A Theory of Downward Wage Rigidity*

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Abstract

Why don't firms decrease wages in recessions? To answer this question, I develop a labor search model where workers are averse to losses *a la* Kahneman and Tversky (1979), and can decide whether to quit their current firm and look for a new job. The model implies that in a recession, firms are reluctant to cut wages because workers are more likely to leave when faced with an income loss. The model is consistent with a number of well-established empirical facts: (1) wage changes are rare, and wage increases are much more likely than wage decreases; (2) wages are more sensitive to aggregate than to idiosyncratic shocks, but the passthrough is incomplete; (3) macroeconomic variables respond asymmetrically to negative and positive shocks.

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1 Introduction

Two stylized facts on wage rigidity stand out in the data (Grigsby et al. (2021), Hazel and Taska (2024)):

- 1. Nominal wages rarely change, and when they do, they are much more likely to increase than to decrease;
- 2. Nominal wages are much more likely to increase in expansions than to decrease in recessions.

These facts are so well-documented that economists have long recognized the importance of incorporating them in macroeconomic models (Schmitt-Grohé and Uribe (2016), Benigno and Eggertsson (2023)). However, standard models with rational agents have made little progress in explaining why downward wage rigidity arises in the first place. In this paper, I provide a simple explanation with roots in the cognitive psychology literature: firms are reluctant to cut wages because workers are averse to losses.

In their seminal work on "prospect theory", Kahneman and Tversky (1979) recognized that standard expected utility preferences are not consistent with many properties of the decisions people make. Figure 1 presents the value function proposed by Kahneman and Tversky (1979). This value function has two main properties. First, economic agents evaluate lotteries not only according to their absolute utility, but also according to the utility gain relative to a "status quo". Second, they are more sensitive to losses than to gains, with a kink at zero gain. Moreover, prospect theory asserts that these utility gains are tied to emotional issues such as regret or anger, rather than related to pure utility gains from consumption (see Barberis (2013) for an excellent review).

There is plenty of anecdotal evidence that this phenomenon is at play in the labor market. In his hundreds of interviews, Bewley (1999) found that managers are reluctant to cut pay because of the adverse effects on morale and turnover. Words like "insult" and "rage" are prevalent in the typical answer, suggesting that the utility loss from a pay cut is not consumption-related.

In this spirit, I develop a labor search model with loss-averse workers and an endogenous quitting decision. After observing the wage offered by the incumbent, workers draw an idiosyncratic search cost, and decide whether to quit the firm and search for a new job. The probability that the worker leaves the firm decreases with

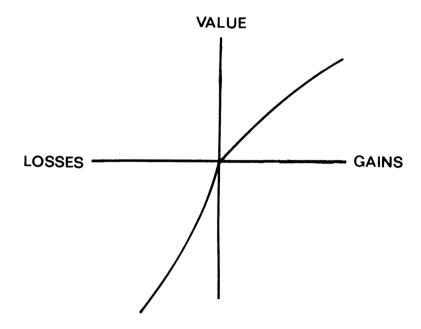


Figure 1: Prospect theory value function, Kahneman and Tversky (1979), p. 279

the current wage, but has a kink at the past wage and is steeper below that point. As a result, when aggregate productivity goes down, most firms fully absorb the shock and keep the wage constant to avoid the steeper region of the quitting probability schedule.

The model is consistent with two additional facts. First, labor market tightness responds differently to negative and positive productivity shocks (McKay and Reis (2008)). In the model, most incumbents absorb the negative productivity shock when the shock is small. Therefore entrants need to pay a high wage in order to attract the worker. The expected returns from posting a vacancy are low, and labor market tightness goes down.

Second, wages are more responsive to aggregate than to idiosyncratic shocks, but the passthrough is partial (Carlsson et al. (2016)). In the benchmark without loss aversion, wages are already more sensitive to aggregate than to idiosyncratic shocks. The model has this property because the quitting decision is based on the wage offer relative to the entire wage distribution. When aggregate productivity goes up, the entire wage distribution of entrants shifts upwards, and incumbents' wages increase one for one. On the other hand, when idiosyncratic productivity goes up, the probability that the worker finds a more productive firm goes down,

and so it is optimal for incumbents to increase the wage less than one for one. Loss aversion plays a role in bringing the model close to the data because it decreases the passthrough of shocks to wages; conditional on a wage decrease, firms do not cut wages by as much as in the benchmark without loss aversion.

The paper is structured as follows. In Section 2, I review the empirical facts on the labor market. In Section 3 I develop the model, and in Section 4 I present the main results. In Section 5 I discuss the relation to alternative theories and further issues. I conclude in Section 6.

2 Empirical Facts

Downward wage rigidity has been extensively documented both in the US and in other advanced economies (see Dickens et al. (2007) for a review of the international evidence). More recently, there have been two studies of nominal wage rigidity that use high-quality datasets from the US. Grigsby et al. (2021) study the several components of the compensation of millions of workers using administrative data from a payroll-processing company. Figure 2 exhibits the most important feature of the data. For continuing workers, changes in nominal base pay have a spike at zero, and almost never decrease. For job-switchers, the authors uncover similar patterns. These results are consistent with Hazel and Taska (2024), who study data on posted wages for new vacancies in the US.¹

While these results relate to unconditional wage changes, these two papers also find that wage decreases are much less likely in a recession than wage increases are in an expansion. Figure 3 illustrates this result, which is confirmed by regression analysis in Hazel and Taska (2024), and by a quasi-experimental analysis of the Great Recession in Grigsby et al. (2021).

The empirical literature has documented more widespread state-dependence in wage setting. Grigsby et al. (2021) estimate that the probability of a wage decline is time-varying. Using a structural general equilibrium model and administrative data from Denmark, Chan et al. (2023a) estimate that the passthrough of TFP shocks to wages is (1) small for negative shocks, and increasing with the size of the shock,

¹A concern about the relevance of downward wage rigidity is that many workers earn performance-based compensation. However, Grigsby et al. (2021) show that bonus or overtime pay is largely acyclical and, conditional on receiving that type of compensation, i.i.d. over time. The authors argue that base pay is therefore the single most important factor in determining the cyclicality of a firm's marginal cost of labor.

