ANSWER KEY- XII/XIII(ASEEM/ANANT) - (PST-4) DATE : 10-01-2016

CODE-1

PHYSICS															
QUS.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ANS.	2	4	1	4	2	1	3	2	4	3	3	3	1	2	2
QUS.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
ANS.	4	1	1	1	2	4	2	1	4	1	3	3	1	1	4
CHEMISTRY															
QUS.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
ANS.	1	3	2	4	4	3	3	4	1	2	3	2	3	2	3
QUS.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
ANS.	3	3	2	2	3	3	3	3	4	2	3	1	3	2	2
MATHEMATICS															
QUS.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
ANS.	1	4	3	4	2	1	1	1	4	2	1	1	2	2	1
QUS.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
ANS.	3	3	2	1	1	4	1	4	3	2	4	2	2	1	2

Note :-
Solutions can be *downloaded* from the website or will be displayed on the notice board !

ANSWER KEY- XII/XIII(ASEEM/ANANT) - (PST-4) DATE : 10-01-2016

CODE-2

PHYSICS															
QUS.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ANS.	4	1	1	1	2	4	2	1	4	1	3	3	1	1	4
QUS.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
ANS.	2	4	1	4	2	1	3	2	4	3	3	3	1	2	2
CHEMISTRY															
QUS.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
ANS.	3	3	2	2	3	3	3	3	4	2	3	1	3	2	2
QUS.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
ANS.	1	3	2	4	4	3	3	4	1	2	3	2	3	2	3
MATHEMATICS															
QUS.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
ANS.	3	3	2	1	1	4	1	4	3	2	4	2	2	1	2
QUS.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
ANS.	1	4	3	4	2	1	1	1	4	2	1	1	2	2	1

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PART SYLLABUS TEST [PST–04]

TARGET-JEE MAIN TEST SERIES 2016

CLASS-XII/XIII- (ASEEM/ANANT) DATE : 10-01-2016

Hints & Solution

PART – I (PHYSICS)

1. The phase difference between two ....

[Sol. Phase of two SHM's at 0.5 s is0

$\phi_1 = \left(5\pi + \frac{\pi}{3}\right)$ and $\phi_2 = \left(4\pi + \frac{\pi}{3}\right)$

Phase difference = $\phi_1 - \phi_2 = \pi + \frac{\pi}{12}$

$= \frac{13\pi}{12}$]

2. A block of mass 1 kg kept over ....

[Sol. For 1-D motion

$t = \frac{20}{2} = 10$ sec

For the part of SHM

$\frac{T}{2} = \frac{2\pi}{2}\sqrt{\frac{M}{k}} = \pi\sqrt{\frac{1}{1}} = \pi$

$(10 + \pi)$ sec **Ans.**]

3. Two identical blocks P and Q have ....

[Sol. $\omega_P = \omega_Q = \sqrt{\frac{k}{m}}$ where k = stiffness of each spring. Conserving $\vec{p}$ at collision

$m\omega_P\frac{A}{2} - m\omega_Q A = 2mv_f$

$\Rightarrow \quad v_f = \sqrt{\frac{k}{m}}\ \frac{A}{4}; \ v_f = \omega_f A_f = \sqrt{\frac{2k}{2m}}$

$A_f = \sqrt{\frac{k}{m}}\ A_f \quad \Rightarrow \quad A_f = \frac{A}{4}$]

4. Four springs have been compressed ....

[Sol. c > b > a = d. Energy conservation

$\frac{1}{2}kA^2 = \frac{1}{2}m(v_{max})^2$ gives $v_{max} = \sqrt{k/m}\,A$.

k or m has to be increased or decreased by a factor of 4 to have the same effect as increasing or decreasing A by a factor of 2.]

5. If the length of a simple pendulum ....

[Sol. $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{\ell} + \frac{1}{R}\right)}} = 2\pi\sqrt{\frac{R}{2g}}$]

6. A block of mass 1kg is connected ....

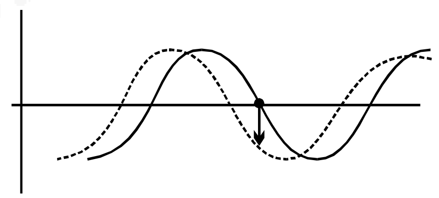
[Sol. $x = 1\cos(\omega t) - \frac{1}{2} = 1\cos(\omega t)$

$\omega t = \frac{2\pi}{3}$

$\Rightarrow \quad t = \frac{2\pi}{3\omega} = \frac{T}{3} = \frac{2\pi}{3}\sqrt{\frac{m}{k}} = \frac{2}{3}$ sec]

8. The transverse wave shown ....

[Sol. $v_p = -v_w\frac{\partial y}{\partial x}$; as $\frac{\partial y}{\partial x} < 0$; $v_w < 0$



$v_p < 0$; particle moves down

9. A wire having a linear density ....

[Sol. $n\frac{v}{2\ell} = 400$ Hz

$(n + 1)\frac{v}{2\ell} = 450$ Hz

$\Rightarrow \quad \frac{n}{n+1} = \frac{400}{450}$

$9n = 8n + 8$

$n = 8$

Now $8 \times \frac{1}{2\ell}\sqrt{\frac{490}{0.1}} = 400$

$\ell = \frac{8}{2} \times \frac{70}{400} = 0.7$]

