Lecture 4: Understanding the small world phenomena

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Social Networks (Soc 204)
Princeton University

Wednesday, February 5, 2025

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- Precept is not a time for lecture; it is a time for new learning.

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- Submit quiz 1 by 11:59pm tonight.

1. Watts, Chapter 3.
2. Watts and Strogatz (1998). Collective dynamics of 'small-world' networks. Nature 393, 440-442.
3. Victor (2011). Scientific Communication As Sequential Art.
4. Watts (1999). Networks, dynamics, and the small world phenomenon. American Journal of Sociology, 105(2):493-527.

Review:

- empirical vs modeling approaches

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- empirical approach runs into difficulties

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- models are different
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Today we will see two different small world models and then an empirical assessment

Duncan says that they wanted to capture four main ideas:

- small overlapping groups that are linked by people who belong to multiple groups

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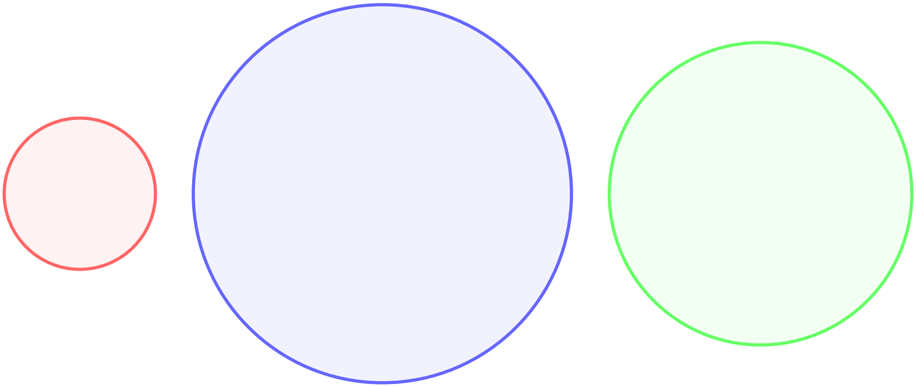
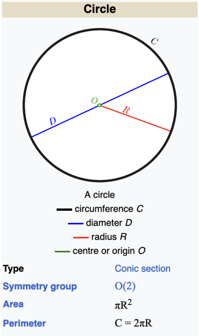
- small overlapping groups that are linked by people who belong to multiple groups
- social network evolve
- not all relationships are equally likely
- occasionally we do things that are not determined by existing network structure

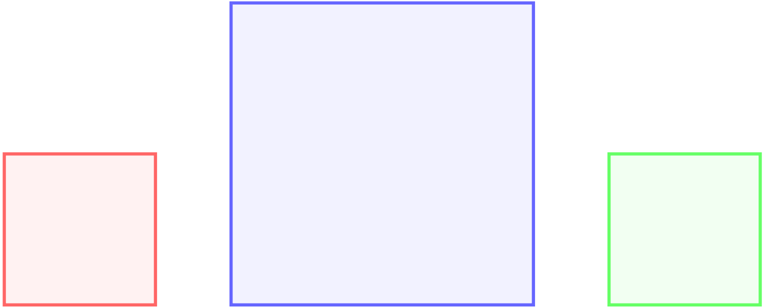
Can we parameterize these ideas?



These circles are all different, but they are from the same “family”

- The family of circles is defined by one parameter: usually the radius
- If something is a circle with radius r, then you know other things about it

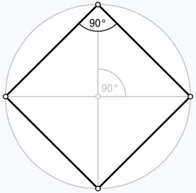




These squares are all different, but they are from the same “family”.