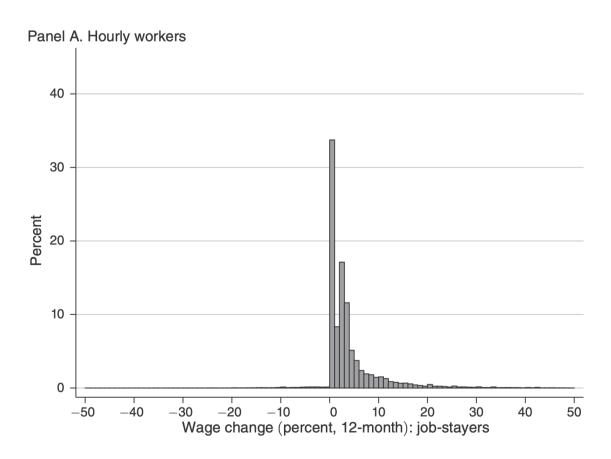


Figure 2: Figure 2, Panel A in Grigsby et al. (2021), "Twelve-Month Nominal Base Wage Change Distribution, Job-Stayers"

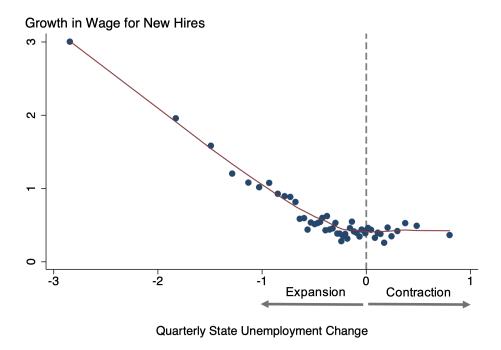


Figure 3: Figure 1 in Hazel and Taska (2024), "Wage Growth for New Hires and Quarterly State Unemployment Changes"

but (2) large for small positive shocks, and decreasing with the size of the shock. The model I develop in this paper is consistent with these observations for negative shocks. For a small negative shock, most firms do not change their wages. For a large negative shock, more and more firms decrease their wages, and the size of the wage change is increasing with the size of the shock.

There are two important dimensions of heterogeneity in what concerns the passthrough of technology shocks to wages. First, the literature has extensively established that wages are more responsive to aggregate than to idiosyncratic shocks, but the passthrough is incomplete. Examples of studies establishing this stylized fact are Guiso et al. (2005), Carneiro et al. (2012), Carlsson et al. (2016), Chan et al. (2023b).

The second important dimension is the identity of firms that cut wages. Here the evidence is mixed. Using administrative data from Denmark, Chan et al. (2023b) estimate that the passthrough of TFP shocks to wages is larger for low-productivity firms. Carneiro et al. (2012), estimate that low-paying jobs in Portugal are more likely to be destroyed in recessions. Under the assumptions that (1) low-paying jobs are more prevalent in low-productivity firms, and (2) job destruction is partly ex-

plained by higher quitting due to wage decreases, these two papers suggest that the incidence of wage decreases falls mainly on low-productivity firms. However, Juhn et al. (2018) estimate that for the US, the volatility of income is larger for high-productivity workers. I show in Section 3 that depending on the productivity distribution, the model is consistent with both of these observations.

Finally, many empirical studies have documented asymmetry in the response of macroeconomic variables to positive and negative shocks. Abbritti and Fahr (2013) document significant skewness in the distribution of growth rates of several macro variables, with recessions being stronger than expansions. McKay and Reis (2008) find that decreases in employment are briefer but more violent than increases. Dupraz et al. (2019) find similar results. In Section 4, I argue that loss aversion implies an asymmetry in the response of labor market tightness, the sole macroeconomic variable of interest in this model.

3 The Model

The model is static. At the beginning of the period, there is a unit mass of firms that employ a worker – the incumbents –, and a measure v of firms opening a vacancy – the entrants. I assume that incumbents cannot post vacancies. The productivity of each match is y + z, where y is an aggregate component, and z a match-specific component. For both incumbents and entrants, z follows a Pareto distribution, i.e.,

$$F_{Z}\left(z
ight) =1-\left(rac{b}{z}
ight) ^{ au}$$
 , for $z\geq b$,

where $\tau > 1$ and b > 0.2 I assume that home production has the minimum productivity, y + b.

At the beginning of the period, incumbents post wage offers w_n , and workers can decide whether to stay, or quit and look for a new job. Strictly speaking, workers cannot search on the job, but if they leave they receive an unemployment compensation equal to a fraction ι of w_n . This compensation is financed by a lump-sum tax, and for now I set $\iota = 1.3$ In order to search, workers must pay a search cost ζ drawn

²The assumption of a Pareto distribution is not necessary, but is helpful in making sure that second-order conditions are satisfied.

³When $\iota = 1$, the decision to search depends only on the expected utility gain from finding an offer that is better than the wage posted by the incumbent. Therefore it is as if there were search on

from an exponential distribution, i.e.,

$$F_{\zeta}(\zeta) = 1 - e^{-\gamma \zeta}$$
, for $\zeta \ge 0$,

where $\gamma > 0$. The search cost is *i.i.d.* across workers.⁴

I assume that incumbents have full bargaining power, but that the wages of entrants are determined through Nash bargaining. This assumption ensures that the problem of incumbents is analytically tractable.

The timing is as follows:

- 1. Incumbent firms post their wage offers w_n , and v entrants post vacancies;
- 2. Workers draw the search cost, and decide whether to quit and search for a new job;
- 3. Search and matching takes place, and entrants' wages w_v are determined through Nash bargaining;
- 4. Production occurs.

Search friction Let n be the measure of job seekers. Given n and v, the economy generates m matches given by

$$m = \mathcal{M}(n, v)$$
,

where $\ensuremath{\mathcal{M}}$ is a matching function satisfying the following standard properties:

- $\mathcal{M}(n,v) \in [0,\min\{n,v\}];$
- \mathcal{M} is increasing in both arguments;
- $\lim_{n\to\infty} \mathcal{M}(n,v) = v$, $\lim_{v\to\infty} \mathcal{M}(n,v) = n$;
- $\mathcal{M}(\vartheta n, \vartheta v) = \vartheta \mathcal{M}(n, v)$, for $\vartheta > 0$.

