T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2016

Paper I – Code 0

PART I

1	2	3	4	5
A	**B**	**D**	**D**	**D**

6	7	8	9	10	11	12	13
A, D	**B, C**	**A, B, D**	**B, D**	**B, C, D**	**A, C, D**	**A, B, C, D**	**D**

14	15	16	17	18
6	**9**	**3**	**8**	**9**

Section I

1. For $\alpha = 45° = i$
   $r = 30°$. When ray graces PR after separation
   $c = 45°$
   ext. L = 45 + 30 = 75°
   $\theta = 180 - (75 + 90) = 15°$

2. $eV_0 = \frac{hc}{\lambda} - \phi$

   $e(2) = \frac{hc}{0.3 \times 10^{-6}} - \phi$

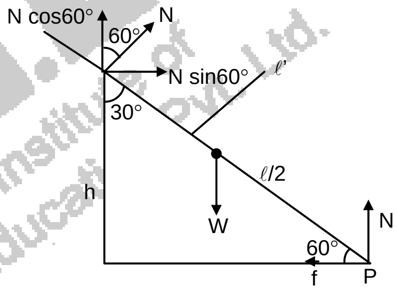
   $e(1) = \frac{hc}{0.4 \times 10^{-6}} - \phi$

   $e = hc\left[\frac{1}{0.3} - \frac{1}{0.4}\right] \times 10^6$

   $e = hc \times \frac{0.1}{0.3 \times 0.4} \times 10^6$

   $h = \frac{1.6 \times 10^{-19} \times 0.3 \times 0.4}{3 \times 10^8 \times 0.1 \times 10^6}$
   $= 0.64 \times 10^{-33}$
   $= 6.4 \times 10^{-34}$

3. $\frac{mc\Delta\theta}{t} = \frac{120 \times 4200 \times (30 - 10)}{3 \times 60 \times 60}$
   = 933 W
   P = 3 kW + 933 W
   = 3933 W

4. 

   $\cos 30° = \frac{h}{\ell'} = \frac{\sqrt{3}}{2}$

   $\ell' = \frac{2h}{\sqrt{3}}$

   $N\cos 60° + N = 16 = \frac{3N}{2} \Rightarrow N = \frac{32}{3}$

   $N \sin 60° = f$

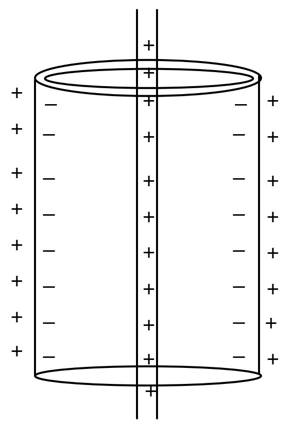
   $16 \times \frac{2}{3} \times \frac{\sqrt{3}}{2} = f \Rightarrow f = \frac{16\sqrt{3}}{3}$ newton

   Taking torque about P,

   $N \times \ell' = W \times \frac{\ell}{2} \cos 60°$

   $\frac{32}{3} \times \frac{2h}{\sqrt{3}} = 16 \times \frac{\ell}{2} \times \frac{1}{2} \Rightarrow \frac{h}{\ell} = \frac{3\sqrt{3}}{16}$

5.



It is a case of discharging of a capacitor in the resistor.

$Q = Q_0 e^{-t/RC}$

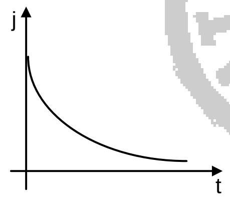
Dividing by ℓ,

$\frac{Q}{\ell} = \frac{Q_0}{\ell} e^{-t/RC}$

$\lambda' = \lambda e^{-t/RC}$

$j = \sigma E = \frac{\sigma \times \lambda'}{2\pi\varepsilon r} = \frac{\sigma\lambda e^{-t/RC}}{2\pi\varepsilon r}$ (at a point)

Hence



Section II

6. $\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right) = \frac{1}{f}$

$\frac{1}{60} - \frac{-1}{-30} = \frac{1}{f}$ $\qquad$ $\frac{1}{f} = \frac{60+30}{60\times 30}$

f = 20 cm

(D) correct

$\frac{1}{+10} + \frac{1}{-30} = \frac{1}{f'} \Rightarrow f' = 15$ cm

Since convex mirror, (c) wrong

Radius of curvature $R_1 = 2f' = 30$ cm

$\Rightarrow$ (B) wrong

$(\mu - 1)\times\left(\frac{1}{R_1}\right) = \frac{1}{f}$

$(\mu - 1)\times\frac{1}{30} = \frac{1}{20}$

$\Rightarrow \mu - 1 = \frac{3}{2} = 1.5 \Rightarrow \mu = 2.5$

$\Rightarrow$ (A) correct

7. B, C

8. $\bar{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$\bar{v} = 3t^2\alpha\hat{i} + 2t\beta\hat{j}$

$\bar{a} = 6t\alpha\hat{i} + 2\beta\hat{j}$

$\bar{v} = 3\times 1^2 \times \frac{10}{3}\hat{i} + 2\times 1\times 5\hat{j}$

$= 10\hat{i} + 10\hat{j}$ (A) correct

$\bar{F} = m\bar{a} = 0.1\times\left[6\times 1\times\frac{10}{3}\hat{i} + 2\times 5\hat{j}\right]$

$= \left(2\hat{i} + \hat{j}\right)$ (C) Wrong

$\bar{L} = \bar{r}\times\bar{p} = \left(\alpha\hat{i} + \beta\hat{j}\right)\times 0.1\left(3\alpha\hat{i} + 2\beta\hat{j}\right)$

