

ME46060 “Engineering Optimization – Concepts and Applications”

Exercise 6

Subject: “Valve spring”

Objectives

- Use SQP (fmincon) to solve constrained optimization problem.

Course material

- Section 5.5 and Section 6.7 of Papalambros and Wilde [1]

Introduction

Consider the following constrained optimization problem of the valve spring:

Minimize $f(\mathbf{x}) = s_{mass}$ (spring mass)

Subject to:

$$L2 \geq L_{min}$$

$$F1 \geq F1_{min}$$

$$F2 \geq F2_{min}$$

$$\tau_{2} \leq \tau_{12max} (= 600e+6 \text{ N/m}^2)$$

$$freq1 \geq freq1b$$

Set constraint:

$$0.02 \leq x(1) \leq 0.04 \text{ (m)}$$

$$0.002 \leq x(2) \leq 0.005 \text{ (m)}$$

This problem is similar as in exercise 2 and 5, except some modifications of the input parameters (now given in the file `springparams2.m`). Row vector \mathbf{x} represents the design variables, being the coil diameter $D (= x(1))$ and the wire diameter $d (= x(2))$ respectively.

The problem is visualized by running the (given) program `springw6ex1`, using the files `springobj2.m` (mass objective function) and `springcon2.m` (constraints).

Exercise 6.1: Visualization of constrained spring optimization problem.

Look `springw6ex1.m` and see how the visualization is realized. Run `springw6ex1.m` and indicate (by hand) the objective function, constraints and the feasible domain resulting Figure 1. Note that the dotted contours lie in the infeasible region. Estimate the expected optimum.

Exercise 6.2: Constrained optimization by SQP.

Extend the file `springw6ex1.m` to `springw6ex2.m` in order to solve the problem mentioned above using the SQP algorithm `fmincon` of MATLAB, and the given files `springobj2.m` and `springcon2.m`. Note that the file `springcon2.m` has to be modified a little to incorporate the (empty) vector of equality constraints (name the modified file `springcon3.m`). Do **not** provide the

gradients of the objective function and the constraints by means of a MATLAB-function. Question: how does the `fmincon` algorithm compute the mentioned gradients in that situation?

Use the point $x_0 (= [D \ d]) = [0.034 \ 0.0045]$ as initial design point. Solve the problem for the output parameters `[x, fval, exitflag, output, lambda]`. Give also output of the constraint values. Plot a marker in the initial point and in the final solution point.

Interpret the results `[x, fval, exitflag, output, lambda]`, and the constraints; check whether the used convergence criteria are satisfied. Which constraints are active? Check the Lagrange multipliers with respect to constraint activity.

Also try some other initial design points.

References

- [1] Papalambros, P.Y. and Wilde, D.J., *Principles of optimal design: modeling and computation*, 3rd edition, Cambridge University Press, 2017.