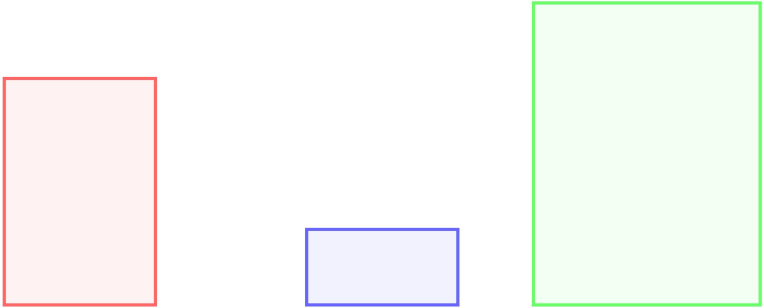
- The family of squares is defined by one parameter: usually the length
- If something is a square with length l, then you know other things about it

Square



A regular quadrilateral

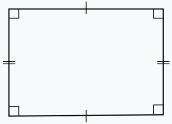
Type	Regular polygon
Edges and vertices	4
Schläfli symbol	{4}
Coxeter–Dynkin diagrams	4
Symmetry group	Dihedral (D_4), order 2×4
Internal angle (degrees)	90°
Properties	Convex, cyclic, equilateral, isogonal, isotoxal
Dual polygon	Self



These rectangles are all different, but they are from the same "family".

- The family of rectangle is defined by two parameters: length and width
- If something is a rectangle with length l and width w then you know other things about it

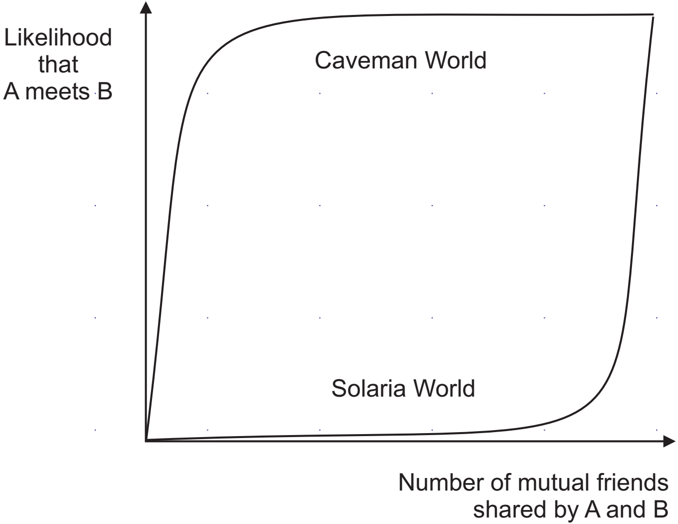
Rectangle

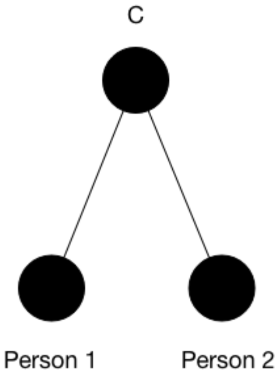


Rectangle

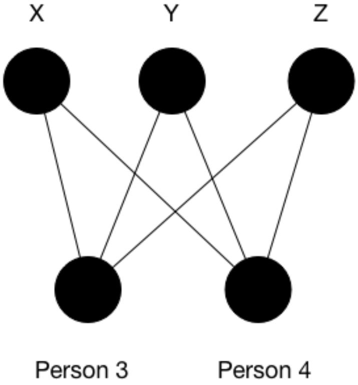
Type	quadrilateral, trapezium, parallelogram, orthotope
Edges and vertices	4
Schläfli symbol	{ } × { }
Coxeter–Dynkin diagrams	
Symmetry group	Dihedral (D_2), [2], (*22), order 4
Properties	convex, isogonal, cyclic Opposite angles and sides are congruent
Dual polygon	rhombus

3.1


Likelihood
that
A meets B
Caveman World
Solaria World
Number of mutual friends
shared by A and B

