Class slides for Thursday, October 24: Weather, Empirical

Matthew J. Salganik

COS 597E/SOC 555 Limits to prediction Fall 2020, Princeton University

$$x' = \sigma(y - x)$$

$$y' = x(\rho - z) - y$$

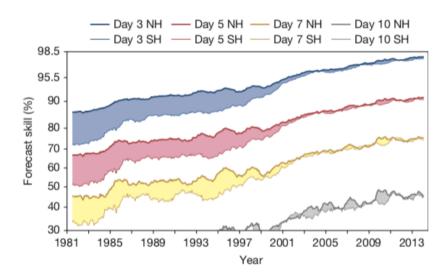
$$z' = xy - \beta z$$

$$\sigma = 10, \rho = 28, \beta = 8/3$$



The quiet revolution of numerical weather prediction

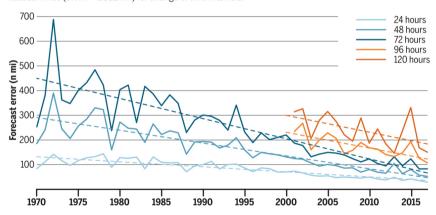
Peter Bauer¹, Alan Thorpe¹ & Gilbert Brunet²



This is impressive.

Advances in hurricane prediction

Data from the NOAA National Hurricane Center (NHC) (13) show that forecast errors for tropical storms and hurricanes in the Atlantic basin have fallen rapidly in recent decades. The graph shows the forecast error in nautical miles (1 n mi = 1.852 km) for a range of time intervals.



This is impressive. Source: Alley et al 2019

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- Many groups make public predictions every day at multiple time scales (5-day forecast, 10-day forecast), and we can all see how accurate they are
- ► No self-fulfilling or self-defeating processes
- ► No concerns about causality
- Predictions based a real physical model
- Business and governments invests in improved performance

So even though this system is fundamentally unpredictable it has a lot going for it.

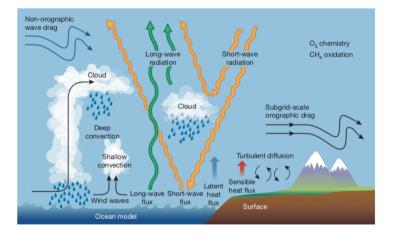
They don't give up despite unpredictability. Here are 4 key ideas that we might learn

from them.



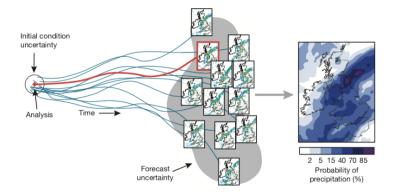
Parameterization: we don't have to model all sub-systems to include their impacts.

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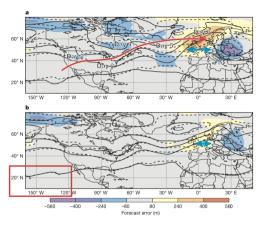
Example: multiple systems impacting the spread of COVID

Ensemble forecasting: we want to average over many models and many initial conditions.



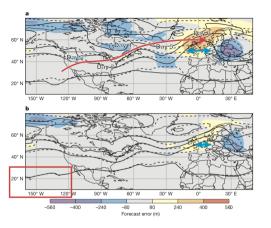
Example: pre-trial risk assessment

Model initialization is key (not just measuring the outcome)



► Contrast with Tetlock and Canadian CIA where we just focused on measuring the outcome

Model initialization is key (not just measuring the outcome)



- ► Contrast with Tetlock and Canadian CIA where we just focused on measuring the outcome
- ► Crudely, errors come from two places: 1) model equations don't match true atmospheric equations, 2) bad initial conditions.

Massive global cooperation



Contrast with the scale we normally work at in sociology

- 4 key ideas that we might learn from them
 - parameterization
 - ensemble forecasting
 - model initialization
 - massive global cooperation

CHAPTER 3 Our Chaotic Weather

ldea 1

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ldea 1

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Might this be related to simple vs complex models?

