
CBSE Sample Paper-05 (Solved)
Mathematics
Class – XII

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- d) Use of calculators is not permitted.

Section A

1. If a matrix has 12 elements, what are the possible orders it can have?
2. Prove that $A - A'$ is skew symmetric.
3. Find the values of x, y, z s.t $\begin{bmatrix} x-y & 0 & 0 \\ z & 6 & 0 \\ 0 & 0 & 2y \end{bmatrix}$ is a scalar matrix?
4. Is R defined on the set $A = \{1, 2, 3, \dots, 14, 15\}$ defined as $R = \{(x, y) : 3x - y = 0\}$ reflexive?
5. Find the angle between the vectors $\vec{a} = 4i + 4j$ and $\vec{b} = 4i - 2j$.
6. Evaluate $\sin^{-1}\left(\frac{1}{6}\right) + \cos^{-1}\left(\frac{1}{6}\right)$?

Section B

7. If $A = \begin{bmatrix} 0 & -\tan \alpha / 2 \\ \tan \alpha / 2 & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that
$$I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$
8. Find the equations of the tangent and the normal to the curve $x = \cos t, y = \sin t$ at $t = \frac{\pi}{4}$.

9. Show that the relation R in the set \mathbb{Z} of integers given by $R=\{(a,b):7 \text{ divides } a-b\}$ is an equivalence relation.
10. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots}}}$ prove that $(2y-1) \frac{dy}{dx} = \frac{1}{x}$.
11. Solve: $3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1-x^2}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$
12. Find a unit vector perpendicular to each of the vectors $(\vec{a}-\vec{b})$ and $(\vec{a}+\vec{b})$ where $\vec{a} = i + j + k$ and $\vec{b} = i + 2j + 3k$.
13. If A and B are independent events such that $P(A \cup B) = 0.6$, $P(A) = 0.2$. Find $P(B)$
14. The relation between the total cost y and the total output x is given by $y = \frac{3x(x+7)}{x+5} + 5$. Prove that the marginal cost continuously falls as output increases.
15. Solve $\frac{dy}{dx} + \frac{2y}{3} = \frac{x}{\sqrt{y}}$
16. For any two vectors \vec{a} and \vec{b} , prove that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.
17. Integrate $\int \frac{e^x}{e^{2x} - 4} dx$. Write ant points for promoting national integration.
18. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2i + 2j - 3k) = 7$, $\vec{r} \cdot (2i + 5j + 3k) = 9$ and the point $(2, 1, 3)$.
19. Find the equation of the plane passing through the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point $(0, 7, -7)$. Show that the line $x = \frac{7-y}{3} = \frac{z+7}{2}$ lies on the plane.

Section C

20. Prove that the volume of the largest cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ of the volume of the sphere.

21. Solve the following system of equations using matrix method

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

22. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per kg of vitamin A and 2 unit per kg of vitamin C. Food 1 costs Rs.50 per kg and Food 2 costs Rs.70 per kg. Using linear programming, find the minimum cost of such a mixture.
23. Draw a rough sketch of the region $\{(x, y) : y^2 < 4x, 4x^2 + 4y^2 \leq 9\}$ and find the area enclosed.
24. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assume that a student who guesses the answer would answer correctly with probability $\frac{1}{4}$. What is the probability that a student knows the answer, given that he has answered it correctly.

25. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

26. Integrate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x) dx$.