Simulated Multivariate Kriging

June 10, 2020

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This notebook implements the Geostatistics Lesson . This code is provided for educational purposes and should be reviewed jointly with the lesson Collocated Cokriging.

Learning Objectives - Review simple cokriging. - Understand the why the Markov Models where developed. - Explore the differences between Markov model and Markov model . - Formulated the Kriging equations using the Markov models. - Implement the Markov model and Markov model . - Understand the Markov model and Markov model work flow(source code available).

[10]:
```
print('Package Versions:')
import matplotlib as matplotlib; print("  matplotlib:", matplotlib.__version__)
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import matplotlib.gridspec as gridspec
from mpl_toolkits.axes_grid1 import make_axes_locatable
import pandas as pd; print("  pandas:", pd.__version__)
import sys; print("  python:", sys.version_info)
import numpy as np; print("  numpy:", np.__version__)
import sklearn as sklearn; print("  sklearn:", sklearn.__version__)
import os
import scipy; print("  scipy:", scipy.__version__)
from scipy import stats
from tqdm import tqdm
from scipy.spatial import distance_matrix
from sklearn.metrics import mean_squared_error
np.set_printoptions(precision=3)
```

```
Package Versions:
  matplotlib: 2.2.2
  pandas: 0.23.0
  python: sys.version_info(major=3, minor=6, micro=10, releaselevel='final',
serial=0)
  numpy: 1.18.2
  sklearn: 0.19.1
  scipy: 1.1.0
```

1 Import Data

Consider two varable Z(**u**), the primary variable - Grade, and Y(**u**), the secondary variable - seismic. We have sample for the primary variable Z(**u**) at 64 locations. The secondary variable

Y(**u**) is measured at all location within our domain. It is considered in a probabilistic sense to use Y(**u**) to inform the prediction of Z(**u**).

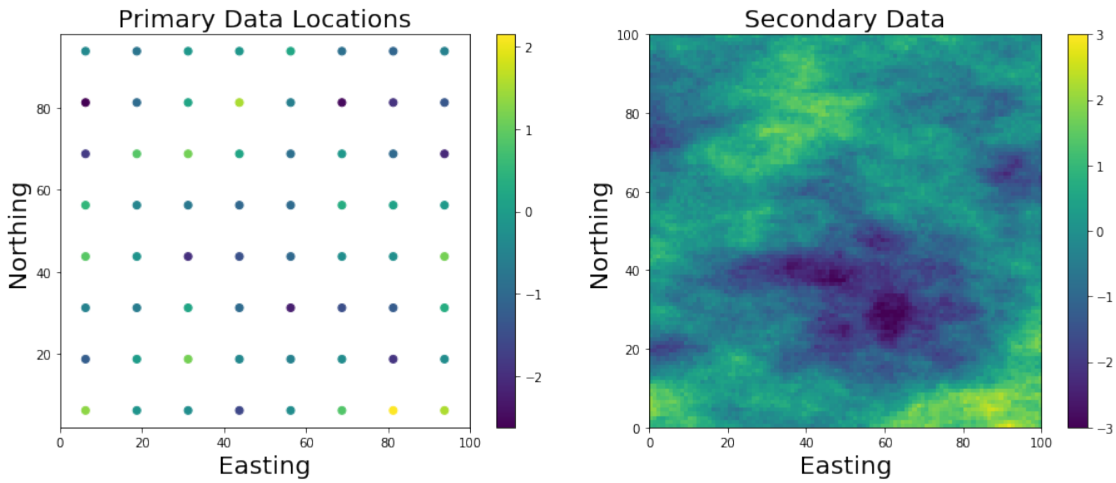
[11]:
```
datafl = pd.read_csv('Data/Cluster1.out')
datafl_sec = pd.read_csv('Data/Ydata.out')
truth = pd.read_csv('Data/true.out')
x = np.asarray(pd.read_csv('Data/X.out')).reshape(len(datafl_sec))
y = np.asarray(pd.read_csv('Data/Y.out')).reshape(len(datafl_sec))
print(datafl.describe())
print(datafl_sec.describe())
```

```
               X          Y    Primary  Secondary
count  64.000000  64.000000  64.000000  64.000000
mean   50.000000  50.000000  -0.424719  -0.197425
std    28.867513  28.867513   1.032475   0.912744
min     6.250000   6.250000  -2.623800  -2.020500
25%    28.125000  28.125000  -1.015100  -0.767850
50%    50.000000  50.000000  -0.387650  -0.224450
75%    71.875000  71.875000   0.176975   0.254700
max    93.750000  93.750000   2.148700   1.889200
                  X             Y     Secondary
count  10000.000000  10000.000000  10000.000000
mean      50.000000     50.000000     -0.250033
std       28.867513     28.867513      0.936804
min        0.500000      0.500000     -3.272810
25%       25.250000     25.250000     -0.817388
50%       50.000000     50.000000     -0.183005
75%       74.750000     74.750000      0.327832
max       99.500000     99.500000      2.781670
```

1.1 Map the Data

[12]:
```
vlim = (-3,3)
f, ax = plt.subplots(1,2,figsize=(13,5.5))
XMIN, XMAX = 0, 100
YMIN, YMAX = 0, 100
SMIN, SMAX = -3,3
gridd = pd.DataFrame()
gridd['Y'] = y
gridd['X'] = x
gridd['Estimate'] = datafl_sec['Secondary']
gridded = np.reshape(gridd.sort_values(by=['Y', 'X'], axis=0,␣
 ↪ascending=True)['Estimate'].values,
                    [100, 100], order='C',)
img0 = ax[0].scatter(datafl['X'],datafl['Y'], c = datafl['Primary'].values)
ax[0].set_title('Primary Data Locations',size = 20)
ax[0].set_xlabel('Easting',size = 20)
ax[0].set_ylabel('Northing',size = 20)
```

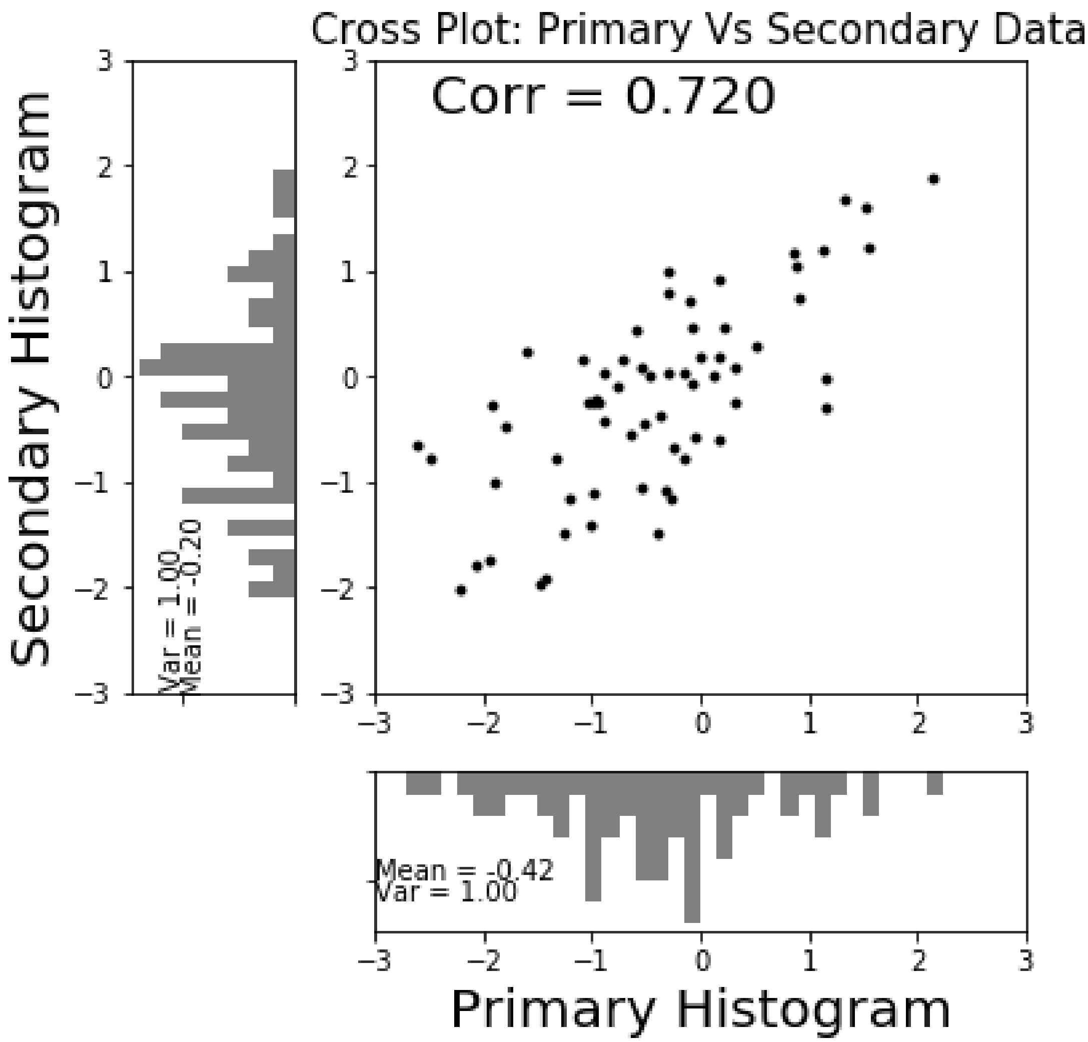
```
ax[0].axis('equal')
ax[0].set(xlim=(0, 100), ylim=(0, 100))
img1 = ax[1].imshow(gridded, origin='lower', extent=[XMIN, XMAX, YMIN, YMAX],␣
 ↪aspect='equal',
                interpolation='none', vmin=SMIN, vmax=SMAX, cmap='viridis')
ax[1].set_xlabel('Easting',size = 20)
ax[1].set_ylabel('Northing',size = 20)
ax[1].set_title('Secondary Data',size = 20)
f.colorbar(img0,ax = ax[0])
f.colorbar(img1,ax = ax[1])
plt.tight_layout()
plt.savefig('../0-Figures/Truth',bbox_inches = 'tight',pad_inches = 0)
```



1.2 Check the Distribution and Correlation of the Data

[13]:
```
# Set up the axes with gridspec
corr = np.corrcoef(datafl['Primary'],datafl['Secondary'])[0,1]
vlim= (-3,3)
fig = plt.figure(figsize=(6, 6))
grid = plt.GridSpec(4, 4, hspace=0.5, wspace=0.5)
main_ax = fig.add_subplot(grid[:-1, 1:])
y_hist = fig.add_subplot(grid[:-1, 0], xticklabels=[], sharey=main_ax)
x_hist = fig.add_subplot(grid[-1, 1:], yticklabels=[], sharex=main_ax)

# scatter points on the main axes
main_ax.plot(datafl['Primary'], datafl['Secondary'], 'ok', markersize=3)
main_ax.set_xlim(vlim)
main_ax.set_ylim(vlim)
main_ax.set_title('Cross Plot: Primary Vs Secondary Data',size = 15)
```

```
main_ax.text(-2.5,2.5,'Corr = {0:.3f}'.format(np.
 ↪corrcoef(datafl['Primary'],datafl['Secondary'])[0,1]),size=20)

# histogram on the attached axes
x_hist.hist(datafl['Primary'], 40, histtype='stepfilled',label = 'Primary',
            orientation='vertical', color='gray',range=vlim)
x_hist.set_xlabel('Primary Histogram',size = 20)
x_hist.invert_yaxis()
x_hist.text(-3,5,'Mean = {0:.2f}'.format(np.average(datafl['Primary'])),size=10)
x_hist.text(-3,6,'Var = {0:.2f}'.format(1.00),size=10)


y_hist.hist(datafl['Secondary'], 40, histtype='stepfilled',
orientation='horizontal', color='gray',range=vlim)
y_hist.set_ylabel('Secondary Histogram',size = 20)
y_hist.invert_xaxis()
y_hist.text(5,-1.5,'Mean = {0:.2f}'.format(np.
 ↪average(datafl['Secondary'])),rotation=90,size=10)
y_hist.text(6,-1.8,'Var = {0:.2f}'.format(1.00),rotation=90,size=10)
plt.savefig('../0-Figures/Cross_plt')
```