- ► Forecasting tides: we are trying to predict the highly predictable regular response.
- ► Forecasting weather: we are trying to predict things beyond the highly predictable regular response (e.g., summer is warmer than winter)

- Might this be related to simple vs complex models?
- ► This reminds me of the question of whether we care about absolute prediction accuracy or prediction accuracy improvement relative to some benchmark

- ► Forecasting tides: we are trying to predict the highly predictable regular response.
- ► Forecasting weather: we are trying to predict things beyond the highly predictable regular response (e.g., summer is warmer than winter)

- Might this be related to simple vs complex models?
- ► This reminds me of the question of whether we care about absolute prediction accuracy or prediction accuracy improvement relative to some benchmark
- When do we want to oceanographers and when do we want to be meteorologists?

The unperformable experiment: Change one atmosphere and leave another atmosphere unchanged, see how they diverge. Alternatives

► Look for twins

Atmospheric Predictability as Revealed by Naturally Occurring Analogues

EDWARD N. LORENZ

Dept. of Meteorology, Massachusetts Institute of Technology, Cambridge, Mass.¹ (Manuscript received 2 April 1969)

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The unperformable experiment: Change one atmosphere and leave another atmosphere unchanged, see how they diverge. Alternatives

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- Physical models (dishpans)
- Computer simulations of mathematical models (To be continued . . .)

Global Atmospheric Research Program (GARP). Goal/selling point:

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What if we made that switch for other problems?

Even in a chaotic atmosphere some things are predictable (winds at high level in equatorial region)

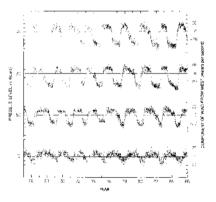


Figure 34. The points, some of which are too closely packed to be individually distinguishable, show daily values of the eastward component of the wind at the 70-, 50-, 30-, and 20-millibar surfaces over Singapore, from 1965 through 1985 as indicated by the scale at the hase. The solid lines are zero-lines. Values above zero indicate winds from the west. The approximate two-vear periodicity is evident.

Four points:

► Sometimes we want to predict the typical outcome and when do we want to do better than predict the typical outcome

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Four points:

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- ▶ Sometimes we want to make accurate predictions and sometimes we should study whether accurate predictions are possible
- Even in a chaotic atmosphere some things are predictable (winds at high level in equatorial region)

Atmospheric predictability experiments with a large numerical model

By E. N. LORENZ, European Centre for Medium Range Weather Forecasts, Reading RG2 9AX,

England

Upper and lower bound on predictability with no extra computing!



model from the European Center for Medium Range Weather Forecasts (ECMWF)

▶ How accurate are predictions? This gives lower bound on accuracy.

We want to understand the upper bound and lower bound for accuracy using the model from the European Center for Medium Range Weather Forecasts (ECMWF)

▶ How accurate are predictions? This gives lower bound on accuracy.

► How much do two similar initial conditions diverge? This gives an upper bound on accuracy.

We'd like to do this without running the model many times because running the model is expensive.

Model produces "prognoses":

- ▶ 1 Dec: 0 day, 1 day, 2 days 10 days
- ▶ 2 Dec: 0 day, 1 day, 2 days . . . , 10 days
- ▶ 3 Dec: 0 day, 1 day, 2 days . . ., 10 days

- ▶ 10 March: 0 day, 1 day, 2 days . . . , 10 days

▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ..., 10 days (11 Dec)

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . ., 10 days (11 Dec)
- ▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)

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- ▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)
- ▶ 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)
- ▶ 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) . . ., 10 days (14 Dec)

How accurate are 1 day forecasts? This is equivalent to asking: What is E_{01} ?

▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ..., 10 days (11 Dec)

▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)

▶ 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)

▶ 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), ..., 10 days (14 Dec)

How accurate are 1 day forecasts? This is equivalent to asking: What is E_{01} ?