10. Both the strings, shown in figure are.........

[Sol. $v = \sqrt{\dfrac{T}{\mu}}$ $v_1 = v_{AB} = \sqrt{\dfrac{T_{AB}}{\mu_{AB}}}$;

$v_{CD} = \sqrt{\dfrac{T_{CD}}{\mu_{CD}}} = v_2$

$2T_{AB} = T_{CD}$; $\mu_{CD} = 4\mu_{AB}$

$\dfrac{v_1}{v_2} = \sqrt{\dfrac{T_{AB}}{T_{CD}}} \times \sqrt{\dfrac{\mu_{CD}}{\mu_{AB}}}$

$R_{CD} = 2R_{AB}$

$\dfrac{v_1}{v_2} = \sqrt{\dfrac{1}{2}} \times \sqrt{4} = \dfrac{2}{\sqrt{2}}$

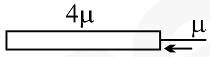
$\mu_{CD} = [\pi\,(2R)2 \times 1]\rho$

$\dfrac{v_1}{v_2} = \sqrt{2}$

$\mu_{AB} = [\pi R^2 \times 1]\rho \quad \Rightarrow \mu_{CD} = 4\,\mu_{AB}$]

11. A composite string is made up ....

[Sol. $V_1 = \sqrt{\dfrac{T}{\mu}}$; $V_2 = \sqrt{\dfrac{T}{4\mu}}$



$V_2 < V_1$

$\Rightarrow$ 2^{nd} is denser $\Rightarrow$ phase change of π wave reflected from denser medium

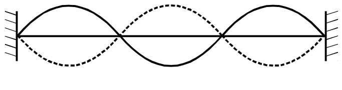
$\Rightarrow \quad A_r = \dfrac{V_2 - V_1}{V_2 + V_1} \times 6 = \dfrac{\dfrac{V_1}{2} - V_1}{\dfrac{V_2}{2} + V_1} \times 6$

$= -\,2\text{mm} \quad \Rightarrow \quad eq^n$

$\Rightarrow -\,(2\text{mm})\sin(5t - 40x)$ **Ans.**]

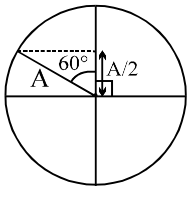
12. A string of length 3L is fixed at both ....

[Sol.



$3\dfrac{\lambda}{2} = 3L \quad \Rightarrow \quad \lambda = 2L$

$y = A_0 \sin kx \sin\left(\omega t + \dfrac{\pi}{2} + \dfrac{\pi}{3}\right)$



$= A_0 \sin\left(\dfrac{2\pi}{2L}\right) \times \left(\dfrac{L}{2}\right) \sin\left(\omega t + \dfrac{5\pi}{6}\right)$

$= A_0 \sin\left(\omega t + \dfrac{5\pi}{6}\right)$

13. An open pipe is suddenly closed ....

[Sol. $\dfrac{1}{2L} \times v + 100 = \dfrac{3}{4L}v$

$100 = \dfrac{3v}{4L} - \dfrac{2v}{4L} = \dfrac{v}{4L}$

$\Rightarrow \quad \dfrac{v}{2L} = 200$ Hz]

14. The fundamental frequency of ....

[Sol. $n_0 = \dfrac{v}{2\ell}$

$n_1 = \dfrac{v}{2(\ell/2 - \Delta\ell)} \qquad n_2 = \dfrac{v}{2(\ell/2 + \Delta\ell)}$

beat freq. = $n_1 - n_2$

$\Rightarrow \quad v\left[\dfrac{1}{\ell - 2\Delta\ell} - \dfrac{1}{\ell + 2\Delta\ell}\right]$

$= \left[\dfrac{(\ell + 2\Delta\ell) - (\ell - 2\Delta\ell)}{\ell^2 - 4\Delta\ell^2}\right] = v\dfrac{4\Delta\ell}{\ell^2 - \Delta\ell^2}$

$= \dfrac{8}{\ell}\dfrac{\Delta\ell v}{2\ell} = \dfrac{8\Delta\ell n_0}{\ell}$]

15. A glass tube of 1.0 meter length ....

[Sol. $\lambda = \dfrac{v}{f} = \dfrac{330}{500} = 0.66 \text{ m} = \dfrac{4\ell}{2n-1}$

$\Rightarrow \quad n = 3$]

16. The intensity of sound 10 m from ....

[Sol. $I \propto 1/d^2$

$130 = 10\,ln(I_1/I_0)$

$90 = 10\,ln(I_2/I_0)$

$4 = ln(I_1/I_2)$

$\dfrac{I_1}{I_2} = 10^4 = \dfrac{d_2^2}{d_1^2}$

$\Rightarrow \quad 10^4 = \dfrac{d_2^2}{10^2}$

$\Rightarrow \quad d_2 = 10^3$ m = 1000 m **]**

17. In a resonance tube experiment ....

[Sol. At the open end pressure node is present. So that
at $x = 0$ P should be zero and $v = f\lambda$,

for first over tone $\frac{3\lambda}{4} = 0.80$

$\Rightarrow$ $f = 300$

$\Rightarrow$ $\omega = 2\pi f = 600\ t$

Hence $P = A \sin \frac{15\pi}{8} x \cos 600\ t$]

18. A car blowing a horn of frequency ....

[Sol. Frequency observed by man is same as "observed" by wall and it reflects the same and as man and wall are relatively at rest, hence man observers same frequency of reflected sound. Hence no beat frequency]

