Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) $2 + 3 = 5$.
d) $5 + 7 = 10$.
e) $x + 2 = 11$.
f) Answer this question.

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propositions a b c d
Truth a b c

What is the negation of each of these propositions?

a) Mei has an MP3 player.
b) There is no pollution in New Jersey.
c) $2 + 1 = 3$.
d) The summer in Maine is hot and sunny.

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a) it's not the case that Mei has an MP3 player
Mai doesn't have MP3 player

b) There is pollution in new Jersey

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c) $2+1 \neq 3$

d) is not

Let p and q be the propositions

p : I bought a lottery ticket this week.
q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a) $\neg p$ **b)** $p \vee q$ **c)** $p \rightarrow q$
d) $p \wedge q$ **e)** $p \leftrightarrow q$ **f)** $\neg p \rightarrow \neg q$
g) $\neg p \wedge \neg q$ **h)** $\neg p \vee (p \wedge q)$

$P \rightarrow (p \wedge q)$

a) I did not buy a lottery ticket this week

b) either I bought ... or I won million dollar

c) If I bought a lottery ticket this week
I won jackpot

c) If I bought a lottery ticket this week, then I won the million dollar jackpot

d) and

e) I if and only if —

f) I did not buy then I did not win

g) I did buy and I did not win

h) I didn buy or (I bought and don)

Let p, q, and r be the propositions

p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.
b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
c) To get an A in this class, it is necessary for you to get an A on the final.
d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

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a) $r \wedge \neg q$ b) $r \wedge p \wedge q$

c) $r \rightarrow p$ d) $(p \wedge \neg q) \wedge r$

e) $(p \wedge q) \rightarrow r$ f) $r \leftrightarrow (p \vee q)$

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Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 3$, then unicorns exist.
b) If $1 + 1 = 3$, then dogs can fly.
c) If $1 + 1 = 2$, then dogs can fly.
d) If $2 + 2 = 4$, then $1 + 2 = 3$.

a) $F \rightarrow F$ b) $F \rightarrow F$ c) $T \rightarrow F$

False

a) $F \rightarrow F$ True b) $F \rightarrow F$ True c) $T \rightarrow F$ False

d) $T \rightarrow T$ True.

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Construct a truth table for each of these compound propositions.

a) $p \rightarrow \neg p$ **b)** $p \leftrightarrow \neg p$

c) $p \oplus (p \vee q)$ **d)** $(p \wedge q) \rightarrow (p \vee q)$

e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

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a)

p	$\neg p$	$p \rightarrow \neg p$
T	F	F
F	T	T

b) $p \leftrightarrow \neg p$

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d) $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \rightarrow p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

e)

p	q	$\neg p$	$q \to \neg p$	$p \leftrightarrow q$	
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

f)

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Construct a truth table for each of these compound propositions.

a) $p \to (\neg q \vee r)$
b) $\neg p \to (q \to r)$
c) $(p \to q) \vee (\neg p \to r)$
d) $(p \to q) \wedge (\neg p \to r)$
e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

a)

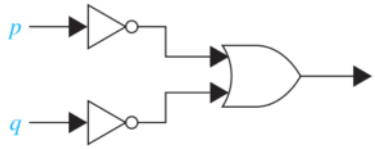
p	q	r	$\neg q$	$(\neg q \vee r)$	$p \to (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

b) $\neg p \rightarrow (q \rightarrow r)$

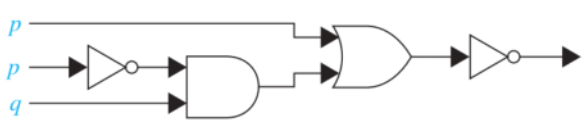
p	q	r	$\neg p$	$q \rightarrow r$	$\neg p \rightarrow (q \rightarrow r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Find the output of each of these combinatorial circuits.

a)



b)



a) $p \rightarrow \neg p$, $q \rightarrow \neg q$ $\wedge \rightarrow \neg p \wedge \neg q$

b) $p \rightarrow p$, $p \rightarrow \neg p$, $q \rightarrow q$ $\wedge \rightarrow (\neg p \wedge q)$ $\vee$ $p \vee (\neg p \wedge q)$ $\rightarrow \neg$ $\neg(p \vee (\neg p \wedge q))$

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Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits p, q, and r.

