Modelos de Mortalidad, aplicación en R

Riskcenter

Universidad de Barcelona

http://www.ub.edu/riskcenter

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Base de datos

Los datos usados en este caso práctico han sido obtenidos de:

- `www.mortality.org`

o bien de

- `www.ine.es`

Datos para la exposición al riesgo de www.mortality.org

Spain, Exposure to risk (period 1x1)
Last modified: 11-Feb-2014, MPv5 (May07)

Year	Age	Female	Male	Total
1908	0	293136.40	305312.99	598449.39
1908	1	265517.70	269814.83	535332.53
1908	2	249101.89	253585.43	502687.32
1908	3	243622.12	247645.45	491267.57
1908	4	243224.34	245105.81	488330.15
1908	5	241190.99	244505.48	485696.48
1908	6	228430.37	232363.62	460794.00
1908	7	225766.30	227997.71	453764.01
.				
.				
1908	109	0.33	0.00	0.33
1908	110+	0.04	0.00	0.04
.				
.				
.				
2012	107	49.58	9.95	59.53
2012	108	21.54	5.30	26.84
2012	109	9.78	2.11	11.90
2012	110+	4.04	3.60	7.64

Tasas brutas de mortalidad www.mortality.org

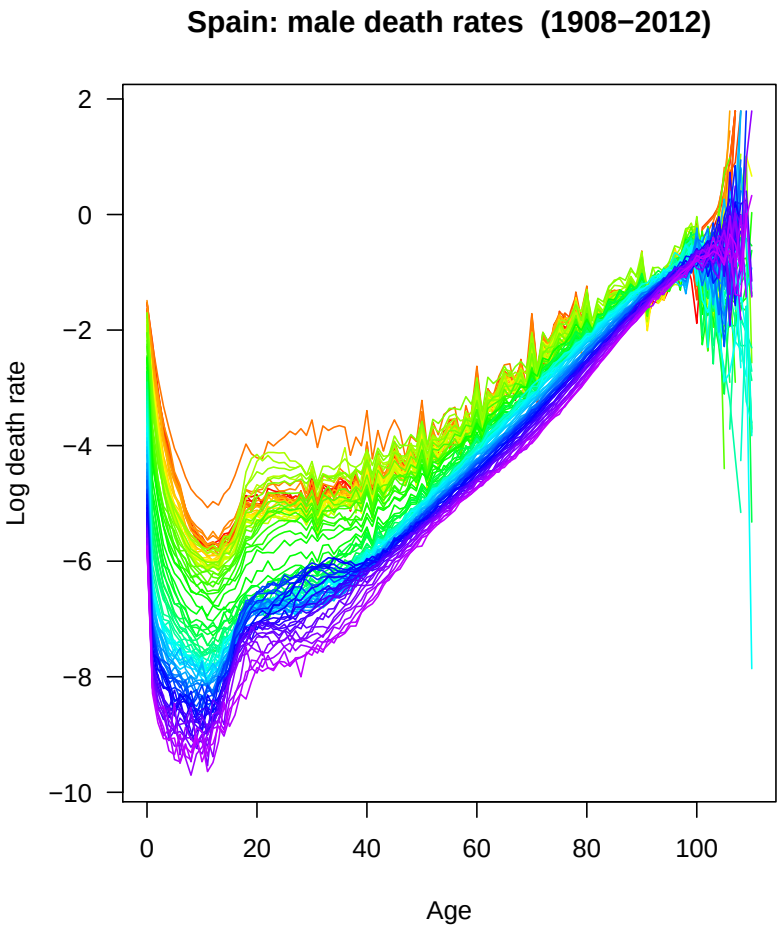
Spain, Death rates (period 1x1)
Last modified: 11-Feb-2014, MPv5 (May07)

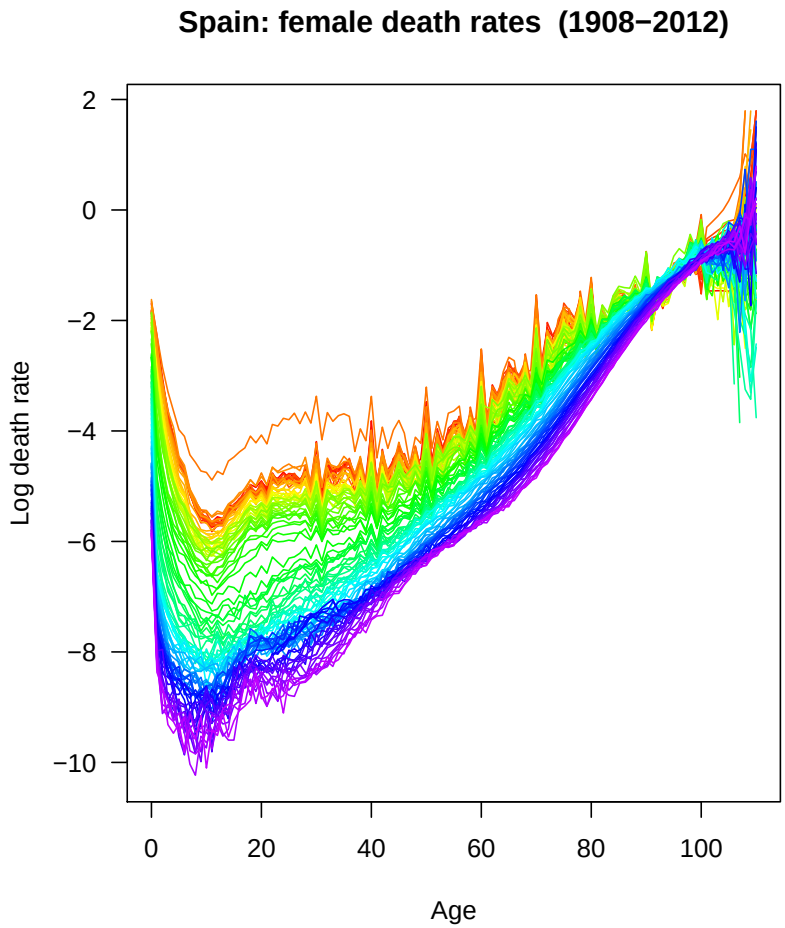
Year	Age	Female	Male	Total
1908	0	0.161112	0.190865	0.176291
1908	1	0.082262	0.085870	0.084081
1908	2	0.045397	0.045655	0.045527
1908	3	0.024595	0.025669	0.025137
1908	4	0.016370	0.016680	0.016526
1908	5	0.011140	0.010852	0.010995
1908	6	0.009253	0.008692	0.008970
1908	7	0.006117	0.005696	0.005905
.				
.				
1908	109	2.467005	.	2.467005
1908	110+	6.000000	.	6.000000
.				
.				
2012	107	0.746344	0.603015	0.722386
2012	108	0.464360	0.377003	0.447094
2012	109	0.919932	0.946372	0.924629
2012	110+	1.238646	1.388889	1.309472

Lectura de los datos y gráfico

```
#install.packages('demography')
library(demography)

#Directamente de la web a R
spain <- hmd.mx("ESP", 'usuario', 'password', "Spain")

#Bajando previamente el *.txt
spain<-read.demogdata("Mx_1x1SPAIN.txt",
    "Exposures_1x1.txt", type="mortality", label="Spain",
    skip = 2, popskip = 2)

#Gráfico de las tasas brutas de mortalidad por sexo
par(mfrow=c(1,2),las=1)
plot(spain,series="male", plot.type="functions")
plot(spain,series="female", plot.type="functions")
```

Gráficos de tasa brutas

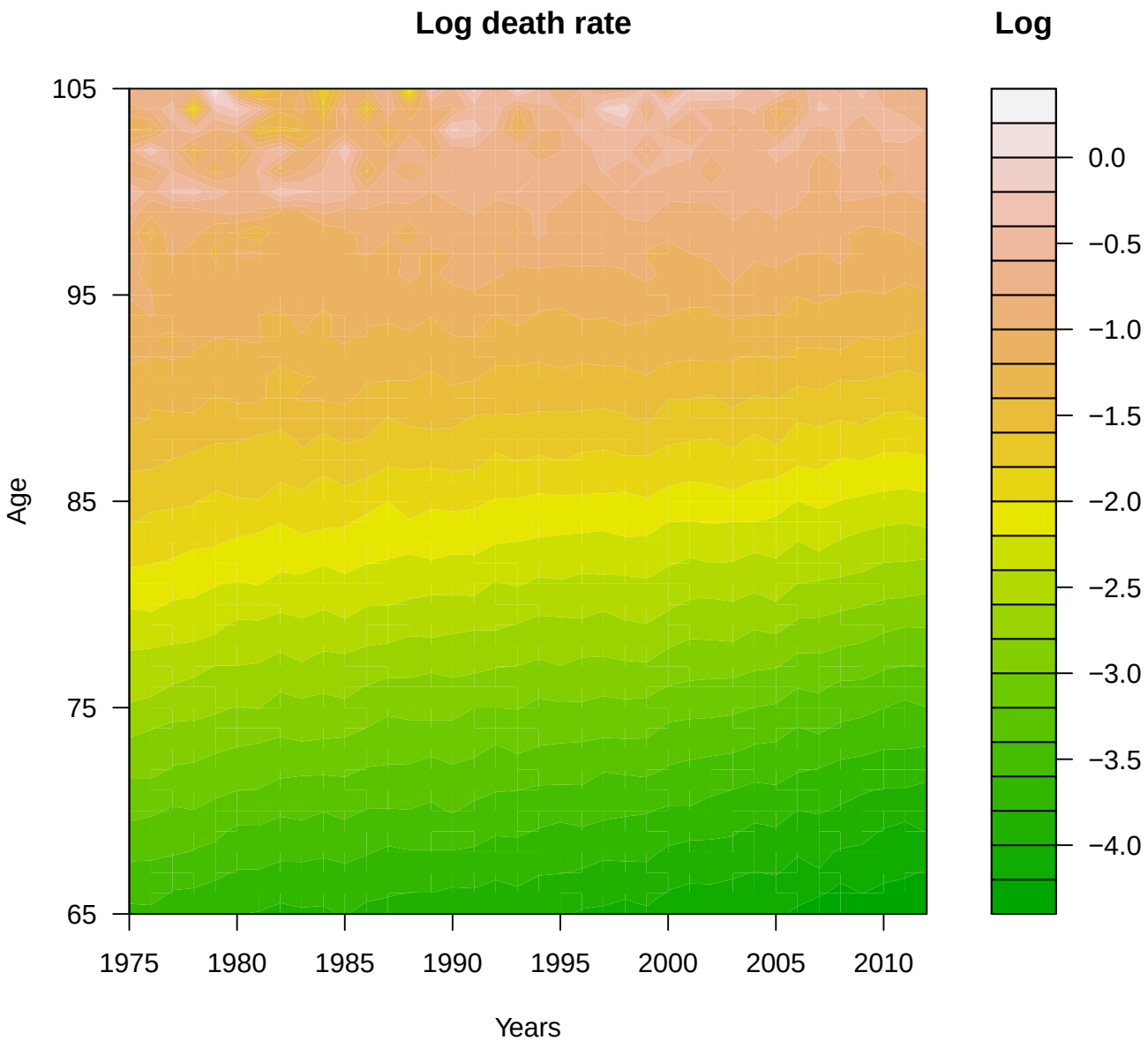




Subset por edad y año

```
#Seleccionando poblaciòn con edad mayor o igual a 1965 y
    tasas de 1975 a 2012
spain.mas65 <- extract.ages(spain,65:105,FALSE)
spain.mas65.desde1975 <-
    extract.years(spain.mas65,1975:2012)

#Otra manera de ver el Log de las tasas brutas
year<-c(1975:2012)      #years
age<-c(65:105)      #ages
z<-log(t(as.matrix(spain.mas65.desde1975$rate$male)))

#Gráfico del  logarítmo de las tasas de mortalidad
par(las=1)
filled.contour(year, age, z, color = terrain.colors,
    nlevels=15,
    plot.title = title(main = "Log death rate ",
    xlab = "Years", ylab = "Age"),
    plot.axes = { axis(1, seq(year[1], max(year), by = 5))
                  axis(2, seq(min(age), max(age), by = 10))
                      },
    key.title = title(main="Log"),
    key.axes = axis(4, seq(-10, 1, by = 0.5)))
```

