

# On the Stability of Isostatically Compensated Mountain Belts

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The formation of convergent mountain belts is invariably accompanied by an increase in gravitational potential energy due to part of the work done by the forces driving convergence. The evolution of potential energy stored in an orogen is dependent on (1) the density structure, (2) the thermal evolution, and (3) the way convergent deformation is partitioned between crust and mantle lithosphere. It is now well recognized that this increase in potential energy associated with the mountain building process raises the possibility that significant extension, or collapse, may accompany the relaxation of the forces driving convergence provided the lithosphere is thermally weakened (e.g., England, 1987). In this paper we evaluate the stability of isostatically compensated mountain belts under the assumption that the strength of continental lithosphere is governed by a combination of frictional sliding and creep processes using the "Brace-Goetze" model for the rheology of the lithosphere. The reference lithosphere, defined to be in potential energy and isostatic balance with the mid-ocean ridges, changes with different thermal parameters of the lithosphere. The instantaneous extensional strain rate for thermally mature mountain belts is calculated by balancing the horizontal buoyancy force stored in the mountain belts (measured relative to the reference state) with the vertically integrated strength of the lithosphere for initial strengths spanning the probable natural range. It is shown that horizontal buoyancy forces arising in isostatically balanced mountain belts are sufficiently large to induce the collapse at significant rates (greater than a few times  $10^{-16} \text{ s}^{-1}$ ) and leading to significant finite extension providing the Moho temperatures exceed about  $650-700^\circ\text{C}$ , a condition only likely to be attained if the mantle lithosphere has not been thickened to the same extent as the overlying crust. Consequently, processes that thin the mantle lithosphere as a consequence of convergent deformation such as the convective instability of a thickened lower thermal boundary layer greatly increase the possibility of collapse. The calculations presented here suggest that near complete destruction of the mountain system by extensional collapse may be possible if such processes can reduce total lithospheric thickening to less than half the contemporary crustal thickening (i.e.,  $f_l \leq f_c/2$ , where  $f_l$  and  $f_c$  are the lithospheric and crustal thickening factors).

## INTRODUCTION

Topography generated on density interfaces during continental deformation affects the potential energy of the lithosphere and therefore has important implications for the mechanics of orogenic belts [e.g., Artyushkov, 1973; Molnar and Tapponier, 1978; England and McKenzie, 1982]. A number of authors have recently investigated the isostatic and thermal consequences of convergent mountain building with variable mantle lithospheric deformation [e.g., England and Houseman, 1988, 1989; Sonder *et al.*, 1987; Molnar and Lyon-Caen, 1988; Sandiford and Powell, 1990, 1991]. Using various simplifications and approximations concerning the density structure, these studies suggest that the active thinning of mantle lithosphere during convergence as proposed, among others, by McKenzie [1978] and Houseman *et al.* [1981] could control the termination of convergent deformation in an orogen and, in some circumstances, induce

extensional collapse. The possibility of collapse is clearly dependent on a number of factors, of which the most important are undoubtedly the thermal state and density structure of the orogen (which together constrain the potential energy of the orogen) as well as the magnitude of the forces supporting the mountain belt [e.g., Molnar and Tapponier, 1978; England, 1987; Gaudemer *et al.*, 1988]. In the most important contribution to date, England [1987] has investigated the possibility of collapse when the forces driving convergence in mountain belts are relaxed. The main purpose of this paper is to provide a quantitative way to evaluate the possibility of mountain collapse, following relaxation of the compressional driving force for mountain building. The work advances that of England [1987] in as much as we employ a general method for evaluating the potential energy of an orogen that can be used to examine the consequences of lithospheric deformation with explicit consideration of both the crust and the mantle lithosphere. In addition, a detailed analytical description of the thermal and isostatic consequences of convergent deformation of the continental lithosphere appropriate to isostatically compensated mountain belts is provided.

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## HORIZONTAL BUOYANCY FORCES IN MOUNTAIN BELTS

The formation of mountain belts is invariably associated with an increase in gravitational potential energy that is created by part of the work done by the forces driving the formation of the mountain system. The potential energy per unit area of a column of material,  $Pe$ , above a given depth,  $z$ , is given by the integral of the vertical stress,  $\sigma_{zz}$ , from the Earth's surface to that depth [Molnar and Lyon-Caen, 1988]:

$$Pe = \int_0^z \sigma_{zz}(\tau) d\tau = g \int_0^z \int_0^\tau \rho(\tau') d\tau' d\tau \quad (1)$$

where  $\rho(z)$  is the density at depth  $z$ . The change in potential energy resulting from deformation can then be represented as the horizontal buoyancy force per unit length of orogen,  $F_b$ , in  $N m^{-1}$ , which arises from lateral variations in the vertical stress contributed by differences in density distribution with depth between the deformed and the undeformed or reference lithosphere as discussed below.

The magnitude of the horizontal buoyancy force arising from convergent deformation is essential to understanding the mechanics of mountain belts, since it limits the magnitude of crustal thickening and ultimately provides the impetus for the potential collapse of mountain belts following relaxation of the compressional driving forces [e.g., England and Houseman, 1988, 1989; Molnar and Lyon-Caen, 1988; Sandiford and Powell, 1990]. In its most general form the horizontal buoyancy force arising as a consequence of an isostatically compensated deformation of a reference lithosphere is given by

$$\begin{aligned} F_b &= Pe(f_c z_c, f_l z_l, H_0, f_c D_0) - Pe(z_c, z_l, H_0, D_0) \\ &\quad - \frac{1}{2} \rho_m g (f_l z_l - z_l - h)^2 \\ &\quad - Pr(z_c, z_l, H_0, D_0) (f_l z_l - z_l - h) \end{aligned} \quad (2)$$

where  $Pe(f_c z_c, f_l z_l, H_0, f_c D_0)$  and  $Pe(z_c, z_l, H_0, D_0)$  represent the potential energies of the deformed and reference lithospheres, respectively,  $Pr(z_c, z_l, H_0, D_0)$  is the vertical stress at the base of the reference lithosphere,  $z_c$  is the reference crust thickness,  $z_l$  is the reference lithosphere thickness,  $f_c$  is the crustal thickening factor defined by the ratio of deformed crustal thickness to the reference crustal thickness,  $f_l$  is the lithospheric thickening factor defined by the ratio of deformed lithosphere thickness to the reference lithosphere thickness,  $h$  is isostatically supported surface elevation contrast between the deformed and reference lithospheres as defined below, and  $H_0$  and  $D_0$  characterize the thermal state of the lithosphere as defined below.

In order to define the differences in vertical stress contributed by differences in the density distribution between the deformed and reference lithosphere as required in determining the excess potential energy of the deformed lithosphere it is necessary to define the thermal structure of the

orogen. Previous workers [e.g., Turcotte, 1983; England, 1987; England and Houseman, 1989; Sandiford and Powell, 1990] have discussed horizontal buoyancy forces under the assumption that the lithosphere has no internal heat source, and here we provide a generalized description of buoyancy force and allow both constant and variable heat sources present in the crust. Importantly, we show that the neglect of internal heat sources leads to significant and important discrepancies in the magnitude of the calculated buoyancy forces arising during mountain belt formation.

## POTENTIAL THERMAL STRUCTURE OF MOUNTAIN BELTS

The thermal evolution of mountain belts is a complex function of the deformation history and is particularly sensitive to processes that are initiated as a response to deformation, for example the erosion induced by the uplift of the deformed and thickened lithosphere. This is in part due to the poor thermal conductivity of rocks and implies that only in maturity may mountain belts attain thermal conditions that approach a steady state temperature distribution. Nevertheless, an important constraint on the range of the thermal conditions likely to be experienced by the mountain belt is provided by the potential thermal structure defined as the steady state temperature distribution for any arbitrary deformation from the reference state [e.g., Sandiford and Powell, 1990, 1991]. We note that the potential thermal structure is only likely to be a useful approximation when considering processes affecting the terminal stages of the mountain building process, such as the initiation of extension following relaxation of the compressive driving forces as considered here, particularly in broad orogenic belts [e.g., Gaudemer et al., 1988].

Under the assumption of steady state thermal conditions and depth and temperature independent thermal conductivity, the temperature profile in the continental lithosphere is dependent on the distribution of heat sources and can be given variously by

$$T^o(z) = T_s + (T_l - T_s) \frac{z}{f_l z_l} \quad (3)$$

$$T^c(z) = T^o(z) + Tl(z) \quad (4)$$

$$\begin{aligned} T^e(z) &= T^o(z) + \frac{H_0 f_c^2 D_0}{k} \\ &\quad \cdot \left[ (1 - e^{(-z/f_c D_0)}) - (1 - e^{(-f_l z_l / f_c D_0)}) \frac{z}{f_l z_l} \right] \end{aligned} \quad (5)$$

where  $T_s$  is surface temperature,  $T_l$  is the temperature at base of lithosphere,  $k$  is the thermal conductivity,  $D_0$  is the characteristic length scale of the internal heat production distribution, and  $H_0$  is the surface heat production.  $T^o$  is the temperature profile in the lithosphere in which no heat source is present,  $T^c$  is the temperature distribution when constant heat source is present throughout the crust, while

$T^e$  is the temperature distribution when the crustal heat source varies exponentially with depth such that at depth  $D_0$  the heat production is given by

$$H(z = D_0) = \frac{H_0}{e}$$

$T1(z)$  is an additional term associated with constant heat source in the crust. In the crust  $T1(z)$  is given by

$$\frac{H_0 f_l z_l z}{2k} \left( \frac{2f_c z_c}{f_l z_l} - \frac{(f_c z_c)^2}{(f_l z_l)^2} - \frac{z}{f_l z_l} \right)$$

while in the mantle lithosphere  $T1(z)$  is given by

$$\frac{H_0 f_c^2 z_c^2}{2k} \left( 1 - \frac{z}{f_l z_l} \right)$$

Here  $f_c = f_l = 1$  refers to the reference lithosphere which is in potential energy and isostatic equilibrium with mid-ocean ridges (see below).

