Lecture 5: Degree distributions and power laws

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Sociology 204: Social Networks
Princeton University

Monday, February 10, 2025

- This is the last week we will post copies of *Six Degrees*

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- First assignment is due this week: Wed 11am

Review:

- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks

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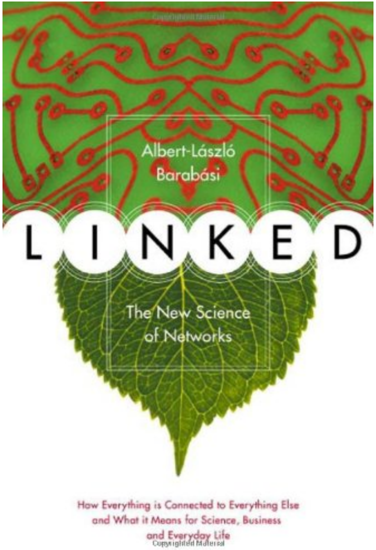
- abstract model helps us understand many types of networks
- these network structural properties are important for dynamics happening on the network (e.g., disease spread)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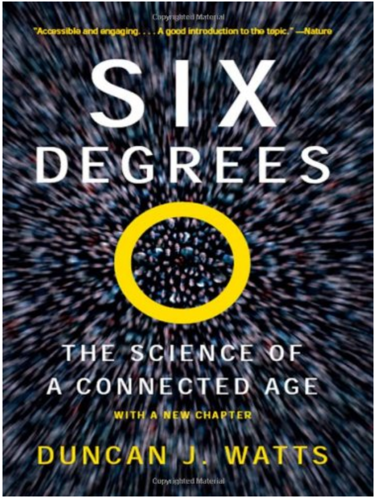
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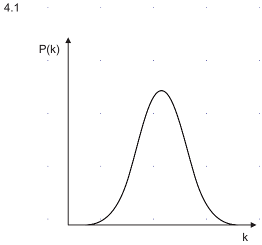
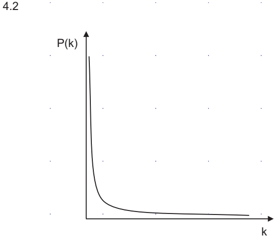
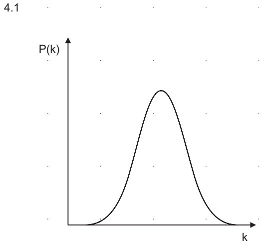
- degree: number of connections that a node has to other nodes (not related to degrees of separation)
- degree distribution: distribution of degrees



(a) Normal



(b) Power law



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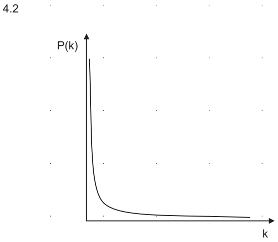
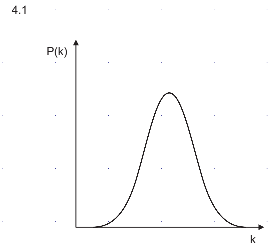
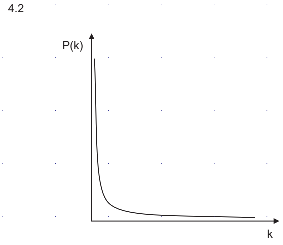


(b) Power law

Is the distribution of heights more similar to normal or scale-free?

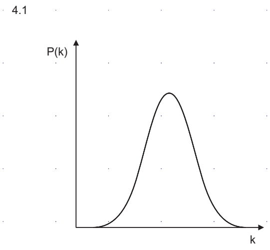


(a) Normal



(b) Power law

Is the distribution of heights more similar to normal or scale-free? normal



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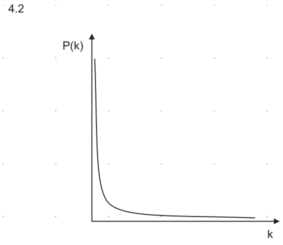
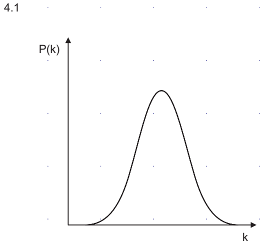


(b) Power law

Is the distribution of wealth more similar to normal or scale-free?



