Sample Paper-02 (Solved)
Mathematics
Class – XII

Time allowed: 3 hours | ANSWERS | Maximum Marks: 100

Section A

1. Solution:

Let the order of Y be nxp.

$\because$ XY is defined $\Rightarrow$n=3, $\because$ YZ is defined $\Rightarrow$p=5

$\because$ Y has order 3x5.

2. Solution:

$$Area=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}=\frac{15}{2} sq.units$$

3. Solution:

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix}=\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix}\Rightarrow 3-x^2=3-8\Rightarrow x^2=8\Rightarrow x=\pm 2\sqrt{2}$$

4. Solution:

$f(x)=x^2$, is neither one-one nor onto.

f(3)=f(-3)=9, hence not one-one. Also f(x) does not assume any negative values, hence it is not onto.

5. Solution:

$$\left|\vec{a}\right|=\sqrt{(3)^2+(-2)^2+(5)^2}=\sqrt{38}$$

$$\therefore l=\frac{3}{\sqrt{38}}, m=\frac{-2}{\sqrt{38}}, n=\frac{5}{\sqrt{38}}$$

6. Solution:

Yes, since every line is parallel to itself, thus the above relation is reflexive.

Section B

7. Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} (C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$$

$$= (a-b)(b^3-c^3) - (b-c)(a^3-b^3)$$
$$= (a-b)(b-c)(b^2+bc+c^2) - (b-c)(a-b)(a^2+ab+b^2)$$
$$= (a-b)(b-c)(\cancel{b^2}+bc+c^2-a^2-ab-\cancel{b^2})$$
$$= (a-b)(b-c)(c-a)(a+b+c)$$

8. Solution:

$$A = \pi r^2$$
$$\therefore \frac{dA}{dt} = \left(\frac{dA}{dr}\right)\left(\frac{dr}{dt}\right) = 2\pi r \frac{dr}{dt}$$
$$\frac{dr}{dt} = 2cm/\sec$$
$$\therefore if\ \ r = 14, \frac{dA}{dt} = 2\pi(14)(2) = 176cm^2/\sec$$

Harmful effects of poisoning water bodies:

Spread of epidemic, death of animals drinking water from the water source.

9. Solution:

Let z$\in$ C

$\because$ g is onto there exists b$\in$ B s.t g(b)=z.

Now, $\because$ b$\in$ B and f is onto there exists a$\in$ A s.t. f(a)=b.

$$\because g(b) = z \Rightarrow g(f(a)) = z \Rightarrow (g \circ f)(a) = z$$
$$\therefore g \circ f \text{ is onto.}$$

10. Solution:

$\because$ f(x) is a polynomial$\Rightarrow$f(x) is continuous on [-4,2].

$\because$ f(x) is a polynomial$\Rightarrow$f(x) is differentiable on]-4,2[.

f(-4)=$(-4)^2$ + 2(-4) – 8 =0

f(2)=4+4-8=0

$\therefore$ f(-4)=f(2)

$\therefore$ all the conditions of Rolle's theorem are satisfied.

So, $\exists\, c \in\,]-4,2[$ s.t. f '(c)=0.

f '(x)=2x+2, f '(x)=0 $\Rightarrow$ 2x+2=0 $\Rightarrow$ x=-1

$\therefore$ for c=-1 $\in$]-4,2[, f '(c)=0.

Thus, Rolle 's Theorem is verified.

11 Solution:

$$Let\ x=\sin^{-1}\left(\frac{5}{13}\right), y=\cos^{-1}\left(\frac{3}{5}\right)$$

$$\Rightarrow \sin x=\frac{5}{13}, \cos x=\sqrt{1-\sin^2 x}=\frac{12}{13}, \tan x=\frac{5}{12}$$

$$\cos y=\frac{3}{5}, \sin y=\sqrt{1-\cos^2 y}=\frac{4}{5}, \tan y=\frac{4}{3}$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{5/12+4/3}{1-(5/12)(4/3)}=\frac{63}{16}$$

