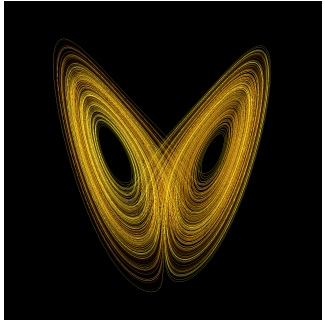
Pre-read for Thursday, October 24: Weather, Empirical

Matthew J. Salganik

COS 597E/SOC 555 Limits to prediction
Fall 2020, Princeton University

$$x' = \sigma(y - x)$$
$$y' = x(\rho - z) - y$$
$$z' = xy - \beta z$$

$$\sigma = 10, \rho = 28, \beta = 8/3$$



https://commons.wikimedia.org/wiki/File:Lorenz_system_r28_s10_b2-6666.png

CHAPTER 3
Our Chaotic Weather

Reading notes

- Forecasting the weather vs forecasting the tides, think about how this relates to absolute measures of predictability vs measures relative some baseline.

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- Forecasting the weather vs forecasting the tides, think about how this relates to absolute measures of predictability vs measures relative some baseline.
- In the section "The Unperformable Experiment" he describes the problems of studying the real atmosphere, and that leads to the reliance of physical models and computer models. To what extent do we see these same tricks in other areas?

The quiet revolution of numerical weather prediction

Peter Bauer[1], Alan Thorpe[1] & Gilbert Brunet[2]

Things I like about numerical weather prediction:

- Many groups make public predictions every day at multiple time scales (5-day forecast, 10-day forecast), and we can all see how accurate they are

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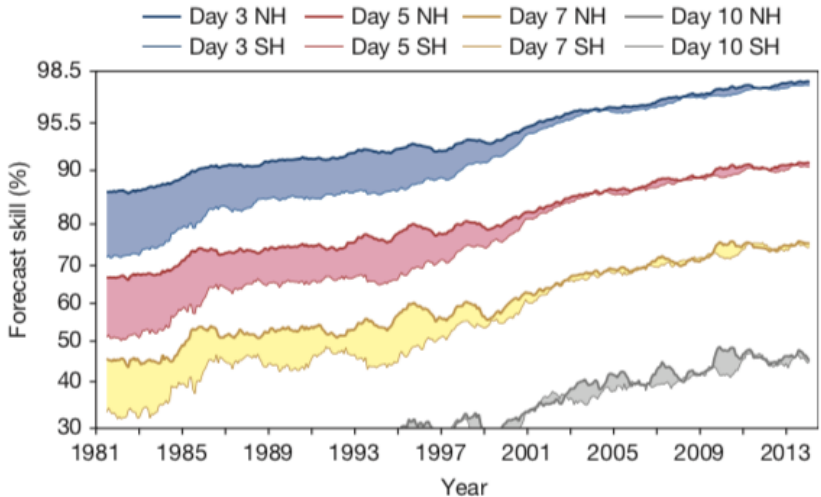
Things I like about numerical weather prediction:

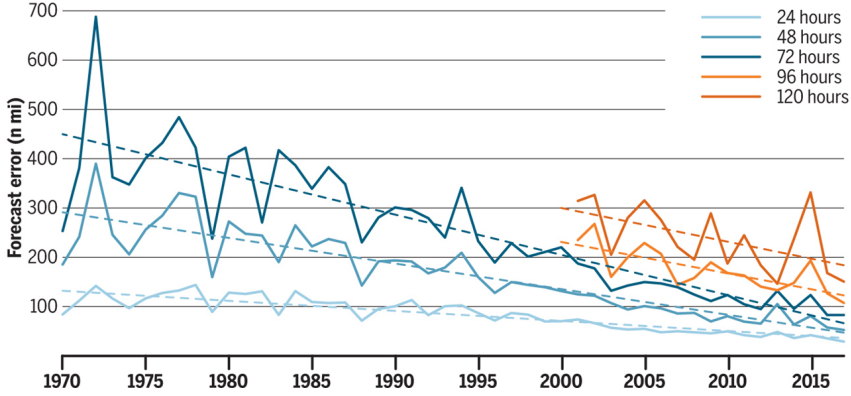
- Many groups make public predictions every day at multiple time scales (5-day forecast, 10-day forecast), and we can all see how accurate they are
- No self-fulfilling or self-defeating processes
- No concerns about causality
- Predictions based a real physical model
- Business and governments invests in improved performance



