SOLUTIONS

SAMPLE QUESTION PAPER - 6

Self Assessment________________________________

Time: 3 Hours **Maximum Marks: 100**

SECTION — A

1. Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Comparing with $X = x\hat{i} + y\hat{j} + z\hat{k}$, we get

$x = 1,$

$y = -2,$

$z = 2$

Magnitude
$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$
$$= \sqrt{1^2 + (-2)^2 + 2^2}$$
$$= \sqrt{1+4+4}$$
$$= 3 \text{ units.}$$

$\therefore$ Unit vector in the direction of given vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$
$$= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

Hence, the vector in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ which has magnitude 9 units is given by

$$9\hat{a} = 9\left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}\right)$$
$$= 3\hat{i} - 6\hat{j} + 6\hat{k}.$$

2. Given,
$$\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-4}{4}$$
$$\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-2}{2}$$
$$\vec{i} = (3\hat{i} - 4\hat{j} + 2\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$$

3. Given differential equation is
$$\left(\frac{dy}{dx}\right)^4 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$$

Here, highest order derivative is d^2y/dx^2, Whose degree is one. So, degree of differential equation is 1.

4. **(i)** Transpose of $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -2 \end{bmatrix}$ is $\begin{bmatrix} 5 & \frac{1}{2} & -2 \end{bmatrix}$

(ii) Transpose of $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ is $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ ½

(iii) Transpose of $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$ is $\begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

½

5. $$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$$
$$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right) dx$$
$$= \int x^{5/2} dx + 3\int x^{1/2} dx + 4\int x^{-1/2} dx$$
$$= \frac{x^{5/2+1}}{\left(\frac{5}{2}+1\right)} + 3.\frac{x^{1/2+1}}{\left(\frac{1}{2}+1\right)} + 4.\frac{x^{-1/2+1}}{\left(-\frac{1}{2}+1\right)} + c \quad ½$$
$$= \frac{x^{7/2}}{\left(\frac{7}{2}\right)} + 3.\frac{x^{3/2}}{\left(\frac{3}{2}\right)} + 4.\frac{x^{1/2}}{\left(\frac{1}{2}\right)} + c$$
$$= \frac{2}{7}x^{7/2} + 3.x^{3/2} + 8.x^{1/2} + c \quad ½$$

6.
$$\vec{a} = \hat{i} - \hat{j} + \hat{k}$$
$$\vec{b} = \hat{i} + \hat{j} - \hat{k}$$
$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|} \quad ...(1)$$
$$\vec{a}.\vec{b} = (\hat{i} - \hat{j} + \hat{k}).(\hat{i} + \hat{j} - \hat{k}) \quad ½$$
$$= 1 - 1 - 1$$
$$= -1$$
$$|\vec{a}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$
$$|\vec{b}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

Hence from (1)
$$\cos\theta = \frac{-1}{3}$$
$$\theta = \cos^{-1}\left(-\frac{1}{3}\right) \quad ½$$

SECTION — B

7. Given, $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

We have to find the value of $A^2 - 3A + 2I$.

Now, $A^2 = A \cdot A$
$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

[Multiplying row by column]
$$A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$
$$3A = 3\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{bmatrix}$$

and
$$2I = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$
$$\Rightarrow \quad A^2 - 3A + 2I$$
$$= \begin{bmatrix} 5-6+2 & -1-0+0 & 2-3+0 \\ 6-6+0 & -2-3+2 & 5-9+0 \\ 0-3+0 & -1+3+0 & -2-0+2 \end{bmatrix}$$
$$\Rightarrow \quad A^2 - 3A + 2I = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{bmatrix}$$

8. $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)$
$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right]$$
$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{25}{169}} + \frac{5}{13}\sqrt{1-\frac{16}{25}}\right] + \sin^{-1}\left(\frac{16}{65}\right) \quad \mathbf{1}$$
$$= \sin^{-1}\left[\frac{4}{5}\times\frac{12}{13} + \frac{5}{3}\times\frac{3}{5}\right] + \sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left(\frac{63}{65}\right) + \sin^{-1}\left(\frac{16}{65}\right)$$
$$= \sin^{-1}\left[\frac{63}{65}\sqrt{1-\left(\frac{16}{65}\right)^2} + \frac{16}{65}\sqrt{1-\left(\frac{63}{65}\right)^2}\right] \quad \mathbf{1}$$
$$= \sin^{-1}\left[\frac{63}{65}\sqrt{1-\frac{256}{4225}} + \frac{16}{65}\sqrt{1-\frac{3969}{4225}}\right]$$
$$= \sin^{-1}\left[\left(\frac{63}{65}\right)\times\left(\frac{63}{65}\right) + \left(\frac{16}{65}\right)\times\left(\frac{16}{65}\right)\right]$$
$$= \sin^{-1}\left[\frac{3969}{4225} + \frac{256}{4225}\right] \quad \mathbf{1}$$
$$= \sin^{-1}\left[\frac{4225}{4225}\right] = \sin^{-1}1 = \sin^{-1}\left(\sin\frac{\pi}{2}\right) = \frac{\pi}{2} \quad \mathbf{1}$$

Or

$$\tan^{-1}3x + \tan^{-1}2x = \frac{\pi}{4}$$
$$= \tan^{-1}\left[\frac{3x+2x}{1-(3x)(2x)}\right] \quad \mathbf{1}$$

.....provided that $2x \times 3x < 1$
$$\Rightarrow \quad x^2 < \frac{1}{6}$$
$$\Rightarrow \quad \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4} \quad \mathbf{1}$$
$$\Rightarrow \quad \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \tan\frac{\pi}{4}\tan^{-1}(1)$$
$$\frac{5x}{1-6x^2} = 1$$
$$\Rightarrow \quad 5x = 1 - 6x^2$$
$$\Rightarrow \quad 6x^2 + 5x - 1 = 0$$
$$\Rightarrow 6x^2 + 6x - x - 1 = 0$$

$\Rightarrow \quad 6x(x+1) - 1(x+1) = 1$

$\Rightarrow \quad (6x-1)(x+1) = 0$

$\Rightarrow \quad x = -1, \frac{1}{6}$ **1**

$x = -1$ is rejected as $(-1)^2 > \frac{1}{6}$

$x = \frac{1}{6}$ is accepted as $\left(\frac{1}{6}\right)^2 < \frac{1}{6}$

Hence, $x = \frac{1}{6}$ is the answer. **1**

9. $\Delta = \begin{bmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{bmatrix}$

Using $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{bmatrix} 5x+\lambda & 2x & 2x \\ 5x+\lambda & x+\lambda & 2x \\ 5x+\lambda & 2x & x+\lambda \end{bmatrix}$$

$$= (5x+\lambda)\begin{bmatrix} 1 & 2x & 2x \\ 1 & x+\lambda & 2x \\ 1 & 2x & x+\lambda \end{bmatrix} \quad \textbf{1}$$

Using $R_2 \to R_2 - R_1, \to R_3 - R_1$

$$= (5x+\lambda)\begin{bmatrix} 1 & 2x & 2x \\ 0 & -x+\lambda & 0 \\ 0 & 0 & -x+\lambda \end{bmatrix} \quad \textbf{2}$$

Expanding along C_1, we get

$= (5x+\lambda)\{(\lambda - x)^2 - 0 + 0)\}$

$\Delta = (5x+\lambda)(\lambda - x)^2$ **1**

[CBSE Marking Scheme 2014]

Or

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$, we get

$$\Delta = \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (c+a)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix} \quad \textbf{1}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix} \quad \textbf{½}$$

Applying $R_3 \to R_3 - (R_1 + R_2)$, we get

$$\Delta = (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix} \quad \textbf{1}$$

Applying $C_1 \to aC_1$ and $C_2 \to bC_2$, we get

$$\Delta = \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ 0 & b(c+a-b) & b^2 \\ -2ba & -2ab & 2ab \end{vmatrix}$$

½

Applying $C_1 \to C_1 + C_3$, $C_2 \to C_2 + C_3$, we get

$$\Delta = \frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c) & a^2 & a^2 \\ b^2 & b(a+c) & b^2 \\ 0 & 0 & 2ab \end{vmatrix} \quad \textbf{½}$$

$$= \frac{(a+b+c)^2}{ab} \times ab \times 2ab \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ 0 & 0 & 1 \end{vmatrix}$$

$= 2ab(a+b+c)^2[(b+c)(c+a) - ab]$

$= 2abc(a+b+c)^3$ **½**

Hence Proved

[CBSE Marking Scheme 2010]

10. Given, $\quad (x^2 + y^2)^2 = xy$

or $\quad x^4 + y^4 + 2x^2y^2 - xy = 0$

Differentiating w.r.t. x, we get

$$4x^3 + 4y^3 \frac{dy}{dx}$$

$$+ (2x^2.2y\frac{dy}{dx} + y^2.4x) - \left(x\frac{dy}{dx} + y.1\right) = 0 \quad \textbf{1}$$

$$= 4x^3 + 4y^3\frac{dy}{dx} + 4x^2y\frac{dy}{dx} + 4xy^2 - x\frac{dy}{dx} - y = 0$$

$$\Rightarrow \frac{dy}{dx}(4y^3 + 4x^2y - x) + 4x^3 + 4xy^2 - y = 0 \quad \textbf{2}$$

$$\frac{dy}{dx} = \frac{-4x^3 - 4xy^2 + y}{4y^3 + 4x^2y - x}$$

or $$\frac{dy}{dx} = -\left(\frac{4x^3 + 4xy^2 - y}{4y^3 + 4x^2y - x}\right) \quad \textbf{1}$$

11. $y = 3\cos(\log x) + 4\sin(\log x)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{3.(-\sin\log x)}{x} + \frac{4\cos(\log x)}{x}$$

Again differentiating,

$$\frac{d^2y}{dx^2} = -3\left[\frac{\frac{x(\cos\log x)}{x} - \sin\log x}{x^2}\right] + 4\left[\frac{\frac{x(-\sin\log x)}{x} - \cos(\log x)}{x^2}\right] \quad \mathbf{1}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \left(\frac{-3\cos\log x + 3\sin\log x}{x^2}\right) + \left(\frac{-4\sin\log x - 4\cos\log x}{x^2}\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-7\cos\log x - \sin\log x}{x^2}$$

Now, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ **2**

$$\text{L.H.S.} = x^2\left(\frac{-7\cos\log x - \sin\log x}{x^2}\right) + x\left(\frac{-3\sin\log x}{x} + \frac{4\cos\log x}{x}\right) + 3\cos\log x + 4\sin\log x$$

$= -7\cos\log x - \sin\log x - 3\sin\log x + 4\cos\log x + 3\cos\log x + 4\sin\log x$

$= 0 =$ R.H.S. **1**

12. Let $y = \cos x.\cos 2x.\cos 3x$

Taking log on both sides

$\log y = \log(\cos x.\cos 2x.\cos 3x)$

$= \log\cos x + \log\cos 2x + \log\cos 3x$

[Note : $\log_a mn = \log_a m + \log_a n$] **1**

Differentiating both sides w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}\frac{d}{dx}(\cos x) + \frac{1}{\cos 2x}\frac{d}{dx}(\cos 2x) + \frac{1}{\cos 3x}\frac{d}{dx}(\cos 3x) \quad \mathbf{1}$$

$$= \frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}$$

$= -[\tan x + 2\tan 2x + 3\tan 3x]$ **1**

$$\therefore \quad \frac{dy}{dx} = -\cos x.\cos 2x.\cos 3x \times [\tan x + 2\tan 2x + 3\tan 3x] \quad \mathbf{1}$$

13. Let $\text{I} = \int \frac{\sin(x-a)}{\sin(x+a)}dx$

Put $x + a = t$

$\Rightarrow \quad x = t - a$

$\Rightarrow \quad dx = dt$

$$\therefore \quad \text{I} = \int \frac{\sin(t-2a)}{\sin t}dt$$

$$\Rightarrow \quad \text{I} = \int \frac{\sin t\cos 2a - \cos t\sin 2a}{\sin t}dt$$

[$\because$ $\sin(x-y) = \sin x\cos y - \cos x\sin y$]

$$\Rightarrow \quad \text{I} = \int \frac{\sin t\cos 2a}{\sin t}dt - \int \frac{\cos t\sin 2a}{\sin t}dt$$

$$\Rightarrow \quad \text{I} = \cos 2a\int dt - \sin 2a\int \cot t\, dt$$

$= t\cos 2a - \sin 2a\,|n|\sin t| + c$

$= (x + a)\cos 2a - \sin 2a|n|\sin(x + a)| + c$

14. $$\int \frac{dx}{\sqrt{5-4x-2x^2}} = \int \frac{dx}{\sqrt{2\left(\frac{5}{2}-2x-x^2\right)}} \quad \mathbf{1}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\frac{5}{2}-2x-x^2+1-1}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\frac{5}{2}+1\right)-(x^2+2x+1)}} \quad \mathbf{1}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\frac{7}{2}-(x+1)^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2-(x+1)^2}}$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x+1}{\sqrt{\frac{7}{2}}}\right) + C \quad \mathbf{1}$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\sqrt{\frac{2}{7}}(x+1) + C \quad \mathbf{1}$$

Or

$\int x\sin^{-1}x\,dx$

$$\text{I} = \int x\sin^{-1}x\,dx$$

$$= \int \sin^{-1}x.x\,dx \quad \mathbf{1}$$

$$= \sin^{-1}x\int x\,dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int x\,dx\right\}dx$$

$$= \sin^{-1}x.\left(\frac{x^2}{2}\right) - \int \frac{1}{\sqrt{1-x^2}}.\frac{x^2}{2}dx$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\text{I}_1$$

where $\text{I}_1 = \int \frac{x^2}{\sqrt{1-x^2}}dx$

Put $x = \sin\theta$, $dx = \cos\theta\, d\theta$ **1**

$\therefore \quad I_1 = \int \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}.\cos\theta d\theta$

$= \int \frac{\sin^2\theta}{\cos\theta}.\cos\theta d\theta = \int \sin^2\theta\, d\theta$

$= \frac{1}{2}\int (1-\cos 2\theta)\, d\theta$

$= \frac{1}{2}\int d\theta - \frac{1}{2}\int \cos 2\theta d\theta$ **1**

$= \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta + C_1$

$= \frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2} + C_1$

$\left[\because \sin\theta = x, \cos\theta = \sqrt{1-x^2}\right]$

$I = \frac{x^2}{2}\sin^{-1}x$

$-\frac{1}{2}\left[\frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}\right] + C$

$I = \frac{1}{4}\sin^{-1}x.(2x^2-1) + \frac{x\sqrt{1-x^2}}{4} + C$ **1**

15. **(i)** Given matrix equation is

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

On applying $C_2 \to C_2 - C_1$, we get

$$\begin{bmatrix} 4 & 2 & -8 \\ 3 & 3 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$$

Which is the required answer.