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▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . . , 10 days (11 Dec)
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- ▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . . , 10 days (12 Dec)
- ▶ 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)
- ▶ 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), ..., 10 days (14 Dec)

How accurate are 2 day forecasts? This is equivalent to asking: What is E_{02} ?

▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ..., 10 days (11 Dec)

2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)

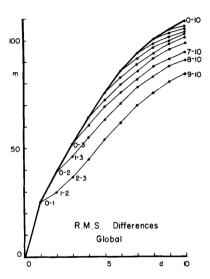
▶ 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)

▶ 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), ..., 10 days (14 Dec)

How accurate are 2 day forecasts? This is equivalent to asking: What is E_{02} ?

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 $E_{01}, E_{02}, \dots E_{010}$ is the heavy curve at the top

about	two	trajectories	that start	close	together?	How f	ast do t	hey div	erge?

What

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- ▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . .
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Distance between 0 day (2 Dec) and 1 day (2 Dec): ϵ

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- ▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . .
- ▶ 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) . . .
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Distance between 0 day (2 Dec) and 1 day (2 Dec): ϵ Distance between 1 day (3 Dec) and 2 days (3 Dec): ϵ

Distance between 1 day (3 Dec) and 2 days (3 Dec): $c \cdot \epsilon$ for (c > 1)

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . .
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Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . .
- ▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . .
- ▶ 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) . . .
- ▶ 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) . . .

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ

Distance between 1 day (4 Dec) and 2 days (4 Dec): $c \cdot \epsilon$ for (c > 1)

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . .
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- •

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ (E_{01}) Distance between 1 day (4 Dec) and 2 days (4 Dec): $c \cdot \epsilon$ for (c > 1)

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . .
- ▶ 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . .
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Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ (E_{01}) Distance between 1 day (4 Dec) and 2 days (4 Dec): $c \cdot \epsilon$ for (c > 1) (E_{12})

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec) . . .
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Distance between 2 day (5 Dec) and 3 days (5 Dec): $c \cdot \epsilon$ for (c > 1)

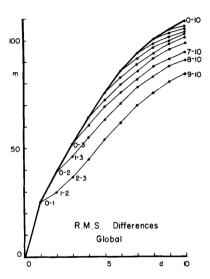
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Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ (E_{01})

Distance between 2 day (5 Dec) and 3 days (5 Dec): $c \cdot \epsilon$ for (c > 1) (E_{23})



 $E_{12}, E_{23}, \dots E_{910}$ is the light curve at the bottom

What about two trajectories that start close together? How fast do they diverge?	

What about two trajectories that start close together? How fast do they diverge? What about two trajectories that start a bit further apart? How fast do they diverge

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) . . .
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Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ

- ▶ 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) . . .
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Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ (E_{02}) Distance between 1 day (4 Dec) and 3 days (4 Dec): $c \cdot \epsilon$ for (c > 1)

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Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ (E_{02}) Distance between 1 day (4 Dec) and 3 days (4 Dec): $c \cdot \epsilon$ for (c > 1) (E_{13})

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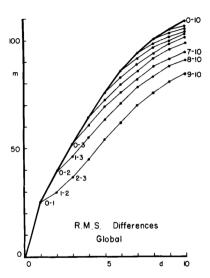
Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ Distance between 2 day (5 Dec) and 4 days (5 Dec): $c \cdot \epsilon$ for (c > 1)

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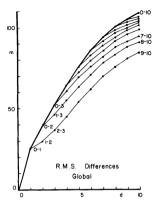
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- ▶ 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) . . .
- •

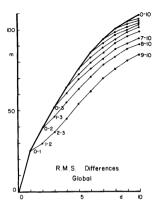
Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ (E_{02}) Distance between 2 day (5 Dec) and 4 days (5 Dec): $c \cdot \epsilon$ for (c > 1) (E_{24})