Gráficos de tasa brutas

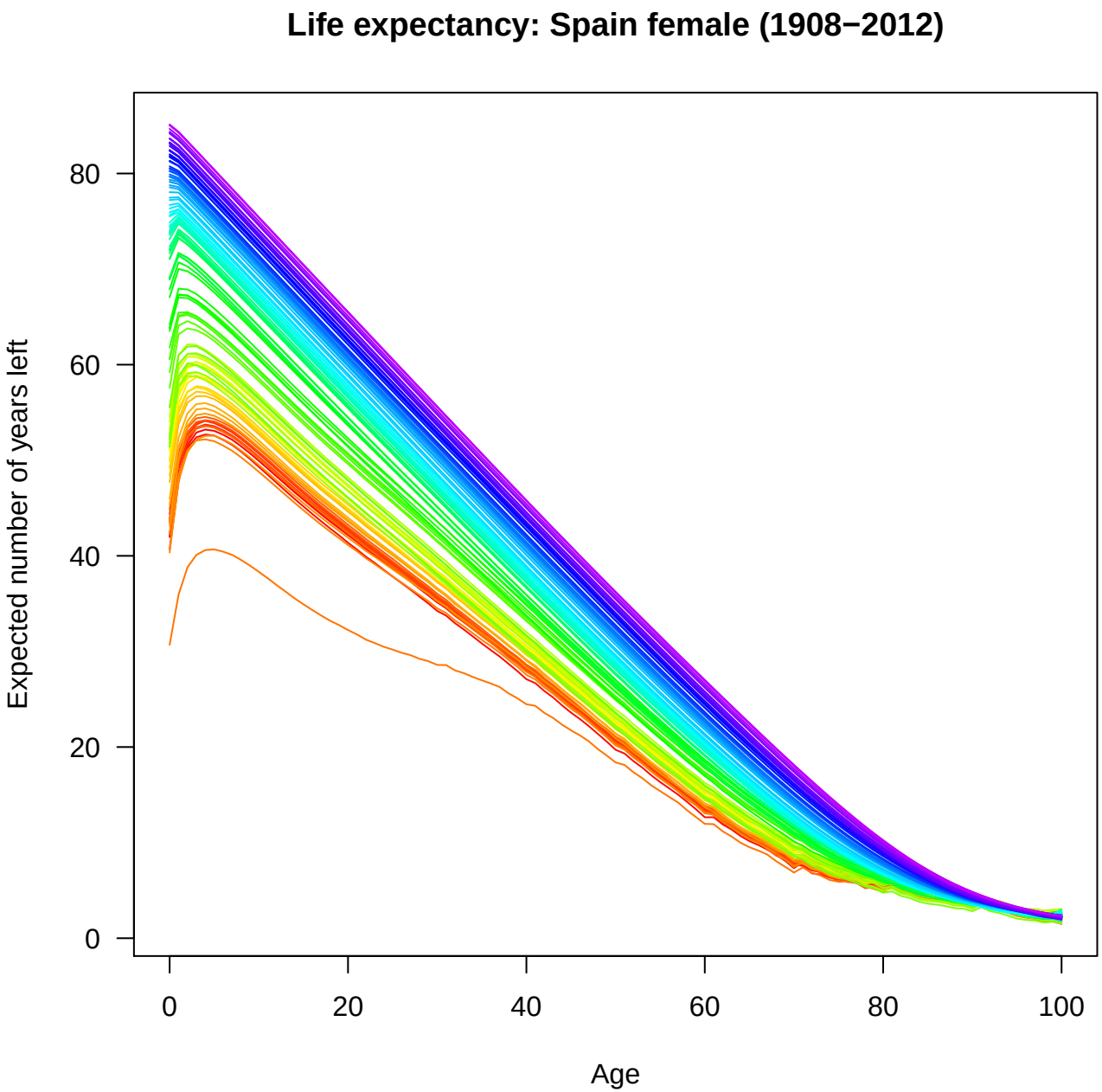


Tasas brutas de mortalidad www.mortality.org

```
#Life table
lifetable(spain, series="male", year= 2012)

par(las=1)
```

```
Period lifetable for Spain : male          Year: 2012
         mx      qx      lx      dx      Lx       Tx       ex
0    0.0032  0.0032  1.0000  0.0032  0.9970  79.3311  79.3311
1    0.0003  0.0003  0.9968  0.0003  0.9967  78.3341  78.5838
2    0.0002  0.0002  0.9966  0.0002  0.9965  77.3374  77.6034
3    0.0001  0.0001  0.9964  0.0001  0.9964  76.3409  76.6150
4    0.0001  0.0001  0.9963  0.0001  0.9963  75.3445  75.6238
5    0.0001  0.0001  0.9962  0.0001  0.9961  74.3483  74.6323
.
.
98   0.3880  0.3250  0.0244  0.0079  0.0204   0.0559   2.2946
99   0.4183  0.3460  0.0164  0.0057  0.0136   0.0355   2.1585
100  0.4912  1.0000  0.0108  0.0108  0.0219   0.0219   2.0358
```

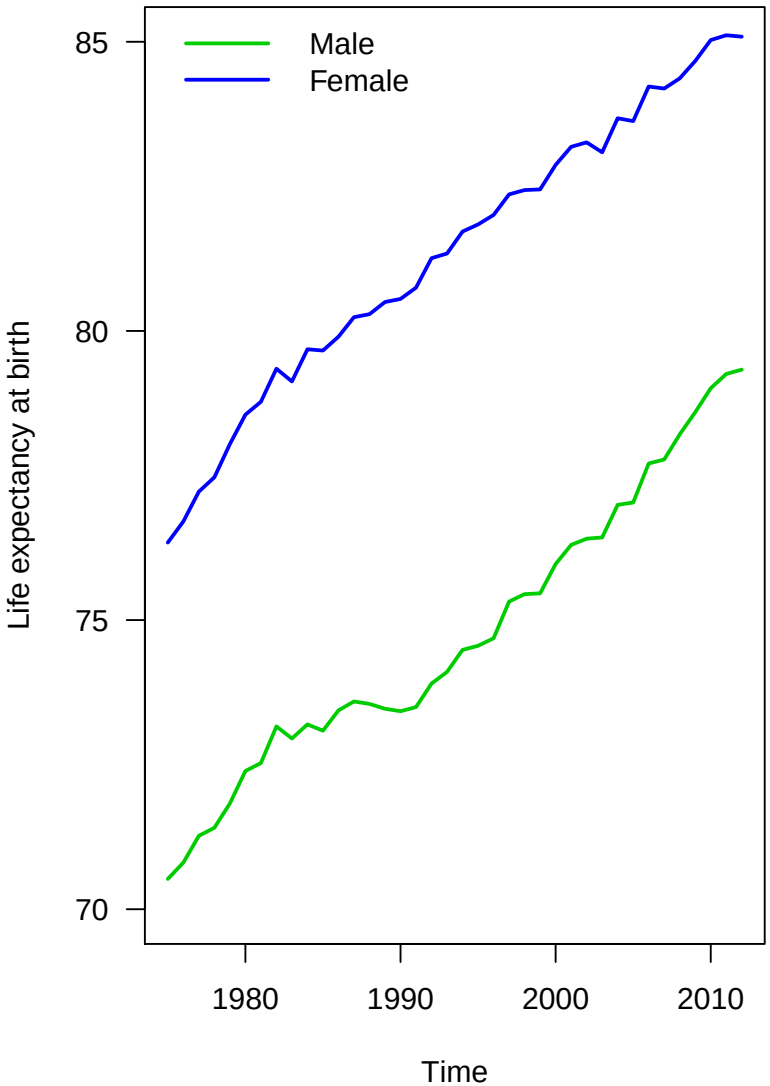
Esperanza de vida

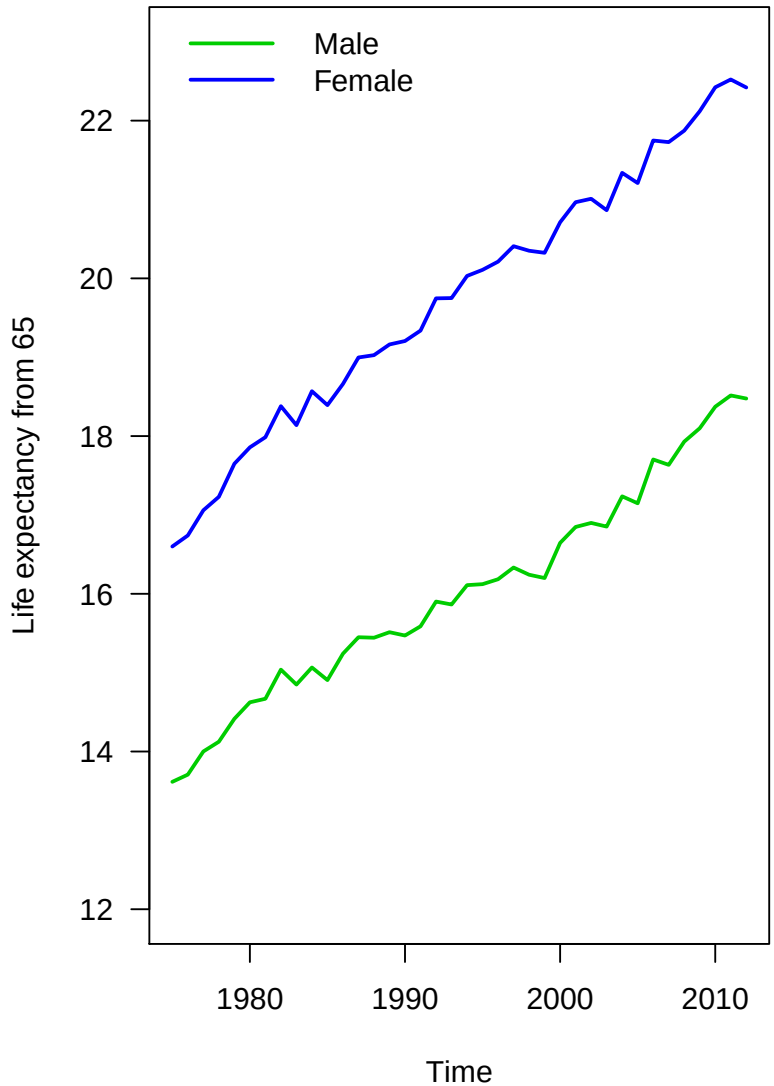


Esperanza de vida

```
###caso alejado del resto
plot(lifetable(spain, series="female", year=1918))
    #pandemia de gripe en 1908


#Esperanza de Vida desde el nacimiento
par(mfrow=c(1,2), las=1)
plot(life.expectancy(spain.desde1975,
    series="male"),ylab="Life expectancy at birth", col=3,
    lwd=2, ylim=c(70,85))
lines(life.expectancy(spain.desde1975, series="female"),
    col=4, lwd=2)
```