$$x+y=\tan^{-1}\left(\frac{63}{16}\right)$$

12. Solution:

$$\vec{a}=3i-4j-4k, \vec{b}=2i-j+k, \vec{c}=i-3j-5k$$

$$\therefore \overrightarrow{AB}=(2i-j+k)-(3i-4j-4k)=-i-3j-5k$$

$$\overrightarrow{BC}=(i-3j-5k)-(2i-j+k)=-i-2j-6k$$

$$\overrightarrow{CA}=(3i-4j-4k)-(i-3j-5k)=2i-j+k$$

$$\left|\overrightarrow{AB}\right|^2=35, \left|\overrightarrow{BC}\right|^2=41, \left|\overrightarrow{CA}\right|^2=6$$

$$\therefore \left|\overrightarrow{AB}\right|^2+\left|\overrightarrow{CA}\right|^2=\left|\overrightarrow{BC}\right|^2$$

Thus, A, B,C form the vertices of a right angled triangle.

13. Solution:

(A) Let A denote the event that problem is solved by A and let B denote the event that problem is solved by B.

$\therefore P(A) = 1/2, P(B) = 1/3, P(\overline{A}) = 1 - 1/2 = 1/2, P(\overline{B}) = 2/3$

P(Problem is solved)= 1- P(Problem is not solved)=1- $P(\overline{A}\overline{B}) = 1 - (1/2)(2/3) = 2/3$

(b) P(exactly one of them solves the problem)= $P(\overline{A}B or A\overline{B}) = (1/2)(2/3) + (1/2)(1/3) = 1/2$

14. Solution:

The function is defined for all points of the real line.

Case I: If c < -3, f(c)= c^3-c+1,

$$\lim_{x \to c}(f(x)) = \lim_{x \to c}\left(x^3 - x + 1\right) = c^3 - c + 1 = f(c)$$

$$\therefore f \text{ is continuous } \forall\ x < -3$$

Case II: If c > -3

$$f(c) = 3c + 2$$

$$\lim_{x \to c}(f(x)) = \lim_{x \to c}(3x + 2) = 3c + 2 = f(c)$$

$$\therefore f \text{ is continuous } \forall\ x > 3$$

Case II: If c = -3

$$\lim_{x \to -3^-}(f(x)) = \lim_{x \to -3^-}(x^3 - x + 1) = -23$$

$$\lim_{x \to -3^+}(f(x)) = \lim_{x \to -3^+}(-2x) = 6$$

Since, L.H.L ≠ R.H.L at x=-3, f(x) is not continuous at x=-3.
Similarly if c=3, L.H.L=-6, R.H.L=11. Thus f is not continuous at x=3

15. Solution:

$$x\frac{dy}{dx} - y + x\cos ec \quad \frac{y}{x} \quad = 0 \qquad \frac{dy}{dx} \quad \frac{y}{x} \quad \cos ec \quad \frac{y}{x} \qquad 0$$

$$\frac{dy}{dx} \quad \frac{y}{x} \quad \cos ec \quad \frac{y}{x}$$

$$Let\ \frac{y}{x} = v \qquad y \quad vx$$

$$Differentiating\ w.r.t\ x, we\ get$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = v - \cos ec(v)$$

$$\frac{-dv}{\cos ecv}=\frac{dx}{x}$$

$$\frac{-dv}{\cos ecv}=\frac{dx}{x}$$

$$-\sin vdv=\frac{dx}{x}$$

$$\therefore \cos v=\log x+c \qquad \cos\frac{y}{x} \quad \log x \quad c$$

$$At\ x=1, y=0 \therefore 1=0+c \qquad c \quad 1$$

$$\therefore \cos\frac{y}{x}=\log x+1$$

16. Solution:

$$\left(\left|\vec{a}\right|\vec{b}+\left|\vec{b}\right|\vec{a}\right).\left(\left|\vec{a}\right|\vec{b}-\left|\vec{b}\right|\vec{a}\right)=\left|\vec{a}\right|\left|\vec{a}\right|\vec{b}.\vec{b}+\left|\vec{b}\right|\left|\vec{a}\right|\vec{a}.\vec{b}-\left|\vec{a}\right|\left|\vec{b}\right|\vec{b}.\vec{a}-\left|\vec{b}\right|\left|\vec{b}\right|\vec{a}.\vec{a}$$

$$=\cancel{\left|\vec{a}\right|^2\left|\vec{b}\right|^2}+\cancel{\left|\vec{b}\right|\left|\vec{a}\right|\vec{a}.\vec{b}}-\cancel{\left|\vec{a}\right|\left|\vec{b}\right|\vec{b}.\vec{a}}-\cancel{\left|\vec{b}\right|^2\left|\vec{a}\right|^2}=0(\because \vec{a}.\vec{b}=\vec{b}.\vec{a})$$

