Gradient Inelastic Force-Based Beam-Column Element in OpenSees

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OpenSees Implementation

Element Command

The Tcl command to define a GI element is:

```
element gradientInelasticBeamColumn eleTag? iNode? jNode? numIntgrPts?
endSecTag1? intSecTag? endSecTag2? lambda1? lambda2? lc? transfTag?
<-integration integrType?> <-iter maxIter? minTol? maxTol?>
```

The necessary arguments are:

- `eleTag` element tag
- `iNode` near (first) node tag
- `jNode` far (end) node tag
- `numIntgrPts` total number of integration points – recommended to exceed $1.5L/l_c + 1$ when default integration method is used, with L = beam length, and l_c = characteristic length
- `endSecTag1` near-end part's section tag (Figure 1)
- `intSecTag` intermediate part's sections tag (Figure 1)
- `endSecTag2` far-end part's section tag (Figure 1)
- `lambda1` fraction of beam length, L, at near end represented by `endSecTag1`
- `lambda2` fraction of beam length, L, at far end represented by `endSecTag2`
- `lc` characteristic length, l_c – can be taken as plastic hinge length
- `transfTag` geometric transformation tag

The optional arguments are:

- `-integration` used to select integration type
  - `integrType` `NewtonCotes`, `Simpson`, or `Trapezoidal` (default: `Simpson`) – if Simpson's rule is used, `numIntgrPts` should be an odd number
- `-iter` used to set iterative solution algorithm parameters
  - `maxIter` maximum number of iterations (default: 50)
  - `minTol` minimum tolerance (default: 1E-10)
  - `maxTol` maximum tolerance (default: 1E-8)

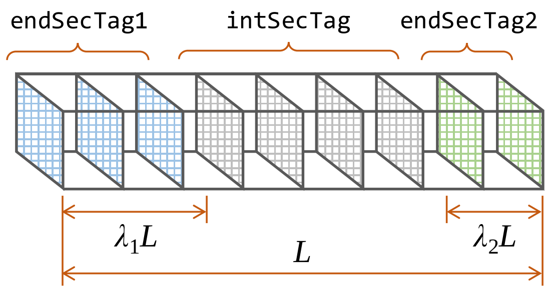

Figure 1. Assignment of pre-defined sections to integration points at different parts of element

Element Recorder Arguments

The valid response parameters that could be recorded through element recorders for a GI FB element are:

- `force/globalForce` nodal forces in global coordinate system
- `localForce` nodal forces in local coordinate system
- `basicForce` nodal forces in basic (reference) coordinate system
- `section $sectionNumber $arg1...` section response parameters
- `dampingForce` damping forces
- `nonlocalStrain` *macroscopic* section strains at *all* integration points

It is noted that section responses shall be expressed in terms of section forces/moments vs. *macroscopic* section strains/curvatures and the section strains/curvature distributions shall be obtained from *macroscopic* section strains/curvatures. The section strains/curvatures obtained via "`section $sectionNumber deformations`" are *material* section strains (i.e. internal parameters) and shall not be used in place of *macroscopic* section strains.

User Input Example

The element command for the GI FB element simulating the RC beam in Figure 2, assuming L = 5, λ_1 = 0.2, λ_2 = 0.3, and l_c = 0.5, while the tag for the predefined geometric transformation object is 20, may take the form below:

```
element gradientInelasticBeamColumn 10 1 2 21 1 2 1 0.2 0.3 0.5 20
-integration Simpson -iter 20 1E-8 1E-6
```

Observations and Recommendations:

- The number of integration points, N, was selected to be equal to 21 to achieve $l_c/\Delta x \geq 1.5$ – or more simply, $N \geq 1.5L/l_c + 1$. This condition has been found to result in discretization convergence from most common applications (Sideris and Salehi 2016; Salehi and Sideris 2017). *Yet, users are recommended to perform their own discretization convergence study, as dictated by their applications, and as they would do for conventional force-based elements.*
- As illustrated in Figure 2, section tags 1 and 2 refer to fiber sections representing the RC beam's cross-sections A-A and B-B. These fiber sections are defined with two different longitudinal reinforcement layouts and two different confined concrete material models because of their transverse reinforcement differences.

- Each member shall be modeled by a single `gradientInelasticBeamColumn` element. If more than one are used, they are recommended to be larger than $4l_c$.

