

Polarimetric Analysis of Sucrose Solutions: Examining Concentration and Wavelength Effects on Optical Rotation via Intensity Monitoring

Matías Santacruz Vallejo, Lucio F. Vecchia

Faculty of Science and Engineering, University of Groningen, Netherlands

Abstract

The optical activity of sucrose solutions varying in relatively low concentrations were tested with a series of LEDs with wavelengths across the visible spectrum. Using linearly plane polarized light, the rotation in the plane of linear polarization caused by the sucrose solutions were determined using the voltage from a photodiode detector, implementing methodology based on Malus' law. It was confirmed following theory, that higher concentrations produced larger rotation at smaller wavelengths; the rotation and dispersion constants for sucrose were also determined. Rotary dispersion using a concentration of 0.7 g/ml was also demonstrated.

Introduction

Sugar polarization was introduced in the 17th century; it is a technique to measure the rotation of the plane of linearly polarized light caused by propagation through a medium. Polarimetric saccharimeters are devices used to ensure quality control and regulatory compliance in the modern food industry, and rely on fundamental physical principles related to polarization and rotation [1]. This phenomenon, called optical activity, occurs as a consequence of having different refractive indices within the same medium due to asymmetrical molecular compositions (i.e. optical isomers). The extent to which optical rotation occurs is dependent on various factors, including the wavelength of light, temperature, and the material's composition [2]. Hence, the research question in this article is to determine to what extent sucrose concentration, and wavelength rotate the plane of polarization of linearly polarized light?

Theory

Polarimetry

Polarimetry is a technique to determine the plane of polarization of a an electromagnetic (EM) wave, usually light. Light is the propagation of an electric and magnetic wave oscillating orthogonally along a specific direction \mathbf{k} . If it travels through a linearly polarized filter, then the transmitted wave will likewise be linearly polarized (i.e. it oscillates in the plane parallel to the transmission axis of the linear polarizer). If the direction of propagation is parallel to $\hat{\mathbf{z}}$, then the plane wave is described as [2]

$$\mathbf{E} = E_0 e^{i(\mathbf{k} \cdot \mathbf{z} - \omega t)} \hat{\mathbf{k}} = (E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}}) e^{i(\mathbf{k} \cdot \mathbf{z} - \omega t)}, \quad (1)$$

where ω is the angular frequency of the wave, and E_0 is the magnitude of the field¹. A phase shift of

θ rad may be introduced to either the magnetic or electric component, by introducing a an imaginary exponential factor accordingly. For example

$$\mathbf{E} = \frac{E_0}{2} \hat{\mathbf{x}} e^{i(\mathbf{k} \cdot \mathbf{z} - \omega t)} + \frac{E_0}{2} \hat{\mathbf{y}} e^{i(\mathbf{k} \cdot \mathbf{z} - \omega t)} \cdot e^{i\theta}, \quad (2)$$

corresponds to a circular polarization since $E_x = E_y$, and a clockwise rotation of θ rad viewed in the direction of propagation. Note that in the case of linearly polarized light, changes in both the left hand and right hand components can be manipulated independently, leading to a change in the plane of polarization.

Optical Activity

Optical activity is a characteristic of some materials which causes the plane of linearly polarized light to rotate. In sucrose, this occurs due to the various spatial arrangements of the molecules, which results in different electromagnetic interactions between the molecules and the components of polarized light. This interaction introduces phase differences by altering the speed of the components in the medium (i.e. its refractive indices) differently, thus introduce changes in the plane of polarization [3].

The relation between a material's left n_L and right hand n_R refractive indices, determines weather the mixture is a dextrorotatory ($n_L > n_R$) or levorotatory ($n_L < n_R$) mixture [2]. Sucrose is a dextrorotatory substance, meaning it rotates the plane of polarization in a clockwise direction relative to the direction of propagation [2].

The Drude Expression

The number of molecules light interacts with affects the extent of rotation that occurs. Provided the concentration of sucrose is low (i.e. < 0.7 g/ml), then the sucrose concentration c will rotate the plane of linear polarization by θ degrees relative to its initial plane. Quantitatively this is described as [2]

$$\theta = [\alpha]_{\lambda}^T \cdot l \cdot c, \quad (3)$$

¹ Hence, E_x, E_y are the amplitudes of the electric field along their respective directions

where l is the path length traversed by light in the solution, and c is the concentration of sugar. The quantity $[\alpha]_{\lambda}^T$ is a material property, though it also depends on the wavelength of incident light and the solution's temperature. It is given by the Drude expression:

$$[\alpha]_{\lambda}^T = \frac{A}{\lambda^2 - \lambda_0^2}, \quad (4)$$

where A is the rotation constant, λ_0 is the dispersion constant. Evidently, if light composed of various wavelengths is used, each component will be rotated by a different amount and separated like in a prism, a phenomena called *rotary dispersion*.