17. Solution:

$$\int\frac{dx}{x(x^4-1)}=\int\frac{x^3dx}{x^4(x^4-1)}$$

$$Let\ x^4=t \Rightarrow 4x^3dx=dt \Rightarrow x^3dx=dt/4$$

$$\therefore \frac{1}{4}\int\frac{dt}{t(t-1)}=\frac{1}{4}\left[\int\left(\frac{1}{t-1}-\frac{1}{t}\right)dt\right]$$

$$=\frac{1}{4}[\log(t-1)-\log t]+c$$

$$=\frac{1}{4}\log\left(\frac{x^4-1}{x^4}\right)+c$$

18. Solution:

Clearly, the above lines are parallel.

$$\therefore Dis\tan ce=\left|\frac{\vec{b}\times(\overrightarrow{a_2}-\overrightarrow{a_1})}{\left|\vec{b}\right|}\right|$$

$$\vec{b}=2i-3j+k, \overrightarrow{a_1}=i+3j-2k, \overrightarrow{a_2}=2i+4j-k$$

$$\therefore \overrightarrow{a_2}-\overrightarrow{a_1}=i+j+k$$

$$\therefore \vec{b}\times(\vec{a_2}-\vec{a_1})=\begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix}=i(-3-1)-j(2-1)+k(2+3)=-4i-j+5k$$

$$\therefore distance=\frac{\left|\sqrt{16+1+25}\right|}{\left|\sqrt{4+9+1}\right|}=\frac{\sqrt{42}}{\sqrt{14}}$$

19. Solution:

$\vec{n_1}=2i+2j-3k, d_1=7$

$\vec{n_2}=2i+5j+3k, d_2=9$

Equation of plane:

$\vec{r}.(\vec{n_1}+\lambda\vec{n_2})=d_1+\lambda d_2$

$\vec{r}.(2i+2j-3k+\lambda(2i+5j+3k))=7+9\lambda$

$Let\ \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

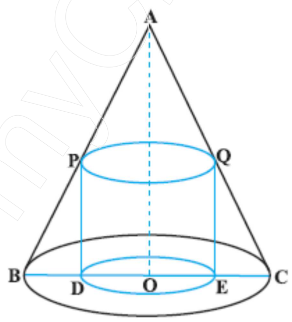
$\therefore x(2+2\lambda)+y(2+5\lambda)+z(-3+3\lambda)=7+9\lambda$

$Putting\ (x,y,z)=(2,1,3)\ we\ get\ \lambda=\frac{10}{9}$

$Substituting\ the\ value\ of\ \lambda\ we\ get, \vec{r}.(38i+68j+3k)=153$

Section C

20. Solution:



Let OC=r be the radius of the cone and OA=h be its height.

Let a cylinder with radius OE = x and height h’ be inscribed in the cone.

Surface Area = 2 π xh’

$\because \triangle QEC \sim \triangle AOC,$

$$\frac{QE}{AO}=\frac{CE}{CO}\Rightarrow\frac{h'}{h}=\frac{r-x}{r}\Rightarrow h'=h\left(\frac{r-x}{r}\right)$$

$$\therefore S=S(x)=2\pi xh'=2\pi xh\left(\frac{r-x}{r}\right)=\frac{2\pi h}{r}\left(rx-x^2\right)$$

$$S'(x)=\frac{2\pi h}{r}\left(r-2x\right)$$

$$S''(x)=\frac{2\pi h}{r}\left(-2\right)$$

$$S'(x)=0\Rightarrow x=r/2$$

$$Also,\ S''(r/2)=\frac{-4\pi h}{r}<0$$

Hence, x=r/2 is a point of maxima.