In order to evaluate the isostatic adjustment of mountain belts, the density structure of the lithosphere is required. We assume a thermally stabilized mantle lithosphere and use a simple density distribution with the density in the crust defined by

$$\rho(z) = \rho_c [1 + \alpha_c (T_l - T(z))]$$

and in the mantle lithosphere

$$\rho(z) = \rho_m [1 + \alpha_m (T_l - T(z))]$$

where  $\rho_c$  and  $\rho_m$  are the crustal density and mantle density at  $T_l$ , respectively, and  $\alpha_c$  and  $\alpha_m$  are the thermal expansion coefficient of expansion of the crust and mantle, respectively. Following Turcotte [1983] and Sandiford and Powell [1990] we make the simplifying assumption that  $\alpha_c \rho_c = \alpha_m \rho_m$ .

The isostatically supported surface elevation  $h$  of a mountain is then given by

$$\frac{h_o}{z_l} = \delta\psi (f_c - 1) + \frac{\alpha (T_l - T_s)}{2} (1 - f_l) \quad (6)$$

$$\frac{h_c}{z_l} = \frac{h_o}{z_l} + \beta_c \left[ \frac{\psi}{6} (1 - f_c^3) + \frac{(f_l f_c^2 - 1)}{4} \right] \quad (7)$$

$$\begin{aligned} \frac{h_e}{z_l} &= \frac{h_o}{z_l} + f_l f_c^2 \beta_e \\ &\cdot \left[ \frac{1 + e^{-f_l z_l / f_c D_o}}{2} - \frac{f_c}{f_l} \psi_1 (1 - e^{-f_l z_l / f_c D_o}) \right] \\ &- \beta_e \left[ \frac{1 + e^{-z_l / D_o}}{2} - \psi_1 (1 - e^{-z_l / D_o}) \right] \end{aligned} \quad (8)$$

where  $h_o$  is the elevation associated with the lithosphere which has no internal heat source,  $h_c$  is the elevation associated with constant heat source in the whole crust and  $h_e$  the elevation associated with an exponentially reducing heat source and the dimensionless parameters  $\delta$ ,  $\psi$ ,  $\psi_1$ ,  $\beta_c$  and  $\beta_e$  are given by

$$\delta = \frac{\rho_m - \rho_c}{\rho_m}$$

$$\begin{aligned} \psi &= \frac{z_c}{z_l} \\ \psi_1 &= \frac{D_o}{z_l} \\ \beta_c &= \frac{\alpha H_o z_c^2}{k} \\ \beta_e &= \frac{\alpha H_o D_o^2}{k} \end{aligned}$$

As discussed by Sonder and England [1986] and England [1987] and illustrated below, the vertically averaged mechanical properties of the lithosphere are particularly sensitive to the thermal regime which can be usefully characterised by the Moho temperature. Following Sandiford and Powell (1990, 1991) we employ the potential Moho temperature, given by equations (3)-(5) with  $z = z_c f_c$ . In addition, the potential surface heat flow is given by

$$q_i^o = k \frac{T_l - T_s}{f_l z_l} \quad (9)$$

$$q_i^c = q_i^o + H_o f_c z_c \left( 1 - \frac{f_c z_c}{2 f_l z_l} \right) \quad (10)$$

$$q_i^e = q_i^o + H_o f_c D_o \left( 1 - \frac{f_c D_o}{f_l z_l} (1 - e^{-f_l z_l / f_c D_o}) \right) \quad (11)$$

in which  $q_i^o$  is the surface heat flow of the lithosphere with no internal heat source and  $q_i^c$  is the surface heat flow of the lithosphere with constant heat source throughout the whole crust, while  $q_i^e$  is the surface heat flow when the heat source varies exponentially with depth.

#### DEFINITION OF THE REFERENCE LITHOSPHERE

In order to evaluate the force balance system in mountain belts it is necessary to define a reference lithosphere, to which the deformation of continental lithosphere associated with mountain belts refer. This lithosphere would remain undeformed if no other external or internal force is present. Also, the lithosphere should be isostatically compensated at its base. Since isostasy does not specify a complete force balance, one obvious constraint on the reference lithosphere is to balance its potential energy with mid-ocean ridges [e.g., Houseman and England, 1986; Sonder et al., 1987].

The vertical stress  $P_r$  measured from the sea level to depth  $z_l - E$  beneath mid-ocean ridges is given by

$$P_{r\text{mid}} = \rho_w g h_w + \rho_b g h_b + \rho_m g (z_l - (E + h_w + h_b)) \quad (12)$$

and its associated potential energy  $P_e$  is given by

$$\begin{aligned} P_{e\text{mid}} &= \frac{\rho_w g h_w^2}{2} + \frac{\rho_b g h_b^2}{2} + \rho_w g h_w h_b \\ &+ (\rho_w g h_w + \rho_b g h_b) (z_l - (E + h_w + h_b)) \\ &+ \frac{\rho_m g}{2} (z_l - (E + h_w + h_b))^2 \end{aligned} \quad (13)$$

where  $E$  is the elevation of the reference lithosphere above sea level,  $\rho_w$  and  $\rho_b$  are the densities of water and oceanic

crust, respectively,  $h_w$  is the depth of the ridge crest beneath sea level, and  $h_b$  is the thickness of the oceanic crust (note that in calculating  $P_{emid}$  we neglect the effect of temperature variation with depth on the density structure in the oceanic crust at the ridges and thus  $\rho_b$  can be regarded as the average density of the oceanic crust).

If the reference lithosphere contains no internal heat source, then the vertical stress at its base is given by

$$\frac{Pr_{ref}}{\rho_m g z_l} = 1 + \frac{\alpha(T_l - T_s)}{2} - \delta\psi \quad (14)$$

and its associated potential energy can be expressed as

$$\frac{Pe_{ref}}{\rho_m g z_l^2} = \frac{1}{2} + \frac{\alpha(T_l - T_s)}{3} - \delta\left(\psi - \frac{\psi^2}{2}\right) \quad (15)$$

If the reference lithosphere contains an internal heat source which is uniformly distributed in the crust and no heat source in the mantle, then at thermal equilibrium,

$$\frac{Pr_{ref}}{\rho_m g z_l} = 1 + \frac{\alpha(T_l - T_s)}{2} - \delta\psi + \beta_l \left( \frac{\psi^3}{6} - \frac{\psi^2}{4} \right) \quad (16)$$

and

$$\begin{aligned} \frac{Pe_{ref}}{\rho_m g z_l^2} &= \frac{1}{2} + \frac{\alpha(T_l - T_s)}{3} - \delta\left(\psi - \frac{1}{2}\psi^2\right) \\ &\quad + \frac{\beta_l \psi^2}{6} \left( \psi - \frac{\psi^2}{4} - 1 \right) \end{aligned} \quad (17)$$

where

$$\beta_l = \frac{\alpha H_o z_l^2}{k}$$

If the reference lithosphere contains an internal heat source which decreases exponentially with depth, then at thermal equilibrium,

$$\begin{aligned} \frac{Pr_{ref}}{\rho_m g z_l} &= 1 + \frac{\alpha(T_l - T_s)}{2} - \delta\psi \\ &\quad - \beta_e \left[ \left( \frac{1 + e^{(-z_l/D_o)}}{2} \right) - \psi_1 (1 - e^{(-z_l/D_o)}) \right] \end{aligned} \quad (18)$$

and

$$\begin{aligned} \frac{Pe_{ref}}{\rho_m g z_l^2} &= \frac{1}{2} + \frac{\alpha(T_l - T_s)}{3} - \delta\left(\psi - \frac{\psi^2}{2}\right) \\ &\quad + \beta_e \left[ \psi\psi_1 + \psi_1 e^{(-z_c/D_o)} (1 - \psi) - \frac{1}{2} + \psi \right] \\ &\quad + \beta_e \left[ (1 - e^{(-z_l/D_o)}) \left( \frac{1}{6} - \frac{\psi^2}{2} (1 - \psi) - \psi_1^2 \right) \right] \end{aligned} \quad (19)$$

Once the internal heat production  $H_o$ , the characteristic length scale of heat production distribution  $D_o$ , the reference crust thickness  $z_c$ , and densities  $\rho_c$  and  $\rho_m$  are specified, then the elevation  $E$  and total thickness  $z_l$  of the reference lithosphere are defined by solving

$$Pr_{mid} = Pr_{ref} \quad (20)$$

$$Pe_{mid} = Pe_{ref} \quad (21)$$

Our preference for the appropriate crustal thickness in the reference lithosphere is  $z_c = 34$  km, and Table 1 lists a few models with different distributions of internal heat sources predicting surface elevations around 1 km and lithosphere thickness of around 125 km, both of which seem appropriate. Figure 1 shows the way in which  $E$ ,  $z_l$ , and  $q_i$  vary with  $z_c$  for a reference lithosphere in potential and isostatic balance with the mid-ocean ridge system assuming an exponentially decreasing heat production distribution with  $H_o = 3.5 \times 10^{-6} \text{ W m}^{-3}$  and  $D_o = 10 \text{ km}$ . In the following discussion we only explicitly consider a reference lithosphere with an exponential heat source distribution with  $H_o$  and  $D_o$  modified to give an initial surface heat flow of 63 mW  $\text{m}^{-2}$  (i.e., model 4 in Table 1).