2 Correlograms

2.1 Initialize Correlogram Types

[14]:
```
def covar ( t, d, r ):
    h = d / r
    if t == 1: #Spherical
        c = 1 - h * (1.5 - 0.5 * np.square(h))
        c[h > 1] = 0
    elif t == 2: #Exponential
        c = np.exp( -3 * h )
    elif t == 3: #Gaussian
        c = np.exp( -3 * np.square(h) )
    return c
```

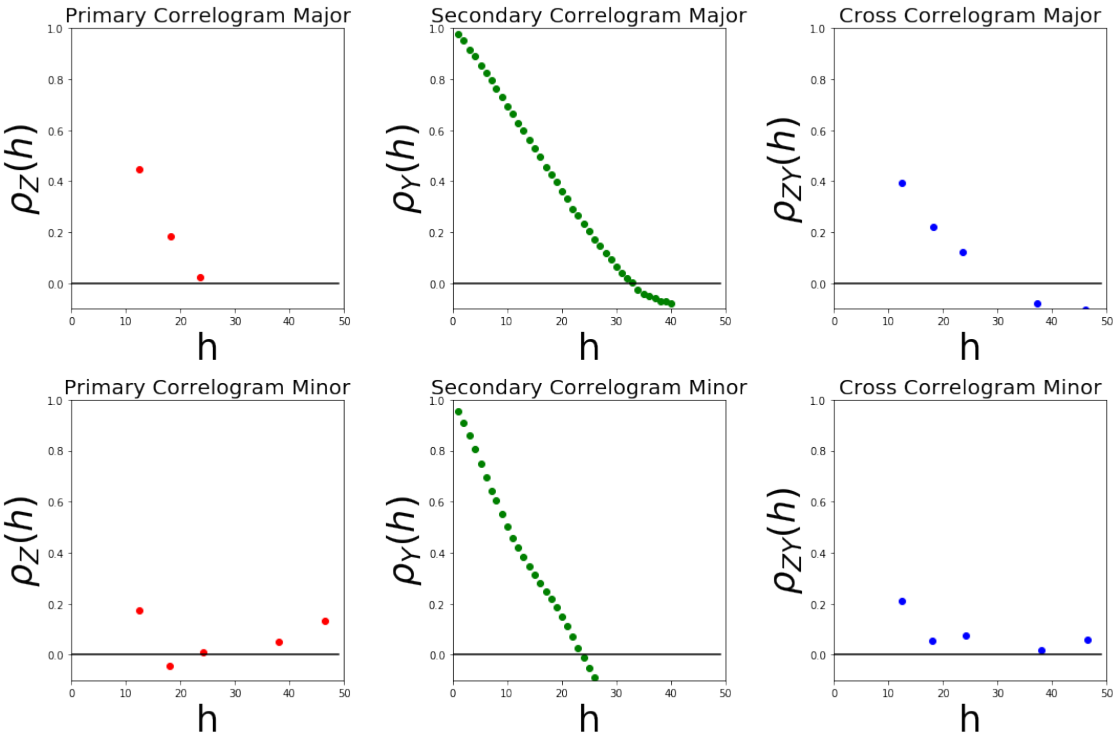
2.2 Fit Experimental Correlogram Points

Experimental variogram points where pre calculated.

[15]:
```python
varcalcfl_1 = pd.read_csv('2-vargfls/varcalc_Cluster.out')
varcalcfl_2 = pd.read_csv('2-vargfls/varcalc_YDATA.out')
varcalcfl_3 = pd.read_csv('2-vargfls/varcalc_Cross.out')
```

[16]:
```python
ones = np.zeros(shape=(50))
Cross_ones = np.zeros(shape=(50))
H = np.zeros(shape=(50))
for h in range(1,50):
    H[h] = h

fig, axes = plt.subplots(2,3, figsize=(15,10))
axes[0,0].plot(varcalcfl_1['Lag Distance'][varcalcfl_1['Variogram Index']==1]
               ,1-varcalcfl_1['Variogram Value'][varcalcfl_1['Variogram␣
 ↪Index']==1]
               ,'ro',color ='Red')
axes[0,1].plot(varcalcfl_2['Lag Distance'][varcalcfl_2['Variogram Index']==1]
               ,1-varcalcfl_2['Variogram Value'][varcalcfl_2['Variogram␣
 ↪Index']==1]
               ,'ro',color ='Green')
axes[0,2].plot(varcalcfl_3['Lag Distance'][varcalcfl_3['Variogram Index']==1]
               ,corr-varcalcfl_3['Variogram Value'][varcalcfl_3['Variogram␣
 ↪Index']==1]
               ,'ro',color ='Blue')
axes[0,0].plot(H,ones,color = 'Black')
axes[0,1].plot(H,ones,color = 'Black')
axes[0,2].plot(H,ones,color = 'Black')
axes[0,0].set_xlim(0,50)
axes[0,0].set_ylim(-0.1,1)
axes[0,0].set_ylabel('$\u03C1_{Z}(h)$',size=35)
axes[0,0].set_xlabel('h',size=35)
axes[0,1].set_xlim(0,50)
axes[0,1].set_ylim(-0.1,1)
axes[0,1].set_ylabel('$\u03C1_Y(h)$',size=35)
axes[0,1].set_xlabel('h',size=35)
axes[0,2].set_xlim(0,50)
axes[0,2].set_ylim(-0.1,1)
axes[0,2].set_ylabel('$\u03C1_{ZY}(h)$',size=35)
axes[0,2].set_xlabel('h',size=35)
axes[0,0].set_title('Primary Correlogram Major',size = 20)
axes[0,1].set_title('Secondary Correlogram Major',size = 20)
axes[0,2].set_title('Cross Correlogram Major',size = 20)

axes[1,0].plot(varcalcfl_1['Lag Distance'][varcalcfl_1['Variogram Index']==2]
```

```
               ,1-varcalcfl_1['Variogram Value'][varcalcfl_1['Variogram␣
↪Index']==2]
               ,'ro',color ='Red')
axes[1,1].plot(varcalcfl_2['Lag Distance'][varcalcfl_2['Variogram Index']==2]
               ,1-varcalcfl_2['Variogram Value'][varcalcfl_2['Variogram␣
↪Index']==2]
               ,'ro',color ='Green')
axes[1,2].plot(varcalcfl_3['Lag Distance'][varcalcfl_3['Variogram Index']==2]
               ,corr-varcalcfl_3['Variogram Value'][varcalcfl_3['Variogram␣
↪Index']==2]
               ,'ro',color ='Blue')
axes[1,0].plot(H,ones,color = 'Black')
axes[1,1].plot(H,ones,color = 'Black')
axes[1,2].plot(H,ones,color = 'Black')
axes[1,0].set_xlim(0,50)
axes[1,0].set_ylim(-0.1,1)
axes[1,0].set_ylabel('$\u03C1_{Z}(h)$',size=35)
axes[1,0].set_xlabel('h',size=35)
axes[1,1].set_xlim(0,50)
axes[1,1].set_ylim(-0.1,1)
axes[1,1].set_ylabel('$\u03C1_Y(h)$',size=35)
axes[1,1].set_xlabel('h',size=35)
axes[1,2].set_xlim(0,50)
axes[1,2].set_ylim(-0.1,1)
axes[1,2].set_ylabel('$\u03C1_{ZY}(h)$',size=35)
axes[1,2].set_xlabel('h',size=35)
axes[1,0].set_title('Primary Correlogram Minor',size = 20)
axes[1,1].set_title('Secondary Correlogram Minor',size = 20)
axes[1,2].set_title('Cross Correlogram Minor',size = 20)
plt.tight_layout()
#plt.savefig('../0-Figures/True_vargs')
```



2.3 Calculate Rotation Matrix

Using a major direction of 90 degrees east of north, this is farily obvious from the primary and secondary data.

[17]:
```
def Rot_Mat(Azimuth,a_max,a_min):
    theta = (Azimuth/180.) * np.pi
    Rot_Mat = np.dot(np.array([[1/a_max,0],[0,1/a_min]]),
                np.array([[np.cos(theta),np.sin(theta)],[-np.sin(theta),np.
 ↪cos(theta)]]))
    return Rot_Mat
```

2.4 Primary Correlogram

[18]:
```
# h1 = Set of points X,Y
# h2 = Set of points X,Y
# k = 0 used for calculating the distance between the same points
# k = 1 used for calculationg distance between different points
# k = 2 used for plotting in the major direction
# k = 3 used for plotting in the minor direction
def C_Z(h1,h2,k):
    C = []
    nstruct = 1
```

```
    vtype = [3]
    a_max = [24]
    a_min = [16]
    Azimuth = 90
    cc = [1]
    c= 0
    for i in range(nstruct):
        Q1 = h1.copy()
        Q2 = h2.copy()
        if k == 0:
            d = distance_matrix(np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])),
                                    np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i])))
        elif k == 1:
            d = np.sqrt(np.square((np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])))-
                                  np.tile((np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i]))),(k,1))).sum(axis=1))
            d = np.asarray(d).reshape(len(d))
        elif k == 2:
            d = Q1/a_max[i]
        elif k == 3:
            d = Q1/a_min[i]
        c = c + covar(vtype[i],d,1)*cc[i]
    return c
```

2.5 Secondary Correlogram

[19]:
```
# h1 = Set of points X,Y
# h2 = Set of points X,Y
# k = 0 used for calculating the distance between the same points
# k = 1 used for calculationg distance between different points
# k = 2 used for plotting in the major direction
# k = 3 used for plotting in the minor direction
def C_Y(h1,h2,k):
    C = []
    nstruct = 2
    vtype = [1,3]
    a_max = [42,43]
    a_min = [28.5,30]
    Azimuth = 90
    cc = [0.9,0.1]
    c= 0
    for i in range(nstruct):
        Q1 = h1.copy()
```

```
        Q2 = h2.copy()
        if k == 0:
            d = distance_matrix(np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])),
                                np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i])))
        elif k == 1:
            d = np.sqrt(np.square((np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])))-
                              np.tile((np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i]))),(k,1)))).sum(axis=1)
            d = np.asarray(d).reshape(len(d))
        elif k == 2:
            d = Q1/a_max[i]
        elif k == 3:
            d = Q1/a_min[i]
        c = c + covar(vtype[i],d,1)*cc[i]
    return c
```

