UZH Zürich
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FS 2018

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Formale Grundlagen der Informatik I - Assignment 5

Hand out: 27.04.2018 - Due to: 17.05.2018

Upload the solutions to the Olat system.

5.1 Relations und Functions

a) Let R be a relation which describes a date with a week day. For example: (2018-05-01, Tuesday) $\in R$, because May 1. 2018 is a Tuesday.

i. Is R a function?

ii. Is the inverse, R^{-1}, a function?

b) Let $S_1 = \{a, b, c, d, e\}$ be a set and $R_1 \subseteq S_1 \times S_1$ a binary relation where the following applications hold:

$c\,R_1\,b$, $e\,R_1\,a$, $a\,R_1\,a$, $c\,R_1\,c$, $d\,R_1\,b$, $d\,R_1\,d$, $b\,R_1\,a$, $e\,R_1\,e$, $b\,R_1\,b$

Is this relation

i. asymmetric?

ii. antisymmetric?

iii. transitive?

iv. reflexive?

c) Let $A := \{1, 2, 3, \ldots, 8\}$ and R a relation defined as

$$R = \{(x, y) \mid x = 5^i \mod 9,\ y = i,\ i \in A\}.$$

Is R a function of A to A? Argue why or why not.

d) Let $\mathbf{O}$ be the set of all odd integers. Prove that $\mathbf{O}$ has the same cardinality as $2\mathbf{Z}$, the set of all even integers.

e) Let R be a relation on a set A and suppose R is symmetric and transitive. Prove the following: If for every $x \in A$ there is a $y \in A$ such that xRy, then R is an equivalence relation.

5.2 Linear homogeneous recursive equations of 3. order

Given a recursive equation

$$a_k = 2a_{k-1} + a_{k-2} - 2a_{k-3}$$

and the starting conditions

$$a_0 = 6 \text{ and } a_1 = 6 \text{ and } a_2 = 12.$$

Derive a closed formula for a_k.

Hint: Use an extension of the approach for recursive equations of second order. This means, determine the roots r_1, r_2, r_3 of the characteristic equation

$$t^3 - 2t^2 - t + 2 = 0.$$

Then $a_k = Ar_1^k + Br_2^k + Cr_3^k$ holds, where A, B and C can be determined by the starting conditions.