(ii) The given differential equation is

$2xy\, dx + (x^2 + 2y^2)\, dy = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{2xy}{x^2+2y^2}$ ...(1)

This is a linear homogeneous differential equation

Put $\quad y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v.1 + \frac{dv}{dx}.x$ ...(2)

From (1) and (2), we get

$v + x\frac{dv}{dx} = -\frac{2x(vx)}{x^2+2v^2x^2}$

$= \frac{-2v}{1+2v^2}$ **1**

$\Rightarrow \quad x\frac{dv}{dx} = -v - \frac{2v}{1+2v^2}$

$= \frac{-(3v+2v^3)}{1+2v^2}$

$\Rightarrow \quad \frac{1+2v^2}{3v+2v^3}dv + \frac{dx}{x} = 0$

Integrating, we get

$\int \frac{1+2v^2}{3v+2v^3}\, dv + \int \frac{dx}{x} = 0$ **1**

$\Rightarrow \quad \frac{1}{3}\int \frac{dt}{t} + \log x = \log C_1,$

[Put $3v + 2v^3 = t$]

$\Rightarrow \quad \frac{1}{3}\log t + \log x = \log C_1$

$\Rightarrow \quad t^{1/3}.\, x = C_1$

$\Rightarrow \quad tx^3 = C, \quad$ where $C = C_1^{\,3}$

$\Rightarrow \quad (3v + 2v^3)\, x^3 = C$

$\Rightarrow \quad 3vx^3 + 2v^3x^3 = C$

$\Rightarrow \quad 3x^2y + 2y^3 = C, \quad [\because vx = y]$ **1**

Which is the required solution of (1).

16. $\frac{dy}{dx} + 2\tan xy = \sin x$ ...(i)

This is a linear differential equation

Here, $\quad P = 2\tan x$ and $Q = \sin x$

$\text{I.F.} = e^{\int P dx} = e^{\int 2\tan x\, dx} = e^{2\log\sec x}$

$= e^{\log\sec^2 x} = \sec^2 x$ **2**

So solution of equation (i) is

$y.\sec^2 x = \int \sin x.\sec^2 x\, dx + c$ **1**

$= \int \tan x.\sec x\, dx + c$

$= \sec x + c$

$\Rightarrow \quad y = \cos x + c\cos^2 x.$ **1**

17. Given,

$\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$

$\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

$\vec{a}+\vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) + (5\hat{i} - \hat{j} + \lambda\hat{k})$ **1**

$= 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$

$\vec{a}-\vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) - (5\hat{i} - \hat{j} + \lambda\hat{k})$

$= -4\hat{i} + (7-\lambda)\hat{k}$ **1**

Since $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular vectors,

so $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$

$\{6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}\}\cdot\{-4\hat{i} + (7-\lambda)\hat{k}\} = 0$

$\Rightarrow \quad -24 + (7+\lambda)(7-\lambda) = 0$ **1**

$\Rightarrow \quad 49 - \lambda^2 - 24 = 0$

$\Rightarrow \quad \lambda^2 = 49 - 24 = 25$

$\Rightarrow \quad \lambda = \pm 5$ units. **1**

18. $\vec{r} = (\hat{i}+\hat{j}) + \lambda\,(2\hat{i}-\hat{j}+\hat{k})$

and $\vec{r} = (2\hat{i}+\hat{j}-\hat{k}) + \mu\,(3\hat{i}-5\hat{j}+2\hat{k})$

here $\vec{a_1} = (\hat{i}+\hat{j}),\ \vec{b_1} = (2\hat{i}-\hat{j}+\hat{k})$

$\vec{a_2} = (2\hat{i}+\hat{j}-\hat{k}),\ \vec{b_2} = (3\hat{i}-5\hat{j}+2\hat{k})$ ½

$\vec{a_2} - \vec{a_1} = (2\hat{i}+\hat{j}-\hat{k}) - (\hat{i}+\hat{j})$

$= \hat{i}-\hat{k}$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$\vec{b_1}\times\vec{b_2} = 3\hat{i}-\hat{j}-7\hat{k}$ **1**

Shortest distance between the line is given by

$$d = \left|\frac{(\vec{a_1}\times\vec{a_2}).(\vec{b_1}\times\vec{b_2})}{|b_1-b_2|}\right|$$ **1**

or $$d = \left|\frac{(\hat{i}-\hat{k}).(3\hat{i}-\hat{j}-7\hat{k})}{\sqrt{(3)^2+(-1)^2+(-7)^2}}\right|$$ **1**

or $$d = \frac{3+7}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$ ½

19. Let X denote the number of successes in 6 trials. The X is a binomial variate with parameters

$n = 6$ and

$p = \frac{2}{3}$ such that

$$P(X = r) = {}^6C_r\left(\frac{2}{3}\right)^{6-r}\left(\frac{1}{3}\right)^r$$ **1**

$r = 0, 1, 2, ...6$ ...(i)

$\therefore$ Probability of at least 4 successes

$= P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

$$= {}^6C_4\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_5\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5 + {}^6C_6\left(\frac{2}{3}\right)^0\left(\frac{2}{3}\right)^6$$ **1**

$$= 15\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + 6\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5 + \left(\frac{1}{3}\right)^6$$ **1**

$$= \left(\frac{1}{3}\right)^6[15\times4+6\times2+1] = 73\left(\frac{1}{3}\right)^6$$ **1**

Or

We have, mean $= np = 4$ ...(1)
and variance $= npq = 4/3$ ...(2)
dividing (2) by (1) we get,

$$\frac{npq}{np} = \frac{\frac{4}{3}}{4} \Rightarrow q = 1/3$$ ½

Or $p = 1-q = 1-1/3$

$\Rightarrow$ $p = 2/3$ ½

From (1), $n(2/3) = 4$

$\Rightarrow$ $n = 6$ ½

$\therefore$ **Binomial distribution is,**

$$(q+p)n = \left(\frac{1}{3}+\frac{2}{3}\right)^6$$ ½

$P(x \geq 1) = 1 - P(x = 0)$

$$= 1 - 6\,C_0\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)$$ **1**

$$= 1 - \frac{1}{729}$$

$$P(x \geq 1) = \frac{728}{729}$$ **1**

SECTION — C

20. Given function is $f : N \to N$ such that

$$f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$$

one-one from the given function, we observe that

Case-I : When x is odd.

Let $f(x_1) = f(x_2)$

$x_1 + 1 = x_2 + 1$

$\Rightarrow$ $x_1 = x_2$

$\because$ $f(x_1) = f(x_2)$

$x_1 = x_2,\ \forall\ x_1, x_2 \in N.$

So, $f(x)$ is one-one. **2**

Case-II : When x is even.

Let $f(x_1) = f(x_2)$

$x_1 - 1 = x_2 - 1$

$\Rightarrow$ $x_1 = x_2$

$\because$ $f(x_1) = f(x_2)$

$\because$ $x_1 = x_2,\ \forall\ x_1, x_2 \in N.$

So, $f(x)$ is one-one.

Hence, from Case I and Case II, we observe that $f(x_1) = f(x_2)$

$\because$ $x_1 = x_2,\ \forall\ x_1, x_2 \in N.$

Therefore $f(x)$ is one-one. **2**

Onto : To show $f(x)$ is onto, we show that its range and codomain are same.

From the definition of given function, we observe that

$f(1) = 1 + 1 = 2$

$f(2) = 2 - 1 = 1$

$f(3) = 3 + 1 = 4$

$f(4) = 4 - 1 = 3$ and so on.

So, we get set of natural numbers as the set of values of $f(x)$.

$\Rightarrow$ Range of $f(x) = N$

Also, given that codomain = N

$$\left[\because \begin{array}{ccc} f: N & \to & N \\ \text{domain} & & \text{codomain} \end{array}\right]$$

Thus range = codomain. Therefore, $f(x)$ is an onto function.

Hence, the function $f(x)$ is bijective. **2**

Or

(i) Commutative : let $x, y \in R - \{-1\}$ then

$x*y = x + y + xy = y + x + yx = y*x$ $\therefore$ * is commutative. **1**

(ii) Associative : let $x, y \in R - \{-1\}$ then

$x*(y*z) = x*(y + z + yz) = x + (y + z + yz) + x(y + z + yz)*x$

$= x + y + z + xy + yz + zx + xyz$ **1½**

$(x*y)*z = (x + y + xy)*z = (x + y + xy) + z + (x + y + xy)\cdot z$

$= x + y + z + xy + yz + zx + xyz$ **1**

$x*(y*z) = (x*y)*z$ $\therefore$ * is Associative.

(iii) Identity Element : let $e \in R - \{-1\}$ such that $a*e = e*a = a \ \forall \ a \in R - \{-1\}$ **½**

$\therefore \quad a + e + ae = a \Rightarrow e = 0$

(iv) Inverse : let $a*b = b*a = e = 0$; $a, b \in R - (-1)$ **½**

$a + b + ab = 0 \ \therefore \ b = \dfrac{-a}{1+a}$ or $a^{-1}\dfrac{-a}{1+a}$ **1**

21. Let $P(t^2, 2t)$ be the point on $y^2 = 4x$, nearest to the point A(2, –8), then

$$AP^2 = (t^2 - 2)^2 + (2t + 8)^2 \quad \textbf{1}$$

Taking $y = AP^2$, we have

$$\begin{aligned} y &= (t^2 - 2)^2 + (2t + 8)^2 \\ &= (t^2 - 2)^2 + (2t + 8)^2 \\ &= (t^4 - 4t^2 + 4) + (4t^2 + 32t + 64) \\ &= t^4 + 32t + 68 \end{aligned} \quad \textbf{1}$$

Differentiate y w.r.t. t,

$$\frac{dy}{dt} = 4t^3 + 32$$

Again differentiate w.r.t. t,

$$\frac{d^2y}{dt^2} = 12t^2.$$

For max. or min. value of y, (and hence of AP) **1**

$$\frac{dy}{dx} = 0$$

$\Rightarrow \quad 4(t^3 + 8) = 0$

$\Rightarrow \quad (t + 2)(t^2 - 2t + 4) = 0$

$\Rightarrow \quad t = -2$ **1**

$[t^2 - 2t + 4 = 0$ has imaginary values of $(t)]$

For $\quad t = -2$,

$$\frac{d^2y}{dt^2} = 12(-2)^2 = 48 > 0 \quad \textbf{1}$$

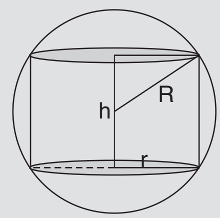
$\Rightarrow \quad y$ is minimum

Hence, AP is minimum

$\Rightarrow P(t^2, 2t) = (4, -4)$ is the point on $y^2 = 4x$, nearest to the given point A(2, –8). **1**

Or

Let the radius and height of the cylinder be r and h respectively.



½

(Volume) $V = r^2h$ ...(i) **½**

But, $\quad r^2 = R^2 - \dfrac{h^2}{4}$

$$\therefore \quad \pi h\left(R^2 - \frac{h^2}{4}\right) = \pi\left(R^2h - \frac{h^3}{4}\right) \quad \textbf{1}$$

$$\frac{dV}{dh} = \pi\left(R^2 - \frac{3h^2}{4}\right) \quad \textbf{½}$$

$$\therefore \quad \frac{dV}{dh} = 0$$

$$\Rightarrow \quad h^2 = \frac{4R^2}{3} \quad \textbf{1}$$

$$\text{or} \quad h = \frac{2R}{\sqrt{3}} \quad \textbf{½}$$

$$\text{and} \quad \frac{d^2V}{dh^2} = \pi\left(\frac{-6h}{4}\right) < 0,$$

$\therefore$ Volume is maximum

$$\text{Maximum volume} = \pi\left[R^2 \cdot \frac{2R}{\sqrt{3}} - \frac{1}{4}\left(\frac{2R}{\sqrt{3}}\right)^3\right]$$

$$= \frac{4\pi R^3}{3\sqrt{3}} \text{ cumbe units}$$

[CBSE Marking Scheme 2014]

22. The parabola and the line are

$$4y = 3x^2 \qquad ...(1)$$

$$3x - 2y + 12 = 0 \text{ respectively} \qquad ...(2)$$

Multiplying eqn. (2) by 2 and subtracting from (1)

$$0 = 3x^2 - 6x - 24$$

$$x^2 - 2x - 8 = 0$$

$$x^2 - 4x + 2x - 8 = 0$$

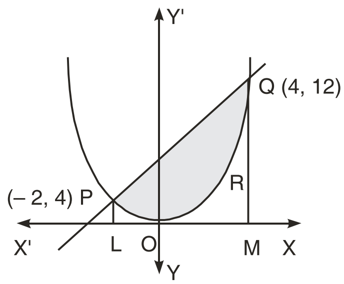
$\Rightarrow x(x-4) + 2(x-4) = 0$

or $(x-4)(x+2) = 0$

$\Rightarrow \qquad x = 4, -2$

From (2), $\qquad y = 12, 3$ **1**

The graph of the parabola and lines are shown in the figure. They intersect at P(–2, 3) and Q (4, 12).