#### MECHANICAL CONSIDERATIONS

Any explicit reference to the strength of the continental lithosphere requires assumptions about the rheological structure of the lithosphere, about which there is considerable uncertainty and, very probably, considerable natural variation. Despite this uncertainty it has become common to model failure of the continental lithosphere as a combination of frictional sliding and strongly temperature dependent ductile creep processes (see the appendix), assuming a simple compositional structure consisting of a quartz-dominated crust and an olivine-dominated mantle. Following the suggestion of P. Molnar (manuscript in preparation, 1992), we refer to this rheological model of the lithosphere as the "Brace-Goetze" lithosphere.

An important, and well recognized problem, with the Brace-Goetze rheology outlined in the appendix concerns the uncertainties in the rheological parameters [Patterson, 1987]. Uncertainties in the activation energies for creep of at least 10% seem within experimental bounds [see Sonder and England, 1986]. For example, the strength of olivine

TABLE 1. Calculated Values for the Definition of Reference Lithosphere With  $z_c = 34$  km.

Model	$E$ , km	$z_l$ , km	$H_o$ , $\mu\text{W m}^{-3}$	$D_o$ , km	$q_0$ , $\text{mW m}^{-2}$
1	1.04	114.75	0	-	33.4
2	1.03	118.6	1.15 (exponential)	10	42.89
3	1.18	125.53	1.15 (constant)	-	64.39
4	0.99	126.23	3.5 (exponential)	10	62.69
1'	0.99	126	0	-	30.47

Note that model 1' is not in isostatic and potential energy balance with mid-ocean ridges, but it is employed for a comparison with model 4.

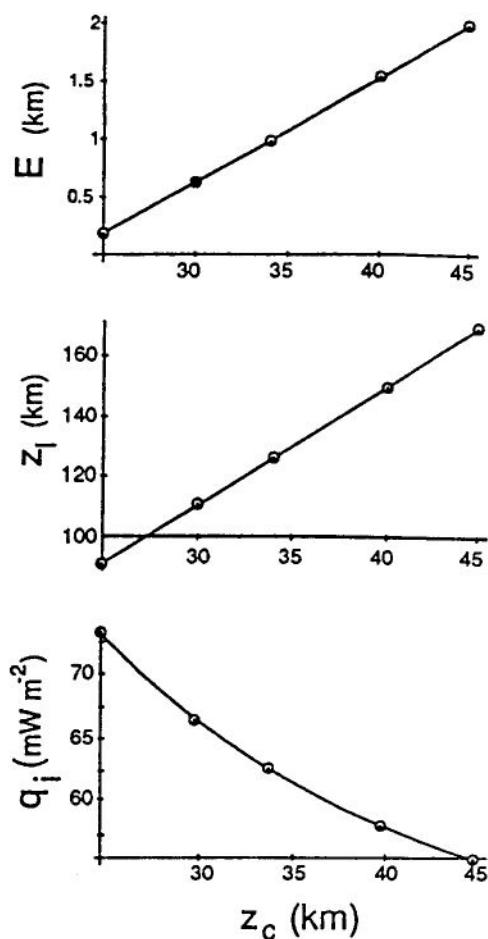


Fig. 1. The variation in elevation  $E$  above sea level, lithospheric thickness  $z_l$ , and surface heat flow  $q_i$ , shown as a function of crustal thickness  $z_c$ , for the reference lithosphere in potential energy and isostatic balance with the mid-ocean ridges. The physical properties of the reference lithosphere and the ocean ridge system are as shown in Table 2, with surface heat production  $H_o = 3.5 \times 10^{-6} \text{ W m}^{-3}$  and an exponentially decreasing heat source distribution with a characteristic length scale  $D_o = 10 \text{ km}$ .

deforming by power law creep at  $600^\circ\text{C}$  varies by a factor of 10 due to a 10 % uncertainty in the activation energy. Moreover, olivine strength decreases by a factor of 10 between  $600^\circ\text{C}$  and  $700^\circ\text{C}$ . Figure 2 shows that a uncertainty in thermal conductivity of 33%, again within experimental bound lead to an uncertainty in Moho temperature of the order of  $80^\circ\text{C}$ . In terms of the bulk strength of the lithosphere, a 10% change in the activation energies for creep, or a  $100^\circ\text{C}$  change in the Moho temperature, leads to a change in the strength of the lithosphere by a factor of 2 at a given strain rate. These considerations imply that with the current data, the calculation of absolute stresses and strengths is futile. However, the relative changes in strength due to changes in thermal structure may be much more precisely defined, thus rendering significance to the evaluation of the mechanical consequences of processes which change the thermal structure of the lithosphere.

In the following section we examine the variation in lithospheric strength due to variation in thermal stucture as-

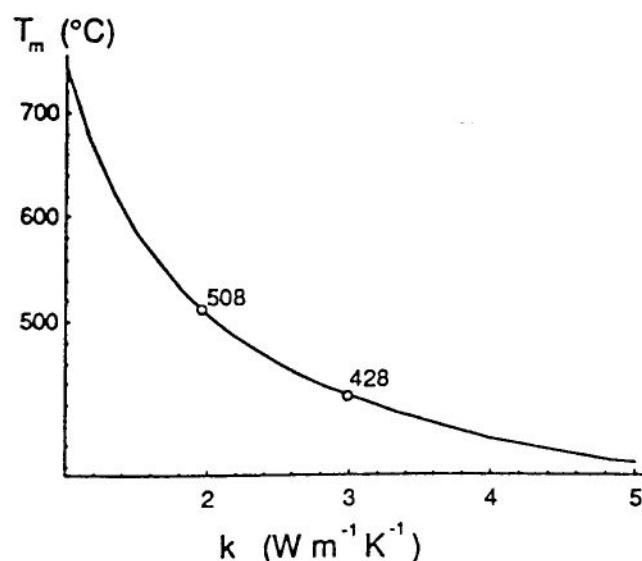


Fig. 2. A reduction in the thermal conductivity  $k$  of 33%, from 3 to  $2 \text{ W m}^{-1} \text{ K}^{-1}$  leads to a change in the  $80^\circ\text{C}$  increase in the Moho temperature for the same surface heat flow.

sociated with the formation of thermally mature mountain systems. We compare the behavior of lithospheres assuming a range of different initial strengths which would seem to bound the plausible range for the strength of lithosphere. These plausible bounds are provided by the following considerations:

1. The reference lithosphere must be capable of deforming at rates appropriate to the formation of modern mountain belts when subject to the forces that drive deformation. The lower bound on crustal thickening rates in belts such as the Himalayas is a few times  $10^{-2} \text{ M yr}^{-1}$ . For the parameter range listed in Table 2 and a crustal thickening rate of  $10^{-2} \text{ M yr}^{-1}$  the strength of our reference lithosphere in compression is  $4 \times 10^{13} \text{ N m}^{-1}$ . Estimates of the forces that drive continental deformation, for example those generated by the subduction of oceanic lithosphere range up to a few times  $10^{13} \text{ N m}^{-1}$  [e.g., Turcotte, 1983], and are consistent with the suggestion of Houseman and England [1986] that the force required to form Tibet is in the range  $1.5 - 2.5 \times 10^{13} \text{ N m}^{-1}$ .

2. As outlined below, the observation that the excess potential energy above that of the reference lithosphere of the largest known mountain systems, with  $f_c = 2$ , is of the order of  $0.8 - 1.15 \times 10^{13} \text{ N m}^{-1}$ . This provides a lower bound on the magnitude of those forces in addition to ridge push that drive convergence, for example slab pull.

These constraints suggest to us that the bulk compressional strength of the lithosphere at strain rates of  $10^{-2} \text{ M yr}^{-1}$  is in the range  $1.4 \times 10^{13} \text{ N m}^{-1}$ , and we show results for this range obtained by modifying the activation energies of creep by an appropriate factor. It is emphasized that these constraints do not test the validity of the specific rheological model used here in terms of the way in which strength varies with depth through the lithosphere.

TABLE 2. Values of Parameters Used in Calculations

Symbol	Definition	Value
$T_l$	temperature at base of lithosphere	1280°C
$T_s$	temperature at surface of lithosphere	0°C
$h_w$	water depth above mid-ocean ridge	2.5 km
$h_b$	oceanic crust thickness above mid-ocean ridge	6 km
$\rho_w$	water density	1.03 g/cm <sup>3</sup>
$\rho_b$	oceanic crust density	2.96 g/cm <sup>3</sup>
$\rho_c$	crustal density at $T_l$	2.7 g/cm <sup>3</sup>
$\rho_m$	mantle density at $T_l$	3.2 g/cm <sup>3</sup>
$\rho_a$	asthenosphere density at $T_l$	3.2 g/cm <sup>3</sup>
$\alpha$	coefficient of thermal expansion	$3 \times 10^{-5} \text{ K}^{-1}$
$k$	thermal conductivity	$3.0 \text{ W m}^{-1} \text{ K}^{-1}$
$D_0$	length of heat source distribution	10 km
$\mu_c$	coefficient of friction in crust	0.6
$\mu_m$	coefficient of friction in mantle	0.8
$\lambda_c$	ratio of crustal pore pressure to $\sigma_n$	0.4
$\lambda_m$	ratio of mantle pore pressure to $\sigma_n$	0
$\sigma_c$	crustal cohesion	0 MPa
$\sigma_m$	mantle cohesion	60 MPa
$A_q$	preexponential constant (quartz)	$5 \times 10^{-6} \text{ MPa}^{-3} \text{ s}^{-1}$
$A_o$	preexponential constant (olivine)	$7 \times 10^4 \text{ MPa}^{-3} \text{ s}^{-1}$
$Q_q$	activation energy for power law creep (quartz)	$1.9 \times 10^5 \text{ J/mol}$
$Q_o$	activation energy for power law creep (olivine)	$5.2 \times 10^5 \text{ J/mol}$
$Q_d$	activation energy for Dorn law creep (olivine)	$5.4 \times 10^5 \text{ J/mol}$
$n$	exponent for power law creep	3
$\sigma_d$	threshold stress for Dorn law creep	8500 MPa
$\varepsilon_d$	strain rate for Dorn law creep	$3.05 \times 10^{11} \text{ s}^{-1} *$
$\dot{\epsilon}$	critical strain rate	$3.16 \times 10^{-16} \text{ s}^{-1}$

Sources of rheological constants are Byerlee [1968], Brace and Kohlstedt [1980], Goetze [1978], Jaoul et al. [1984] and England [1987].

