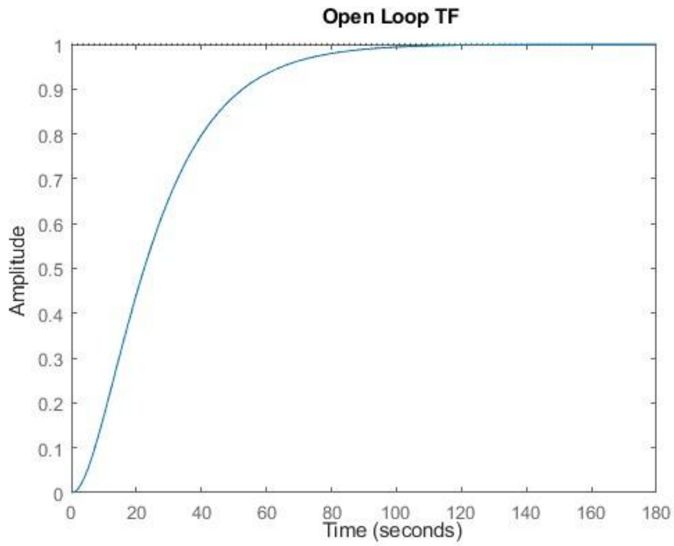
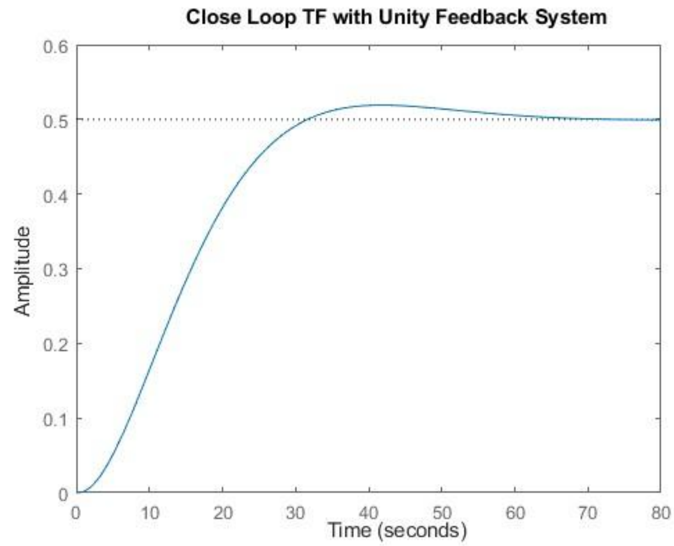
Case 1:





The above transfer function plots the step response of the system with plant and no controller with the system. The Step Response of the system improved upon the incorporation of Unity Feedback system. From the step response of **"Close Loop TF with Unity Feedback System"**, the settling time as well as overshoot of the system was much improved with the unity feedback system.

Case 2a:

Here gain value of Max Overshoot of 2% and settling time of 9 seconds, was obtained using both the Routh stability Criterion and the Root Locus Method.

Case 2: $G_c(s) = \dfrac{k(s+0.3)(s+1.5)}{s}$, $G_p(s) = \dfrac{0.018}{(s+0.06)(s+0.1)(s+3)}$

$$G(s) = G_p(s)\,G_c(s)$$

$$G = \frac{0.018k(s+0.3)(s+1.5)}{s(s+0.06)(s+0.1)(s+3)}$$

$$1 + G(s) = 0$$

$$s(s+0.06)(s+0.1)(s+3) + 0.018k(s+0.3)(s+1.5) = 0$$

$$s^4 + 3.16s^3 + 0.486s^2 + 0.018s + 0.018ks^2 + 0.0324ks + 0.0081k = 0$$

$$s^4 + 3.16s^3 + (0.486 + 0.018k)s^2 + (0.018 + 0.0324k)s + 0.0081k = 0$$

s^4	1	$0.486 + 0.018k$	$0.0081k$
s^3	3.16	$0.018 + 0.0324k$	0
s^2	$0.48 - 0.084k$	$0.0081k$	
s^1	$\dfrac{-0.0027k^2 - 0.0116k + 0.00864}{0.48 - 0.084k}$	0	
s^0	$0.0081k$		