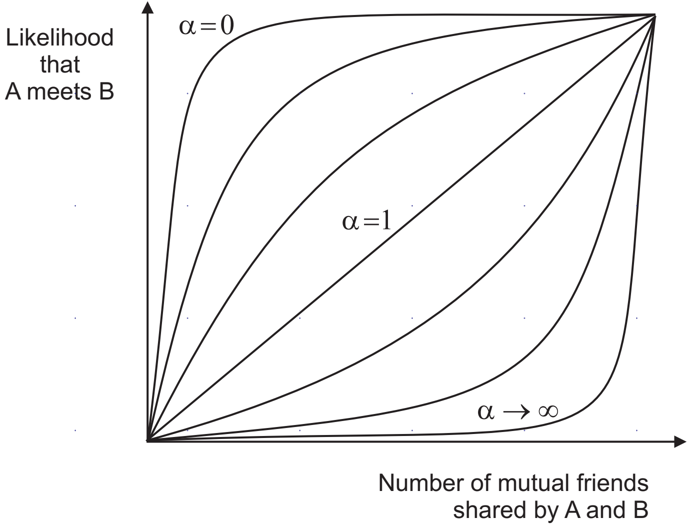


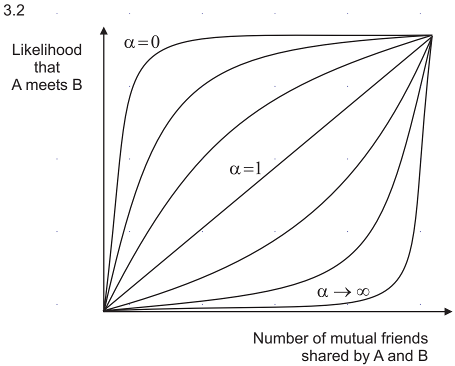
1 mutual friend



3 mutual friend

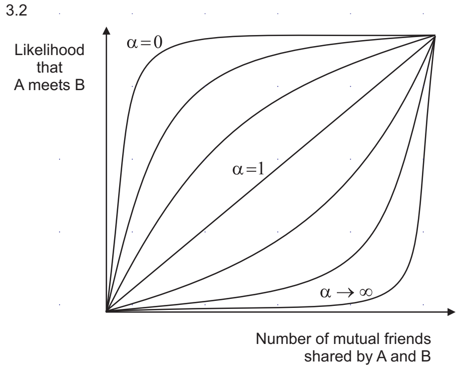
3.2


Likelihood that A meets B
$\alpha = 0$
$\alpha = 1$
$\alpha \to \infty$
Number of mutual friends shared by A and B



As technology changes do you think we are moving more toward:

1. caveman world ($\alpha = 0$)
2. solaria world ($\alpha \to \infty$)



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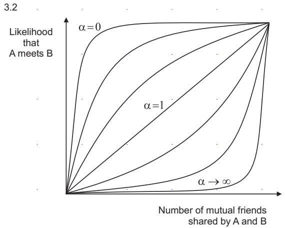
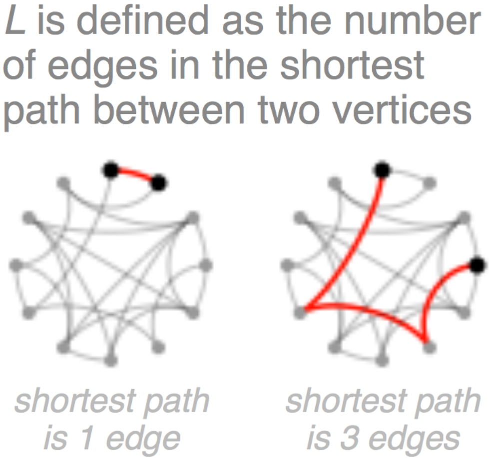


$$R_{i,j} = \begin{cases} 1 & m_{i,j} \geq k \\ \left[\dfrac{m_{i,j}}{k}\right]^{\alpha}(1-p)+p & k > m_{i,j} > 0, \\ p & m_{i,j} = 0 \end{cases} \tag{5}$$

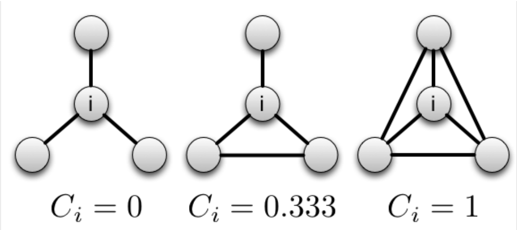
Note on this process of modeling: the graph came before the equation.