Let $\theta \equiv v/n$ be labor market tightness. The constant returns to scale assumption implies that the probability that an entrant finds a job-seeker, m/v, and the probability that a job-seeker finds an entrant, m/n, can be written as functions of θ only, *i.e.*,

$$\alpha_{f}\left(\theta\right)\equiv\frac{m}{v}=\mathcal{M}\left(\theta^{-1},1\right)$$
,

the job.

⁴The assumption of an exponential distribution is for simplicity. The model would exhibit the same qualitative properties as long as the distribution of search costs has a decreasing hazard rate.

$$\alpha_w(\theta) \equiv \frac{m}{n} = \mathcal{M}(1, \theta).$$

It follows that $\alpha'_{f}(\theta) \leq 0$ and $\alpha'_{w}(\theta) \geq 0$.

Worker's preferences As in Kőszegi and Rabin (2006), the worker has preferences over labor income m and a reference point r that are characterized by the utility function

$$u\left(m\mid r\right) = \begin{cases} m + \eta\left(m - r\right), & \text{if } m \geq r\\ m + \lambda\eta\left(m - r\right), & \text{if } m < r \end{cases},$$

where $\eta \geq 0$ and $\lambda \geq 1$. $\eta = 0$ is the standard model where labor income gains are irrelevant, and $\lambda = 1$ is the benchmark case where gains and losses are equally valued. This utility function is a simplified version of Kahneman and Tversky (1979), since I abstract from excess sensitivity and probability weighting. To isolate the effect of loss aversion on the wage setting decision, I assume that utility is additive in the search cost and in the lump-sum tax.

3.1 Entrants' Wages

For any variable x, let $\tilde{x} \equiv x - y$. Then utility from income m at reference r can be written as

$$u(m \mid r) = y + u(\tilde{m} \mid \tilde{r}).$$

Consider a worker with an incumbent wage offer w_n , who has found a match with productivity z_v that pays a wage equal to w_v . The utility gain of accepting the new match is

$$u\left(w_{v}\mid r\right)-u\left(w_{n}\mid r\right)=u\left(\tilde{w}_{v}\mid \tilde{r}\right)-u\left(\tilde{w}_{n}\mid \tilde{r}\right)$$
,

and the surplus to the entrant is

$$y + z_v - w_v = z_v - \tilde{w}_v.$$

The normalized real wage \tilde{w}_v maximizes the generalized Nash product⁵

$$\mathcal{B}\left(\tilde{w}_{v}, z_{v}, \tilde{w}_{n} \mid \tilde{r}\right) = \left[u\left(\tilde{w}_{v} \mid \tilde{r}\right) - u\left(\tilde{w}_{n} \mid \tilde{r}\right)\right]^{\phi} \left(z_{v} - \tilde{w}_{v}\right)^{1 - \phi}, \ \phi \in (0, 1).$$

⁵It is straightforward to show that even though $u(\cdot | \cdot)$ has a kink, the utility possibility frontier is convex. Therefore the Nash bargaining solution is valid.

The problem is only well-defined when $z_v \geq \tilde{w}_n$, so that when a job-seeker with incumbent offer \tilde{w}_n meets an entrant with match productivity $z_v < \tilde{w}_n$, both parties agree not to produce.

The first-order conditions for \tilde{w}_v imply the following Lemma.

Lemma 1. Assume that $z_v \geq \tilde{w}_n$. If $\tilde{w}_n \geq \tilde{r}$,

$$\tilde{w}_v(z_v, \tilde{w}_n \mid \tilde{r}) = \phi z_v + (1 - \phi) \tilde{w}_n.$$

If $\tilde{w}_n < \tilde{r}$,

$$\tilde{w}_{v}\left(z_{v}, \tilde{w}_{n} \mid \tilde{r}\right) = \begin{cases}
\tilde{r} + \phi \left[z_{v} - z_{v,\ell}\left(\tilde{w}_{n}, \tilde{r}\right)\right], & \text{if } z_{v} < z_{v,\ell}\left(\tilde{w}_{n}, \tilde{r}\right) \\
\tilde{r}, & \text{if } z_{v,\ell}\left(\tilde{w}_{n}, \tilde{r}\right) \leq z_{v} < z_{v,h}\left(\tilde{w}_{n}, \tilde{r}\right), \\
\tilde{r} + \phi \left[z_{v} - z_{v,h}\left(\tilde{w}_{n}, \tilde{r}\right)\right], & \text{if } z_{v} \geq z_{v,h}\left(\tilde{w}_{n}, \tilde{r}\right)
\end{cases} (1)$$

where

$$z_{v,\ell}\left(\tilde{w}_{n},\tilde{r}\right)\equiv\left(\frac{1}{\phi}\right)\tilde{r}-\left(\frac{1-\phi}{\phi}\right)\tilde{w}_{n},\tag{2}$$

and

$$z_{v,h}\left(\tilde{w}_{n},\tilde{r}\right)=z_{v,\ell}\left(\tilde{w}_{n},\tilde{r}\right)+\left(\frac{1-\phi}{\phi}\right)\frac{\eta\left(\lambda-1\right)}{1+\eta}\left(\tilde{r}-\tilde{w}_{n}\right). \tag{3}$$

The intuition for this Lemma is as follows. When $\tilde{w}_n \geq \tilde{r}$, the entrant's wage must also be above the reference (since $\tilde{w}_v \geq \tilde{w}_n$). Therefore the utility gain is reference-independent,

$$u\left(\tilde{w}_{v}\mid\tilde{r}\right)-u\left(\tilde{w}_{n}\mid\tilde{r}\right)=\left(1+\eta\right)\left(\tilde{w}_{v}-\tilde{w}_{n}\right),$$

and the solution to the entrant's wage is standard. On the other hand, when $\tilde{w}_n < \tilde{r}$ the utility gain is reference-dependent:

$$u\left(\tilde{w}_{v}\mid\tilde{r}\right)-u\left(\tilde{w}_{n}\mid\tilde{r}\right)=\begin{cases}\left(1+\eta\lambda\right)\left(\tilde{w}_{v}-\tilde{w}_{n}\right), & \text{if } \tilde{w}_{v}<\tilde{r}\\ \left(1+\eta\right)\left(\tilde{w}_{v}-\tilde{w}_{n}\right)+\eta\left(\lambda-1\right)\left(\tilde{r}_{n}-\tilde{w}_{n}\right), & \text{if } \tilde{w}_{v}\geq\tilde{r}\end{cases}.$$