19. A source of sound is moving with ....

[Sol. $\lambda_A = \left(10u - \frac{u}{2}\right)\frac{1}{f} = \frac{9.5u}{f}$

$\lambda_B = \left(10u + \frac{u}{2}\right)\frac{1}{f} = \frac{10.5u}{f}$

$\frac{\lambda_A}{\lambda_B} = \frac{\left(\frac{19}{2}\right)\left(\frac{u}{f}\right)}{\left(\frac{21}{2}\right)\left(\frac{u}{f}\right)} = \frac{19}{21}$ **Ans.]**

21. A coaxial cable having radius "a" of ....

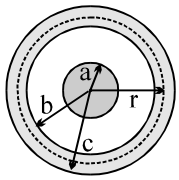
[Sol. By Ampere's law

$B(2\pi r) = \mu_0(i - i')$

Now, current density in the outer wire is constant

$$\frac{i}{\pi(c^2 - b^2)} = \frac{i'}{\pi(r^2 - b^2)}$$

$$i' = i\left(\frac{r^2 - b^2}{c^2 - b^2}\right)$$

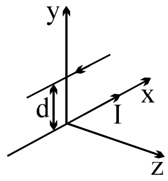


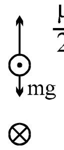
$$\therefore B = \frac{\mu_0 i}{2\pi r}\left[1 - \frac{r^2 - b^2}{c^2 - b^2}\right]$$

$$= \frac{\mu_0 i}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)]$$

22. A very long wire carrying current I ....

[Sol.





$\frac{\mu_0}{2\pi}\frac{i_1 i}{d}\hat{j}$

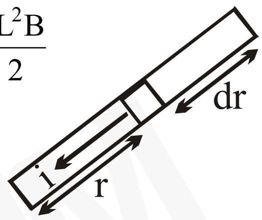
in y direction stable
in x direction neutral
in z direction unstable]

23. A thin uniform rod with negligible mass ......

[Sol. Torque due to magnetic force

$|d\vec{\tau}| = \int_0^L (idrB)r = \frac{iL^2B}{2}$

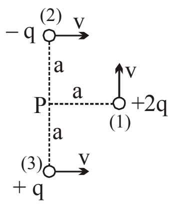
In equilibrium



$\frac{iL^2B}{2} = (kx) \times L \sin 30°$ or $x = \frac{5iLB}{8k}$]

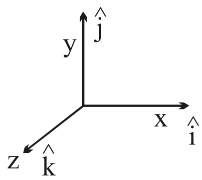
24. A charge $+2q$ moves vertically ....

[Sol. $B_1 = \frac{\mu_0}{4\pi}\frac{2qv}{a}(+\hat{k})$ $\vec{B}$ due to charge (1)



$B_2 = \frac{\mu_0}{4\pi}\frac{qv}{a}(+\hat{k})$

$B_3 = \frac{\mu_0}{4\pi}\frac{qv}{a}(+\hat{k})$



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\ $\vec{B} = \frac{4\pi_0 qv}{4\pi a^2} = \frac{\mu_0 qv}{\pi a^2}$]

25. A wire carrying current I has ....

[Sol. $-\frac{\mu I}{2a}\left(\frac{\theta}{2\pi}\right) + \frac{\mu_0 I}{4a\pi} = 0$ $\theta = 2$ radian]

26. Figure shows the path of an electron ....

[Sol. The magnetic for a on electron between (1) & (2) is downwards and between (3) & (4) plates upwards. (From the shape of the curve) so electric force must be opposite the magnetic force as the path is straight line.]

27. A particle of specific charge σ (q / m) ....

[Sol. Radius should be $r_2 - r_1$

$r = \frac{mu}{qB}$; $(r_2 - r_1) = \frac{u}{\sigma B}$ thus $u = \sigma B\,(r_2 - r_1)$]

28. A particle of mass m and charge q ....

[Sol. KE of particle = $\frac{1}{2}mv^2$ = qV

F_B = qvB (upward)

Net downward force = qE

As particle moves with constant velocity

$\therefore$ qvB = qE $\Rightarrow$ $v = \frac{E}{B}$

$\therefore V = \frac{m}{2q}\left(\frac{E}{B}\right)^2$]

29. Two particles having the same ....

[Sol. For a uniform helical path,

$T = \frac{2\pi m}{qB}$; pitch = $\frac{2\pi mv\cos\theta}{qB}$

$R = \frac{mv\sin\theta}{qB}$]

30. A bar magnet has coercivity ....

Sol. The bar magnet has coercivity $4\times10^3\,Am^{-1}$ i.e., it requires a magnetic intensity $H = 4\times10^3\,Am^{-1}$ to get demagnetised. Let i be the current carried by solenoid having n number of turns per metre length, then by definition H = ni. Here $H = 4\times10^3$ Amp turn metre-1

$n = \frac{N}{l} = \frac{60}{0.12} = 500$ turn metre-1

$\Rightarrow i = \frac{H}{n} = \frac{4\times10^3}{500} = 8.0\ A$

Hints & Solution

PART- II (CHEMISTRY)

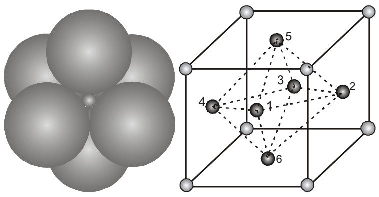
31. An element occurring …..