$= 0.1\left[2\alpha\beta\hat{k} - 3\alpha\beta\hat{k}\right]$

$= 0.1\alpha\beta\ \left(-\hat{k}\right)$

$= 0.1\times\frac{10}{3}\times 5\left(-\hat{k}\right) = -\frac{5}{3}\hat{k}$

(B) correct

$\bar{\tau} = \bar{r}\times\bar{F} = \left(\alpha\hat{i} + \beta\hat{j}\right)\left(2\hat{i} + \hat{j}\right)$

$= \alpha\hat{k} - 2\beta\hat{k} = \left(\frac{10}{3} - 2\times 5\right)\hat{k}$

$= -\frac{20}{3}\hat{k}$

(D) correct

9. $E = \text{farad/m} = \frac{\text{coulomb}}{\text{volt m}} = \frac{\text{coulomb}^2}{\text{J m}}$

k_BT = energy = joule

A. $\sqrt{\frac{\text{coloumb}^2}{m^3\times\frac{\text{coulomb}^2}{\text{J m}}\times J}} = \frac{1}{m}$ $\Rightarrow$ WRONG

B. $\sqrt{\frac{\text{coloumb}^2}{\text{J m}}} = \frac{J}{\frac{1}{m^3}\times\text{coulomb}^2} = m$

$\Rightarrow$ CORRECT

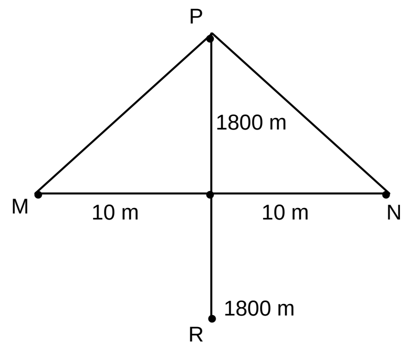
C. $\sqrt{\frac{\text{coloumb}^2}{\frac{\text{coulomb}^2}{J\times m}\times\left(\frac{1}{m^3}\right)^{2/3}\times J}} = m^{3/2}$

$\Rightarrow$ WRONG

D. $\sqrt{\frac{\text{coloumb}^2}{\frac{\text{coulomb}^2}{\text{J m}}\times\left(\frac{1}{m^3}\right)^{1/3}\times J}} = m$

$\Rightarrow$ CORRECT

10.



The component of the car's velocity along PM and PN are to be considered.

$f_1' = f_1\left[\frac{c+v_0'}{c}\right]$ $f_2' = f_2\left[\frac{c+v_0'}{c}\right]$

Since the distances PQ and RQ are large, the component velocity is nearly equal to the car's velocity.

$v_P = f_1' - f_2' = (f_1 - f_2)\left(\frac{c+v_0'}{c}\right)$ (approaching)

$v_R = f_1'' - f_2'' = (f_1 - f_2)\left(\frac{c-v_0'}{c}\right)$ (Receding)

$v_Q = (f_1 - f_2)$ since motion is perpendicular to MN

$v_P + V_R = (f_1 - f_2)\left[\frac{c+v_0}{c} + \frac{c-v_0'}{c}\right]$

$= v_Q \times 2 \Rightarrow$ (c) correct

(D) Correct since component velocity is nearly the same as car's velocity. Hence beat frequency is nearly constant.

(B) correct since approaching car suddenly becomes receding car.

11. Snell's law

$n_1 \sin\theta_i = n_2 \sin\theta_f \Rightarrow$ (C) correct

θ_f depends on n_2, but ℓ is independent of $n_2 \Rightarrow$ (D) correct.

(A) correct as shift depends on R.I of slab.

(B) wrong since if $n_1 = n_2$, $n_1\sin\theta_i = 0$ which need not be the case

12. $r_n = r_0\frac{n^2}{Z}$

$\frac{r_{n+1}-r_n}{r_n} = \frac{(n+1)^2 - n^2}{n^2}$ independent of Z

$\Rightarrow$ (A) correct

$= \frac{2n+1}{n^3}$

$\approx \frac{2n}{n^2} \approx \frac{1}{n} \Rightarrow$ (B) correct

$E_n = -Rhc\frac{Z^2}{n^2}$

$$\frac{E_{n+1}-E_n}{E_n} = \frac{\frac{-1}{(n+1)^2}+\frac{1}{n^2}}{\frac{1}{n^2}} = \frac{-n^2+(n+1)^2}{n^2(n+1)^2}$$

$= \frac{2n+1}{n^2(n+1)^2} \approx \frac{2n}{n^2n^2} \approx \frac{1}{n^3}$ (C) correct

$L_n = \frac{nh}{2\pi}\ \frac{L_{n+1}-L_n}{L_n} = \frac{(n+1)-n}{n} = \frac{1}{n}$

(D) correct

13. Due to non-uniform evaporation, the resistance of the filament is not uniform along its length. Hence heating is not uniform $\Rightarrow$ (A) WRONG

$R = \frac{\rho\ell}{A}$, A decreases with evaporation $\Rightarrow$ R increases $\Rightarrow$ (B) WRONG.

(D) $P = \frac{V^2}{R}$, R increases with time $\Rightarrow$ (D) correct.

(C) As it consumes less power, lesser heating and lesser temperature. Hence by Wien's law λ corresponding to maximum intensity increases. That is, frequency decreases $\Rightarrow$ (C) wrong.