$k > 0$, $k > 5.7142$, $k_1 = 0.6011$, $k_2 = -4.84$

$$\boxed{0 < k < 5.7142}$$

$$G = \frac{0.018K(s+0.3)(s+1.5)}{s(s+0.06)(s+0.1)(s+3)}$$

$$\varepsilon = \frac{-\ln(M_{os})}{\sqrt{\pi^2 + \ln(M_{oc})}} = 0.7797$$

$$\theta = \cos^{-1}(\varepsilon) = 38.76^\circ \quad , \quad \sigma = 0.5$$

$$\omega_d = \sigma \tan\theta = 0.4015$$

$$K = -\frac{s(s+0.06)(s+0.1)(s+3)}{0.018(s+0.3)(s+1.5)}$$

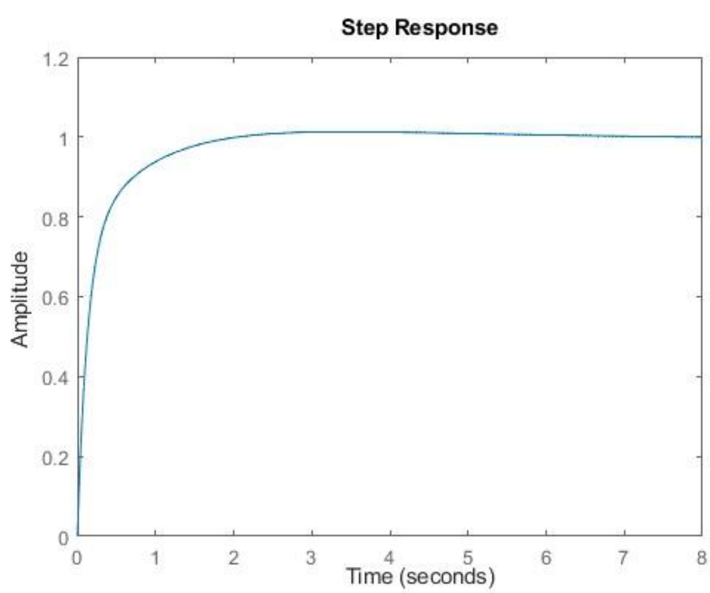
$$s = -0.5 \pm j0.4$$

$$K = \frac{-(-0.5+j0.4)(-0.5+j0.4+0.06)(-0.5+j0.4+0.1)(-0.5+j0.4+3)}{0.018(-0.5+j0.4+0.3)(-0.5+j0.4+1.5)}$$

$$|K| = \frac{\left(\sqrt{0.5^2+0.4^2}\right)\left(\sqrt{0.44^2+0.4^2}\right)\left(\sqrt{0.4^2+0.4^2}\right)\left(\sqrt{2.5^2+0.4^2}\right)}{0.018\left(\sqrt{0.2^2+0.4^2}\right)\left(\sqrt{1^2+0.4^2}\right)}$$

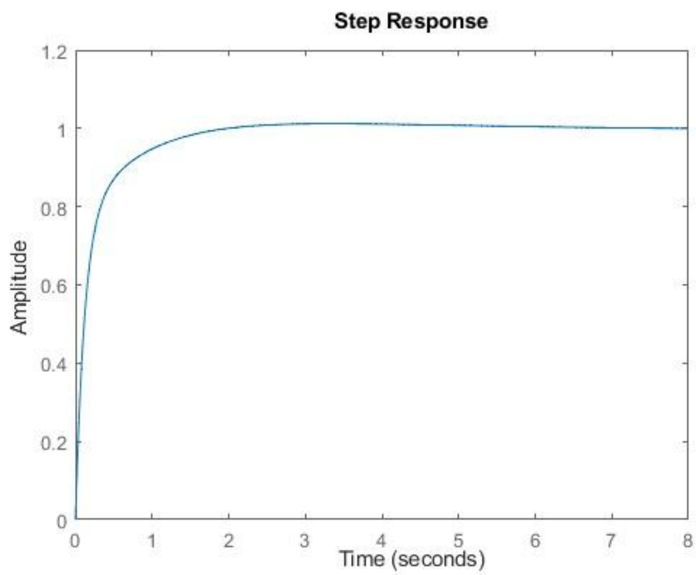
$$|K| = 62.89$$

Additionally, using MATLAB the Closed loop poles were determined. Using PD controller, the step response of the given was obtained. Sisotool was used to adjust the gain value that satisfies the system performance parameters of Max Overshoot of 4% and Settling time of 9 seconds.



Case 2b:

Using root locus method for Max Overshoot of 4% and Settling time of 9 seconds, the design point as well as value of gain, k, was calculated. PID controller was utilized to achieve the required performance parameters of the system as depicted in the figure below.



