
DISCRETE MATHEMATICS

MATH 381

BASIC CONCEPTS AND EXAMPLES EXPLAINING THE FUNDAMENTALS OF
DISCRETE MATHEMATICS.

By
MANSI SAKARVADIA
The University of North Carolina
Chapel Hill

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Recall:

- Functions $f: A \rightarrow B$
- Image $Im(f) = f(A)$

If $f(a) = b$, say "a is a preimage of b"

- $Gr(f) = \{(a, b) | f(a) = b\} \subseteq A \times B$
- $Gr(f) = \{(a, f(a)) | a \in A\}$

Graph $Gr(f)$ is a relation between A and B

Which binary relations (subsets of $A \times B$) are graphs of functions?

- A subset $s \subseteq A \times B$ is the graph of a function if for every element $a \in A$, there is a unique element $b \in B$ such that $(a, b) \in S$.
- key Can't have (a, b_1) and $(a, b_2) \in S$ where $b_1 \neq b_2$ and expect S to be a graph
- (abstraction of "straight line test" about graphs $f: \mathbb{R} \rightarrow \mathbb{R}$)

1.1 Restriction of Domain

suppose $f: A \rightarrow B$

Consider $A' \subseteq A$

DEFINITION: the restriction of f to A' is $f|_{A'}: A' \rightarrow B$ defined by $(f|_{A'})(a) = f(a), a \in A'$

KEY POINT: What does it mean for 2 functions $f: A \rightarrow B$ and $g: C \rightarrow D$ to be equal?

- NEED: $A=C$, $B=D$, and $f(a) = g(a) \forall a \in A$

1.2 Restriction of Codomain

If B' is a set with $Im(f) \subseteq B' \subseteq B$, then we consider:

$$f' : A \rightarrow B'$$

defined by $f'(a) = f(a)$ for all $a \in A$

$$EX. Im(f|_{A'}) = f(A')$$

- $Im(f|_{A'}) \leftarrow$ image of the restriction of f to A'
- $f(A') \leftarrow$ image of the subset $A' \subseteq A$ under f

1.3 Arithmetic of functions

- A function is called real-valued if its codomain is $\in \mathbb{R}$
- A function is called inter-valued if its codomain is $\in \mathbb{Z}$
- DEFINITION: Suppose that f_1 and f_2 are two real-valued function both w/ domain A . Then we have $f_1 + f_2$ and $f_1 f_2$ (the sum and product), two real-valued functions on A , defined by:

$$- f_1 + f_2(x) = f_1(x) + f_2(x)$$

$$- f_1 f_2(x) = f_1(x) * f_2(x)$$

$$- f_1, f_2 : \mathbb{R} \rightarrow \mathbb{R} \text{ given by } f_1(x) = x^2, f_2(x) = x - x^2$$

* Then $f_1 + f_2, f_1 * f_2 : \mathbb{R} \rightarrow \mathbb{R}$ are defined by:

$$f_1 + f_2(x) = f_1(x) + f_2(x) = x$$

$$f_1 f_2(x) = f_1(x) * f_2(x) = x^3 - x^4$$

* Note: $f_1 + f_2(x) = \iota_{\mathbb{R}}$ (hard to read symbol is iota subscript real-number symbol)

1.4 Injective, Surjective, Bijective

These are properties that a function may or may not have.

Consider $f : A \rightarrow B$

Injective

DEFN: Say that f is injective if for any $x, y \in A$, $f(x) = f(y) \rightarrow x = y$

- Equivalent Statements:
 - Whenever $x \neq y$ belong to A we must have $f(x) \neq f(y)$
 - * contra-positive, "Distinct points of A map to distinct values of B "

- For every b in B , there is at most one a in A which $f(a) = b$.
 - * "every point of B has at most one preimage."
- Eg. $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(n) = n^2$
 - * not injective because $f(-1) = f(1)$.
 - * $f|_{\mathbb{Z}^{>0}}$ is injective (function where domain is integer's greater than zero)
- How to prove?
 - * Criteria for injectivity
 - Suppose $f : A \rightarrow B$ where B, A are subsets of real numbers
 - f is increasing if $x < y \rightarrow f(x) \leq f(y)$
 - f is strictly increasing if $x < y \rightarrow f(x) < f(y)$
 - f is decreasing if $x < y \rightarrow f(x) \geq f(y)$
 - f is strictly decreasing if $x < y \rightarrow f(x) > f(y)$
 - * Proposition: A function that is strictly increasing or decreasing is injective

Surjective

DEFN: A function $f : A \rightarrow B$ is surjective if $\text{Im}(f) = B$

"surjective" = "onto"

- Equivalent descriptions:
 - For every value of b in B there is at least one element a in A such that $f(a) = b$
 - "every point of B has at least one preimage"
 - $f : \mathbb{Z} \rightarrow \mathbb{Z}$ not surjective because $\text{Im}(f) = \mathbb{Z}^{\text{greater than or } = 0} \not\subseteq \mathbb{Z}$
 - $f(n) = n + 1$ is surjective because any value m is mapped to by $m-1$.