Tutorial

problem ①

$$\left.\begin{array}{l} P_1, P_2, \ldots, P_n \\ \hline (q \rightarrow r) \end{array}\right\} \text{vaild}$$

$$(P_1 \wedge P_2 \ldots P_n) \longrightarrow (q \rightarrow r)$$

$$\equiv \neg(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \vee (q \rightarrow r)$$

$$\equiv \neg(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \vee (\neg q \vee r)$$

$$\equiv [\neg(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \vee \neg q] \vee r$$

D'morgen $\boxed{\neg(P_1 \wedge P_2) \equiv \neg P_1 \vee \neg P_2}$

$$\equiv \neg(P_1 \wedge P_2 \wedge \cdots \wedge P_n \wedge q) \vee r$$

$$\equiv (P_1 \wedge P_2 \wedge \cdots \wedge P_n \wedge q) \longrightarrow r$$

$\Longrightarrow$ argument

$$\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \\ q \\ \hline \therefore r \end{array}$$

problem 27

p : I play hockey
q : I am sore
s : I use whirlpool

$$\left.\begin{array}{l} p \rightarrow q \\ q \rightarrow s \end{array}\right\} \; p \rightarrow s \equiv \neg s \rightarrow \neg p$$

$\neg s$

$\neg s \rightarrow \neg p$
$\neg s$

$\therefore \neg p$

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

$\{(p \rightarrow q) \wedge p\} \rightarrow q$

$p(x)$: x is an insect
$q(x)$: x has six legs
$S(x,y)$: x eat y.

Domain : Creatures

Dargonfilue := D
sparden := SP

$\forall x\,[p(x) \rightarrow q(x)]$

$P(D)$
$\neg q(SP)$
$S(SP, D)$

Let a be an arbitrary element

$$\left.\begin{array}{l} P(a) \rightarrow q(a) \\ P(D) \end{array}\right\} \Rightarrow q(D)$$

$$\left\{\begin{array}{l} P(a) \longrightarrow q(a) \\ P(D) \\ \neg q(SP) \\ S(SP, D) \end{array}\right\} \Rightarrow q(D)$$

$$\left\{\begin{array}{l} P(a) \longrightarrow q(a) \equiv \neg q(a) \longrightarrow \neg P(a) \\ \neg q(SP) \end{array}\right\} \neg P(SP)$$

$$q(D) \wedge \neg P(SP)$$

$$S(SP, D)$$

$$\exists (x, y)\left[S(x, y) \wedge P(y)\right]$$

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problem ③

a) $p(x)$: x is a + real

$q(x)$: x^2 is a positive real

$$\forall x\left[p(x) \longrightarrow q(x)\right]$$

$$p(a) \longrightarrow q(a)$$

$$q(a) \longrightarrow p(a)$$

$$\boxed{a = -2}$$

b) $p(x)$: $x^2 \neq 0$

$q(x)$: $x \neq 0$

$$\forall x\left[p(x) \longrightarrow q(x)\right]$$

$$\frac{\forall x[p(x) \longrightarrow q(x)]}{p(a) \longrightarrow q(a).}$$

$\boxed{\text{problem (21)}}$

$$\frac{\left\{\begin{array}{l}\forall x(P(x) \longrightarrow (Q(x) \wedge S(x))) \\ \forall x(P(x) \wedge R(x))\end{array}\right\}}{\forall x(R(x) \wedge S(x))}$$

Let (a) be an arbitrary element.

- $P(a) \longrightarrow (Q(a) \wedge S(a))$
- $P(a) \wedge R(a)$

$P(a) \wedge R(a)$

$P(a)$

$R(a)$

$$\frac{\begin{array}{l}P(a) \\ P(a) \longrightarrow (Q(a) \wedge S(a))\end{array}}{Q(a) \wedge S(a)}$$

$$\left.\begin{array}{l}P(a) \\ R(a) \\ Q(a) \wedge S(a)\end{array}\right\} \quad \begin{array}{l}P(a) \\ R(a) \\ Q(a) \\ S(a)\end{array}$$

$(R(a) \wedge S(a))$

$$(R(a) \land S(a))$$

$$\Rightarrow \quad \forall x \, [R(x) \land S(x)]$$

$\boxed{\text{problem } 5}$

$$\forall x \, [P(x) \lor Q(x)]$$
$$\forall x \, [\neg Q(x) \lor S(x)]$$
$$\forall x \, [R(x) \rightarrow \neg S(x)]$$
$$\exists x \, \neg P(x)$$

$$\exists x \, \neg R(x)$$

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choose element "c" such that
$\neg P(c)$ is true.

$$P(c) \lor Q(c) \equiv \neg P(c) \rightarrow Q(c)$$
$$\neg Q(c) \lor S(c) \equiv Q(c) \rightarrow S(c)$$
$$R(c) \rightarrow \neg S(c) \equiv S(c) \rightarrow \neg R(c)$$
$$\neg P(c)$$

$$\neg P(c) \rightarrow \neg R(c)$$

$\neg p(c) \longrightarrow \neg R(c)$

$\neg p(c)$

$\neg R(c)$

$\Rightarrow \boxed{\exists x \quad \neg R(x)}$

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problem 6

Def: $n \in \mathbb{Z}^+$ n is odd if $n = 2m+1$
$m \in \mathbb{Z}^+$

$$n = p^2 - L^2 \qquad p, L \in \mathbb{Z}^+$$

if p, L both are even, ✗
$p = 2k$, $L = 2q$

$$p^2 - L^2 = (2k)^2 - (2q)^2 = 4k^2 - 4q^2$$
$$= 2[2k^2 - 2q^2]$$

if p, L both are odd ✗

$p = 2k+1$, $L = 2q+1$

$$p^2 - L^2 = (2k+1)^2 - (2q+1)^2$$
$$= 4k^2 + 4k \cancel{+1} - 4q^2 - 4q \cancel{-1}$$
$$= 2[2k^2 + 2k - 2q^2 - 2q]$$