When $\tilde{w}_v \geq \tilde{r}$, there is an extra term $\eta (\lambda - 1) (\tilde{r}_n - \tilde{w}_n) > 0$ that accounts for the fact that by accepting the entrant's offer, the worker avoids a real income loss. For matches with high productivity $-z_v \geq z_{v,h} (\tilde{w}_n, \tilde{r})$ –, it is optimal to pay above the reference. For matches with low productivity $-z_v < z_{v,\ell} (\tilde{w}_n, \tilde{r})$ – it is optimal to pay below the reference. Moreover, substituting the thresholds (2) and (3) in the wage

equation (1), \tilde{w}_v can be written as

$$\tilde{w}_{v}\left(z_{v},\tilde{w}_{n}\mid\tilde{r}\right) = \begin{cases} \phi z_{v} + \left(1-\phi\right)\tilde{w}_{n}, & \text{if } z_{v} < z_{v,\ell}\left(\tilde{w}_{n},\tilde{r}\right)\\ \tilde{r}, & \text{if } z_{v,\ell}\left(\tilde{w}_{n},\tilde{r}\right) \leq z_{v} < z_{v,h}\left(\tilde{w}_{n},\tilde{r}\right)\\ \phi z_{v} + \left(1-\phi\right)\tilde{w}_{n} - \left(1-\phi\right)\frac{\eta(\lambda-1)}{1+\eta}\left(\tilde{r}-\tilde{w}_{n}\right), & \text{if } z_{v} \geq z_{v,h}\left(\tilde{w}_{n},\tilde{r}\right) \end{cases}$$

Matches with low productivity pay below the reference, so the utility gain from switching jobs is again reference-independent. Therefore the solution to \tilde{w}_v is standard. On the other hand, firms with a high-productivity match are able to extract additional $(1-\phi)\frac{\eta(\lambda-1)}{1+\eta}\left(\tilde{r}-\tilde{w}_n\right)$ units of surplus due to the fact that they pay above the reference point.

It follows that in this model, wages of new hires are not fully flexible either.

3.2 Worker's Problem

Let

$$\mathcal{O}\left(\tilde{w}_{n} \mid \tilde{r}\right) \equiv \int_{z_{v} \geq \tilde{w}_{n}} \left[u\left(\tilde{w}_{v}\left(z_{v}, \tilde{w}_{n} \mid \tilde{r}\right) \mid \tilde{r}\right) - u\left(\tilde{w}_{n} \mid \tilde{r}\right)\right] F_{Z}\left(dz_{v}\right) \tag{4}$$

be the expected returns from search, and

$$S(x) \equiv \int_{z_v \ge x} \overline{F}_Z(z_v) dz_v = \Pr(z_v \ge x) \mathbb{E}[z_v - x \mid z_v \ge x]$$

where $\overline{F}_{Z}(z) \equiv 1 - F_{Z}(z)$, and the second equality follows from the fact that for a positive random variable X,

$$\mathbb{E}\left[X\right] = \int_0^\infty \overline{F}_X\left(x\right) dx.$$

Substituting equation (1) in (4), it is straightforward to show that

$$\mathcal{O}\left(\tilde{w}_{n} \mid \tilde{r}\right) = \begin{cases} \left(1 + \eta\right) \phi S\left(\tilde{w}_{n}\right), & \text{if } \tilde{w}_{n} \geq \tilde{r} \\ \left(1 + \eta\right) \phi S_{\lambda}\left(\tilde{w}_{n} \mid \tilde{r}\right), & \text{if } \tilde{w}_{n} < \tilde{r} \end{cases}$$
(5)

where

$$S_{\lambda}\left(ilde{w}_{n}\mid ilde{r}
ight)\equiv S\left[z_{v,h}\left(ilde{w}_{n}, ilde{r}
ight)
ight]+\left(rac{1+\eta\lambda}{1+\eta}
ight)\left\{ S\left(ilde{w}_{n}
ight)-S\left[z_{v,\ell}\left(ilde{w}_{n}, ilde{r}
ight)
ight]
ight\} .$$

When $\lambda = 1$, the option value of search is $(1 + \eta) \phi S(\tilde{w}_n)$, which is proportional to the expected difference between the match productivity z_v and the outside option \tilde{w}_n . When $\lambda > 1$, the option value is reference-dependent. These following Lemma establishes three useful properties of $\mathcal{O}(\cdot \mid \cdot)$.

Lemma 2. $\mathcal{O}\left(\tilde{w}_{n} \mid \tilde{r}\right)$ satisfies the following properties.

- 1. $\mathcal{O}(\tilde{w}_n \mid \tilde{r})$ is continuous, and strictly decreasing in \tilde{w}_n ;
- 2. $\mathcal{O}(\tilde{w}_n \mid \tilde{r})$ is non-differentiable at \tilde{r} ;

3.
$$\left|\frac{\partial \mathcal{O}(\tilde{w}_n|\tilde{r})}{\partial \tilde{w}_n}\right| > (1+\eta) \phi \left|S'\left(\tilde{w}_n\right)\right| \text{ whenever } \tilde{w}_n < \tilde{r}.$$

The first property is standard: the option value of decrease is low if the wage offered by the incumbent is high. The second and third properties are due to loss aversion.

The second property says that the option value of search has a kink at the reference point \tilde{r} . The third property says that with loss aversion, the marginal effect of a wage decrease on the option value of search is higher than in the benchmark. This property is due to the fact that when $\tilde{w}_n < \tilde{r}$, by searching the worker earns an income level that is above the reference.