First metric:

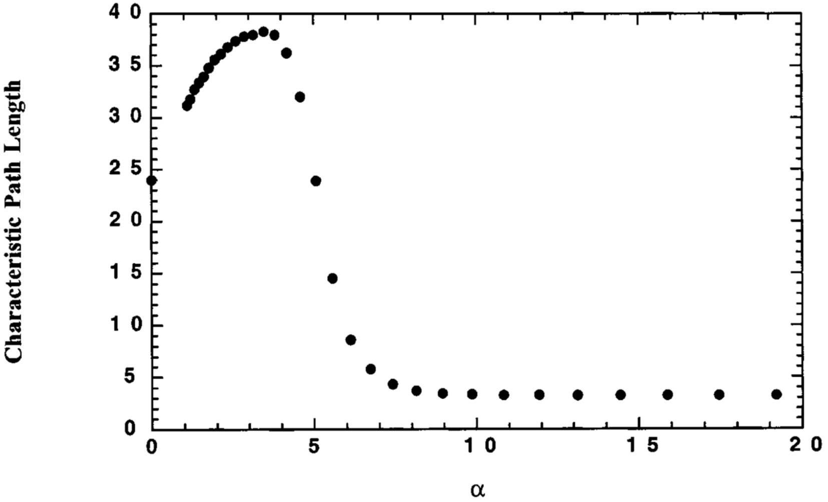
Characteristics path length L: number of edges in shortest path, averaged over all paths



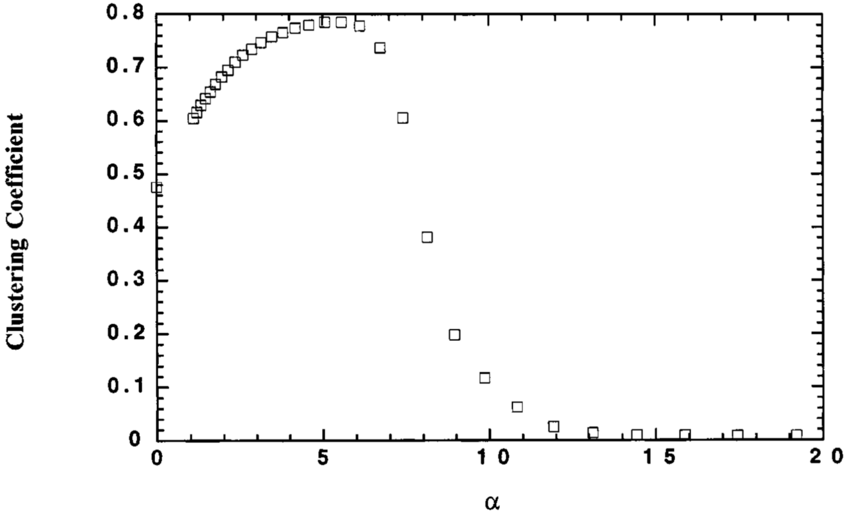
Second metric:
Clustering coefficient C: probability that two friends of a randomly chosen person are friends



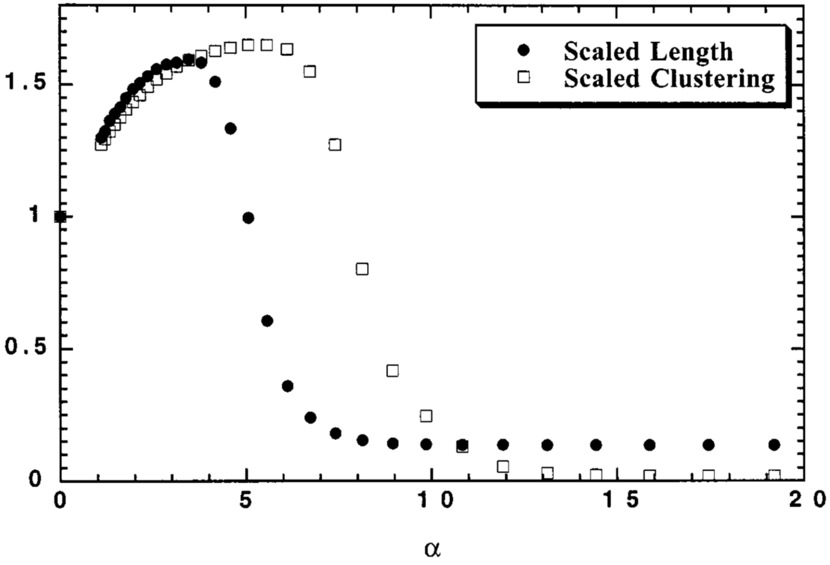
If we simulate lots of people following these rules, what kinds of networks get created?

