**CBSE Sample Paper-06**
**Mathematics**
**Class – XII**

Time allowed: 3 hours **ANSWERS** Maximum Marks: 100

## Section A

1. Solution:

   $[2.1]=2,[2.3]=2$, thus it is not one-one.

   Since it takes only integral values, hence it is not onto also.

2. Solution:

   $Consider\ (AB-BA)^{'}=(AB)^{'}-(BA)^{'}=B^{'}A^{'}-A^{'}B^{'}=BA-AB=-(AB-BA)$

   Thus, AB-BA is skew symmetric.

3. Solution:

   The operation * is not a binary operation as 2*3=2-3=-1$\notin Z^{+}$.

4. Solution:

   $$(AB)^{-1}=B^{-1}A^{-1}=\begin{bmatrix}3 & 0\\ -1 & 6\end{bmatrix}\begin{bmatrix}1 & 3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}3 & 9\\ -7 & 9\end{bmatrix}$$

5. Solution:

   $\vec{a}=3i+4j\ and\ \vec{b}=4i+3j$

   $$\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\frac{12+12}{5.5}=\frac{24}{25}\Rightarrow\theta=\cos^{-1}\left(\frac{24}{25}\right)$$

6. Solution:

   $$\begin{vmatrix}1 & a & b+c\\ 1 & b & a+c\\ 1 & c & a+b\end{vmatrix}=\begin{vmatrix}1 & a & a+b+c\\ 1 & b & b+a+c\\ 1 & c & c+a+b\end{vmatrix}(C_3\to C_2+C_3)$$

   $$=(a+b+c)\begin{vmatrix}1 & a & 1\\ 1 & b & 1\\ 1 & c & 1\end{vmatrix}=0(\because C_3=C_1)$$

## Section B

7. Solution:

   Let x = $\cos 2\theta$

   $$\theta=\frac{1}{2}\cos^{-1}x$$

$$\therefore \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$Now, \frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}=\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$$\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\tan^{-1}(\tan(\frac{\pi}{4}-\theta))=\frac{\pi}{4}-\theta$$

$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

8. Solution:

$\vec{a}=5i+2j-4k, \overline{N}=2i+3j-k$

The equation of a plane is given by $(\vec{r}-\vec{a}).\overline{N}=0$

$\Rightarrow\left[\vec{r}-(5i+2j-4k)\right].(2i+3j-k)=0$

Transforming into Cartesian form we get, [(x-5)i+(y-2)j+(z+4)k].(2i+3j-k)=0, i.e. 2x+3y-z=20.

9. Solution:

(A) Let A denote the event that problem is solved by A and let B denote the event that problem is solved by B.

$\therefore P(A)=1/2, P(B)=1/3, P(\overline{A})=1-1/2=1/2, P(\overline{B})=2/3$

P(Problem is solved)= 1- P(Problem is not solved)=1- $P(\overline{A}\overline{B})=1-(1/2)(2/3)=2/3$

(B) P(exactly one of them solves the problem)= $P(\overline{A}B or A\overline{B})=(1/2)(2/3)+(1/2)(1/3)=1/2$

10. Solution:

Since f is constant for x<2, x>10, f(x) is continuous for x<2, x>10.

At x=2,

$\lim\limits_{x\to 2^-}(f(x))=\lim\limits_{x\to 2^-}(5)=5$

$\lim\limits_{x\to 2^+}(f(x))=\lim\limits_{x\to 2^-}(ax+b)=2a+b$

$\therefore 2a+b=5$

At x=10,

$\lim\limits_{x\to 10^-}(f(x))=\lim\limits_{x\to 10^-}(ax+b)=10a+b$

$\lim\limits_{x\to 10^+}(f(x))=\lim\limits_{x\to 2^-}(21)=21$

$\therefore 10a+b=21$

Solving the above two equations we get a=2, b=1.