Section III

14. $E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{970\times10^{-10}\times1.6\times10^{-19}}$

= 12 eV

$\Delta E = 13.6 - 12 = 1.6$

$\frac{13.6}{n^2} = 1.6 \Rightarrow n = 3$

Transitions = 6

15. ${}^{12}_{5}B \rightarrow {}^{12}_{6}C + e^-$

12.014u   12u

Energy released due to mass defect of

0.014u = 931.5 × 0.014 = 13.041 MeV

$\Rightarrow$ Energy of e^-

= 13.041 MeV – excitation energy of 4.041 MeV

= 9 MeV

16. $v = \frac{2}{9}\frac{[\rho-\sigma]ga^2}{\eta}$

$v_P = \frac{2}{9}\frac{[8-0.8]g\times1}{3}$

$v_Q = \frac{2}{9}\frac{[8-1.6]g\times(0.5)^2}{2}$

$\frac{V_P}{V_Q} = \frac{7.2\times1\times2}{6.4\times(0.5)^2\times3}$

$= \frac{14.4}{4.8} = 3$

17. $I_{min} = \frac{5}{12}$

$\frac{1}{R_{eff}} = \frac{1}{3}+\frac{1}{4}+\frac{1}{12}$

$\therefore R_{eff} = \frac{12}{8} = 1.5$

$\therefore I_{max} = \frac{5}{1.5}$

So $\frac{I_{max}}{I_{min}} = \frac{12}{1.5} = 8$

18. $P \propto T^4$

$\log_2\left(\frac{P_1}{P_0}\right) = 1 = \log_2\left(\frac{760}{T_0}\right)^4$

$$\log_2\left(\frac{P_2}{P_0}\right) = x = \log_2\left(\frac{3040}{T_0}\right)^4$$

$$\Rightarrow 2' = \left(\frac{760}{T_0}\right)^4$$

$$2^x = \left(\frac{3040}{T_0}\right)^4$$

$$\frac{2}{2^x} = \left(\frac{760}{3040}\right)^4 = \left(\frac{1}{4}\right)^4 = \left(\frac{1}{2^2}\right)^4$$

$$\frac{2}{2^x} = \frac{2}{2.2^{x-1}} = \frac{1}{2^8}$$

$\Rightarrow 2^{x-1} = 2^8$

$x - 1 = 8$

$x = 9$

PART II

19	20	21	22	23
B	**B**	**A**	**A**	**C**

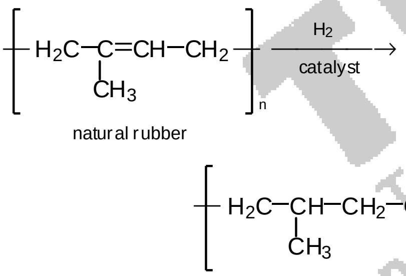
24	25	26	27	28	29	30	31
B, C	**B, C**	**B**	**B, C, D**	**A, C, D**	**A**	**A, B, C**	**B, D**

32	33	34	35	36
4	**5**	**9**	**4**	**6**

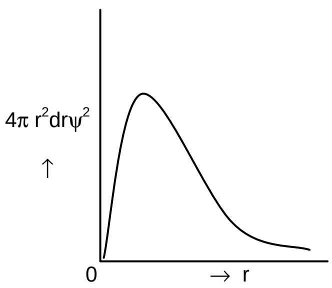
Section I

19. Atomic radii increases in the order
Ga < Al < In < Tl

20. $[NiCl_4]^{2-}$, $Na_3[CoF_6]$ and CsO_2 are paramagnetic Compounds.

21. 

22. The plot of radial probability function ($4\pi r^2 dr\psi^2$) against distance from the nucleus (r) for 1s orbital is



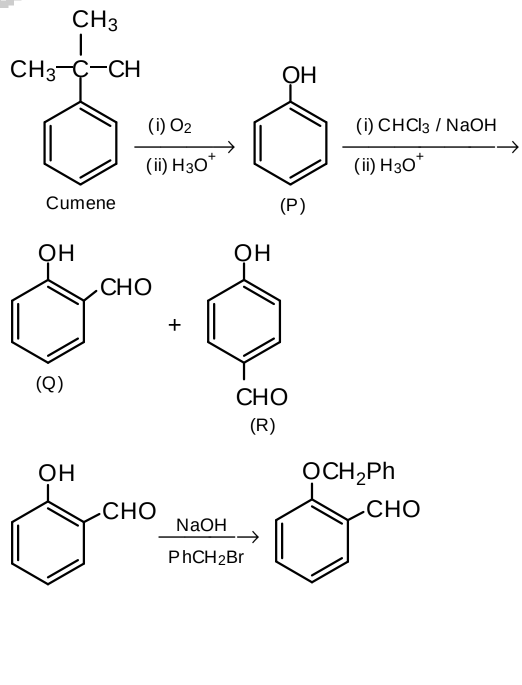
23. $q_{rev} = P_{ext}.\Delta V$
$= 3 \times 1$ L atm
$= 3 \times 101.3$ J

$$\Delta S_{surr} = \frac{-q_{rev}}{T}$$

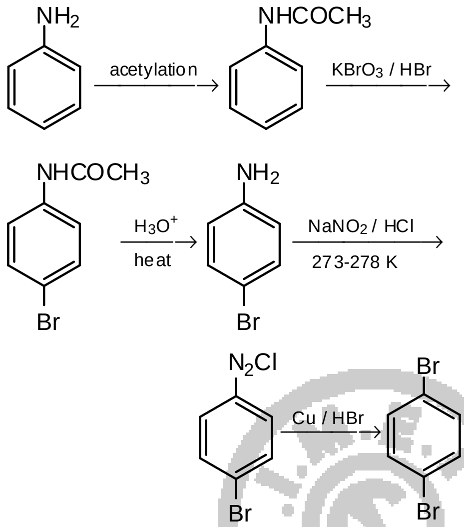
$$= \frac{-3\times101.3}{300}$$

$= -1.013$ J K^{-1}

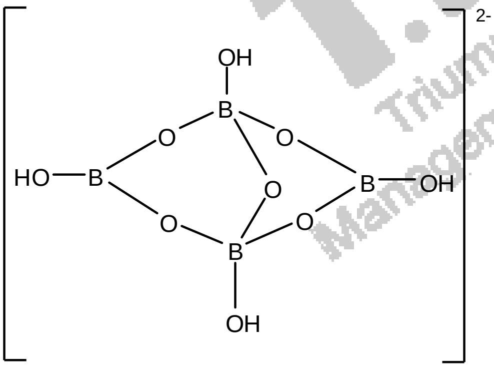
Section II

24. 