1

The area enclosed by the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$

= Area of the region PORQP

= Area of trapezium PLMQP

– Area of the region LMQROP**1**

$$= \int_{-2}^{4} (y_1 - y_2)\,dx$$

$$= \int_{-2}^{4} \left(\frac{3x+12}{2} - \frac{3x^2}{4}\right) dx \qquad \mathbf{1}$$

where $y_1 = \dfrac{3x+12}{2}$ and $y_2 = \dfrac{3x^2}{4}$

Required area $= \dfrac{1}{2}\displaystyle\int_{-2}^{4} (3x+12)\,dx - \dfrac{3}{4}\int_{-2}^{4} x^2\,dx$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4} \qquad \mathbf{1}$$

$$= \frac{1}{2}[(24+48) - (6-24)] - \frac{3}{4}\left[\frac{64}{3} + \frac{8}{3}\right]$$

$$= \frac{90}{2} - \frac{3 \times 72}{4 \times 3}$$

$= 45 - 18$

$= 27$ square units **1**

23. The given diff. equation can be written as :

$$\frac{dx}{dy} + (\cot y)x = \cos y \qquad \mathbf{1}$$

This is linear differential equation

$\Rightarrow \qquad$ I.F. $= e^{\int \cot y\, dy} = e^{\log \sin y} = \sin y$ **1**

$\therefore$ The solution is :

$$x \sin y = \int \sin y \cos y\, dy + c$$

$$= \frac{1}{2}\int \sin 2y\, dy + c$$

$\Rightarrow x \sin y = \dfrac{-1}{4}\cos 2y + c$ **1**

It is given that $y = 0$, when $x = 0$ **1**

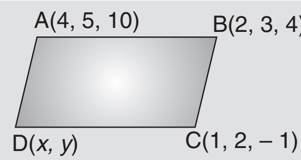
$$c - \frac{1}{4} = 0 \Rightarrow c = \frac{1}{4}$$

$\therefore \; x \sin y = \dfrac{1}{4}(1 - \cos 2y) = \dfrac{1}{2}\sin^2 y$ **1**

$\Rightarrow \qquad 2x = \sin y$

Which is the required solution. **1**

24.



P.V. of A $= 4\hat{i} + 5\hat{j} + 10\hat{k}$

and $\qquad$ B $= 2\hat{i} + 3\hat{j} + 4\hat{k}$

Vector equation of AB is

$$\vec{r} = \left(4\hat{i} + 5\hat{j} + 10\hat{k}\right)$$

$$+\lambda\left[\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) - \left(4\hat{i} + 5\hat{j} + 10\hat{k}\right)\right] \qquad \mathbf{1}$$

$$= 4\hat{r} + 5\hat{j} + 10\hat{k} + \lambda\left(-2\hat{i} - 2\hat{j} - 6\hat{k}\right) \qquad \mathbf{1}$$

or $\qquad \vec{r} = 4\hat{r} + 5\hat{j} + 10\hat{k} + \lambda\left(\hat{i} + \hat{j} + 5\hat{k}\right)$ **1**

Similarly, vector equation BC is **1½**

$$\vec{r} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + \mu\left[-\hat{i} - \hat{j} - 5\hat{k}\right]$$

or $\qquad \vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k} + \mu\left[\hat{i} + \hat{j} + 5\hat{k}\right]$

Mid-point of AC is $\dfrac{5}{2}\hat{i} + \dfrac{7}{2}\hat{j} + \dfrac{9}{2}\hat{k}$

Mid-point of BD is $\dfrac{x+2}{2}\hat{i} + \dfrac{3+y}{2}\hat{j} + \dfrac{4+z}{2}\hat{k}$ **1**

coordinates of D are (3, 4, 5). **½**

[CBSE Marking Scheme 2010]

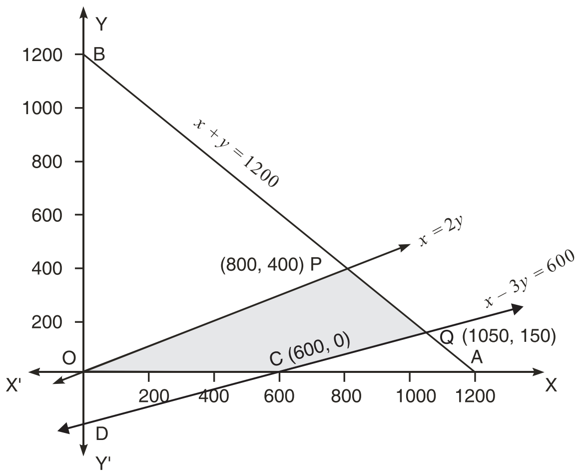
25. Let x dolls of type A and y dolls of type B are produced to have the maximum profit.

Company makes profits of ₹ 12 and ₹ 16 per doll respectively on doll A and B.

$\Rightarrow$ Profit $\qquad Z = 12x + 16y$

The production level of $x + y$ should not exceed 1200

$$x + y \leq 1200 \qquad \mathbf{1}$$



½

The production level of dolls of type A exceeds three times the production of dolls of type B by at most 600.

$\Rightarrow \quad x - 3y \leq 600$

Demand for dolls of type B is atmost half of that for dolls of type A

$$y \leq \frac{x}{2}$$ **1**

Thus, L.P.P. may be stated as

To maximize $\quad z = 12x + 16y$

Constraints, are $x + y \leq 1200$, $x - 3y \leq 600$,

$$y \leq \frac{x}{2},\ x, y \geq 0$$ ½

(i) The line $\quad x + y = 1200$

passes through A(1200, 0), B(0, 1200)

Putting $x = 0$, $y = 0$ in $x + y \leq 1200$, we get $0 \leq 1200$ which is true.

$\Rightarrow x + y \leq 1200$ lies on and below AB.

(ii) The line $x - 3y = 600$ passes through **1**
C(400, 0), D(0, – 200).

Putting $x = 0$, $y = 0$ in $x - 3y \leq 600$.

We get $0 \leq 600$, which is true.

$\Rightarrow$ Origin lies in the region.

i.e., $x - 3y \leq 600$ lies on and above CD.

(iii) $y = \frac{x}{2}$ passes through (400, 200) and (0, 0),

Put $x = 200$, $y = 0$ in $y \leq \frac{x}{2}$, $0 \leq \frac{200}{2}$, which is true.

$\Rightarrow$ (200, 0) lies in this region *i.e.,* the region lies on and below OP.

(iv) $x \geq 0$ lies on and to the right of y-axis.

(v) $y \geq 0$ lies on and above x-axis.

The shaded area PQCO represents the feasible region.

The point P is in the intersection of the lines

$$x + y = 1200,\ y = \frac{x}{2}$$

Solving, we get the coordinates of P(800, 400). **1**

The point Q is the intersection of the lines

$$x + y = 1200 \qquad ..(1)$$

$$x - 3y = 600 \qquad ...(2)$$

Subtracting $\quad 4y = 600$, $y = 150$

$\Rightarrow \quad x = 1200 - 150 = 1050$

$\therefore$ The point Q is (1050, 150).

The point C is (600, 0).

Objective function is

$$z = 12x + 16y$$

At P (800, 400), $\quad z = 9600 + 6400 = 16000$

At Q (1050, 150), $z = 12 \times 1050 + 16 \times 150$
$= 12600 + 2400 = 15000$

At C (600, 0), $\quad z = 12 \times 600 + 0$
$= 7200 + 0 = 7200$

At O (0, 0), $\quad z = 0$

$\Rightarrow$ Z is maixmum at P (800, 400). The maximum value of Z is ₹. 16000.

Thus, to maximize the profit, 800 dolls of type A and 400 dolls of type B should be produced to get a maximum profit of ₹ 16000. **1**

26. Let E_1, E_2 and E_3 be the events that the insured vehicle is a scooter, a car or a truck. Let A be the event that an insured vehicle meets an accident, then

$$P(E_1) = \frac{3000}{3000+4000+5000}$$

$$= \frac{3}{12}$$

$$P(E_2) = \frac{4}{12}$$ **1**

and $\quad P(E_3) = \frac{5}{12}$

Also, $\quad P(A/E_1) = 0.02$, $P(A/E_2) = 0.03$,
$P(A/E_3) = 0.04$.

Now, $\displaystyle\sum_{i=1}^{3} P(E_i).P\left(\frac{A}{E_i}\right)$ **1**

$$= \frac{3}{12} \times 0{\cdot}02 + \frac{4}{12} \times 0{\cdot}03 + \frac{5}{12} \times 0{\cdot}04$$

$$= \frac{0{\cdot}38}{12}$$ **1**

$\therefore$ By Baye's theorem, we have

(i) $$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{\sum_{i=1}^{3} P(E_i).P(A/E_i)}$$

$$= \frac{\frac{3}{12}\times 0\cdot 02}{\frac{0\cdot 38}{12}} = \frac{3}{19}$$ **1**

Similarly,

(ii) $$P(E_2/A) = \frac{\frac{4}{12}\times 0\cdot 03}{\frac{0\cdot 38}{12}} = \frac{6}{19}$$ **1**

(iii) $$P(E_3/A) = \frac{\frac{5}{12}\times 0\cdot 04}{\frac{0\cdot 38}{12}} = \frac{10}{19}$$ **1**

■■■

SOLUTIONS

SAMPLE QUESTION PAPER - 7

Self Assessment

Time: 3 Hours **Maximum Marks: 100**

SECTION — A

1. Order = 2

2. Degree = 5

3. (i) $18 = 1 \times 18 = 2 \times 9 = 3 \times 6$

There are 6 matrices of order

1×18, 18×1, 2×9, 9×2, 3×6, 6×3. ½

(ii) $5 = 1 \times 5$

There are two matrices with 5 elements of order 1×5, 5×1 ½

4. The vector equation of a line parallel to Z-axis is $\vec{m} = 0\hat{i} + 0\hat{j} + \hat{k}$. Then, the required line passes through the point A(α, β, γ) whose position vector $\vec{r_1} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ and is parallel to the vector $\vec{m} = \left(0\hat{i} + 0\hat{j} + \hat{k}\right)$.

½

$\therefore$ The equation is $\vec{r} = \vec{r_1} + \lambda\vec{m}$

$= (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}) + \lambda(0\hat{i} + 0\hat{j} + \hat{k})$

$= (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}) + \lambda(\hat{k})$ ½

5. Given vectors are $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$.

Also, $\vec{a}$ and $\vec{b}$ are parallel vectors.

So, $\vec{a} \times \vec{b} = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 9 \\ 1 & p & 3 \end{vmatrix} = \vec{0}$$

$$\Rightarrow \hat{i}(6 - 9p) - \hat{j}(9 - 9) + \hat{k}(3p - 2) = \vec{0}$$

$$\Rightarrow \hat{i}(6 - 9p) + \hat{k}(3p - 2) = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

On comparing the coeffcients of $\hat{i}$ or $\hat{k}$ from both sides, we get

$$6 - 9p = 0$$

$$\Rightarrow p = \frac{2}{3}$$ 1

6. $\vec{a} \cdot \vec{b} = 8$

$\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is given by

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{\sqrt{2^2 + 6^2 + 3^2}}$$ ½

$$= \frac{8}{\sqrt{49}} = \frac{8}{7}$$ ½

SECTION — B

7. $\Rightarrow$ $\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ **1**

Applying $R_1 \to R_1 - R_2$, we get

$\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ **1**

Applying $R_2 \to R_2 - 5R_1$, we get

$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} A$ **1**

Applying $R_1 \to R_1 + R_2$, we get

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} A$ **½**

Now, applying $R_2 \to (-1)R_2$, we get

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} A$ **½**

8. $\sin\left[\left(\frac{\pi}{3}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = \theta$ **1**

$\sin\theta = -\frac{1}{2}$

$\sin\theta = -\sin\left(\frac{\pi}{6}\right)$

$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$ **1**

$\theta = -\frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\sin\left[\left(\frac{\pi}{3}\right) - \sin^{-1}\left(-\frac{\pi}{6}\right)\right]$ **1**

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{2} = 1$ **1**

9. L.H.S. $= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$; $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ **1**

$= 1 \cdot \begin{vmatrix} b-a & c-a \\ b^3-a^3 & c^3-a^3 \end{vmatrix}$

$= (b-a)(c-a) \begin{vmatrix} 1 & 1 \\ b^2+ab+a^2 & c^2+ca+a^2 \end{vmatrix}$ **1**

