Class slides for Tuesday, October 20: Weather, Lorenz attractor

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COS 597E/SOC 555 Limits to prediction
Fall 2020, Princeton University

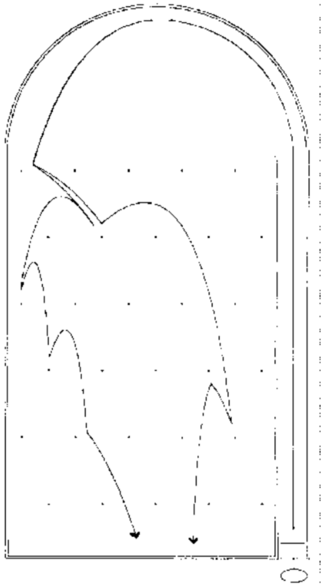
“My other classes make me feel smarter and this class makes me feel dumber.”
- An undergraduate at Columbia about 10 years ago.

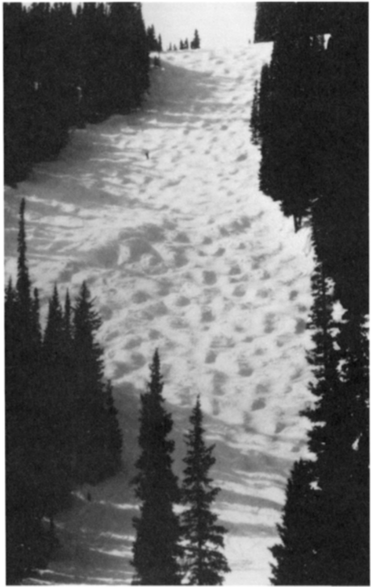
"My other classes make me feel smarter and this class makes me feel dumber."
- An undergraduate at Columbia about 10 years ago.

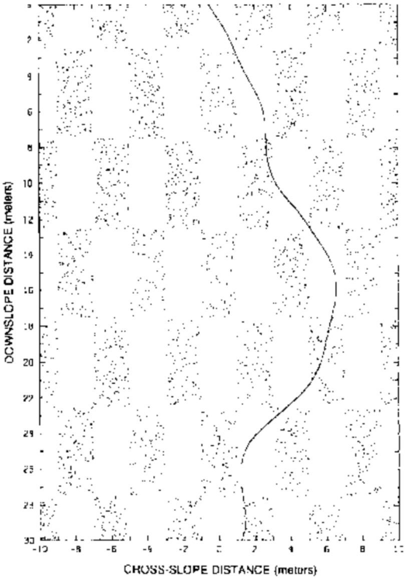
- classes that help you answer questions
- classes that help you ask questions

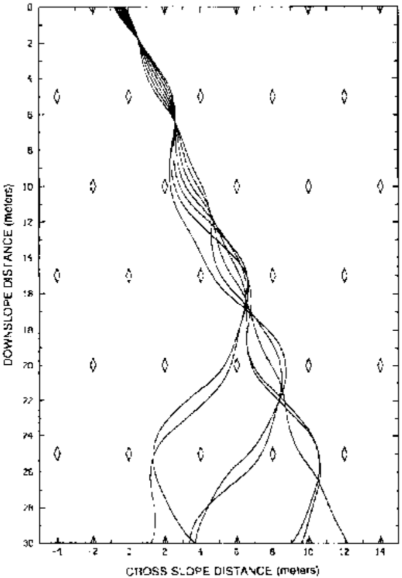
Lorenz's work on deterministic chaos "profoundly influenced a wide range of basic sciences and brought about one of the most dramatic changes in mankind's view of nature since Sir Isaac Newton."
-1991 Kyoto Prize for basic sciences in the field of earth and planetary sciences.

Deterministic and unpredictable






DOWNSLOPE DISTANCE (meters)
CROSS-SLOPE DISTANCE (meters)



DOWNSLOPE DISTANCE (meters)
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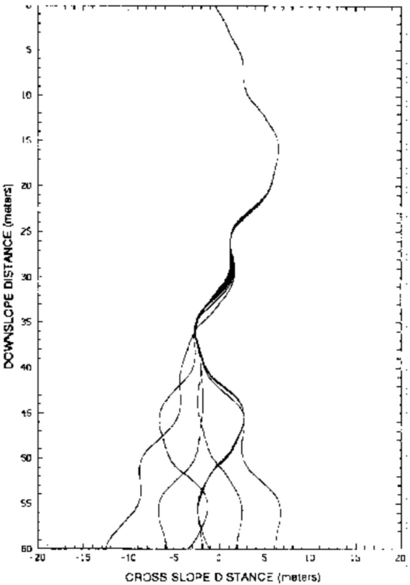
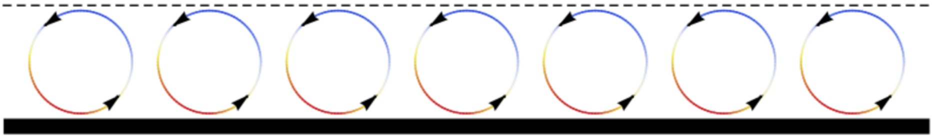

