DISCRETE MATHEMATICS
MATH 381

Basic concepts and examples explaining the fundamentals of Discrete Mathematics.

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Contents

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Recall:

- Functions f: A→B
- Image $Im(f) = f(a)$

If f(a) = b, say "a is a preimage of b"

- $Gr(f) = \{(a,b)|f(a) = b\} \subseteq AxB$
- $Gr(f) = \{(a, f(a))|a \in A\}$

Graph $Gr(f)$ is a relation between A and B

Which binary relations (subsets of AxB) are graphs of functions?

- A subset $s \subseteq AxB$ is the graph of a function if for every element $a \in A$, there is a unique element $b \in B$ such that $(a,b) \in S$.
- key Can't have (a, b_1) and $(a, b_2) \in$ S where $b_1! = b_2$ and expect S to be a graph
- (abstraction of "straight line test" about graphs $f : \mathbb{R} \to \mathbb{R}$)

1.1 Restriction of Domain

suppose $f : A \to B$
Consider $A' \subseteq A$

DEFINITION: the restriction of f to A' is $f|_{A'} : A' \to B$ defined by $(f|_{A'}) = f(a), a \in A$

KEY POINT: What does it mean for 2 functions $f : A \to B$ and $g : C \to D$ to be equal?

- NEED: A=C, B=D, and $f(a) = g(a)a \in A$

1.2 Restriction of Codomain

If B' is a set with $Im(f) \subseteq B' \subseteq B$, then we consider:
$f' : A \to B'$
defined by $f'(a) = f(a) forall a \in A$

EX. $Im(f|_{A'}) = f(A')$

- $Im(f|_{A'}) \leftarrow$ image of the restriction of f to A'
- $f(A') \leftarrow$ image of the subset $A' \subseteq A$ under f•

1.3 Arithmetic of functions

- A function is called real-valued if its codomain is $\in \mathbb{R}$
- A function is called inter-valued if its codomain is $\in \mathbb{Z}$
- DEFINITION: Suppose that f_1 and f_2 are two real-valued function both w/ domain A. Then we have $f_1 + f_2$ and $f_1 f_2$ (the sum and product), two real-valued functions on A, defined by:
  - $f_1 + f_2(x) = f_1(x) + f_2(x)$
  - $f_1 f_2(x) = f_1(x) * f_2(x)$
  - $f_1, f_2 : \mathbb{R} \to \mathbb{R}$ given by $f_1(x) = x^2, f_2(x) = x - x^2$
    - Then $f_1 + f_2, f_1 * f_2 : \mathbb{R} \to \mathbb{R}$ are defined by:
      $f_1 + f_2(x) = f_1(x) + f_2(x) = x$
      $f_1 f_2(x) = f_1(x) * f_2(x) = x^3 - x^4$
    - Note: $f_1 + f_2(x) = \iota_{\mathbb{R}}$ (hard to read symbol is iota subscript real-number symbol)

1.4 Injective, Surjective, Bijective

These are properties that a function may or may not have.
Consider $f : A \to B$

Injective

DEFN: Say that f is injective if for any $x, y \in A$, $f(x) = f(y) \to x = y$

- Equivalent Statements:
  - Whenever x!=y belong to A be must have f(x)!=f(y)
    - contra-positive, "Distinct points of A map to distinct values of B"

  - For every b in B, there is at most one a in A which f(a)= b.
    - "every point of B has at most one preimage."
  - Eg. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = n^2$
    - <u>not injective</u> because f(-1) = f(1).
    - $f|_{\mathbb{Z}^{>\nless}}$ is injective (function where domain is integer's greater than zero)
  - How to prove?
    - Criteria for injectivity
      Suppose $f : A \to B$ where B, A are subsets of real numbers
      - f is increasing if $x < y \to f(x) <= f(y)$
      - f is strictly increasing if $x < y \to f(x) < f(y)$
      - f is decreasing if $x < y \to f(x) >= f(y)$
      - f is strictly decreasing if $x < y \to f(x) > f(y)$
    - Proposition: A function that is strictly increasing or decreasing is injective