CSCE 420 - Fall 2023 Homework 2 (HW2)

due: Tues, Oct 24, 11:59 pm

Turn-in answers as a Word document (HW2.docx or .pdf) and commit/push it to your class github repo.

When pushing the final version of your HW1, also run the command:

git tag "HW1" && git push origin "HW1"

This will be used to record time of submission for late penalty when applicable.

All homeworks must be typed, *not* hand-written and scanned/photo-shot.

1a. Prove that "Implication Introduction" (the opposite of Implication Elimination) is a **sound rule of inference** (ROI) using a **truth table**. If you have a Horn clause, with 1 positive literal and *n-1* negative literals, like (¬XvZv¬Y), you can transform it into a conjunctive rule by collecting the negative literals as positive antecedents, e.g. X^Y→Z. It is sufficient to prove this for *n-1*=2 antecedents. (In fact, this is a truth-preserving operation, hence sound.)

X	Y	Z	-X	-Y	-X v Z v -Y	X ^ Y	X ^ Y -> Z
T	T	T	F	F	T	T	T
T	T	F	F	F	F	T	F
T	F	T	F	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	F	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

The truth values for these sentences are the same so they are truth preserving meaning it is a sound rule of inference

1b. Prove that (A^B→C^D) |- (A^B→C) ("conjunctive rule splitting") is a **sound rule-ofinference** using a **truth table**.

A	B	C	D	A^B	C^D	A^B->C^D	A^B->C
T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T
T	T	F	T	T	F	F	F
T	T	F	F	T	F	F	F
T	F	T	T	F	T	T	T
T	F	T	F	F	F	T	T
T	F	F	T	F	F	T	T

T	F	F	F	F	F	T	T
F	T	T	T	F	T	T	T
F	T	T	F	F	F	T	T
F	T	F	T	F	F	T	T
F	T	F	F	F	F	T	T
F	F	T	T	F	T	T	T
F	F	T	F	F	F	T	T
F	F	F	F	F	F	T	T

This is rule is sound because all the models that satisfy the premise also satisfy the derived sentence.

1c. Also prove (A^B→C^D) |= (A^B→C) using **Natural Deduction.** (hint: use 1a above)

1 -(A^B) v C^D implication elimination
2 -Av-B v C^D demorgans law (1)
3 -A v -B v C distribution (2)
4 -A v -B v D distribution (2)
5 A ^ B -> C implication introduction (4)

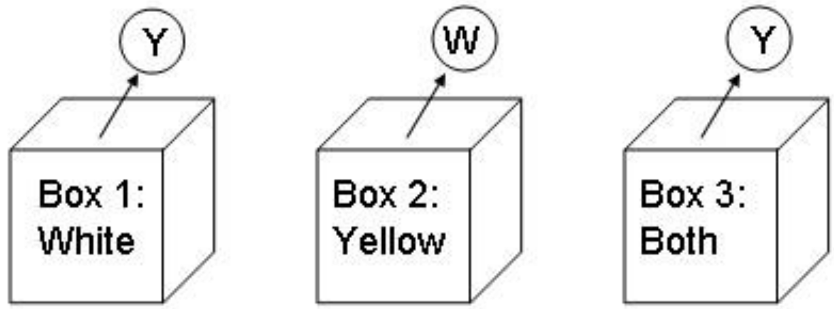
1d. Also prove (A^B→C^D) |= (A^B→C) using **Resolution**.

1 -(A^B) v C rewrote derived sentence
2 -(-(A^B) v C) negate derived sentence (1)
3 A^B^-C Demorgans law (2)
4 A And elimination (3)
5 B And elimination (3)
6 -C And elimination (3)
7 A^B -> C^D from premise
8 -(A^B) v C^D implication elimination (7)
9 -Av-B v C^D Demorgans law (8)
10 -A v -B v C distribution (9)
11 -A v -B v D distribution (9)
12 -B v C Resolution (10 and 4)
13 C Resolution (12 and 6)
14 {} Resolution (13 and 6)

By proving the negated derived sentence results in an empty sentence though resolution, the premise is proved to entail the derived sentence.

2. <u>**Sammy's Sport Shop**</u>

You are the proprietor of *Sammy's Sport Shop*. You have just received a shipment of three boxes filled with tennis balls. One box contains only yellow tennis balls, one box contains only white tennis balls, and one contains both yellow and white tennis balls. You would like to stock the tennis balls in appropriate places on your shelves. Unfortunately, the boxes have been labeled incorrectly; the manufacturer tells you that you have exactly one box of each, but that **each box is definitely labeled wrong**. You draw one ball from each box and observe its color. Given the initial (incorrect) labeling of the boxes above, and the three observations, use Propositional Logic to infer the correct contents of the middle box.



Use propositional symbols in the following form: O1Y means a yellow ball was drawn (observed) from box 1, L1W means box 1 was initially labeled white, C1W means box 1 contains (only) white balls, and C1B means box 1 actually contains both types of tennis balls. Note, there is no 'O1B', etc, because you can't directly "observe both". When you draw a tennis ball, it will either be white or yellow.