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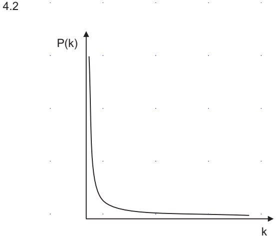
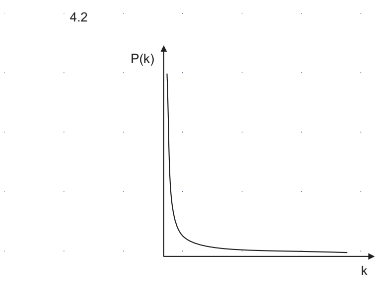


(b) Power law

Is the distribution of wealth more similar to normal or scale-free? scale-free



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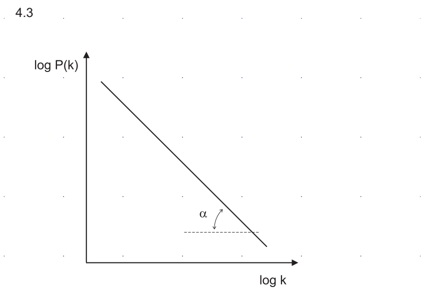
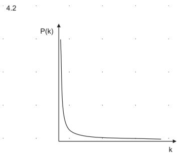


(b) log-log Power law



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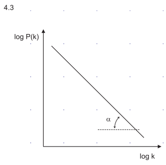

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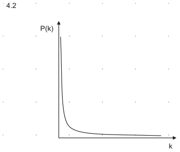

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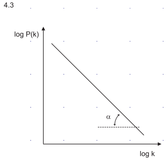
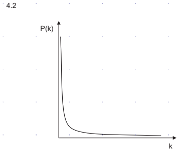


(b) log-log Power law

$$p(k) \sim \frac{1}{k^n}$$



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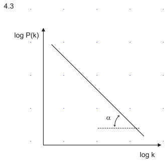
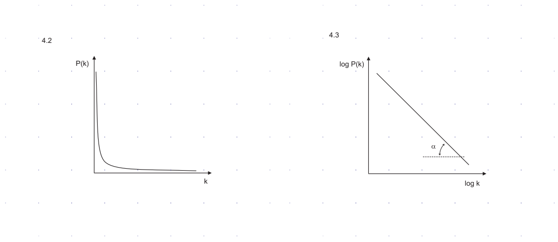


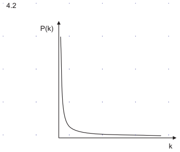
(b) log-log Power law

$p(k) \sim \frac{1}{k^n}$
$logp(k) \sim log(\frac{1}{k^n})$



(a) Power law (b) log-log Power law

$p(k) \sim \frac{1}{k^n}$
$logp(k) \sim log(\frac{1}{k^n})$
$logp(k) \sim log(1) - log(k^n)$



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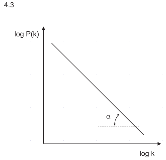


(b) log-log Power law

$$p(k) \sim \frac{1}{k^n}$$
$$logp(k) \sim log(\frac{1}{k^n})$$
$$logp(k) \sim log(1) - log(k^n)$$
$$logp(k) \sim -nlog(k)$$

It turns out that many degree distributions follow a power law distribution (which Barabasi calls "scale-free")

$p(k) \sim \frac{1}{k^{\gamma}}$

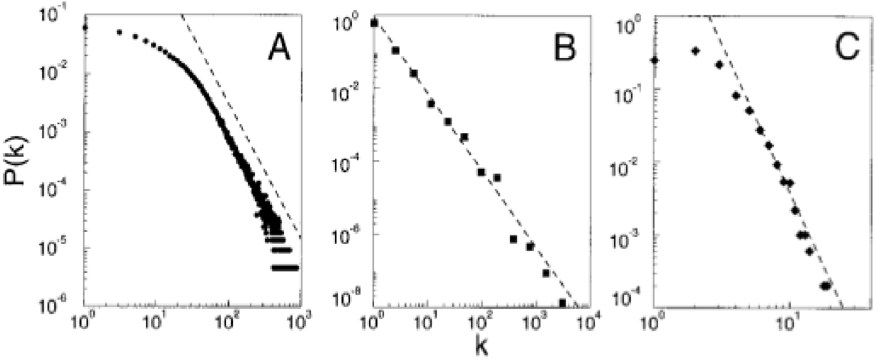
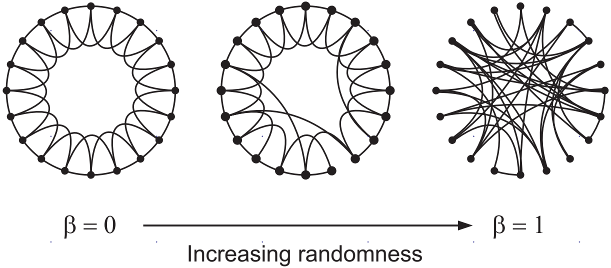


Fig. 1. The distribution function of connectivities for various large networks. (**A**) Actor collaboration graph with $N = 212{,}250$ vertices and average connectivity $\langle k \rangle = 28.78$. (**B**) WWW, $N = 325{,}729$, $\langle k \rangle = 5.46$ (6). (**C**) Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{actor} = 2.3$, (B) $\gamma_{www} = 2.1$ and (C) $\gamma_{power} = 4$.