\*This value is modified to allow smooth transition between power law creep and Dorn law creep at 200 MPa.

## RESULTS

In what follows, all results are presented in  $f_c - f_l$  space since this enables the clear illustration of the relationship between changes in the mantle lithosphere thickness and the various physical properties discussed here [e.g., Sandiford and Powell, 1990]. We consider it unlikely that mountain belts evolve with  $f_l > f_c$ , and therefore we only explicitly consider the region in  $f_c - f_l$  space bounded by the homogeneous thickening line ( $f_c = f_l$ ) and the line that defines the maximum thickening of the crust with complete detachment of the mantle lithosphere ( $f_l = \psi f_c$ ). In view of the observation that crustal thickness in modern mountain belts does not seem to exceed about 70 km we consider an upper limit to  $f_c$  of 2. The results presented here are based on the assumption that thermal equilibrium has been achieved during the orogenic episode as may be appropriate to mature mountain systems (see earlier discussion). The isostatically supported elevation, potential surface heatflow and potential Moho temperatures are shown in Figures 3-5 for reference lithosphere model 4 (Table 1).

### Horizontal Buoyancy Force (Figures 6, 7, and 8)

Figures 6 and 7 illustrate the horizontal buoyancy forces in  $f_c - f_l$  space assuming potential thermal structure for

the heat production distributions given by model 4 (exponential) and model 1' (no heat production), respectively. We note that the two models give rise to significantly different estimates of the magnitude of the buoyancy forces resulting from deformation, particularly at large  $f_l$  and  $f_c$  (at  $f_l = f_c = 2$  the difference in the estimates is  $8 \times 10^{12} \text{ N m}^{-1}$ ), and the implication is that when considering the force balance operating in thermally mature mountain belts it is important to consider the effect of internal heat sources. The following are some important features: (1) all deformations of the reference lithosphere with  $f_c \geq f_l$  cause an increase in the potential energy stored in the orogen, (2) for a given crustal thickening the magnitude of the potential energy varies with the inverse of the lithosphere thickness, and (3) the natural limit to crustal thickness of about 70 km suggested by observations in modern mountain belts suggests the maximum potential energy excess over and above that of mid-ocean ridges is no greater than  $0.8-1.15 \times 10^{13} \text{ N m}^{-1}$ . As illustrated by Figure 7c, the horizontal buoyancy force is relatively insensitive to the changes in thermal conductivity. A 33% reduction in the thermal conductivity (from 3 to  $2 \text{ W m}^{-1} \text{ K}^{-1}$ ) results in noticeable increases of buoyancy forces only at large  $f_c$  and  $f_l$ . The maximum difference is about  $4 \times 10^{12} \text{ N m}^{-1}$ , equivalent to 57% increase of buoy-

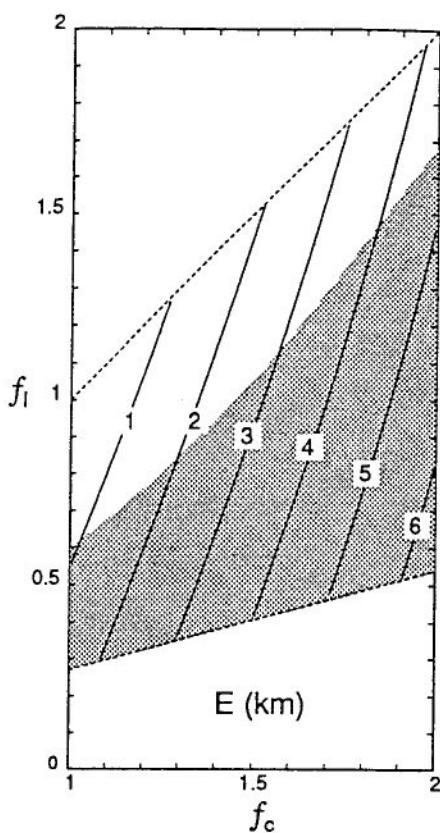


Fig. 3. The  $f_c - f_l$  plane contoured for the isostatically supported surface elevation relative the reference lithosphere. The contours are calculated using equation (8) in which internal heat source varies exponentially with depth. The contour interval is 1 km. The shaded area indicates the region in which instantaneous extensional strain rates greater than  $\approx 10^{-2} \text{ Myr}^{-1}$ , which may develop as a consequence of the buoyancy forces following relaxation of the forces driving convergence, assuming a rheology governed by the parameter range in Table 2 with the activation energies for creep reduced by 8% (see Figure 10b).

ancy force, at  $f_l = f_c = 2$ . For  $f_l < 1.0$ , there are basically negligible changes in buoyancy forces compared with Figure 6. Figure 7c also indicates that the buoyancy force, associated with deformation when  $k = 2 \text{ W m}^{-1} \text{ k}^{-1}$ , is virtually independent of  $f_l$  when  $f_c > 1.8$ .

Figures 8a and 8b show that the horizontal buoyancy force is sensitive to the changes in thermal expansion coefficient. In general, the lower  $\alpha$  is, the lower the buoyancy forces are. An increase of the thermal expansion coefficient by a factor of 2 results in a maximum 36% increase of potential energy at  $f_c = 2$  and  $f_l = \psi f_c$ . A decrease of the thermal expansion coefficient by a factor of 2 results in a maximum 18% reduction in potential energy at  $f_c = 2$  and  $f_l = \psi f_c$ , which is half of that gained by an increase of the thermal expansion coefficient by a factor of 2. However, for both cases there are negligible changes in buoyancy forces at  $f_c = f_l = 2$ .

#### Vertically Integrated Strength of the Lithosphere (Figure 9)

Assuming the rheology outlined in the appendix with the parameter range as listed in Table 2 and assuming the po-

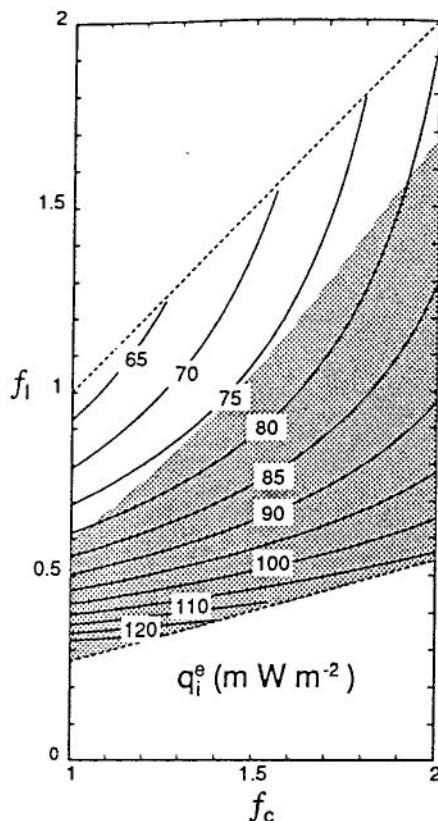


Fig. 4. The  $f_c - f_l$  plane contoured for the potential surface heatflow. The contours are calculated using equation (11). The contour interval is  $5 \text{ mW m}^{-2}$ . The shaded area is as explained in Figure 3.

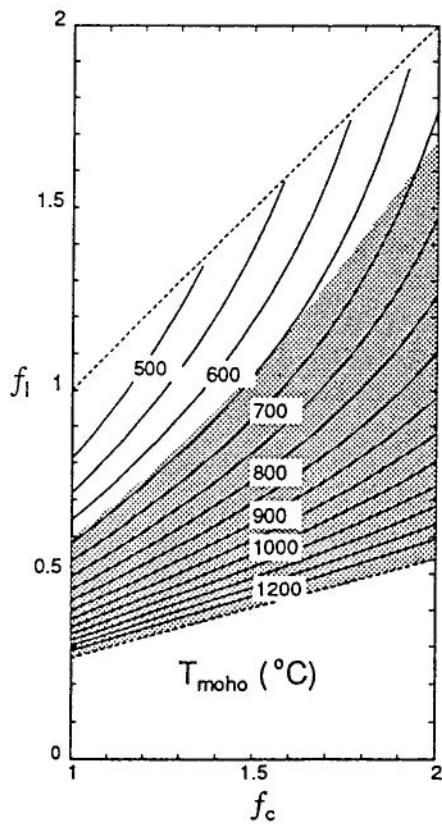


Fig. 5. The  $f_c - f_l$  plane contoured for potential Moho temperature. The contour interval is  $50^\circ\text{C}$ . The shaded area is as explained in Figure 3.