Gráficos de tasa brutas





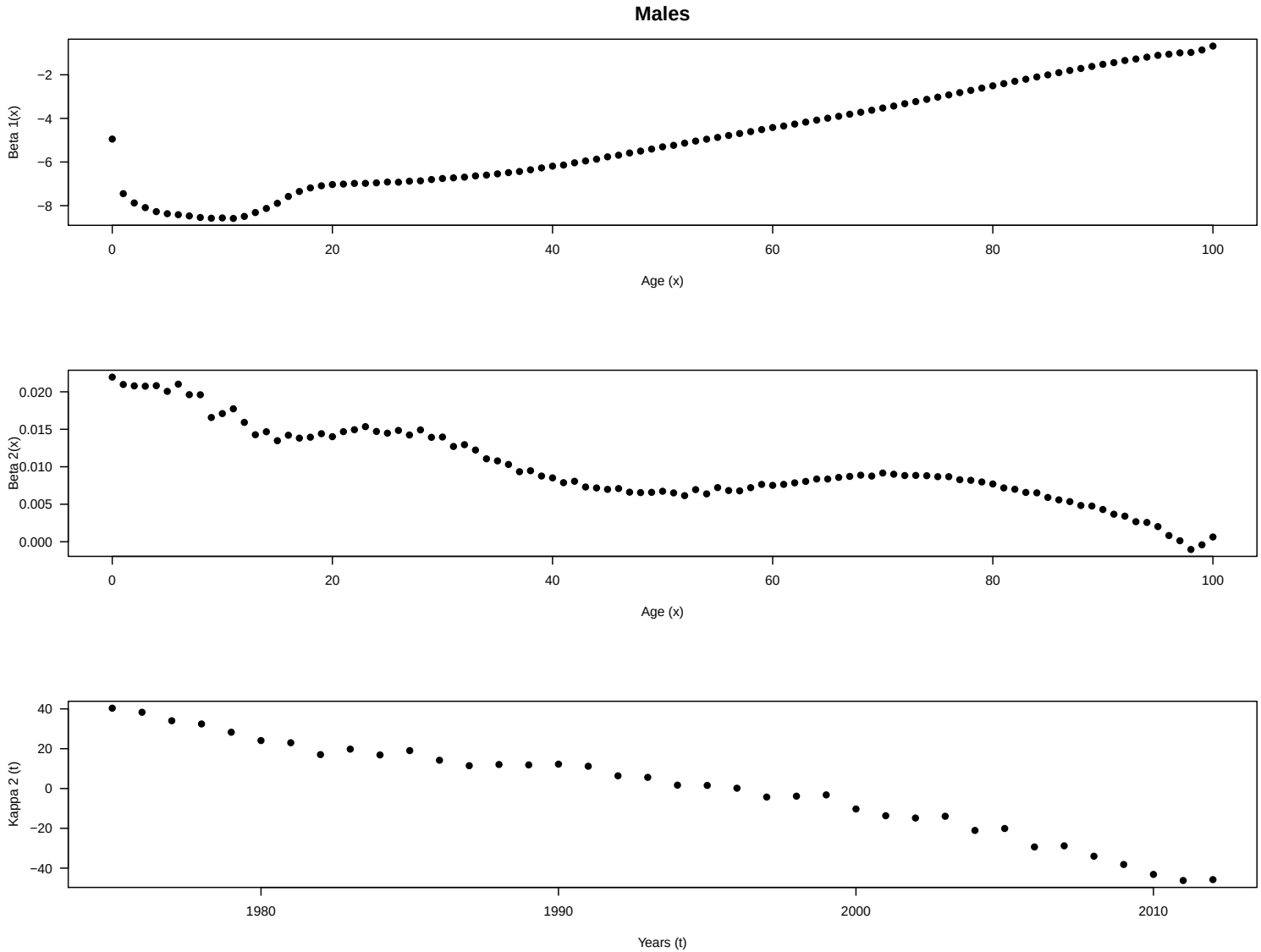
Modelo Lee-Carter

```
#install.packages('demography')
library(demography)

#Directamente de la web a R
spain <- hmd.mx("ESP", 'usuario', 'password', "Spain")

#Bajando previamente el *.txt
spain<-read.demogdata("Mx_1x1SPAIN.txt",
    "Exposures_1x1.txt", type="mortality", label="Spain",
    skip = 2, popskip = 2)

#Gráfico de las tasas brutas de mortalidad por sexo
par(mfrow=c(1,2),las=1)
plot(spain,series="male", plot.type="functions")
plot(spain,series="female", plot.type="functions")
```

Modelo Lee-Carter - Parámetros estimados

```
#Seleccionando tasas de 1975 a  2012
spain.desde1975 <- extract.years(spain,1975:2012)


#Modelo Lee Carter
spain.desde1975.LC<-lca(spain.desde1975,
    series="male",adjust="dt", max.age=105)
```

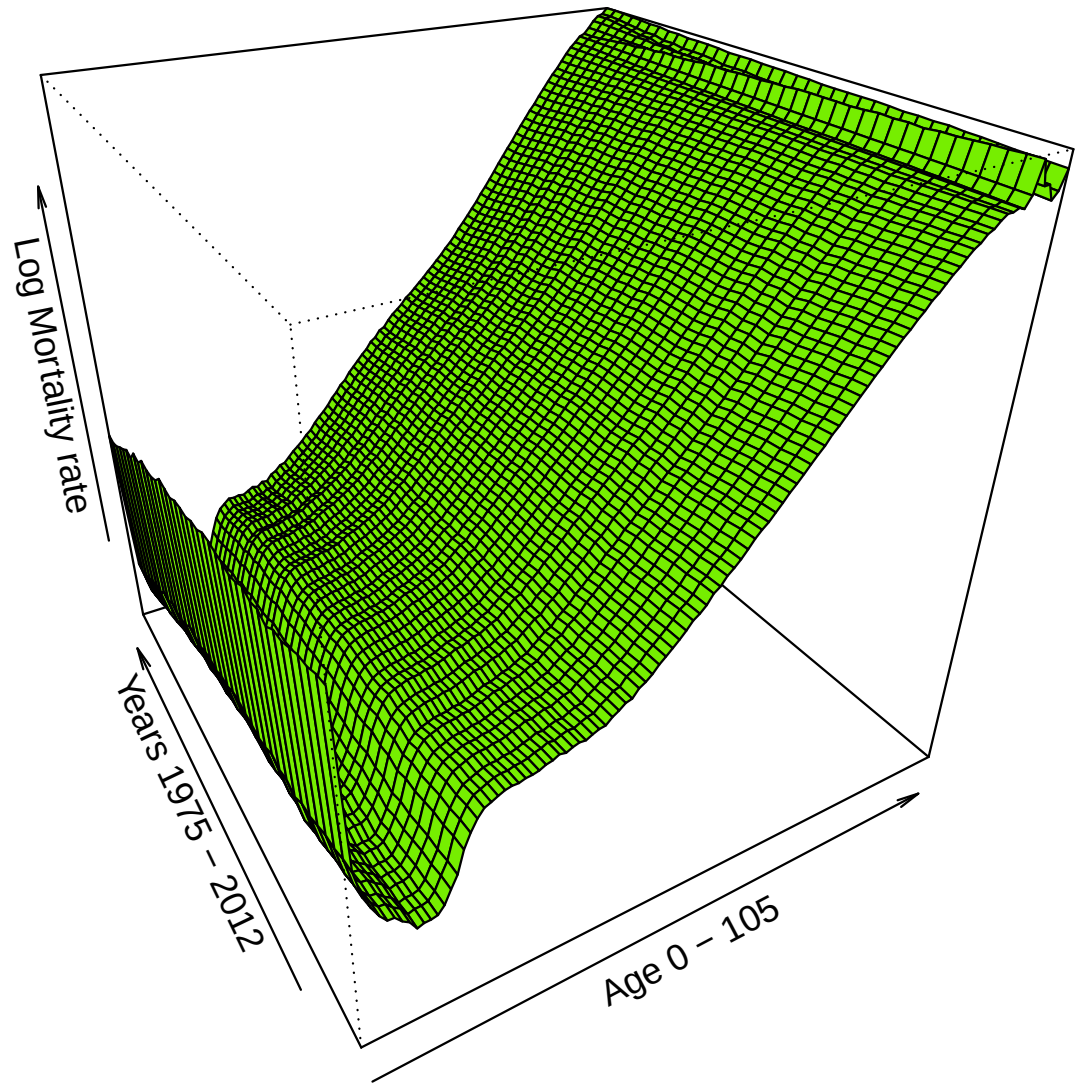
Modelo Lee-Carter - Parámetros estimados



Modelo Lee-Carter - Tasas de Mortalidad estimadas

```
#"LABEL" "AGE" "YEAR" "MALE" "AX" "BX" "KT" "RESIDUALS"
    "FITTED" "VARPROP" "Y" "MDEV" "CALL" "ADJUST" "TYPE"

#PARÁMETROS ESTIMADOS
spain.desde1975.LC$ax
```