Let $W_{\overline{q}}(\tilde{w}_n \mid \tilde{r})$ be the utility of a worker who does not quit, and $W_q(\tilde{w}_n \mid \tilde{r})$ the utility of a worker who quits. Then

$$W_{\overline{q}}\left(\tilde{w}_{n}\mid\tilde{r}\right)=y+u\left(\tilde{w}_{n}\mid\tilde{r}\right)-T$$
,

where T is a lump-sum tax, and

$$W_{q}\left(\tilde{w}_{n}\mid\tilde{r}\right)=y+u\left(\tilde{w}_{n}\mid\tilde{r}\right)+\alpha_{w}\left(\theta\right)\mathcal{O}\left(\tilde{w}_{n}\mid\tilde{r}\right)-\zeta-T.$$

It follows that the worker stays if

$$W_{q}\left(\tilde{w}_{n} \mid \tilde{r}\right) \leq W_{\overline{q}}\left(\tilde{w}_{n} \mid \tilde{r}\right)$$

$$\iff \zeta \geq \alpha_{w}\left(\theta\right) \mathcal{O}\left(\tilde{w}_{n} \mid \tilde{r}\right).$$

Taking the expectation over ζ , the retention probability, $\rho\left(\tilde{w}_{n}\mid\tilde{r}\right)$, is

$$\rho\left(\tilde{w}_{n}\mid\tilde{r}\right)\equiv\overline{F}_{\zeta}\left[\alpha_{w}\left(\theta\right)\mathcal{O}\left(\tilde{w}_{n}\mid\tilde{r}\right)\right]=e^{-\gamma\alpha_{w}\left(\theta\right)\mathcal{O}\left(\tilde{w}_{n}\mid\tilde{r}\right)}.$$

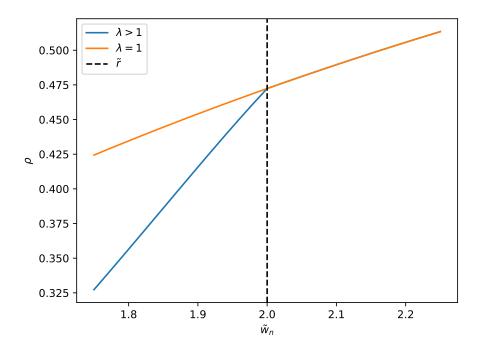


Figure 4: Retention probability schedule.

By the law of large numbers, the pool of job seekers is $n=1-\overline{\rho}\equiv 1-\mathbb{E}\left[\rho\left(\tilde{w}_n\mid \tilde{r}\right)\right]$.

The shape of ρ drives most results. As illustrated in Figure 4, it has a kink at \tilde{r} and is steeper when $\tilde{w}_n < \tilde{r}$. When productivity goes down, incumbents are reluctant to cut pay because in the loss region, workers are more inclined to leave the firm.

3.3 Incumbent's Problem

An incumbent with match productivity z_n chooses \tilde{w}_n to solve

$$\max_{\tilde{w}_n \in [b, z_n]} J\left(\tilde{w}_n, z_n \mid \tilde{r}\right) \equiv \rho\left(\tilde{w}_n \mid \tilde{r}\right) \left(z_n - \tilde{w}_n\right). \tag{6}$$

The following Lemma establishes regularity conditions under which the problem is well-behaved.

Lemma 3. If $\phi \geq \frac{1}{2}$, J is strictly log-concave in \tilde{w}_n . Therefore the second-order conditions are satisfied.

Proof. (Sketch) The proof follows from taking the second derivative of $\ln J$ in each of the regions.

I assume that $\phi \ge \frac{1}{2}$ throughout. The first-order conditions to (6) imply the following Proposition.

Proposition 1. Let $\tilde{w}_n(z_n \mid \tilde{r})$ be the solution to (6). There are two cases of interest.

Case I: $b \ge \tilde{r}$. In that case, $\tilde{w}_n(z_n) = b$ if

$$z_n \leq b + \frac{1}{\phi(1+\eta)\gamma\alpha_w(\theta)}.$$

Otherwise, $\tilde{w}_n(z_n \mid \tilde{r})$ solves the first-order condition for an interior solution involving $\tilde{w}_n > \tilde{r}$.

Case II: $b < \tilde{r}$. In that case, $\tilde{w}_n(z_n) = b$ if

$$z_{n} \leq b + \frac{1}{\gamma \alpha_{w}\left(\theta\right) \phi\left(1 + \eta \lambda\right) \left\{1 + \left(\frac{1 - \phi}{\phi}\right) \left\{\overline{F}_{Z}\left[z_{v, \ell}\left(b, \tilde{r}\right)\right] - \overline{F}_{Z}\left[z_{v, h}\left(b, \tilde{r}\right)\right]\right\}\right\}}.$$

Otherwise, $\tilde{w}_n(z_n \mid \tilde{r})$ satisfies the following property:

$$\tilde{w}_{n}(z_{n} \mid \tilde{r}) \begin{cases} <\tilde{r}, & \text{if } g(z_{n}, \tilde{r}) < g_{\ell}(\theta) \\ =\tilde{r}, & \text{if } g_{\ell}(\theta) \leq g(z_{n}, \tilde{r}) < g_{h}(\theta), \\ >\tilde{r}, & \text{if } g(z_{n}, \tilde{r}) > g_{h}(\theta) \end{cases}$$

where

$$g(z_n, \tilde{r}) = \overline{F}_Z(\tilde{r})(z_n - \tilde{r}),$$

and

$$g_{\ell}\left(\theta\right) \equiv \frac{1}{\gamma \alpha_{w}\left(\theta\right) \phi\left(1+\eta\lambda\right)}; \ g_{h}\left(\theta\right) \equiv \frac{1}{\gamma \alpha_{w}\left(\theta\right) \phi\left(1+\eta\right)}.$$

When deciding the wage offer, the incumbent weighs two forces. On the one hand, a unit increase in the wage has a marginal cost of $\rho\left(\tilde{w}_{n}\mid\tilde{r}\right)$ units of expected surplus. On the other hand, it has a marginal benefit of $\rho_{\tilde{w}_{n}}\left(\tilde{w}_{n}\mid\tilde{r}\right)\left(z_{n}-\tilde{w}_{n}\right)$ due to the increase in the retention probability. For low-productivity firms, the marginal cost of increasing the wage from b is higher than the marginal benefit, so they pay just enough so that the worker is indifferent between staying in the firm and producing at home.