11. Solution:

$$F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}, F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\therefore F(x)F(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\sin x\cos y-\cos x\sin y & 0\\ \sin x\cos y+\cos x\sin y & \cos x\cos y-\sin x\sin y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

12. Solution:

$$\frac{d}{dx}\left(\cos^{-1}\sqrt{\frac{1+x}{2}}\right)=\frac{-1}{\sqrt{1-\left(\sqrt{\frac{1+x}{2}}\right)^2}}\frac{d}{dx}\sqrt{\frac{1+x}{2}}=\frac{-1}{\sqrt{\frac{1-x}{2}}}\left(\frac{1}{2}\right)\left(\frac{1+x}{2}\right)^{-1/2}\frac{1}{2}$$

$$=\frac{-1}{2\sqrt{(1-x)(1+x)}}=\frac{-1}{2\sqrt{(1-x^2)}}$$

13. Solution:

$$f\circ g(x)=f\left(\frac{x+3}{2}\right)=2\left(\frac{x+3}{2}\right)-3=x=I_R(x)\Rightarrow f\circ g=I_R$$

$$g\circ f(x)=g(2x-3)=\frac{2x-3+3}{2}=x=I_R(x)\Rightarrow g\circ f=I_R$$

14. Solution:

$$y=x^3-11x+5$$

$$\Rightarrow\frac{dy}{dx}=3x^2-11$$

Also, from equation of tangent we get dy/dx=1

$\therefore 3x^2-11=1\Rightarrow x=\pm 2$.

Solving y=x-11 for y we get the possible points are (2,-9),(-2,-13).

But (-2,-13) does not lie on the curve, hence required point is (2,-9).

15. Solution:

$$\frac{dT}{dt}=-c(T-S)$$

$$\int\frac{dT}{T-S}=\int -cdt$$

$$\therefore \log(T-S)=-ct+k$$

$$\Rightarrow e^{-ct+k}=T-S$$

Putting the condition T(0)=40, we get $(40-S)e^{-ct}=T-S$.

Measures to control global warming:

(1) Planting more trees
(2) Car pools to prevent emission of carbon dioxide which in turn causes global warming.

16. Solution:

$\because \vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a} \Rightarrow \vec{a}\perp\vec{c}, \vec{b}\perp\vec{c}, \vec{a}\perp\vec{b}, \vec{a}\perp\vec{c} \Rightarrow \vec{a},\vec{b},\vec{c}$ are mutually perpendicular.

$$\left|\vec{a}\times\vec{b}\right|=\left|\vec{c}\right|, \left|\vec{b}\times\vec{c}\right|=\left|\vec{a}\right| \Rightarrow \left|\vec{a}\right|\left|\vec{b}\right|\sin 90=\left|\vec{c}\right| \text{ and } \left|\vec{b}\right|\left|\vec{c}\right|\sin 90=\left|\vec{a}\right| \left(\because \vec{a},\vec{b},\vec{c} \text{ are mutually perpendicular}\right)$$

$$\Rightarrow \left|\vec{a}\right|\left|\vec{b}\right|=\left|\vec{c}\right| \text{ and } \left|\vec{b}\right|\left|\vec{c}\right|=\left|\vec{a}\right|$$

$$\Rightarrow \left|\vec{b}\right|\left|\vec{c}\right|\left|\vec{b}\right|=\left|\vec{c}\right| \Rightarrow \left|\vec{b}\right|^2\left|\vec{c}\right|=\left|\vec{c}\right| \Rightarrow \left|\vec{b}\right|=1 \Rightarrow \left|\vec{a}\right|=\left|\vec{c}\right|$$

17. Solution:

$$\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=\int e^x\left(\frac{1}{x}\right)dx-\int e^x\left(\frac{1}{x^2}\right)dx$$

$$=\frac{1}{x}\int e^x dx-\int\left(\frac{d}{dx}\left(\frac{1}{x}\right)\int e^x dx\right)dx-\int e^x\left(\frac{1}{x^2}\right)dx$$

$$=\frac{1}{x}e^x-\int\frac{-1}{x^2}e^x-\int e^x\left(\frac{1}{x^2}\right)dx \qquad \text{(using integration by parts)}$$

$$=\frac{1}{x}e^x+\int\frac{1}{x^2}e^x-\int e^x\left(\frac{1}{x^2}\right)dx$$

$$=\frac{1}{x}e^x$$

18. Solution:

Projection vector of $\vec{a}$ along $\vec{b}$ is given by $\left(\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|^2}\right)\vec{b}$.

$$\vec{a}=2i+3j-3k, \vec{b}=5j-k$$

$$\left(\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|^2}\right)\vec{b}=\left(\frac{(2i+3j-3k).(5j-k)}{\left(\sqrt{26}\right)^2}\right)5j-k=\frac{9}{13}(5j-k)$$

19. Solution:

$$x_1=-3, y_1=1, z_1=5, x_2=-1, y_2=2, z_2=5$$
$$a_1=-3, b_1=1, c_1=5, a_2=-1, b_2=2, c_2=5,$$

The lines are coplanar iff $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$

Substituting the values, we get the value of determinant as 0.
Hence, the lines are co-planar.

