

- $T_{p,q}(a, b) = (bq + aq + ap, bp + aq)$
- $T_{p,q}(bq + aq + ap, bp + aq) = ((bp + aq)q + (bq + aq + ap)q + (bq + aq + ap)p, pe(bp + aq)p + (bq + aq + ap)q)) = (bq' + aq' + ap', bp' + aq')$
- If we expand the first parameter:

$$\begin{aligned} &bpq + aqq + bqq + aqq + apq + bpq + apq + app \\ &= bq' + aq' + ap' \end{aligned}$$

Which can be written in the form:

$$\begin{aligned} &b(2pq + qq) + a(2pq + qq) + a(pp + qq) \\ &= bq' + aq' + ap' \end{aligned}$$

Therefore:

$$\begin{aligned} p &= (p^2 + q^2) \\ q &= (2pq + q^2) \end{aligned}$$

As desired.