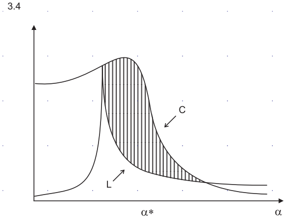


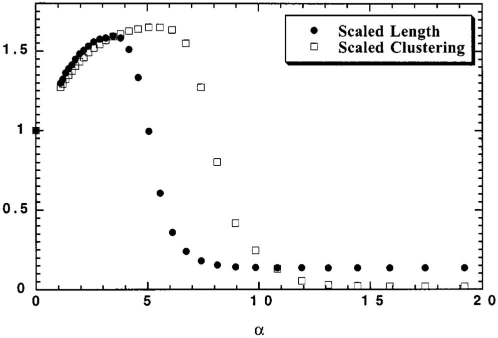
Characteristic Path Length
4 0
3 5
3 0
2 5
2 0
1 5
1 0
5
0
0
5
1 0
1 5
2 0
α



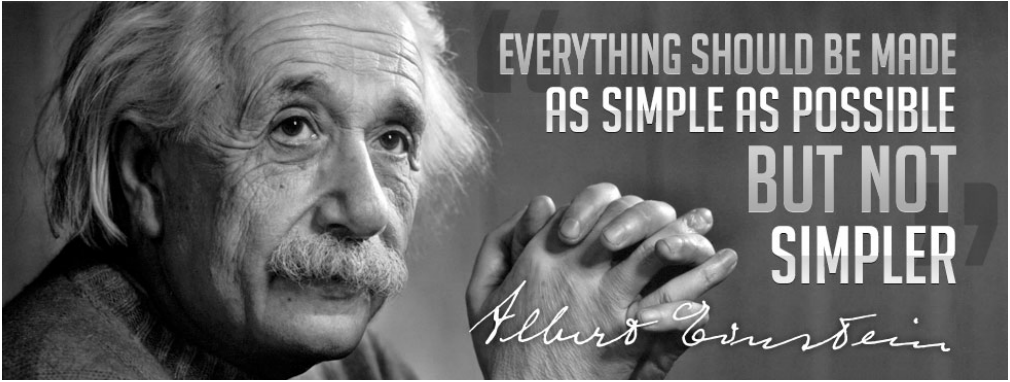
Clustering Coefficient
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
α


Scaled Length
Scaled Clustering
1.5
1
0.5
0
0
5
10
15
20
α



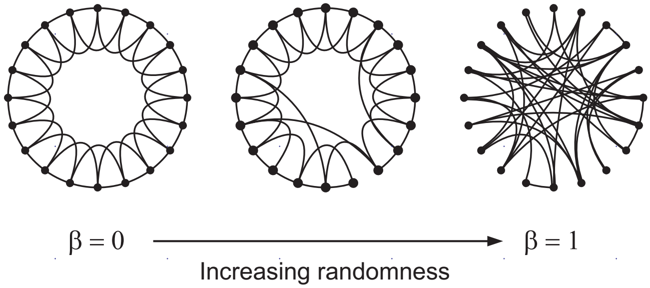


Paper and book look different because paper does not include the disconnected region