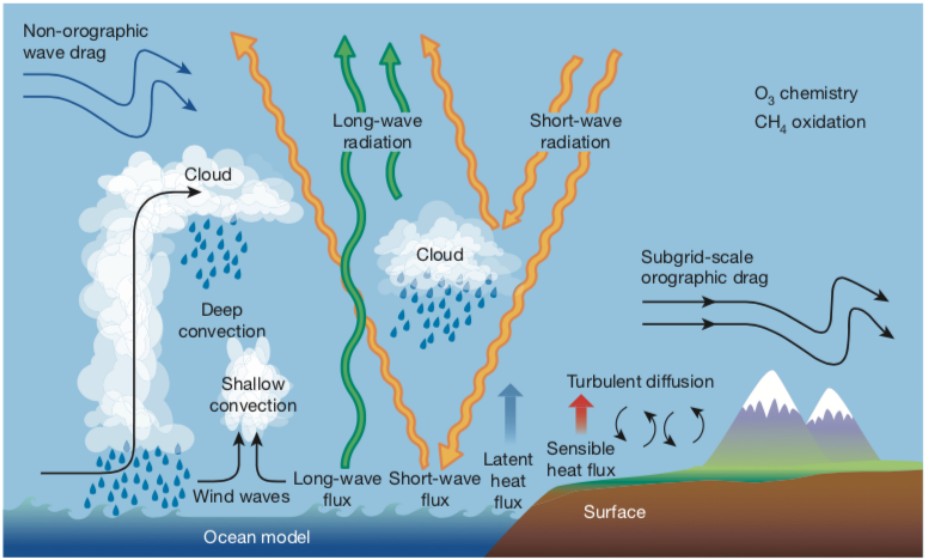
This is impressive.

Advances in hurricane prediction

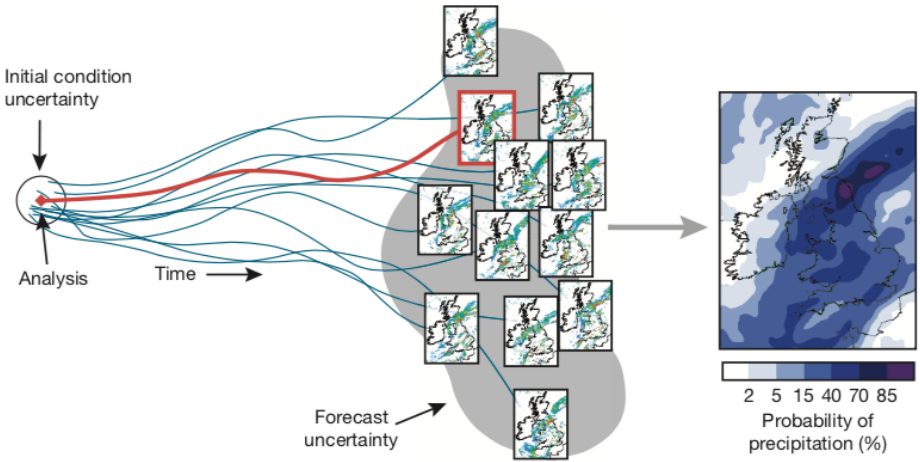
Data from the NOAA National Hurricane Center (NHC) (*13*) show that forecast errors for tropical storms and hurricanes in the Atlantic basin have fallen rapidly in recent decades. The graph shows the forecast error in nautical miles (1 n mi = 1.852 km) for a range of time intervals.



This is impressive. Source: Alley et al 2019



Note the many interacting systems involved.



Sensitive dependences does not make them quit.

Atmospheric predictability experiments with a large numerical model

By E. N. LORENZ,[1] *European Centre for Medium Range Weather Forecasts, Reading RG2 9AX, England*

Upper and lower bound on predictability with no extra computing!

We want to understand the upper bound and lower bound for accuracy using the model from the European Center for Medium Range Weather Forecasts (ECMWF)

- How accurate are predictions? This gives lower bound on accuracy.

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- How accurate are predictions? This gives lower bound on accuracy.
- How much do two similar initial conditions diverge? This gives an upper bound on accuracy.