$= (b-a)(c-a)$

$\times \begin{vmatrix} 1 & 0 \\ b^2+ab+a^2 & (c^2-b^2)+(ca+ab) \end{vmatrix}$

$C_2 \to C_2 - C_1$ **1**

$= (b-a)(c-a)\{(c-b)(c+b) + a(c-b)\}$

$= (b-a)(c-a)(c-b)(c+b+a)$

$= (a-b)(b-c)(c-a)(a+b+c)$

= R.H.S. **1**

Or

$A = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$; $C_2 \to C_2 - C_3$

$A = \begin{vmatrix} 4-x & 0 & 2x \\ x-4 & 4-x & 2x \\ 0 & x-4 & x+4 \end{vmatrix}$ **1**

$A = (4-x^2)\begin{vmatrix} 1 & 0 & 2x \\ -1 & 1 & 2x \\ 0 & -1 & x+4 \end{vmatrix}$ **1**

$A = (4-x)^2\,[x+4+2x) + 1(2x)]$

$= (4-x)^2\,(3x+4+2x)$ **1**

$= (4-x)^2\,(5x+4)$. **1**

10. $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

or $y\sqrt{1-x^2} = \sin^{-1}x$

Differentiating w.r.t. x, we get

$\sqrt{1-x^2}\,.\frac{dy}{dx} + y.\frac{1}{2}\,(1-x^2)^{-½}\,(-2x)$ **1**

$= \dfrac{1}{\sqrt{1-x^2}}$

or $(1-x^2)\frac{dy}{dx} - xy = 1$ **1**

Again differentiating w.r.t x, we get

$(1-x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(-2x) - \left(x\frac{dy}{dx} + y.1\right) = 0$ **1**

or $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$ **1**

11. Let $y = x^x - 2^{\sin x}$

$y = u - v$

Differentiting. w.r.t. x

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} \quad \mathbf{1}$$

Now $u = x^x$

Taking log on both sides, **1**

$$\log u = \log x^x = x \log x$$

$[\log m^n = n \log m]$

Differentiating

$$\frac{1}{u}\frac{du}{dx} = 1.\log x + x.\frac{1}{x} = (1 + \log x)$$

$$\frac{du}{dx} = u\,(1 + \log x) = x^x\,(1 + \log x)$$

Putting $u = 2^{\sin x}$,

$$\log u = \log 2^{\sin x} = \sin x \log 2 \quad \mathbf{1}$$

$$\frac{1}{u}\frac{du}{dx} = \cos x \log 2$$

$$\frac{du}{dx} = u \cos x \log 2 = 2^{\sin x} \cos x \log 2$$

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{du}{dx}$$

$$= x^x\,(1 + \log x) - 2^{\sin x} \cos x \log 2. \quad \mathbf{1}$$

12. Let $g(x) = |x - 2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$ ½

$$\text{LHL} = \lim_{x \to 2^-} (2 - x) = 0$$

$$\text{RHL} = \lim_{x \to 2^+} (x - 2) = 0 \text{ and } g(2) = 0 \quad \mathbf{1}$$

$\therefore$ $g(x)$ is continuous at $x = 2$ ½

$$\text{LHD} = \lim_{h \to 0} \frac{g(2) - g(2-h)}{h}$$

$$= \lim_{h \to 0} \frac{0 - (2 - 2 + h)}{h} = -1 \quad \mathbf{1}$$

$$\text{RHD} = \lim_{h \to 0} \frac{g(2+h) - g(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2 + h - 2) - 0}{h} = -1 \quad ½$$

LHD $\neq$ RHD $\therefore$ $g(x)$ is not differentiable at $x = 2$ ½

13. $\int_{-1}^{2} |x^3 - x|\,dx$

$$= \int_{-1}^{0}(x^3 - x)dx - \int_{0}^{1}(x^3 - x)dx + \int_{1}^{2}(x^3 - x)dx \quad \mathbf{1}$$

$$= \int_{-1}^{0}(x^3 - x)dx + \int_{0}^{1}(x - x^3)dx + \int_{1}^{2}(x^3 - x)dx \quad ½$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^4}{2}\right]_{1}^{2} \quad \mathbf{1}$$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right) \quad ½$$

$$= \frac{11}{4} \quad \mathbf{1}$$

[CBSE Marking Scheme 2011]

Or

$$\text{Let } \mathrm{I} = \int \frac{x^2}{x^4 + 3x^2 + 2} \cdot x\,dx$$

$$= \frac{1}{2}\int \frac{t\,dt}{t^2 + 3t + 2}, \quad \text{where } t = x^2 \quad \mathbf{1}$$

$$= \frac{1}{2}\left[\int\left(\frac{2}{t+2} - \frac{1}{t+1}\right)dt\right] \quad \mathbf{1}$$

$$= \frac{1}{2}\left[2\log|t+2| - \log|t+1|\right] + C \quad \mathbf{1}$$

$$= \log\left|\frac{t+2}{\sqrt{t+1}}\right| + C$$

$$= \log\left|\frac{x^2+2}{\sqrt{x^2+1}}\right| + C \quad \mathbf{1}$$

[CBSE Marking Scheme March 2014 (Outside)]

14. We have $y = \mathrm{A}\cos x - \mathrm{B}\sin x$ ...(i)

$$\frac{dy}{dx} = -\mathrm{A}\sin x - \mathrm{B}\cos x$$

$$\frac{d^2y}{dx^2} = -\mathrm{A}\cos x + \mathrm{B}\sin x \quad \mathbf{1}$$

or

$$\frac{d^2y}{dx^2} = -(\mathrm{A}\cos x - \mathrm{B}\sin x) \quad \mathbf{1}$$

$$\frac{d^2y}{dx^2} = -y \quad \mathbf{1}$$

$$\frac{d^2y}{dx^2} + y = 0 \quad \textbf{Hence proved} \quad \mathbf{1}$$

Or

$$(y^2 - x^2)\,dy = 3xy\,dx \quad \text{...(i)}$$

$$\frac{dy}{dx} = \frac{3xy}{y^2 - x^2}$$

Put $y = ux$

$$\frac{dy}{dx} = u + x\frac{du}{dx} \quad \mathbf{1}$$

From equation (i), we have

$$u + x\frac{du}{dx} = \frac{3xvu}{u^2x^2 - x^2}$$

$$u + x\frac{du}{dx} = \frac{3u}{u^2 - 1}$$

$$x\frac{du}{dx} = \frac{3u}{u^2-1} - u$$

$$x\frac{du}{dx} = \frac{4u - u^3}{u^2 - 1}$$

$$\int \frac{u^2-1}{4u-u^3}du = \int \frac{1}{x}dx \qquad 1$$

Now, $$\frac{u^2-1}{4u-u^3} = \frac{u^2-1}{u(2-u)(2+u)}$$

$$= \frac{A}{u} + \frac{B}{2-u} + \frac{C}{2+u}$$

$\therefore$ $u^2 - 1 = A(2-u)(2+u) + B(2+u)u + Cu(2-u)$

Putting $2 - u = 0$

$\therefore$ $B = \frac{3}{8}$

Put $u = 0$

$\therefore$ $A = -\frac{1}{4}$

Again, put $2 + u = 0$ **1**

$\therefore$ $C = -\frac{3}{8}$

Now from $\int \frac{u^2-1}{u(4-u^2)}du = \int \frac{1}{x}dx$

$$= -\frac{1}{4}\int\frac{1}{u}du + \frac{3}{8}\int\frac{1}{(2-u)}du - \frac{3}{8}\int\frac{1}{2+u}du$$

$$= \int\frac{1}{x}dx = -\frac{1}{4}\log|u| - \frac{3}{8}|u-2| - \frac{3}{8}$$

$\log|u+2| = \log|x| + \log c$

$$-\frac{1}{4}\log\left|\frac{y}{x}\right| - \frac{3}{8}\log\left|\frac{y}{x} - 2\right| - \frac{3}{8}\log\left|\frac{y}{x} + 2\right|$$

$= \log|x| + \log c.$ **1**

15. (i) Let y denotes the number of bacteria at any instant t, then according to the problem,

$$\frac{dy}{dt} \propto y \Rightarrow \frac{dy}{y} = k\,dt \qquad ...(1)$$

where k is the constant of proportionality, taken to be positive.

On integrating (1), we get **½**

$$\log y = kt + C \qquad ...(2)$$

where C is a parameter.

Let y_0 be the initial number of bacteria *i.e.*, at $t = 0$.

Using this in (2), we get

$$C = \log y_0 \qquad ½$$

So, we have $\log y = kt + \log y_0$

$$\Rightarrow \log\frac{y}{y_0} = kt \qquad ...(3)$$

According to the problem,

$$y = \left[y_0 + \frac{10}{100}y_0\right]$$

$$= \frac{11y_0}{10}, \text{ when } t = 2 \qquad ½$$

So, from (3), we get

$$\log\frac{\frac{11y_0}{10}}{y_0} = k(2)$$

$$\Rightarrow k = \frac{1}{2}\log\frac{11}{10} \qquad ...(4)$$

Using (4) in (3), we get

$$\log\frac{y}{y_0} = \frac{1}{2}\left[\log\frac{11}{10}\right]t \qquad ...(5)\ ½$$

Let the number of bacteria becomes 1,00,000 to 2,00,000 in t_1 hours *i.e.*, $y = 2y_0$, when $t = t_1$ hours.

Using (5), we get

$$\log\frac{2y_0}{y_0} = \frac{1}{2}\left[\log\frac{11}{10}\right]t_1$$

$$\therefore t_1 = \frac{2\log 2}{\log\frac{11}{10}}$$

Hence, the required numbers of hours

$$= \frac{2\log 2}{\log\frac{11}{10}} \qquad 1$$

(ii) Differential equation **½**

Value : Bacteria is harmful to health. **½**

Or

Let $$l = \int (x-3)\sqrt{x^2+3x-18}\,dx$$

Here, we can write $(x - 3)$ as

$$x - 3 = A\frac{d}{dx}(x^2 + 3x - 18) + B$$

$$\Rightarrow x - 3 = A(2x + 3) + B$$

On equating the coefficients of x and constant term from both sides, we get

$$2A = 1 \text{ and } 3A + B = -3$$

$\Rightarrow \quad A = \frac{1}{2}$ and $3 \times \frac{1}{2} + B = -3$

$\Rightarrow \quad A = \frac{1}{2}$ and $B = -\frac{3}{2} - 3$

$\Rightarrow \quad A = \frac{1}{2}$ and $B = -\frac{9}{2}$ **1**

Thus, the given integral reduces in the following form

$$l = \int \left\{ \frac{1}{2}(2x+3) - \frac{9}{2} \right\} \sqrt{x^2+3x-18}\,dx$$

$$= \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}\,dx - \frac{9}{2}\sqrt{x^2+3x-18}\,dx$$

$$= \frac{1}{2}l_1 - \frac{9}{2}l_2 \quad \text{...(i)}$$

where, $l_1 = \int (2x+3)\sqrt{x^2+3x-18}\,dx$

Put $x^2 + 3x - 18 = t$

$\Rightarrow \quad (2x+3)dx = dt$

$\therefore \quad l_1 = \int t^{1/2} dt = \frac{2}{3}t^{3/2} + C_1$

$$= \frac{2}{3}(x^2+3x-18)^{3/2} + C_1$$

and $\quad l_2 = \int \sqrt{x^2+3x-18}\,dx$

$$= \int \sqrt{\left(x+\frac{3}{2}\right)^2 - 18 - \frac{9}{4}}dx$$

$$= \int \sqrt{\left(x+\frac{3}{2}\right)^2 - \frac{81}{4}}\,dx$$

$$= \int \sqrt{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\,dx$$

$$= \frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x-18} \quad \mathbf{1}$$

$$-\frac{81}{8}\log\left|\left(x+\frac{3}{2}\right) + \sqrt{x^2+3x-18}\right| + C_2$$

$$\left[\because \int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|\right]$$

$$= \frac{2x+3}{4}\sqrt{x^2+3x-18}$$

$$= -\frac{81}{8}\log\left|\frac{2x+3}{2} + \sqrt{x^2+3x-18}\right| + C_2$$

1

On putting the value of l_1 and l_2 in Eq. (i), we get

$$l = \frac{1}{2}\left[\frac{2}{3}(x^2+3x-18)^{3/2} + C_1\right]$$

$$-\frac{9}{2}\left[\frac{2x+3}{4}\sqrt{x^2+3x-18} - \frac{81}{8}\log\left|\frac{2x+3}{2} + \sqrt{x^2+3x-18}\right| + C_2\right]$$

$$\Rightarrow \quad l = \frac{1}{3}(x^3+3x-18)^{3/2}$$

$$-\frac{9}{8}(2x+3)\sqrt{x^2+3x-18}$$

$$+\frac{729}{16}\log\left|\frac{2x+3}{2} + \sqrt{x^2+3x-18}\right| + C$$

1

where, $C = \frac{C_1}{2} - \frac{9C_2}{2}$

16. Let $\quad \vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$

and $\quad \vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$

Now, $\quad |\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$$= \frac{1}{7}\sqrt{4+9+36} \quad \mathbf{1}$$

$$= \frac{1}{7}\sqrt{49} = \frac{1}{7}\times 7 = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2}$$

$$= \frac{1}{7}\sqrt{9+36+4}$$

$$= \frac{1}{7}\sqrt{49} = \frac{1}{7}\times 7 = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2}$$

$$= \frac{1}{7}\sqrt{36+4+9}$$

$$= \frac{1}{7}\sqrt{49} = \frac{1}{7}\times 7 = 1$$ **1**

$\therefore$ $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vector

Now, $\vec{a}\,.\,\vec{b} = \frac{1}{49}$ [2 (3) + (3) (– 6) + (6) (2)]

$$= \frac{1}{49}(6 - 18 + 12) = 0$$ **1**

$\Rightarrow$ $\vec{a} \perp \vec{b}$

$\vec{b}\,.\,\vec{c} = \frac{1}{49}$ [(3) (6) + (– 6) (2) + (2)(–3)]

$$= \frac{1}{49}(18 - 12 - 6) = 0$$

$\Rightarrow$ $\vec{b} \perp \vec{c}$

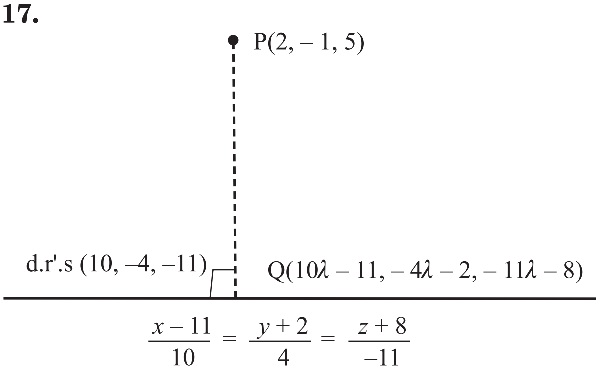
and $\vec{c}\,.\,\vec{a} = \frac{1}{49}$ [(6) (2) + (2) (3) + (– 3))6)]

$$= \frac{1}{49}[12 + 6 - 18] = 0$$

$\Rightarrow$ $\vec{c} \perp \vec{a}$

Hence $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. **1**

17.