Modelo Lee-Carter - Tasas de Mortalidad estimadas

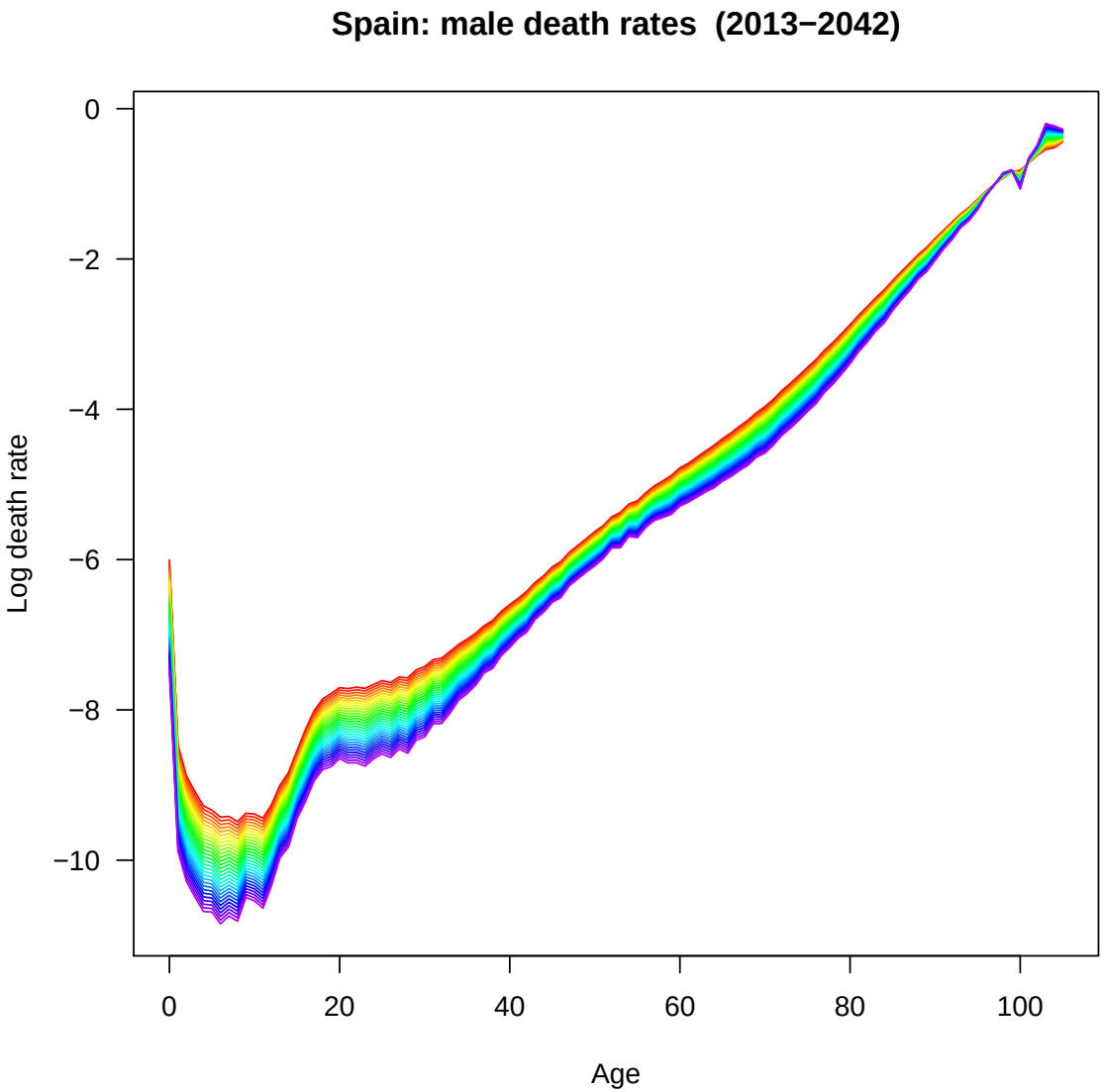


Modelo Lee-Carter - Forecasts

```
spain.desde1975.LC$kt

Forecasts for Spain
Data type: mortality

  Call: forecast.lca(object = spain.desde1975.LC, h = 30)
  Based on model: lca(data = spain.desde1975, series =
      "male", max.age = 105, adjust = "dt")
  Adjustment method: dt
  Jump-off method: fit

    Years: 2013 - 2042
    Ages:   0 - 105
```

Modelo Lee-Carter - Tasas de Mortalidad estimadas

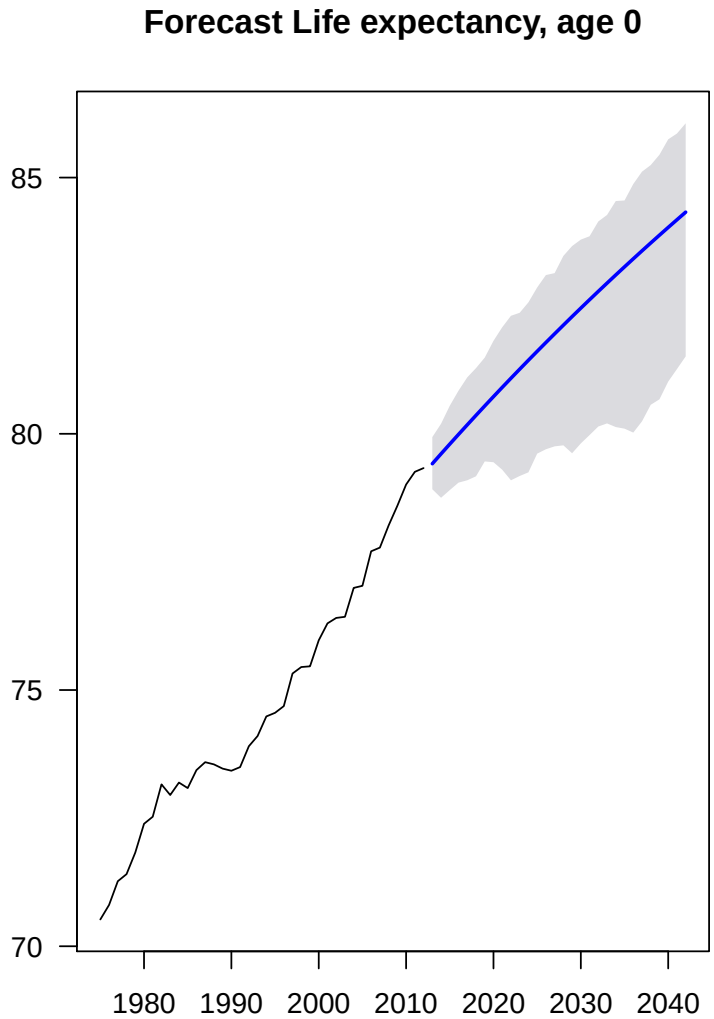
```
spain.desde1975.LC$residuals$y
```

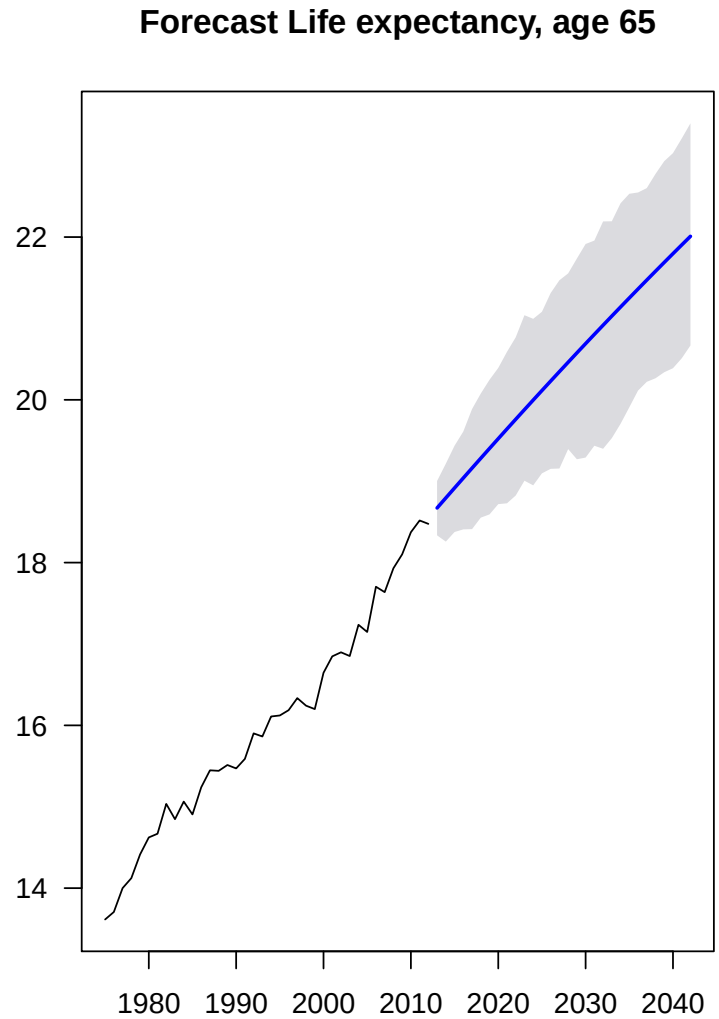


Predicción Lee-Carter de la esperanza de vida

```
op<-par(mfrow=c(3,1), las=1)
 plot(spain.desde1975.LC$age, spain.desde1975.LC$ax,
      xlab='Age (x)', ylab='Beta 1(x)', pch=19)
 plot(spain.desde1975.LC$age, spain.desde1975.LC$bx,
      xlab='Age (x)', ylab='Beta 2(x)', pch=19)
 plot(spain.desde1975.LC$year, spain.desde1975.LC$kt,
      xlab='Years (t)', ylab='Kappa 2 (t)', pch=19)
par(op)
 mtext('Males', side=3, line=2, font=2)
```