25. BrF_5 – sp^3d^2 hybridisation
– 5 bp and 1 lp
ClF_3 – sp^3d hybridisation
– 3 bp and 2 lp
XeF_4 – sp^3d^2 hybridisation
– 4 bp and 2 lp
SF_4 – sp^3d hybridisation
– 4 bp and 1 lp

26. 

27. Higher the activation energy, slower the reaction. Rate constant increases with increase in temperature as the rate of collision increases which in turn increases the number of activated molecules.
Larger the activation energy, greater the influence of change in temperature on rate constant.
The pre-exponentitial factor (A) is a measure of the number of collisions.

28. 

29. Only CuS is precipitated selectively.

30. Ketones containing –OH group on α-carbon and all aldehydes answer Tollens' test.

31. Nuclides that lie below the band of stability have too few neutrons for stability and decay by positron-emission or electron capture. In either case the number of neutrons increases and decreases the number of protons.

Section III

32. $D \propto \lambda.\bar{c}$

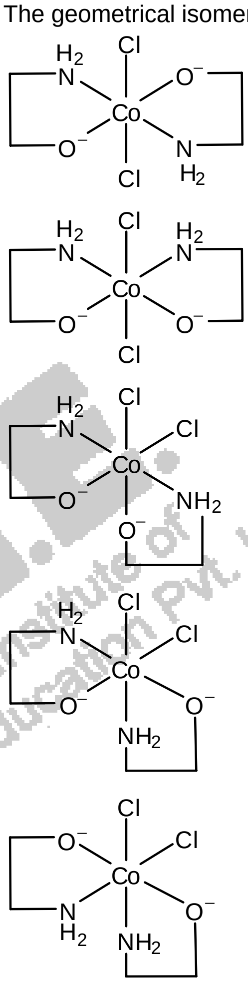
But $\lambda \propto \dfrac{T}{P}$

and $\bar{c} \propto \sqrt{T}$

$\therefore D \propto \dfrac{T^{3/2}}{P}$

$\dfrac{D_2}{D_1} = \dfrac{(2^2)^{3/2}}{2} = 4$

33. The geometrical isomers are



34. $\dfrac{n_2}{n_1} = \dfrac{1}{9}$

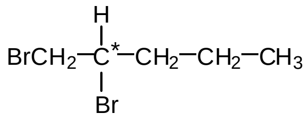
$m = \dfrac{1\times1000}{9\times M_1}$

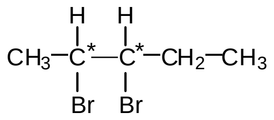
$M = \dfrac{1\times2\times1000}{1\times M_2 + 9\times M_1}$

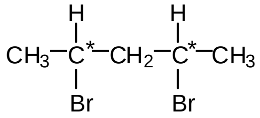
Given, $\dfrac{1000}{9M_1} = \dfrac{2000}{M_2 + 9M_1}$

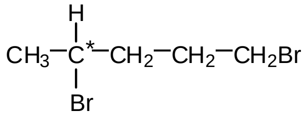
$\therefore \dfrac{M_2}{M_1} = 9$

35. Possible chiral products are









36. $\underset{(n=3)}{8MnO_4^-} + \underset{(n=8)}{3S_2O_3^{2-}} \rightarrow MnO_2 + 6SO_4^{2-}$

PART III

37	38	39	40	41
C	**A**	**D**	**D**	**C**

42	43	44	45	46	47	48	49
A, D	**B, C, D**	**A, B, C**	**B, C**	**B, C**	**A, C**	**A, D**	**A, B, C, D**

50	51	52	53	54
5	**3**	**1**	**2**	**0**

Section I

37. $P(T_1) = \frac{20}{100}$ $P(T_2) = \frac{80}{100}$ D – Defective

Let $P\left(D/T_2\right) = x$ $P\left(D/T_1\right) = 10x$

P(D) = 0.07

$P(D) = P(T_1)\ P\left(D/T_1\right) + P(T_2) \times P\left(D/T_2\right)$

$.07 = \frac{20}{100} \times 10x + \frac{80}{100} \times x$

$\Rightarrow 7 = 200x + 80x$

$\Rightarrow x = \frac{7}{280} = \frac{1}{40}$

$\Rightarrow P\left(D/T_2\right) = \frac{1}{40} \Rightarrow P\left(D'/T_2\right) = \frac{39}{40}$

$P\left(D/T_1\right) = \frac{1}{4} \Rightarrow P\left(D'/T_1\right) = \frac{3}{4}$

Required Probability = $P\left(T_2/D'\right)$

$$= \frac{P(T_2)\ P\left(D'/T_2\right)}{P(T_1)\ P\left(D'/T_1\right) + P(T_2)\ P\left(D'/T_1\right)}$$

$$= \frac{\frac{80}{100} \times \frac{39}{40}}{\frac{20}{100} \times \frac{3}{4} + \frac{80}{100} \times \frac{39}{40}}$$

$$= \frac{78}{93}$$

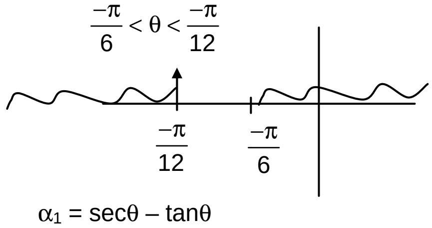
38. 6G 4B ⇒ atmost one boy
(i) Boy + 3G ⇒ $^4C_1 \times {}^6C_3$
(ii) 4G ⇒ $^6C_4$
Total ways of selection of team
= 4 × 20 + 15 = 95
From each team the captain may be chosen in $^4C_1$ ways
∴ (Team + Captain) = 95 × 4 = 380