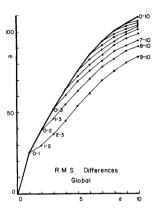
 $E_{12}, E_{23}, \dots E_{910}$ is the light curve second from the bottom



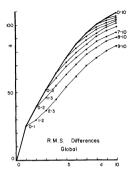
► Heavy curve (top) compares two different equations: "true" atmosphere and ECMWF



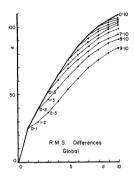
- ► Heavy curve (top) compares two different equations: "true" atmosphere and ECMWF
- ▶ Thin curves compare the same equations: ECMWF and ECMWF



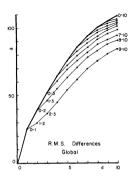
- ► Heavy curve (top) compares two different equations: "true" atmosphere and ECMWF
- ▶ Thin curves compare the same equations: ECMWF and ECMWF
- "The excess slope of the heavy curve over that of an intersection thin curve may therefore be regarded as a measure of the maximum amount by which the model may still be improved." Have you seen this anywhere else?



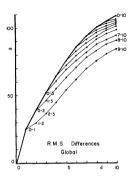
► The curves should level off at the same value: RMS difference between two randomly chosen weathers ("analyses"), RMS difference between two randomly chosen forecasts ("prognoses"). Call this leveling off point "the ceiling".



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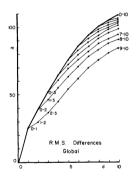


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- ▶ $E_{01} = 25$ which takes about 3.5 days to double.

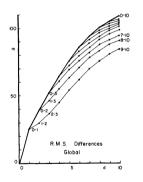


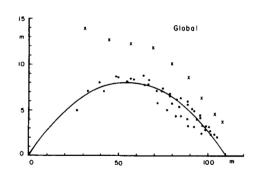
- ► The curves should level off at the same value: RMS difference between two randomly chosen weathers ("analyses"), RMS difference between two randomly chosen forecasts ("prognoses"). Call this leveling off point "the ceiling".
- ► How quickly do we get to the ceiling?
- $ightharpoonup E_{01} = 25$ which takes about 3.5 days to double.
- ▶ Under some assumptions, time for errors to go from $\frac{1}{3}$ to $\frac{1}{2}$ = time from $\frac{1}{2}$ to $\frac{2}{3}$ = the time that small errors double. Based on Fig 1, this is 2.5 days.

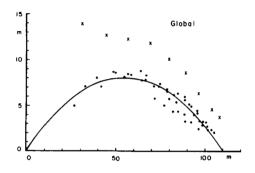
What is the rate of change of the error (dE/dt) as a function of the error (E)? This is related to how long it will take us to get to the ceiling.



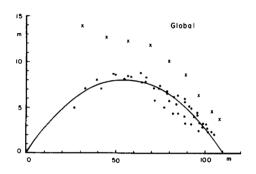
What is the rate of change of the error (dE/dt) as a function of the error (E)? This is related to how long it will take us to get to the ceiling.







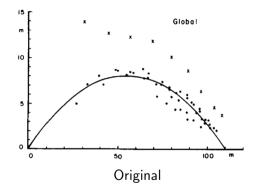
x-axis: error, y-axis: rate of growth of error

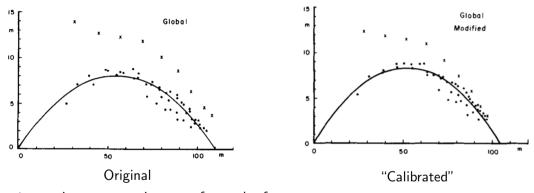


- x-axis: error, y-axis: rate of growth of error
- ▶ difference between crosses (prognosis vs analysis) and dots (prognosis vs prognosis) shows room for improvement in prognosis