The initial facts describing this particular situation are: {O1Y, L1W, O2W, L2Y, O3Y, L3B}

2a. Using these propositional symbols, <u>write a propositional knowledge base</u> (sammy.kb) that captures the knowledge in this domain (i.e. implications of what different observations or labels mean, as well as constraints inherent in this problem, such as that all boxes have different contents). *Do it in a complete and general way*, writing down *all* the rules and constraints, not just the ones needed to make the specific inference about the middle box. *Do not include derived knowledge* that depends on the particular labeling of this instance shown above; stick to what is stated in the problem description above. Your KB should be general enough to reason about any alternative scenario, not just the one given above (e.g. with different observations and labels and box contents).

```
// labels are incorrect
1       L1Y -> -C1Y
2       L1W -> -C1W
3       L1B -> -C1B
4       L2Y -> -C2Y
5       L2W -> -C2W
6       L2B -> -C2B
7       L3Y -> -C3Y
8       L3W -> -C3W
9       L3B -> -C3B

// all contents different
10      C1W -> -C2W ^ -C3W
11      C1Y -> -C2Y ^ -C3Y
```

12 C1B -> -C2B ^ -C3B
13 C2W -> -C1W ^ -C3W
14 C2Y -> -C1Y ^ -C3Y
15 C2B -> -C1B ^ -C3B
16 C3W -> -C1W ^ -C2W
17 C3Y -> -C1Y ^ -C2Y
18 C3B -> -C1B ^ -C2B

// observed color implies either box is that color or contains both
19 O1Y -> C1Y v C1B
20 O1W -> C1W v C1B
21 O2Y -> C2Y v C2B
22 O2W -> C2W v C2B
23 O3Y -> C3Y v C3B
24 O3W -> C3W v C3B

// labels are incorrect (another way to label)
25 L1Y -> C2Y v C3Y
26 L1W -> C2W v C3W
27 L1B -> C2B v C3B
28 L2Y -> C1Y v C3Y
29 L2W -> C1W v C3W
30 L2B -> C1B v C3B
31 L3Y -> C1Y v C2Y
32 L3W -> C1W v C2W
33 L3B -> C1B v C2B

// cant have multiple labels
34 C1Y -> -C1W
35 C1Y -> -C1B
36 C1W -> -C1Y
37 C1W -> -C1B
38 C1B -> -C1Y
39 C1B -> -C1W

34 C2Y -> -C2W
35 C2Y -> -C2B
36 C2W -> -C2Y
37 C2W -> -C2B
38 C2B -> -C2Y
39 C2B -> -C2W

34 C3Y -> -C3W
35 C3Y -> -C3B
36 C3W -> -C3Y

37	C3W -> -C3B
38	C3B -> -C3Y
39	C3B -> -C3W

2b. Prove that box 2 must contain white balls (**C2W**) using **Natural Deduction**.

a	L2Y	observed
b	O2W	observed
c	L1W	observed
d	O1Y	observed
e	L3B	observed
f	O3Y	observed
g	-C2Y	(a and 4)
h	-C1W	(c and 2)
i	-C3B	(e and 9)
j	C2W v C2B	(b and 22)
k	C1Y v C1B	(d and 19)
l	C3Y v C3B	(f and 23)
m	C3Y	resolution (i and l)
n	-C1Y ^ -C2Y	(m and 17)
o	-C1Y	and elimination (n)
p	-C2Y	and elimination (n)
q	C1B	resolution (k and o)
r	-C2B ^ -C3B	(q and 12)
s	-C2B	and elimination (r)
t	-C3B	and elimination (r)
u	C2W	resolution (j and s)

2c. Convert your KB to CNF.

// labels are incorrect

1	-L1Y v -C1Y
2	-L1W v -C1W
3	-L1B v -C1B
4	-L2Y v -C2Y
5	-L2W v -C2W
6	-L2B v -C2B
7	-L3Y v -C3Y
8	-L3W v -C3W
9	-L3B v -C3B

// all contents different

10	-C1W v -C2W
11	-C1W v -C3W

12 -C1Y v -C2Y
13 -C1Y v -C3Y
14 -C1B v -C2B
15 -C1B v -C3B
16 -C2W v -C1W
17 -C2W v -C3W
18 -C2Y v -C1Y
19 -C2Y v -C3Y
20 -C2B v -C1B
21 -C2B v -C3B
22 -C3W v -C1W
23 -C3W v -C2W
24 -C3Y v -C1Y
25 -C3Y v -C2Y
26 -C3B v -C1B
27 -C3B v -C2B

// observed color implies either box is that color or contains both
28 -O1Y v C1Y v C1B
29 -O1W v C1W v C1B
30 -O2Y v C2Y v C2B
31 -O2W v C2W v C2B
32 -O3Y v C3Y v C3B
33 -O3W v C3W v C3B

