Tutorial 3: Proofs and Rules of Inference

Ahmed Attia
Mohamed Salaheldeen

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Problem 1

Show that the argument form with premises $p_1, p_2, \cdots, p_n$ and conclusion $q \to r$ is valid if the argument form with premises $p_1, p_2, \cdots, p_n, q$ and conclusion r is valid.

Problem 2

For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises

1. "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirlpool."
2. "All insects have six legs." "Dragonflies are insects." "Spiders do not have six legs." "Spiders eat dragonflies.
3. "Every student has an Internet account." "Homer does not have an Internet account." "Maggie has an Internet account."

Problem 3

Determine whether these are valid arguments.

1. If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.
2. If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.

Problem 4

Use rules of inference to show that if $\forall x(P(x) \to (Q(x) \land S(x)))$ and $\forall x(P(x) \land R(x))$ are true, then $\forall x(R(x) \land S(x))$ is true.

Problem 5

Use rules of inference to show that if $\forall x(P(x) \lor Q(x))$, $\forall x(\neg Q(x) \lor S(x))$, $\forall x(R(x) \to \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true

Problem 6

Use a direct proof to show that every odd integer is the difference of two squares.

Problem 7

Prove that if n is a perfect square, then $n + 2$ is not a perfect square

Problem 8

Pove or disprove that the product of two irrational numbers is irrational.

Problem 9

Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

Problem 10

Prove that if x is irrational, then $1/x$ is irrational.

Problem 11

Prove that if m and n are integers and mn is even, then m is even or n is even.

Problem 12

Show that if n is an integer and $n^3 + 5$ is odd, then n is even using a) a proof by contraposition. b) a proof by contradiction.

Problem 13

Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks

Problem 14

Show that at least ten of any 64 days chosen must fall on the same day of the week.

Problem 15

Use a proof by contradiction to show that there is no rational number r for which $r^3 + r + 1 = 0$.

Problem 16

Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.