## Section C

20. Solution:

Let $B_1$ denote the event that the item is produced by A. $P(B_1)=60/100=.6$

Let $B_2$ denote the event that the item is produced by B. $P(B_2)=40/100=.4$

Let E denote the event that the item is defective.

$P(B_2/E)=?$ , $P(E/B_1)=0.02$, $P(E/B_2)=0.01$

By Baye's theorem,

$$P(B_2/E)=\frac{P(E/B_2)P(B_2)}{P(E/B_2)P(B_2)+P(E/B_2)P(B_2)}$$

$$=\frac{(1/100)(40/100)}{(2/100)(60/100)+(1/100)(40/100)}=\frac{1}{4}$$

21. Solution:

$$Let\ y=\tan^{-1}\frac{2\sqrt{x}}{1-x}$$
$$Let\ x=\tan^2\theta$$
$$\therefore y=\tan^{-1}\frac{2\tan\theta}{1-\tan^2\theta}=\tan^{-1}(\tan 2\theta)=2\theta=2\tan^{-1}\sqrt{x}$$
$$\frac{dy}{dx}=2\frac{1}{1+\left(\sqrt{x}\right)^2}\frac{1}{2\sqrt{x}}=\frac{1}{(1+x)\sqrt{x}}$$

$$Let\ z=\sin^{-1}\frac{2\sqrt{x}}{1+x}$$
$$Let\ x=\tan^2\theta$$
$$\therefore z=\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}=\sin^{-1}(\sin 2\theta)=2\theta=2\tan^{-1}\sqrt{x}$$
$$\frac{dz}{dx}=\frac{1}{(1+x)\sqrt{x}}$$
$$\frac{dy}{dz}=\frac{dy}{dx}\div\frac{dz}{dx}=\frac{1}{(1+x)\sqrt{x}}\div\frac{1}{(1+x)\sqrt{x}}=1$$

22. Solution :

$$I=\int_0^{\pi}\frac{xdx}{4\cos^2 x+9\sin^2 x}=\int_0^{\pi}\frac{(\pi-x)dx}{4\cos^2 x+9\sin^2 x}$$

$$\therefore 2I=\pi\int_0^{\pi}\frac{dx}{4\cos^2 x+9\sin^2 x}=2\pi\int_0^{\frac{\pi}{2}}\frac{dx}{4\cos^2 x+9\sin^2 x}$$

$$= 2\pi\left[\int_0^{\frac{\pi}{4}} \frac{dx}{4\cos^2 x + 9\sin^2 x} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{4\cos^2 x + 9\sin^2 x}\right]$$

$$= 2\pi\left[\int_0^{\frac{\pi}{4}} \frac{\sec^2 x dx}{4 + 9\tan^2 x} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos ec^2 x dx}{4\cot^2 x + 9}\right]$$

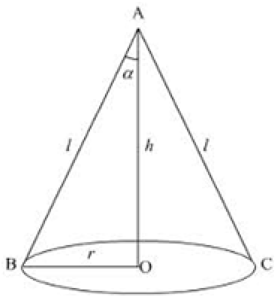
Putting tanx=t and cotx=u, we get

$$2I = 2\pi\left[\int_0^1 \frac{dt}{4 + 9t^2} - \int_1^0 \frac{du}{4u^2 + 9}\right] = 2\pi\left[\frac{1}{9}\left(\frac{3}{2}\right)\tan^{-1}\frac{t}{2/3}\Bigg|_0^1 - \frac{1}{4}\left(\frac{2}{3}\right)\tan^{-1}\frac{u}{3/2}\Bigg|_1^0\right]$$

$$= 2\pi\left[\frac{1}{6}\tan^{-1}\left(\frac{3}{2}\right) + \frac{1}{6}\tan^{-1}\left(\frac{2}{3}\right)\right] = \frac{2\pi}{6}\left(\frac{\pi}{2}\right) = \frac{\pi^2}{6}$$

$$\therefore I = \frac{\pi^2}{12}$$

23. Solution:




Let l,h,r, $\alpha$ denote the slant height, height, radius and semi-vertical angle of the cone respectively.