Equation of given line is,

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

$\therefore$ Direction ratio's of lines are l, m and n = 10, – 4 and – 11 respectively.

Let, $$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$$ **1**

$\therefore$ Coordinates of general point Q is given by $x = 10\lambda + 11$, $y = -4\lambda - 2$, $z = -11\lambda - 8$

$\therefore$ Direction ratio's of the line joining this point Q to P are

$10\lambda + 11 - 2, -4\lambda - 2 - (-1), -(11\lambda) - 8 - 5$ or $10\lambda + 9, -4\lambda - 1, -11\lambda - 13$.

If PQ is perpendicular to the given line, then

$l_1l_2 + m_1m_2 + n_1n_2 = 0$ **1**

$\Rightarrow$ $10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$

$\Rightarrow$ $100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$

$\Rightarrow$ $237\lambda + 237 = 0$

$\Rightarrow$ $273\lambda = -273$

$\Rightarrow$ $\lambda = -1$ **1**

$\therefore$ Co-ordinate of foot of $\perp$Q are

$x = 10(-1) + 11 = 1$

$y = -4(-1) - 2 = 2$

$z = -11(-1) - 8 = 3$

$\therefore$ Q are (1, 2, 3) and $\perp$ distance of P from the line = PQ

$$= \sqrt{(1+2)^2 + (2+1)^2 + (3+5)^3}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units.}$$ **1**

18. Given matrix in A = $\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}$

Let A = IA

$$\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$ **1**

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$ **½**

Applying $R_2 \to R_2 - 5R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -5 & 6 \end{bmatrix} A$$ **½**

Applying $R_1 \to R_1 + R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ -5 & 6 \end{bmatrix} A$$ **½**

Now applying $R_2 \to (-1)R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} A$$ **½**

Hence, $$A^{-1} = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} [\because A^{-1}A = 1]$$ **1**

19. Let E_1, E_2 and A be the events defined as follows :

E_1 = 5 occurs

E_2 = 5 does not occur

A = a report that it is a 5. **1**

We have,

$$P(E_1) = \frac{1}{6}$$

and $P(E_2) = 1 - \frac{1}{6} = \frac{5}{6}$ **1**

Now, $P\left(\frac{A}{E_1}\right)$ = Probability that A speak truth

$= \frac{8}{10} = \frac{4}{5}$

and, $P\left(\frac{A}{E_2}\right)$ = Probability that A does not speak truth

$= 1 - \frac{4}{5} = \frac{1}{5}$ **1**

We have to find $P\left(\frac{E_1}{A}\right)$ *i.e.*, the probability that there is 5 on the dice given that A has reported that there is 5. By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \times \frac{1}{5}} = \frac{4}{9}$$ **1**

SECTION — C

20. Given, $f(x) = \frac{4x+3}{6x-4}$

Let $x_1, x_2 \in A = R - \left\{\frac{2}{3}\right\}; x_1 \neq x_2$

One-one Consider, $f(x_1) = f(x_2)$

$\therefore \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$ **1**

$\Rightarrow (4x_1 + 3)(6x_2 - 4) = (4x_2 + 3)(6x_1 - 4)$

$\Rightarrow 24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 - 16x_2 + 18x_1 - 12$

$\Rightarrow -34x_1 = -34x_2$

$\Rightarrow x_1 = x_2$

So, f is one-one. **1½**

Onto Let $y = \frac{4x+3}{6x-4}$

$\Rightarrow 6xy - 4y = 4x + 3$

$\Rightarrow (6y - 4)x = 3 + 4y$

$\Rightarrow x = \frac{3+4y}{6y-4}$

and $y \neq \frac{4}{6}$, *i.e.*, $y \neq \frac{2}{3}$

$\therefore y \in R - \left\{\frac{2}{3}\right\}$

Thus, f is onto. **1½**

Since, f is one-one and onto.

$\therefore x = f^{-1}(y) = \frac{3+4y}{6y-4}$

$\Rightarrow f^{-1}(x) = \frac{3+4x}{6x-4}$ **2**

Or

Given, $f(x) = \frac{x-1}{x-2}$

$\Rightarrow f : A \to B$, where $A = R - \{2\}$ and $B = R - \{1\}$.

One-one Let $f(x_1) = f(x_2)$, $\forall x_1, x_2 \in A$

$\Rightarrow \frac{x_1-1}{x_1-2} = \frac{x_2-1}{x_2-2}$ **1**

$\Rightarrow (x_1 - 1)(x_2 - 2) = (x_2 - 1)(x_1 - 2)$

$\Rightarrow x_1x_1 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

$\because f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall x_1, x_2 \in A$ **2**

Therefore, $f(x)$ is one-one.

Onto Let $y = \frac{x-1}{x-2}$

$\Rightarrow xy - 2y = x - 1$ **1**

$\Rightarrow x = \frac{2y-1}{y-1}$ ...(i)

Since, $x \in R - \{2\}$, $\forall y \in R - \{1\}$

So, range of $f(x) = R - \{1\}$

$\therefore$ Range = Codomain

Therefore, $f(x)$ is onto. **1**

Also, from Eq. (i), we get

$f^{-1}(y) = \frac{2y-1}{y-1}$ $[x = f^{-1}(y)]$

$\Rightarrow f^{-1}(x) = \frac{2x-1}{x-1}$ **1**

21. Let the line through P(3, 4) meets x-axis at A(a, 0), then equation of line PA

$\Rightarrow (y - 0) = \frac{4-0}{3-a}(x - a)$

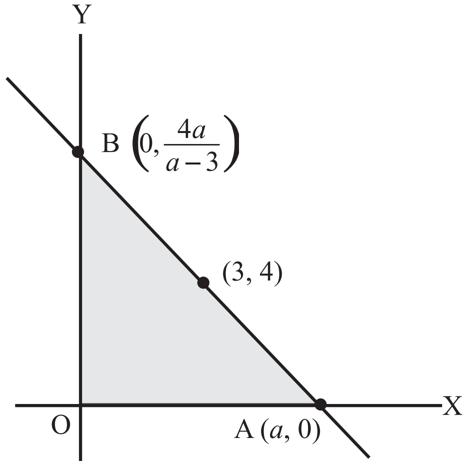
$\Rightarrow (3 - a)y = 4(x - a)$ ...(1)

Line (1) meets y-axis when $x = 0$

$\therefore (3 - a)y = 4(0 - a)$ **1**

$$\Rightarrow \qquad y = \frac{4a}{a-3}$$

So, line (1) meets y-axis at B $\left[0, \frac{4a}{a-3}\right]$



Area of triangle, OAB

$$= \frac{1}{2}\text{ (OA)(OB)} \qquad \textbf{1}$$

$$\Delta = \frac{2a^2}{a-3}$$

$$\frac{d\Delta}{da} = \frac{(a-3)4a - 2a^2}{(a-3)^2} \qquad \textbf{1}$$

$$= \frac{2a^2 - 12a}{(a-3)^2}$$

Now, $\dfrac{d\Delta}{da} = 0$

$\Rightarrow \quad 2a^2 - 12a = 0$

$\Rightarrow \quad a = 6$

$$\frac{d^2\Delta}{da^2} = \frac{(a-3)^2(4a-12)-(2a^2-12a)(a-3)2}{(a-3)^4} \qquad \textbf{1}$$

$$= \frac{36}{(a-3)^3}$$

At a = 6, $\dfrac{d^2\Delta}{da^2} = \dfrac{36}{27} = \dfrac{4}{3} > 0$

$\therefore$ Δ is minimum at a = 6

Equation of line is $(3 - 6)y = 4(x - 6)$ **1**

or $4x + 2y = 24$

and minimum area of triangle

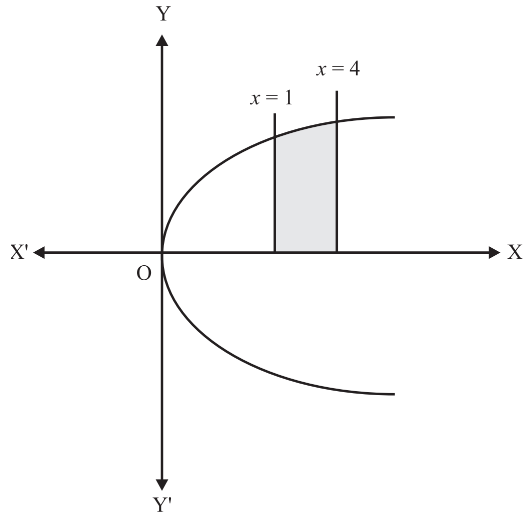
$$= \frac{2(6)^2}{6-3} = \frac{2\times 36}{3}$$

= 24 sq. units. **1**

22. The required area

= Area of the shaded portion

$= \int_1^4 y\,dx = \int_1^4 \sqrt{4x}\,dx$ **1**



2

$$= 2.\left(\frac{x^{3/2}}{3/2}\right)_1^4 = \frac{4}{3}\,[4^{3/2} - 1] \qquad \textbf{2}$$

$$= \frac{28}{3}\text{ sq. units.} \qquad \textbf{1}$$

Or

Here, $f(x) = x^2 + x + 1,$

$a = 0, b = 2$

$nh = b - a = 2$

By the formula,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\ [f(a) + f(a+h) + \ldots$$

$$+ f(a + (n-1)h)] \quad (\text{where } nh = b - a.)$$

$$\Rightarrow \int_0^2 (x^2 + x + 1)dx \qquad \textbf{1}$$

$= \lim\limits_{h\to 0} h\ [(0 + 0 + 1) + (h^2 + h + 1) + (2^2h^2 + 2h$

$+ 1) + \ldots. + (n-1)^2h^2 + (n-1)h + 1],$ **1**

(where $nh = 2$)

$= \lim\limits_{h\to 0} h\ [\{12 + 2^2 + \ldots + (n-1)^2\}\ h^2$ **1**

$+ (1 + 2 + \ldots.. + n + 1)\ h + n],$

where $nh = 2$

$$= \lim_{h\to 0}\ h\left(\frac{1}{6}(n-1)n(2n-1)h^2 + \frac{(n-1)n}{2}h + n\right) \qquad \textbf{1}$$

$$= \lim_{h\to 0}$$

$$\left(\frac{1}{6}(nh-h)nh(2nh-h) + \frac{1}{2}(nh-h)nh + nh\right) \qquad \textbf{1}$$

$$= \frac{1}{6}\ (2-0)\times 2(2\times 2 - 0) + \frac{1}{2}(2-0)\times 2 + 2$$

$$= \frac{8}{3} + 2 + 2 = \frac{20}{3} \qquad \textbf{1}$$

23. $\int_0^{\pi} \frac{dx}{5+4\cos x}$

$$= \int_0^{\pi} \frac{dx}{5\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)+4\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)} \quad \mathbf{1}$$

$$= \int_0^{\pi} \frac{dx}{9\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}$$

Dividing by $\cos^2\frac{x}{2}$ numerator and denominator

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx \quad \mathbf{1}$$

Now, let $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

Also, when $x = 0$, then $t = \tan 0 = 0$ and when $x = \pi$, then $t = \tan\frac{x}{2} = \infty$ **1**

$$\therefore \quad I = 2\int_0^{\infty}\frac{dt}{3^2+t^2} \quad \mathbf{1}$$

$$= \frac{2}{3}\left(\tan^{-1}\frac{t}{3}\right)_0^{\infty} \quad \mathbf{1}$$

$$= \frac{2}{3}[\tan^{-1}\infty - \tan^{-1}0]$$

$$= \frac{2}{3}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{3} \quad \mathbf{1}$$

24. First two points A and B are, (0, 0, 0) and (2, 1, 1) respectively. **1**

$\therefore$ Direction ratios of AB are 2, 1, 1.