For firms paying $\tilde{w}_n > b$, whether the optimal wage is above or below the reference depends on the function $g(z_n, \tilde{r})$. In particular, firms with low enough $g(z_n, \tilde{r})$ pay below the reference.

 $g\left(z_{n}, \tilde{r}\right)$ can be low for two reasons. Either the ex-post surplus at the reference, $z_{n}-\tilde{r}$, is low, or the probability of finding a new match with productivity higher than the reference, $\overline{F}_{Z}\left(\tilde{r}\right)$, is low. When $z_{n}-\tilde{r}$ is low, firms cannot afford to pay above the reference. When $\overline{F}_{Z}\left(\tilde{r}\right)$ is low, workers search with very low probability, and therefore a wage decrease has little influence in their search decisions. This property suggests that in response to a negative aggregate productivity shock, the firms that decrease their wages are at the tails of the productivity distribution. In section (4), I show that with the Pareto distribution only low-productivity firms lower their wages.

3.4 Vacancy Posting

Let $V(z_n \mid \tilde{r}(z_n))$ be the expected surplus from meeting a worker with incumbent match productivity z_n and reference \tilde{r} . It is easy to show that

$$\begin{split} V\left(z_{n}\mid\tilde{r}\right) &\equiv \mathbb{E}_{z_{v}}\left[z_{v}-\tilde{w}_{v}\left(z_{v},z_{n}\mid\tilde{r}\right)\right] \\ &= \begin{cases} \left(1-\phi\right)S\left[\tilde{w}_{n}\left(z_{n}\mid\tilde{r}\right)\right]+\phi\left\{S\left[z_{v,\ell}\left(z_{n}\mid\tilde{r}\right)\right]-S\left[z_{v,h}\left(z_{n}\mid\tilde{r}\right)\right]\right\}, & \text{if } \tilde{w}_{n}\left(z_{n}\mid\tilde{r}\right)<\tilde{r} \\ \left(1-\phi\right)S\left[\tilde{w}_{n}\left(z_{n}\mid\tilde{r}\right)\right], & \text{if } \tilde{w}_{n}\left(z_{n}\mid\tilde{r}\right)\geq\tilde{r} \end{cases}. \end{split}$$

If the incumbent pays above the reference, the entrant extracts a fraction $1-\phi$ of the option value of search, as in the benchmark without loss aversion. When the incumbent pays below the reference, the entrant extracts more surplus in expected value because the worker is willing to accept a lower wage in order to avoid an income loss.

Let *c* be the unit cost of posting a vacancy. The expected value of posting a vacancy is

$$\mathcal{V} = -c + \alpha_f(\theta) \mathbb{E}_{z_n} \left[\frac{1 - \rho \left(z_n \mid \tilde{r} \right)}{1 - \overline{\rho}} V\left(z_n \mid \tilde{r} \right) \right], \tag{7}$$

where the weights $(1-\rho)$ / $(1-\overline{\rho})$ account for the fact that workers in low-paying jobs are more likely to quit their firms.

3.5 Equilibrium

The equilibrium in this economy is defined as follows.

Definition 1. Given reference points $(\tilde{r}(z_n))_{z_n \geq b}$, an equilibrium is a wage function $\tilde{w}_n(z_n)$, a lump-sum tax function $T(z_n)$, and a labor market tightness θ such that:

- 1. Given θ and $\tilde{r}(z_n)$, $\tilde{w}_n(z_n)$ solves (6);
- 2. Given θ , $\tilde{w}_n(z_n)$, and $\tilde{r}(z_n)$, $T(z_n)$ satisfies the government's budget constraint:

$$\int T(z_n) F_Z(dz_n) = \int \left[1 - \rho(z_n)\right] \left[1 - \alpha_w(\theta) \overline{F}_Z(z_n)\right] \left[y + \tilde{w}_n(z_n)\right] F_Z(dz_n);$$

3. θ is such that V = 0.

The following Proposition establishes that under mild conditions on parameters, the equilibrium exists and is unique.

Proposition 2 (Existence and uniqueness.). *Suppose that* $c \leq (1 - \phi) \frac{b}{\tau - 1}$. *Then the equilibrium exists and is unique.*

Proof. (Sketch) As $\theta \to 0$, $\tilde{w}_n \to b$ and

$$\lim_{\theta \to 0} \alpha_f\left(\theta\right) \mathbb{E}_{z_n} \left[\frac{1 - \rho\left(z_n \mid \tilde{r}\right)}{1 - \overline{\rho}} V\left(z_n \mid \tilde{r}\right) \right] = (1 - \phi) \frac{b}{\tau - 1}.$$

By increasing differences of J in θ , \tilde{w}_n is increasing in θ . The expected surplus for an entrant is decreasing in the incumbent's wage. By monotonicity of expectation, the right-hand-side of (7) is therefore decreasing with θ . The result follows.

4 Downward Wage Rigidity

I start by establishing the properties of the equilibrium when $\lambda = 1$.

Proposition 3 (Benchmark Equilibrium). Let $\lambda = 1$, let θ^* be the equilibrium labor market tightness, $\tilde{w}_n^*(z_n)$ be the equilibrium normalized wage function. Then

$$\frac{d\theta^*}{dy}=0,$$

and

$$\frac{d\tilde{w}_n^*\left(z_n\right)}{dy}=0;\ \frac{d\tilde{w}_n^*\left(z_n\right)}{dz_n}\in\left[0,1\right).$$

It follows that equilibrium incumbent wages, $w_n^*(z_n)$, respond one-for-one to y.

Proof. (Sketch) (6) shows that when $\lambda=1$, the problem of the incumbent does not depend on y. (7) shows that the expected value to the entrant does not depend directly on y either. Therefore θ^* does not depend on y, and neither does $\tilde{w}_n^*(z_n)$. $d\tilde{w}_n^*(z_n)/dz_n$ can be obtained by totally differentiating the first-order condition for an interior solution.