Malus' Law

Malus' law describes the change in intensity of linearly polarized light when it passes through a polarizer. The transmitted intensity I , is given by

$$I = I_0 \cos^2 \theta, \quad (5)$$

where I_0 is the initial intensity of the beam, and θ is the angle between the plane of polarization of the light and the transmission axis of the polarizer [2]. Hence, a detector can be used to monitor the intensity of incident light; following from Malus' law, the intensity of light reaching its maximum indicates alignment with the plane of polarization, while minimum intensity signifies a perpendicular orientation to the initial plane of polarization. This is a common detection method for determining planes of linear polarization.

Experimental Set-up

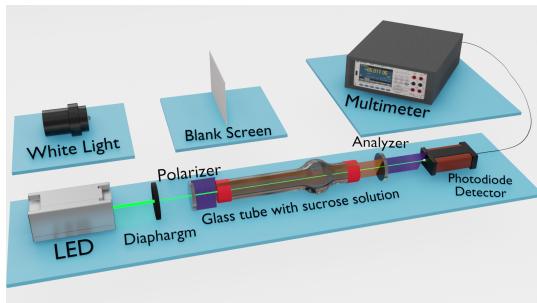


Figure 1: Schematic of experimental set-up. Purple planes are planes of linear polarization due to the polarizer/analyzer. The orange plane within the glass tube is the plane of polarization in the solution.

The setup was built on an optical rail; an LED of known wavelength is used to generate light, which is incident on a diaphragm to control the width of the beam. A linear polarizer is fitted before a glass polarimeter tube containing a sucrose solution of known concentration. A second linear polarizer (the analyzer), is fitted between the test tube and a photodiode. The photodiode produces a voltage with a magnitude proportional to the intensity of incident light, and relays it to multimeter for measurements.

With the tube fitted and optical alignment achieved, the analyzer was rotated in intervals of 101° and the voltage was recorded. This was performed for sucrose solution concentrations of: 0.00 , 0.12053 ± 0.00008 , and 0.2406 ± 0.0002 [g/ml]. Moreover, LEDs of wavelengths of 405 ± 10 , 465 ± 20 , 528 ± 30 , 591 ± 20 , 631 ± 20 , and 780 ± 30 [nm] were used².

The LEDs were replaced by a source of white light to photograph rotary dispersion patterns on the reflective screen. Likewise, this was performed at various analyzer orientations using a high concentration sucrose solution of 0.7011 ± 0.0005 g/ml.

Results

The voltage generated by the photodiode at various corresponding angles of the analyzer were plotted for all wavelengths (see Figure 5 in the Appendix). The solution of pure water was taken as a reference for the plane of polarization for each LED, so the data points were shifted horizontally, to ensure no rotation polarization was observed with the solution of distilled water (F). Moreover, the voltage was normalized to ensure a common scale. From the peaks in these graphs (see Figure 5), the rotation in the plane of linear polarization were found for each wavelength; they were plotted in Figure 2.

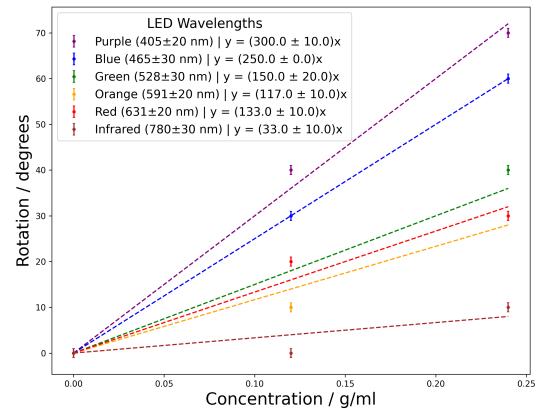


Figure 2: The concentration-rotation relation plotted for the wavelengths of LEDs; errors in the concentration are too small to be visible. A linear equation was fit through each series.

Using the gradients from Figure 2, the specific rotation factors $[\alpha]_{\lambda}^T$ at various wavelengths were calculated (the length of the tube l was measured at 200.0 ± 0.5 mm). They were plotted against wavelength (see Figure 3), and fit using the Scipy's 'curve-fit' to find the rotation and dispersion constants; $A = (18 \pm 5) \times 10^5$ m⁴/kg, and $\lambda_0 = 220 \pm 80$ nm, respectively. Note that the point corresponding to 631 ± 30 nm (the red LED) was not used, as it provided a specific rotation that was inconsistent with the remaining values obtained from equation 4.