We'd like to do this without running the model many times because running the model is expensive.

Model produces "prognoses":

- 1 Dec: 0 day, 1 day, 2 days ..., 10 days
- 2 Dec: 0 day, 1 day, 2 days ..., 10 days
- 3 Dec: 0 day, 1 day, 2 days ..., 10 days
- ⋮
- 10 March: 0 day, 1 day, 2 days ..., 10 days

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . ., 10 days (11 Dec)

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . ., 10 days (11 Dec)
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . ., 10 days (12 Dec)

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ..., 10 days (11 Dec)
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . ., 10 days (11 Dec)
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . ., 10 days (12 Dec)
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) . . ., 10 days (13 Dec)
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) . . ., 10 days (14 Dec)
- $\vdots$

How accurate are 1 day forecasts? This is equivalent to asking: What is E_{01}?

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ..., 10 days (11 Dec)
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ..., 10 days (14 Dec)
- $\vdots$

How accurate are 1 day forecasts? This is equivalent to asking: What is E_{01}?

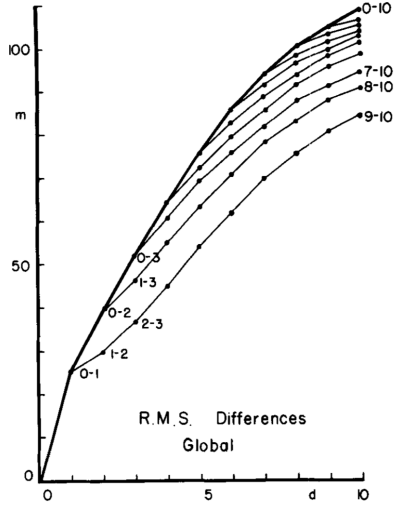
- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ..., 10 days (11 Dec)
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ..., 10 days (14 Dec)
- ⋮

How accurate are 2 day forecasts? This is equivalent to asking: What is E_{02}?

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) . . ., 10 days (11 Dec)
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) . . ., 10 days (12 Dec)
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) . . ., 10 days (13 Dec)
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) . . ., 10 days (14 Dec)
- ⋮

How accurate are 2 day forecasts? This is equivalent to asking: What is E_{02}?

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ..., 10 days (11 Dec)
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ..., 10 days (12 Dec)
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ..., 10 days (13 Dec)
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ..., 10 days (14 Dec)
- ⋮



$E_{01}, E_{02}, \ldots E_{010}$ is the heavy curve at the top

What about two trajectories that start close together? How fast do they diverge?

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- ⋮

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- $\vdots$

Distance between 0 day (2 Dec) and 1 day (2 Dec): ϵ

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- ⋮

Distance between 0 day (2 Dec) and 1 day (2 Dec): ϵ
Distance between 1 day (3 Dec) and 2 days (3 Dec): $c \cdot \epsilon$ for $(c > 1)$

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- ⋮

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ
Distance between 1 day (4 Dec) and 2 days (4 Dec): $c \cdot \epsilon$ for $(c > 1)$

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ (E_{01})
Distance between 1 day (4 Dec) and 2 days (4 Dec): $c \cdot \epsilon$ for ($c > 1$)

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec) ...
- ⋮

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ (E_{01})
Distance between 1 day (4 Dec) and 2 days (4 Dec): $c \cdot \epsilon$ for ($c > 1$) (E_{12})

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) 3 days (6 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec) ...
- ⋮

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) 3 days (6 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) 3 days (6 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec) ...
- ⋮

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ
Distance between 2 day (5 Dec) and 3 days (5 Dec): $c \cdot \epsilon$ for $(c > 1)$

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec) 3 days (6 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ (E_{01})
Distance between 2 day (5 Dec) and 3 days (5 Dec): $c \cdot \epsilon$ for ($c > 1$)

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec) ...
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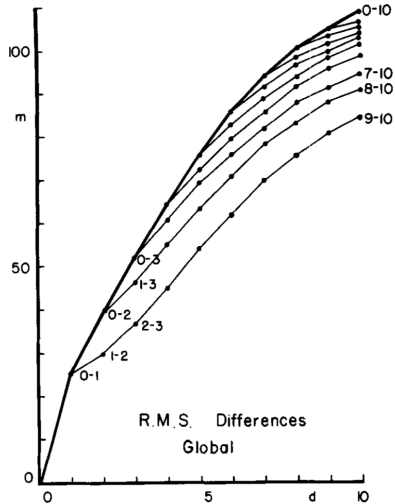
- ⋮