2.6 Scaling Correlogram

[20]:
```
# h1 = Set of points X,Y
# h2 = Set of points X,Y
# k = 0 used for calculating the distance between the same points
# k = 1 used for calculationg distance between different points
# k = 2 used for plotting in the major direction
# k = 3 used for plotting in the minor direction
def C_r(h1,h2,k):
    C = []
    nstruct = 1
    vtype = [3]
    a_max = [18]
    a_min = [13]
    Azimuth = 90
    cc = [1]
    c= 0
    for i in range(nstruct):
        Q1 = h1.copy()
        Q2 = h2.copy()
        if k == 0:
            d = distance_matrix(np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])),
                                np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i])))
        elif k == 1:
```

```
            d = np.sqrt(np.square((np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])))-
                              np.tile((np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i]))),(k,1))).sum(axis=1))
            d = np.asarray(d).reshape(len(d))
        elif k == 2:
            d = Q1/a_max[i]
        elif k == 3:
            d = Q1/a_min[i]
        c = c + covar(vtype[i],d,1)*cc[i]
    return c
```

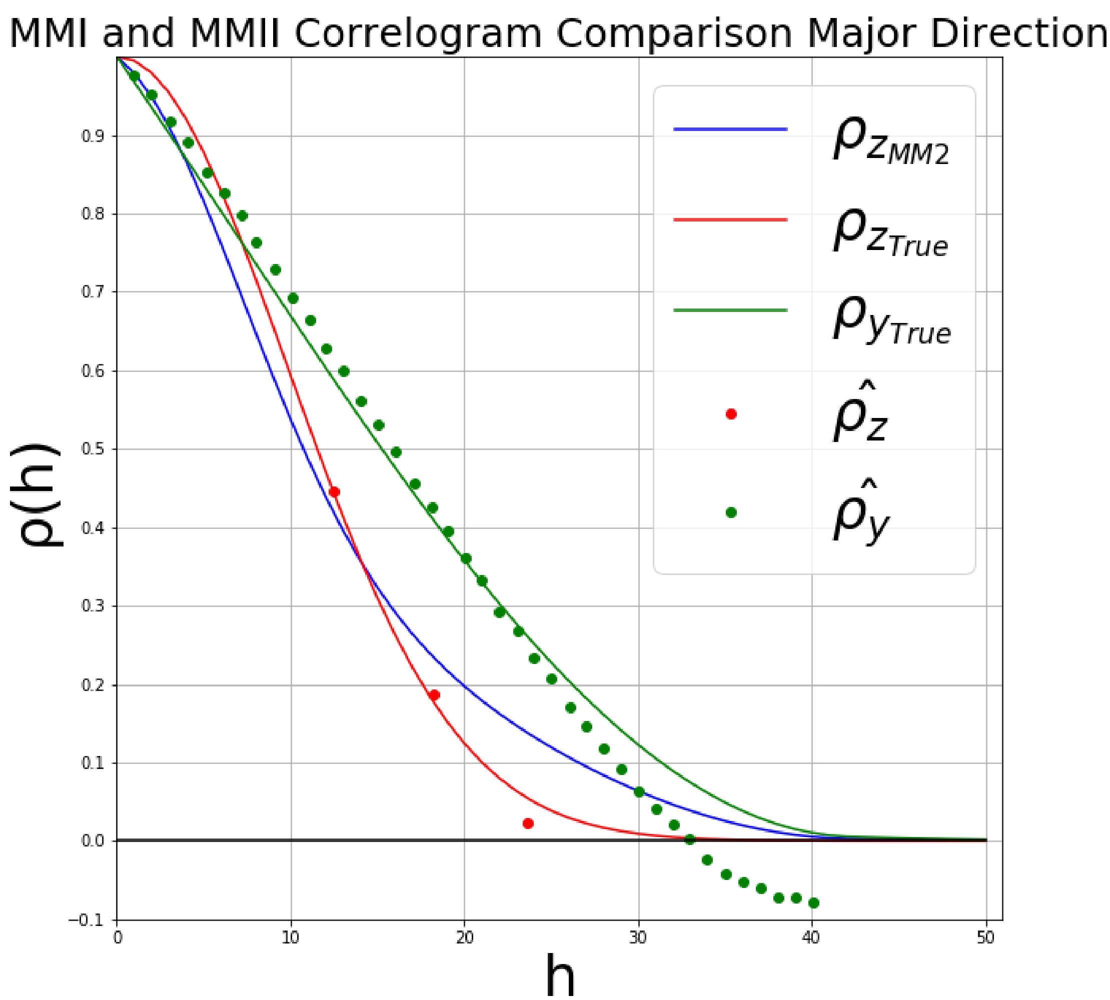
2.7 C_Z Correlogram MM2

[21]:
```
# h1 = Set of points X,Y
# h2 = Set of points X,Y
# Corr = correlation between primary and secondary data
# k = 0 used for calculating the distance between the same points
# k = 1 used for calculationg distance between different points
# k = 2 used for plotting in the major direction
# k = 3 used for plotting in the minor direction
def C_Z_MM2(h1,h2,k,corr):
    return ((C_Y(h1,h2,k) * corr**2) + ((1-corr**2) * C_r(h1,h2,k)))
```

2.8 Plots Correlogram Models

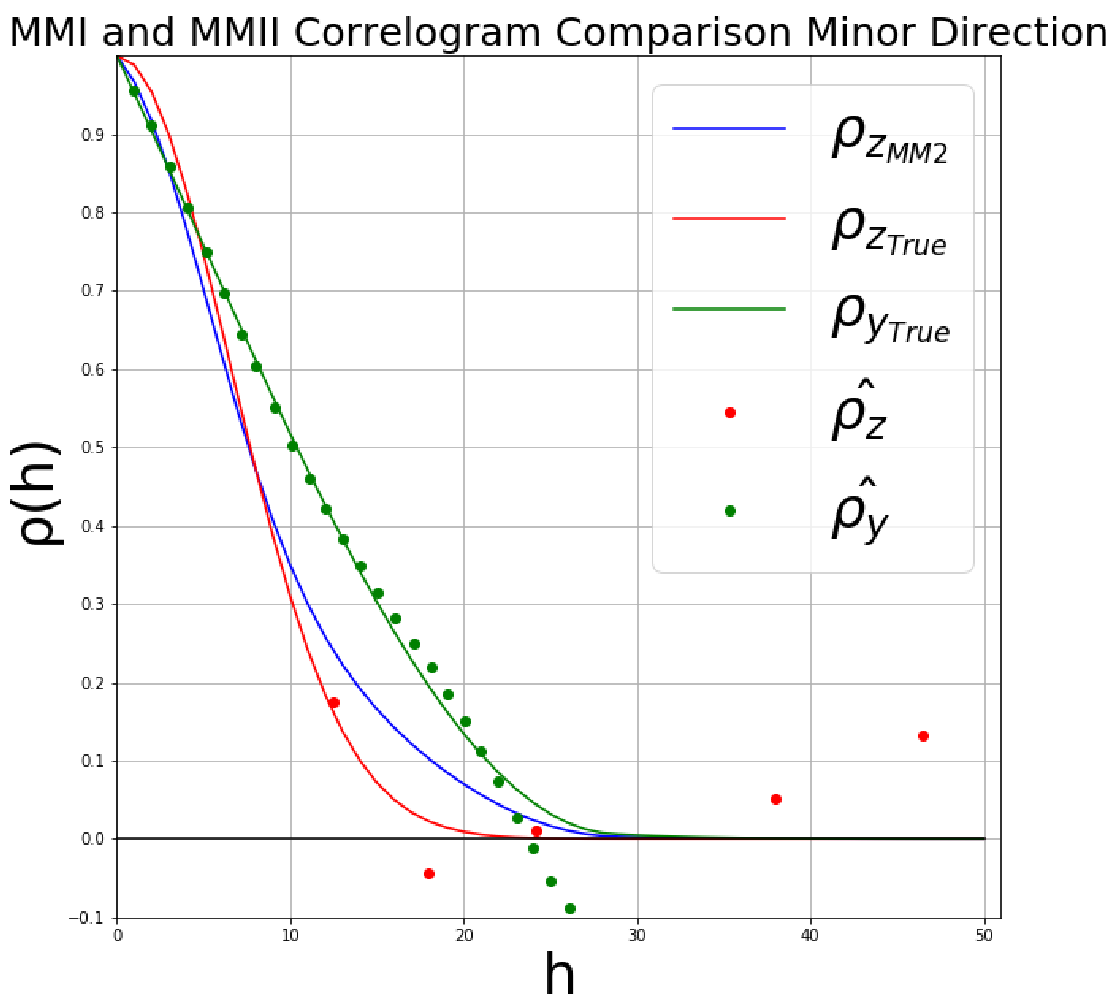
[22]:
```
corr = np.corrcoef(datafl['Primary'],datafl['Secondary'])[0,1]
cy = np.zeros(shape=(51))
cz_True = np.zeros(shape=(51))
cr = np.zeros(shape=(51))
cz = np.zeros(shape=(51))
H =np.zeros(shape=(51))
ones = np.zeros(shape=(51))
for h in range(0,51):
    cy[h] = C_Y(np.matrix(h),np.matrix(h),2)
    cz_True[h] = C_Z(np.matrix(h),np.matrix(h),2)
    cz[h] = C_Z_MM2(np.matrix(h),np.matrix(h),2,corr)
    cr[h] = C_r(np.matrix(h),np.matrix(h),2)
    H[h] = h
fig, axes = plt.subplots(1,1, figsize=(10,10))
axes.plot(H,cz,color = 'Blue',label = '$\u03C1_{z_{MM2}}$')
axes.plot(H,cz_True,color= 'red',label = '$\u03C1_{z_{True}}$')
axes.plot(H,cy,color = 'Green',label = '$\u03C1_{y_{True}}$')
axes.plot(varcalcfl_1['Lag Distance'][varcalcfl_1['Variogram Index']==1]
               ,1-varcalcfl_1['Variogram Value'][varcalcfl_1['Variogram␣
↪Index']==1]
```

```
                ,'ro',color ='Red',label = '$\hat{\u03C1_{z}}$')
axes.plot(varcalcfl_2['Lag Distance'][varcalcfl_2['Variogram Index']==1]
                ,1-varcalcfl_2['Variogram Value'][varcalcfl_2['Variogram␣
 ↪Index']==1]
                ,'ro',color ='Green',label = '$\hat{\u03C1_{y}}$')
axes.plot(H,ones,color = 'Black')
axes.grid()
plt.xlim(0,51)
plt.xticks(np.arange(min(H), max(H)+1, 10))
plt.ylim(-0.1,1)
plt.yticks(np.arange(-0.1, 1, 0.1))
plt.ylabel('\u03C1(h)',size=35)
plt.xlabel('h',size=35)
plt.title('MMI and MMII Correlogram Comparison Major Direction',size = 25)
axes.legend(loc='best', prop={"size":35})
plt.savefig('../0-Figures/MM1_MM2_var_major')
```