The major sources of error include ambient temperature, and the path length l . However, the latter was

² Errors were found using the full width at half maximum of the LEDs measured spectra, found in the Appendix.

likely too small to significantly influence the results, while the temperature stayed within a small range of $20.0 \pm 0.7^\circ\text{C}$.

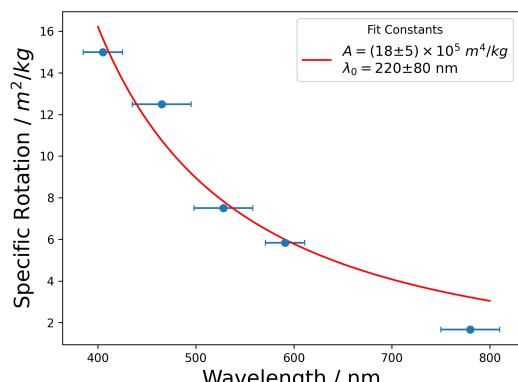


Figure 3: The specific rotation vs wavelength found using the gradients of Figure 2 and equation 4.

The rotary dispersion patterns were also photographed at various angles of the polarizer. They are shown in Figure 4.

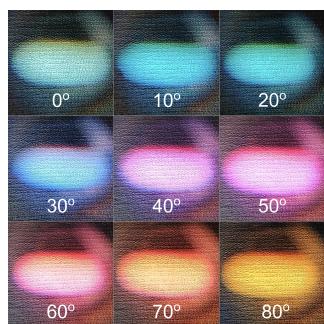


Figure 4: Rotary dispersion images at various (primary) polarizer orientations. Captured with a mobile phone camera.

Discussion

Examining Figure 2, it is evident that the concentration of sucrose in the solution is linearly proportional to the rotation in the plane of linear polarization, as was predicted by equation 3. It should be noted that the number of data points is relatively small considering the range, and further confidence in the data would greatly benefit from additional measurements at smaller concentration intervals. Moreover, the precision of the equipment ensured that the errors in both the concentration and the rotation were small at around 0.1%, and 3% respectively. Nevertheless, error in the gradient which is relevant for determining the specific rotation, ranged in percentage error between 0.25% and 30.31%; this was due to the large spread in the data.

Furthermore, Figure 2 also demonstrates the dependence of optical rotation on the wavelength of light; clearly, light with a smaller wavelength was more strongly rotated in its plane of linear polarization. This is reflected by steeper gradients, and follows as a direct consequence of the Drude expression (equation 4).

Indeed, linearly increasing wavelengths should lead to exponentially decreasing rotation; this is confirmed in Figure 3. The values obtained values for A and λ_0 were not satisfactorily close to their literature counterparts of $(22 \pm 1) \times 10^7 \text{ m}^4/\text{kg}$ and $\lambda_0 = 131 \pm 10 \text{ nm}$ [4] as they lie outside the margin of error. Although it is possible that temperature fluctuations altered the value of the the $[\alpha]_\lambda^T$ factor, they are unlikely enough to account for the magnitude of error. This suggest there were large systematic errors, most likely in measuring the rotation in the plane of polarization, which propagated forward and reduced the accuracy (but thus not the precision) of the results.

Rotary Dispersion

As expected, the various wavelengths composing the white light source experienced distinct levels of optical activity within the sucrose solution, causing them to rotate at various angles. When the light exits the solution and is analyzed with a polarizer, the differently rotated wavelengths combine again leading to an interference pattern. Hence, the combination of the sucrose's optical activity and the polarizer's selective filtering produced a spectrum of colors similar to a prism, with larger polarization angles resulting in spectra of longer wavelengths.

Conclusion

The aim of this investigation was to determine to what extent sucrose concentration, and wavelength rotate the plane of polarization of linearly polarized light. It was found, using LEDs with narrow peaks in their spectra, that smaller wavelengths experienced stronger rotary effects due to the optical activity of sucrose. Similarly, more concentrated sucrose solutions were found to cause stronger rotations. The quantitative nature of this relationship is wavelength dependent, thus, no single values were found to describe the relationships. However, the rotation and dispersion constants were found to be $(18 \pm 5) \times 10^5 \text{ m}^4/\text{kg}$, and $220 \pm 80 \text{ nm}$, respectively. These were useful in proving the Drude expression, though far from the literature values due to propagated systematic errors in measuring the rotation. Moreover, rotary dispersion at high sucrose concentrations for white light sources was shown.