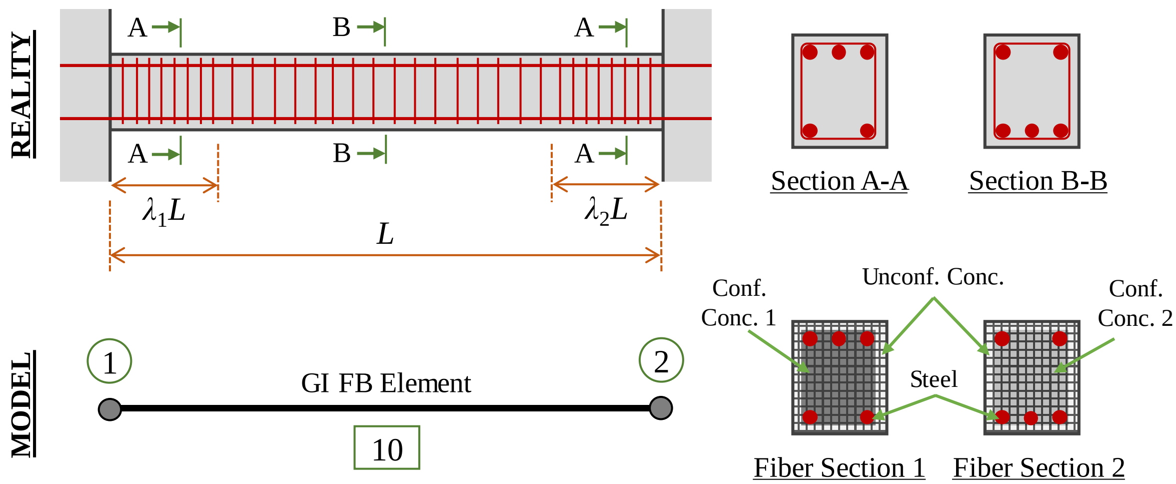

Figure 2. Example for using GI FB element in OpenSees

Why Gradient Inelastic Beam-Column Elements?

In the presence of softening section constitutive relations (e.g., for reinforced concrete members that experience significant damage), conventional force-based (FB) beam-column element formulations suffer from strain localization. The strain localization phenomenon, which represents concentration/localization of strains at a single integration point, further leads to loss of response objectivity, i.e., divergence, instead of convergence, of the element response with increasing number of integration points along the element length (see Figure 3(c)). These problems significantly compromise the accuracy of computational predictions of the damage and failure of members simulated using conventional FB elements.

To tackle the above issues of conventional FB element formulations, the *gradient inelastic* (GI) FB element formulation, `gradientInelasticBeamColumn` element in OpenSees, was developed. This element formulation is based on the GI beam theory (Sideris and Salehi 2016; Salehi and Sideris 2017), which eliminates the strain localization and response objectivity problems by utilizing a set of gradient-based nonlocality relations that ensure the continuity of section strains (e.g., curvature) over the element length, upon the occurrence of softening at any section. The GI element does not necessitate any certain form of constitutive relations and permits users to use the same constitutive relations used in conventional FB element formulations.

From the **user's perspective**, the `gradientInelasticBeamColumn` element has similar input to other force-based fiber elements' and the only additional parameter that this element requires is a characteristic length, l_c, which controls the spread of plasticity/damage in the vicinity of a softening location. In the simulation of RC beams/columns, this parameter can be taken equal to the plastic hinge length. If l_c equals zero, the GI beam element formulation turns into a conventional FB element formulation (i.e., as if the classical beam theory is used).

Validation Examples

Two examples of using GI FB beam-column elements are presented in the subsequent sections. For more examples on the application of the GI FB elements in the analysis of structures, see Salehi et al. (2017) and Salehi et al. (2020).

Example 1

An example of the ability of the GI FB element formulation to produce mesh-convergent softening responses is shown in Figure 3(a) for a column subjected to constant vertical load and monotonically increasing lateral displacement (Sideris and Salehi 2016). The entire column is modeled by a single GI FB element with a varying number of integration points, N, while l_c is equal to either zero (representing a conventional FB element) or cross-section depth (40 cm). The uniaxial material response considered herein is shown in Figure 3(b), while each integration point is represented by a fiber section. The numerical integration scheme used herein is the trapezoidal rule.