This Proposition establishes that in the benchmark case, labor market tightness is invariant to aggregate productivity. Therefore, any effects of y on equilibrium θ when $\lambda > 1$ are solely due to loss aversion.

This Proposition also establishes that the equilibrium wage function, $w_n^*(z_n) = y + \tilde{w}_n^*(z_n)$ is more sensitive to aggregate than to idiosyncratic productivity. This property, which does not depend on distributional assumptions, arises due to the fact that when idiosyncratic productivity increases, the probability that the worker finds a better offer goes down. Therefore the monopsony power of the incumbent increases, and it is optimal not to increase the wage one-for-one. Figure 5 illustrates this result.

Why wages don't decrease in recessions I now present the main result. Suppose that in the past, aggregate productivity was constant and equal to y_{-1} . In that case, it is optimal for firms to set a constant wage and the reference point is irrelevant. Therefore I set

$$r\left(z_{n}\right)=y_{-1}+\tilde{w}_{n}^{*}\left(z_{n}\right).$$

Now suppose that

$$y = y_{-1} + \sigma \epsilon$$
, $\sigma > 0$,

where ϵ is drawn from a symmetric distribution with mean 0 and variance normalized to 1. Then

$$\tilde{r}(z_n) = r(z_n) - y = \tilde{w}_n^*(z_n) - \sigma \epsilon.$$

In the presence of a negative aggregate productivity shock $\epsilon < 0$, the normalized reference point rises above the benchmark normalized optimal wage. This fact implies that at $\tilde{w}_n^*(z_n) < \tilde{r}(z_n)$, workers leave the firm with a higher probability. Setting

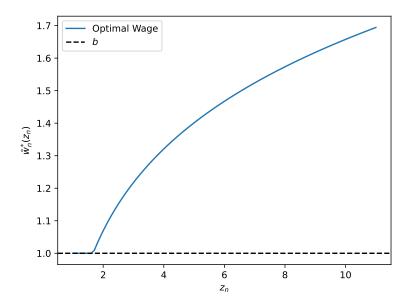


Figure 5: Optimal incumbent's normalized wage as a function of z_n when $\lambda = 1$.

 $\tilde{w}_n = \tilde{w}_n^*(z_n)$ is no longer optimal. The following Proposition summarizes these results.

Proposition 4 (Downward Wage Rigidity). Suppose $\theta = \theta^*$, and let $\tilde{w}_n^{\lambda}(z_n)$ be the optimal wage when $\lambda > 1$.

- 1. If $\epsilon > 0$, $\tilde{w}_n^{\lambda}(z_n) = \tilde{w}_n^*(z_n)$. The equilibrium wage function and labor market tightness are the same as when $\lambda = 1$;
- 2. If $\epsilon < 0$, there is a threshold \overline{z} such that

$$w_n^{\lambda}(z_n)$$
 $\begin{cases} < r(z_n), & \text{if } z_n < \overline{z} \\ = r(z_n), & \text{if } z_n > \overline{z} \end{cases}$.

Proof. (Sketch) The proof makes use of Proposition 1. If $\epsilon \geq 0$, it is easy to show that $g(z_n, \tilde{r}(z_n)) > g_h(\theta^*)$, implying that loss-aversion is irrelevant. Therefore it is optimal to set the same normalized wage, and $w_n = y + \tilde{w}_n$ responds one-for-one with productivity. If $\epsilon < 0$, it is easy to show that $g(z_n, \tilde{r}(z_n)) \leq g_h(\theta^*)$ for all firms, so that they either decrease the wage or keep it. Taking the total derivative of $g(z_n, \tilde{r}(z_n))$ with respect to g_h establishes that g_n is strictly increasing in z_n , and

takes negative values if z_n is low enough. Since $g_\ell(\theta^*)$ is strictly positive, it follows that there is a threshold \overline{z} below which firms decrease wages.

This Proposition establishes that for the same labor market tightness θ^* , firms with high-productivity matches keep their wage unchanged, while firms with low-productivity matches decrease their wages.

Recall from the discussion in subection 3.3 that there are two reasons why a firm might find it optimal to set the wage below the reference: either the ex-post surplus at the reference is low, or the probability of finding a better offer at the reference is low. The Pareto distribution has fat tails. Therefore, as z_n increases, the probability of finding a better offer does not decrease fast enough to make it worthwhile for firms with high-productivity matches to decrease their wages. The model implies that consistent with Carneiro et al. (2012), low-paying jobs are more likely to be destroyed in a recession.

Asymmetry in the Response of Macroeconomic Variables While the previous Proposition relates to the partial equilibrium effect of an aggregate productivity shock, the model implies that θ responds asymmetrically to negative and positive shocks. As established in Proposition 4, equilibrium tightness is unchanged in the event of a positive shock. However, if $\epsilon < 0$, a fraction of firms fully absorb the shock.

If the shock is small, most incumbents absorb the shock. Because entrants need to pay a high enough wage to attract the worker, entrants also need to (at least partially) absorb the shock. The expected surplus from posting a vacancy is low, and labor market tightness must decrease in equilibrium.

As the size of the shock increases, more and more incumbent decrease their wages. The probability that an entrant needs to match a low outside offer increases and, from Lemma 1, so does the probability that an entrant is able to extract more surplus from the worker by paying above the reference. For large enough shocks, the expected surplus from posting a vacancy increase, and labor market tightness also increases in equilibrium.

These results are summarized in Figure 6.

Aggregate vs. Idiosyncratic Shocks Suppose now that the reference point of a particular firm is

$$\tilde{r}\left(z_{n}\right)=\tilde{w}^{*}\left(z_{n}-\sigma\epsilon\right).$$

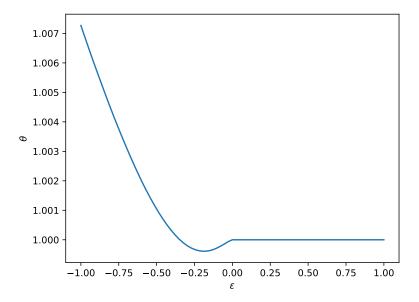


Figure 6: Equilibrium tightness as a function of the size of the shock.

