Current interval arthmatic

An identity I, such that for all intervals x, $x \times I = x$. Does exit and has a value of $I = (1, 1)$.

Proof. For some positive-positive (x, y). The expression $(a, b) \times (x, y)$ is given by:

- If (a, b) is positive-positive, then $(a, b) \times (x, y) = (a \cdot x, b \cdot y)$
- If (a, b) is negative-negative, then $(a, b) \times (x, y) = (a \cdot y, b \cdot x)$
- If (a, b) is negative-positive, then $(a, b) \times (x, y) = (a \cdot y, b \cdot y)$

Now, substituting $(x, y) = (1, 1)$. We see that all of the expressions result in (a, b). Thus, $(1, 1)$ is the multiplicative identity of intervals. $\square$

However, there does not exist an inverse interval x^{-1}, such that $x \times x^{-1} = (1, 1)$.

Proof. Consider the multiplication of (lx, ux) and (ly, uy).

- If x is positive-positive, then
  - If y is positive-positive $\rightarrow (lx \cdot ly, ux \cdot uy)$.
  - If y is negative-negative $\rightarrow (ux \cdot ly, lx \cdot uy$.
  - If y is negative-positive $\rightarrow (ux \cdot ly, ux \cdot uy$.

Examining the first case, if $x \times y = (1, 1) \implies (lx \cdot ly, ux \cdot uy) = (1, 1)$. This implies that $y = (\frac{1}{lx}, \frac{1}{ux})$. Since a must be less than b in any given interval of the form (a, b). The positive-positive inverse interval y does not exist.

As for the rest of the cases, it can be shown that the inverse intervals do not exist due to the fact that they can not produce a positive-positive interval such as $(1, 1)$. Due to their sign.

Thus, for any given interval x. An inverse does not necessarily exit. $\square$

Since the intervals do not produce a field. It's not possible to treat them like real number.

After some further research. I have encountered the following: wiki page

It seems that my approach was more mathematical, and I forgot what it means to do asthmatic on intervals. In any case, Using the same symbol more than one time, increases the error. See this