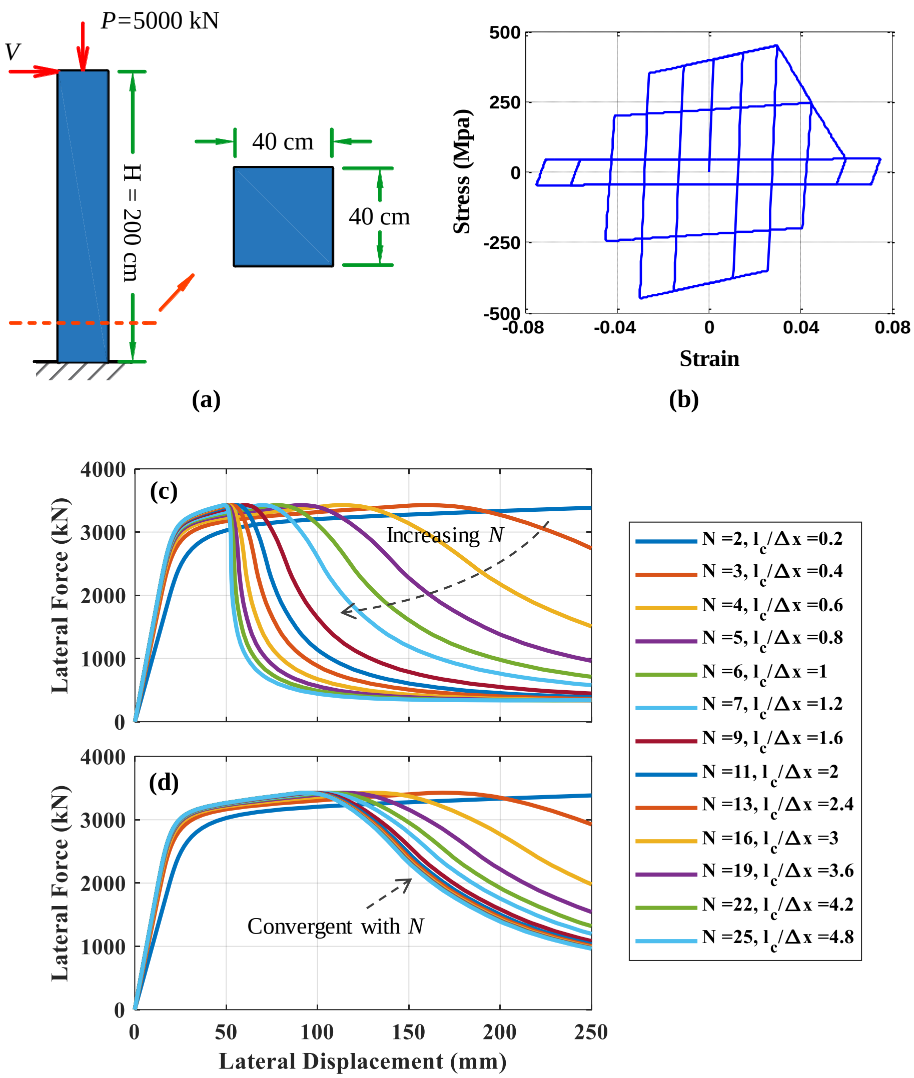

Figure 3. Response objectivity demonstration (Sideris and Salehi 2016): (a) selected column dimensions; (b) selected material response; (c) force-displacement response predictions with $l_c = 0$; (d) force-displacement response predictions with l_c = 40 cm (N = number of integration points; l_c = characteristic length; Δx = constant spacing of integration points – $l_c/\Delta x$ not applicable when $l_c = 0$)

According to Figure 3(c), when $l_c = 0$ (i.e., for conventional FB elements), the predicted column response does not converge with N, but exhibits faster and progressively more abrupt softening as N increases. However, for $l_c > 0$ (i.e., for GI FB element), the predicted column response converges to a single response as N increases, thereby showing response objectivity (Figure 3(d)). For the integration scheme considered herein (i.e., trapezoidal rule), the responses become convergent for values of N that lead to $l_c/\Delta x \geq 3$, where Δx denotes the constant spacing between the integration points ($\Delta x = L / (N - 1)$). If integration schemes of higher order of accuracy are used, such as the composite Simpson's rule (i.e., the default choice in `gradientInelasticBeamColumn` element in OpenSees) or Newton-Cotes, solution convergence is faster, usually for $l_c/\Delta x \geq 1.5$.