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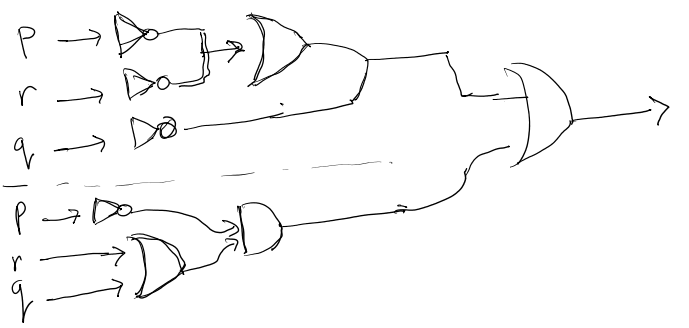

Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \rightarrow p$
b) $p \rightarrow (p \vee q)$
c) $\neg p \rightarrow (p \rightarrow q)$
d) $(p \wedge q) \rightarrow (p \rightarrow q)$
e) $\neg(p \rightarrow q) \rightarrow p$
f) $\neg(p \rightarrow q) \rightarrow \neg q$

a) $(p \wedge q) \rightarrow p$

p	q	$p \wedge q$	$p \wedge q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$(p \wedge q) \rightarrow p \equiv \neg(p \wedge q) \vee p$

$\equiv (\neg p \vee \neg q) \vee p$

$\equiv T \vee \neg q \equiv T$

b) $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$$p \to (p \vee q) \equiv \neg p \vee p \vee q$$
$$\equiv T \vee q \equiv T$$

---

c) $\neg p \to (p \to q)$

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

$$\neg p \to (p \to q) \equiv p \vee (p \to q)$$
$$\equiv p \vee (\neg p \vee q)$$
$$\equiv T \vee q \equiv T$$

---

d) $(p \wedge q) \to (p \to q)$

$$\equiv \neg(p \wedge q) \vee (p \to q)$$
$$\equiv \neg p \vee \neg q \vee (\neg p \vee q)$$
$$\equiv (\neg p \vee \neg p) \vee (\neg q \vee q) \equiv T$$
$$\qquad\qquad\qquad\qquad\quad T$$

---

e) $\neg(p \to q) \to p$

$$\equiv (p \to q) \vee p \equiv \neg p \vee q \vee p \equiv T$$

f) $\neg(p \rightarrow q) \rightarrow \neg q$

$\equiv (p \rightarrow q) \vee \neg q \equiv \neg p \vee q \vee \neg q \equiv T$

Determine whether each of these compound propositions is satisfiable.

a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
b) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

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a)

p	q	¬p	¬q	p ∨ ¬q	¬p ∨ q	¬p ∨ ¬q
T	T	F	F	T	T	F
T	F	F	T	T	F	
F	T	T	F	F		
F	F	T	T	T	T	T

b)

p	q	¬p	¬q	p → q	p → ¬q	¬p → q
T	T	F	F	T	F	
T	F	F	T	F		
F	T	T	F	T	T	F
F	F	T	T	T	T	F

c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

p	q	¬p	p ↔ q	¬p ↔ q
T	T	F	T	
T	F	F	F	
F	T	T	F	
F	F	T	T	F

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Show that the negation of an unsatisfiable compound proposition is a tautology and the negation of a compound proposition that is a tautology is unsatisfiable.

① negation of an unsatisfiable compound proposition is a tautology.

if a compound proposition is unsatisfiable
⇒ it's false for all values of proposition
⇒ it's a contradiction
negation of a contradiction is a tautology.

② negation of a compound proposition that is a tautology is unsatisfiable

∀ values of proposition it's true
⇒ for every assignment the value is true
⇒ negation of this true value is always false for every assignment.
⇒ system is unsatisfiable

$\neg[(p \rightarrow q) \wedge (\neg p)]$

$\neg[(\neg p \vee q) \wedge \neg p] = \neg[(\neg p \wedge \neg p) \vee (q \wedge \neg p)]$

$= \neg[\neg p \vee (q \wedge \neg p)]$ satisfiable

$= \neg[(\neg p \vee q) \wedge (\neg p \vee \neg p)]$

($\neg p \vee \neg p$ = $\neg p$)

$$\neg \left[(p \wedge q) \wedge (p \wedge \neg q) \right]$$

$$= \neg \left[p \wedge F \right] = \neg \left[F \right] = T$$