39. $x^2 - 2x\sec\theta + 1 = 0$

$$x = \frac{(2\sec\theta) \pm \sqrt{4\sec^2\theta - 4}}{2}$$

$= \sec\theta \pm \tan\theta$

$\frac{-\pi}{6} < \theta < \frac{-\pi}{12}$



$\alpha_1 = \sec\theta - \tan\theta$
$\beta_1 = \sec\theta + \tan\theta$

$x^2 + 2x\tan\theta - 1 = 0$

$$x = \frac{(2\tan\theta) \pm \sqrt{4\tan^2\theta + 4}}{2}$$

$= \tan\theta \pm \sec\theta$

$\alpha_2 = \tan\theta + \sec\theta$

$\beta_2 = \tan\theta - \sec\theta$

$\alpha_1 + \beta_2 = \sec\theta - \tan\theta + \tan\theta - \sec\theta$

$= 0$

40. $\frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x} + 2\left[\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right] = 0$

$\Rightarrow 2\cos\left(\frac{\pi}{6} - x\right) + 2\cos 2x = 0$

$\Rightarrow \cos\left(\frac{\pi}{6} - x\right) + \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{\pi}{12} + \frac{x}{2}\right)\cos\left(\frac{\pi}{12} - \frac{3x}{2}\right) = 0$

$\Rightarrow \frac{\pi}{12} + \frac{x}{2} = \frac{\pi}{2}$ or $\frac{\pi}{12} - \frac{3x}{2} = \frac{\pi}{2}$

$\Rightarrow x = \frac{5\pi}{6}$ $\Rightarrow x = \frac{-5\pi}{18}$

$\therefore$ Required sum $= \frac{5\pi}{6} - \frac{5\pi}{18}$

$= \frac{5\pi}{9}$

41. $4\alpha x^2 + \frac{1}{x} \geq 1$

Since $x > 0$, $4\alpha + \frac{1}{x^3} \geq \frac{1}{x^2}$

$\Rightarrow \alpha \geq \frac{1}{4}\left(\frac{1}{x^2} - \frac{1}{x^3}\right)$

Minimum value of α is the maximum value of

$f(x) = \frac{1}{4}\left(\frac{1}{x^2} - \frac{1}{x^3}\right)$

$\frac{1}{4}(y^2 - y^3)$, where $y = \frac{1}{x}$

$f' = \frac{1}{4}(2y - 3y^2)$

$f'' = \frac{1}{4}(2 - 6y)$

$f' = 0 \Rightarrow y = 0$ or $\frac{2}{3}$

But $y = \frac{1}{x} \neq 0$

$\therefore y = \frac{2}{3}$

$\Rightarrow$ Also, $f'' < 0$ at $y = \frac{2}{3}$

$\therefore$ Maximum of $f = \frac{1}{27}$

Section II

42. $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0$

$[(x+2)^2 + y(x+2)]\frac{dy}{dx} - y^2 = 0$

$X = x + 2$

$Y = y$

$\frac{dY}{dX} = \frac{dy}{dx}$

$(X^2 + YX)\frac{dY}{dX} - Y^2 = 0$

$Y = VX$

$\frac{dY}{dX} = V + X\frac{dV}{dX}$

$(X^2 + X^2V)\left(V + X\frac{dV}{dX}\right) - V^2X^2 = 0$

$(1 + V)\left(V + X\frac{dV}{dX}\right) - V^2 = 0$

$V + X\frac{dV}{dX} + V^2 + V \times \frac{dV}{dV} - V^2 = 0$

$X(1 + V)\frac{dV}{dX} + V = 0$

$\frac{(1+V)dV}{V} + \frac{dX}{X} = 0$

Integrating $\log V + V + \log X = C$

$\log Y + V = C$

$\Rightarrow \log y + \frac{y}{x+2} = C$

Curve passes through (1, 3)

$\log 3 + \frac{3}{3} = C$

$C = 1 + \log 3$

Solution curve is

$\log y + \frac{y}{x+2} = 1 + \log 3$

$y = x + 2$ intersects at (1, 3)

(A) true

$2\log x + 2 = 1 + \log 3 - (x + 2)$

$= \log 3 - x - 1$

$x + \log x^2 = -3 + \log 3$

$x + 2\log x = \log 3 - 3$

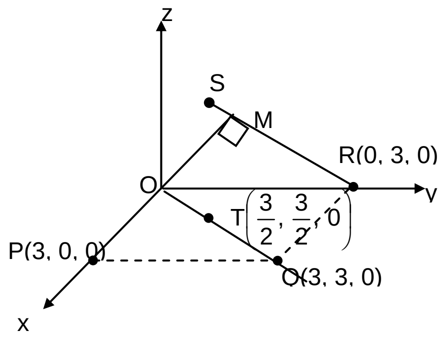
$2\log(y + 3) + \frac{(x+3)^2}{x+2} = 1 + \log 3$

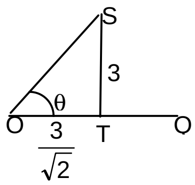
$\therefore y = (x + 3)^2$ do not intersect

$\therefore$ D is correct

$\therefore$ A, D correct

43.