$M_{os} = 4\% \qquad t_s = 9\,\text{sec}$

$\xi = 0.715 \quad , \quad \theta = 44.3^\circ \quad , \quad \xi\omega_n = 0.44$

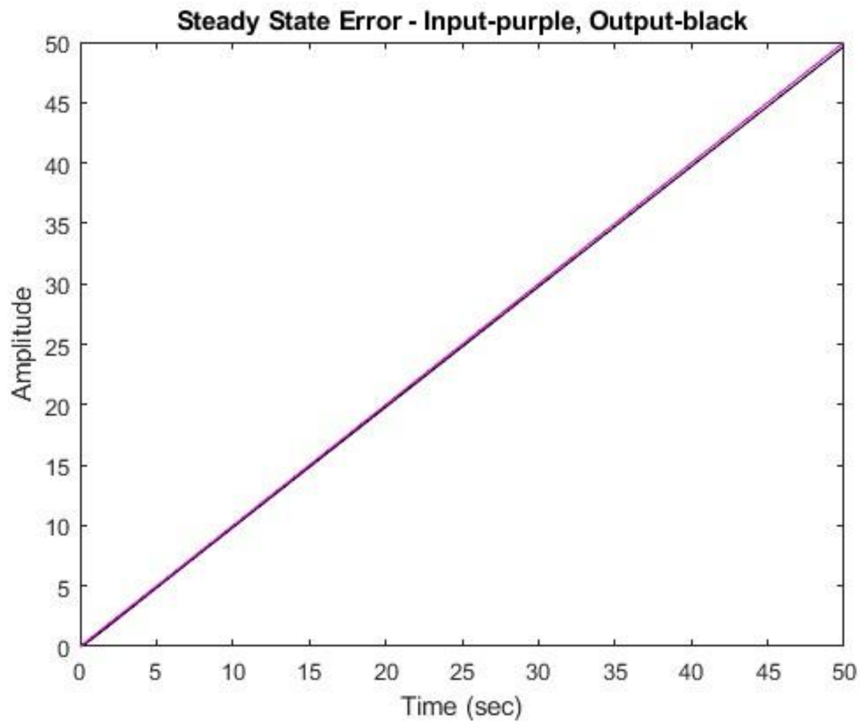
$\omega_d = 0.434 \quad , \quad s_d = -0.44 \pm j0.43$

$$k = \frac{-(-0.4+j0.4)(-0.44+j0.43+0.06)(-0.44+j0.43+0.1)(-0.44+j0.43+3)}{0.018(-0.44+j0.43+0.3)(-0.44+j0.43+1.5)}$$

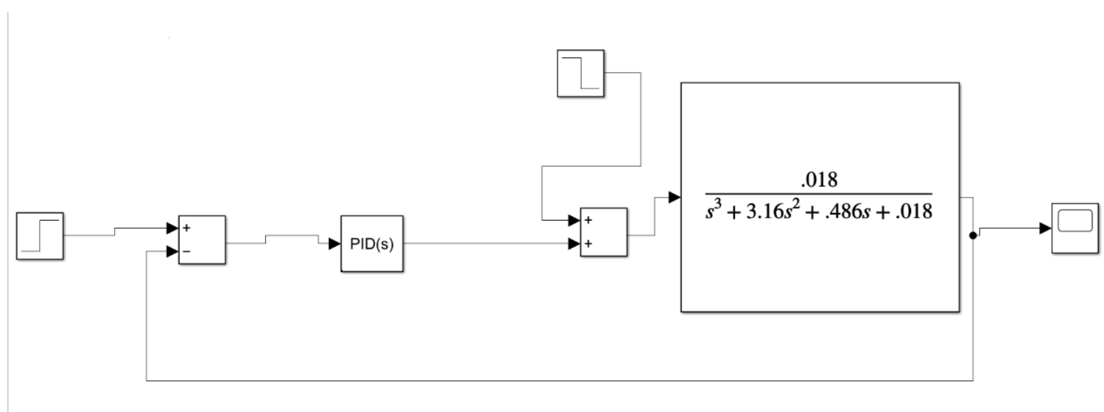
$$k = \frac{\left(\sqrt{0.4^2+0.4^2}\right)\left(\sqrt{0.38^2+0.43^2}\right)\left(\sqrt{0.34^2+0.43^2}\right)\left(\sqrt{2.56^2+0.43^2}\right)}{0.018\left(\sqrt{0.14^2+0.43^2}\right)\left(\sqrt{1.06^2+0.43^2}\right)}$$

$k = 51.61$

The calculated value of gain, k, was adjusted using the sisotool to achieve the required system performance parameters. This adjusted gain value was used to make sure that the system had the steady state error of zero, as depicted in the Figure on next page.



Finally, using SIMULINK the unity negative disturbance rejection was checked. The Simulink model and graph are given below. This satisfied the requirements for overshot, settling time and steady state error of zero. The PID Controller was tuned using linearization plant.



SIMULINK Graph: