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Formale Grundlagen der Informatik I - Assignment 5

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Upload the solutions to the Olat system.

5.1 Relations und Functions

a) Let R be a relation which describes a date with a week day. For example: (2018-05-01, Tuesday) $\in R$, because May 1. 2018 is a Tuesday.

i. Is R a function?
R is a function because every date gets assigned to a weekday, and there are never two weekdays for one date.

ii. Is the inverse, R^{-1}, a function?
The inverse is not a function because for one weekday there are several dates.

b) Let $S_1 = \{a, b, c, d, e\}$ be a set and $R_1 \subseteq S_1 \times S_1$ a binary relation where the following applications hold:

$c\,R_1\,b$, $e\,R_1\,a$, $a\,R_1\,a$, $c\,R_1\,c$, $d\,R_1\,b$, $d\,R_1\,d$, $b\,R_1\,a$, $e\,R_1\,e$, $b\,R_1\,b$

Is this relation

i. asymmetric? No. Example: cR_1c

ii. antisymmetric? Yes. $\forall x, y \in S_1$: if xRy and $yRx \rightarrow x = y$

iii. transitive? No. Example: dRb, bRa but $d \not Ra$

iv. reflexive? Yes. $\forall x \in S_1 : xRx$

c) Let $A := \{1, 2, 3, \ldots, 8\}$ and R a relation defined as

$$R = \{(x, y) \mid x = 5^i \mod 9,\ y = i,\ i \in A\}.$$

Is R a function of A to A? Argue why or why not.
It is not a function because $x = 3$ has no value y.

d) Let **O** be the set of all odd integers. Prove that **O** has the same cardinality as $2\mathbf{Z}$, the set of all even integers.
$x \in O, y \in 2Z, f(x) = x + 1 = y$
Injective: $x_1, x_2 \in O, f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$ thus it is injective.
Surjective: $\forall y \in 2Z, x \in O, y - 1 = x = f(x)$ so it is surjective.
There is a bijective function from O to $2Z$ which proves that they have the same cardinality.

e) Let R be a relation on a set A and suppose R is symmetric and transitive. Prove the following: If for every $x \in A$ there is a $y \in A$ such that xRy, then R is an equivalence relation.
The relation needs to be symmetric, reflexive and transitive in order to be an equivalence relation. So it needs to be shown that the relation is reflexive.
$\forall x \in A \exists y \in A : xRy$. Because of symmetry it applies that xRy and yRx. Because of transitivity it follows that xRy and $yRx \Rightarrow xRx$. The relation is reflexive and therefore it is an equivalence relation.

5.2 Linear homogeneous recursive equations of 3. order

Given a recursive equation

$$a_k = 2a_{k-1} + a_{k-2} - 2a_{k-3}$$

and the starting conditions

$$a_0 = 6 \text{ and } a_1 = 6 \text{ and } a_2 = 12.$$

Derive a closed formula for a_k.
Hint: Use an extension of the approach for recursive equations of second order. This means, determine the roots r_1, r_2, r_3 of the characteristic equation

$$t^3 - 2t^2 - t + 2 = 0.$$

Then $a_k = Ar_1^k + Br_2^k + Cr_3^k$ holds, where A, B and C can be determined by the starting conditions.
$t^3 - 2t^2 - t + 2 = (t + 1)(t - 1)(t - 2) = 0 \Rightarrow r_1 = 1, r_2 = 2, r_3 = -1$
$a_k = A + 2^k B + (-1)^k C$
$a_0 = A + B + C = 6$
$a_1 = A + 2B - C = 6$
$a_2 = A + 4B + C = 12$
It follows that $A = 3, B = 2, C = 1$ and so $a_k = 3 + 2^{k+1} + (-1)^k$