CBSE Sample Paper-03
Mathematics
Class – XII

Time allowed: 3 hours | Maximum Marks: 100

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. Is R defined on the set A={1,2,3,4,5,6} as R={(x,y): y is divisible by x} symmetric.

2. Calculate the direction cosines of the vector $\vec{a}=3i-2j-5k$.

3. What is the principal value branch of $\cos^{-1}x$?

4. Find X and Y if $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}, X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$.

5. Evaluate without expanding $\begin{vmatrix}102 & 18 & 36\\ 1 & 3 & 4\\ 17 & 3 & 6\end{vmatrix}$.

6. Find A'=[-2 4 5], $B'=\begin{bmatrix}1\\ 3\\ 6\end{bmatrix}$ find (AB)'.

Section B

7. Using properties of determinants prove that

$$\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c})=abc+bc+ca+ab$$

8. Find the equations of all lines having slope 0 and that are tangent to the curve $y=\frac{1}{x^2-2x+2}$.

9. If $f(x)=\frac{x-1}{x+1},(x\neq 1,-1)$, show that $f\circ f^{-1}$ is an identity function.

10. Find $\frac{dy}{dx}$ if log(xy)= x^2+y^2.

11. Prove that $\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right),|x|<\frac{1}{\sqrt{3}}$

12. If A, B, C have the co-ordinates (2,0,0) , (0,1,0) , (0,0,2) , then show that ABC is an isosceles triangle.

13. (a)If A and B are two events defined on a sample space s.t.

$P(A\cup B)=\frac{5}{6},P(A\cap B)=\frac{1}{3},P(B^c)=\frac{1}{3}, find\ P(A).$

(b) If A and B are two events defined on a sample space s.t.

$P(A)=\frac{1}{4},P(B)=\frac{1}{2},P(A and\ B)=\frac{1}{8}, find\ P(notA\ and\ notB).$

14. Find all intervals on which the function $f(x)=-2x^3-9x^2-12x+1$ is (a)strictly increasing (b) strictly decreasing.

15. Solve the differential equation $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$

16. Show that $\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right).\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)=0$

17. Integrate $\int\log(1+x^2)dx$.

18. Find theshortest distance between the lines l_1 and l_2 given by :

$\vec{r}=(i+2j+k)+\lambda(i-j+k)$
$\vec{r}=(2i-j-k)+\mu(2i+j+2k)$

19. Find the vector equation of the plane passing through the points i+j-k and 2i+6j+k and parallel to the line $\vec{r}=(3i-5j+k)+\lambda(i-2j+k)$

Section C

20. A factory can hire two tailors A and B in order to stich pants and shirts. Tailor A can stich 6 shirts and 4 pants in a day. Tailor B can stich 10 shirts and 4 pants in a day. Tailor A charges 15 per day and tailor B charges 20 per day. The factory has to produce minimum 60 shirts and 32 pants. State as a linear programming problem and minimize the labour cost.

21. Find the area of the region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$

22. A, Band C play a game and the chances of winning in it in an attempt are 2/3, 1/2, ¼ respectively. A has the first chance, followed by Band C. The cycle is repeated till one of them wins the game. Find their respective chances of winning the game.

23. If $x = a\sec^3\theta, y = a\tan^3\theta$, *find* $\frac{d^2y}{dx^2}$ *at* $\theta = \frac{\pi}{4}$.

24. Integrate $\int \frac{x^2 dx}{(x+3)\sqrt{3x+4}}$

25. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

26. Solve the following system of equations using matrix method

$$x + y + z = 4$$
$$2x - y + z = -1$$
$$2x + y - 3z = -9$$