Distance between 0 day (3 Dec) and 1 day (3 Dec): ϵ (E_{01})
Distance between 2 day (5 Dec) and 3 days (5 Dec): $c \cdot \epsilon$ for ($c > 1$) (E_{23})



$E_{12}, E_{23}, \ldots E_{910}$ is the light curve at the bottom

What about two trajectories that start close together? How fast do they diverge?

~~What about two trajectories that start close together? How fast do they diverge?~~
What about two trajectories that start a bit further apart? How fast do they diverge?

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- ⋮

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
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- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
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- ⋮

Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ
Distance between 1 day (4 Dec) and 3 days (4 Dec): $c \cdot \epsilon$ for $(c > 1)$

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ (E_{02})
Distance between 1 day (4 Dec) and 3 days (4 Dec): $c \cdot \epsilon$ for ($c > 1$)

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ (E_{02})
Distance between 1 day (4 Dec) and 3 days (4 Dec): $c \cdot \epsilon$ for $(c > 1)$ (E_{13})

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- ⋮

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- $\vdots$

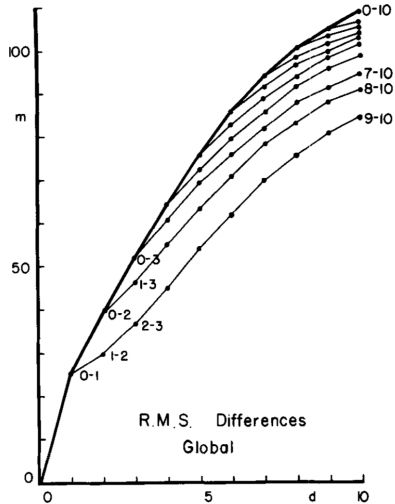
Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ
Distance between 2 day (5 Dec) and 4 days (5 Dec): $c \cdot \epsilon$ for $(c > 1)$

- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ (E_{02})
Distance between 2 day (5 Dec) and 4 days (5 Dec): $c \cdot \epsilon$ for ($c > 1$)

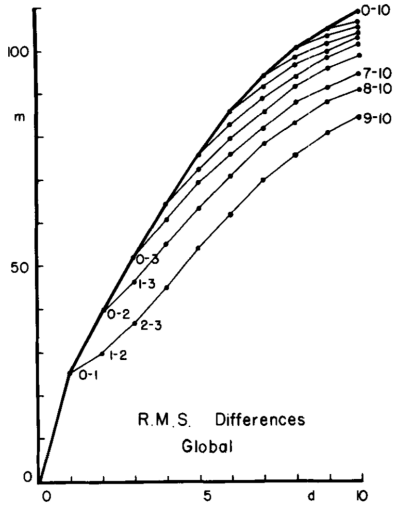
- 1 Dec: 0 day (1 Dec), 1 day (2 Dec), 2 days (3 Dec), 3 days (4 Dec), 4 days (5 Dec) ...
- 2 Dec: 0 day (2 Dec), 1 day (3 Dec), 2 days (4 Dec), 3 days (5 Dec), 4 days (6 Dec) ...
- 3 Dec: 0 day (3 Dec), 1 day (4 Dec), 2 days (5 Dec), 3 days (6 Dec), 4 days (7 Dec) ...
- 4 Dec: 0 day (4 Dec), 1 day (5 Dec), 2 days (6 Dec), 3 days (7 Dec), 4 days (8 Dec) ...
- $\vdots$

Distance between 0 day (3 Dec) and 2 day (3 Dec): ϵ (E_{02})
Distance between 2 day (5 Dec) and 4 days (5 Dec): $c \cdot \epsilon$ for $(c > 1)$ (E_{24})



$E_{12}, E_{23}, \ldots E_{910}$ is the light curve second from the bottom

What can we learn from this figure?



Pre-read for Thursday, October 24: Weather, Empirical

Matthew J. Salganik

COS 597E/SOC 555 Limits to prediction
Fall 2020, Princeton University