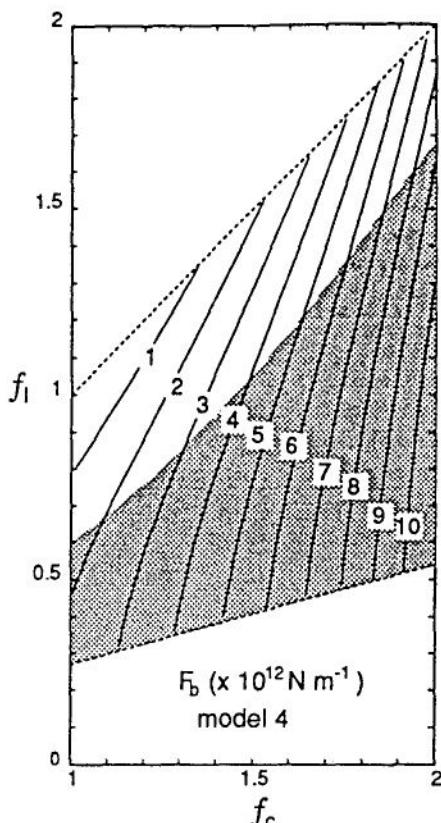


Fig. 6. The  $f_c - f_l$  plane contoured for the induced horizontal buoyancy force assuming the potential thermal structure appropriate to an exponential heat source distribution (equation (5)) for the reference lithosphere model 4 in Table 1. The contour interval is  $1 \times 10^{12} \text{ N m}^{-1}$ . The shaded area is as explained in Figure 3.

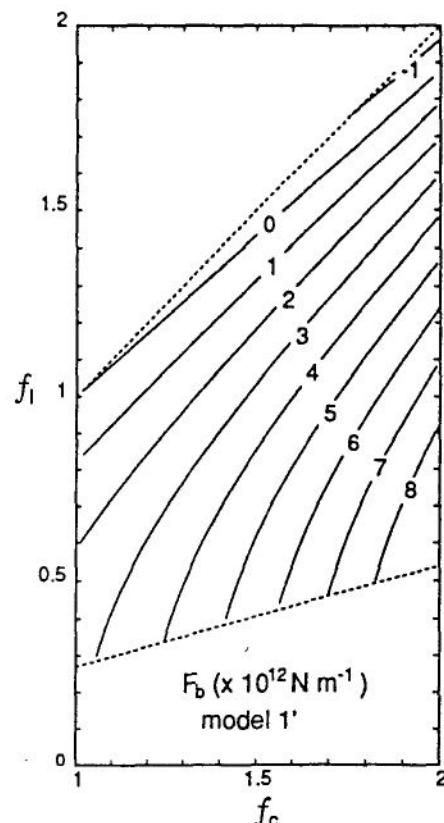


Fig. 7a. The  $f_c - f_l$  plane contoured for the induced horizontal buoyancy force assuming the potential thermal structure appropriate to no internal heat source (i.e., the linear geotherm given by equation (3)) for the reference lithosphere model 4 in Table 1. The contour interval is  $1 \times 10^{12} \text{ N m}^{-1}$ .

tential thermal structure the strength of the lithosphere  $F_1$  at a strain rate of  $10^{-2} \text{ M yr}^{-1}$  ( $= 3.16 \times 10^{-16} \text{ s}^{-1}$ ) calculated by numerical integration over the thickness of lithosphere using a 100-m stepping interval is shown in Figures 9a-9d. An important feature illustrated by Figure 9 is that under the assumption of potential thermal structure, lithospheric strength reduction occurs in response to both crustal thickening and mantle lithospheric thinning. Comparison between Figure 5 suggests that this is principally due to the dependence of strength on Moho temperature [see also Sonder and England, 1986; England, 1987].

Due to the uncertainties of the strength of the lithospheric material, we present models (shown in Figures 9a-9c) based on a range of equivalent reference strengths of  $1 - 4 \times 10^{13} \text{ N m}^{-1}$  at compression, consistent with the range in the magnitude of forces during convergence proposed by Turcotte [1983] and England and Houseman [1988, 1989]. Figure 9d illustrates that the effect of a 33% reduction in thermal conductivity is equivalent to about 7% reduction in activation energy.

#### Instantaneous Extensional Strain Rate (Figures 10 and 11)

Based on the range of the integrated strengths discussed above, several results are presented to illustrate the possibil-

ity of mountain extension. Figures 10a-10d show the instantaneous extensional strain rate that would result from the application of the extensional buoyancy force, as shown in Figures 6 and 7c, to the potential thermal structure, as for example, illustrated by the potential Moho temperature in Figure 5. This situation would apply to a thermally mature mountain system with the relaxation of the forces driving convergence and thus these figures can be used to evaluate the potential stability of mountain systems to collapse. We note that geologically significant strain rates during collapse (that is greater than about  $10^{-2} \text{ M yr}^{-1}$ ) will be generated providing the configuration of the lithosphere is governed by  $f_l < f_c - 0.5$  for any  $f_c$  from 1 to 2 for all models. Of particular interest is the striking correspondence between the calculated extensional strain rates and the potential Moho temperatures (Figure 5) with geologically significant extensional strain rates only developing for potential Moho temperatures higher than approximately  $650 - 700^\circ\text{C}$  for the range of initial strengths considered here.

One advantage of the  $f_c - f_l$  parameterisation is that the deformation-denudation paths for orogens, or at least parts thereof, can be mapped onto the  $f_c - f_l$  plane [e.g., Sandiford and Powell, 1990] as shown in Figure 11. The mechanical calculations presented above suggest that extensional de-

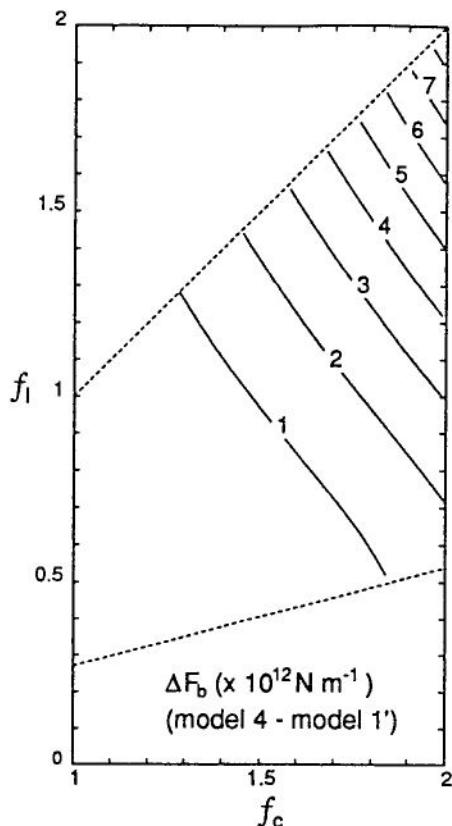


Fig. 7b. The  $f_c - f_l$  plane contoured for the difference of induced horizontal buoyancy forces between two models (one is with internal heat source, i.e., model 4 in Table 1, the other (model 1') is without internal heat source). The contour interval is  $1 \times 10^{12}$   $\text{N m}^{-1}$ .

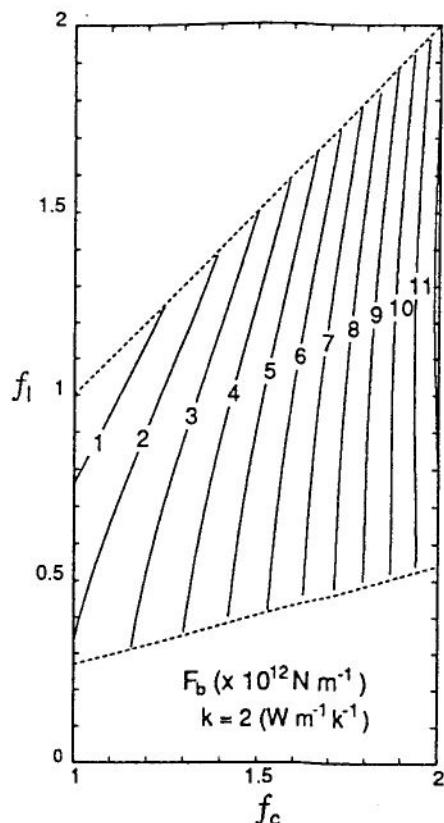


Fig. 7c. Same as in Figure 6, except that the average thermal conductivity of the lithosphere is given by  $2 \text{ W m}^{-1} \text{K}^{-1}$ .

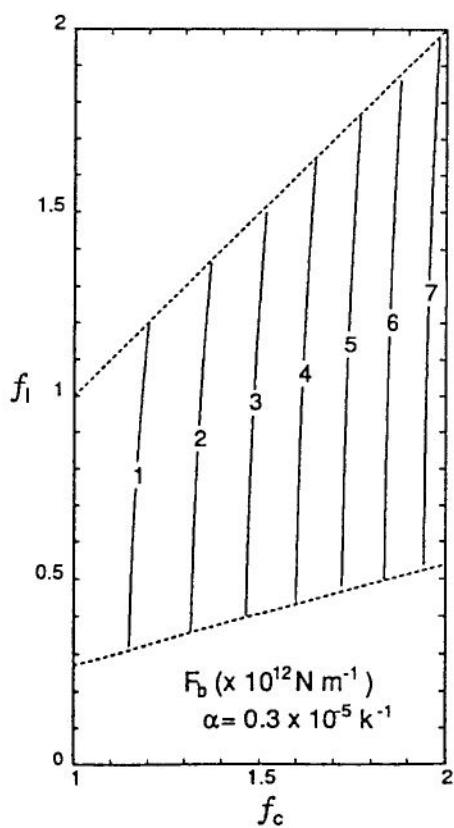


Fig. 8a. Same as in Figure 6, except that the average thermal expansion coefficient is reduced by a factor of 2.

