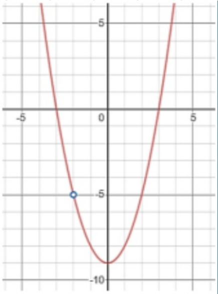
How to find limits using your calculator!

1. Type the given equation into your calculator.
2. Use the table feature & do in & out tables.
3. find the # were finding the limit of " $\lim_{x \to 3}$

4. Since limits are looking for the "y" of when the graph aproaches x=a, in this case 3, we need to find the y values from both sides.
5. Subtract & add .001, .01, .1 to the #, input these in the in & out table (in #2)
6. Look for what y value is being aproached.
7. That is your limit.

You can also type a # very close to the in this case "3" in order to make the calculator round it for ex $2.\overline{99}$ or 3.000001, This is your limit.

Consider the function, $f(x) = \frac{x^3+2x^2-9x-18}{x+2}$ where x≠-2



Look at its graph.
What is $f(x)$ approaching as x approaches -2?
Use your fingers and trace along the graph on both sides of -2.

The question above can be rewritten symbolically as $\lim_{x\to -2} \frac{x^3+2x^2-9x-18}{x+2}$ or as $\lim_{x\to -2} f(x)$.

Numerically (using a table of values)

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
f(x)							

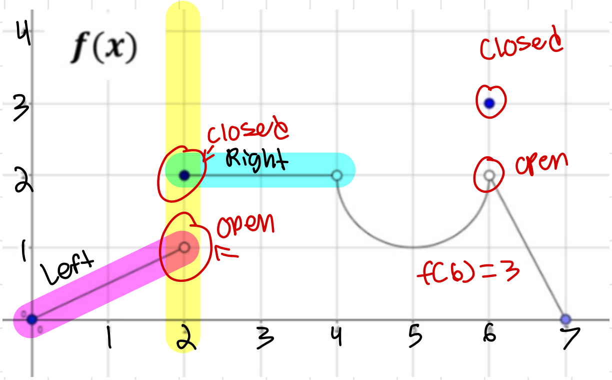
What happen to values of $f(x)$ as x approaches -2 from the left? ________

What happen to values of $f(x)$ as x approaches -2 from the right? ________

Therefore $\lim_{x\to -2} f(x) =$________

Verbally: The limit of the function, $f(x)$, as x approaches close to -2 is __________.

How to find limits using a graph



For this example we'll use the limit $\lim_{x \to 2} f(x)$

As you can see, when x is aproaching 2 there are two points (2,1) & (2,2) this means that this limit DNE.

This is not the case for one sided limits! $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$. Pay attention, if the # is raised to a "+" simbol then its a right sided limit, a "-" symbol is used for a left sided limit.

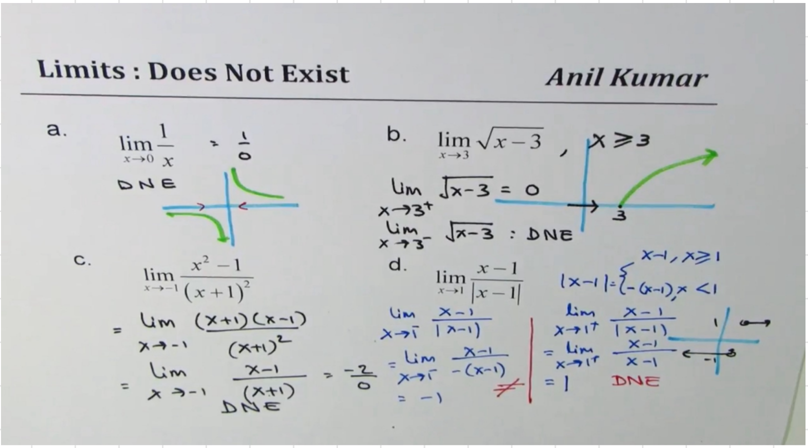
When x aproaches 2 from the left side, the limit is 1. When x aproaches 2 from the right side the limit is 2

There being two different limits is an indicator that the limit DNE.

f(2) can be tricky as there are two points when x=2, but look at both, when the circle is not filled it means that theres a "hole" in ur graph, so theres no true #. The filled circle is the correct answer. f(2)=2.

When theres only an open circle then f(2)= undifined.

Limits don’t exit when…

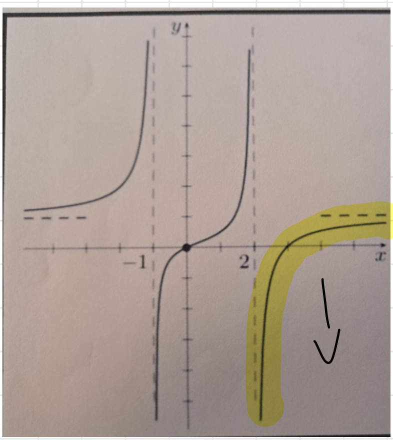


a. the denominator equals zero / verticle asymptotes looks like a.

b. Square root when radical equals zero, there wont be either left or right side limit, & if limits aren't the same then they DNE.

