+m)a$$

$$a = \frac{mg}{2M+m}$$

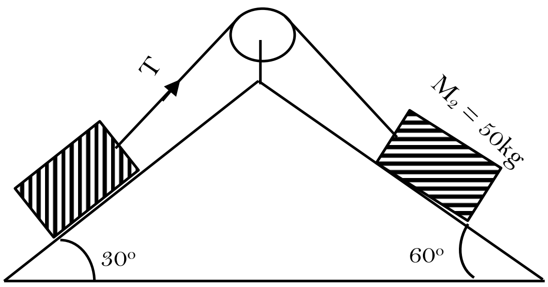
$$0.048672 = \frac{0.01Mg}{2M+0.01M}$$

On solving $g = 9.7344\text{m/s}^2$

ASSIGNMENT NO 3.

1. NECTA 2006/P1/2(b)

   Two blocks connected to a string over a small frictionless pulley rest on frictionless plane as shown in the figure below



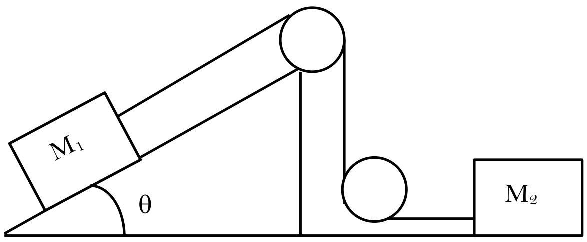
   Calculate: -

   (i) Acceleration of the block system

   (ii) Tension in the string.

2. Blocks of masses M_1 and M_2 are connected to each other via a string and pulley as shown in the arrangement. Assuming that no friction show that the acceleration of mass M_2 is

$$a = \frac{M_1 g \sin\theta}{M_1 + M_2}$$



3. TAHOSSA FV 2011/3
   (a) State Newton's laws of motion

   (b) Three blocks of masses 2.0, 4.0 and 6.0kg are arranged in order of lower middle and upper respectively are connected by string on frictionless plane of 60^o to the horizontal. A force 120N is applied causing an upward movement of the blocks. The connecting cords are light
      (i) What is the acceleration of the blocks?
      (ii) What are the tensions between upper and middle blocks and the lower and middle blocks?
      Ans. (i) $1.51 m/s^2$ (ii) 60.02N, 20N

4. Calculate the tension and acceleration of the system shown on the figure below