Sol. [1] There are two atoms in a *bcc* unit cell.

So, number of atoms in 12.08×10^{23} unit cells $=2\times12.08\times10^{23}=24.16\times10^{23}$ atom.

32. In octahedral holes (voids)……….

Sol. **Octahedral void (3-Dimensional 6 coordinate void)** The octahedral void is formed whenever two spheres are placed, one on top and the other below a square arrangement of spheres



33. If $C+O_2\rightarrow CO_2+94.2\,kcal$ ………….

Sol. [2] $C+O_2\rightarrow CO_2+94.2\ Kcal.$

…..(i)

$H_2+\frac{1}{2}O_2\rightarrow H_2O+68.3\ Kcal.$

….(ii)

On multiplication of *eq.* (ii) by 2 and than adding in *eq.* (i)

$C+2H_2+2O_2\rightarrow CO_2+2H_2O+230.8\ Kcal$ …(iii)

On subtracting *eq*. (iii) by following *eq*.

$CH_4+2O_2\rightarrow CO_2+2H_2O+210.8\ Kcal.$ we get,

$C+2H_2\rightarrow CH_4\ \ \Delta H=20\ Kcal.$

34. One mole of water at ……..

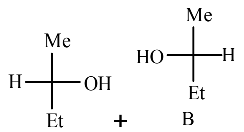
Sol. [4] The entropy change $=\frac{\text{heat of vaporis ation}}{\text{temperatur e}}$

Here, heat of vaporisation $=540\,cal/gm$

$=540\times18\,cal\,mol^{-1}$

Temperature of water $=100+273=373\,K$

$\therefore$ entropy change $=\frac{540\times18}{373}=26.06\,cal\,mol^{-1}K^{-1}$

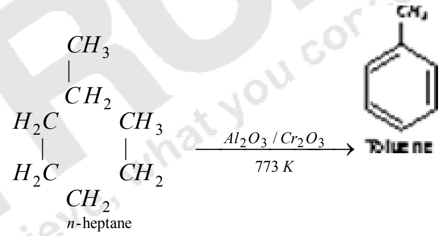
35. 

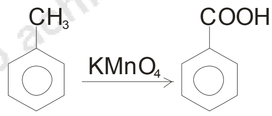
………..

Sol. I $\rightarrow$ only one product it is formed and inversion at chiral centre takes place hence it is S_N2

II $\rightarrow$ Racemic mixture is formed hence it is S_N1 reaction

36. *n*-heptane by ……………..

Sol [3] 



37. acetylene and ……………..

Sol. [3] Acetylene reacts with ammonical cuprous chloride to give brown *ppt* where as ethylene does not give this reaction.

38. The entropy values …….

Sol. [4] $\Delta S^{o}=2S^{o}_{HCl}-(S^{o}_{H_2}+S^{o}_{Cl_2})$

$=2\times186.7-(130.6+223.0)=19.8\,JK^{-1}mol^{-1}$

Entropy change for the reaction can be calculated

39. An exothermic ………..

Sol. [1] For exothermic reactions $H_p<H_R$.

For endothermic reactions $H_p>H_R$.

40. ΔH°_{f} (298 *K*) of …………

Sol. The standard enthalpy of formation of every element in its stable state of aggregation at one bar pressure and at specified temperature is assigned a zero value”.The specified temperature is usually taken as 25 °C.

A few exmaples are ΔH_f° (O_2, g) = 0

ΔH_f° (C, graphite) = 0 ΔH_f° (C, diamond) $\neq$ 0

ΔH_f° (Br_2, ℓ) = 0 ΔH_f° (S, rhombic) = 0

ΔH_f° (S, monoclinic) $\neq$ 0

ΔH_f° (P, white) = 0 ΔH_f° (P, black) $\neq$ 0

41. The bond …………

Sol. [3] Aim: $\frac{1}{2}H_2 + \frac{1}{2}Cl_2 \rightarrow HCl$

$$\Delta H = \sum B.E._{(Products)} - \sum B.E._{(Reactants)}$$

$$= B.E.(HCl) - \left[\frac{1}{2}B.E.(H_2) + \frac{1}{2}B.E.(Cl_2)\right]$$

$$= -103 - \left[\frac{1}{2}(-104) + \frac{1}{2}(-58)\right]$$

$$= -103 - (-52 - 29) = -22 \text{ kcal}.$$

42. At 300 *K*, the ……………..

Sol. [2] $\Delta G = \Delta H - T\Delta S$

For a sportaneous reaction, $\Delta G < 0$

When $\Delta H = +ve$ and $\Delta S = -ve$ than the reaction is non-spontaneous.

43. An ionic compound has ……

Sol [3] A atoms are at eight corners of the cube.

Therefore, the no. of A atoms in the unit cell = $\frac{8}{8} = 1$

. B atoms are at the face centre of six faces.

Therefore, its share in the unit cell = $\frac{6}{2} = 3$. C is present in octahedral voids Therefore, its share in the unit cell = The formula is AB_3C_4.

44. In CsCl structure, ……

Sol. [2] Cl^- ions in $CsCl$ adopt *BCC* type of packing.