Predicción Lee-Carter de la esperanza de vida

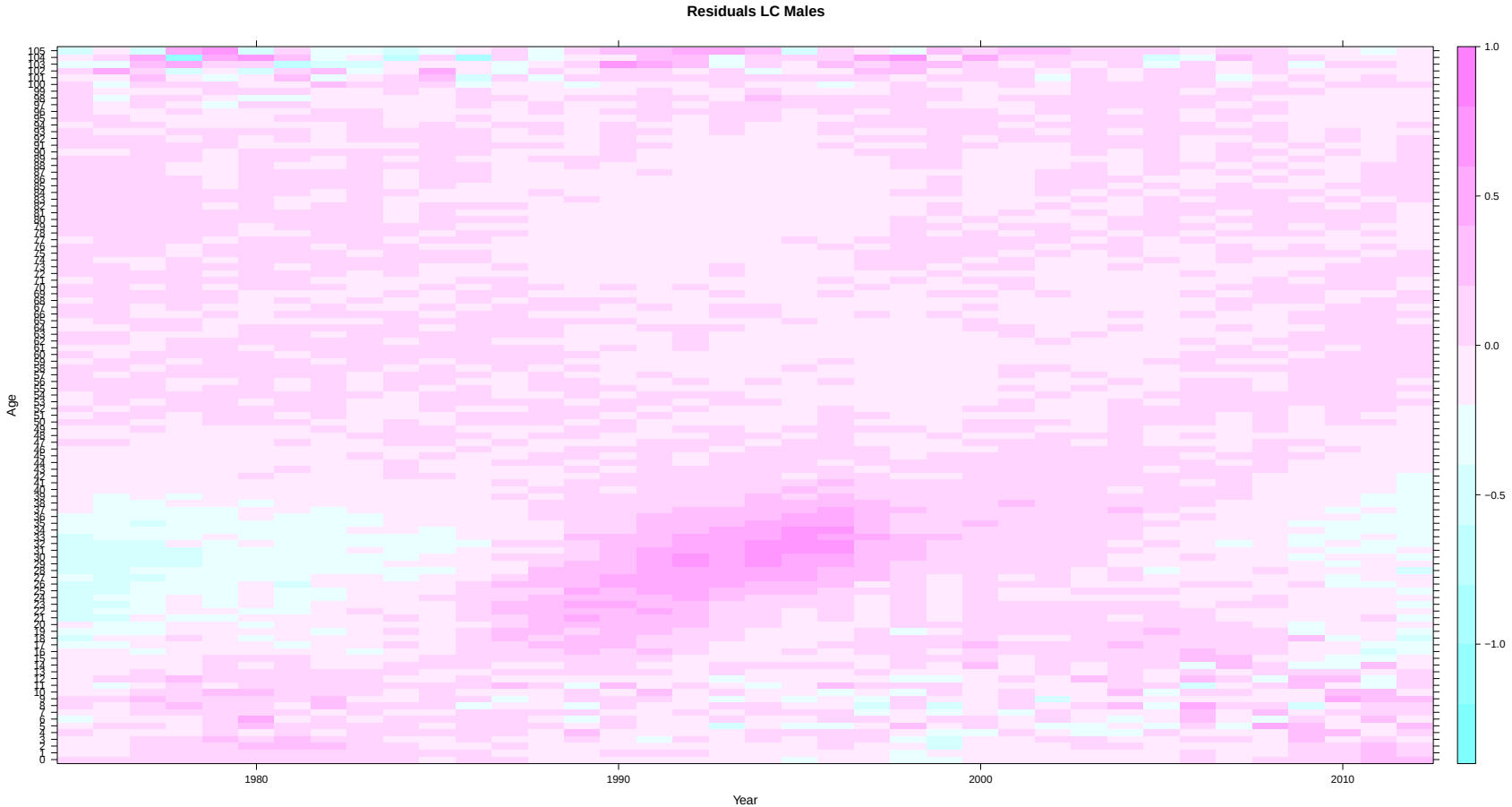




Modelo Lee-Carter - Residuales

```
persp(spain.desde1975.LC$age, spain.desde1975.LC$year,
    spain.desde1975.LC$fitted$y, , phi=30, theta=-30, col
    ="chartreuse2",
      main=paste('Log Mortality rates LC, Males', min(year)
          , '-', max(year)),
      xlab=paste('Age',0,'-',105),
          ylab=paste('Years',min(year),'-',max(year)),
          zlab='Log Mortality rate')

#Forecasts a 50 años
spain.desde1975.LC.fcast<-forecast(spain.desde1975.LC,
    h=30, level=95)
```

Modelo Lee-Carter - Residuales



LifeMetrics Open Source R code for Stochastic Mortality Modelling

http://www.macs.hw.ac.uk/~andrewc/lifemetrics/

```
#Cargando Lifemetrics
source("fitModels.r")
```

Funciones R:

a) fit701 ← Lee-Carter (LC) ← M1
b) fit702 ← Renshaw-Haberman (RH) ← M2
c) fit703 ← Currie Age-Period-Cohort (APC) ← M3
d) fit705 ← Cairns-Blake-Dowd (CBD) ← M5
e) fit706 ← 1ra Generalización CBD ← M6
f) fit707 ← 2da Generalización CBD ← M7
g) fit708 ← 3ra Generalización CBD ← M8

Para hacer simulaciones de los modelos LC y CDB:

```
source("simModels.r")
```

LifeMetrics Open Source R code for Stochastic Mortality Modelling

Todos las funciones necesitan los siguientes argumentos:

```
# Inputs:
#    xv = vector of ages, length n
#    yv = vector of years, length m
#    etx = m x n matrix of exposures
#    dtx = m x n matrix of deaths
#    wa = m x n matrix of weights (0 or 1)
```

Internamente la función calcula:

```
mtx=dtx/etx    # matrix of death rates
qtx=1-exp(-mtx) # matrix of mortality rates
```

LifeMetrics Open Source R code for Stochastic Mortality Modelling

Usando los datos de España de 1975 a 2012 y edades de 0 a 105, se tiene:

```
#Argumentos de las Funciones
age<-spain.1975.0.105$age
year<-spain.1975.0.105$year
etx<-t(spain.1975.0.105$pop$male)
dtx<-t(spain.1975.0.105$pop$male*spain.1975.0.105$rate$male)
wa<-etx*0+1
```

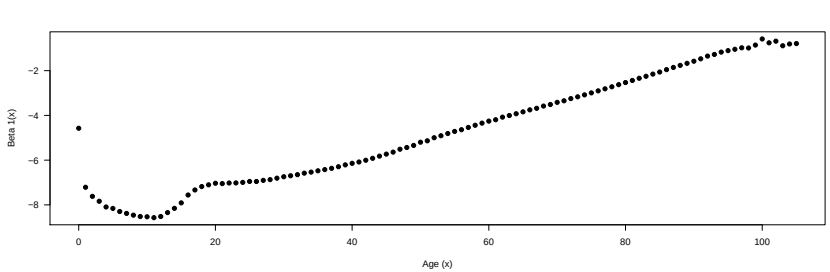
Modelo Renshaw-Haberman

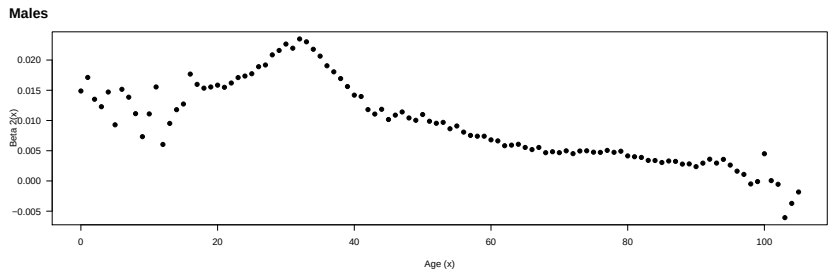
```
############### Modelo M2 - Renshaw-Haberman ##############
mod.M2<-fit702(age,year,etx,dtx,wa)

names(mod.M2)
#"beta1" "beta2" "beta3" "kappa2" "gamma3" "x" "y" "cy"
    "wa" "epsilon" "mhat" "ll" "BIC" "npar"

#Parámetros estimados
mod.M2$beta1
mod.M2$beta2
mod.M2$beta3
mod.M2$kappa2
mod.M2$gamma3

#Tasas estimadas y residuales
mod.M2$mhat
```

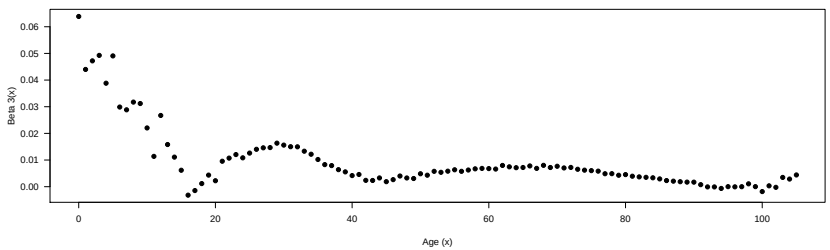
Modelo Renshaw-Haberman - Parámetros estimados

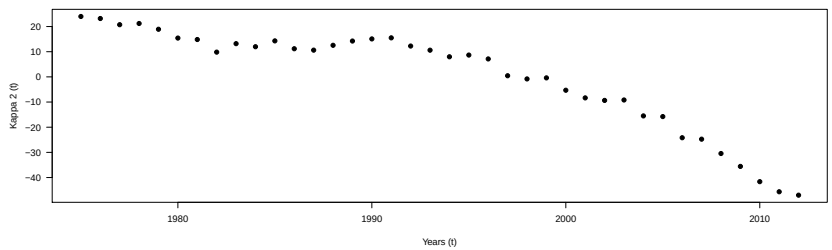
```
#Gráfico de los parámetros estimados
op<-par(mfrow=c(3,2), las=1)
 plot(mod.M2$x, mod.M2$beta1, xlab='Age (x)', ylab='Beta
     1(x)', pch=19)
 plot(mod.M2$x, mod.M2$beta2, xlab='Age (x)', ylab='Beta
     2(x)', pch=19)
 plot(mod.M2$x, mod.M2$beta3, xlab='Age (x)', ylab='Beta
     3(x)', pch=19)
 plot(mod.M2$y, mod.M2$kappa2, xlab='Years (t)',
     ylab='Kappa 2 (t)', pch=19)
 plot(mod.M2$cy, mod.M2$gamma3, xlab='Years', ylab='Gamma 3
     (t)', pch=19)
par(op)
```

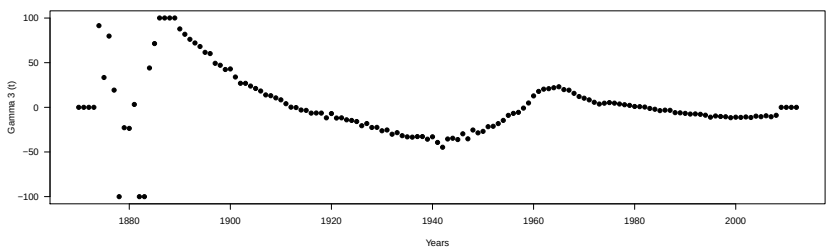
Modelo Renshaw-Haberman - Parámetros estimados