Direction ratios of CD joining the points C(3, 5, – 1) and D(4, 3, – 1) are (1, – 2, 0).

If AB $\perp$ CD, then $a_1.a_2 + b_1.b_3 + c_1.c_2 = 0$ **2**

i.e., 2.1 + 1. (– 2) + 1.0 = 2 – 2 + 0 = 0

which is true **2**

$\Rightarrow$ AB is perpendicular to CD. **1**

25. Let x kg of fertilizer F_1 and y kg of fertilizer F_2 is to be required. We are given that :

Fertilizer	Nitrogen	Phosphoric Acid	Quantity	Cost
F_1 F_2	10% 5%	6% 10%	x kg y kg	₹ 6 ₹ 5
Total	14 kg	14 kg		

Quantity of nitrogen in fedilizers F_1 and F_2 = 10% of x + 5% of y = $\frac{x}{10}+\frac{x}{20}$

Least quantity of nitrogen = 14 kg **1**

$\Rightarrow \frac{x}{10}+\frac{y}{20} \geq 14$ or $2x + y \geq 280$

Quantity of phosphoric acid in fertilizers F_1 and F_2

$$= 6\% \text{ of } x + 10\% \text{ of } y = \frac{6x}{100}+\frac{y}{10}$$

Least quantity of phosphoric acid = 14 kg

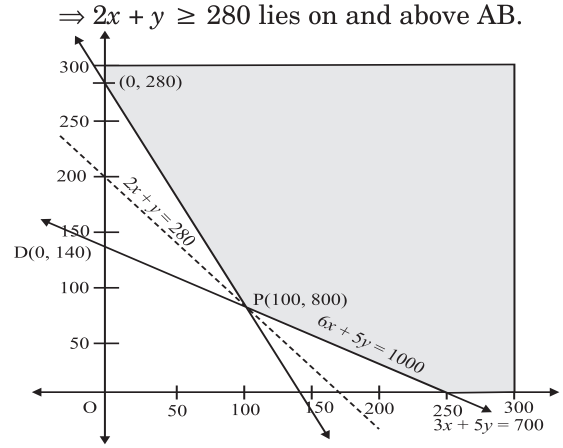
$\Rightarrow \frac{6x}{100}+\frac{y}{10} \geq 14$

or $6x + 10y \geq 1400$ or $3x + 5y \geq 700$

$\therefore$ We have to minimize the cost $Z = 6x + 5y$ and constraints are : $2x + y \geq 280$, $3x + 5y \geq 700$; $x, y \geq 0$.

(i) The line $2x + y = 280$ passes through A (140, 0), B(0, 280). Putting $x = 0, y = 0$, in $2x + y \geq 280$, $0 \geq 280$ which is not true. **1**

$\Rightarrow 2x + y \geq 280$ lies on and above AB.



1

(ii) The line $3x + 5y = 700$ passes through C $\left(\frac{700}{3},0\right)$, D(0, 140). Putting $x = 0, y = 0$, is $3x + 5y \geq 700$, $0 \geq 70$ which in not true.

$\Rightarrow 3x + 5y \geq 700$ lies on and above CD.

(iii) $x \geq 0$ lies on and to the right of y-axis.

(iv) $y \geq 0$ lies on and above the x-axis. **1**

The shaded area $YBPCX$ is the feasible region where P is the intersection of AB and CD *i.e.*,

$$2x + y = 280 \quad \text{...(1)}$$
$$3x + 5y = 700 \quad \text{...(2)}$$

Multiplying (1) by 5 and subtracting (2) from it, $7x = 700$.

$\therefore \quad x = 100$

From (1), $\quad y = 80$

$\therefore$ The point P is (100, 80).

The value of Z at B, P are

At B (0, 280), $Z = 6x + 5y = 0 + 5 \times 280$
$= 1400$

At P(100, 80), $Z = 6 \times 100 + 5 \times 80$
$= 600 + 400$
$= 1000$

At C $\left(\frac{700}{3}, 0\right)$, $Z = 6 \times \frac{700}{3} + 0$
$= 1400$

$\therefore$ Z is minimum at P(100, 80) where $Z = 1000$

But, the feasible region is unbounded.

Consider the inequality $6x + 5y < 1000$. **1**

$6x + 5y = 1000$ passes through $\left(\frac{500}{3}, 0\right)$ and (0, 200).

Putting $x = 0, y = 0$ in $6x + 5y < 1000$, $0 < 1000$ which is true.

$\therefore$ $6x + 5y < 1000$ lies below the line $6x + 5y = 1000$.

There is no point in common between feasible region and $6x + 5y < 1000$.

Hence, minimum cost of Z is ₹ 1000 when quantities of fertilizer F_1 and F_2 are 100 kg and 80 kg respectively. **1**

Value : Good fertitizer increases crop yields.

26. Events A fails and B fails are denoted by $\overline{A}$ and $\overline{B}$ respectively.

We have $P(\overline{A}) = 0.2$

and P(A and B fail) $= 0.15$

i.e., $P(\overline{A} \cap \overline{B}) = 0.15$

$P(\overline{B} \text{ alone}) = P(\overline{B}) - P(\overline{A} \cap \overline{B})$
$= 0.15$ **1**

Now, $0.15 = P(\overline{B}) - 0.15$ **1**

$\therefore$ $P(\overline{B}) = 0.30$

(i) $P(\overline{A}/\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}$ **1**

$= \frac{0.15}{0.30} = \frac{1}{2} = 0.5$ **1**

(ii) P(A fails alone) $= P(\overline{A} \text{ alone})$
$= P(\overline{A}) - P(\overline{A} \cap \overline{B})$ **1**
$= 0.20 - 0.15 = 0.05.$ **1**

SOLUTIONS

SAMPLE QUESTION PAPER - 8

Self Assessment______________________________

Time: 3 Hours **Maximum Marks: 100**

SECTION — A

1. Given

$$x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$$

The highest order derivative is $\left(\frac{d^2y}{dx^2}\right)$ and its Power is 2.

So, the degree of differential equation is 2.

2.

$$y = mx + c$$

$$\frac{dy}{dx} = m$$

$$\frac{d^2y}{dx^2} = 0$$

3. $A = \begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$, $B = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

A = B, if $3x + 7 = 0$

$\therefore \quad x = -\frac{7}{3}$ ½

$5 = y - 2$ or $y = 7$

$y + 1 = 8.$

$\therefore \quad y = 7$

$2 - 3x = 4,$

$x = -\frac{2}{3}$

Here, x has two values viz $-\frac{7}{3}, -\frac{2}{3}$ ½

$\therefore$ Not possible to find values of x and y.

4. Since, $\sqrt{(2)^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3 \neq 1$

$\therefore$ Required direction cosines are :

$$\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

5. Since given vectors $\vec{a}$ and $\vec{b}$ are parallels, then :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$$

$$\Rightarrow \quad p = \frac{-1}{3}$$

6. Since, the given vectors are coplanar,

$[\vec{a}\ \vec{b}\ \vec{c}] = 0$ ½

$$\Rightarrow \quad \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0$$

Expanding along R_3, we get

$$3\lambda = 21$$

$\Rightarrow \quad \lambda = 7$ ½

SECTION — B

7. We have $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = [-1\ 2\ 1]$

Now, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}[-1\quad 2\quad 1]$

$= \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$ **1**

$\therefore \quad (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ ...(i) **1**

Also, $B'A' = [-1\quad 2\quad 1]' \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}'$

$= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}[1\quad -4\quad 3]'$

$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ ...(ii) **1½**

$\therefore$ From (i) and (ii), we observe that

$(AB)' = A'B'$ **½**

[CBSE Marking Scheme 2010]

8. $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \frac{\pi}{4}$ **1**

$\frac{5x}{1-6x^2} = 1$

$1 - 6x^2 = 5x$ **1**

$6x^2 + 5x - 1 = 0$

$6x^2 + 6x - x - 1 = 0$

$6x(x + 1) - 1(x + 1) = 0$ **1**

$(6x - 1)(x + 1) = 0$

$x = \frac{1}{6}, x = -1$

(Not possible)

$\Rightarrow \quad x = \frac{1}{6}$ **1**

9. Let $\Delta = \begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$ **1**

Operating $R_1 \to R_1 + (R_2 + R_3)$, we get

$\Delta = \begin{vmatrix} 2(x+y+z) & x+y+z & x+y+z \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$

$\Rightarrow \Delta = (x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$ **1**

Operating $C_1 \to C_1 - (C_2 + C_3)$; $C_2 \to C_2 - C_3$, we get

$\Delta = (x+y+z)\begin{vmatrix} 0 & 0 & 1 \\ 0 & z-x & x \\ x-z & y-z & z \end{vmatrix}$ **1**

$= (x + y + z)\,[1\,\{0 - (z - x)(x - z)\}]$

$= (x + y + z)(x - z)^2.$ **1**

Or

Let $\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$, we get **1**

$\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b+a \\ 1 & 1 & c+b \end{vmatrix}$ ...(1) **1**

Since a, b, c are in A.P., then ...(2)

$b - a = c - b$ **1**

Using (2) in (1), we get

$R_2 = R_3$

$\therefore \quad \Delta = 0.$ **1**

10. We have,

$2A - 3B + 5C = 0$

$\Rightarrow \quad 2A = 3B - 5C$

$\Rightarrow \quad 2A = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

$\Rightarrow \quad 2A = \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} -10 & 0 & 10 \\ -35 & -5 & -30 \end{bmatrix}$

$\Rightarrow \quad 2A = \begin{bmatrix} -6-10 & 6+0 & 0+10 \\ 9-35 & 3-5 & 12-30 \end{bmatrix}$

$\Rightarrow \quad 2A = \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$

$\Rightarrow \quad A = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$

11. $(x-y)e^{\frac{x}{x-y}} = a$

$\Rightarrow \quad \log(x-y) + \frac{x}{x-y} = \log a$ **1**

Differentiate w.r.t. 'x'

$\Rightarrow \frac{1-y'}{x-y} + \frac{1\cdot(x-y) - x(1-y')}{(x-y)^2} = 0$ **2**

$\Rightarrow (x-y)(1-y') + x - y - x(1-y') = 0$ **½**

$\Rightarrow \quad yy' + x - 2y = 0$

or $\quad y\frac{dy}{dx} + x = 2y$ **½**

[CBSE Marking Scheme 2014]

12. Let $\quad y = u.v.w = u.\,(vw)$

Differentiating both sides

(i) $\quad \frac{du}{dx} = u'.(vw) + u\,\frac{d}{dx}\,(vw)$

$= u'.(vw) + u[v'w + vw']$ **1**

$= u'.v.w + uv'w + uvw'$

$= \frac{du}{dx}.v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$

(ii) $\quad y = u.v.w$

Taking log on both sides

$\log y = \log u + \log v + \log w,$

$[\because \log mn = \log m + \log n]$ **1**

Differentiating both sides

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{du}{dx} + \frac{1}{w}\frac{dw}{dx}$

$\frac{du}{dx} = y\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{du}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$

$= uvw\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{du}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$ **1**

$= vw\frac{du}{dx} + uw\frac{du}{dx} + uv\frac{dw}{dx}$

$= \frac{du}{dx}.v.w + u.\frac{du}{dx}.w + u.v.\frac{dw}{dx}$ **1**

13. $\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$

$\Rightarrow A\,(x+1)\,(x+3) + B\,(x+3) + C\,(x+1)^2 = 3x - 2$

$\Rightarrow A\,(x^2 + 4x + 3) + B\,(x+3) + C(x^2 + 1 + 2x) = 3x - 2$

$\Rightarrow x^2\,(A + C) + x\,(4A + B + 2C) + (3A + 3B + C) = 3x - 2$

Comparing the coefficients of both sides, we get

$A + C = 0$ ...(1)

$4A + B + 2C = 3$ ...(2)

$3A + 3B + C = -2$ ...(3)

From equation (1), $A = -C$ **1**

Substituting it in equation (2) and equation (3), we get

$-4C + B + 2C = 3$

$\Rightarrow \quad B - C = 3$ ...(4)

and $\quad -C + 3B + C = -2$

$\Rightarrow \quad 3B - 2C = -2$ ...(5)

From equation (4) and equation (5), we get

$-2B = 5$

$B = -\frac{5}{2}$

Substituting it in equation (4), we get

$-\frac{5}{2} - 2C = 3$

$2C = -\frac{5}{2} - 3 = -\frac{11}{2} \Rightarrow C = -\frac{11}{4}$ **1**

$A = -C = \frac{11}{4}$

$I = \int\left(\frac{11}{4(x+1)} - \frac{5}{2(x+1)^2} - \frac{11}{4(x+3)}\right)dx$ **1**

$= \frac{11}{4}\int\frac{dx}{(x+1)} - \frac{5}{2}\frac{dx}{(x+1)^2} - \frac{11}{4}\int\frac{dx}{(x+3)}$

$= \frac{11}{4}\,\log\,(x+1) + \frac{5}{2}.\frac{1}{(x+1)}$

$= \frac{11}{4}\,\log\,(x+3) + C$ **1**

14. Let, $\quad I = \int_0^{\pi/4}\frac{2\sin\theta\cos\theta}{\sin^4\theta + \cos^4\theta}\,d\theta$

$= \int_0^{\pi/4}\frac{2\sec^2\theta\tan\theta}{\tan^4\theta + 1}\,d\theta$

[divide by $\cos^4\theta$]