In CsCl, simple cubic until cell is formed by Cl^- and cubical void is occupied by Cs^+ ion. Hence coordination number is 8

45. What is the correct ……………..

Sol [3]

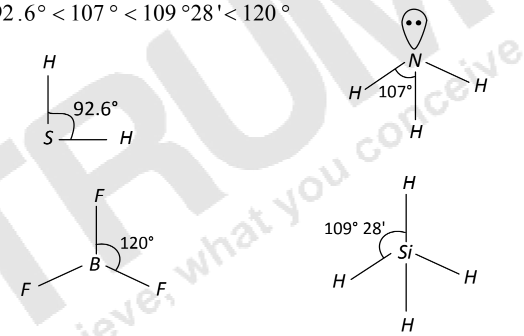
Atom/Ion	Hybridisation
NO_2^+	sp
SF_4	sp^3d with one lone pair of electron
PF_6^-	sp^3d^2

46. The correct order …………

Sol[3] The correct order of bond angle (Smallest first) is

$H_2S < NH_3 < SiH_4 < BF_3$

$92.6° < 107° < 109°28' < 120°$



47. 1-butyne reacts …………….

Sol. $CH_3 - CH_2 - C \equiv CH \xrightarrow[\text{alk. } KMnO_4]{\text{Cold}} CH_3CH_2COOH + CO_2$

48. N_2 and O_2 are converted ……………….

Sol. [2]In the conversion of O_2 into O_2^- bond order decreases

	Bond order
N_2	3
O_2	2
N_2^-	2.5
O_2^-	1.5

49. In a reaction …..

Sol $CH_2 = CH_2 \xrightarrow{HOCl} \underset{Cl}{\underset{|}{CH_2}} - \underset{OH}{\underset{|}{CH_2}} \xrightarrow{aq\ NaHCO_3}$

$$\underset{\text{Glycol}}{\begin{array}{l} CH_2 - OH \\ | \\ CH_2 - OH \end{array}}$$

50. In which of the following, ……….

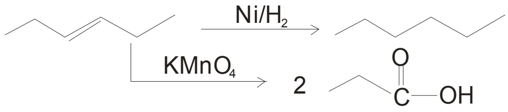
Sol $CH_3 - CH = CH - CH_3 + HBr \xrightarrow{\text{Peroxide}}$

$$\underset{\text{2-Bromobutane}}{CH_3 - CH_2 - \underset{Br}{\underset{|}{C}}H - CH_3}$$

Anti-markownikoff’s rule is not applicable to symmetrical alkenes.

51. A hydrocarbon X adds on ……..

Sol. As hydrocarbon uses 1 mole H_2 hence it has one π - bond, and also on reaction with $KMnO_4$ it provides only one carboxylic acid hence it is symmetrical hydrocarbon



52. The product of following reaction ……………

Sol. $CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH=CH_2 \xrightarrow[ii)\ NaBH_4]{I)Hg(OAc)_2;H_2O}$

$CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-\underset{\underset{OH}{|}}{CH}-CH_3$

$CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH=CH_2 \xrightarrow[(ii)\ H_2O_2/OH^-]{(i)\ B_2H_6/THF}$

$CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_2-CH_2OH$

53. What is the end product ……..

Sol[3] $CaC_2 \xrightarrow{H_2O} \underset{\text{Acetylene}}{HC\equiv CH} \xrightarrow[Hg^{+2}]{Dil.\ H_2SO_4} \underset{\text{Acetaldelyde}}{CH_3CHO}$

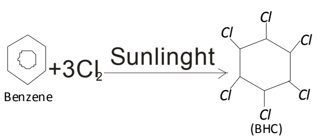
$\xrightarrow[H_2]{Ni} \underset{\text{Ethyl alcohol}}{CH_3CH_2-OH}$

54. Benzene reacts with ……

Sol Benzene reacts with CH_3COCl in the presence of $AlCl_3$ to give $C_6H_5COCH_3$

by friedal craft rection. now ring is strongly deactivated due to strong electron withdrawing nature of CH_3CO- group hence there is no further reaction with CH_3COCl in the presence of $AlCl_3$.

55. Gammexane is …………….



Sol.

56. Considering entropy (S) …………….

Sol. [3] $\Delta S_{system} + \Delta S_{surroundings} > O$ (for spontaneity)

Because of $\Delta S = R \ln \frac{V_2}{V_1}$

Here the volume of gas increase from V_1 to V_2 at constant temperature T.

The total increase in entropy of the system and its surrounding during the spontaneous process of expansion considered above is, thus $R \ln\left(\frac{V_2}{V_1}\right)$

since $V_2 > V_1$ it is obvious that the spontaneous (irreversible) isothermal expansion of a gas is accompanied by an increase in the entropy of the system and its surrounding considered together.

$\Delta S_{system} + \Delta S_{surroundings} > 0$.

57. The absolute ………………..

Sol. [1] Heat of neutralisation will be less than $-57.33\ kJ/mole$ because some amount of this energy will be required for the dissociation of weak base (MgO)

$H^+ + OH^+ \rightarrow H_2O$, $\Delta H = -57.33$ kJ as MgO + $H_2O \rightarrow Mg(OH)_2$ is a weak base hence on neutralization it should release energy which is less than (2 × 57.33) kJ as some of energy is utilized in the dissociation if base.

58. Due to Frenkel defect…….

Sol. [3] Since no ions are missing from the crystal as a whole, there is no effect on density.