$$\sin\alpha = \frac{r}{l} \Rightarrow r = l\sin\alpha, \cos\alpha = \frac{h}{l} \Rightarrow h = l\cos\alpha$$

$$Volume = V = \frac{\pi}{3}r^2 h = \frac{\pi}{3}l^3 \sin^2\alpha\cos\alpha$$

$$\therefore \frac{dV}{d\alpha} = \frac{\pi}{3}l^3[2\sin\alpha\cos^2\alpha + \sin^2\alpha(-\sin\alpha)] = \frac{\pi}{3}l^3[\sin\alpha(2\cos^2\alpha - \sin^2\alpha)]$$

$$\frac{dV}{d\alpha} = 0 \Rightarrow \alpha = \tan^{-1}(\sqrt{2})$$

$$\frac{d^2V}{d\alpha^2} = \frac{\pi}{3}l^3[\cos\alpha(2\cos^2\alpha - \sin^2\alpha) + \sin\alpha(4\cos\alpha(-\sin\alpha) - 2\sin\alpha\cos\alpha)] = \frac{\pi}{3\cos^3\alpha}l^3[2 - 7\tan^2\alpha]$$

$$If\ \tan\alpha = \sqrt{2} \Rightarrow \frac{d^2V}{d\alpha^2} < 0$$

Thus, for maximum volume $\tan\alpha = \sqrt{2}$.

24. Solution:

Let the three numbers be x,y,z. We can formulate the above as the mathematical problem:

$$x+y+z=6$$
$$y+3z=11$$
$$x-2y+z=0$$

$$Let\ A=\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}, b=\begin{bmatrix}6\\11\\0\end{bmatrix}$$

$$|A|=9, A^{-1}=\frac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}, X=A^{-1}b=\frac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}\begin{bmatrix}6\\11\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\therefore x=1, y=2, z=3$$

25. Solution:

Suppose the factory produces x units of machine A and y units of machine B.

Then, Profit Z= 10,500x + 9000y

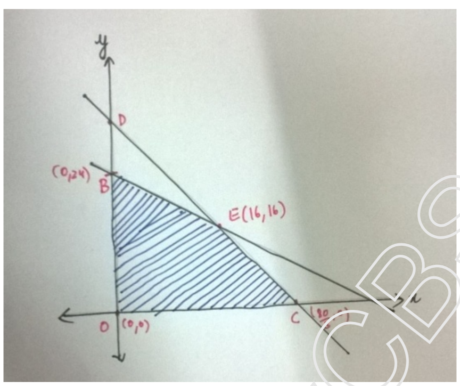
The mathematical formulation of the problem is as follows:

Max Z= 10,500x+9000y

s.t 10x+20y$\le$ 480, x+2y$\le$ 48 (metal constraint)

15x+10y$\le$ 400, 3x+2y$\le$ 80 (painting constraint)

x$\ge$ 0, y$\ge$0




We graph the above inequalities. The feasible region is as shown in the figure. We observe the feasible region is bounded and the corner points are O,B,E and C. The co-ordinates of the corner points are (0,0), (0,24),(16,16),(80/3,0).

| Corner Point | Z=10,500x + 9000y |
|---|---|
| (0,0) | 0 |
| (0,24) | 2,16,000 |
| (16,16) | **3,12,000** |
| (80/3,0) | 2,80,000 |

Thus profit is maximized by producing 16 units each of machine A and B.

26. Solution:

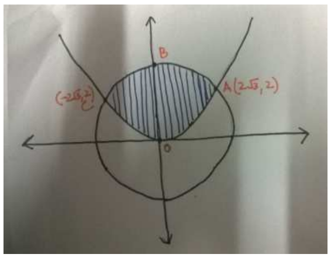
The point of intersection of the two curves:

$x^2 + y^2 = 16, x^2 = 6y$

$\Rightarrow y^2 + 6y - 16 = 0$

$\Rightarrow y = 2, -8$

Rejecting y=-8, we get $x = \pm 2\sqrt{3}$.




Shaded area= Required area=Ar(OAB)+Ar(OBC)=2 Ar(OAB)

$$Area = 2\int_0^{2\sqrt{3}} (y_1 - y_2)dx = 2\int_0^{2\sqrt{3}} \left(\sqrt{16 - x^2} - \frac{x^2}{6}\right)dx$$

$$= 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right) - \frac{1}{6}\left(\frac{x^3}{3}\right)\Bigg|_0^{2\sqrt{3}}\right]$$

$$= 2\left[2\sqrt{3} + \frac{8\pi}{3} - \frac{4\sqrt{3}}{3}\right] = \frac{4}{3}[4\pi + \sqrt{3}]$$