(A) 

$OT = \dfrac{3}{\sqrt{2}}$

$ST = 3$

$\tan\theta = \sqrt{2}$

$\therefore$ A is incorrect

(B) Equation of line OQ is $x = y$

As S lies directly above T, equation of the plane containing ΔOQS is $x - y = 0$

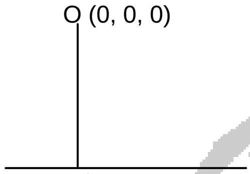
(B) correct

(C) Length of the $\perp$ar from $P(3, 0, 0)$ to $x - y = 0$

Is $\dfrac{3}{\sqrt{2}}$

$\therefore$ (C) correct

(D)



$M\left(\dfrac{3}{2}\lambda, \dfrac{-3}{2}\lambda + 3, 3\lambda\right)$

Equation of the line joining R and S is

$$\frac{x-0}{\frac{3}{2}} = \frac{y-3}{\frac{-3}{2}} = \frac{z-0}{3}$$

M lies on this line

$\Rightarrow$ M is $\left(\dfrac{3}{2}\lambda, \dfrac{-3}{2}\lambda + 3, 3\lambda\right)$

D. r' s of OM are $\dfrac{3}{2}\lambda, \dfrac{-3}{2}\lambda + 3, 3\lambda$

D. r's of RS are $\dfrac{3}{2}, \dfrac{-3}{2}, 3$

Since OM $\perp$ RS,

$$\frac{3}{2}\lambda, \times \frac{3}{2} + \left(\frac{-3}{2}\lambda + 3\right) \times \frac{-3}{2} + 3\lambda \times 3 = 0$$

$\Rightarrow \lambda = \dfrac{1}{3}$

$\Rightarrow$ M is $\left(\dfrac{1}{2}, \dfrac{5}{2}, 1\right)$

$\therefore OM = \sqrt{\dfrac{15}{2}}$

(D) correct

44. $x^2 + y^2 = 3$

$x^2 = 2y$

$y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0$

$y = -3, 1$

Since P is in the first quadrant, $y = 1$

$x^2 = 2 \Rightarrow x = \sqrt{2}$

P is $\left(\sqrt{2}, 1\right)$

Circle C_2 : $x^2 + (y - p)^2 = 12$ $\quad Q_2 (0, p)$

Circle C_3 : $x^2 + (y - q)^2 = 12$ $\quad Q_3 (0, q)$

Tangent at P to $C_1 \Rightarrow x\sqrt{2} + y = 3 \quad \rightarrow) \cancel{(x)}$

$\cancel{(x)}$ touches C_2

$$\frac{p-3}{\sqrt{2+1}} = \pm 2\sqrt{3}$$

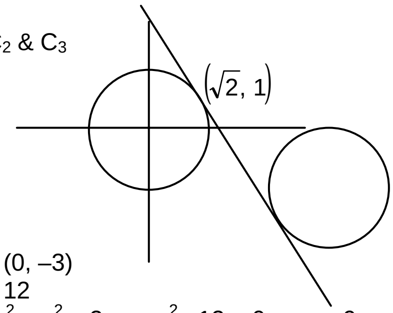
$p = 3 = \pm 6$

$p = 9, -3$

$p = 9$

$q = -3$

centres of C_2 & C_3



$(0, 9) \quad (0, -3)$

$Q_2Q_3 = 12$

Circle C_2 : $x^2 + y^2 - 2py + p^2 - 12 = 0 \rightarrow p = 9$

$x^2 + y^2 - 18y + 69 = 0$

$xx_1 + yy_1 - 9(y + y_1) + 69 = 0$

$xx_1 + (y_1 - 9)y + 69 - 9y_1 = 0$

$x\sqrt{2} + y = 3$

$$\frac{x_1}{\sqrt{2}} = \frac{y_1 - 9}{1} = \frac{9y_1 - 69}{3}$$

$3y_1 - 27 = 9y_1 - 69$

$6y_1 = 69 - 27$

$= 42$

$y_1 = 7$

$$\frac{x_1}{\sqrt{2}} = \frac{7-9}{1} = -2$$

$x_1 = -2\sqrt{2}$

R_2 is $\left(-2\sqrt{2}, 7\right)$

Circle C_3

$x^2 + y^2 + 6y - 3 = 0$

$xx_1 + yy_1 + 3(y + y_1) - 3 = 0$

$xx_1 + (y_1 + 3)y + 3y_1 - 3 = 0$

$x\sqrt{2} + y = 3$

$$\frac{x_1}{\sqrt{2}} = \frac{y_1 + 3}{1} = \frac{3 - 3y_1}{3}$$

$$\frac{x_1}{\sqrt{2}} = +2$$

$x_1 = +2\sqrt{2}$

R_3 is $\left(2\sqrt{2}, -1\right)$

$3y_1 + 9 = 3 - 3y_1$

$6y_1 = -6$

$y_1 = -1$

$=$ R_1 is $\left(-2\sqrt{2}, 7\right)$

R_2 is $\left(2\sqrt{2}, -1\right)$

$R_1R_2^2 = 32 + 64 = 96$

$R_1R_2 = \sqrt{16 \times 6} = 4\sqrt{6}$

ΔOR_2R_3

$(0, 0) \quad \left(-2\sqrt{2}, 7\right) \quad \left(2\sqrt{2}, -1\right)$

Area of $\Delta OR_2R_3 = \frac{1}{2}\left|\begin{matrix}-2\sqrt{2}\times(-1)\\ -7\times 2\sqrt{2}\end{matrix}\right|$

$$= \frac{1}{2}\left|-12\sqrt{2}\right| = 6\sqrt{2}$$

Q_2 (0, 9)
Q_3 (0, –3)
$P\left(\sqrt{2}, 1\right)$ (0, 9) (0, –3)

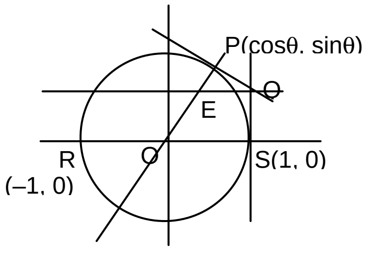
Area of ΔPQ_2Q_3

$$= \frac{1}{2}\left\{\sqrt{2}(12)\right\} = 6\sqrt{2}$$

45. $g(f(x)) = x \Rightarrow f(x) = g^{-1}(x)$ ——— (1)
Also, $g'(f(x))\ f'(x) = 1$
$\therefore g'(x^3 + 3x + 2) \times (3x^2 + 3) = 1$
Put x = 0
Then $g'(2) = \frac{1}{3}$
$\therefore$ (A) incorrect
Now, $h(g(g(x))) = x$
$\Rightarrow h(x) = [g(g(x))]^{-1} = g^{-1}(g^{-1}(x)) = f(f(x))$, by (1)
$\therefore h'(x) = f'(f(x)) \times f'(x)$
$\therefore h'(1) = f'(6) \times f'(1)$
$= 111 \times 6 = 666$
(B) correct
Also, $h(x) = f(f(x)) \Rightarrow h(0) = f(f(0))$
$f(2) = 16$
$\therefore$ (C) correct
$h(x) = f(f(x)) \Rightarrow h(g(3)) = f[f(g(3))]$
$= f(3)$ [since $f = g^{-1}$]
$= 38$
(D) incorrect