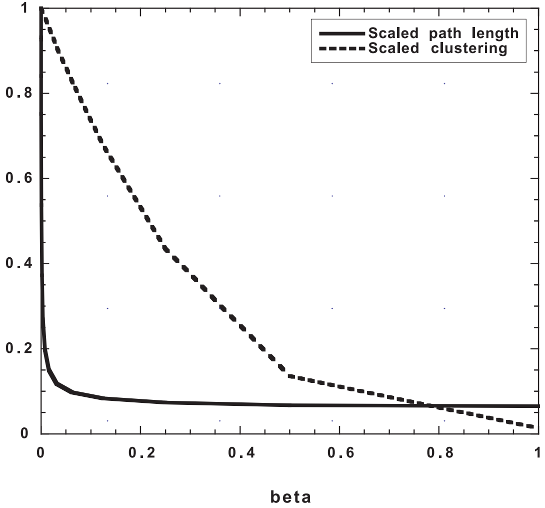


http://vireomd.net/blog/dhc/einstein-kiss.html

3.6

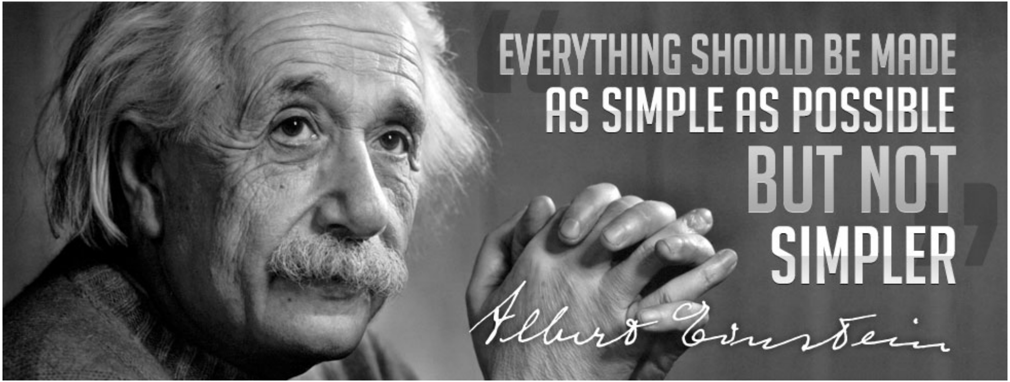


3.7



Scaled path length
Scaled clustering
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
beta

Demo: `http://mathinsight.org/small_world_network`



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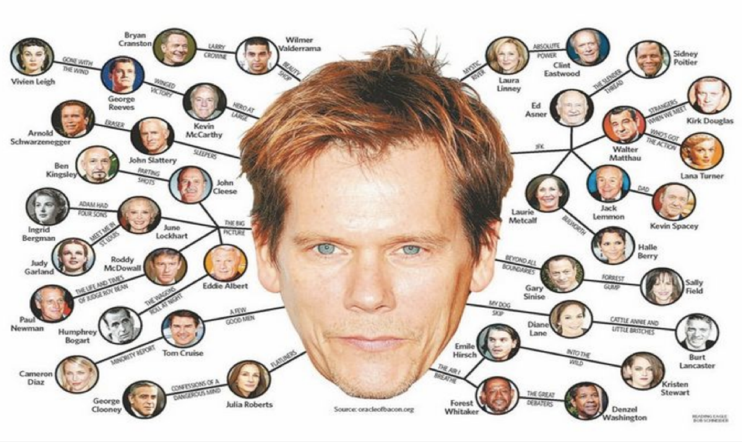
Are real networks small world networks?



Small world means:

- $L_{actual} \approx L_{random}$
- $C_{actual} >> C_{random}$

	L_{actual}	L_{random}	C_{actual}	C_{random}

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Movie actors				

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Power Grid				

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Network ‘Small-World-Ness’: A Quantitative Method for Determining Canonical Network Equivalence

Mark D. Humphries*, Kevin Gurney

Adaptive Behaviour Research Group, Department of Psychology, University of Sheffield, Sheffield, United Kingdom

Abstract

Background: Many technological, biological, social, and information networks fall into the broad class of ‘small-world’ networks: they have tightly interconnected clusters of nodes, and a shortest mean path length that is similar to a matched random graph (same number of nodes and edges). This semi-quantitative definition leads to a categorical distinction (‘small/not-small’) rather than a quantitative, continuous grading of networks, and can lead to uncertainty about a network’s small-world status. Moreover, systems described by small-world networks are often studied using an equivalent canonical network model – the Watts-Strogatz (WS) model. However, the process of establishing an equivalent WS model is imprecise and there is a pressing need to discover ways in which this equivalence may be quantified.

