Pre-read for Tuesday, October 20: Weather, Lorenz attractor

Matthew J. Salganik

COS 597E/SOC 555 Limits to prediction
Fall 2020, Princeton University

Interested in system that are not random but looks random. Could also be said as systems that are deterministic but do not look deterministic.

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- tumbling of a rock down a mountainside
- breaking of waves on the shore

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For our class these are interesting examples because

- there are precise laws governing behavior
- they are unpredictable in some ways, but predictable in others

sensitive dependence on initial conditions: two nearly identical initial conditions diverge until they bear not more resemblance than two states chosen randomly

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- pinball machine

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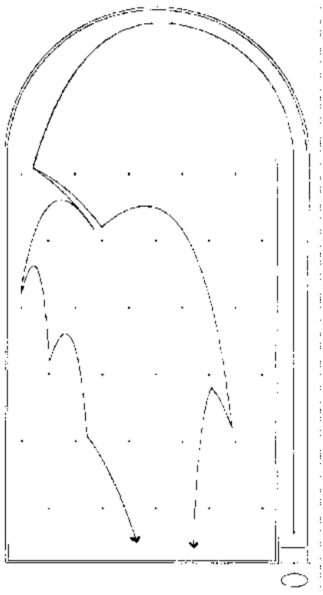
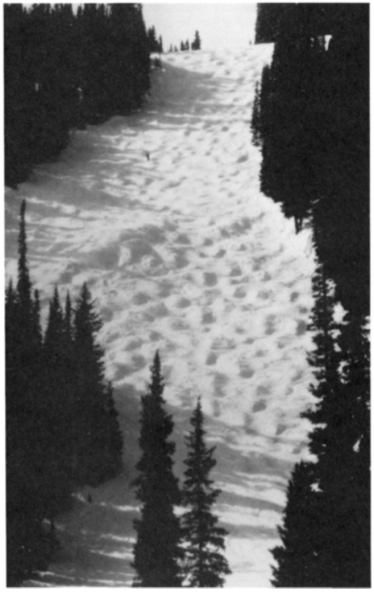
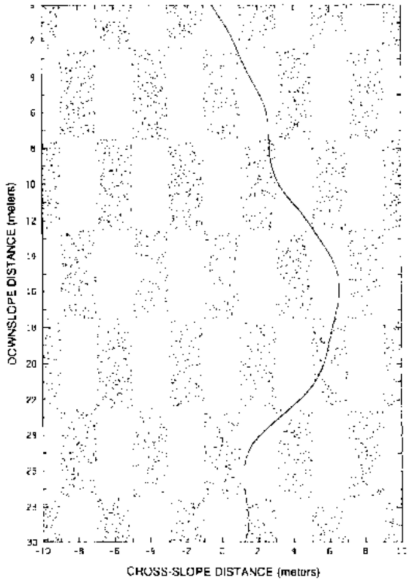
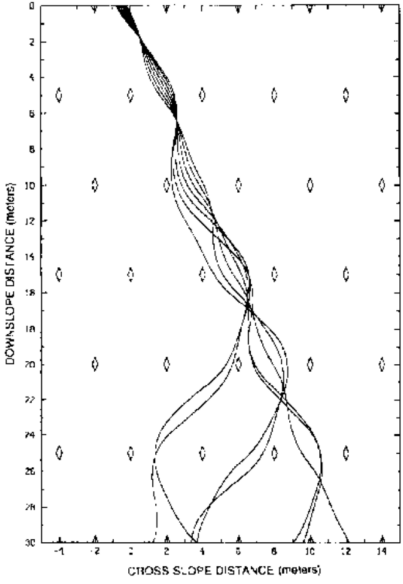
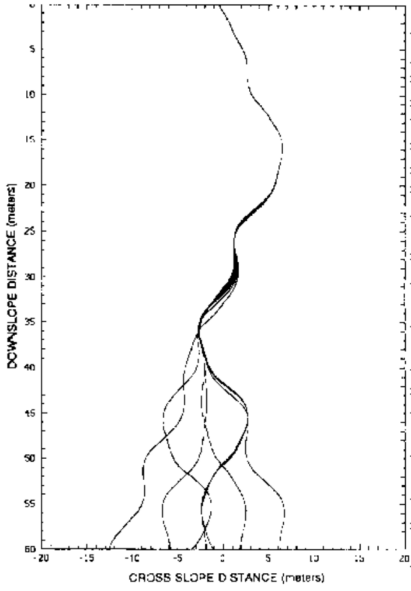
- pinball machine
- ski slope






DOWNSLOPE DISTANCE (meters)
CROSS-SLOPE DISTANCE (meters)


DOWNSLOPE DISTANCE (meters)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
-4
-2
0
2
4
6
8
10
12
14
CROSS SLOPE DISTANCE (meters)


DOWNSLOPE DISTANCE (meters)
0
5
10
15
20
25
30
35
40
45
50
55
60
-20
-15
-10
-5
0
5
10
15
20
CROSS SLOPE DISTANCE (meters)

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

$$x' = \sigma(y - x)$$
$$y' = x(\rho - z) - y$$
$$z' = xy - \beta z$$

- x: the rate of convective motion - i.e. how fast the rolls are rotating,
- y: the temperature difference between the ascending and descending currents, and
- z: the distortion (from linearity) of the vertical temperature profile.

$\sigma = 10, \rho = 28, \beta = 8/3$

For more on how he came to study this system see Lorenz (1993) page 130 - 146

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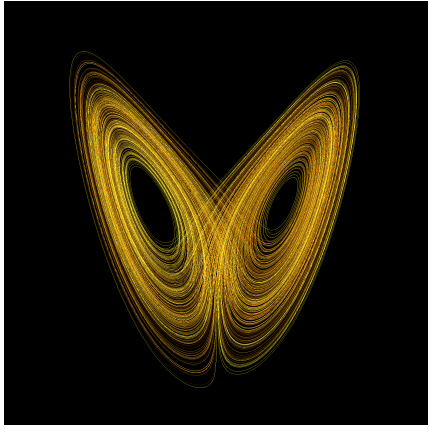

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