if p is odd L is even

$p = 2k+1 \qquad L = 2q$

$p^2 - L^2 = 4k^2 + 4k + 1 - 4q^2$

$= 2[2k^2 + 2k - 2q^2] + 1$

$= n = 2m + 1$

$m = \cancel{2k^2} + 2k - \cancel{2q^2}$

$k = q$

$p = k+1 \quad , \quad L = k$

proof:

since n is odd $\Rightarrow n = 2m+1$

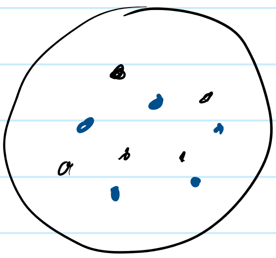
$n = 2m+1+0$

$= (2m+1+m^2) - m^2$

$= (m+1)^2 - m^2$

$= p^2 - L^2$

$\boxed{\text{problem 13}}$



proof by contradiction

$p \rightarrow q$

- you pick 3 socks
- you did not get ~~any~~ pair.

$\Rightarrow$ 1 black or blue — 2 black or blue — 3 must be neither

problem 7

$\underline{n}$ is a perfect square
$n+2$ is not a perfect square

$$\{ n = k^2 \}$$

(k^2) $(k+1)^2$

n s

$1^2, 2^2 \to 3^2, 4^2, \ldots$
$(k)^2 \; (k+1)^2$

$(n+2 > \underline{n})$

$(n = k^2)$

$$(k+1)^2 = k^2 + 2k + 1$$
$$= n + 2k + 1$$

$k = 0$

$n = 1^2 = 1$
$n+2 = 3$ ⊗

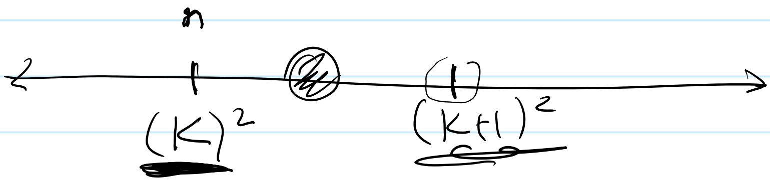
$k > 0$

$\underline{n+2k+1} > \underline{n+2}$

$n+2 = 3$ ⊗

$n + 2k+1 > n+2$

$(k+1)^2 > n+2 \qquad k \neq 0$



$n < \underline{n+2} < (k+1)^2$

$n+2$ cannot be a perfect square !!

problem 8

$x \in \mathbb{Q}'$, $y \in \mathbb{Q}'$

$\Rightarrow \boxed{xy \in \mathbb{Q}'}$

$\sqrt{3}$, $\sqrt{5}$, e

$x, y \qquad x = y = \sqrt{3}$

$(\sqrt{3})^2 = 3 \in \mathbb{Q}$

problem 9

Let $\left(\frac{a}{b}\right) \in \mathbb{Q}$ s.t $b \neq 0$ let $\underline{x \in \mathbb{Q}'}$

$\left(\frac{a}{b} x\right) \in \mathbb{Q}'$?!

if $\frac{a}{b}, \frac{c}{d} \in \mathbb{Q}$ s.t $b, d \neq 0$

if $\frac{a}{b}, \frac{c}{d} \in Q$ s.t $b, d \neq 0$

$\frac{a}{b} \times \frac{c}{d} \in Q$

$\frac{a}{b} \in Q$, $x \in Q'$

$\Rightarrow \frac{b}{a} \in Q$

$\left(x \frac{a}{b}\right)\left(\frac{b}{a}\right) = x$

irrational

problem (10)

$x \in Q' \Rightarrow 1/x \in Q'$

$1/x \in Q \longrightarrow x \in Q$

let $\frac{1}{x} = \frac{a}{b}$ $\Rightarrow x = \left(\frac{b}{a}\right) \in Q$

$b \neq 0$

problem (16)

$n \in \mathbb{Z}^+$

n odd $\Longleftrightarrow$ $5n+6$ odd

$(n \text{ odd} \longrightarrow 5n+6 \text{ odd}) \wedge (5n+6 \text{ odd} \longrightarrow n \text{ is odd})$

$(\underline{n} \text{ odd} \rightarrow \underline{5n+6} \text{ odd}) \wedge (\underline{5n+6} \text{ odd} \rightarrow \underline{n} \text{ is odd})$

$\Rightarrow$ $n = 2m+1$

$\underline{5n+6} = 5(2m+1)+6 = 10m+5+6$

$= 10m + 5+5+1 = 10m+10+1$

$= 2(\underbrace{5m+5}) + 1 \Rightarrow$

5n+6 is odd

$\Leftarrow$ 5n+6 is odd $\longrightarrow$ n is odd

n is even $\longrightarrow$ 5n+6 is even

$n = 2m$

$5n+6 = 5(2m)+6 = 10m+6$

$= 2(\underbrace{5m+3})$

$\boxed{5n+6 \text{ is even}}$