Tracking Systems
Viterbi Algorithm HMM

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Lab 7 Report

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1 Introduction

This lab deals with application of viterbi algorithm to find the most probable sequence of states that produce the sequence. The system consists of states that the system can be in and individual probabilities of observations each state can produce.

2 Methods

The Viterbi Algorithm states that to get the maximum probability of the nth element in the sequence, knowledge of the maximum probability of the n-1th element is enough. Recursively iterating this algorithm over the entire sequence gives the sequence with maximum probability.

Or for a two state problem, the maximum probability of producing a A in the 3rd position (with C in the 2nd position) for state H is given by

$$p(A,3)_H = e(A)_H \max(p(C,2)_H p_{HH}, p(C,2)_L p_{LH})$$

While the same probability for state L is given by

$$p(A,3)_L = e(A)_L \max(p(C,2)_H p_{HL}, p(C,2)_L p_{LL})$$

Iterating this process over each element in the sequence gives us the maximum probability of sequence of states that generate that sequence.

3 Results

3.1 Input Sequence I

For the input sequence:GGCACTGAA

The probability table for each state is given by

Input Sequence	G	G	C	A	C	T	G	A	A
State 'H'	**-2.737**	**-5.474**	**-8.211**	-11.533	-14.002	-17.324	-19.535	-22.857	-25.648
State 'L'	-3.322	-6.059	-8.796	**-10.943**	**-14.002**	**-16.476**	**-19.535**	**-22.004**	**-24.473**

Table 1: Input Sequence GGCACTGAA

The boldfaced numbers show the maximum probabilities for each input sequence. The state that corresponds to the boldfaced probability would be the sequence of the most probable sequence of states.

Listing 1: Output sequence for GGCACTGAA

```
Value:-2.737 State:H
Value:-5.474 State:H
```

```
Value:-8.211 State:H
Value:-10.943 State:L
Value:-14.002 State:L
Value:-16.476 State:L
Value:-19.535 State:L
Value:-22.004 State:L
Value:-24.473 State:L
```

The most probable path for the input sequence is **HHHLLLLLL**. The value here would be log_2 of the actual value. Thus the probability of that sequence of states is

$$2^{-24.473} = 4.2943\text{e} - 08$$

3.2 Input Sequence II

For the input sequence:TCAGCGGCT

The probability table for each state is given by

Input Sequence	T	C	A	G	C	G	G	C	T
State 'H'	-3.322	-5.796	-9.118	**-11.324**	**-14.061**	**-16.798**	**-19.535**	**-22.272**	-25.594
State 'L'	**-2.737**	**-5.796**	**-8.265**	-11.324	-14.383	-17.383	-20.120	-22.857	**-25.009**

Table 2: Input Sequence TCAGCGGCT

The boldfaced numbers show the maximum probabilities for each input sequence. The state that corresponds to the boldfaced probability would be the sequence of the most probable sequence of states.

Listing 2: Output sequence for TCAGCGGCT

```
Value:-2.737 State:L
Value:-5.796 State:L
Value:-8.265 State:L
Value:-11.324 State:H
Value:-14.061 State:H
Value:-16.798 State:H
Value:-19.535 State:H
Value:-22.272 State:H
Value:-25.009 State:L
```

The most probable path for the input sequence is **LLLHHHHHL**. The value here would be log_2 of the actual value. Thus the probability of that sequence of states is

$$2^{-25.009} = 2.9617\text{e} - 08$$