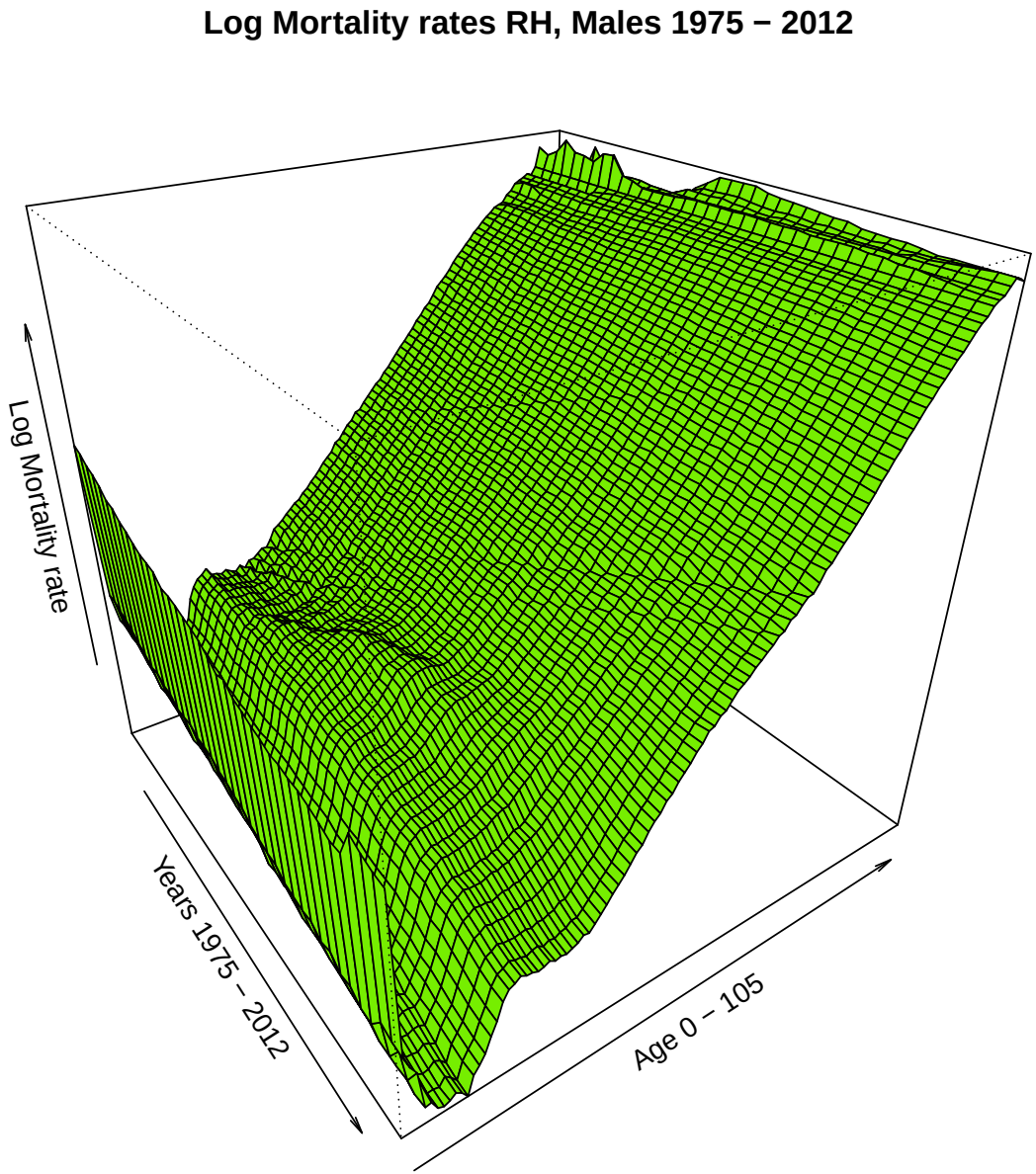




Modelo Renshaw-Haberman - Tasas de Mortalidad estimadas

```
#Gráfico de las Tasas Estimadas
persp(mod.M2$y, mod.M2$x, log(mod.M2$mhat), phi=30,
    theta=55, col ="chartreuse2",
      main=paste('Log Mortality rates RH, Males',
          min(year), '-', max(year)),
```

Modelo Renshaw-Haberman - Tasas de Mortalidad estimadas

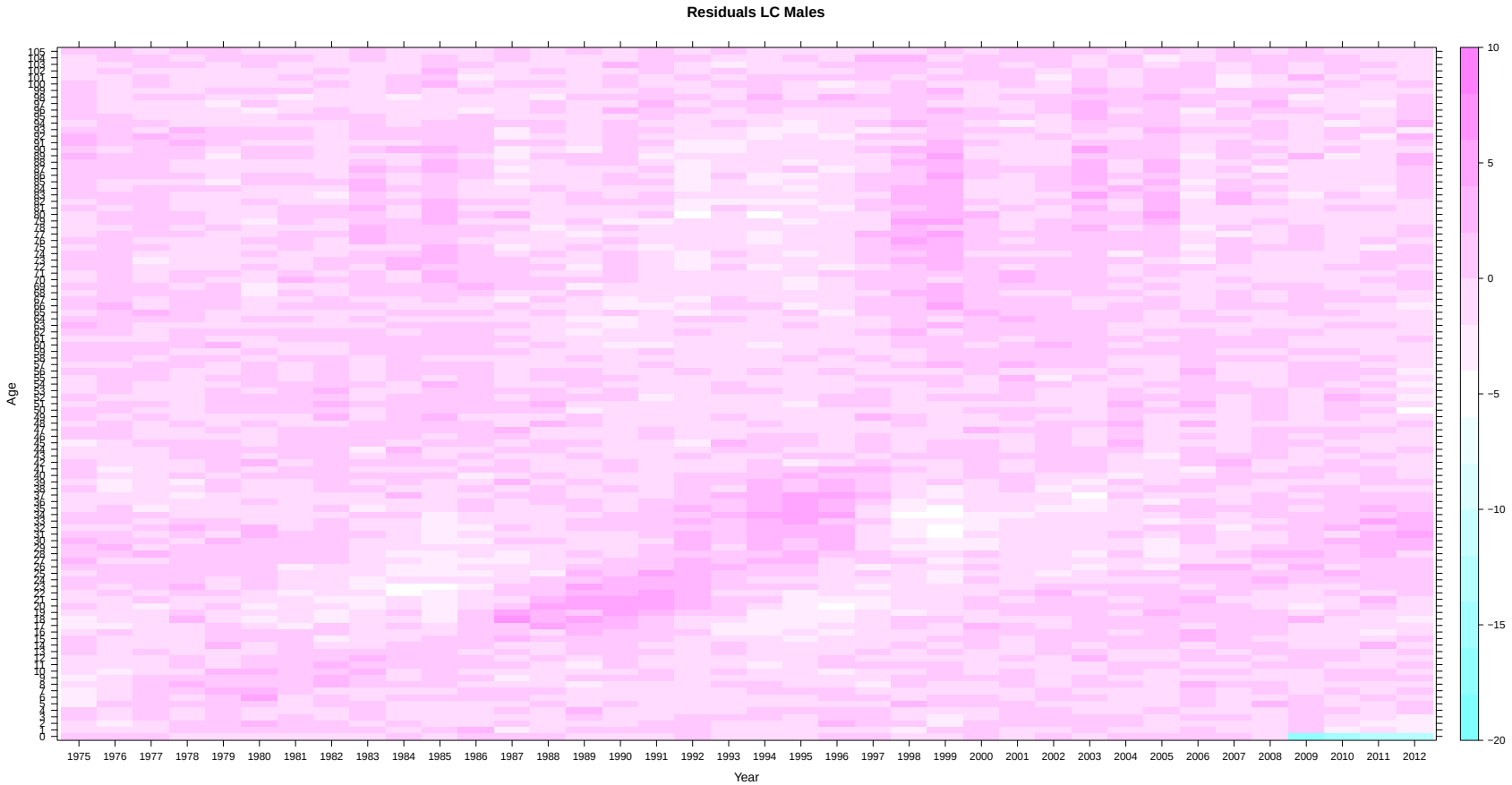


Modelo Renshaw-Haberman - Goodness of fit

```
#Residuales
levelplot(mod.M2$epsilon, colorkey=list(space='right'),
    xlab='Year', ylab='Age',
          col.regions=cm.colors, main='Residuals LC Males',
              pretty=T, aspect='fill')

#Bayes Information Criterion (BIC) & Log-likelihood (ll)
mod.M2$BIC
```

```
> mod.M2$BIC
[1] -22694.03
> mod.M2$ll
[1] -20640.76
```

Modelo Renshaw-Haberman - Residuales



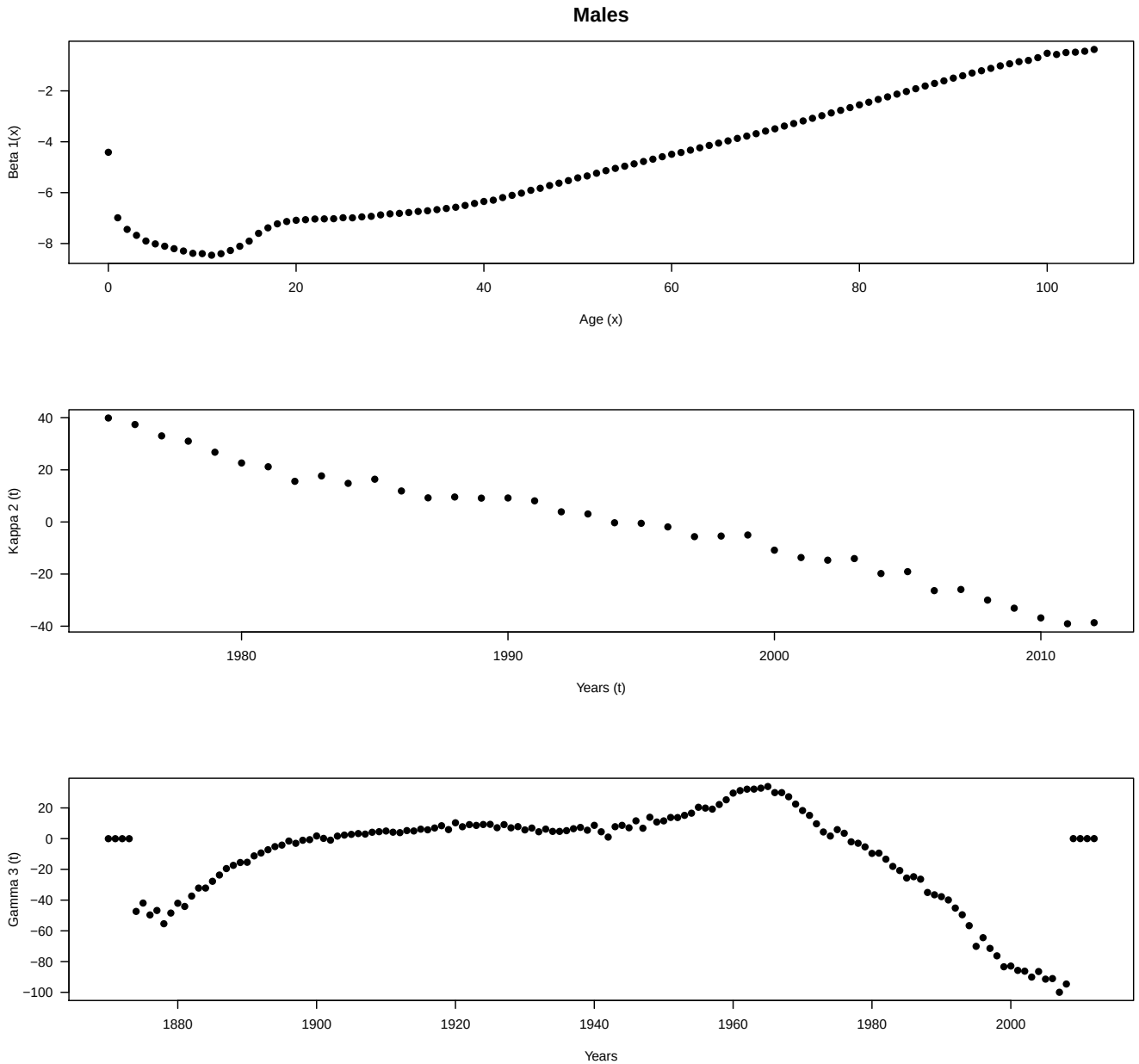
Modelo Age-Period-Cohort

```
########### Modelo M3 - Currie Age-Period-Cohort ##########
mod.M3<-fit703(age,year,etx,dtx,wa)

names(mod.M3)
#"beta1" "beta2" "beta3" "kappa2" "gamma3" "x" "y" "cy"
    "wa" "epsilon" "mhat" "ll" "BIC" "npar"

#Parámetros estimados
mod.M3$beta1
mod.M3$kappa2
mod.M3$gamma3

#Tasas estimadas y residuales
mod.M3$mhat
mod.M3$epsilon
```

