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Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n + 1 > n)$ **b)** $\exists n(2n = 3n)$

c) $\exists n(n = -n)$ **d)** $\forall n(3n \leq 4n)$

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a) True

b) True

c) True

d) False (n negative)

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Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\exists x(x^2 = 2)$ **b)** $\exists x(x^2 = -1)$

c) $\forall x(x^2 + 2 \geq 1)$ **d)** $\forall x(x^2 \neq x)$

a) True

b) False

c) True

d) False

Suppose that the domain of the propositional function $P(x)$ consists of the integers 1, 2, 3, 4, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

a) $\exists x P(x)$ **b)** $\forall x P(x)$

c) $\neg\exists x P(x)$ **d)** $\neg\forall x P(x)$

e) $\forall x((x \neq 3) \rightarrow P(x)) \vee \exists x \neg P(x)$

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a) $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

b) $P(1) \wedge \ldots \wedge P(5)$

c) $\neg (P(1) \vee \ldots P(5)) \equiv \neg(P(1)) \wedge \ldots \wedge \neg P(5)$

$\equiv \forall x \ \neg P(x)$

d) $\neg \forall x P(x) \equiv \exists x \ \neg P(x)$

e) $\underbrace{\forall x ((x \neq 3) \rightarrow P(x))}_{} \vee \underbrace{\exists x \ \neg P(x)}_{}$

1, 2, 3, 4, 5

$[(1 \neq 3) \rightarrow P(1)] \wedge \ldots \wedge [5 \neq 3 \rightarrow P(5)]$

$\equiv P(1) \wedge P(2) \wedge P(4) \wedge P(5)$

$\exists x \ \neg P(x)$

$\neg P(1) \vee \neg(P(2)) \vee \ldots \vee \neg P(5)$

$[P(1) \wedge P(2) \wedge P(4) \wedge P(5)] \vee [\neg P(1) \vee \neg P(2) \ldots \vee \neg P(5)]$

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Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) Something is not in the correct place.

b) All tools are in the correct place and are in excellent condition.

c) Everything is in the correct place and in excellent condition.

d) Nothing is in the correct place and is in excellent condition.

e) One of your tools is not in the correct place, but it is in excellent condition.

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Domain $x \in$ all things

a) $p(x)$: x is in the correct place

$\exists x \ \neg p(x)$

b) $Q(x)$: x is a tool

$S(x)$: x is in excellent condition.

$\forall x \ [Q(x) \to p(x) \wedge S(x)]$

problem is with $\exists$?!

why not $Q(x) \wedge p(x) \wedge S(x)$?!

c) $\forall x \ p(x) \wedge S(x)$

d) $\forall x \ \neg(p(x) \wedge S(x))$

why not $\neg p(x) \wedge \neg S(x)$?

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e) $\exists! x, [Q(x) \wedge \neg p(x) \wedge S(x)]$

Determine whether $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ are logically equivalent. Justify your answer.

what does the first statment say

$\forall x\, [P(x) \rightarrow Q(x)]$

That means if $P(x)$ is satisfied $\rightarrow$ $Q(x)$ is satisfied

if a student studies $\rightarrow$ student will get an A.

All ↓ will get an A.

However it did not imply anything about students who does not study.

$\forall x\, P(x) \rightarrow \forall x\, Q(x)$ T

P(x) is true $\forall$ x

Q(x) is true $\forall$ x

x_0

P(x) is false for some x $\rightarrow$

only

$P(x)$: x is divisible by 2

$Q(x)$: x is divisible by 4

$[\forall x\, P(x)] \rightarrow [\forall x\, Q(x)]$

is False

because $\exists$ number that are not divisible by 2

This statment is always True

However,

$\forall x\, [P(x) \rightarrow Q(x)]$ is false since

$\exists x \; P(x) \land \neg Q(x)$

2 is divisible by 2 by not divisible by 4.

Let $Q(x, y)$ be the statement "Student x has been a contestant on quiz show y." Express each of these sentences in terms of $Q(x, y)$, quantifiers, and logical connectives, where the domain for x consists of all students at your school and for y consists of all quiz shows on television.

a) There is a student at your school who has been a contestant on a television quiz show.

b) No student at your school has ever been a contestant on a television quiz show.

c) There is a student at your school who has been a contestant on *Jeopardy!* and on *Wheel of Fortune*.

d) Every television quiz show has had a student from your school as a contestant.

e) At least two students from your school have been contestants on *Jeopardy!*.

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a) $\exists x \exists y \; Q(x,y)$

b) $\neg(\exists x \exists y \; Q(x,y)) \equiv \forall x \forall y \; \neg Q(x,y)$

c) $\exists x \; Q(x, J) \land Q(x, W)$

d) $\forall y \exists x \; Q(x,y)$

e) $\exists x, \exists y \; [\, Q(x, J) \land Q(y, J) \land (x \neq y) \,]$

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Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\forall x \exists y (x^2 = y)$

b) $\forall x \exists y (x = y^2)$

c) $\exists x \forall y (xy = 0)$

d) $\exists x \exists y (x + y \neq y + x)$

e) $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$

f) $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$

g) $\forall x \exists y (x + y = 1)$

h) $\exists x \exists y (x + 2y = 2 \land 2x + 4y = 5)$

i) $\forall x \exists y (x + y = 2 \land 2x - y = 1)$

j) $\forall x \forall y \exists z (z = (x + y)/2)$

a) True

b) False (x is negative)

c) True (0)

d) False (counterexample)

e) $\forall x\,[(x=0) \vee (\exists y\,(xy=1))]$
(multiplicative inverse True)

f) False

g) True (additive inverse)

h) $(x+2y=2)$ $\quad x = 2-2y$
$2x+4y=5$

$4-4y+4y=5 \Rightarrow 4=5$ contradiction

$\Rightarrow$ False

i) $X = 2-y$

$2(2-y) - y = 1$
$4-3y=1 \quad \Rightarrow \boxed{y=1}$
$\Rightarrow \boxed{X=1}$

$\boxed{X=2 \quad y=0 \quad \text{contradict}}$

False

j) $\forall x\,\forall y\,\exists z\,[z = \frac{1}{2}(x+y)]$

True.