> [Linear Algebra Cheatsheet]

Distance Formulae

- Distance between $P_{(x,y)}$ and origin O in `2D` axes -

$$D = \sqrt{x^2 + y^2}$$

- Distance between $P_{(x_1,y_1)}$ and $Q_{(x_2,y_2)}$ in `2D` axes -

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- Distance between $P_{(x,y,z)}$ and origin O in `3D` axes -

$$D = \sqrt{x^2 + y^2 + z^2}$$

- Distance between $P_{(x_1,y_1,z_1)}$ and $Q_{(x_2,y_2,z_2)}$ in `3D` axes -

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Note - Geometrically we can visualize these points and distances till `3D` not beyond that. But with the Linear Algebra, we can mathematically solve `ND` related problems easily.

- Distance between
  - $P \rightarrow (x_1, x_2, x_3, \ldots, x_n)$
  - $Q \rightarrow (y_1, y_2, y_3, \ldots, y_n)$ in `ND` axes -

$$D = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

In [ ]:

Dot product

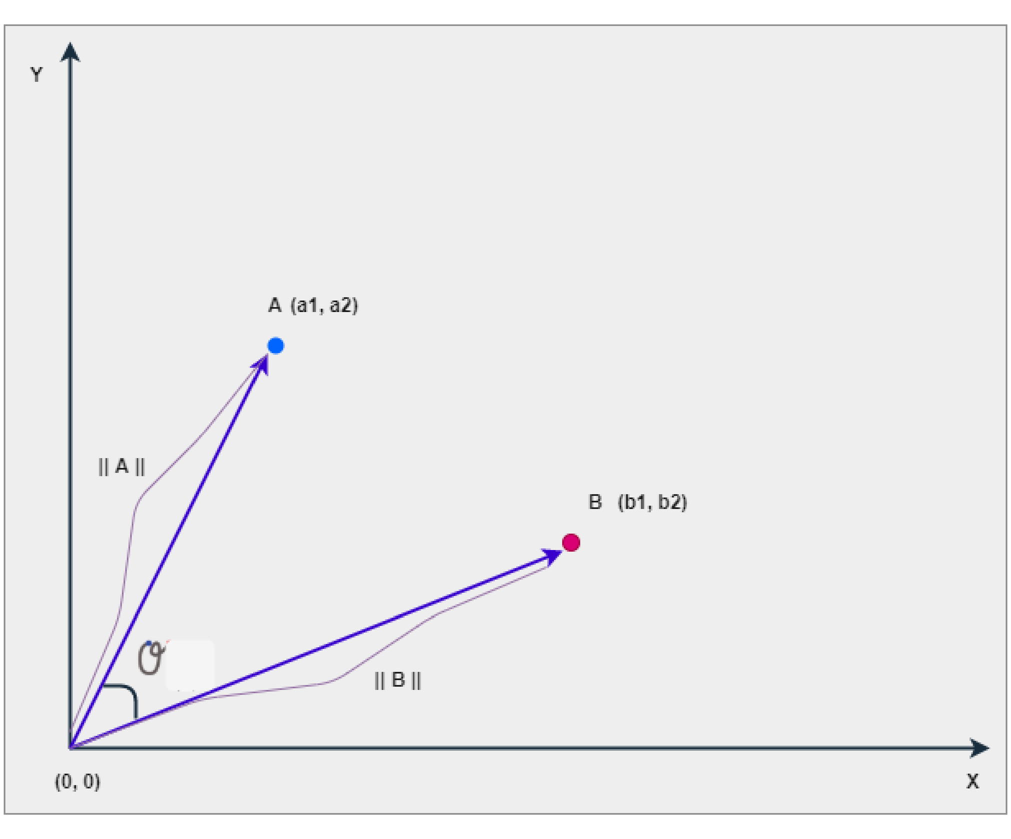
Let `A` and `B` be two matrices

```
A = [1 2 3]
B = [4 5 6]

B' = [[4]
      [5]
      [6]]
```