Example 2

The GI FB element's response prediction capabilities are further validated through the simulation of an RC column tested under cyclic lateral displacement (Salehi et al. 2020). The column dimensions, reinforcement details, and loading are depicted in Figure 4(a) (Goodnight et al. 2015; Goodnight et al. 2016) and it is modeled via a single GI FB beam-column element and a rigid (very stiff elastic) beam-column element (Figure 4(b)). The GI FB element consists of 15 equally-spaced integration points and the composite Simpson's integration rule is used – leading to $l_c/\Delta x \geq 1.5$. Each integration point was represented by a fiber section comprising longitudinal steel fibers, unconfined concrete fibers, and confined concrete fibers. The compressive stress-strain backbone curves considered for unconfined and confined concrete are shown in Figure 4(c). The characteristic length, l_c, is taken as plastic hinge length (~15 in).

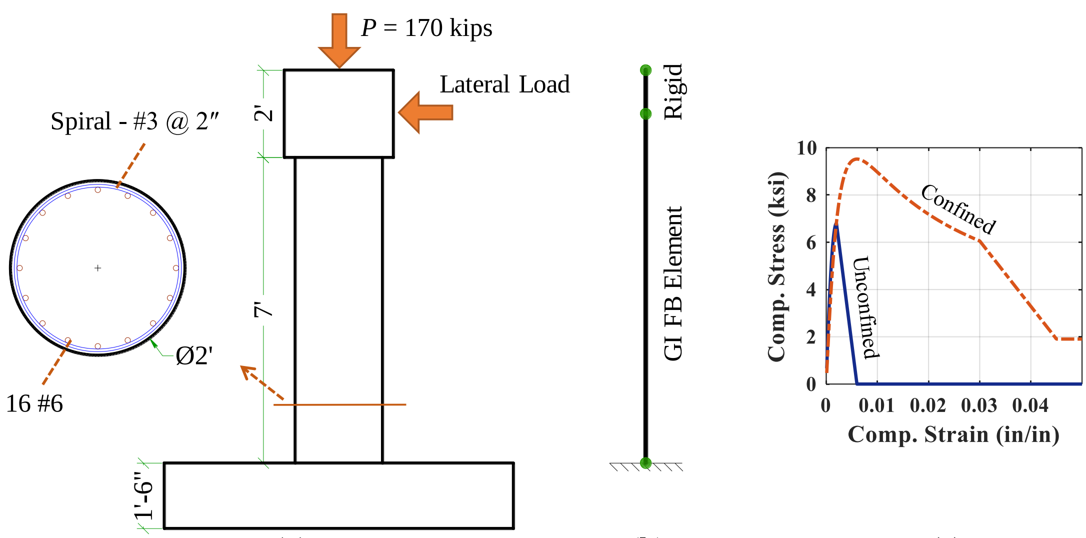

Figure 4. Comparison with experimental data (Salehi et al. 2020): (a) RC column dimensions, reinforcement details, and loading; (b) element configuration; (c) concrete stress-strain backbone curves; (d) force vs. displacement responses; (e) curvature distributions at different displacement demands

As shown in Figure 5(a), the experimental and simulated force-displacement responses are in a very good agreement. The *macroscopic* curvature distributions obtained from the simulation model are also very close to those measured during the test at different displacement demands (Figure 5(b)). These results show the ability of the GI FB element formulation to capture experimental test data in terms of both local (e.g., section forces/strains) and global quantities (e.g., nodal forces).

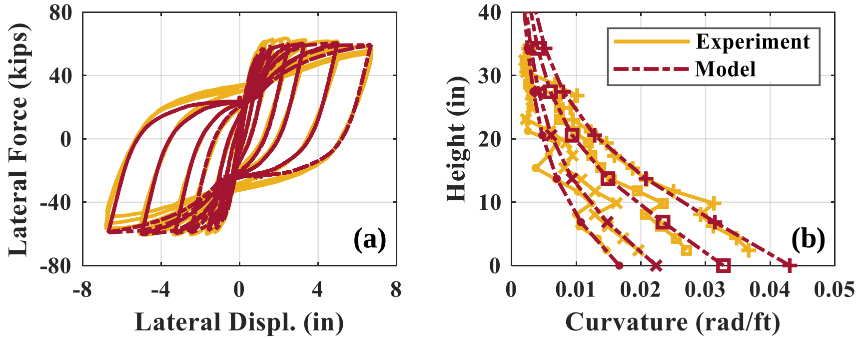

Figure 5. Comparison with experimental data (Salehi et al. 2020): (a) force vs. displacement responses; (b) curvature distributions at different displacement demands

References

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