[23]:

```
corr = np.corrcoef(datafl['Primary'],datafl['Secondary'])[0,1]
cy = np.zeros(shape=(51))
cz_True = np.zeros(shape=(51))
cr = np.zeros(shape=(51))
cz = np.zeros(shape=(51))
H =np.zeros(shape=(51))
ones = np.zeros(shape=(51))
for h in range(0,51):
    cy[h] = C_Y(np.matrix(h),np.matrix(h),3)
    cz_True[h] = C_Z(np.matrix(h),np.matrix(h),3)
    cz[h] = C_Z_MM2(np.matrix(h),np.matrix(h),3,corr)
    cr[h] = C_r(np.matrix(h),np.matrix(h),3)
    H[h] = h
fig, axes = plt.subplots(1,1, figsize=(10,10))
axes.plot(H,cz,color = 'Blue',label = '$\u03C1_{z_{MM2}}$')
axes.plot(H,cz_True,color= 'red',label = '$\u03C1_{z_{True}}$')
axes.plot(H,cy,color = 'Green',label = '$\u03C1_{y_{True}}$')
axes.plot(varcalcfl_1['Lag Distance'][varcalcfl_1['Variogram Index']==2]
               ,1-varcalcfl_1['Variogram Value'][varcalcfl_1['Variogram␣
 ↪Index']==2]
               ,'ro',color ='Red',label = '$\hat{\u03C1_{z}}$')
axes.plot(varcalcfl_2['Lag Distance'][varcalcfl_2['Variogram Index']==2]
               ,1-varcalcfl_2['Variogram Value'][varcalcfl_2['Variogram␣
 ↪Index']==2]
               ,'ro',color ='Green',label = '$\hat{\u03C1_{y}}$')
axes.plot(H,ones,color = 'Black')
axes.grid()
plt.xlim(0,51)
plt.xticks(np.arange(min(H), max(H)+1, 10))
plt.ylim(-0.1,1)
plt.yticks(np.arange(-0.1, 1, 0.1))
plt.ylabel('\u03C1(h)',size=35)
plt.xlabel('h',size=35)
plt.title('MMI and MMII Correlogram Comparison Minor Direction',size = 25)
axes.legend(loc='best', prop={"size":35})
plt.savefig('../0-Figures/MM1_MM2_var_Minor')
```



2.9 Cross Correlogram

[25]:
```
def C_ZY(h1,h2,k,corr):
    C = []
    nstruct = 1
    vtype = [1]
    a_max = [45]
    a_min = [30]
    Azimuth = 90
    cc = [corr]
    c= 0
    for i in range(nstruct):
        Q1 = h1.copy()
        Q2 = h2.copy()
        if k == 0:
```

```
            d = distance_matrix(np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])),
                                    np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i])))
        elif k == 1:
            d = np.sqrt(np.square((np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])))-
                                  np.tile((np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i]))),(k,1))).sum(axis=1))
            d = np.asarray(d).reshape(len(d))
        elif k == 2:
            d = Q1/a_max[i]
        elif k == 3:
            d = Q1/a_min[i]
        c = c + covar(vtype[i],d,1)*cc[i]
    return c
```

2.10 LMC

The new variogram that will be used for full cokriging, these variograms will be slightly differnt the variograms modelled above. For LMC variograms the sill should be the variance of the variable for the primary and secondary variables. The correlations is the sill of the cross-correlogram

2.10.1 Primary

[16]:
```
# h1 = Set of points X,Y
# h2 = Set of points X,Y
# k = 0 used for calculating the distance between the same points
# k = 1 used for calculationg distance between different points
# k = 2 used for plotting in the major direction
# k = 3 used for plotting in the minor direction
def C_Z_LMC(h1,h2,k):
    C = []
    nstruct = 2
    vtype = [1,1]
    a_max = [33,40]
    a_min = [15,30]
    Azimuth = 90
    cc = [0.85,0.15]
    c = 0
    for i in range(nstruct):
        Q1 = h1.copy()
        Q2 = h2.copy()
        if k == 0:
            d = distance_matrix(np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])),
```

```
                                    np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i])))
        elif k == 1:
            d = np.sqrt(np.square((np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])))-
                                  np.tile((np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i]))),(k,1))).sum(axis=1))
            d = np.asarray(d).reshape(len(d))
        elif k == 2:
            d = Q1/a_max[i]
        elif k == 3:
            d = Q1/a_min[i]
        c = c + covar(vtype[i],d,1)*cc[i]
    return c
```

2.10.2 Secondary

[17]:
```
# h1 = Set of points X,Y
# h2 = Set of points X,Y
# k = 0 used for calculating the distance between the same points
# k = 1 used for calculationg distance between different points
# k = 2 used for plotting in the major direction
# k = 3 used for plotting in the minor direction
def C_Y_LMC(h1,h2,k):
    C = []
    nstruct = 2
    vtype = [1,1]
    a_max = [33,40]
    a_min = [15,30]
    Azimuth = 90
    cc = [0.25,0.75]
    c= 0
    for i in range(nstruct):
        Q1 = h1.copy()
        Q2 = h2.copy()
        if k == 0:
            d = distance_matrix(np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])),
                                    np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i])))
        elif k == 1:
            d = np.sqrt(np.square((np.
↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])))-
                                  np.tile((np.
↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i]))),(k,1))).sum(axis=1))
            d = np.asarray(d).reshape(len(d))
```

```
        elif k == 2:
            d = Q1/a_max[i]
        elif k == 3:
            d = Q1/a_min[i]
        c = c + covar(vtype[i],d,1)*cc[i]
    return c
```

2.10.3 Cross

[18]:
```
# h1 = Set of points X,Y
# h2 = Set of points X,Y
# Corr = correlation between primary and secondary data
# k = 0 used for calculating the distance between the same points
# k = 1 used for calculationg distance between different points
# k = 2 used for plotting in the major direction
# k = 3 used for plotting in the minor direction
def C_ZY_LMC(h1,h2,k,corr):
    C = []
    nstruct = 2
    vtype = [1,1]
    a_max = [33,40]
    a_min = [15,30]
    Azimuth = 90
    cc = [corr*0.6,corr*0.4]
    c= 0
    for i in range(nstruct):
        Q1 = h1.copy()
        Q2 = h2.copy()
        if k == 0:
            d = distance_matrix(np.
 ↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])),
                                   np.
 ↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i])))
        elif k == 1:
            d = np.sqrt(np.square((np.
 ↪matmul(Q1,Rot_Mat(Azimuth,a_max[i],a_min[i])))-
                                 np.tile((np.
 ↪matmul(Q2,Rot_Mat(Azimuth,a_max[i],a_min[i]))),(k,1))).sum(axis=1))
            d = np.asarray(d).reshape(len(d))
        elif k == 2:
            d = Q1/a_max[i]
        elif k == 3:
            d = Q1/a_min[i]
        c = c + covar(vtype[i],d,1)*cc[i]
    return c
```

[19]:
```
s_1 = np.array([[0.0001, 0],
               [0, 0.0001]])
s_2 = np.array([[0.85, corr*0.6],
               [corr*0.6,0.25]])
s_3 = np.array([[0.15, corr*0.4],
               [corr*0.4,0.75]])

print(np.linalg.det(s_1))
print(np.linalg.det(s_2))
print(np.linalg.det(s_3))
print(s_1+s_2+s_3)
```

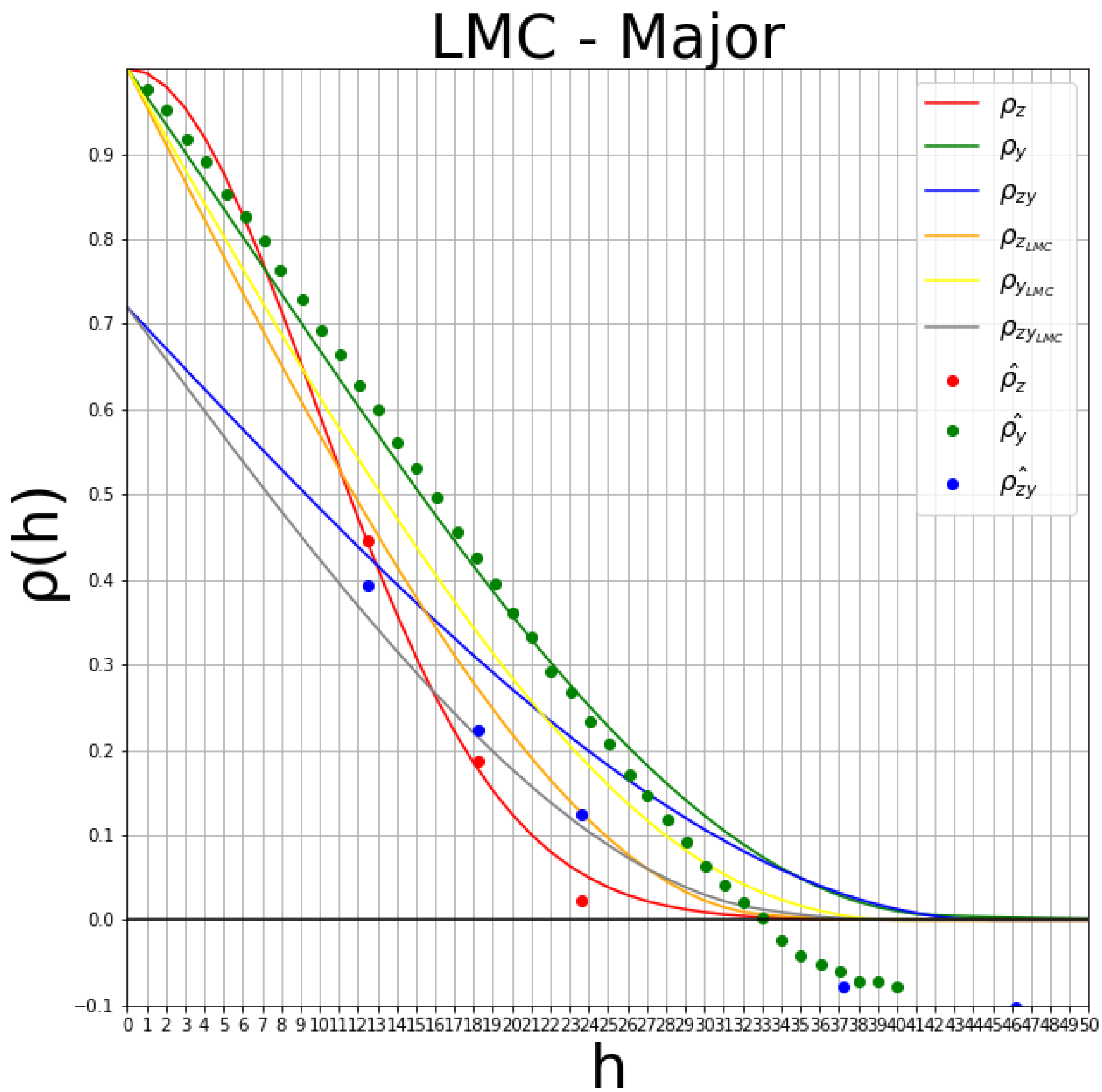
```
1.0000000000000018e-08
0.02601118558622158
0.02961608248276517
[[1.   0.72]
 [0.72 1.  ]]
```