Thus, the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

21. Solution:

Let $\frac{1}{x}=u,\frac{1}{y}=v,\frac{1}{z}=w$

$\therefore$ the system of equations becomes,

$$3u-2v+3w=8$$
$$2u+v-w=1$$
$$4u-3v+2w=4$$

$$Let\ A=\begin{bmatrix}3&-2&3\\2&1&-1\\4&-3&2\end{bmatrix},b=\begin{bmatrix}8\\1\\4\end{bmatrix}$$

$$|A|=-17\neq 0, A^{-1}=\frac{-1}{17}\begin{bmatrix}-1&-5&-1\\-8&-6&9\\-10&1&7\end{bmatrix}, U=A^{-1}b=\frac{-1}{17}\begin{bmatrix}-1&-5&-1\\-8&-6&9\\-10&1&7\end{bmatrix}\begin{bmatrix}8\\1\\4\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\therefore u=1\Rightarrow x=1, v=2\Rightarrow y=\frac{1}{2}, w=3\Rightarrow z=\frac{1}{3}$$

22. Solution:

Suppose tailor A works for x days and tailor B works for y days.

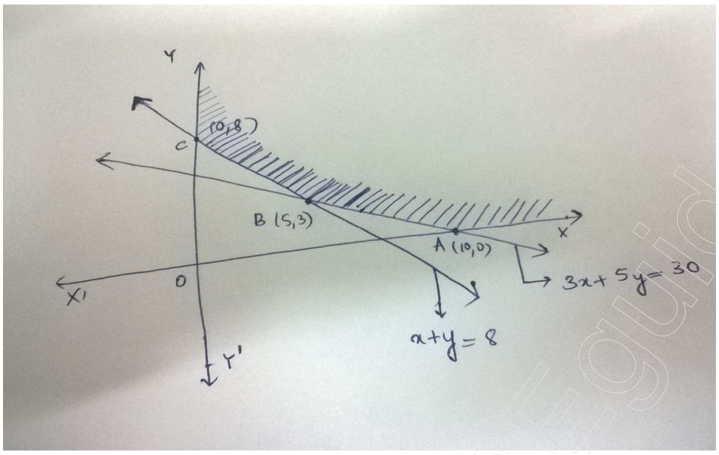
Then, Cost Z= 15x + 20y

	Tailor A	Tailor B	Min Requirement
Shirts	6	10	60
Pants	4	4	32
Cost per day	15	20	

The mathematical formulation of the problem is as follows:

Min Z= 15x+20y

$$\text{s.t} \quad \begin{aligned} &6x+10y \geq 60 \Rightarrow 3x+5y \geq 30 \\ &4x+4y \geq 32 \Rightarrow x+y \geq 8 \\ &x \geq 0, y \geq 0 \end{aligned}$$



We graph the above inequalities. The feasible region is as shown in the figure. We observe the feasible region is unbounded and the corner points are A,B and C. The co-ordinates of the corner points are (10,0), (5,3),(0,8).

Corner Point	Z=15x +20y
(10,0)	150
(5,3)	**135**
(0,8)	160

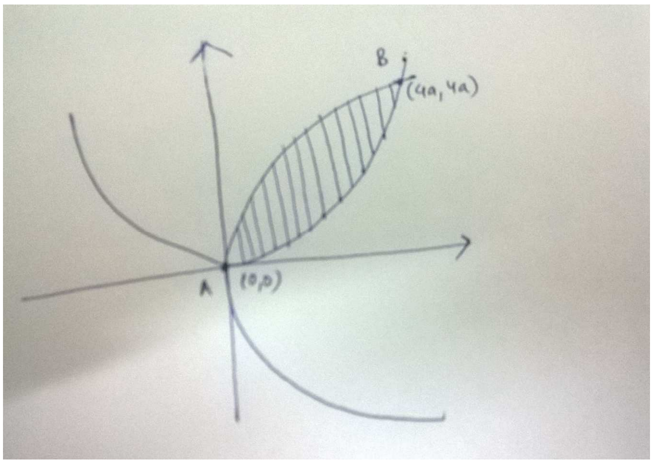
Thus cost is minimized by hiring A for 5 days and hiring B for 3 days.