References

- [1] M. McMahon, *What is a saccharometer?* 2023. Accessed: Dec. 13, 2023. [Online]. Available: <https://www.allthescience.org/what-is-a-saccharometer.htm>.
- [2] University of Groningen Faculty of Science and Engineering, *Physics laboratory 3 & 4 sugar polarization rotation*.
- [3] X. Liu, *Organic Chemistry I*. Kwantlen Polytechnic University, 2023. Accessed: Dec. 13, 2023. [Online]. Available: <https://kpu.pressbooks.pub/organicchemistry/>.
- [4] R. N. Compton, S. M. Mahurin, and R. N. Zare, "Demonstration of optical rotatory dispersion of sucrose," *J. Chem. Educ.*, vol. 76, no. 9, p. 1234, Sep. 1999.

Appendix

Determination of Polarization Rotation from Voltage Measurements

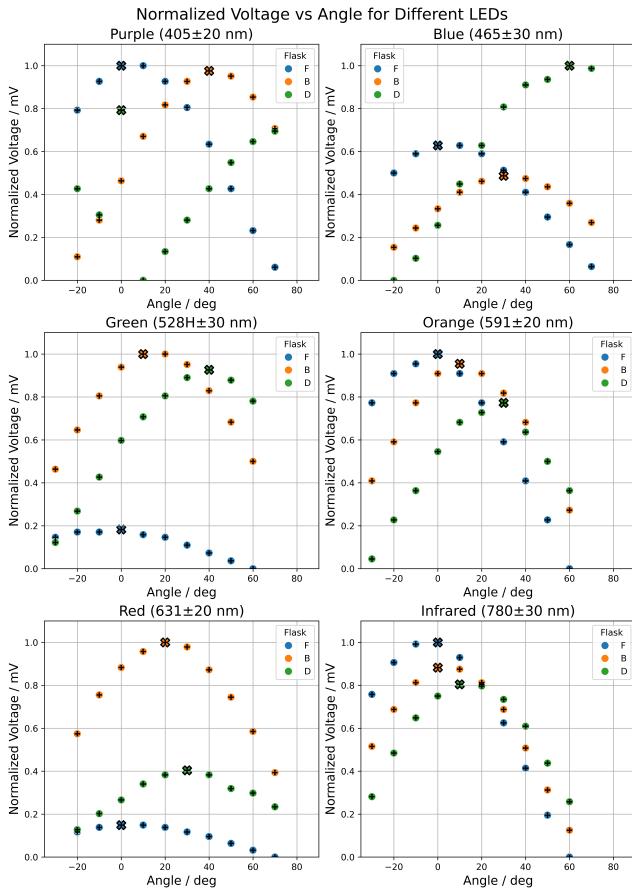


Figure 5: Graphs for the normalized voltage at varying angular positions of the analyzer relative to the (initial) polarizer. Each graph corresponds to a specific LED, tested the solutions of concentrations 0.00 g/ml (F), 0.12053 ± 0.00008 g/ml (B), and 0.2406 ± 0.0002 g/ml (D). The maxima of each series is marked with a bold cross.

Note that all graphs share a common vertical (Voltage) scale, which was achieved by normalizing each series individually, the aim being to prevent altering the appearance of the results by the magnitude of the generated electric signal, as it is largely dependent on the settings of the multimeter and the diaphragm of the photodiode. Evidently from the crosses, the solution with highest concentration (D) consistently caused the most rotation, while the pure water caused no rotation. It should be noted that these plots are evidence of Malus' law. Although no equation was fit to the data, the general relation tended to follow the expected sinusoidal behaviour.

Concentration Uncertainties

The following describes how the errors in the concentrations of the sucrose solution were obtained. Firstly, knowing that the relation to calculate a solution concentration c from the mass m of the solute and volume

of the solvent V is

$$c = \frac{m}{V}, \quad (6)$$

the expression for the absolute error in the concentration is found using the standard error relation to be

$$\Delta c = c \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}. \quad (7)$$

This formula was applied to flasks B, and D, with their corresponding values mass values (since flask F has no sucrose, it has no concentration uncertainty). Hence, the values for sucrose concentrations in the flasks were found to be:

- B: 0.12053 ± 0.00008 g/ml
- D: 0.2406 ± 0.0002 g/ml

Uncertainty in Specific Rotation

The errors in the gradient shown in Figure 2 were calculated using the covariance in the parameter found during the Scipy's fit (i.e. a non-linear least squares fit). These values can be found in Table 1. Note the large percentage errors with increasing

λ / nm	$[\alpha]_{\lambda}^T / \text{m}^2/\text{kg}$	% Error
405	300	3.34
465	250	0.25
528	150	13.30
591	117	8.55
780	33	30.31

Table 1: Values and errors in the Specific rotation calculated from the gradient of the concentration-rotation relation.