2.10.4 Plot LMC

[20]:
```
cy = np.zeros(shape=(51))
cz_True = np.zeros(shape=(51))
cz = np.zeros(shape=(51))
czy = np.zeros(shape=(51))
cy_LMC = np.zeros(shape=(51))
cz_LMC = np.zeros(shape=(51))
czy_LMC = np.zeros(shape=(51))
H =np.zeros(shape=(51))
ones = np.zeros(shape=(51))
for h in range(0,51):
    cy[h] = C_Y(np.matrix(h),np.matrix(h),2)
    cz_True[h] = C_Z(np.matrix(h),np.matrix(h),2)
    czy[h] = C_ZY(np.matrix(h),np.matrix(h),2,corr)
    H[h] = h
fig, axes = plt.subplots(1,1, figsize=(10,10))
axes.plot(H,cz_True,color= 'red',label = '$\u03C1_{z}$')
axes.plot(H,cy,color = 'Green',label = '$\u03C1_{y}$')
axes.plot(H,czy,color = 'Blue',label = '$\u03C1_{zy}$')


for h in range(0,51):
    cy_LMC[h] = C_Y_LMC(np.matrix(h),np.matrix(h),2)
    cz_LMC[h] = C_Z_LMC(np.matrix(h),np.matrix(h),2)
    czy_LMC[h] = C_ZY_LMC(np.matrix(h),np.matrix(h),2,corr)

axes.plot(H,cz_LMC,color= 'Orange',label = '$\u03C1_{z_{LMC}}$')
axes.plot(H,cy_LMC,color = 'Yellow',label = '$\u03C1_{y_{LMC}}$')
```

```
axes.plot(H,czy_LMC,color = 'Grey',label = '$\u03C1_{zy_{LMC}}$')


axes.plot(varcalcfl_1['Lag Distance'][varcalcfl_1['Variogram Index']==1]
               ,1-varcalcfl_1['Variogram Value'][varcalcfl_1['Variogram␣
↪Index']==1]
               ,'ro',color ='Red',label = '$\hat{\u03C1_{z}}$')
axes.plot(varcalcfl_2['Lag Distance'][varcalcfl_2['Variogram Index']==1]
               ,1-varcalcfl_2['Variogram Value'][varcalcfl_2['Variogram␣
↪Index']==1]
               ,'ro',color ='Green',label = '$\hat{\u03C1_{y}}$')
axes.plot(varcalcfl_3['Lag Distance'][varcalcfl_3['Variogram Index']==1]
               ,corr-varcalcfl_3['Variogram Value'][varcalcfl_3['Variogram␣
↪Index']==1]
               ,'ro',color ='Blue',label = '$\hat{\u03C1_{zy}}$')
axes.plot(H,ones,color = 'Black')
axes.grid()
plt.xlim(0,29)
plt.xticks(np.arange(min(H), max(H)+1, 1.0))
plt.ylim(-0.1,1)
plt.yticks(np.arange(-0.1, 1, 0.1))
plt.ylabel('\u03C1(h)',size=35)
plt.xlabel('h',size=35)
plt.title('LMC - Major',size = 35)
axes.legend(loc='best', prop={"size":15})
#plt.savefig('../0-Figures/MM1_MM2_var')
```

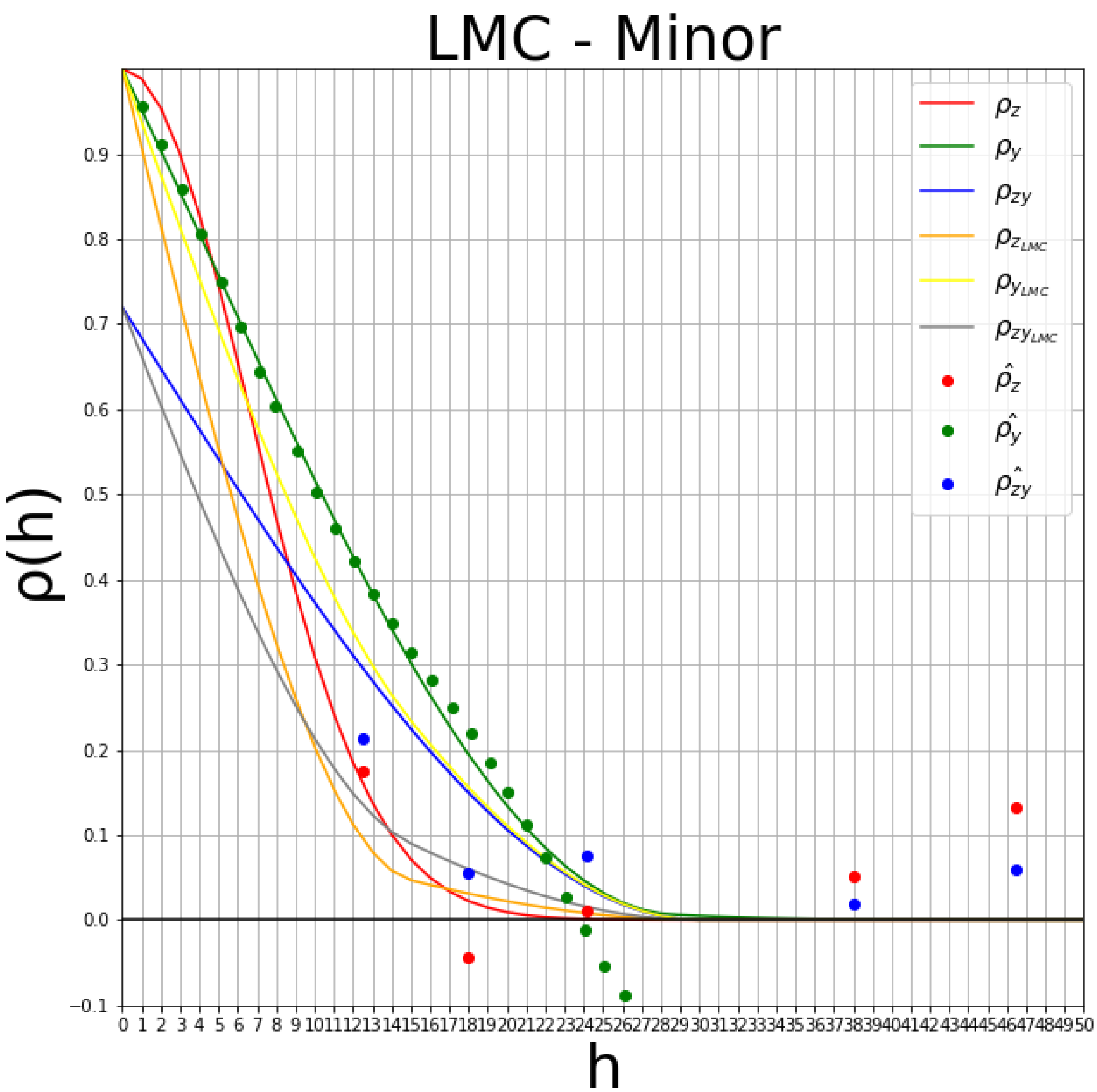
[20]: <matplotlib.legend.Legend at 0x2d1a12f6668>




```
[21]: cy = np.zeros(shape=(51))
cz_True = np.zeros(shape=(51))
cz = np.zeros(shape=(51))
czy = np.zeros(shape=(51))
cy_LMC = np.zeros(shape=(51))
cz_LMC = np.zeros(shape=(51))
czy_LMC = np.zeros(shape=(51))
H =np.zeros(shape=(51))
ones = np.zeros(shape=(51))
for h in range(0,51):
    cy[h] = C_Y(np.matrix(h),np.matrix(h),3)
    cz_True[h] = C_Z(np.matrix(h),np.matrix(h),3)
    czy[h] = C_ZY(np.matrix(h),np.matrix(h),3,corr)
```

```
    H[h] = h
fig, axes = plt.subplots(1,1, figsize=(10,10))
axes.plot(H,cz_True,color= 'red',label = '$\u03C1_{z}$')
axes.plot(H,cy,color = 'Green',label = '$\u03C1_{y}$')
axes.plot(H,czy,color = 'Blue',label = '$\u03C1_{zy}$')


for h in range(0,51):
    cy_LMC[h] = C_Y_LMC(np.matrix(h),np.matrix(h),3)
    cz_LMC[h] = C_Z_LMC(np.matrix(h),np.matrix(h),3)
    czy_LMC[h] = C_ZY_LMC(np.matrix(h),np.matrix(h),3,corr)

axes.plot(H,cz_LMC,color= 'Orange',label = '$\u03C1_{z_{LMC}}$')
axes.plot(H,cy_LMC,color = 'Yellow',label = '$\u03C1_{y_{LMC}}$')
axes.plot(H,czy_LMC,color = 'Grey',label = '$\u03C1_{zy_{LMC}}$')


axes.plot(varcalcfl_1['Lag Distance'][varcalcfl_1['Variogram Index']==2]
               ,1-varcalcfl_1['Variogram Value'][varcalcfl_1['Variogram␣
 ↪Index']==2]
               ,'ro',color ='Red',label = '$\hat{\u03C1_{z}}$')
axes.plot(varcalcfl_2['Lag Distance'][varcalcfl_2['Variogram Index']==2]
               ,1-varcalcfl_2['Variogram Value'][varcalcfl_2['Variogram␣
 ↪Index']==2]
               ,'ro',color ='Green',label = '$\hat{\u03C1_{y}}$')
axes.plot(varcalcfl_3['Lag Distance'][varcalcfl_3['Variogram Index']==2]
               ,corr-varcalcfl_3['Variogram Value'][varcalcfl_3['Variogram␣
 ↪Index']==2]
               ,'ro',color ='Blue',label = '$\hat{\u03C1_{zy}}$')
axes.plot(H,ones,color = 'Black')
axes.grid()
plt.xlim(0,29)
plt.xticks(np.arange(min(H), max(H)+1, 1.0))
plt.ylim(-0.1,1)
plt.yticks(np.arange(-0.1, 1, 0.1))
plt.ylabel('\u03C1(h)',size=35)
plt.xlabel('h',size=35)
plt.title('LMC - Minor',size = 35)
axes.legend(loc='best', prop={"size":15})
#plt.savefig('../0-Figures/MM1_MM2_var')
```

```
[21]: <matplotlib.legend.Legend at 0x2d1a2ad1e80>
```



3 Kriging

3.1 Data Statistics

[22]:
```
Mean_Z = np.average(datafl['Primary'])
STD_Z = 1.0
print(Mean_Z)
print(STD_Z)
```

```
-0.42471875000000003
1.0
```

[23]:
```
Mean_Y = np.average(datafl['Secondary'])
STD_Y = 1.0
print(Mean_Y)
print(STD_Y)
```

```
-0.19742499999999996
1.0
```

[24]:
```
corr = np.corrcoef(datafl['Primary'],datafl['Secondary'])[0,1]
print(corr)
```

```
0.7197391780935075
```

3.2 Create a KDTree to Quickly Get Nearest Points

[25]:
```
from sklearn.neighbors import KDTree
```

[26]:
```
datafl_XY = datafl.as_matrix(['X', 'Y'])
tree = KDTree(datafl_XY)
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:1:
FutureWarning: Method .as_matrix will be removed in a future version. Use
.values instead.
  """Entry point for launching an IPython kernel.
```

[27]:
```
Pred_grid_xy = np.matrix([x,y]).T
```

[28]:
```
k = 60 #number of data to use
X_Y = np.zeros((len(x),k,2))
X_Y_Star = np.zeros((len(x),k,2))
closematrix_Primary = np.zeros((len(x),k))
closematrix_Secondary = np.zeros((len(x),k))
neardistmatrix = np.zeros((len(x),k))
for i in range (0,len(x)):
    nearest_dist, nearest_ind = tree.query(Pred_grid_xy[i:i+1,:], k=k)
    a  = nearest_ind.ravel()
    group = datafl.iloc[a,:]
    closematrix_Primary[i,:] = group['Primary']
    closematrix_Secondary[i,:] = group['Secondary']
    neardistmatrix[i,:] = nearest_dist
    X_Y[i,:,:] = group[['X','Y']]
```