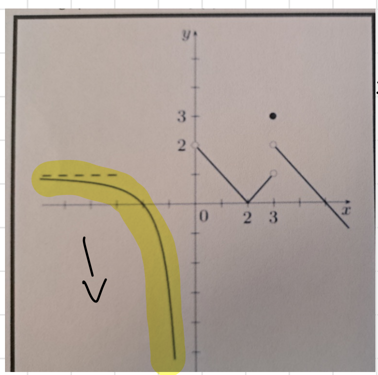
C. Again when the denominator equals zero it DNE.

When theres a one sided limit, it is prefered to use ∞ & $-\infty$ when theyre going to infinity & not DNE.



$\lim_{x\to 2^+} f(x) = -\infty$

When x aproaches 2 from the right side its going towards negative infinity.



$\lim_{x\to 0^-} f(x) = -\infty$

When x is apraching 0 from the left its going towards negative infinity.

The Property of Limits

b & c are real #'s, f & g are functions. $f(x) = L$ & $g(x) = K$

1. Scalar Multiple $\lim_{x \to c} bf(x) = bL$

This is used to move the "b" infront, to be multiplied by the answer of $f(x) = L \cdot b$

2. Sum / Difference $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$

* Remember L & K are possible answers to <u>Both</u> functions, you literally just add them.

3. Product: $\lim_{x \to c} [f(x) \cdot g(x)] = L \cdot K$

4. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$

5. Power $\lim_{x \to c} [f(x)]^n = L^n$

1. Use the table for each of the to find the given limits.

$\lim_{x \to 3} f(x) = 4$	$\lim_{x \to -3} f(x) = 2$	$\lim_{x \to 3} g(x) = 1$	$\lim_{x \to -3} g(x) = 5$

a. $\lim_{x \to 3} (2f(x) + g(-x)) =$

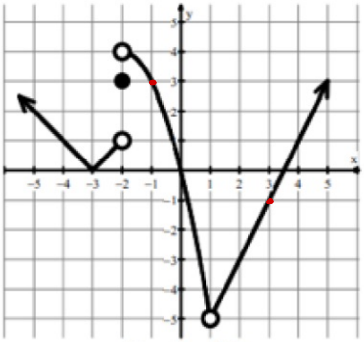
b. $\lim_{x \to -3} \left(\frac{g(x)}{f(-x)}\right) =$

1. input the value of $f(x)$ as $x \to 3$, which is 4 (on table)
2. now multiply it by 2
3. input the value of $g(x)$ as $x \to -3$ * We used the $x \to -3$ function because of the "-" in $g(-x)$, which makes it -3.
4. add & solve

1. $(2(4) + g(-x))$
2. $(8 + g(-x))$
3. $(8 + 5)$
3. $= 13$

Use the graph for each problem to find the given limits.

3.

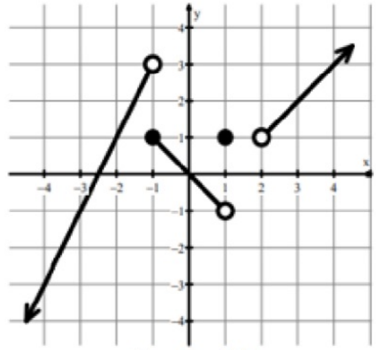


Graph of f

a. $\lim_{x\to 3} f(f(x)) =$ 3

b. $\lim_{x\to 1} f(f(x)) =$

4.



Graph of f

a. $\lim_{x\to 2} f(f(x)) =$

b. $\lim_{x\to 4} f(f(x)) =$

3a. f(f(x)). Focus on the parentheses in the inside first.
f(-1), as x-> 3 on the f(x) graph y="-1", this is now our new limit. we're now looking for f(-1).
f(-1) as x->-1 on the f(x) graph y="3"

5. Use the table for each problem to find the given limits.

$f(1) = 4$	$g(1) = -2$	$h(1) = -3$
$\lim_{x\to 1} f(x) = -1$	$\lim_{x\to 1} g(x) = 3$	$\lim_{x\to 1} h(x) = 6$

a. $\lim_{x\to 1}\left((f(x))^2 - h(x)\right) - g(1) =$ -3

b. $f(1) + \lim_{x\to 1}(-g(x)) =$ 1

5a. f(x) as x->1 = -1
Square it = 1
h(x) as x->1 = 6
g(1) = -2
now, put it together.
1-6+2 * --2=+2
= -3

5b. f(1) = 4
g(x), as x->1 = 3
now put it together
4-3,
=1

Limits algebraically

1. Aways input the x value & pray you get a # ≠ $\frac{0}{0}$
2. If you get $\frac{0}{0}$ then there are different paths for you to take.