DOWNSLOPE DISTANCE (meters)
CROSS SLOPE DISTANCE (meters)

Sensitive dependence on initial conditions: two nearly identical initial conditions diverge until they bear not more resemblance than two states chosen randomly

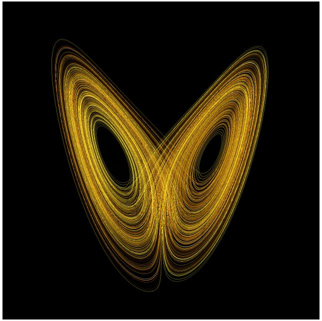


- x: the rate of convective motion - i.e. how fast the rolls are rotating,
- y: the temperature difference between the ascending and descending currents
- z: the distortion (from linearity) of the vertical temperature profile.

https://marksmath.org/visualization/LorenzExperiment/

$$x' = \sigma(y - x)$$
$$y' = x(\rho - z) - y$$
$$z' = xy - \beta z$$

$$\sigma = 10, \rho = 28, \beta = 8/3$$



https://commons.wikimedia.org/wiki/File:Lorenz_system_r28_s10_b2-6666.png

DEMO: `https://marksmath.org/visualization/LorenzExperiment/`

BREAK INTO GROUPS

If we reduce measurement error how much time does that buy us?

If we reduce measurement error how much time does that buy us? Not much

Simulation results, switch to R

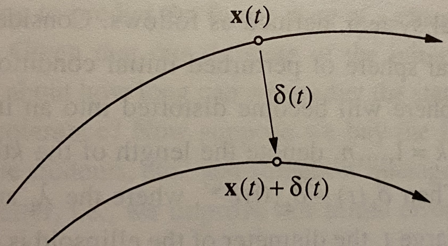


Figure 9.3.4

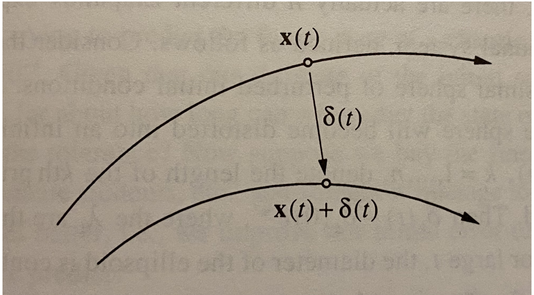


Figure 9.3.4

It turns out that:

$$\underbrace{\|\delta(t)\|}_{\text{divergence at time } t} \sim \underbrace{\|\delta_0\|}_{\text{initial divergence}} \underbrace{e^{\lambda t}}_{\text{rate of growth}}$$

where:

λ is called the Liapunov exponent and it varies from system to system

Strogatz (1994), Sec 9.3

Let a be a measure of tolerance and $t_{horizon}$ be the time that the discrepancy between two trajectories exceeds our tolerance, it turns out that

$$t_{horizon} \sim O\left(\frac{1}{\lambda} ln \frac{a}{\|\delta_0\|}\right)$$

Because of the logarithmic dependence on $\|\delta_0\|$, we need order of magnitude decreases in $\|\delta_0\|$ in order to produce one extra multiple of $\frac{1}{\lambda}$. For Lorenz system $\lambda \approx 0.9$.

Strogatz (1994), Sec 9.3

Compare and contrast to other work

These systems seem to be unpredictable in some ways and predictable in other ways:
https://www.youtube.com/watch?v=6YDHBFVIvIs

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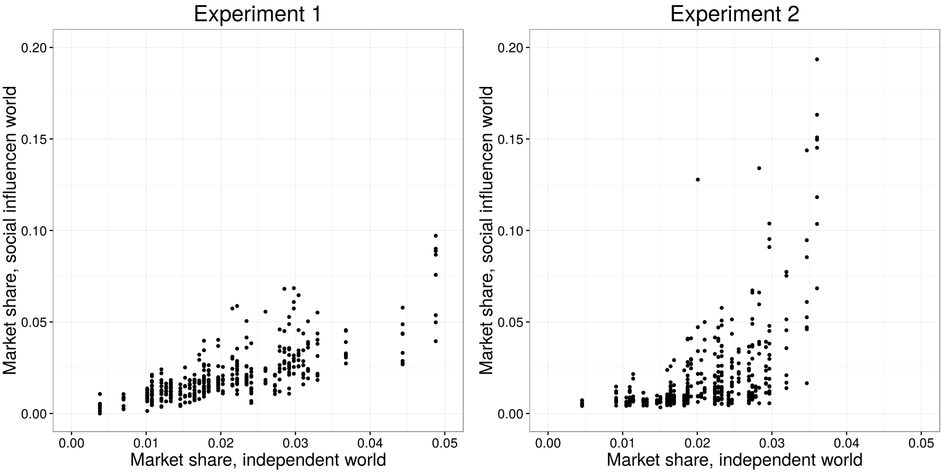