46. $PQ = kI \Rightarrow Q = kP^{-1}$
Equate the (2, 3) element on both sides
$\therefore \frac{-k}{8} = k \times \frac{1}{12\alpha + 20} \times -(3\alpha + 4)$
$\Rightarrow \alpha = -1$. Then $|P| = 8$
Also, $PQ = kI \Rightarrow |P|\ |Q| = k^3 \times 1$
$\Rightarrow 8 \times \frac{k^2}{2} = k^3$
$\Rightarrow k = 4 \quad \therefore |Q| = 8$
$\therefore$ (B) correct and (A) incorrect
$|P \text{ adj } Q| = |P|\ |Q|^2 = 8 \times 8^2 = 2^9$
$\therefore$ (C) correct
$|Q \text{ adj } P| = |Q|\ |P|^2 = 8 \times 8^2 = 2^9$
$\therefore$ (D) incorrect

47.



Tangent at P
$x\cos\theta + y\sin\theta = 1$
Tangent at S $\rightarrow$ x = 1
$y\sin\theta = 1 - x\cos\theta$
$= 1 - \cos\theta$
$y = \frac{1 - \cos\theta}{\sin\theta}$
Q is $\left(1, \frac{1-\cos\theta}{\sin\theta}\right)$
Equation of the line through Q parallel to RS is
$y = \frac{1-\cos\theta}{\sin\theta}$ ——— (1)
Equation of the normal at P is
$y = (\tan\theta)x$ ——— (2)
$x\tan\theta = \frac{1-\cos\theta}{\sin\theta}$
$x = \frac{1-\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\sin\theta}$
$= \frac{\cos\theta - \cos^2\theta}{\sin^2\theta}$
$x = \frac{\cos\theta - \cos^2\theta}{\sin^2\theta}, \quad y = \frac{1-\cos\theta}{\sin\theta}$
$= \frac{(\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}, = \frac{\cos\theta}{1+\cos\theta}$
$x\cos\theta + x = \cos\theta$
$\cos\theta = \frac{x}{1-x}$ ——— (1)
$y\sin\theta = 1 - \cos\theta = 1 - \frac{x}{1-x}$
$= \frac{1-2x}{1-x}$
$\sin\theta = \frac{1-2x}{(1-x)y}$ ——— (2)
Locus of E is
$\frac{(1-2x)^2}{y^2(1-x)^2} + \frac{x^2}{(1-x)^2} = 1$
$(1 - 2x)^2 + y^2x^2 = y^2(1 - x)^2$
$(1 - 2x)^2 \pm y^2\,[1 + x^2 - 2x - x^2]$
$= y^2(1 - 2x)$
$x \neq \frac{1}{2}$
$y^2 = 1 - 2x$ is the locus
$x = \frac{1}{3} \rightarrow y^2 = \frac{1}{3}$, $y = \pm\frac{1}{\sqrt{3}}$

48. $f'(x) + \frac{f(x)}{x} = 2$
General solution is
$xf(x) = C + x^2$
$f(x) = \frac{C}{x} + x$
$\lim_{x\to 0^+} xf\left(\frac{1}{x}\right) = \lim_{x\to 0^+}\left\{Cx^2 + 1\right\}$
$= 1$ (B false)
$\lim_{x\to 0^+} f'\left(\frac{1}{x}\right) = \lim_{x\to 0^+}\left(-Cx^2 + 1\right)$
$= 1 \rightarrow$ (A) is true)

$$\lim_{x\to 0^+} x^2 f'(x) = \lim_{x\to 0}\left(-C + x^2\right)$$

$$= -C \neq 0 \qquad \begin{cases} f(1) \neq 1 \\ C + 1 \neq 1 \\ C \neq 0 \end{cases}$$

$\Rightarrow$ (C) is false

We have

$f(x) = \frac{C}{x} + x$

Taking C = 1

$|f(x)| \le 2$ for $x \in (0, 2)$

(D) is true

49. $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$

$= \frac{3s-2s}{9} = \frac{s}{9}$

$9(s - x) = 4s$

$9x = 5s \Rightarrow x = \frac{5s}{9}$

$\frac{s-y}{3} = \frac{s}{9}$

$3s - 3y = s$

$3y = 2s \Rightarrow y = \frac{2s}{3}$

$\frac{s-z}{2} = \frac{s}{9}$

$9s - 9z = 2s$

$9z = 7s$

$z = \frac{7s}{9}$

Area of XYZ = $\sqrt{s(s-x)(s-y)(s-z)}$

$= \sqrt{s \times \frac{4s}{9} \times \frac{s}{3} \times \frac{2s}{9}}$

$= \frac{s^2 \times 2\sqrt{2}}{9\sqrt{3}}$

$r = \frac{\Delta}{s} \qquad \pi\left(\frac{\Delta^2}{s^2}\right) = \frac{8\pi}{3}$ (given)