That is, the firm has a idiosyncratic shock of size $\sigma \epsilon$. Ideally, the firm would like to face the no-loss-aversion retention probability schedule and set the normalized wage equal to $\tilde{w}^*(z_n)$. Due to the concavity of the wage schedule apparent in Figure 5, when $\epsilon > 0$,

$$\left[\tilde{w}^*\left(z_n\right) - \sigma\epsilon\right] - \tilde{w}^*\left(z_n\right) > \tilde{w}^*\left(z_n - \sigma\epsilon\right) - \tilde{w}^*\left(z_n\right).$$

This inequality says that the distance of the reference point to the ideal wage is higher with an aggregate than with an idiosyncratic shock. Therefore the firm is less reluctant to keep the wage in the event of an idiosyncratic shock.

Figure 7 shows, as a function of z_n and for the same labor market tightness, the optimal wage change in response to an idiosyncratic and to an aggregate shock, with and without loss aversion. Both with and without loss aversion, the wage is more responsive to aggregate than to productivity shocks (conditional on a wage change). With loss aversion, however, the passthrough is significantly reduced for both types of shocks, as the wage is anchored to the reference.

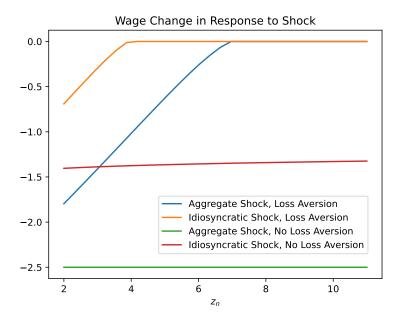


Figure 7: Optimal wage change in response to an idiosyncratic and an aggregate shock of the same amount.

5 Discussion

In this section, I discuss how this model differs from alternative theories of downward wage rigidity, as well as other issues.

Relation to Alternative Theories

Related Models With Loss Aversion This paper is most related to Eliaz and Spiegler (2014). In this model, the output of the firm is reduced to an exogenous fraction γ of aggregate productivity whenever the wage goes below the reference. Therefore it speaks more to the literature on efficiency wages than on the literature on loss aversion. Moreover, the paper does not draw any conclusions for the heterogeneous passthrough of idiosyncratic vs. aggregate shocks, nor on how the passthrough differs across firms. Finally, wages of new hires are still responsive to aggregate productivity, which is at odds with the stylized fact that many jobswitchers also have fully rigid wages. Another paper that uses loss aversion in the context of labor search is DellaVigna et al. (2017). Here, workers have a utility function that is similar to the one I use in this paper, but there is no on-the-job search

or an endogenous quitting decision. The focus is rather on the timing effects of unemployment benefits, and therefore the paper does not draw any conclusions for downward wage rigidity.

Exogenous Downward Wage Rigidity Downward wage rigidity is commonly incorporated in macroeconomic models by assuming exogenously that wages cannot decrease (Schmitt-Grohé and Uribe (2016)). This approach is in general unable to account for how, conditional on a negative aggregate productivity shock, the probability of a wage decrease and the passthrough to wages may depend on the size of the shock. Moreover, the model I propose in this paper has the potential to lead to different policy conclusions because preferences depend explicitly on labor income losses.

Staggered Wage Setting Another popular alternative in modeling wage rigidity is by assuming that in any period, wages can only change with a given probability, as in Erceg et al. (2000) or Gertler and Trigari (2009). Unlike in the present paper, these models imply that the response of wages to shocks is symmetric to first-order, and that the probability of a wage change is time-dependent only.

The Firm as an Insurance Provider Azariadis (1975) starts a long body of work that models risk-neutral firms as insurance-providers to risk-averse workers. In general, this type of models is able to generate a muted response of wages to productivity shocks, but has more difficulty in generating full wage rigidity.

Nominal vs. Real Wages In the data, it is not clear whether real wages are more flexible than nominal wages (see Card and Hyslop (1997) for an example of a paper that answers in the affirmative, and Hazel and Taska (2024) for an example of a paper that answers in the negative). However, this model is easily able to incorporate a nominal rigidity by assuming that the worker is concerned with gains and losses in nominal labor income. In that case, the reference point can be written as

$$\tilde{r}\left(z_{n}\right)=\tilde{w}_{n}^{*}\left(z_{n}\right)-\sigma\epsilon-\pi,$$

where π is the inflation rate. Inflation mitigates the effects of downward wage rigidity, in the sense that only large enough negative productivity shocks, $-\sigma\epsilon > \pi$,

represent a constraint to the firm. Otherwise, the firm is able to grant a small enough nominal wage increase while reducing the real wage to the desired level. However, in order to do a full analysis of how price and wage rigidities interact, this model needs to be expanded to include the goods market explicitly.

Dynamics In this paper, the initial conditions I have assumed imply that wages still respond one-for-one to positive aggregate productivity shocks. If firms were forward-looking, it would be natural to expect that the response of wages to positive shocks would be more muted. If productivity increases, a firm might be reluctant to increase wages due to fear of reducing them in the future. Therefore, introducing dynamics could bring the model closer to the data, as the probability that wages increase in expansions is not 100%.

Choice of Reference Point As in DellaVigna et al. (2017), I have assumed that the reference point corresponds to past wages. There is some evidence that in the context of labor supply, the reference point also has a forward-looking component (Camerer et al. (1997)), and depends on hours worked (Crawford and Meng (2011)). Incorporating these components in the reference point has the potential to mitigate wage rigidity. However, if hours worked are stable, and if the reference point in period t does not vary with the productivity shock of period t, wages will still be rigid in the sense that they will not respond to aggregate productivity shocks (Eliaz and Spiegler (2014)).

6 Conclusion

In this paper, I have developed a static labor search model in which workers are averse to income losses, and decide whether to quit their current firm and look for a new job. Consistent with the data, the model implies that (1) wages are downward rigid in recessions, (2) labor market tightness responds asymmetrically to negative and positive shocks, and (3) wages respond more to aggregate than idiosyncratic shocks, but the passthrough is partial.

As I discussed in Section 5, the model would profit the most from the introduction of dynamics and of the goods market.

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