Computational Fluid Dynamic Assignment (B.S.)

Mechanical Engineering Department, University of Tehran

Project #1

Due date 1402/10/15
@ 23:55

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Background:

Consider the homogeneous nonlinear parabolic PDE:

$$\frac{\partial u}{\partial t} - u_{xx} = -uu_x \qquad 0 \le x \le 1 \qquad t \ge 0$$

Subject to an initial condition:

$$u(x,0) = x$$

with Dirichlet and nonlocal boundary conditions:

$$u(0,t) = 0$$
$$\int_0^1 u(x,t)dx = \frac{1}{2(1+t)}$$

a) Write a computer program to solve the above equation using the Keller Box method. Follow these guidelines:

b) Replace the second and higher derivatives with first derivatives by introducing additional variables

c) Use the Taylor series expansion to develop a finite difference representation.

d) Linearise the system of equations by imposing the following equations:

$$u_i^{k+1} = u_i^k + \delta u_i^k$$

e) Perform the calculation for a uniform mesh
f) Perform the calculation for a non-uniform mesh

g) Check the mesh dependency of your numerical results.
h) Validate your numerical result with the exact solution.
i) Plot the velocity profile in time and space.

Tip 1: The coefficient matrix must be solved using the Thomas algorithm
Tip 2: Refer to the attached paper, lemma1, to convert a non-local boundary condition into a local boundary condition.