This defect is shown by ionic solids. The smaller ion (usually cation) is dislocated from its normal site to an interstitial site It creates a *vacancy defect* at its original site and an **interstitial defect** at its new location. Frenkel defect is also called **dislocation defect**. It does not change the density of the solid. Frenkel defect is shown by ionic substance in which there is a large difference in the size of ions, for example, ZnS, AgCl, AgBr and AgI due to small size of Zn^{2+} and Ag^+ ions

59. Which of the following ………

Sol [2] With calculated amount of Grignard reagent, acetyl chloride forms ketones.

$CH_3COCl + XMgCH_3 \rightarrow CH_3COCH_3 + Mg\begin{smallmatrix} Cl \\ X \end{smallmatrix}$

PART- III (MATHS)

61. A survey shows that 63% ……………..

Sol. Let 100 be the total number of population

Let A be the set of people who like chese and b, the set if people who like butter.

By data n(A) = 63. n(B) = 76, n $(A \cap B) = x$

Now $n(A \cup B) \le 100$

$\Rightarrow$ n(A) + n(B) – $n(A \cap B) \le 100$

i.e., 63 + 76 – x $\le$ 100 $\Rightarrow$ x $\ge$ 39

also $A \cap B \subseteq A \Rightarrow n(A \cap B) \le n(A)$

$\Rightarrow$ x $x \le 63$

Combining (1) and (2), we get

39 $\le$ x $\le$ 63

62. If tanx – $\tan^2$x = 1, ………..

Sol. **[4]** $\tan^4 x - 2\tan^3 x - \tan^2 x + 2\tan x + 1$

$\tan^4 x + \tan^2 x - 2\tan^3 x + 2\tan x - 2\tan^2 x + 1$

= $(\tan^2 - \tan x)^2 + 2(\tan x - \tan^2 x) + 1 = 4$.

63. Joint equation of pair of ………………

Sol. **[3]** Homogeneous part of the given equation is $y^2 + 3xy = 0$, which represents straight lines y = 0 and y + 3x = 0. Now lines perpendicular to these lines are x = 0 & x – 3y = 0

So combined equation of above lines is $x^2 - 3xy = 0$.

64. If $\hat{a}, \hat{b}, \hat{c}$ are ………

Sol. $[\hat{a}\ \vec{p}\ \vec{q}]$ = projection of $\vec{p} \times \vec{q}$ in the direction of $\hat{a}$. Hence the given vector is $\vec{p} \times \vec{q}$

65. Two finite sets ……..

Sol According to the given condition, we have $2^m = 2^n + 56$ which is satisfied if m = 6 and n = 3

66. The minimum value ……………..

Sol **[1]** $3^{\sin^6 x}$ and $3^{\cos^6 x}$ are positive numbers .

And A.M. $\ge$ G. M.

$$\frac{3^{\sin^6 x} + 3^{\cos^6 x}}{2} \ge \sqrt{3^{\sin^6 x + \cos^6 x}} \quad .$$

. . (1)

$$= 2\sqrt{3^{1-\frac{3}{4}\sin^2 2x}}$$

$$\Rightarrow 3^{\sin^6 x} + 3^{\cos^6 x} \ge 2 \cdot \sqrt{3^{\frac{1}{4}}}$$

$= 2.3^{1/8}$

Alternate:

Clearly the equality (1) holds for $3^{\sin^6 x} = 3^{\cos^6 x}$

$$\Rightarrow \sin^6 x = \cos^6 x = \left(\frac{1}{\sqrt{2}}\right)^6 = \frac{1}{8}$$

67. If cos25° + sin 25° = k, …………….

Sol **[1]** cos 20°= cos(45° - 25°)

$$= \frac{1}{\sqrt{2}}(\cos 25° + \sin 25°)$$

68. A non-zero vector $\vec{a}$ ……

Sol Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\vec{a}$, $\hat{i}$, $\hat{i} + \hat{j}$ are coplanar and $\vec{a}, \hat{i} - \hat{j}$, and $\hat{i} + \hat{k}$ are coplanar

$$\Rightarrow \begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0 \text{ and } \begin{vmatrix} x & y & z \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ z = 0 & -x –y + z =0

$\Rightarrow$ z = 0 & y = -x

$\Rightarrow \vec{a} = x\hat{i} - y\hat{j}$

now, clearly angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ is $\frac{\pi}{4}$.

69. A and B are two ....

Sol. The number of elements will be 2^{mm}

Hence the number of subsets of a × B is 2^{12}

70. Consider the family of ....

Sol.[2] If lines (i) and (ii) are same then

$$\frac{2\lambda+1}{\mu+3}=\frac{3\lambda+1}{2\mu+2}=\frac{5\lambda+1}{6\mu+4}$$

Solve it value of $\lambda = -\frac{3}{7}$

Required line x – 2y + 8 = 0

71. Value of $\sin^4\frac{\pi}{8}$ + $\sin^4\frac{3\pi}{8}$ + $\sin^4$...........

Sol **[1]** $\sin^4\frac{\pi}{8}+\sin^4\frac{7\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}$

$= 2\left(\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}\right)=2\left(\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}\right)$

$= 2\left(1-2\sin^2\frac{\pi}{8}.\cos^2\frac{\pi}{8}\right)=2\left(1-\frac{1}{2}\sin^2\frac{\pi}{4}\right)$

$= 2\left(1-\frac{1}{4}\right)=\frac{3}{2}$.