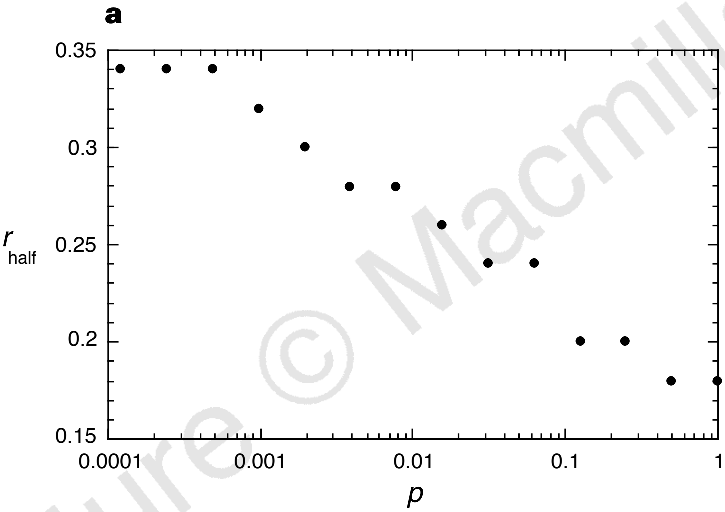
Methodology/Principal Findings: We defined a precise measure of ‘small-world-ness’ S based on the trade off between high local clustering and short path length. A network is now deemed a ‘small-world’ if $S>1$ - an assertion which may be tested statistically. We then examined the behavior of S on a large data-set of real-world systems. We found that all these systems were linked by a linear relationship between their S values and the network size n. Moreover, we show a method for assigning a unique Watts-Strogatz (WS) model to any real-world network, and show analytically that the WS models associated with our sample of networks also show linearity between S and n. Linearity between S and n is not, however, inevitable, and neither is S maximal for an arbitrary network of given size. Linearity may, however, be explained by a common limiting growth process.

Conclusions/Significance: We have shown how the notion of a small-world network may be quantified. Several key properties of the metric are described and the use of WS canonical models is placed on a more secure footing.

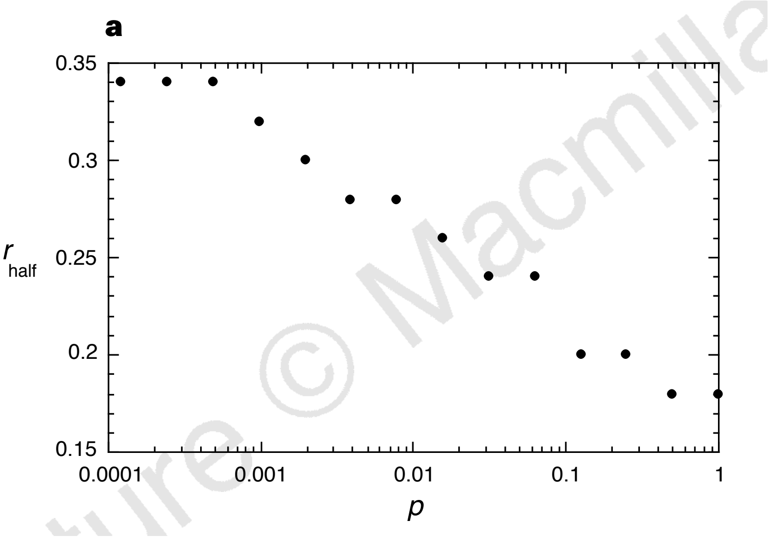
https://dx.doi.org/10.1371/journal.pone.0002051

We started in sidewalk cafe in Tunis, and we ended up in a worm brain.

