MODEL PAPER - I

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 100***

General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION A

Question number 1 to 10 carry one mark each.

1. Find the value of x, if

$$\begin{pmatrix} 5x + y & -y \\ 2y - x & 3 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -3 & 3 \end{pmatrix}$$

2. Let * be a binary operation on N given by a * b = HCF (a, b), a, b $\in$ N. Write the value of 6 * 4.

3. Evaluate : $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1 - x^2}} dx$

4. Evaluate : $\int \frac{\sec^2(\log x)}{x} dx$

5. Write the principal value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$.

6. Write the value of the determinant :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

7. Find the value of x from the following :

$$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$$

8. Find the value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{k} \times \hat{i}) \cdot \hat{j}$

9. Write the direction cosines of the line equally inclined to the three coordinate axes.

10. If $\vec{p}$ is a unit vector and $(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$, then find $|\vec{x}|$.

SECTION B

Question numbers 11 to 22 carry 4 marks each.

11. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of (a) the perimeter, (b) the area of the rectangle.

OR

Find the intervals in which the function f given by f(x) = sin x + cos x, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

OR

12. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

13. Consider $f : R - \left\{\frac{-4}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$ defined as $f(x) = \frac{4x}{3x + 4}$

Show that f is invertible. Hence find f^{-1}.

14. Evaluate : $\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}$.

OR

Evaluate : $\int x \sin^{-1} x \, dx$.

15. If $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$, show that $(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

16. The probability of student A passing an examination is $\frac{3}{5}$ and of student B passing is $\frac{4}{5}$. Find the probability of passing the examination by

(i) both the students A and B

(ii) atleast one of the students A and B.

What ideal conditions a student should keep in mind while appearing in an examination?

17. Using properties of determinants, prove the following :

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

18. Solve the following differential equation :

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right).$$

19. Solve the following differential equation :

$$\cos^2 x \cdot \frac{dy}{dx} + y = \tan x.$$

20. Find the shortest distance between the lines

$$\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k}$$

$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right).$$

21. Prove the following :

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right] = \frac{x}{2},\ x \in \left(0, \frac{\pi}{4}\right).$$

OR

Solve for x :

$$2 \tan^{-1} (\cos x) = \tan^{-1} (2 \operatorname{cosec} x)$$

22. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

OR

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are three coplanar vectors. Show that $\left(\vec{a} + \vec{b}\right)$, $\left(\vec{b} + \vec{c}\right)$ and $\left(\vec{c} + \vec{a}\right)$ are also coplanar.

SECTION C

Question number 23 to 29 carry 6 marks each.

23. Find the equation of the plane determined by the points A (3, –1, 2), B (5, 2, 4) and C (–1, –1, 6). Also find the distance of the point P(6, 5, 9) from the plane.

24. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

25. Evaluate : $\displaystyle\int_0^{\pi} \frac{x\,dx}{a^2 \cos^2 x + b^2 \sin^2 x}$.

26. Two schools A and B decided to award prizes to their students for three values – honesty, regularity and discipline. School A decided to award Rs. 11000 for three values to 5, 4 and 3 students respectively while school B decided to award Rs. 10700 for three values to 4, 3 and 5 students respectively. If all three prizes together amount to Rs. 2700, then

(i) Represent the above situation in matrix form and solve it by matrix method.

(ii) Which value you prefer to be awarded most and why?

OR

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}.$$

27. Coloured balls are distributed in three bags as shown in the following table :

Bag	*Colour of the Ball*		
	Red	*White*	*Black*
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and then two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from bag I?

28. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that be can sell all the items that he can buy, how should he invest his money in order to maximise the profit? Formulate this as a linear programming problem and solve it graphically. What values are being promoted?

29. If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

OR

A tank with rectangular base and rectangular sides open at the top is to be constructed so that it's depts is 2m and volume is $8m^3$. If building of tank cost Rs. 70 per sq metre for the base and Rs. 45 per sq metre for the sides. What is the cost of least expensive tank.