Modelo Age-Period-Cohort - Parámetros estimados

```
#Gráfico de los parámetros estimados
op<-par(mfrow=c(3,1), las=1)
 plot(mod.M3$x, mod.M3$beta1, xlab='Age (x)', ylab='Beta
     1(x)', pch=19)
 plot(mod.M3$y, mod.M3$kappa2, xlab='Years (t)',
     ylab='Kappa 2 (t)', pch=19)
 plot(mod.M3$cy, mod.M3$gamma3, xlab='Years', ylab='Gamma 3
     (t)', pch=19)
par(op)
 mtext('Males', side=3, line=2, font=2)
```

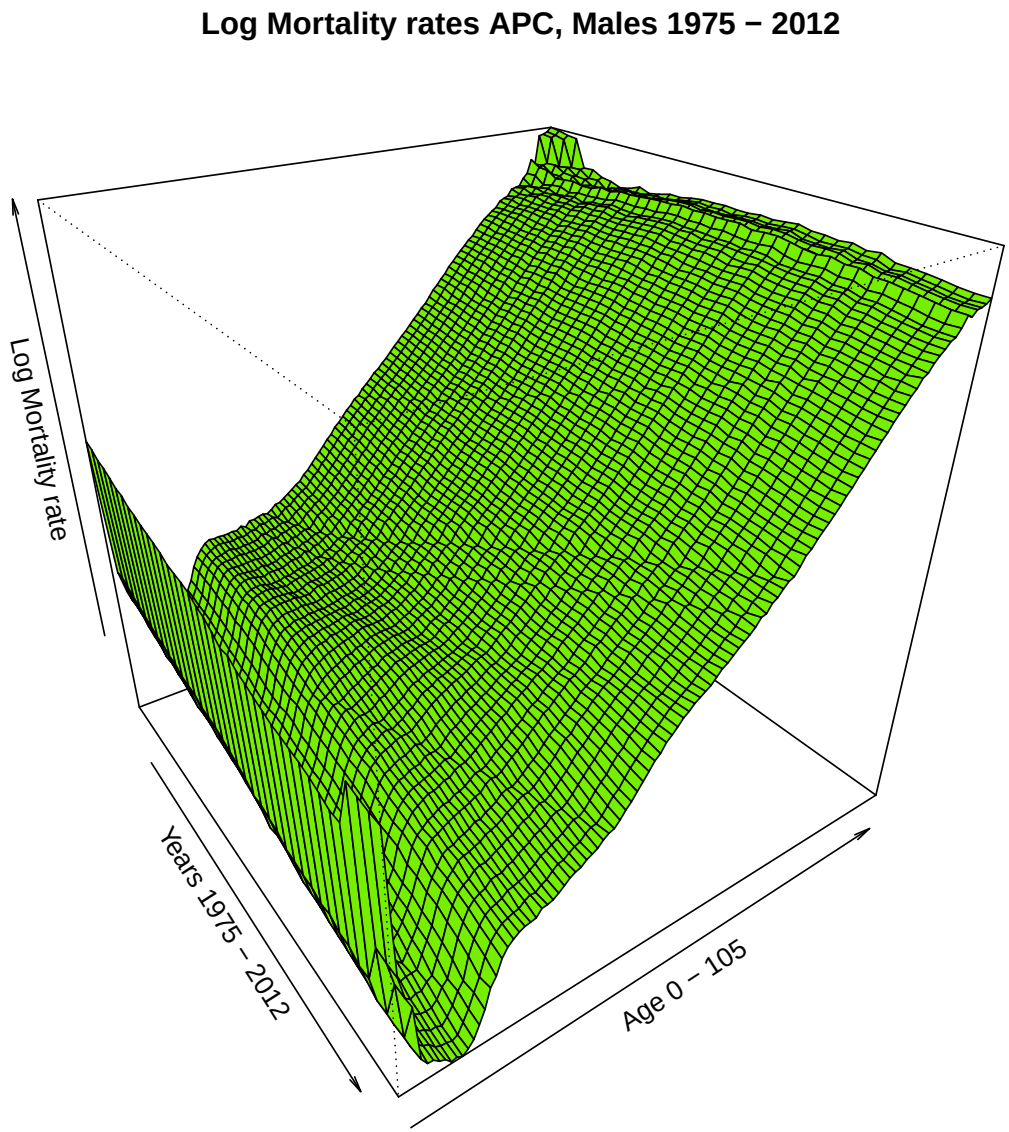
Modelo Age-Period-Cohort - Parámetros estimados



Modelo Age-Period-Cohort - Tasas de Mortalidad estimadas

```
#Gráfico de las Tasas Estimadas
persp(mod.M3$y, mod.M3$x, log(mod.M3$mhat), phi=30,
    theta=55, col ="chartreuse2",
      main=paste('Log Mortality rates APC, Males',
          min(year), '-', max(year)),
      ylab=paste('Age',0,'-',105),
          xlab=paste('Years',min(year),'-',max(year)),
          zlab='Log Mortality rate')
```

Modelo Age-Period-Cohort - Tasas de Mortalidad estimadas

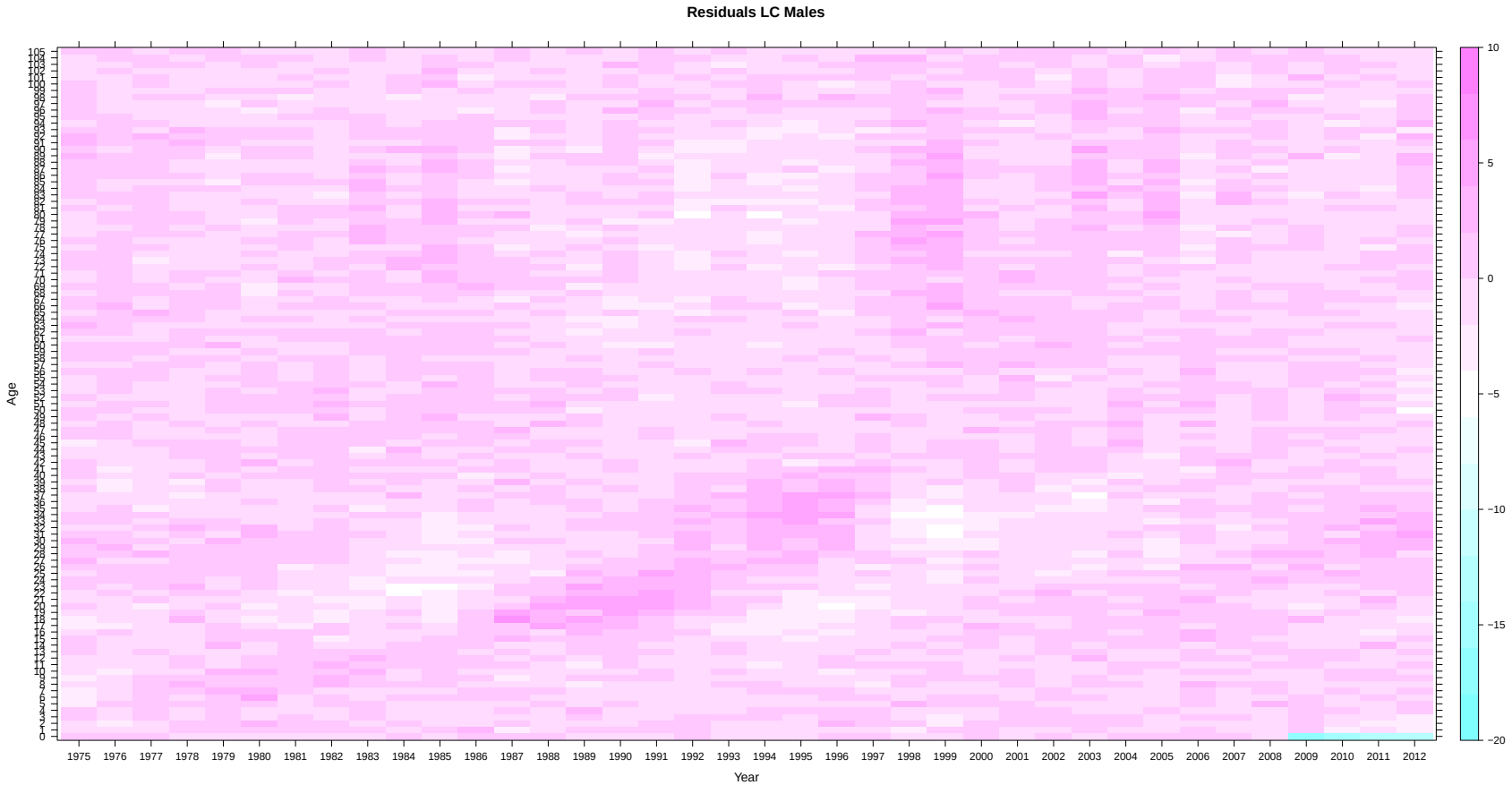


Modelo Age-Period-Cohort - Goodness of fit

```
#Residuales
levelplot(mod.M3$epsilon, colorkey=list(space='right'),
    xlab='Year', ylab='Age',
          col.regions=cm.colors, main='Residuals APC
              Males', pretty=T, aspect='fill')

#Bayes Information Criterion (BIC) & Log-likelihood (ll)
mod.M3$BIC
mod.M3$ll
```

```
> mod.M3$BIC
[1] -30363.34
> mod.M3$ll
[1] -29185.3
```