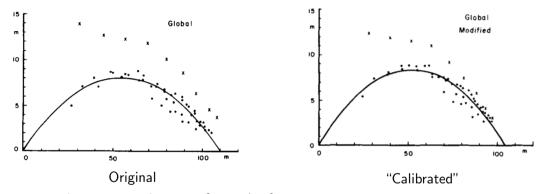
But are these improvements really possible? Yes, he shows in Sec 4 (the part I didn't

assign)

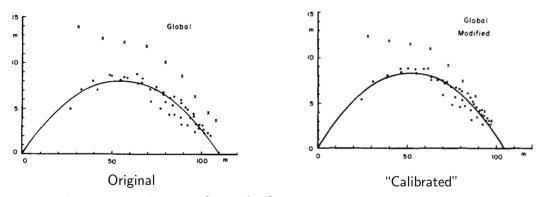




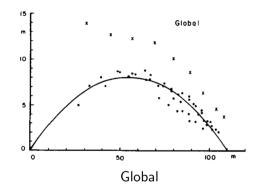
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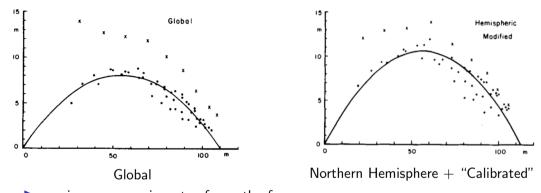


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- "Calibrated" prognoses make crosses closer to dots.

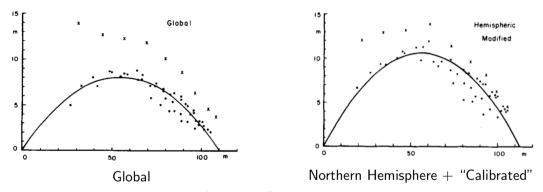


- x-axis: error, y-axis: rate of growth of error
- "Calibrated" prognoses make crosses closer to dots.
- ▶ Improvements likely to come when errors are mid-sized not large



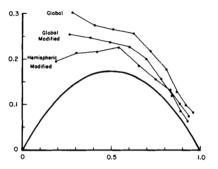


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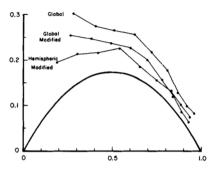


x-axis: error, y-axis: rate of growth of error

► In Norther Hemisphere difference between crosses (prognosis vs analysis) and dots (prognosis vs prognosis) is smaller



"Accepting the modified ECMWF model, applied to the NH, as a state-of-the-art model, we find that we have established upper and lower bounds to atmospheric predictability which are reasonably close together."

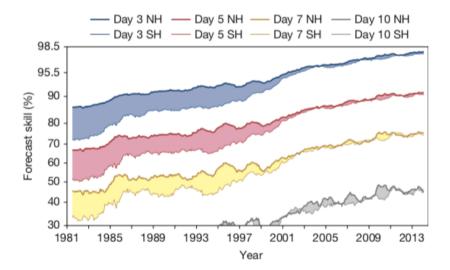


- "Accepting the modified ECMWF model, applied to the NH, as a state-of-the-art model, we find that we have established upper and lower bounds to atmospheric predictability which are reasonably close together."
- ► "Assuming that we have correctly estimated the doubling time, we find that, even without further improvements in one-day forecasting, we may eventually make 10 day forecasts as good as present 7 day forecasts"

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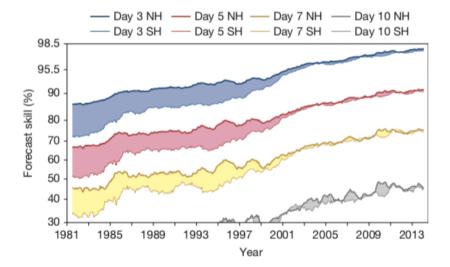
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Atmospheric predictability revisited