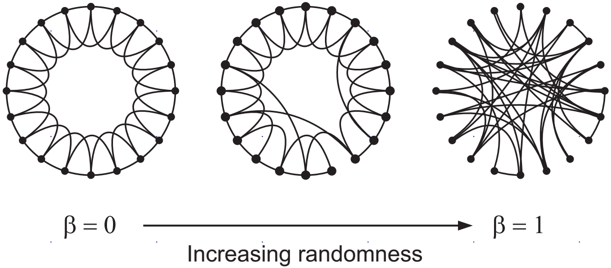
Does β model produce power law degree distribution?

3.6



Does β model produce power law degree distribution? No

3.6



Barabasi and Albert propose a very simple model that generates networks with power law degree distributions

- growth (new nodes enter the system)
- preferential attachment (more likely to connect to high degree nodes)

A brief in class demo

Demo

http://netlogoweb.org/launch#http://netlogoweb.org/assets/modelslib/Sample%20Models/Networks/Preferential%20Attachment.nlogo

Follow up work:

- Implications
- Empirical
- Modeling

Implication

Epidemic Spreading in Scale-Free Networks

Romualdo Pastor-Satorras[1] and Alessandro Vespignani[2]

[1]*Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord, Mòdul B4, 08034 Barcelona, Spain*

[2]*The Abdus Salam International Centre for Theoretical Physics (ICTP), P.O. Box 586, 34100 Trieste, Italy*

(Received 20 October 2000)

Implication

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(Received 20 October 2000)

- Diseases are harder to stop when spreading in scale-free networks

`http://dx.doi.org/10.1103/PhysRevLett.86.3200`

Implication

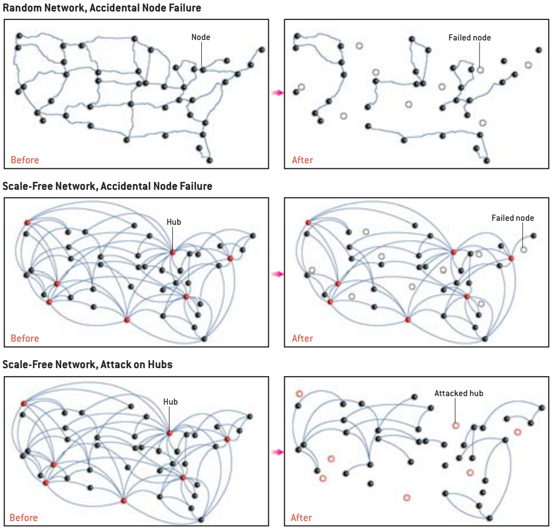
Error and attack tolerance of complex networks

Réka Albert, Hawoong Jeong & Albert-László Barabási

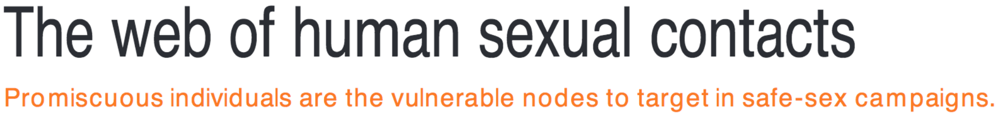
Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame, Notre Dame, Indiana 46556, USA

`http://dx.doi.org/10.1038/35019019`

Implication



Empirical



https://doi.org/10.1038/35082140

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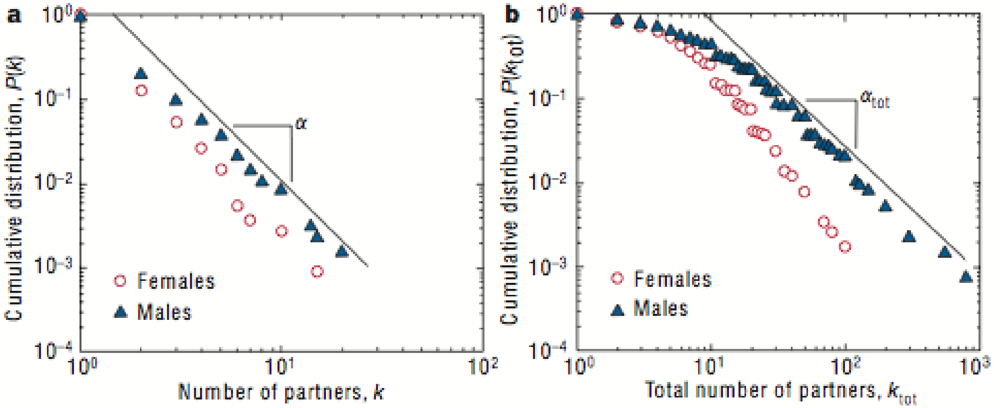