Can Cascades be Predicted?

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Deals with time in a different way

War Is in the Error Term

Erik Gartzke

Lorenz system more clearly shows a limit to prediction in a model that seems reasonable (to me)

Showing sensitive dependence seems like a rigorous way to show a limit to prediction, but it seems detached from how we talked about measuring predictability last class (e.g., R^2, MSE, etc) and how we talk about unpredictability (e.g., Bayes error rate)

Data generating process in
statistics/machine learning

$$y = f(x) + \epsilon$$

Data generating process in dynamical systems

Data generating process in statistics/machine learning

$$y = f(x) + \epsilon$$

$$x' = \sigma(y - x)$$
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$$\sigma = 10, \rho = 28, \beta = 8/3$$

Thinking back to Breiman, I wonder if these are different cultures with different approaches to predictability

Can deep learning predict chaotic systems?

Can ~~deep learning~~ reservoir computing predict chaotic systems?

Model-Free Prediction of Large Spatiotemporally Chaotic Systems from Data: A Machine Learning Approach

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(Dated: December 12, 2017)

We demonstrate the effectiveness of using machine learning for model-free prediction of spatiotemporally chaotic systems *of arbitrarily large spatial extent and attractor dimension* purely from observations of the system's past evolution. We present a parallel scheme with an example implementation based on the reservoir computing paradigm and demonstrate the scalability of our scheme using the Kuramoto-Sivashinsky equation as an example of a spatiotemporally chaotic system.

https://doi.org/10.1103/PhysRevLett.120.024102, about 300 citations

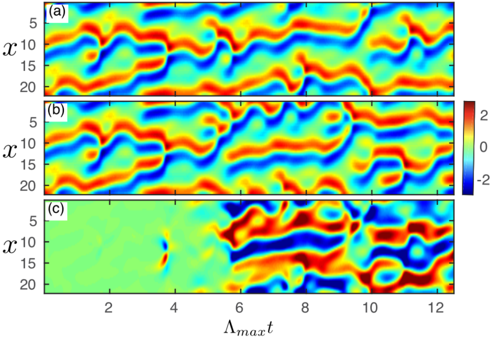

FIG. 2. Prediction of a KS equation with $L = 22$, $\mu = 0$ using a single reservoir of size $D_r = 5000$. (a) Actual data from the KS model. (b) Reservoir prediction. (c) Error (panel (b) minus panel (a)) in the reservoir prediction. We multiply t by the largest Lyapunov exponent (Λ_{max}) of the model, so that each unit on the horizontal axis represents one Lyapunov time, i.e., the average amount of time for errors to grow by a factor of e.

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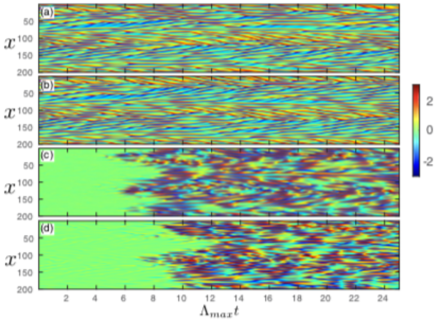

FIG. 4. Prediction of KS equation ($L = 200$, $Q = 512$, $\mu = 0.01$, $\lambda = 100$) with the parallelized reservoir prediction scheme using $g = 64$ reservoirs. (a) Actual KS equation data. (b) Reservoir prediction ($\tilde{\mathbf{u}}(t)$). (c) Error in the reservoir prediction. (d) Error in a prediction made by integrating the KS equation when it uses the reservoir output at $t = 0$, $\tilde{\mathbf{u}}(0)$, as its initial condition.

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- Deterministic systems can be unpredictable
- Sensitive dependence on initial conditions seems like a formal way to show unpredictability, but it seems different from our other measures of predictability and unpredictability
- It now seems hard to imagine any prediction without a time-window attached to it (e.g., a 3-day forecast or a 5-day forecast)

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