Put $\tan^2\theta = t$

$\therefore \quad 2\tan\theta\sec^2\theta\,d\theta = dt$ **1**

When $x = 0$, $t = 0$

When $x = \frac{\pi}{4}$, $t = 1$

$\therefore \quad I = \int_0^1 \frac{dt}{1+t^4}$ **1**

$\Rightarrow \quad I = \left[\tan^{-1} t\right]_0^1$ **1**

$\Rightarrow \quad I = \tan^{-1}(1) - \tan^{-1}(0)$

$I = \frac{\pi}{4}$ **1**

15. Consider,

$y = e^x.\cos x$ ...(i)

$\Rightarrow \quad y = e^x(-\sin x) + \cos x.e^x$

$\Rightarrow \quad y' = -e^x \sin x + y$ ...(ii) **1**

[From (i)]

$\Rightarrow \quad y'' = -[e^x \cos x + e^x \sin x] + y'$

$= -[y + y - y'] + y',$

[From (i) and (ii)] **1**

$\Rightarrow \quad y' - 2y' + 2y = 0$

Differential equation corresponding to the function $y = e^x \cos x$ is

$y'' - 2y' + 2y = 0$ **1**

Therefore, $y = e^x \cos x$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$ **1**

Or

Consider the equation

$(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2}$

$= 1 + 3\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2$ ...(i) **1**

Put $\quad y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

Substituting in (i), we get

$v + x\frac{dv}{dx} = 1 + 3v + v^2$

$\Rightarrow \quad x\frac{dv}{dx} = 1 + 2v + v^2 = (1+v)^2$ **1**

$\Rightarrow \quad \frac{1}{(v+1)^2}\,dv = \frac{dx}{x}$

Integrating both sides, we get

$\int \frac{1}{(v+1)^2} = \int \frac{dx}{x}$

$\Rightarrow \quad \frac{-1}{v+1} = \log|x| + C$ **1**

$\Rightarrow \quad \frac{-1}{\frac{y}{x}+1} = \log|x| + C$

$\Rightarrow \quad \frac{-x}{y+x} = \log|x| + C$ is the required solution. **1**

16. Here,

$2x^2\frac{dy}{dx} - 2xy + y^2 = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$ ...(i) **1**

This is a homogeneous differential equation.

Put $y = vx$, so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we get

$v + x\frac{dv}{dx} = \frac{2vx^2 - v^2x^2}{2x^2} = v - \frac{v^2}{2}$

$\Rightarrow x\frac{dv}{dx} = -\frac{v^2}{2} \Rightarrow 2\frac{dv}{v^2} + \frac{dx}{x} = 0$ **1**

Integrating it, we get

$-\frac{2}{v} + \log|x| = C$

or $-\frac{2x}{y} + \log|x| = C,$ [as $y = vx$]...(ii)

Given that $y = e$, when $x = e$, then by (i)

$-\frac{2e}{e} + \log e = C \Rightarrow -2 + 1 = C \Rightarrow C = -1$ **1**

Put C = – 1 in (i), we get

$-\frac{2x}{y} + \log(x) = -1 \Rightarrow \frac{2x}{y} = 1 + \log|x|$

$\Rightarrow \quad y = \frac{2x}{1 + \log|x|}$ **1**

which is the required solution.

Or

We have,

$f(x) = x^2 + 2x + 3, x \in [4, 6]$

$f(x)$ being a polynomial, is both continuous and differentiable every where and in particular, in [4, 6]

$f(4) = 27$ & $f(6) = 51 \neq f(4)$ **1**

Thus, condition of lagrange's mean value is satisfied.

Hence, there exist a point]4, 6[

such that $f'(c) = \frac{\{f(b) - f(a)\}}{b-a}$ **1**

$\therefore \quad f'(x) = 2x + 2$

$\Rightarrow \quad f'(c) = 2c + 2$ **1**

$$\therefore \qquad 2c + 2 = \frac{\{f(6) - f(4)\}}{6-4}$$

$$\Rightarrow \qquad \frac{51-27}{2} = 2c + 2$$

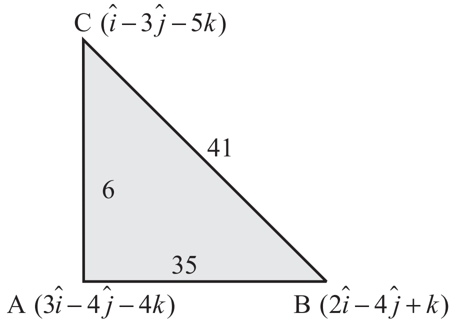
$\Rightarrow \quad 2c + 2 = 12 \Rightarrow c = 5 \in]4, 6[$ **1**

Thus, verified.

17. $$\vec{AB} = \vec{b} - \vec{a}$$

$$= (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$$

$$= \hat{i} - 3\hat{j} - 5\hat{k}$$ **1**



1

$$|\vec{AB}|^2 = (-1)^2 + 3^2 + 5^2$$
$$= 1 + 9 + 25 = 35$$ **1**

$$\vec{BC} = \vec{c} - \vec{b}$$

$$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$= (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k}$$

$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$|\vec{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2$$
$$= 1 + 4 + 36 = 41$$

$$\vec{CA} = \vec{a} - \vec{c}$$

$$= (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} + 5\hat{k})$$

$$= (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k}$$

$$= 2\hat{i} - \hat{j} - \hat{k}$$

$$|\vec{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$$

Now, $|\vec{AB}|^2 + |\vec{BC}|^2 = |\vec{BC}|^2$

or, $(\vec{AB})^2 + (\vec{CA})^2 = (\vec{BC})^2$

Hence, ΔABC is a right angled triangle. **1**

18. Ist method :

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \qquad ...(i)$$

and, $$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \qquad ...(ii)$$

Let (i) passes through (3, 5, 7) and is directly proportional to 1, – 2, 1. **1**

$\therefore$ Its vector equation is,

$$\vec{r} = \vec{a_1} + \lambda \vec{b_1}$$

$$\vec{a_1} = 3\hat{i} + 5\hat{j} + 7\hat{k}$$

and $$\vec{b_1} = \hat{i} + 2\hat{j} + \hat{k}$$

Let line (ii) passes through (–1, – 1, – 1) and directly proportional to 7, – 6, 1.

$\therefore$ Its vector equation is.

$$\vec{r} = \vec{a_2} + \lambda \vec{b_2}$$

$$\vec{a_2} = -\hat{i} - \hat{j} - \hat{k}$$

and $$\vec{b_2} = -7\hat{i} - 6\hat{j} - \hat{k}$$ **1**

Shortest distance (S.D.) = $\left| \dfrac{(\vec{a_2} - \vec{a_1}).(\vec{b_2} \times \vec{b_1})}{|\vec{b_1}| \times |\vec{b_2}|} \right|$

$$(\vec{a_2} - \vec{a_1}) = -\hat{i} - \hat{j} - \hat{k} - (3\hat{i} + 5\hat{j} - 7\hat{k})$$

$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$

$$= -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

$$\Rightarrow \quad \hat{i}\,(-2-6) - \hat{j}\,(1-7) + \hat{k}\,(-6+14)$$

$$= 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \sqrt{4^2 + 6^2 + 8^2}$$
$$= \sqrt{16 + 36 + 64}$$
$$= \sqrt{116}$$ **1**

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})$$

$$= (-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})$$

$$= -16 - 36 - 64$$
$$= -116$$

$$\text{S.D.} = \left| \frac{-116}{\sqrt{116}} \right|$$

$$= \sqrt{116}$$

S.D. $= 2\sqrt{29}$ units. **1**

IInd Method :

Here $\vec{a_1} = 3\hat{i} + 5\hat{j} + 7\hat{k},\ \vec{b_1} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{a_2} = -\hat{i} - \hat{j} - \hat{k},\ \vec{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$ **1**

Shortest distance (S.D.) = $\dfrac{\left|(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})\right|}{|\vec{b_1} \times \vec{b_2}|}$

$\vec{a_2} - \vec{a_1} = 4\hat{i} - 6\hat{j} - 8\hat{k}$ **1**

Finding $\vec{b_1} \times \vec{b_2} = 4\hat{i} + 6\hat{j} + 8\hat{k}$

S.D. $= \dfrac{|-16 - 36 - 64|}{\sqrt{116}}$ **1**

S.D. $= \dfrac{116}{\sqrt{116}} = \sqrt{116}$

S.D. $= 2\sqrt{29}$ units. **1**

Or

Ist method :

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$$

General points, $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$

Its distance from the point A (1, 2, 3) is $3\sqrt{2}$

$\therefore \quad PA^2 = 18$

$(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 2)^2 + (2\lambda + 3 - 3)^2 = 18$

$(3\lambda - 3)^2 + (2\lambda - 3)^2 + (2\lambda)^2 = 18$

$9\lambda^2 + 9 - 18\lambda - 14\lambda^2 + 9 - 12\lambda + 4\lambda^2 = 18$ **1**

$17\lambda^2 - 30\lambda = 18 - 18$

$17\lambda^2 - 30\lambda = 0$

$\lambda(17\lambda - 30) = 0$

$\Rightarrow \quad \lambda = 0, \dfrac{30}{17}$ **1**

When $\lambda = 0, x = (-2, -1, 3)$

When $\lambda = \dfrac{30}{17}$

$x = 3 \times \dfrac{30}{17} - 2 = \dfrac{56}{17}$

$y = 2 \times \dfrac{30}{17} - 1 = \dfrac{43}{17}$ **1**

$z = 2 \times \dfrac{30}{17} + 3 = \dfrac{111}{17}$

$\therefore$ The points are $(-2, -1, 3)$ and $\left(\dfrac{56}{17}, \dfrac{43}{17}, \dfrac{111}{17}\right)$ **1**

IInd Method :

A general point on the line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2} = \lambda$ is

$x = 3\lambda - 2, y = 2\lambda - 1, z = 2\lambda + 3$ **1**

Its distance from (1, 2, 3) = $3\sqrt{2}$

$\therefore \quad (3\sqrt{2})^2 = (3\lambda - 3)^2 + (2\lambda - 3)^2 + (2\lambda)^2$ **1**

$\Rightarrow \quad \lambda = 0$ or $\lambda = \dfrac{30}{17}$ **1**

$\Rightarrow$ The points are $\left(\dfrac{56}{17}, \dfrac{43}{17}, \dfrac{111}{17}\right)$

or $(-2, -1, 3)$ **1**

19. Let E_1, E_2 and A be the events defined as follows :

E_1 = six occurs

E_2 = six does not occur

and A = the man reports that it is a six

We have

$P(E_1) = \dfrac{1}{6}$ and $P(E_2) = 1 - \dfrac{1}{6} = \dfrac{5}{6}$ **1**

Now,

$P\left(\dfrac{A}{E_1}\right)$ = Probability that the man speaks truth = $\dfrac{3}{4}$

and $P\left(\dfrac{A}{E_2}\right)$ = Probability that the man does not speak truth

$= 1 - \dfrac{3}{4} = \dfrac{1}{4}$ **1**

We have to find $P\left(\dfrac{E_1}{A}\right)$ *i.e.*, the probability that there is six on the dice given that the man has reported that there is a six. By Baye's theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$ **1**

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}}$$

$$= \frac{3}{3+5} = \frac{3}{8}$$ **1**

SECTION — C

20. Given, $f(x) = \dfrac{4x+3}{6x-4}$

To show f is one-one :

Let, $f(x_1) = f(x_2)$,

then $\dfrac{4x_1+3}{6x_1-4} = \dfrac{4x_2+3}{6x_2-4}$

$\Rightarrow (4x_1 + 3)(6x_2 - 4) = (6x_1 - 4)(4x_2 + 3)$ **1**

$\Rightarrow 24x_1x_2 - 16x_1 + 18x_2 - 12$

$= 24x_1x_2 + 18x_1 - 16x_24 - 12$

$\Rightarrow - 16x_1 + 18x_2 = 18x_1 - 16x_2$ **1**

$\Rightarrow - 16x_1 - 18x_1 = - 18x_2 - 16x_2$

$\Rightarrow 34x_1 = 34x_2$

$\Rightarrow x_1 = x_2$

$\Rightarrow f_1$ is one-one. **1**

To show f is onto :

Let, $y \in B$

$\therefore y = f(x)$

$\Rightarrow y = \dfrac{4x+3}{6x-4}$

$\Rightarrow y(6x - 4) = 4x + 3$ **1**

$\Rightarrow 6xy - 4y = 4x + 3$

$\Rightarrow 6xy - 4x = 4y + 3$

$\Rightarrow x(6y - 4) = 4y + 3$

$\Rightarrow x = \dfrac{4y+3}{6y-4} \in B$

$= R - \left\{\dfrac{2}{3}\right\}$

$\Rightarrow$ For every value of y except $y = \left\{\dfrac{2}{3}\right\}$, there is a pre-image $x = \dfrac{4y+3}{6y-4} = g(y)$. **1**

$\Rightarrow x \in A$

To find f^{-1} :

Since, f is one-one and onto, therefore f is invertible.