23. Solution:

The point of intersection of the two curves:

$$x^2 = \frac{y^4}{16a^2} \Rightarrow 4ay = \frac{y^4}{16a^2}$$
$$\Rightarrow y(y^3 - 64a^3) = 0 \Rightarrow y = 0, y = 4a$$
$$If\ \ y = 0, x = 0; y = 4a \Rightarrow x = 4a$$

$\therefore$ points of intersection are A(0,0) and B(4a,4a)



$$Area = \int_0^{4a} (y_2 - y_1)dx = \int_0^{4a} \left(\sqrt{4ax} - \frac{x^2}{4a} \right) dx$$

$$= \sqrt{4a}\,\frac{x^{3/2}}{3/2} - \frac{x^3}{12a}\Bigg|_0^{4a} = \frac{16a^2}{3}$$

24. Solution:

Let E denote the event that the ball drawn is red.

Let E_1 denote the event that the ball is drawn from bag X, $P(E_1)=1/3$.

Let E_2 denote the event that the ball is drawn from bag Y, $P(E_2)=1/3$

Let E_3 denote the event that the ball is drawn from bag Z, $P(E_3)=1/3$

$P(E/E_1)=3/5$, $P(E/E_2)=4/9$, $P(E/E_3)=3/5$, $P(E_2/E)=?$

By Baye's theorem,

$$P(E_2/E) = \frac{P(E/E_2)P(E_2)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2) + P(E/E_3)P(E_3)}$$

$$= \frac{(4/9)(1/3)}{(3/5)(1/3) + (4/9)(1/3) + (3/5)(1/3)}$$

$$= \frac{10}{37}$$

25. Solution:

$$x = a(\cos t + t\sin t),\ y = a(\sin t - t\cos t)$$

$$\frac{dx}{dt} = a(\cancel{-\sin t} + \cancel{\sin t} + t\cos t) = at\cos t \Rightarrow \frac{dt}{dx} = \frac{1}{at\cos t}$$

$$\frac{dy}{dt} = a(\cancel{\cos t} - \cancel{\cos t} - t(-\sin t)) = at\sin t$$

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right)\left(\frac{dt}{dx}\right) = \left(at\sin t\right)\frac{1}{at\cos t} = \tan t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = \frac{d}{dt}\left(\tan t\right)\frac{dt}{dx} = \left(\sec^2 t\right)\left(\frac{1}{at\cos t}\right) = \frac{1}{at\cos^3 t}$$

26. Solution :

$$I = \int_0^{\pi} \frac{xdx}{4\cos^2 x + 9\sin^2 x} = \int_0^{\pi} \frac{(\pi - x)dx}{4\cos^2 x + 9\sin^2 x}$$

$$\therefore 2I = \pi\int_0^{\pi} \frac{dx}{4\cos^2 x + 9\sin^2 x} = 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{4\cos^2 x + 9\sin^2 x}$$

$$= 2\pi\left[\int_0^{\frac{\pi}{4}} \frac{dx}{4\cos^2 x + 9\sin^2 x} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{4\cos^2 x + 9\sin^2 x}\right]$$

$$= 2\pi\left[\int_0^{\frac{\pi}{4}} \frac{\sec^2 xdx}{4 + 9\tan^2 x} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos ec^2 xdx}{4\cot^2 x + 9}\right]$$

Putting tanx=t and cotx=u, we get

$$2I = 2\pi\left[\int_0^1 \frac{dt}{4 + 9t^2} - \int_1^0 \frac{du}{4u^2 + 9}\right] = 2\pi\left[\frac{1}{9}\left(\frac{3}{2}\right)\tan^{-1}\frac{t}{2/3}\Big|_0^1 - \frac{1}{4}\left(\frac{2}{3}\right)\tan^{-1}\frac{u}{3/2}\Big|_1^0\right]$$

$$= 2\pi\left[\frac{1}{6}\tan^{-1}\left(\frac{3}{2}\right) + \frac{1}{6}\tan^{-1}\left(\frac{2}{3}\right)\right] = \frac{2\pi}{6}\left(\frac{\pi}{2}\right) = \frac{\pi^2}{6}$$

$$\therefore I = \frac{\pi^2}{12}$$