72. The point $(a^2, a + 1)$ lies in the ....

Sol.[1] Since origin and point $(a^2, a + 1)$ lie on the same side of both the lines , so

$3a^2 - (a + 1) + 1 > 0$, $a(3a - 1) > 0$ gives

$a \in (-\infty, 0) \cup \left(\frac{1}{3},\infty\right)$

and $a^2 + 2(a + 1) - 5 < 0$

$a^2 + 2a - 3 < 0 \Rightarrow (a - 1)(a + 3) < 0 \Rightarrow a \in (-3, 1)$

By both the inequalities $a \in (-3, 0) \cup \left(\frac{1}{3},1\right)$

73. If $\bar{a}$ is a unit ....

Sol $(\bar{a}\times\bar{x})+\bar{b}=\bar{x} \Rightarrow \bar{a}\times(\bar{a}\times\bar{x})+(\bar{a}\times\bar{b})=\bar{a}\times\bar{x}$

$(\bar{a}\cdot\bar{x})\bar{a}-(\bar{a}\cdot\bar{a})\bar{x}+(\bar{a}\times\bar{b})=\bar{x}-\bar{b}$

projection of $\bar{x}$ along $\bar{a}$ is 2 units

$\Rightarrow \frac{(\bar{a}\cdot\bar{x})}{|\bar{a}|}=2 \quad \Rightarrow \bar{a}\cdot\bar{x}=2$

So $\bar{x}=\frac{1}{2}\left[2\bar{a}+\bar{b}+(\bar{a}\times\bar{b})\right]$.

74. The relation R defined ....

Sol. A = $\{x : |x| < 3, x \in Z\}$

= $\{-2,-1,0,1,2\}$

R = $\{(x,y) : y = |x|\}$

R = {(-2,2), (-1,1),(0,0),(1,1), (2,2)} is obviously a relation efined on A.

75. If $\tan\theta=\sqrt{n}$, for some ....

Sol. $\sec 2\theta = \frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+n}{1-n}$, where n is a non-square natural number, so $1 - n \neq 0$. Hence $\sec 2\theta$ is a rational number.

76. If lines x + 2y – 1 = 0, ...

Sol.[3] lines are concurrent

$$\begin{vmatrix} 1 & 2 & -1 \\ a & 1 & 3 \\ b & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow$ 7b – 3a + 5 = 0

locus of (a, b) is 3x – 7y = 5

least distance from (0, 0) = length of perpendicular from (0, 0) $=\frac{5}{\sqrt{58}}$

77. If $0 < \alpha < \frac{\pi}{6}$ …….

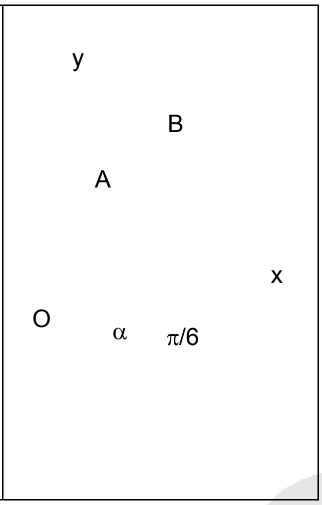
Sol.

On the graph of the y = sin x, let $A \equiv (\alpha, \sin\alpha)$, $B = \left(\frac{\pi}{6}, \sin\frac{\pi}{6}\right)$.

Clearly slope of OA > slope of OB

So, $\frac{\sin\alpha}{\alpha} > \frac{\sin\frac{\pi}{6}}{\frac{\pi}{6}} = \frac{3}{\pi}$

$\Rightarrow \frac{\alpha}{\sin\alpha} < \frac{\pi}{3}$.



78. $\tan\frac{\pi}{8}$ is the …………….

Sol. Let $\tan\frac{\pi}{8} = x$

$\Rightarrow \frac{\pi}{8} = \tan^{-1} x$

$\Rightarrow 4\tan^{-1} x = \frac{\pi}{2}$

$\Rightarrow 2 \times \tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\frac{\frac{4x}{1-x^2}}{1-\left(\frac{2x}{1-x^2}\right)^2} = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\frac{4x(1-x^2)}{1-6x^2+x^4} = \frac{\pi}{2}$

$\Rightarrow x^4 - 6x^2 + 1 = 0$

79. The following ……

Sol. R is not reflexive, for $|a - a| = 0 \Rightarrow |a - a| \not> 0$

R is symmetric, since $|a - b| > 0$

$\therefore$ a R b = b R a $\Rightarrow$ R is symmetric

R is not transitive, for consider $2,5,2 \in R$

Then 2 R 5 $\Rightarrow |2 - 5| = 3 > 0$ and 5 R 2

$\Rightarrow |5 - 2| = 3 > 0$

But 2 R5, 5 R 2 $\not\Rightarrow$ 2 R 2 for $|2 - 2| = 0$ and not > 0.

80. If the ………………

Sol. $\vec{d}$ is the angle bisector of $\vec{a}$ and $\vec{b}$

Thus $m\vec{d} = \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ where m > 0, a scalar

$\Rightarrow m\vec{d} = \frac{-4i+3k}{5} + \frac{14i+2j-5k}{15} = \frac{-12i+9k+14i+2j-5k}{15} = \frac{2i+2j+4k}{15}$

Therefore a unit vector in the direction of $\vec{d}$ is

$\frac{\vec{d}}{|\vec{d}|} = \frac{\frac{2}{15}\left(\hat{i}+\hat{j}+2\hat{k}\right)}{\frac{2}{15}\sqrt{1+1+4}} = \frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

$\Rightarrow \vec{d} = \hat{i}+\hat{j}+2\hat{k}$.