// labels are incorrect (another way to label)
34 -L1Y v C2Y v C3Y
35 -L1W v C2W v C3W
36 -L1B v C2B v C3B
37 -L2Y v C1Y v C3Y
38 -L2W v C1W v C3W
39 -L2B v C1B v C3B
40 -L3Y v C1Y v C2Y
41 -L3W v C1W v C2W
42 -L3B v C1B v C2B

// cant have multiple labels
43 -C1Y v -C1W
44 -C1Y v -C1B
45 -C1W v -C1Y
46 -C1W v -C1B
47 -C1B v -C1Y
48 -C1B v -C1W

49 -C2Y v -C2W

50	-C2Y v -C2B
51	-C2W v -C2Y
52	-C2W v -C2B
53	-C2B v -C2Y
54	-C2B v -C2W
55	-C3Y v -C3W
56	-C3Y v -C3B
57	-C3W v -C3Y
58	-C3W v -C3B
59	-C3B v -C3Y
60	-C3B v -C3W

2d. Prove C2W using **Resolution.**

Facts = { i O1Y,
ii L1W,
iii O2W,
iv L2Y,
v O3Y,
vi L3B }

a	-C2W	negate derived sentence
b	-L1W v C3W	resolution (a and 35)
c	-L3W v C1W	resolution (a and 41)
d	-O2W v C2B	resolution (a and 31)
e	C3W	resolution (b and iii)
f	C3Y v C3B	32 and v
g	-C3Y	e and 55
h	C3B	f and g
i	-C3W	h and 60
j	{ }	e and i

Because the negated derived sentence resulted in an empty sentence, the derived sentence is true through resolution refutation.

3. Do **Forward Chaining** for the *CanGetToWork* KB below.

You don't need to follow the formal FC algorithm (with agenda/queue and counts array). Just indicate which rules are triggered (in any order), and keep going until all consequences are generated.
Show the final list of all inferred propositions at the end. *Is CanGetToWork among them?*

```
KB = { a. CanBikeToWork → CanGetToWork
       b. CanDriveToWork → CanGetToWork
       c. CanWalkToWork → CanGetToWork
       d. HaveBike ^ WorkCloseToHome ^ Sunny → CanBikeToWork
       e. HaveMountainBike → HaveBike
       f. HaveTenSpeed → HaveBike
       g. OwnCar → CanDriveToWork
       h. OwnCar → MustGetAnnualInspection
       i. OwnCar → MustHaveValidLicense
       j. CanRentCar → CanDriveToWork
       k. HaveMoney ^ CarRentalOpen → CanRentCar
       l. HertzOpen→ CarRentalOpen
       m. AvisOpen→ CarRentalOpen
       n. EnterpriseOpen→ CarRentalOpen
       o. CarRentalOpen → IsNotAHoliday
       p. HaveMoney ^ TaxiAvailable → CanDriveToWork
       q. Sunny ^ WorkCloseToHome → CanWalkToWork
       r. HaveUmbrella ^ WorkCloseToHome → CanWalkToWork
       s. Sunny → StreetsDry }
```

Facts: { Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen }

Facts (agenda):

1) Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen

2) Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike

3) Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike, CarRentalOpen

4) Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity,

WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike, CarRentalOpen, CanRentCar, IsNotAHoliday

5) Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike, CarRentalOpen, CanRentCar, IsNotAHoliday, CanDriveToWork

6) Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike, CarRentalOpen, CanRentCar, IsNotAHoliday, CanDriveToWork, CanGetToWork

7) Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, HaveBike, CarRentalOpen, CanRentCar, IsNotAHoliday, CanDriveToWork, CanGetToWork

CanGetToWork is generated from the given facts

4. Do **Backward Chaining** for the *CanGetToWork* KB.

In this case, you should follow the BC algorithm closely (the pseudocode for the propositional version of Back-chaining is given in the lecture slides).

Important: when you pop a subgoal (proposition) from the goal stack, you should systematically go through all rules that can be used to prove it IN THE ORDER THEY APPEAR IN THE KB. In some cases, this will lead to *back-tracking*, which you should show.

Also, the sequence of results depends on order in which antecedents are pushed onto the stack. If you have a rule like A^B→C, and you pop C off the stack, push the antecedents in reverse order, so B goes in first, then A; in the next iteration, A would be the next subgoal popped off the stack.

CanGetToWork	goal
CanBikeToWork	pop CanGetToWork
HaveBike, WorkCloseToHome, Sunny	pop CanBikeToWork
HaveMountainBike, WorkCloseToHome, Sunny	pop HaveBike
HaveTenSpeed, WorkCloseToHome, Sunny	backtrack; not provable

CanDriveToWork	backtrack; not provable
OwnCar	pop CanDriveToWork
CanRentCar	backtrack; not provable
HaveMoney, CarRentalOpen	pop CanRentCar
CarRentalOpen	pop HaveMoney (fact)
HertzOpen	pop CarRentalOpen
AvisOpen	backtrack; not provable
{}	pop AvisOpen

CanGetToWork is true since it resulted in an empty set through back chaining.