ME46060 “Engineering Optimization – Concepts and Applications” Exercise 5

Subject: “Valve spring”

Objectives
- Investigate the influence of the step size on the accuracy of finite difference gradients.
- Understand the KKT conditions for constrained optimization.

Course material
- Section 4.1, 4.2, and 4.3 of Chapter 4, Section 5.1, 5.2, 5.3, 5.4, 5.6 and 5.7 of Chapter 5, and Section 9.2 of Chapter 9 of Papalambros and Wilde [1]

Introduction
Consider again the following optimization problem of the valve spring (see previous week):

Minimize f(**x**) = smass (spring mass)

Subject to:

L2 ≥ Lmin
F1 ≥ F1min
F2 ≥ F2min
Tau2 ≤ Tau12max (= 600e+6 N/m^2)
freq1 ≥ freq1lb
Set constraint:
0.02 ≤ x(1) ≤ 0.04 (m)
0.002 ≤ x(2) ≤ 0.005 (m)

Row vector **x** represents the design variables, being the coil diameter D (= x(1)) and the wire diameter d (= x(2)) respectively. The other parameters are given in the file springparams1.m.

Exercise 5.1: Finite difference gradients

We would like to compute the gradients of objective function and constraints in the design point x=(0.024, 0.004). This can be done analytically or by symbolic computation. But for reasons of generality forward finite differencing is used to estimate the derivatives of objective function and constraints with respect to the design variables (see Section 8.2 in [1]). The objective function and constraints are coded in the given function-files springobj1.m and springcon1.m respectively (the file springanalysis1.m is used, and the file of design parameters (springparams1.m) is read within the function-files springobj1.m and springcon1.m).

The program springw5ex1.m computes the finite difference derivatives of the objective function and the constraints, and plots the gradients of the objective function and (only) constraint g1 as function of the finite difference step size. Run this program and explain how the finite difference step size affects the accuracy of the calculated gradients. Which two types of errors can be identified? What is the usable range of finite difference step sizes?

Add the following expression: g(1)=str2double(num2str(g(1))) at the end of the function-file springcon1.m. This expression causes g(1) to have only five instead of sixteen significant digits. Rerun springw5ex1.m and explain the results. What is now the usable range of finite difference step sizes for constraint g_1?

Modify springw5ex1.m in order to plot the gradients of the constraints g4 and g5 as function of the step size. Discuss the results.

Exercise 5.2: Karush-Kuhn-Tucker conditions

The Karush-Kuhn-Tucker conditions represent the *necessary* conditions for a constrained optimum. Point **x*** = [0.020522 0.003520] is the constraint optimum of the valve spring optimization problem as given above. From the contour plot of exercise 2 you can observe that constraints g_2, g_3 and g_4 are active at this optimum point. Check the KKT conditions for **x***.
Discuss the results and explain how they were obtained.

Hints:

- To compute the Lagrange multipliers μ:
    - Modify the program springw5ex1.m to a program named springw5ex2.m which is able to solve the set of equations $\nabla g \cdot \mu = -\nabla f$ for the unknowns μ (see lecture slide handouts and [1]). Use step size hi = 1e-8 to compute the finite difference gradients.
    - Use the observation that the Lagrange multipliers μ_1 and μ_5 are zero because g_1 and g_5 are inactive.
- Note that g_2 and g_3 are dependent (set of non-regular design points). How do you cope with that?

Check the KKT-conditions also in the following points:
The intersection of constraints g_2 (and g_3) and g_1 (point [0.02462 0.004035])
The intersection of constraints g_4 and g_1 (point [0.022251 0.004075])
Discuss the results.

References

[1] Papalambros, P.Y. and Wilde, D.J., *Principles of optimal design: modeling and computation*, 3rd edition, Cambridge University Press, 2017.