[29]:
```
datafl_XY_2nd = datafl_sec.as_matrix(['X', 'Y'])
tree_2nd = KDTree(datafl_XY_2nd)
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:1:
FutureWarning: Method .as_matrix will be removed in a future version. Use
```

```
.values instead.
  """Entry point for launching an IPython kernel.
```

[30]:
```
k = k #number of neighbours
X_Y_2nd = np.zeros((len(x),k,2))
closematrix_Secondary_2nd = np.zeros((len(x),k))
for i in range (0,len(x)):
    nearest_dist, nearest_ind = tree_2nd.query(Pred_grid_xy[i:i+1,:], k=k)
    a  = nearest_ind.ravel()
    group = datafl_sec.iloc[a,:]
    closematrix_Secondary_2nd[i,:] = group['Secondary']
    X_Y_2nd[i,:,:] = group[['X','Y']]
```

3.3 Simple Kriging

[31]:
```
est_SK = np.zeros(shape = (len(x)))
for z in tqdm(range(0,len(x))):
    Kriging_Matrix = np.zeros(shape=((k,k)))
    #h = distance_matrix(X_Y[z,:,:].tolist(),X_Y[z,:,:].tolist())
    #C_ZZ
    Kriging_Matrix = C_Z(X_Y[z,:,:],X_Y[z,:,:],0)
    #Set up Right Hand Side
    #print(Kriging_Matrix.reshape(((k)),((k))))
    r = np.zeros(shape=(k))
    k_weights = r
    #RHS #C_z*
    r = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)
    Kriging_Matrix.reshape(((k)),((k)))
    #Calculate Kriging Weights
    try:
        k_weights = np.dot(np.linalg.inv(Kriging_Matrix),r)
    except:
        s_m = s_m+1
        sm_idx.append(z)
        k_weights = np.dot(scipy.linalg.pinv(Kriging_Matrix),r)
    #Start Est at zero
    est_SK[z] = 0
    #add in mean_z
    est_SK[z] = est_SK[z] + Mean_Z
    for i in range (0,k):
        #add in Z_i
        est_SK[z] = est_SK[z] + k_weights[i]*(closematrix_Primary[z,i] - Mean_Z)
#print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
```

```
100%|
| 10000/10000 [00:34<00:00, 288.83it/s]
```

3.4 Full Cokriging

[32]:
```
cz = np.zeros(shape = (k,k))
czy = np.zeros(shape = (k,k))
czy_2 = np.zeros(shape = (k,k))
cy = np.zeros(shape = (k,k))
s_m = 0
sm_idx = []
est_Full_CCK = np.zeros(shape = (len(x)))
for z in tqdm(range(0,len(x))):
    Kriging_Matrix = np.zeros(shape=((k*2),(k*2)))
    #C_ZZ
    cz = C_Z_LMC(X_Y[z,:,:],X_Y[z,:,:],0)
    #C_ZY
    czy = C_ZY_LMC(X_Y[z,:,:],X_Y_2nd[z,:,:],0,corr)
    czy_2 = C_ZY_LMC(X_Y_2nd[z,:,:],X_Y[z,:,:],0,corr)
    #C_YY
    cy = C_Y_LMC(X_Y_2nd[z,:,:],X_Y_2nd[z,:,:],0)
    Kriging_Matrix = np.vstack((np.hstack((cz,czy)),np.hstack((czy.T,cy))))
    #print(Kriging_Matrix)
    #Set up Right Hand Sides
    r = np.zeros(shape=(k*2))
    k_weights = np.zeros(shape=(k*2))
    #RHS #C_z*
    r[0:k] = C_Z_LMC(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)
    #RHS #C_zy*
    r[k:k*2] = C_ZY_LMC(X_Y_2nd[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1,corr)
    #Calculate Kriging Weights
    try:
        k_weights = np.dot(np.linalg.inv(Kriging_Matrix),r)
    except:
        s_m = s_m+1
        sm_idx.append(z)
        k_weights = np.dot(scipy.linalg.pinv(Kriging_Matrix),r)
    #Start Est at zero
    est_Full_CCK[z] = 0
    #add in mean_z
    est_Full_CCK[z] = est_Full_CCK[z] + Mean_Z
    for i in range (0,k):
        #add in Z_i
        est_Full_CCK[z] = est_Full_CCK[z] + k_weights[i] *␣
 ↪(closematrix_Primary[z,i] - Mean_Z)/STD_Z
        #add in Y_i
        est_Full_CCK[z] = est_Full_CCK[z] + k_weights[i+k] *␣
 ↪(closematrix_Secondary_2nd[z,i]- Mean_Y)/STD_Y
print('There where {} Singular Matrices'.format(s_m))
#print(Kriging_Matrix.reshape(((2*k)),((2*k))))
```

```
100%|
| 10000/10000 [01:48<00:00, 92.24it/s]

There where 0 Singular Matrices
```

3.5 Simple Collocated Cokriging - MM1

[33]:
```python
est_MM1 = np.zeros(shape = (len(x)))
for z in tqdm(range(0,len(x))):
    Kriging_Matrix = np.zeros(shape=((k+1),(k+1)))
    #C_ZZ
    Kriging_Matrix[0:k,0:k] = C_Z(X_Y[z,:,:],X_Y[z,:,:],0)
    #Set up Right Hand Side
    #print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
    r = np.zeros(shape=(k+1))
    k_weights = np.zeros(shape=(k))
    #RHS #C_z*
    r[0:k] = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)
    #RHS corr
    r[k] = corr
    #c_zy
    Kriging_Matrix[k,0:k+1] = r * corr
    Kriging_Matrix[0:k+1,k] = r * corr
    Kriging_Matrix[k,k] = 1
    #Calculate Kriging Weights
    try:
        k_weights = np.dot(np.linalg.inv(Kriging_Matrix),r)
    except:
        s_m = s_m+1
        sm_idx.append(z)
        k_weights = np.dot(scipy.linalg.pinv(Kriging_Matrix),r)
    #Start Est at zero
    est_MM1[z] = 0
    #add in mean_z
    est_MM1[z] = est_MM1[z] + Mean_Z
    #add in the Y_0
    est_MM1[z] = est_MM1[z] + k_weights[k] *␣
 ↪(datafl_sec['Secondary'][z]-Mean_Y)/STD_Y
    for i in range (0,k):
        #add in Z_i
        est_MM1[z] = est_MM1[z] + k_weights[i] * (closematrix_Primary[z,i] -␣
 ↪Mean_Z)/STD_Z
#print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
```

```
100%|
| 10000/10000 [00:36<00:00, 277.61it/s]
```

3.6 Simple Collocated Cokriging - MM2

[34]:
```
est_MM2 = np.zeros(shape = (len(x)))
s_m = 0
sm_idx =[]
for z in tqdm(range(0,len(x))):
    Kriging_Matrix = np.zeros(shape=((k+1),(k+1)))
    #C_ZZ
    Kriging_Matrix[0:k,0:k] = C_Z_MM2(X_Y[z,:,:],X_Y[z,:,:],0,corr)
    #Set up Right Hand Side
    #print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
    r = np.zeros(shape=(k+1))
    k_weights = np.zeros(shape=(k))
    #RHS #C_z*
    r[0:k] = C_Z_MM2(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1,corr)
    #RHS corr
    r[k] = corr
    #c_zy
    Kriging_Matrix[k,0:k] = C_Y(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1) *␣
 ↪corr
    Kriging_Matrix[0:k,k] = C_Y(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1) *␣
 ↪corr
    Kriging_Matrix[k,k] = 1
    #Calculate Kriging Weights
    try:
        k_weights = np.dot(np.linalg.inv(Kriging_Matrix),r)
    except:
        s_m = s_m+1
        sm_idx.append(z)
        k_weights = np.dot(scipy.linalg.pinv(Kriging_Matrix),r)
    #Start Est at zero
    est_MM2[z] = 0
    #add in mean_z
    est_MM2[z] = est_MM2[z] + Mean_Z
    #add in the Y_0
    est_MM2[z] = est_MM2[z] + k_weights[k] *␣
 ↪(datafl_sec['Secondary'][z]-Mean_Y)/STD_Y
    for i in range (0,k):
        #add in Z_i
        est_MM2[z] = est_MM2[z] + k_weights[i] * (closematrix_Primary[z,i] -␣
 ↪Mean_Z)/STD_Z
#print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
print('There where {} Singular Matrices'.format(s_m))
```

```
100%|
| 10000/10000 [00:51<00:00, 194.81it/s]

There where 0 Singular Matrices
```

3.7 Intrinsic Collocated Cokriging - MM1

[35]:
```
s_m = 0
sm_idx =[]
cz = np.zeros(shape = (k,k))
czy = np.zeros(shape = (k,k))
cy = np.zeros(shape = (k,k))
est_icck_MM1 = np.zeros(shape = (len(x)))
for z in tqdm(range(0,len(x))):
    Kriging_Matrix = np.zeros(shape=((k*2+1),(k*2+1)))
    #C_ZZ
    cz = C_Z(X_Y[z,:,:],X_Y[z,:,:],0)
    #C_ZY
    czy = C_Z(X_Y[z,:,:],X_Y[z,:,:],0) * corr
    #C_YY
    cy = C_Z(X_Y[z,:,:],X_Y[z,:,:],0)
    #Set up Right Hand Side
    Kriging_Matrix[0:k*2,0:k*2] = np.vstack((np.hstack((cz,czy)),np.hstack((czy.
 ↪T,cy))))
    #print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
    r = np.zeros(shape=(k*2)+1)
    k_weights = r
    #RHS #C_z*
    r[0:k] = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)
    #RHS #C_yz*
    r[k:k*2] = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1) * corr
    #RHS corr
    r[k*2] = corr
    #c_zy
    Kriging_Matrix[k*2,0:k] = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)␣
 ↪* corr
    Kriging_Matrix[0:k,k*2] = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)␣
 ↪* corr
    #c_z
    Kriging_Matrix[k*2,k:k*2] = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)
    Kriging_Matrix[k:k*2,k*2] = C_Z(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)
    Kriging_Matrix[k*2,k*2] = 1
    #Calculate Kriging Weights
    try:
        k_weights = np.dot(np.linalg.inv(Kriging_Matrix),r)
    except:
        s_m = s_m+1
        sm_idx.append(z)
        k_weights = np.dot(scipy.linalg.pinv(Kriging_Matrix),r)
    #Start Est at zero
```

```
    est_icck_MM1[z] = 0
    #add in mean_z
    est_icck_MM1[z] = est_icck_MM1[z] + Mean_Z
    #add in the Y_0
    est_icck_MM1[z] = est_icck_MM1[z] + k_weights[k*2] *␣
 ↪(datafl_sec['Secondary'][z]-Mean_Y)/STD_Y
    for i in range (0,k):
        #add in Z_i
        est_icck_MM1[z] = est_icck_MM1[z] + k_weights[i] *␣
 ↪(closematrix_Primary[z,i] - Mean_Z)/STD_Z
        #add in Y_i
        est_icck_MM1[z] = est_icck_MM1[z] + k_weights[i+k] *␣
 ↪(closematrix_Secondary[z,i]- Mean_Y)/STD_Y
#print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
print('There where {} Singular Matrices'.format(s_m))
```

```
100%|
| 10000/10000 [01:45<00:00, 94.56it/s]
```

```
There where 0 Singular Matrices
```