LEDs Spectra Error Analysis: Full Width at Half Maximum

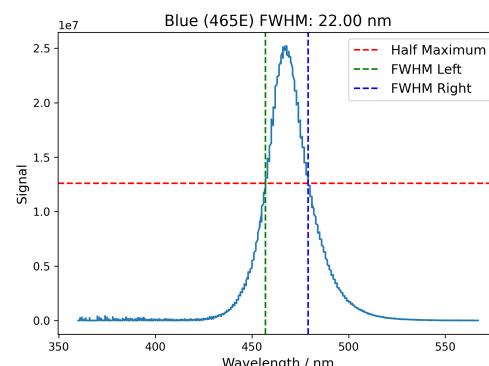
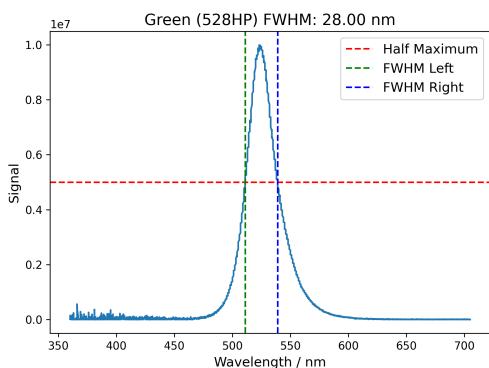
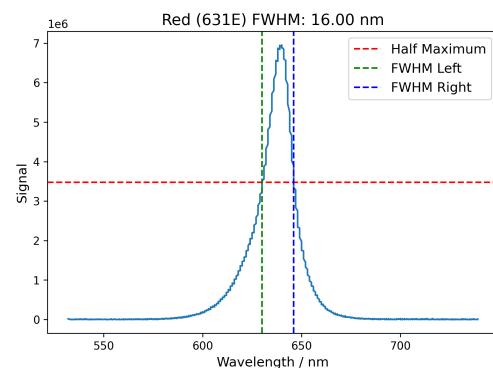
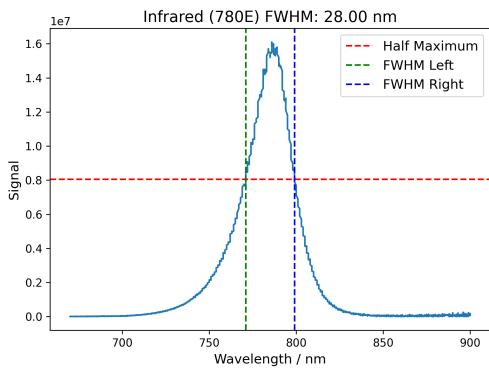
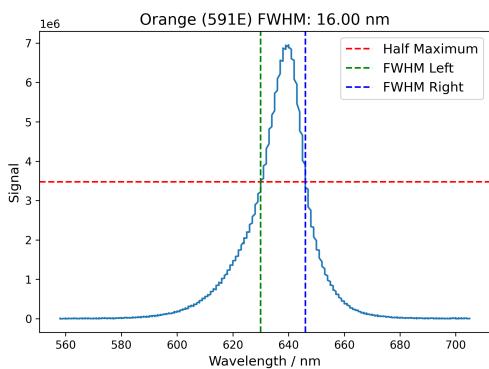
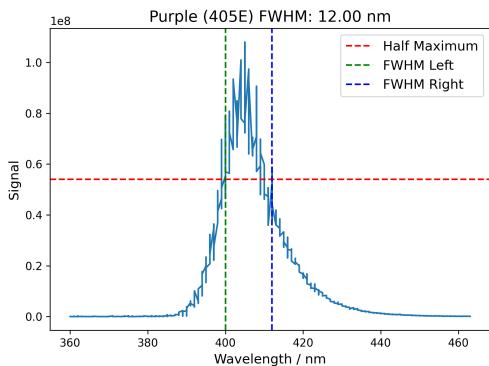


Figure 6: Blue (465E) LED Spectra.

**Figure 7:** Green (528HP) LED Spectra.**Figure 11:** Red (631E) LED Spectra.**Figure 8:** Infrared (780E) LED Spectra.

Links to Raw Data

The raw data was digitized and is accessible here.

**Figure 9:** Orange (591E) LED Spectra.**Figure 10:** Purple (405E) LED Spectra.