81. N is the set of ……….

Sol. Let$(a,b) \in N \times N$

Since ab = ba, this implies (a,b) R (a,b)

$\therefore$ R is reflexive

Let (a,b) (R) (c,d) $\Rightarrow$ ad = bc $\Rightarrow$ da = cb

Or cb = da

$\Rightarrow$ (c,d) R (a,b) $\Rightarrow$ R is symmetric

Let (a,b) R (c,d) and (c,d) R (e,f)

$\Rightarrow$ ad = bc,cf = de $\Rightarrow$ (ad) (cf) = (bc) (de)

$\Rightarrow$ af = be $\Rightarrow$ (a,b) R (e,f)

$\therefore$ R is transitive. Hence R is an equivalence relation

82. The line $3x - 4y + 7 =$ …………

Sol.[1] As $(-1, 1)$ is a point on $3x - 4y + 7 = 0$, the rotation is possible.

Slope of the given line $= \frac{3}{4}$.

Slope of the line in its new position $= \frac{\frac{3}{4}-1}{1+\frac{3}{4}} = -\frac{1}{7}$

The required equation is $y - 1 = -\frac{1}{7}(x + 1)$

or $7y + x - 6 = 0$.

83. $\sin x + \cos x = y^2 - y + \ldots\ldots\ldots\ldots$

Sol. **(4)** $y^2 - y + a = \left(y - \frac{1}{2}\right)^2 + a - \frac{1}{4}$.

Since $-\sqrt{2} \le \sin x + \cos x \le \sqrt{2}$, given equation will have no real value of x for any y if $a - \frac{1}{4} > \sqrt{2}$

i.e. $a \in \left(\sqrt{2} + \frac{1}{4},\ \infty\right) \Rightarrow a \in (\sqrt{3}, \infty)$ (as $\sqrt{2} + 1/4 < \sqrt{3}$)

84. A straight line L with negative ……

Sol. **[3]** Let the equation of the line L be

$y - 2 = m(x - 8), m < 0$

coordinates of P and Q are $P\left(8 - \frac{2}{m}, 0\right)$

and $Q(0, 2 - 8m)$

So $OP + OQ = 8 - \frac{2}{m} + 2 - 8m$

$= 10 + \frac{2}{-m} + 8(-m)$

$\ge 10 + 2\sqrt{\frac{2}{-m} \times 8(-m)} \ge 18$

absolute min. value of $OP + OQ = 18$.

85. The length of the …

Sol. One diagonal vector is $6\vec{a} - \vec{b}$

Length $= \sqrt{36a^2 - b^2 - 12\vec{a}.\vec{b}}$

$= \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times 1/\sqrt{2}} = 15$

Other diagonal is $= 4\vec{a} + 5\vec{b}$, its length is

$\sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

86. Let $A = [-1,1], B = [-1,1], \ldots\ldots\ldots\ldots$

Sol. $R_1 = \{(x,y) \in A \times B : x^2 + y^2 = 1\}$

Since $x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

(or) $y = \pm\sqrt{1 - x^2}$

For $x = 0 \in A, y = \pm 1$. i.e., $(0,1)\ (0,-1) \in R_1$ i.e., one element $0 \in A$ is mapped onto two elements $-1, 1 \in B$. So R_1 is not a function

$R_2 = \{(x,y) \in A \times C : x^2 + y^2 = 1\}$

Here $x^2 + y^2 = 1 \Rightarrow y = \pm\sqrt{1 - x^2}$

But $y \in C$ ie $[0, \infty]$. i.e., $y \ge 0$

$\Rightarrow y = \sqrt{1 - x^2}$

$\therefore R_2$ is a function from A onto C.

87. The values of k, ……..

Sol **[2]** Given equations will be valid when $-1 \le (k^2 - 4)^2 + 1 \le 1$ . . . . (1)

and $-1 \le (k + 2) \le 1$ . . . (2)

From (1), $(k^2 - 4)^2 \le 0 \Rightarrow k^2 - 4 = 0 \Rightarrow k = \pm 2$

But $k = 2$ is not satisfying the equation (2). Therefore $k = -2$.

88. The vertices of a …………….

Sol. **[2]**We have AB = 10, BC = 5. By bisector property $\frac{AD}{DC} = \frac{10}{5} = \frac{2}{1}$

$\Rightarrow$ co-ordinates of D are $\left(\frac{1}{3}, \frac{1}{3}\right)$ whence equation of BD is $y - 1 = \frac{1/3 - 1}{1/3 - 5}(x - 5)$ or $x - 7y + 2 = 0$.

89. If $\vec{a}$ and $\vec{b}$ are two …………

Sol. **(1)** $\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} + \vec{b}\right)$

$= \vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = 2\left(1 + \vec{a} \cdot \vec{b}\right)$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos 60^0$

$= 1 \times 1 \times \frac{1}{2} = 2\left(1 + \frac{1}{2}\right) = 3 > 1$.

90. The set $(A \cap B^c)^c \cup (B \cap C)$ ………….

Sol. **Let S =** $\left(A \cap B^c\right)^c \cup \left(B \cap C\right)$

$\Rightarrow$ **S =** $\left(A^c \cup B\right) \cup \left(B \cap C\right)$

{de Morgan's law}

$\Rightarrow$ **S =** $A^c \cup \left(B \cup \left(B \cup\right)\right)$

$\therefore S = A^c \cup B$