By LIZZIE S. R. FROUDE, LENNART BENGTSSON and

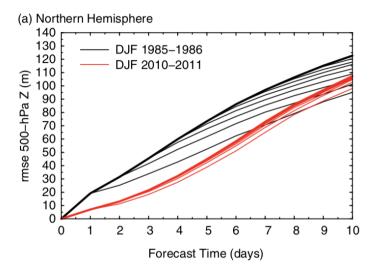
KEVIN I. HODGES*, Environmental Systems Science Centre (ESSC), University of Reading, Harry Pitt Building, Whiteknights P.O. Box 238, Reading, Berkshire RG6 6AL, UK

(Manuscript received 20 June 2012: in final form 21 May 2013)

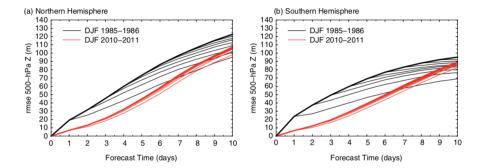
ABSTRACT

This article examines the potential to improve numerical weather prediction (NWP) by estimating upper and lower bounds on predictability by re-visiting the original study of Lorenz (1982) but applied to the most recent version of the European Centre for Medium Range Weather Forecasts (ECMWF) forecast system, for both the deterministic and ensemble prediction systems (EPS). These bounds are contrasted with an older version of the same NWP system to see how they have changed with improvements to the NWP system. The computations were performed for the earlier seasons of DJF 1985/1986 and JJA 1986 and the later seasons of DJF 2010/2011 and JJA 2011 using the 500-hPa geopotential height field. Results indicate that for this field, we may be approaching the limit of deterministic forecasting so that further improvements might only be obtained by improving the initial state. The results also show that predictability calculations with earlier versions of the model may overestimate notential forecast skill, which may be due to insufficient internal variability in the model and because recent versions of the model are more realistic in representing the true atmospheric evolution. The same methodology is applied to the EPS to calculate upper and lower bounds of predictability of the ensemble mean forecast in order to explore how ensemble forecasting could extend the limits of the deterministic forecast. The results show that there is a large potential to improve the ensemble predictions, but for the increased predictability of the ensemble mean, there will be a trade-off in information as the forecasts will become increasingly smoothed with time. From around the 10-d forecast time, the ensemble mean begins to converge towards climatology. Until this point, the ensemble mean is able to predict the main features of the large-scale flow accurately and with high consistency from one forecast cycle to the next. By the 15-d forecast time, the ensemble mean has lost information with the anomaly of the flow strongly smoothed out. In contrast, the control forecast is much less consistent from run to run, but provides more detailed (unsmoothed) but less useful information.

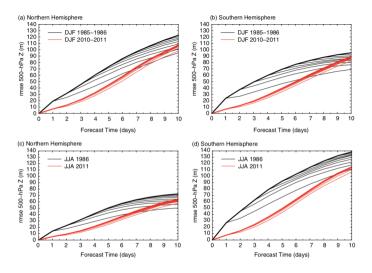
Keywords: numerical weather prediction, predictability, predictive skill, ensemble prediction



Note the compression between thick and think lines. DJF = Dec, Jan, Feb



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Note compression between thick and think lines. DJF = Dec, Jan, Feb; JJA = Jun, Jul, Aug

Rough summary of insight from Lorenz (1982): by simulating weather we can learn how much weather simulation can be improved. That seems surprising and potentially valuable. This seems to require that we attempt to match our predictions to real data.



3 takeways

▶ It is possible to make long-term improvements in predictability so what we can do now is not the limit of what is possible (limitation of brute force approach, could remind you of deep learning and image recognition)

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remind you of deep learning and image recognition)

➤ Two promising approaches that we might borrow for other domains: 1) Approach over Lorenz (1982) seems to offer some way of assessing how much more improvement is possible 2) Ensemble forecasting seems promising

Class slides for Thursday, October 24: Weather, Empirical

Matthew J. Salganik

COS 597E/SOC 555 Limits to prediction Fall 2020, Princeton University