a. Square roots.

$$\lim_{x \to 0} \frac{\sqrt{x+a} - \sqrt{a}}{x}$$

1. multiply the fraction by the $\frac{n}{n}$ if the radical is on the numerator, but $\frac{d}{d}$ if radical on denominator.
2. pay attention to the sign after the first radical $\sqrt{x+a} - \sqrt{a}$, on the $\frac{n}{n}$ or $\frac{d}{d}$ side you want to switch these, so they can cancel out. it should look like below
3. $\lim_{x \to 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} \cdot \frac{\sqrt{x+a} + \sqrt{a}}{\sqrt{x+a} + \sqrt{a}}$
4. When two radicals multiply the radicals cancel out $\sqrt{x+a}^2 = \sqrt{x+a} \cdot \sqrt{x+a}$ Both equal $x+a$.
5. multiply.
6. $\lim_{x \to 0} \frac{x+a-a}{x(\sqrt{x+a} + \sqrt{a})}$

7 the a's in the n cancel out

$x + a - a$

8 the x's cancel out leaving 1 as the n NOT 0.

$$\frac{\cancel{x}}{\cancel{x}(\sqrt{x+a} + \sqrt{a})} = \frac{1}{\sqrt{x+a} + \sqrt{a}}$$

9. input the x, which is zero

$$\frac{1}{\sqrt{0+a} + \sqrt{a}} \to \frac{1}{\sqrt{a} + \sqrt{a}}$$

10. add

$\frac{1}{2\sqrt{a}}$ * its illegal to have a radical in the denominator. multiply it by the reciprocle (can't spell)

$$\frac{1}{2\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}}$$

roots cancel out

$$\frac{\sqrt{a}}{2a}$$

* Note: Sometimes it wont be $\sqrt{x+a} \pm \sqrt{a}$ the operation will be = to that of whats needed to cancel for ex.

$$\frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{x-3} \to \frac{x+1-4}{x-3(\sqrt{x+1} + 2)} \to \frac{\cancel{x-3}}{\cancel{x-3}(\sqrt{x+1} + 2)}$$

* now they can cancel out!

Limits algebraically

b. factoring

$$\lim_{x \to -5} \frac{x^2-25}{x^2+7x+10}$$

1. In order to solve you must factor BOTH the top AND Bottom.
2. $x^2-25 \to (x+5)(x-5)$
3. $x^2+7x+10$ is a bit tricky, in this scenerio you'll mutiply the leading Coeficient by the "c"

   $1x^2+7x+10 = 10$
4. Now what multiplies to 10 but adds up to b, in this case 7.

   10

   $2 + 5$
5. Rewrite this time using your to answers & add them seperately.

   $x^2+2x+5x+10$, technically its still 7x just written differently.
6. You can now factor by grouping.
7. $x^2+2x \to x(x+2)$

   $5x+10 \to 5(x+2)$
8. You know you did it right when whats in the paranthesis is the same "x+2"
9. now drag the x & 5 along with the x+2.

   $x(x+2)\ 5(x+2)$

   $(x+5)\ (x+2)$
10. Put it together

$$\frac{(\cancel{x+5})(x-5)}{(\cancel{x+5})(x+2)}$$

11. Cancel out the x+5's

$$\frac{x-5}{x+2}$$

12. now substitute x by the lim x→#

$$\frac{-5-5}{-5+2}$$

13. evaluate

$-10/3$

infinity limits

$a>b = +\infty, -\infty$

$a=b = \frac{a}{b}$

$a<b = 0$

$$\lim_{x\to\infty} \frac{(3x-2)(6+5x^3)}{(1-7x^2)(5x+6)}$$

$$\frac{15x^4}{-35x^3}$$

$$\frac{15x}{-35}$$

$$\frac{15(\infty)}{-35}$$

$= -\infty$

$$\lim_{x\to\infty} \frac{-7(2+9x^3)}{(6x^3-5)(x+4)}$$

$$\frac{-63x^3}{6x^4}$$

$b>a$

$=0$

$$\lim_{x\to\infty} \frac{12x^2-54x^5-10+45x^3}{7x^5+56x^3+2x^2+16}$$

$$\frac{-54x^5}{7x^5}$$

$a=b, \frac{c}{d}$

$-\frac{54}{7}$

$$\lim_{x\to\infty} \frac{21x^2-58x+21}{54-33x-10x^2}$$

$$\frac{21x^2}{-10x^2}$$

$a=b, \frac{c}{d}$

$-\frac{21}{10}$

$$\lim_{x\to\infty} \frac{(1+10x^3)(4+x^2)}{(10-x^3)(x^2-3)}$$

$$\frac{10\cancel{x^5}}{-\cancel{x^5}}$$

-10