a
b
Cumulative distribution, P(k)
Number of partners, k
α
Females
Males
Cumulative distribution, P(k_tot)
Total number of partners, k_tot
α_tot
Females
Males

Social networks

Sexual contacts and epidemic thresholds

Distributions of the number of sexual partners reported in surveys show a pronounced skew, with most people having had one or no partners in the past year and a small fraction having had many[1,2]. Liljeros and colleagues[3] infer from the results of a Swedish survey that there is a "scale-free" population distribution of sexual contacts, consistent with a preferential-attachment model[3,4], in which "the rich get richer" and epidemics are driven by extremely promiscuous individuals. Here we reanalyse the data from Sweden and from other countries, using more appropriate statistical tools. Our findings support the conventional wisdom that epidemic thresholds exist in these populations, and indicate that current public-health strategies to reduce the spread of HIV and other sexually transmitted infections do not need to be radically refocused.

Empirical

ARTICLE

https://doi.org/10.1038/s41467-019-08746-5 OPEN

Scale-free networks are rare

Anna D. Broido[1] & Aaron Clauset[2,3,4]

- Formal definitions of scale-free networks: Super-weak, weakest, weak, strong, strongest

Empirical

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https://doi.org/10.1038/s41467-019-08746-5 OPEN

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Empirical

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- Analyzed nearly 1,000 social, biological, technological, transportation, and information networks
- Strongest form of scale-free structure is very rare
- Social networks seem least scale-free, whereas technical and biological seem more scale-free

https://doi.org/10.1038/s41467-019-08746-5

Empirical

COMMENT

https://doi.org/10.1038/s41467-019-09038-8

OPEN

Rare and everywhere: Perspectives on scale-free networks

Petter Holme [1]

https://doi.org/10.1038/s41467-019-09038-8

Modeling

Organization of growing random networks

P. L. Krapivsky and S. Redner
Center for BioDynamics, Center for Polymer Studies, and Department of Physics, Boston University, Boston, Massachusetts 02215
(Received 7 November 2000; published 24 May 2001)

- Generalizes preferential attachment process

`https://doi.org/10.1103/PhysRevE.63.066123`

Modeling

Scale-Free Networks from Varying Vertex Intrinsic Fitness

G. Caldarelli,[1] A. Capocci,[2] P. De Los Rios,[3,4] and M. A. Muñoz[5]

[1]*INFM UdR ROMA1 Dipartimento Fisica, Università di Roma "La Sapienza," Piazzale Aldo Moro 2 00185, Roma, Italy*
[2]*Département de Physique, Université de Fribourg-Pérolles, CH-1700 Fribourg, Switzerland*
[3]*Institut de Physique Théorique, Université de Lausanne, CH-1004 Lausanne, Switzerland*
[4]*INFM UdR Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*
[5]*Instituto de Física Teórica y Computacional Carlos I, Universidad de Granada, Facultad de Ciencias, 18071-Granada, Spain*
(Received 15 July 2002; published 3 December 2002)

- power laws can from from "good-get-richer" in addition to "rich-get-richer"

`https://doi.org/10.1103/PhysRevLett.89.258702`

Question from previous year:
“Is it possible for hubs to exist even where a network doesn’t follow a power law distribution? Meaning, the fact that some nodes will be more connected than other nodes, but without the entire network being scale-free?”

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A note on terminology:

- power law
- scale-free
- hubs

- growth $+$ preferential attachment $\rightarrow$ power law degree distribution

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- some (but not all) real networks have a power law degree distribution

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- some (but not all) real networks have a power law degree distribution
- diseases spread more easily on networks with power law degree distribution than on other types of networks
- networks with power law degree distribution are robust to random failure but fragile to targeted attack

Reflection and feedback:

`http://rb.gy/sgib8o`

- Gladwell, M. (1999). Six degrees of Lois Weisberg. *The New Yorker*.
- Watts, Chapter 4, 114-129.
- Feld, S.L. (1981) The focused organization of social ties. *American Journal of Sociology*.

$\rightarrow$