$$A.B = A * B^T = \sum A_i * B_i^T$$

Geometric representation of dot product



From the above figure, we can have A.B as -

$$A.B = (a_1 * b_1 + a_2 * b_2) = ||A|| * ||B|| * \cos\theta$$

where -

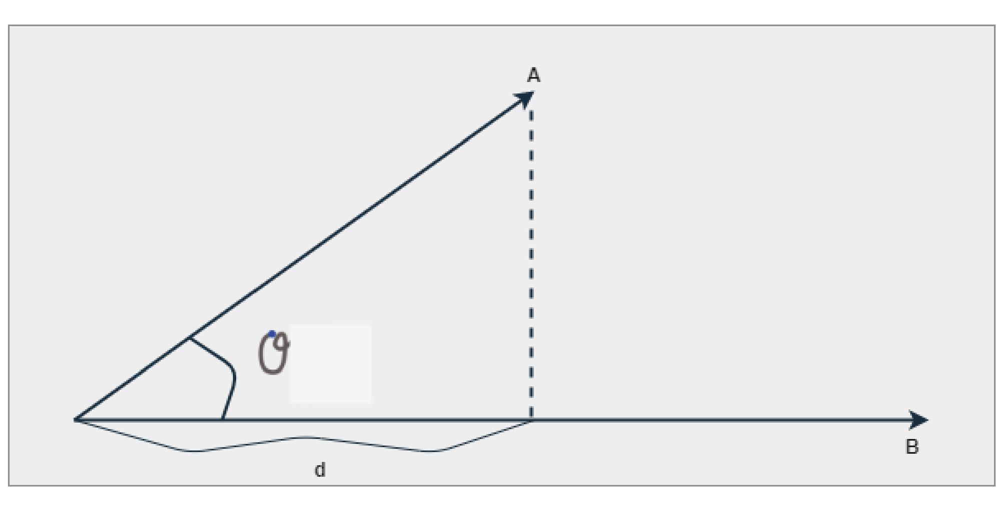
$$\theta = \cos^{-1}\left(\frac{||A|| * ||B||}{(a_1 * b_1 + a_2 * b_2)}\right)$$
$$||A|| = \sqrt{a_1^2 + a_2^2}$$
$$||B|| = \sqrt{b_1^2 + b_2^2}$$

Note - If a dot product between two vectors is 0, then they are perpendicular to each other.

In [ ]:

Projection

Projection of A on B can be visualized like -



Here -

$||d||$ is the projection of A on B

and we can say $||d|| = ||A|| \cos\theta$ (proof is below)

w.k.t

$$A.B = \sum_{i=1}^{n} A_i * B_i = ||A|| * ||B|| * \cos\theta$$

which is

$$d = \frac{A.B}{||B||} = \frac{||A|| * ||B|| * \cos\theta}{||B||} = ||A|| \cos\theta$$

In other words, we can compute projection of A on B without knowing θ by

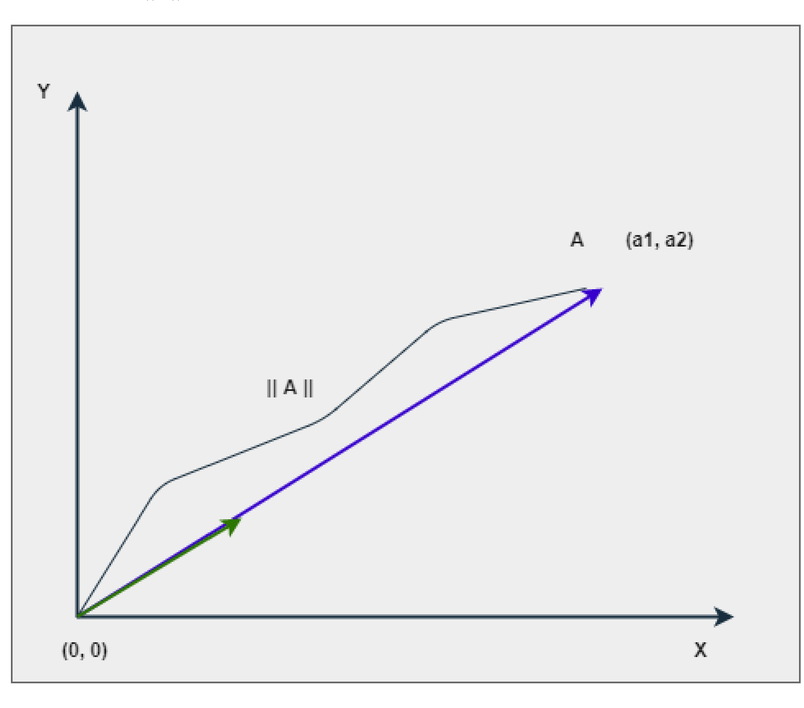
$$d = \frac{A.B}{||B||}$$

In [ ]:

Unit vector

A unit vector is a vector in the same direction the original vector.