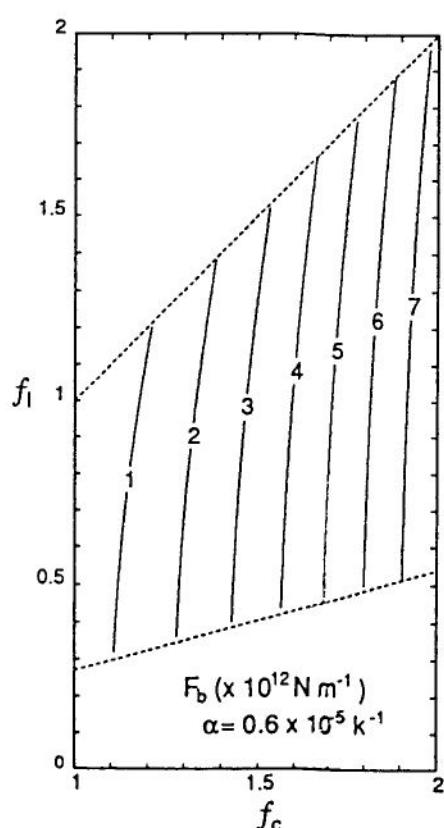


Fig. 8b. Same as in Figure 6, except that the average thermal expansion coefficient is increased by a factor of 2.

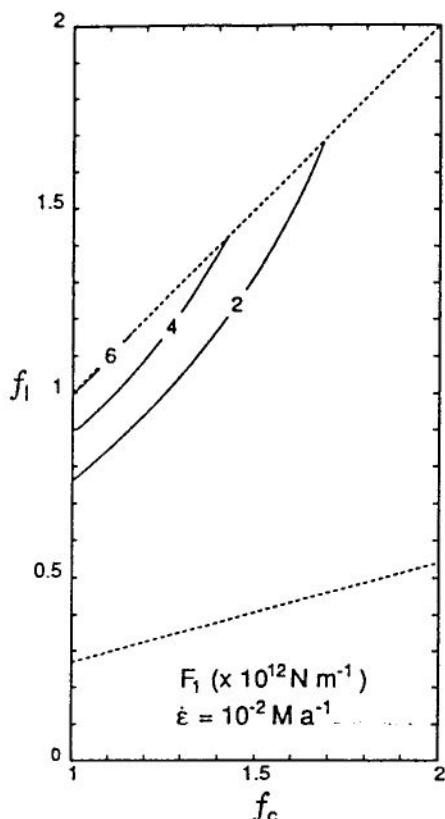


Fig. 9a. The  $f_c - f_l$  plane contoured for the vertically integrated extensional strength of lithosphere calculated using 100-m interval for numerical integration over the lithosphere, assuming the potential thermal structure as given by equation (5), and a strain rate of  $10^{-2} \text{ M yr}^{-1}$  ( $= 3.16 \times 10^{-16} \text{ s}^{-1}$ ). The contour interval is  $2 \times 10^{12} \text{ N m}^{-1}$ . The result is given by a reduction (19%) of the activation energy listed in Table 2. This is equivalent to compressional strength of  $1 \times 10^{13} \text{ N m}^{-1}$  in the initial reference state ( $f_l = f_c = 1$ ).

nudation of thermally mature mountain belts following the relaxation of the forces driving convergence is only likely to be significant if the Moho temperatures exceed about 650–700°C. For the thermal properties considered here, such a situation is only likely to pertain for  $f_l < f_c$  as may occur in response to a convective instability of the lower thermal boundary layer. For deformation geometries in which  $f_l = f_c$ , significant denudation by extension is unlikely, and the denudation regime should therefore be dominated by erosion (upper dashed arrow in Figure 11). If  $f_l < 0.5 \times f_c$  and the potential thermal structure is approached, then almost complete denudation by extension may be possible (lower solid arrow in Figure 11).

#### DISCUSSION

Following the study of Houseman *et al.* [1981] which examined the initiation of convective instability in a thickened lower thermal boundary at the base of the continental lithosphere much has been made of the role of the mantle lithosphere in terminating orogeny and inducing collapse

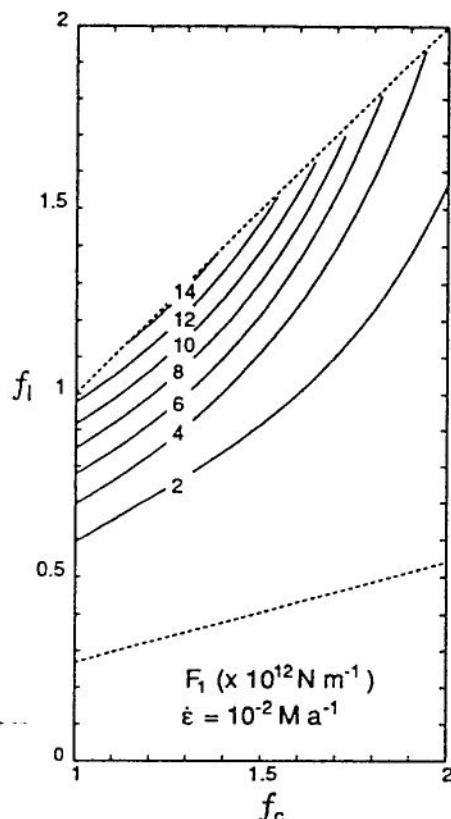


Fig. 9b. Same as in Figure 9a, except that the vertically integrated strength is given by a reduction (8%) of the activation energy listed in Table 2. This is equivalent to compressional strength of  $2.5 \times 10^{13} \text{ N m}^{-1}$  in the initial reference state ( $f_l = f_c = 1$ ).

of mountain belts (among many others, see *England and Houseman*, [1988, 1989]; *Molnar and Lyon-Caen*, [1988]; *Sandiford and Powell*, [1990]). Our intention has not been to further delve into the nature and timing of such processes, constraints on which are now needed from the geological record. Rather our intention has been to investigate, in a more rigorous fashion than previously attempted, the basic physical consequences of the general class of processes that affect the force balance operating in isostatically compensated mountain belts, using a simple parameterization of the deformation, namely, the  $f_c - f_l$  parameterization. The numerical investigations presented in this paper certainly corroborate the notion that the stability of isostatically compensated mountain belts to collapse is inherently dependent on the way in which the mantle lithosphere responds to the convergent deformation and provide a simple graphical means of evaluating stability albeit dependent on uncertainties in the rheological parameters used in calculations of the mechanical strength of the lithosphere. One important example concerns the extent to which extensional collapse may destroy a thermally mature mountain belt. As illustrated schematically in Figure 11, the calculations summarized in Figure 10 suggest that near complete destruction

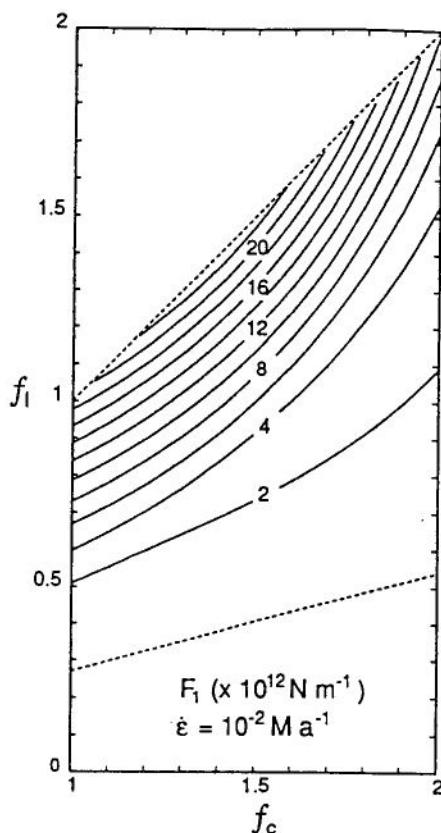


Fig. 9c. Same as in Figure 9a, except that the vertically integrated strength is given by no reduction of the activation energy listed in Table 2. This is equivalent to compressional strength of  $4 \times 10^{13} \text{ N m}^{-1}$  in the initial reference state ( $f_l = f_c = 1$ ).