$\frac{\Delta^2}{s^2} = \frac{8}{3}$

$\frac{8s^4}{81 \times 3s^2} = \frac{8}{3} \Rightarrow s^2 = 81$

$s = 9$

$x = \frac{5}{9} \times 9 = 5$

$y = \frac{2s}{3} = \frac{2 \times 9}{3} = 6$

$z = \frac{7s}{9} = \frac{7 \times 9}{9} = 7$

Area of $\Delta XYZ = \frac{2\sqrt{2}}{9\sqrt{3}} \times 81$

$= \left(2\sqrt{2}\right) 3\sqrt{3} = 6\sqrt{6}$

$\frac{abc}{4R} = \Delta = 6\sqrt{6}$

$R = \frac{abc}{4 \times 6\sqrt{6}} = \frac{5 \times 6 \times 7}{4 \times 6\sqrt{6}}$

$= \frac{35}{4\sqrt{6}}$

$4R \sin\frac{x}{2} \sin\frac{y}{2} \sin\frac{z}{2} = \frac{\Delta}{s} = \frac{6\sqrt{6}}{9} = \frac{2\sqrt{6}}{3}$

$\sin\frac{x}{2} \sin\frac{y}{2} \sin\frac{z}{2} = \frac{2\sqrt{6}}{3 \times 4R}$

$= \frac{2\sqrt{6} \times 4\sqrt{6}}{12 \times 35}$

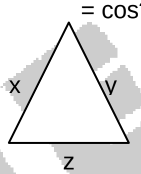
$= \frac{8 \times 6}{12 \times 35} = \frac{4}{35}$

$\text{Sin}^2\left(\frac{x+y}{2}\right)$

$\sin^2\left(\frac{180^\circ - z}{2}\right)$

$= \sin^2\left(90^\circ - \frac{z}{2}\right)$

$= \cos^2\frac{z}{2} = \frac{1}{2}(1 + \cos z)$



$z^2 = x^2 + y^2 - 2xy\cos Z$

$49 = 25 + 36 - 2 \times 5 \times 6 \cos Z$

$-60\cos Z = 49 - 61 = -12$

$\cos Z = \frac{1}{5}$

$\cos^2\frac{Z}{2} = \frac{1}{2}\left(1 + \frac{1}{5}\right) = \frac{1}{2} \times \frac{6}{5} = \frac{3}{5}$

Section III

50. Coefficient of x^2

$= 1 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^{49}C_2 + {}^{50}C_2(m^2)$

$= {}^{50}C_3 + (m^2)\,{}^{50}C_2,$

As ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

${}^{50}C_3 + m^2\,{}^{50}C_2 = (3n + 1)\,{}^{51}C_3$

${}^{51}C_3 + (m^2 - 1)\,{}^{50}C_2 = (3n + 1)\,{}^{51}C_3$

$(m^2 - 1)\,{}^{50}C_2 = 3n\,{}^{51}C_3$

$\frac{50 \times 49}{2}\left(m^2 - 1\right) = 3n\,\frac{51 \times 50 \times 49}{3 \times 2}$

$m^2 - 1 = 51n$

$m^2 = 51n + 1 \qquad 51 \times 5 = 255$

$m = 16 \qquad 255 + 1 = 256$

$n = 5$

51. $\lim_{x\to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x}$

By L' Hospital Rule

$6\beta = 1 \Rightarrow \beta = \frac{1}{6}$ and $\alpha = \frac{1}{3}$

$\therefore 6(\alpha + \beta) = 6\left(\frac{1}{6} + \frac{1}{3}\right)$

$= 6\left(\frac{3}{6}\right) = 3$

52. $z = \frac{-1+\sqrt{3}i}{2}$

$$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} \begin{bmatrix} (-\omega)^2 & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$= \begin{bmatrix} (-\omega)^{2r} + \omega^{45} & (-\omega)^r\,\omega^{2s} + \omega^{2s}\omega^r \\ \omega^{2s}(-\omega)^r + \omega^r\omega^{2s} & \omega^{4s} + \omega^{2r} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$(-\omega)^{2r} + \omega^{4s} = -1$

$\omega^{2s}((-\omega)^r + \omega^r) = 0$ $\qquad \omega^{2s} = 0$(not possible)

$\therefore (-\omega)^r + \omega^r = 0 \Rightarrow$ r should be odd

r = 1 or 3

$-\omega + \omega = 0 \quad -\omega^2 + \omega^2 = 0$

r = 1 $\quad \omega^2 + \omega^{4S} = -1 \qquad \therefore 4s = 4 \Rightarrow s = 1$

r = 3 $\quad 1 + \omega^{4s} = -1$ Not possible

$\therefore$ Only ordered pair is (1, 1)

53. $$\begin{vmatrix} x & x^2 & 1 \\ 2x & (2x)^2 & 1 \\ 3x & (3x)^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 2x & (2x)^2 & (2x)^3 \\ 3x & (3x)^2 & (3x)^3 \end{vmatrix} = 10$$

$$\begin{vmatrix} x & x^2 & 1 \\ x & 3x^2 & 0 \\ 2x & 8x^2 & 0 \end{vmatrix} + 6x^3 \begin{vmatrix} 1 & x & x^2 \\ 1 & 2x & (2x)^2 \\ 1 & 3x & (3x)^2 \end{vmatrix} = 10$$

$\Rightarrow (8x^3 - 6x^3) + 6x^3 \times$

$$x^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 10$$

$\Rightarrow 2x^3 + 6x^6 (-1)(-1)(2) = 10$

$\Rightarrow 2x^3 + 12x^6 = 10$

Let $x^3 = y$

$12y^2 + 2y - 10 = 0$

$(y + 12)(y - 10) = 0$

$y = -12$; $y = 10$

$x^3 = -12$ or $x^3 = 10$

$\therefore$ 2 real roots

54. Given

$$\int_0^x \frac{t^2}{1+t^4}\,dt = 2x - 1$$

Differentiating both sides with respect to x

$$\frac{x^2}{1+x^4} = 2$$

$2x^4 - x^2 + 2 = 0$

Roots of the above are complex

Hence, there is no solution.

Number of distinct solutions in (0, 1) is therefore, 0