3.8 Intrinsic Collocated Cokriging - MM2

[36]:

```
s_m = 0
sm_idx =[]
cz = np.zeros(shape = (k,k))
czy = np.zeros(shape = (k,k))
cy = np.zeros(shape = (k,k))
est_icck_MM2 = np.zeros(shape = (len(x)))
for z in tqdm(range(0,len(x))):
    Kriging_Matrix = np.zeros(shape=((k*2+1),(k*2+1)))
    #C_ZZ
    #1
    cz = C_Z_MM2(X_Y[z,:,:],X_Y[z,:,:],0,corr)
    #C_ZY
    #2,#3
    czy = corr * C_Y(X_Y[z,:,:],X_Y[z,:,:],0)
    #C_YY
    #4
    cy = C_Y(X_Y[z,:,:],X_Y[z,:,:],0)
    #Set up Right Hand Side
    #print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
    Kriging_Matrix[0:k*2,0:k*2] = np.vstack((np.hstack((cz,czy)),np.hstack((czy.
 ↪T,cy))))
    r = np.zeros(shape=(k*2)+1)
    k_weights = r
```

```
    #RHS #C_z*
    #5
    r[0:k] = C_Z_MM2(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1,corr)
    #RHS #C_yz*
    #6
    r[k:k*2] = C_Y(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1) * corr
    #RHS corr
    #7
    r[k*2] = corr
    #c_zy
    #8
    Kriging_Matrix[k*2,0:k] = C_Y(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)␣
↪* corr
    Kriging_Matrix[0:k,k*2] = C_Y(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)␣
↪* corr
    #c_y
    #9
    Kriging_Matrix[k*2,k:k*2] = C_Y(X_Y[z,:,:],np.tile(Pred_grid_xy[z],(k,1)),1)
    Kriging_Matrix[k:k*2,k*2]  = C_Y(X_Y[z,:,:],np.
↪tile(Pred_grid_xy[z],(k,1)),1)
    Kriging_Matrix[k*2,k*2] = 1
    #Kriging_Matrix.reshape(((2*k)+1),((2*k)+1))
    #Calculate Kriging Weights
    try:
        k_weights = np.dot(np.linalg.inv(Kriging_Matrix),r)
    except:
        s_m = s_m+1
        sm_idx.append(z)
        k_weights = np.dot(scipy.linalg.pinv(Kriging_Matrix),r)
    #Start Est at zero
    est_icck_MM2[z] = 0
    #add in mean_z
    est_icck_MM2[z] = est_icck_MM2[z] + Mean_Z
    #add in the Y_0
    est_icck_MM2[z] = est_icck_MM2[z] + k_weights[k*2] *␣
↪(datafl_sec['Secondary'][z]-Mean_Y)/STD_Y
    for i in range (0,k):
        #add in Z_i
        est_icck_MM2[z] = est_icck_MM2[z] + k_weights[i] *␣
↪(closematrix_Primary[z,i] - Mean_Z)/STD_Z
        #add in Y_i
        est_icck_MM2[z] = est_icck_MM2[z] + k_weights[i+k] *␣
↪(closematrix_Secondary[z,i]- Mean_Y)/STD_Y
#print(Kriging_Matrix.reshape(((2*k)+1),((2*k)+1)))
print('There where {} Singular Matrices'.format(s_m))
```

100%|

```
| 10000/10000 [02:02<00:00, 81.31it/s]
```

```
There where 0 Singular Matrices
```

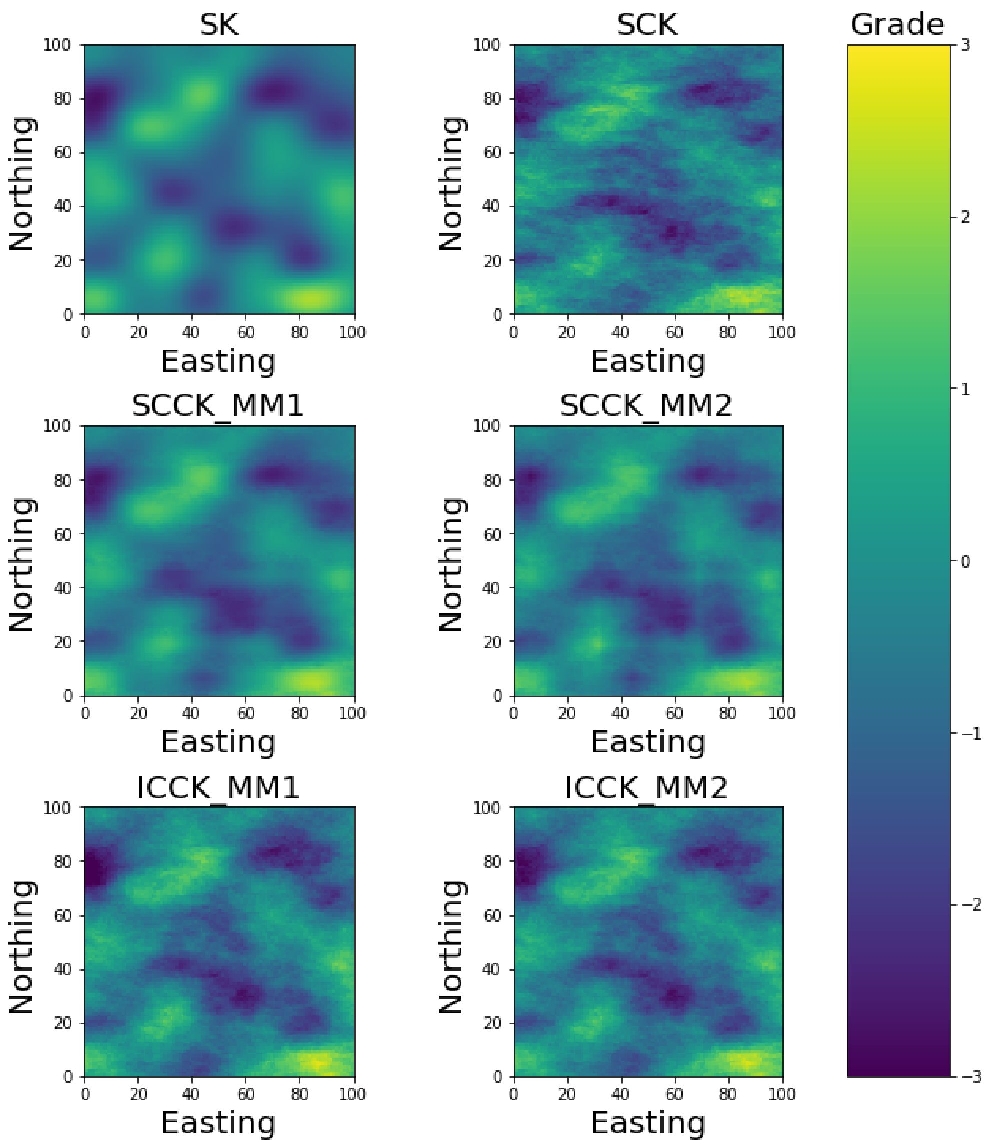
3.9 Results

[37]:
```
ktypes_vals_str = ['SK','SCK','SCCK_MM1','SCCK_MM2','ICCK_MM1','ICCK_MM2']
ktypes_vals = [est_SK,est_Full_CCK,est_MM1,est_MM2,est_icck_MM1,est_icck_MM2]
```

3.9.1 Pixelplt

[38]:
```
#Set up Plotting Grid
fig = plt.figure(constrained_layout=True,figsize=(10,10))
gs = gridspec.GridSpec(3, 6,figure=fig)
gs.update(wspace=1,hspace=0.7)
ax1 = plt.subplot(gs[0, :2])
ax2 = plt.subplot(gs[0, 2:4])
ax3 = plt.subplot(gs[1, :2])
ax4 = plt.subplot(gs[1, 2:4])
ax5 = plt.subplot(gs[2, :2])
ax6 = plt.subplot(gs[2, 2:4])
ax7 = plt.subplot(gs[:,4])
#Generate Pixelplt
axes = [ax1,ax2,ax3,ax4,ax5,ax6]
XMIN, XMAX = 0, 100
YMIN, YMAX = 0, 100
SMIN, SMAX = -3,3
gridd = pd.DataFrame()
gridd['Y'] = y
gridd['X'] = x
for i in range(0,len(ktypes_vals)):
    gridd['Estimate'] = ktypes_vals[i]
    gridded = np.reshape(gridd.sort_values(by=['Y', 'X'], axis=0,␣
 ↪ascending=True)['Estimate'].values,
                      [100, 100], order='C')
    ax = axes[i]
    ax.imshow(gridded, origin='lower', extent=[XMIN, XMAX, YMIN, YMAX],␣
 ↪aspect='equal',
                interpolation='none', vmin=SMIN, vmax=SMAX, cmap='viridis')
    ax.set_title('{}'.format(ktypes_vals_str[i]),size=20)
    ax.set_xlabel('Easting',size=20)
    ax.set_ylabel('Northing',size=20)
norm = matplotlib.colors.Normalize(vmin=-3, vmax=3)
cb1 = matplotlib.colorbar.ColorbarBase(ax7, cmap= plt.get_cmap('viridis'),norm␣
 ↪= norm)
ax7.set_title('Grade',size=20)
```