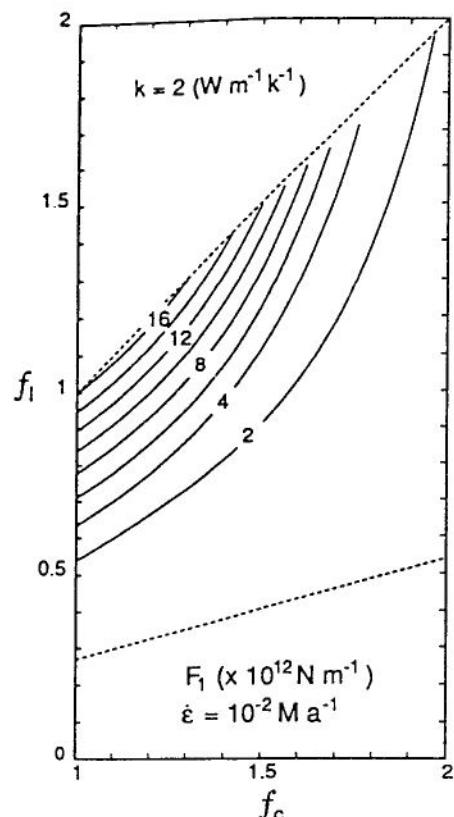
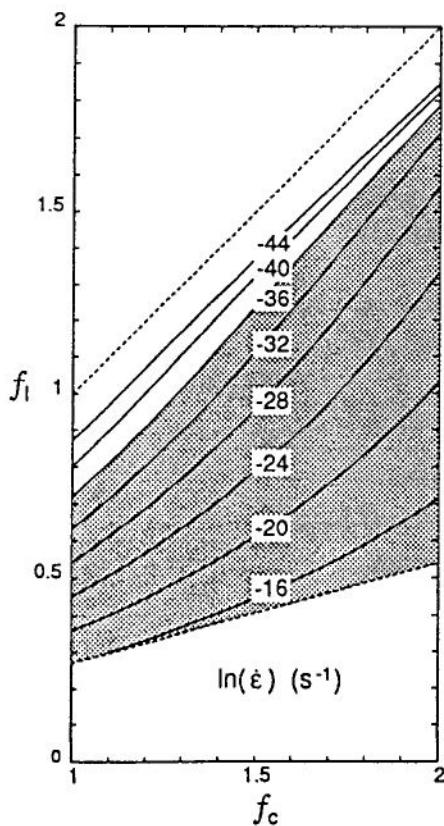


Fig. 9d. Same as in Figure 9c, except that the average thermal conductivity is given by  $2 \text{ W m}^{-1} \text{ k}^{-1}$ .



of the mountain system by extensional collapse may be possible if processes such as convective instability of a lower thermal boundary layer can reduce total lithospheric thickening to less than half the contemporary crustal thickening (i.e.,  $f_l \leq f_c/2$ ).

An important question concerning the role of mantle lithospheric thinning beneath mountain belts by processes such as convective instability of a thermal boundary layer is whether extension can be initiated independently of changes in the driving forces for convergence such as suggested by *England and Houseman* [1989] and *Sandiford and Powell* [1990], among others. As indicated earlier, these workers have assumed a linear geotherm in calculating of the change in potential energy associated with mantle lithospheric thinning, and as illustrated in Figure 7a the maximum increase

Fig. 10a. The  $f_c - f_l$  plane contoured for the instantaneous strain rate of a mountain belt undergoing extension in response to the relaxation of the forces driving convergence. Contours are for  $\ln(\dot{\epsilon})$  in units of  $\text{s}^{-1}$ . The instantaneous strain rate is calculated numerically by equating the buoyancy force (Figure 6) with the vertically integrated strength (Figure 9a) assuming the potential thermal structure given by equation (5) and a reference lithosphere given by model 4 in Table 1. The shaded area indicates the region in which the extensional strain rates are greater than  $10^{-2} \text{ Myr}^{-1}$ .

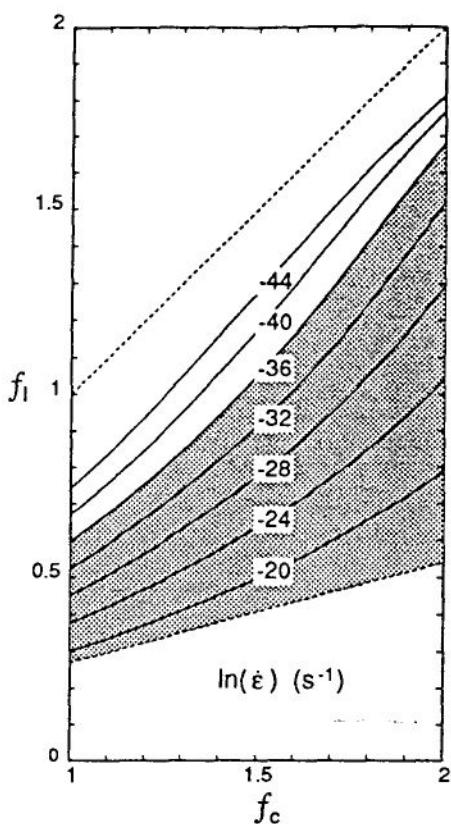


Fig. 10b. Same as in Figure 10a, except that the instantaneous strain rate is given by balancing the horizontal buoyancy force (Figure 6) with the strength (Figure 9b).

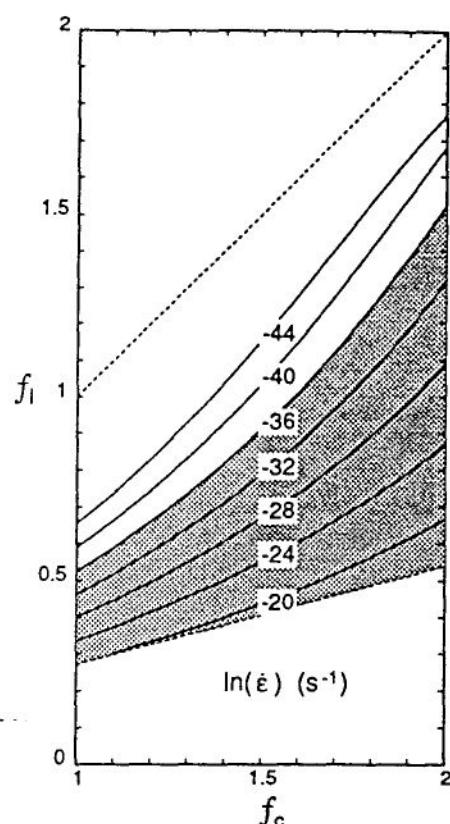


Fig. 10c. Same as in Figure 10a, except that the instantaneous strain rate is given by balancing the horizontal buoyancy force (Figure 6) with the strength (Figure 9c).

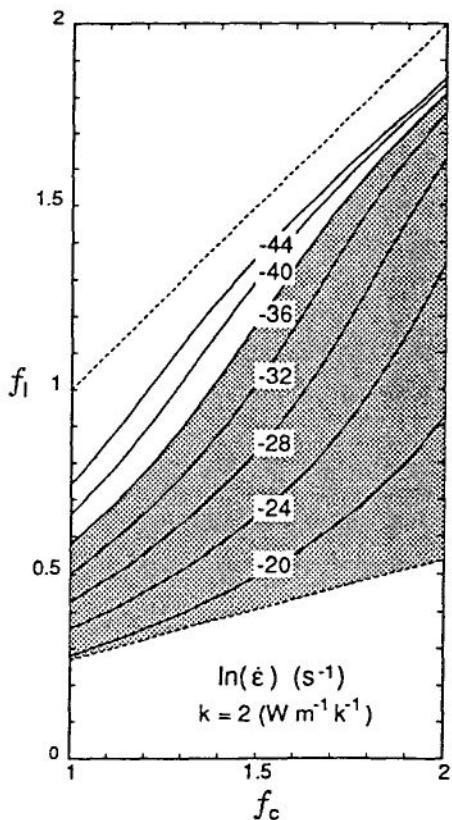


Fig. 10d. Same as in Figure 10c, except that the instantaneous strain rate is given by balancing the horizontal buoyancy force (Figure 7c) with the strength (Figure 9d).

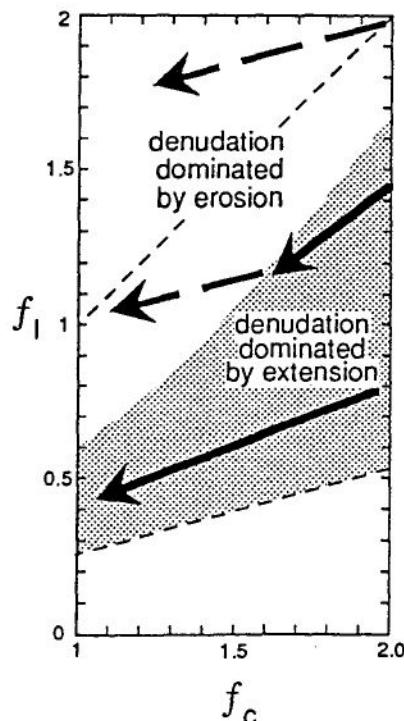


Fig. 11. Schematic  $f_c - f_l$  paths during denudation by extension and/or erosion following the combined effects of the removal of various amounts of mantle lithosphere by a process such as convective instability of the thermal boundary layer and the relaxation of the forces driving convergence. The shaded region is same as in Figure 10b.

in potential energy associated with complete detachment of a previously thickened mantle lithosphere for a linear geotherm is equivalent to  $9 \times 10^{12} \text{ N m}^{-1}$  at  $f_c = 2$ . The calculations presented here show that when internal heat sources are considered, the potential energy increase associated with such mantle lithospheric thinning may be significantly less than calculated assuming a linear geotherm. For example, Figure 6, appropriate to a thermally mature mountain belt, shows that the increase in potential energy associated with internal heat source is less than  $2 \times 10^{12} \text{ N m}^{-1}$  at  $f_c = 2$  when a complete detachment of a previously thickened mantle lithosphere occurs. Under these circumstances it seems unlikely that thinning of a previously thickened mantle lithosphere can lead to significant extension without an associated reduction in the forces driving convergence.

In agreement with previous studies [Sonder *et al.*, 1987; England and Houseman, 1988, 1989], we have shown that the stability of mountain belts is critically dependent on the thermal state and rheological parameters of the lithosphere. Although the vertically integrated strength is more robust than the shear strength within the lithosphere, it should be pointed that the study presented here only presents a limiting case of the complex system in mountain belts, because the results are more or less based on our limited understanding of the thermal structure and mechanical strength of the lithosphere.