Suppose A is a vector, the unit vector $\hat{A} = \frac{A}{||A||}$



- $\hat{A}$ is in the same direction as A.
- $||\hat{A}|| = 1$

In [ ]:

Line

- The general form of a line is

$$ax + by + c = 0 \rightarrow (1)$$

- Other equation which we are familiar is

$$y = mx + c \rightarrow (2)$$

Consider $eq(1)$

- Replace (x, y) with (x_1, x_2).
- Replace coefficients with a and b to w_1 and w_2 respectively.
- Replace c with w_0

We get the line equation as

$$w_1x_1 + w_2x_2 + w_0 = 0 \rightarrow (3)$$

The line in `3D` becomes a plane.

- The general form of a plane is

$$ax + by + cz + d = 0 \rightarrow (4)$$

We can represent (4) in the form of (3) as

$$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0 \rightarrow (5)$$

The line in `ND` is called `Hyper Plane`. The equation can be taken as -

$$w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n + w_0 = 0 \rightarrow (6)$$
$$(6) \implies w_0 + \sum_{i=1}^{n} w_ix_i = 0 \rightarrow (7)$$

(7) can be represented in vector notation

```
w0 + [w1 w2 w3 ... w5] * [[x1] = 0
                          [x2]
                          [x3]
                          [x4]
                          [x5]
                           ..
                          [xn]]
```

> Considering W as row vector and X as column vector

$$w_0 + W.X = 0 \rightarrow (8)$$

- If y intercept (w_0) is equal to 0, then $W.X = 0 \implies$ equation of a line or plane or hyper plane passing through origin.
- When $W.X = 0$, it means that W is $\perp$ to X.

In [ ]:

Distance b/w point to line

$$d = \pm\left(\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right)$$

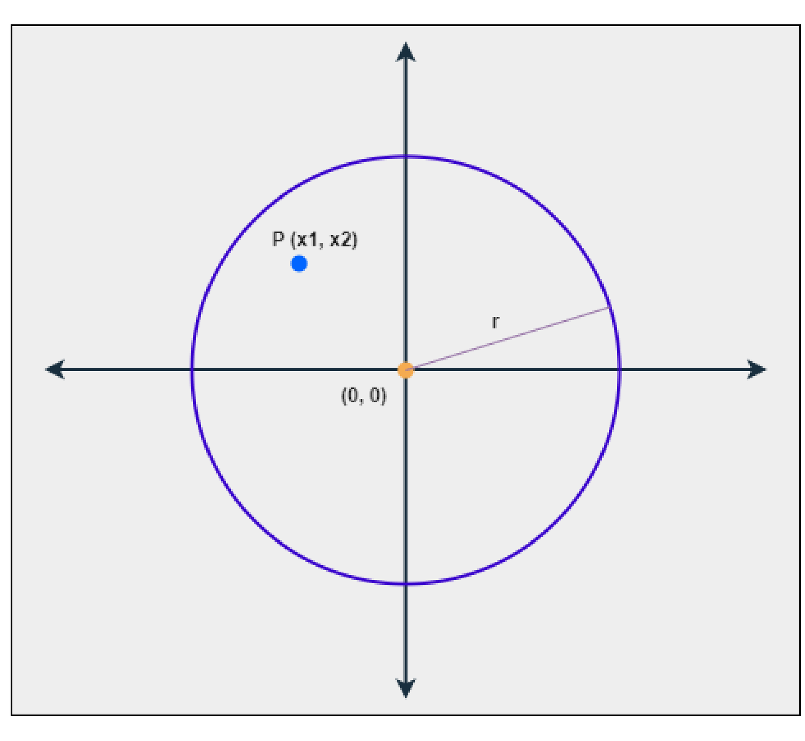
Here (x_1, y_1) is the point and

$ax+by+c = 0$ is the equation of the line

Credits - Image from Internet

In [ ]:

Circles



- From the above figure, we can see a cirlce placed exactly at origin.
- The equation of circle is given (considering $(h, k) = (0, 0)$) as

$$(x - h)^2 + (y - k)^2 = r^2 \implies x^2 + y^2 = r^2 \rightarrow (1)$$

How can we determine if a point lies inside the circle or not?