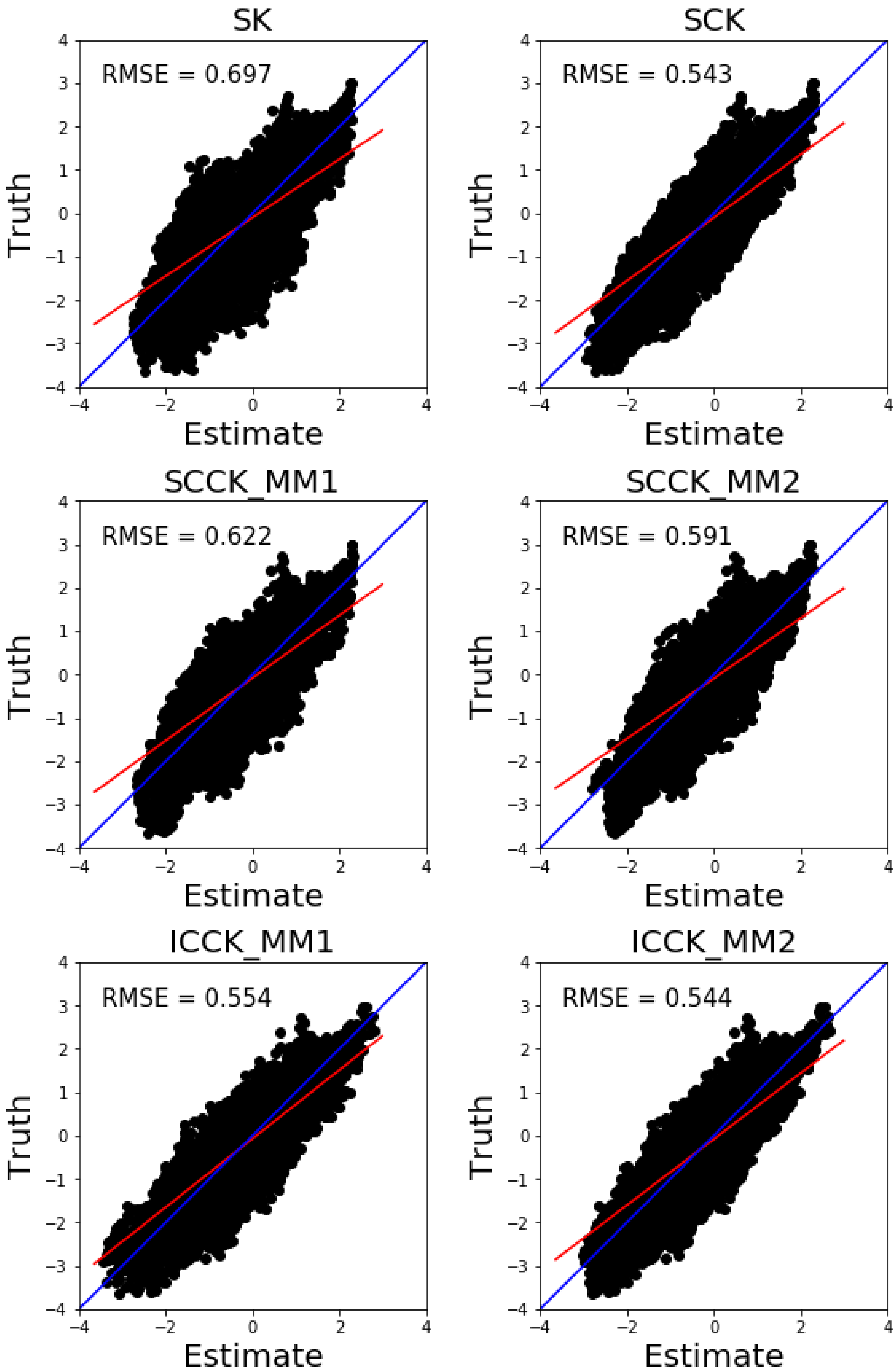
```
plt.show()
fig.savefig('../0-Figures/estimates.png')
```



3.9.2 Data Reproduction

[39]:
```
#Set up Plotting Grid
fig = plt.figure(constrained_layout=True,figsize=(8,12))
gs = gridspec.GridSpec(3, 4,figure=fig)
gs.update(wspace=1,hspace=0.7)
ax1 = plt.subplot(gs[0, :2])
ax2 = plt.subplot(gs[0, 2:4])
ax3 = plt.subplot(gs[1, :2])
```

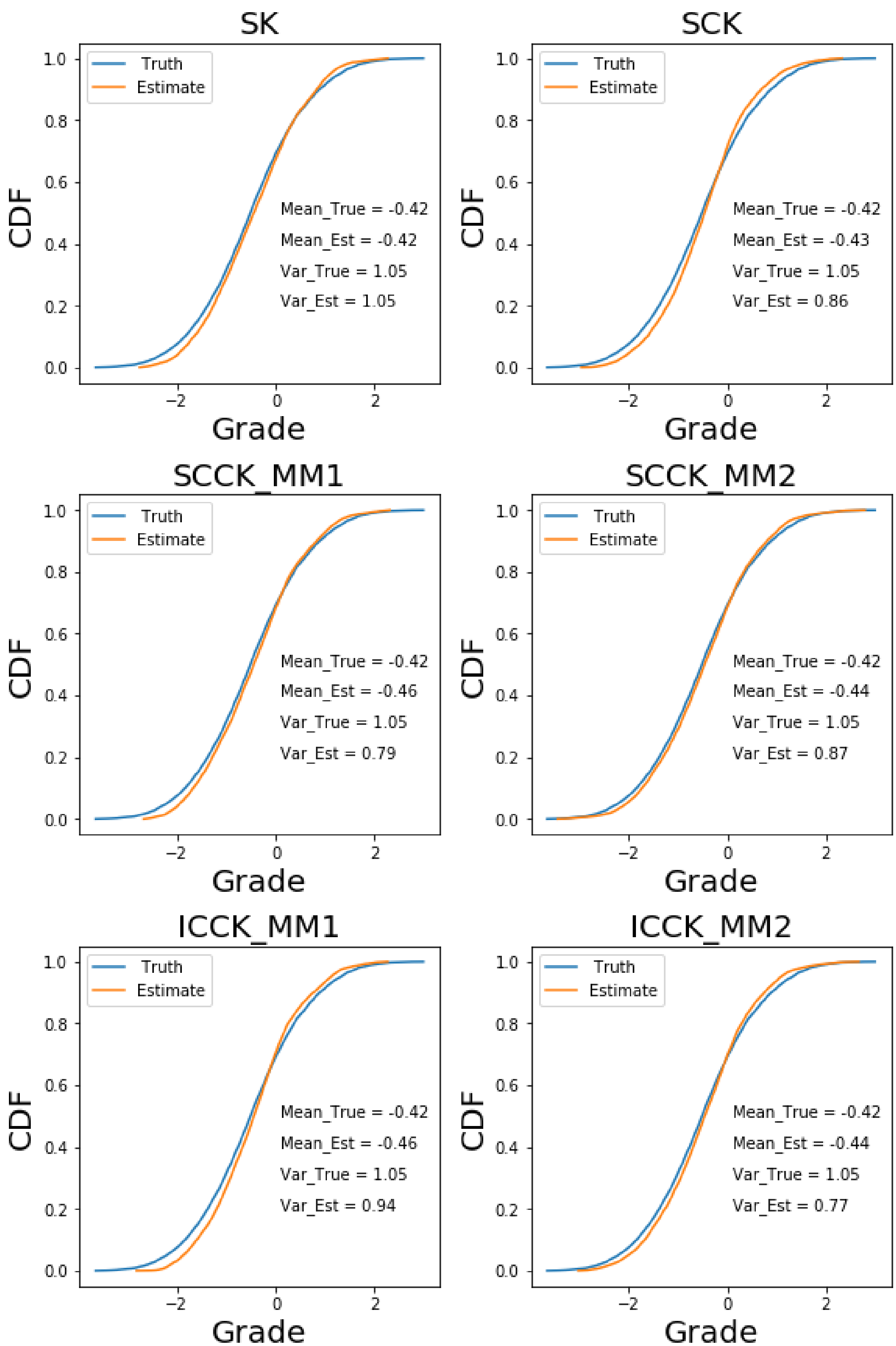
```
ax4 = plt.subplot(gs[1, 2:4])
ax5 = plt.subplot(gs[2, :2])
ax6 = plt.subplot(gs[2, 2:4])
axes = [ax1,ax2,ax3,ax4,ax5,ax6]
for i in range(0,len(ktypes_vals)):
    ax = axes[i]
    ax.scatter(ktypes_vals[i],truth['Primary'],color='Black')
    ax.plot(np.unique(truth['Primary']),
             np.poly1d(np.polyfit(truth['Primary'], ktypes_vals[i], 1))(np.
 ↪unique(truth['Primary'])),
            color = 'Red')
    ax.set_title('{}'.format(ktypes_vals_str[i]),size = 20)
    ax.set_xlim(-4,4)
    ax.set_ylim(-4,4)
    x_45 = np.linspace(*ax.get_xlim())
    ax.plot(x_45, x_45,color = 'blue')
    ax.set_xlabel('Estimate',size = 20)
    ax.set_ylabel('Truth',size = 20)
    ax.set_aspect('equal', 'box')
    ax.text(-3.5,3.0,'RMSE = {:.3f}'.format(np.
 ↪sqrt(mean_squared_error(ktypes_vals[i],truth['Primary']))),size=15)
plt.show()
fig.savefig('../0-Figures/Scatter_true.png')
```


SK
RMSE = 0.697
SCK
RMSE = 0.543
SCCK_MM1
RMSE = 0.622
SCCK_MM2
RMSE = 0.591
ICCK_MM1
RMSE = 0.554
ICCK_MM2
RMSE = 0.544
Truth
Estimate

3.9.3 Histogram Reproduction

[40]:
```
mean = []
var = []
RMSE = []
ktypes_vals = [est_SK,est_Full_CCK,est_MM1,est_icck_MM1,est_MM2,est_icck_MM2]
mean.append(np.mean(datafl['Primary']))
var.append(np.var(datafl['Primary']))
RMSE.append(0)
for ktypes_vals in ktypes_vals:
    mean.append(np.mean(ktypes_vals))
    var.append(np.var(ktypes_vals))
    RMSE.append(np.sqrt(mean_squared_error(ktypes_vals,truth['Primary'])))
```

[41]:
```
ktypes_vals = [est_SK,est_Full_CCK,est_MM1,est_icck_MM1,est_MM2,est_icck_MM2]
fig = plt.figure(constrained_layout=True,figsize=(8,12))
gs = gridspec.GridSpec(3, 4,figure=fig)
gs.update(wspace=1,hspace=0.7)
ax1 = plt.subplot(gs[0, :2])
ax2 = plt.subplot(gs[0, 2:4])
ax3 = plt.subplot(gs[1, :2])
ax4 = plt.subplot(gs[1, 2:4])
ax5 = plt.subplot(gs[2, :2])
ax6 = plt.subplot(gs[2, 2:4])
axes = [ax1,ax2,ax3,ax4,ax5,ax6]
for i in range(0,len(ktypes_vals)):
    ax = axes[i]
    x_cdf, y_cdf = sorted(truth['Primary']), np.arange(len(truth['Primary'])) /␣
↪len(truth['Primary'])
    ax.plot(x_cdf, y_cdf,label = ' Truth')
    est = ktypes_vals[i]
    x_cdf, y_cdf = sorted(est), np.arange(len(est)) / len(est)
    ax.plot(x_cdf, y_cdf, label = 'Estimate')
    ax.set_title('{}'.format(ktypes_vals_str[i]),size = 20)
    ax.set_xlabel('Grade',size = 20)
    ax.set_ylabel('CDF',size = 20)
    ax.legend(loc = 'best')
    ax.text(0.1,0.50,'Mean_True = {:.2f}'.format(mean[0]),size=10)
    ax.text(0.1,0.40,'Mean_Est = {:.2f}'.format(mean[i]),size=10)
    ax.text(0.1,0.30,'Var_True = {:.2f}'.format(var[0]),size=10)
    ax.text(0.1,0.20,'Var_Est = {:.2f}'.format(var[i]),size=10)
fig.savefig('../0-Figures/hist_rep.png')
```


SK
Truth
Estimate
Mean_True = -0.42
Mean_Est = -0.42
Var_True = 1.05
Var_Est = 1.05
CDF
Grade
SCK
Truth
Estimate
Mean_True = -0.42
Mean_Est = -0.43
Var_True = 1.05
Var_Est = 0.86
CDF
Grade
SCCK_MM1
Truth
Estimate
Mean_True = -0.42
Mean_Est = -0.46
Var_True = 1.05
Var_Est = 0.79
CDF
Grade
SCCK_MM2
Truth
Estimate
Mean_True = -0.42
Mean_Est = -0.44
Var_True = 1.05
Var_Est = 0.87
CDF
Grade
ICCK_MM1
Truth
Estimate
Mean_True = -0.42
Mean_Est = -0.46
Var_True = 1.05
Var_Est = 0.94
CDF
Grade
ICCK_MM2
Truth
Estimate
Mean_True = -0.42
Mean_Est = -0.44
Var_True = 1.05
Var_Est = 0.77
CDF
Grade