Modelo Age-Period-Cohort - Residuales



Modelo CBD

```
############### Modelo M5 - Cairns-Blake-Dowd #############
mod.M5<-fit705(age,year,etx,dtx,wa)

names(mod.M5)
#"kappa1" "kappa2" "kappa4" "beta2" "beta3" "beta4"
    "gamma3" "x0" "x" "y" "cy" "epsilon" "mhat" "mtx"
    "npar" "ll" "BIC" "wa"

#Parámetros estimados
mod.M5$beta2
mod.M5$beta3
mod.M5$kappa1
mod.M5$kappa2
```

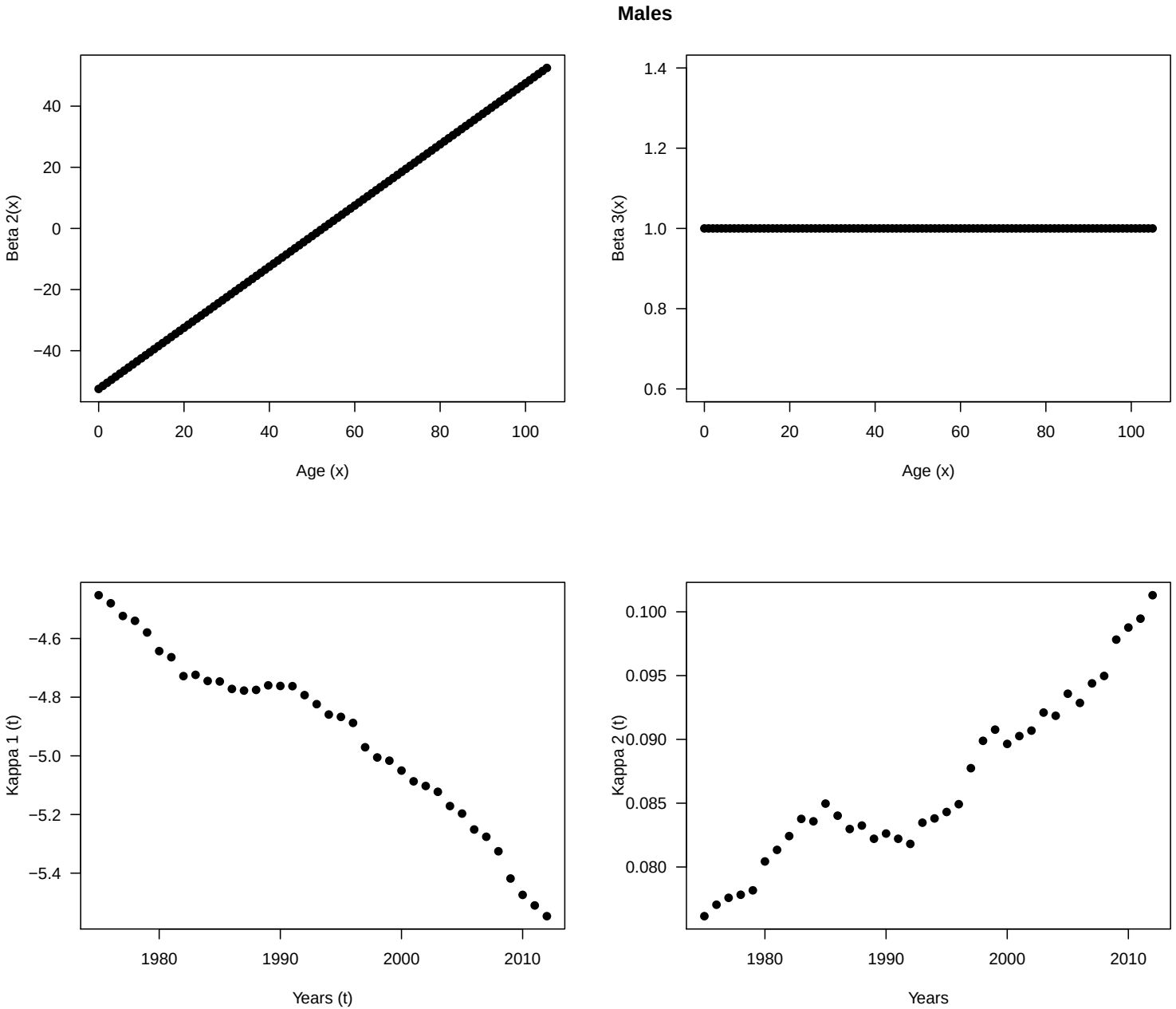
$\hat{\beta}_3 = 1$ y $\hat{\beta}_2 = (x - \bar{x})$

```
#Tasas estimadas y residuales
mod.M5$mhat
mod.M5$epsilon
```

Modelo CBD - Parámetros estimados

```
#Gráfico de los parámetros estimados
op<-par(mfrow=c(2,2), las=1)
 plot(mod.M5$x, mod.M5$beta2, xlab='Age (x)', ylab='Beta
     2(x)', pch=19)
 plot(mod.M5$x, mod.M5$beta3, xlab='Age (x)', ylab='Beta
     3(x)', pch=19)
 plot(mod.M5$y, mod.M5$kappa1, xlab='Years (t)',
     ylab='Kappa 1 (t)', pch=19)
 plot(mod.M5$y, mod.M5$kappa2, xlab='Years', ylab='Kappa 2
     (t)', pch=19)
par(op)
 mtext('Males', side=3, line=2, font=2)
```

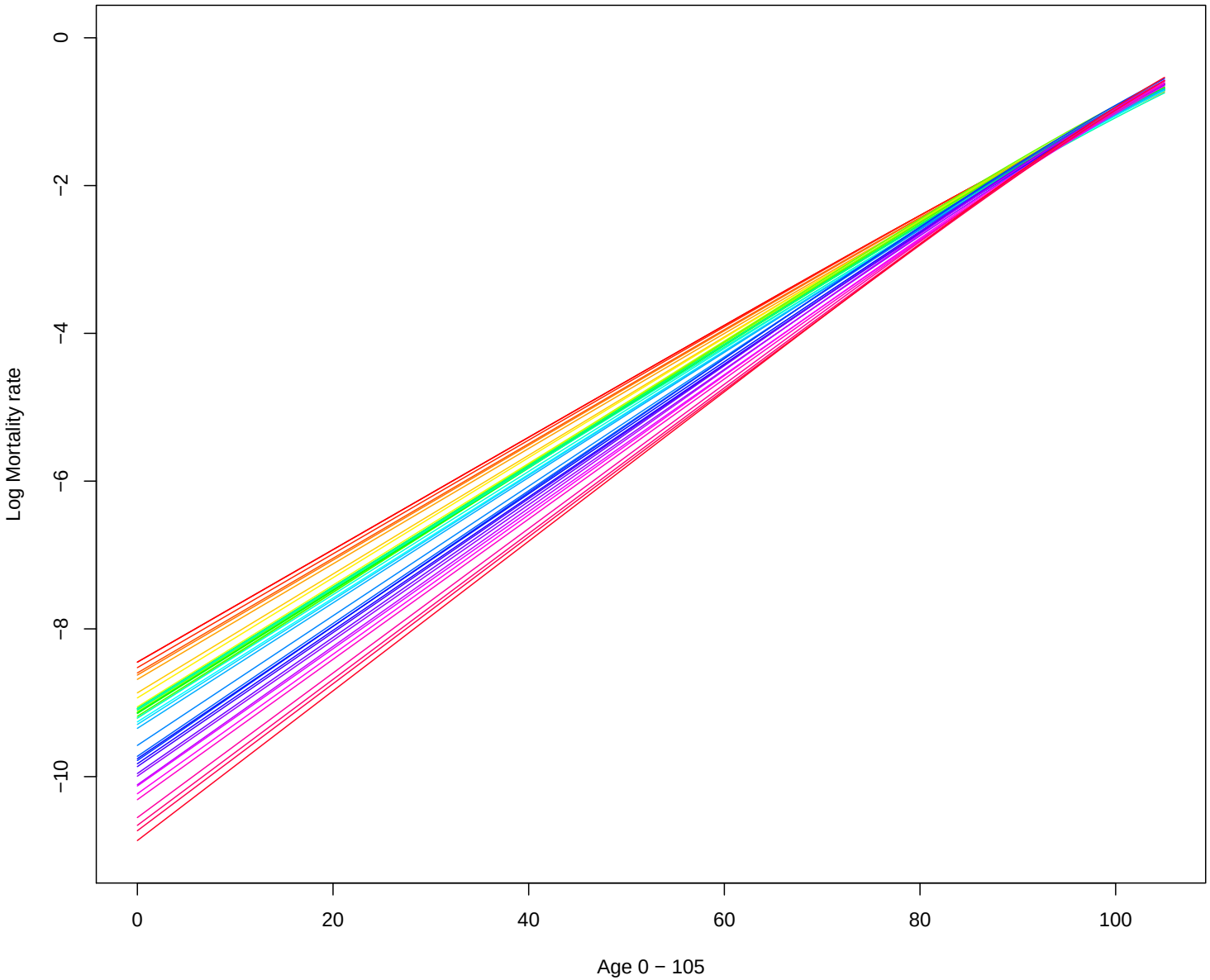
Modelo CBD - Parámetros estimados



Modelo CBD - Tasas de Mortalidad estimadas

```
#Gráfico de las Tasas Estimadas
persp(mod.M5$y, mod.M5$x, log(mod.M5$mhat), phi=30,
    theta=55, col ="chartreuse2",
      main=paste('Log Mortality rates RH, Males',
          min(year), '-', max(year)),
      ylab=paste('Age',0,'-',105),
          xlab=paste('Years',min(year),'-',max(year)),
          zlab='Log Mortality rate')

x11()
plot(age, log(mod.M5$mhat)[1,], col= rainbow(1), type='l',
    ylim=c(-11,0), xlab=paste('Age',0,'-',105), ylab='Log
    Mortality rate')
for(i in 1:38){
      lines(age, log(mod.M5$mhat)[i,], col= rainbow(38)[i],
          type='l')
```

Modelo CBD - Tasas de Mortalidad estimadas



Modelo CBD - Goodness of fit

```
#Residuales
levelplot(mod.M5$epsilon[,21:106],
    colorkey=list(space='right'), xlab='Year', ylab='Age',
          col.regions=cm.colors, main='Residuals LC Males',
              pretty=T, aspect='fill')

#Bayes Information Criterion (BIC) & Log-likelihood (ll)
mod.M5$BIC
```

```
> mod.M5$BIC
[1] -416742.4
> mod.M5$ll
[1] -416427.1
```

Modelo CBD - Residuales