#### APPENDIX: INTEGRATED MECHANICAL STRENGTH OF THE LITHOSPHERE

In this paper we adopt a rheological model for the lithosphere following the work of Brace and Kohlstedt [1980] which we term the "Brace-Goetze" lithosphere.

At low temperature and high strain rate, the failure mechanism in the upper crust is modelled as frictional sliding according to Byerlee's law [Byerlee, 1968; Brace and Kohlstedt, 1980]:

$$\tau_s = \sigma_0 + \mu \sigma_n (1 - \lambda) \quad (22)$$

where  $\tau_s$ ,  $\sigma_0$ ,  $\mu$ ,  $\sigma_n$ , and  $\lambda$  are, the shear stress, the cohesion, the coefficient of friction, normal stress on the failure plane and the ratio of fluid pore pressure to the normal stress, respectively. Assuming  $\lambda$  is constant throughout the brittle upper crust, then for biaxial horizontal extension the maximum and minimum principle stresses are given by

$$\sigma_1 - \sigma_3 = \frac{2(\sigma_0 - \mu \sigma_n(1 - \lambda))}{\sqrt{\mu^2 + 1 + \mu}} \quad (23)$$

At high temperature the failure mechanism for horizontal extension is modelled as (1) power law creep in the crust for

all  $(\sigma_1 - \sigma_3)$  and in the mantle for  $(\sigma_1 - \sigma_3) < 200 \text{ MPa}$  [e.g., Brace and Kohlstedt, 1980],

$$\sigma_1 - \sigma_3 = \left( \frac{\dot{\epsilon}}{A_p} \right)^{1/n} \exp \left( \frac{Q_p}{nRT} \right) \quad (24)$$

where  $A_p$  is a material constant,  $n$  is the power law exponent,  $Q_p$  is the activation energy,  $R$  is the gas constant,  $\dot{\epsilon}$  is the strain rate, and  $T$  is material temperature, or (2) Dorn law creep in the mantle for  $(\sigma_1 - \sigma_3) > 200 \text{ MPa}$  [e.g., Brace and Kohlstedt, 1980],

$$\sigma_1 - \sigma_3 = \sigma_d \left[ 1 - \sqrt{\frac{RT}{Q_d} \ln \left( \frac{\dot{\epsilon}_d}{\dot{\epsilon}} \right)} \right] \quad (25)$$

where  $Q_d$  is the activation energy,  $\sigma_d$  is the threshold stress, and  $\dot{\epsilon}_d$  is the preexponential constant for Dorn law creep.

Under the conditions defined by the above equations the vertically integrated strength of the lithosphere,  $F_1$ , is then given by

$$F_1 = \int_0^{z_1} (\sigma_1 - \sigma_3) dz \quad (26)$$

and is uniquely defined only when the thermal structure and strain rate are specified. In the calculations presented in this paper the strength of the lithosphere is calculated by numerical integration over the spatial domain. However, we provide approximate analytical expressions for the strength of the crustal and mantle parts of the lithosphere below.

Based on the 'Brace-Goetze' rheology described above the vertically integrated strength of the crust can be approximated by

$$F_c = \left( \frac{\dot{\epsilon}}{A_q} \right)^{1/n} \exp \left( \frac{Q_q}{nRT_{bd}} \right) \left[ \frac{1}{2} Z_{bd} + \frac{Q_q}{nRr_c} (y_1 - \exp(x_2 - x_1)y_2) \right] \quad (27)$$

where

$$x_1 = \frac{Q_q}{nRT_{bd}}$$

$$x_2 = \frac{Q_q}{nRT_m}$$

$$y_1 = 1/x_1^2 + 2/x_1^3 + 6/x_1^4$$

$$y_2 = 1/x_2^2 + 2/x_2^3 + 6/x_2^4$$

$$r_c = \frac{T_m - T_{bd}}{z_c - Z_{bd}}$$

in which  $Z_{bd}$  is the depth to the brittle-ductile transition and its temperature is  $T_{bd}$ .  $T_m$  is Moho temperature.  $r_c$  is the thermal gradient in the lower crust, here it is perhaps reasonable to regard  $r_c$  as a constant since we assume that crustal heat production is concentrated mainly in the upper crust. The brittle-ductile transition depth and its temperature are governed by

$$\frac{2(\sigma_0 - \mu \rho_c g Z_{bd}(1 - \lambda))}{\sqrt{\mu^2 + 1 + \mu}} = \left( \frac{\dot{\epsilon}}{A_q} \right)^{1/n} \exp \left( \frac{Q_q}{nRT_{bd}} \right) \quad (28)$$

$Z_{bd}$  is mainly controlled by Moho temperature and to a lesser extent by  $\dot{\epsilon}$ ; the lower the Moho temperature and the higher strain rates, the deeper the transition depth. The transition temperature (for compression) ranges from 190° to 320°C for  $\dot{\epsilon}$  in the range  $10^{-18} - 10^{-14} \text{ s}^{-1}$  and Moho temperatures in the range 350° - 750°C. Sonder and England [1986] presented a much simpler expression for the crustal strength  $F_c$ , in which they discarded the term involved with the Moho temperature and adopted a crust without internal heat sources. For significant heat production in the upper crust (or higher Moho temperature), the expression for  $F_c$  of Sonder and England [1986] can be  $15 \times 10^{12} \text{ N m}^{-1}$  higher than the  $F_c$  given by equation (27) for  $\dot{\epsilon} > 10^{-14} \text{ s}^{-1}$ .

The vertically integrated strength of the mantle part of the lithosphere can be similarly given by

$$F_m = F_m^b + F_m^D + F_m^P \quad (29)$$

and

$$\begin{aligned} F_m^b &= (Z_{bd1} - z_c) \left[ \frac{2(\sigma_0 - \mu \rho_c g z_c (1 - \lambda_c))}{\sqrt{\mu^2 + 1 + \mu}} + \right. \\ &\quad \left. \frac{(\sigma_0 - \mu \rho_m g (Z_{bd1} - z_c) (1 - \lambda_m))}{\sqrt{\mu^2 + 1 + \mu}} \right] \end{aligned} \quad (30)$$

$$\begin{aligned} F_m^D &= \sigma_d (Z_t - Z_{bd1}) - \frac{2\sigma_d}{3r_m} \left[ T_t \sqrt{\frac{RT_t}{Q_d} \ln\left(\frac{\dot{\epsilon}_d}{\dot{\epsilon}}\right)} - \right. \\ &\quad \left. T_{bd1} \sqrt{\frac{RT_{bd1}}{Q_d} \ln\left(\frac{\dot{\epsilon}_d}{\dot{\epsilon}}\right)} \right] \end{aligned} \quad (31)$$

$$\begin{aligned} F_m^P &= \left( \frac{\dot{\epsilon}}{A_o} \right)^{1/n} \exp\left( \frac{Q_o}{nRT_t} \right) \\ &\quad \frac{Q_o}{nRr_m} [y_3 - \exp(x_4 - x_3)y_4] \end{aligned} \quad (32)$$

where

$$x_3 = \frac{Q_o}{nRT_t}$$

$$x_4 = \frac{Q_o}{nRT_t}$$

$$y_3 = 1/x_1^3 + 2/x_3^3 + 6/x_3^4$$

$$y_4 = 1/x_4^2 + 2/x_4^3 + 6/x_4^4$$

$$r_m = \frac{T_t - T_m}{z_t - z_c}$$

in which  $Z_{bd1}$  is the depth to the brittle-ductile transition in the upper mantle and its temperature is  $T_{bd1}$ ,  $Z_t$  is the transition depth between Dorn law creep and power law creep and its temperature is  $T_t$ ,  $r_m$  is the thermal gradient in the mantle.  $F_m^b$  is the integrated strength of brittle part in the upper mantle,  $F_m^D$  and  $F_m^P$  are the integrated strengths of ductile parts in the mantle lithosphere, which are governed by Dorn law and power law creep respectively.

For Moho temperatures higher than a critical temperature  $T_c$  defined by

$$\frac{2(\sigma_0 - \mu \rho_c g z_c (1 - \lambda_c))}{\sqrt{\mu^2 + 1 + \mu}} = \sigma_d \left[ 1 - \sqrt{\frac{RT_c}{Q_d} \ln\left(\frac{\dot{\epsilon}_d}{\dot{\epsilon}}\right)} \right] \quad (33)$$

there will be no brittle failure in the upper mantle, and  $F_m^b = 0$ ,  $Z_{bd1} = z_c$  and  $T_{bd1} = T_m$ . For a 34-km-thick crust with other rheological parameters defined in Table 2, the critical Moho temperature (for compression) is in the range 415 - 497°C for  $\dot{\epsilon}$  in the range  $10^{-17} - 10^{-14} \text{ s}^{-1}$ .

If Moho temperature is still higher than the transition temperature,  $T_t$ , between power law creep and Dorn law creep, which is defined by

$$\sigma_d \left[ 1 - \sqrt{\frac{RT_t}{Q_d} \ln\left(\frac{\dot{\epsilon}_d}{\dot{\epsilon}}\right)} \right] = 200 \quad (34)$$

then the integrated strength in the whole upper mantle is given by  $F_m^p$  in which  $T_t = T_m$ . For the rheological parameters in Table 2 the transition temperature (for compression) ranges from 670 to 780°C for  $\dot{\epsilon}$  in the range  $10^{-17} - 10^{-14} \text{ s}^{-1}$ .

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