- Store the radius (r) value
- Compute the distance between (h, k) and $P_{(x,y)}$ and store in d
  - if $d < r \implies$ P is inside
  - if $d > r \implies$ P is outside
  - if $d = r \implies$ P is on the circle

Let's assume circle's center is at Origin and coordinate axes as $(x_1, x_2, x_3 \ldots, x_n)$

- Equation of circle in `3D` is -

$$x_1^2 + x_2^2 + x_3^2 = r^2 \rightarrow (2)$$

Similarly

- Equation of circle in `ND` is -

$$x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2 = r^2 \rightarrow (3)$$

$eq(3)$ can be represented as -

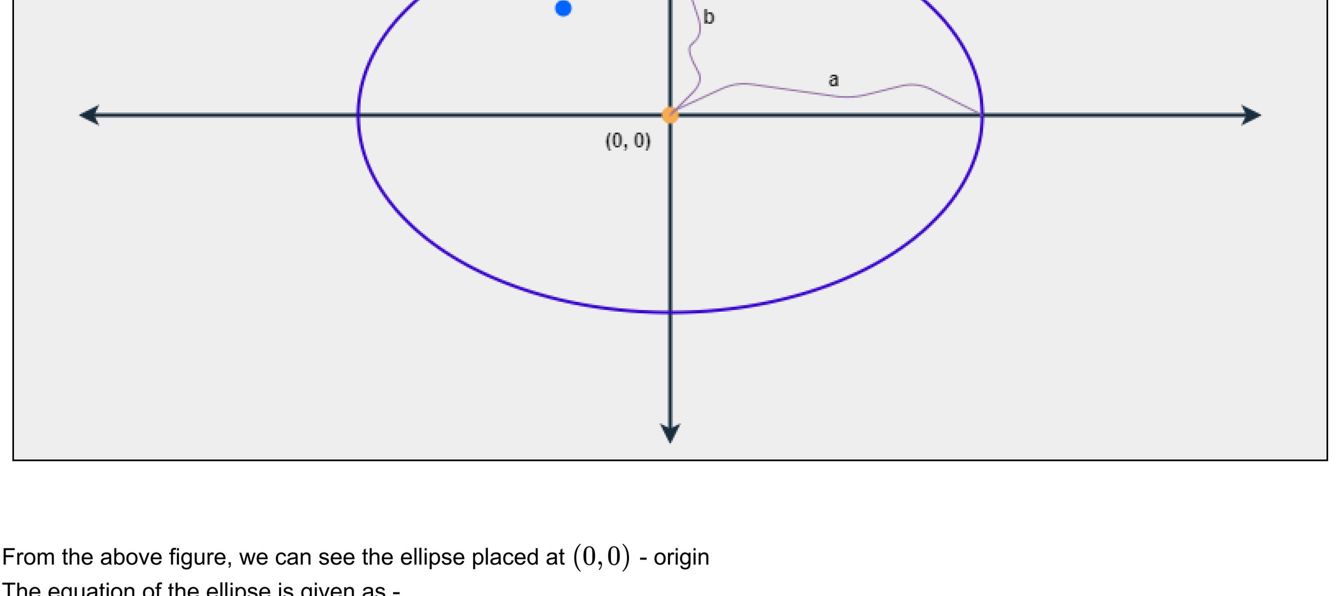
$$d = \sum_{i=1}^{n} x_i^2 = r^2 \rightarrow (4)$$

Take $P_{(x_1, x_2, x_3, \ldots, x_n)}$ and check if P lies inside or outside or on the hyper-sphere

- From $eq(4)$
  - if $d < r \implies$ P is inside
  - if $d > r \implies$ P is outside
  - if $d = r \implies$ P is on the circle

In [ ]:

Ellipse



- From the above figure, we can see the ellipse placed at $(0, 0)$ - origin
- The equation of the ellipse is given as -

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow (1)$$

How can we determine if a point $P_{(x1,x2)}$ lies inside the ellipse or not (`2D`)?

- If $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2}$
  - $< 1 \implies$ inside
  - $> 1 \implies$ outside
  - $= 1 \implies$ on the ellipse

Let's assume circle's center is at Origin and coordinate axes as $(x_1, x_2, x_3 \ldots, x_n)$

- Equation of ellipse in `3D` is -

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1 \rightarrow (2)$$

Similarly

- Equation of ellipse in `ND` is considering $(a_1, a_2, a_3, \ldots, a_n)$ as constants (denominator)-

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} + \cdots + \frac{x_n^2}{a_n^2} = 1 \rightarrow (3)$$

$eq(3)$ can be represented as -

$$d \implies \sum_{i=1}^{n} \frac{x_i^2}{a_i^2} = 1 \rightarrow (4)$$

Take $P_{(x_1, x_2, x_3, \ldots, x_n)}$ and check if P lies inside or outside or on the hyper-ellipsoid

- From $eq(4)$
  - if $d < 1 \implies$ P is inside
  - if $d > 1 \implies$ P is outside
  - if $d = 1 \implies$ P is on the hyper-ellipsoid

In [ ]:

End