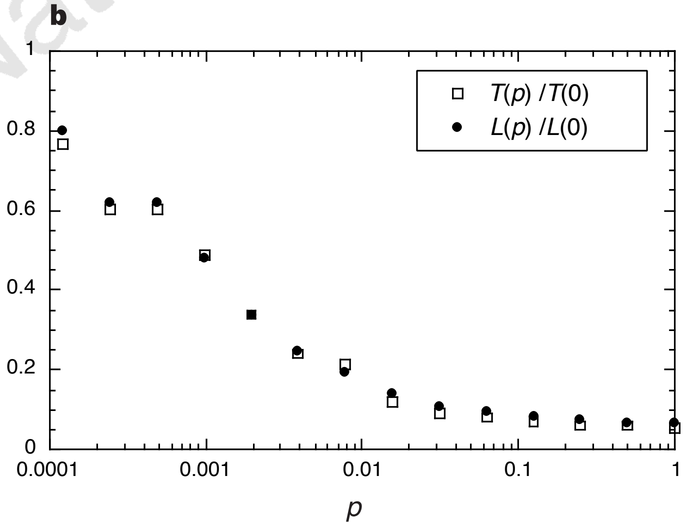
Who cares?

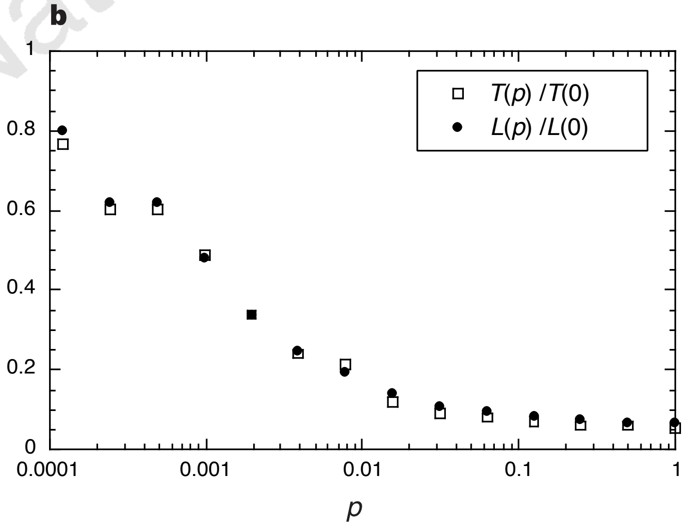


a
0.35
0.3
0.25
0.2
0.15
r half
0.0001
0.001
0.01
0.1
1
p



The more shortcuts the less infectious (r) a disease needs to be to spread


b
1
0.8
0.6
0.4
0.2
0
0.0001
0.001
0.01
0.1
1
p
T(p) /T(0)
L(p) /L(0)



The more shortcuts the faster a disease spreads

Making length contractions concrete . . . .

What we learned

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- small local changes can have global impacts
- similarity across networks of different types
- network structure impacts dynamics

Here's my approach to reading articles. You will get better with practice, and you will find your own way.

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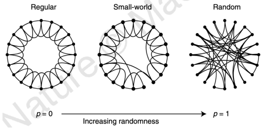
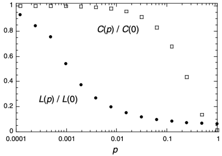
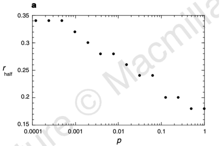
- try to explain the paper to your friend

A second look at the figures/tables in Watts and Strogatz (1998)







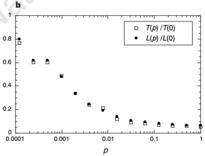


Table 1 Empirical examples of small-world networks

	L_{actual}	L_{random}	C_{actual}	C_{random}
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
C. elegans	2.65	2.25	0.28	0.05

Reflection and feedback:

`http://rb.gy/zczj7r`

Next class. Power law networks

- Watts, Chapter 4, 101-114.
- Barabasi, A.L. and Bonabeau, E. (2003) Scale-free networks. *Scientific American*, 50-59.
- Barabasi, A.L. and Albert, R. (1999) The emergence of scaling in random networks. *Science*, 286:509-512.
- Liljeros, F. et al. (2001). The web of human sexual contacts. *Nature*, 411:907-908 with comment and rejoinder.