Thus, $f^{-1}(x) = \dfrac{4x+3}{6x-4} = g(x)$ **1**

Or

Let y be an arbitrary element of range f, then $y = 4x^2 + 12x + 15$, for some x is N, which implies that

$y - 15 = 4x^2 + 12x$

$\Rightarrow 4(x^2 + 3x) = y - 15$

$\Rightarrow 4\left[\left(x+\dfrac{3}{2}\right)^2 - \dfrac{9}{4}\right] = y - 15$

$\Rightarrow \left(x+\dfrac{3}{2}\right)^2 - \dfrac{9}{4} = \dfrac{y-15}{4}$ **½**

$\Rightarrow \left(x+\dfrac{3}{2}\right)^2 = \dfrac{y-15}{4} + \dfrac{9}{4}$

$\Rightarrow \left(x+\dfrac{3}{2}\right)^2 = \dfrac{y-15+9}{4}$

$\Rightarrow \left(x+\dfrac{3}{2}\right)^2 = \dfrac{y-6}{4}$

$\Rightarrow x+\dfrac{3}{2} = \pm\dfrac{\sqrt{y-6}}{2}$

$\Rightarrow x = \dfrac{-3+\sqrt{y-6}}{2}$ or $\dfrac{-3-\sqrt{y-6}}{2}$

But $x \in N \Rightarrow x = \dfrac{-3+\sqrt{y-6}}{2}$

$\therefore g(x) = \dfrac{-3+\sqrt{x-6}}{2} \in S$

$=$ range of f **½**

To prove invertible, $fog = gof = I_s$

$(fog) = f|g(x)|$ **1**

$= f\left(\dfrac{-3+\sqrt{x-6}}{2}\right)$

$= 4\left(\dfrac{-3+\sqrt{x-6}}{2}\right)^2 + 12\left(\dfrac{-3+\sqrt{x-6}}{2}\right) + 15$

$= 4\left(\dfrac{9+x-6-6\sqrt{x-6}}{4}\right) + 6\left(-3+\sqrt{x-6}\right) + 15$

$= 3 + x - 6\sqrt{x-6} - 18 + 6\sqrt{x-6} + 15$ **1**

$(fog)(x) = x$

$(fog)(x) = g\,|f(x)|$

$= g(4x^2 + 12x + 15)$

$= \dfrac{-3+\sqrt{4x^2+12x+15-6}}{2}$

$= \dfrac{-3+\sqrt{4x^2+12x+9}}{2}$ **1**

$= \dfrac{-3+\sqrt{(2x+3)^2}}{2}$

$= \dfrac{-3+2x+3}{2}$

$= \dfrac{2x}{2}$ **1**

$(fog)\,(x) = x$

$(fog)(x) = (gof)(x) = x$

$fog = gof = I_s$ **1**

21. Given equation of curve is

$$16x^2 + 9y^2 = 144 \quad \text{...(i)}$$

Since (x_1, y_1) lies on (i), then

$$16x_1^2 + 9y_1^2 = 144$$

$\Rightarrow \quad 9y_1^2 = 144 - 16 \times 4,$ [as $x_1 = 2$]

$\Rightarrow \quad 9y_1^2 = 80$

$\Rightarrow \quad y_1 = \pm\frac{4\sqrt{5}}{3}$

$\Rightarrow \quad y_1 = \frac{4\sqrt{5}}{3},$ [as $y_1 > 0$]

$\Rightarrow \quad (x_1, y_1) = \left(2, \frac{4\sqrt{5}}{3}\right)$

Now, differentiating (i) w.r.t. x, we get

$$32x + 18y\frac{dy}{dx} = 0 \quad \textbf{1}$$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{16x}{9y}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{16\times 2}{9\times\frac{4\sqrt{5}}{3}} = -\frac{8}{3\sqrt{5}}$

$\therefore$ The equation of tangent at (x_1, y_1) is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} (x, x_1)$$

i.e., $\quad y - \frac{4\sqrt{5}}{3} = -\frac{8}{3\sqrt{5}}(x-2)$ **1**

$\Rightarrow \quad 3\sqrt{5}\,y - 20 = -8x + 16$

$\Rightarrow 8x + 3\sqrt{5}\,y - 36 = 0$ ...(ii) **1**

Suppose, tangent (ii) meets the x-axis at the point $(k, 0)$, then

$$8k + 0 - 36 = 0$$

$\Rightarrow \quad k = \frac{36}{8} = \frac{9}{2}$

Thus, tangent intersects x-axis at the point $\left(\frac{9}{2}, 0\right)$ **1**

Again, equation of normal at (x_1, y_1) is

$$(y - y_1)\left(\frac{dy}{dx}\right)_{(x_1,y_1)} + (x - x_1) = 0$$

i.e., $\left(y - \frac{4\sqrt{5}}{3}\right)\times\left(-\frac{8}{3\sqrt{5}}\right) + (x-2) = 0$

$\Rightarrow \quad -8(3y - 4\sqrt{5}) + 9\sqrt{5}\,(x-2) = 0$

$\Rightarrow \quad 9\sqrt{5}\,x - 24y + 14\sqrt{5} = 0$ ...(iii) **1**

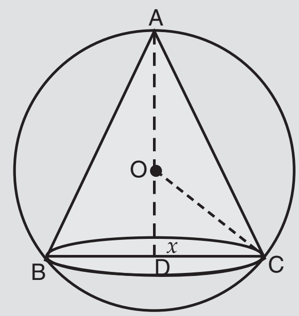
Suppose normal (iii) meets the x-axis at the point $(m, 0)$, then

$$9\sqrt{5}\,m - 0 + 14\sqrt{5} = 0$$

$\Rightarrow \quad m = -\frac{14}{9}$

Thus, normal intersects the x-axis at the point $\left(\frac{-14}{9}, 0\right)$. **1**

Or



½

Let radius of cone be x and its height be h.

$\therefore \quad OD = (h - r)$ ½

Volume of cone (V) $= \frac{1}{3}\pi x^2 h$ ...(i) ½

In ΔOCD, $x^2 + (h-r)^2 = r^2$

(or) $\quad x^2 = r^2 - (h-r)^2$

$\therefore \quad V = \frac{1}{3}\pi h\{r^2 - (h-r)^2\}$

$= \frac{1}{3}\pi(-h^3 + 2h^2 r)$ **1**

$\frac{dV}{dh} = \frac{\pi}{3}(-3h^3 + 4hr)$ **1**

$\frac{dV}{dh} = 0$

$\Rightarrow \quad h = \frac{4r}{3}$ ½

$\frac{d^2V}{dh^2} = \frac{\pi}{3}(-6h + 4r)$

$= \frac{\pi}{3}\left(-6\left(\frac{4r}{3}\right) + 4r\right)$

$= -\frac{4\pi r}{3} < 0$ **1**

$\therefore$ At $h = \frac{4r}{3}$, volume is maximum

Maximum volume

$$= \frac{1}{3}\pi\left\{-\left(\frac{4r}{3}\right)^2 + 2\left(\frac{4r}{3}\right)^3 r\right\}$$

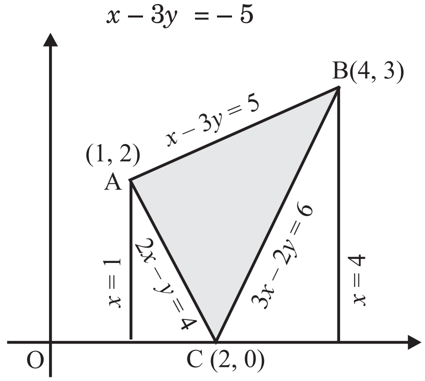
$$= \frac{8}{27}\left(\frac{4}{3}\pi r^3\right) \quad \mathbf{1}$$

$$= \frac{8}{27} \text{ (Volume of Sphere)}$$

[CBSE Marking Scheme 2014]

22. The given lines are

$$2x + y = 4 \quad ...(1)$$
$$3x - 2y = 6 \quad ...(2)$$
$$x - 3y = -5 \quad ...(3)$$



1

Multiply (3) by 2 and subtract from (1)

$$7y = 14$$
$$y = 2$$

From (3), $x - 3 \times 2 = -5$

$\Rightarrow \quad x = 1$

Lines (1) and (3) intersect at (1, 2)

Solving (3) and (2), we get $x = 4$, $y = 3$

Lines (2) and (3) intersect at (4, 3)

Solving (1) and (2), we get $x = 2$, $y = 0$ **1**

Lines (1) and (2) intersect at (2, 0)

Area of ΔABC = Area of trapezium ALMB – Area of ΔALC – Area of ΔBCM

$$= \int_1^4 y_1 dx - \int_1^4 y_2 dx - \int_2^4 y_3 dx$$

where y_1 is for $x - 3y = -5$ or $y = \dfrac{x+5}{3}$ **1**

y_2 is for $2x + y = 4$ or $y = 4 - 2x$

y_3 is for $3x - 2y = 6$ or $y = \dfrac{3x-6}{2}$

Area of ΔABC

$$= \int_1^4 \frac{x+5}{3} dx - \int_1^4 (4-2x)dx - \int_2^4 \frac{3x-6}{2} dx$$

$$= \frac{1}{3}\left(\frac{x^2}{2} + 5x\right)_1^4 - [4x - x^2]_1^2 - \frac{1}{2}\left(\frac{3x^2}{2} - 6x\right)_2^4 \quad \mathbf{1}$$

$$= \frac{1}{3}\left[\left(\frac{16}{2} + 20\right) - \left(\frac{1}{2} + 5\right)\right] - [(8-4) - (4-1)]$$
$$- \frac{1}{2}\left[\left(\frac{3 \times 16}{2} - 24\right) - \left(\frac{12}{2} + 12\right)\right]$$

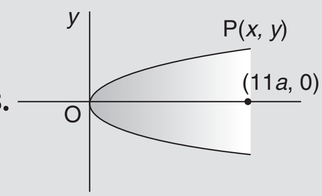
$$= \frac{1}{3}\left(28 - \frac{11}{2}\right) - [4-3] - \frac{1}{2}[0-6] \quad \mathbf{1}$$

$$= \left(\frac{1}{3} \times \frac{45}{2} - 1 - 3\right)$$

$$= \frac{15}{2} - 4$$

$$= \frac{7}{2} \text{ sq. units.} \quad \mathbf{1}$$

23.



1

Let P(x, y) be the nearest point

$$D = \sqrt{(x - 11a)^2 + y^2} \quad \mathbf{½}$$

or $\quad S = (x - 11a)^2 + y^2$

$$= (x - 11a)^2 + 4ax \quad \mathbf{½}$$

$$\frac{dS}{dx} = 2(x - 11a) + 4a \quad \mathbf{1}$$

$$\frac{dS}{dx} = 0 \Rightarrow x = 9a \quad \mathbf{1}$$

$\therefore \quad y = \pm 6a$ **1**

$$\Rightarrow \quad \frac{d^2S}{dx^2} = 2 > 0$$

For minimum distance, coordinates are

P($9a$, $\pm 6a$) **½ + ½**

[CBSE Marking Scheme 2014]

24. The given planes are

$$2x + y + 3z - 2 = 0 \quad ...(1)$$

and $\quad x - 2y + 5 = 0 \quad ...(2)$

Direction ratios of normal of plane (1) are 2, 1, 3. **1**

Directions ratios of normal of plane (2) are 1, 2, 0. **1**

Planes (1) and (2) are perpendicular to each other, then **2**

$a_1a_2 + b_1b_2 + c_1c_2 = 0$ *i.e.*, 2.1 + 1.(–2) + 3.0

= 2 – 2 + 0 = 0 **1**

Hence, the planes (1) and (2) are perpendicular to each other. **1**

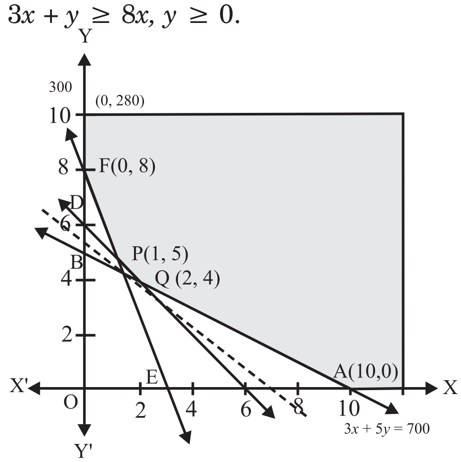
25. Let there be x kg food X and y food Y.

Food X costs ₹ 16 per kg and food Y costs ₹ 20 per kg.

$\therefore$ $Z = 16x + 20y$ is the objective function, where Z is to be minimize.

Now constraints are

$x + 2y \geq 10$, $2x + 2y \geq 12$ or $x + y \geq 6$ and $3x + y \geq 8x$, $y \geq 0$.



1

(i) The line $x + 2y = 10$ passes through A (10, 0) and B (0, 5). Putting $x = 0, y = 0$ in $x + 2y \geq 10$.

$0 \geq 10$ which is not true.

$\Rightarrow$ $x + 2y \geq 10$ region lies on and above AB.

(ii) The line $x + y = 6$ passes through C(6, 0) and D(0, 6). **1**

Putting $x = 0, y = 0$ i.e., $x + y \geq 6$.

$0 \geq 6$ which is not true.

$\Rightarrow x + y \geq 6$ lies on and above CD. **1**

(iii) The line $3x + y = 8$ passes through E $\left(\frac{8}{3}, 0\right)$, F(0, 8).

Putting $x = 0$, $y = 0$ in $3x + y \geq 8$ or $0 \geq 8$ which is not true.

$\Rightarrow 3x + y \geq 8$ lies on and above EF.

(iv) $x \geq 0$ lies on and to the right of y-axis.

(v) $y \geq 0$ lies on and above x-axis. **1**

The shaded area YFPQAY represents the feasible region. Now, P is the point of intersection of the lines CD and EF.

$$x + y = 6 \quad ...(1)$$
$$3x + y = 8 \quad ...(2)$$

Subtracting (1) from (2)

$$2x = 2, x = 1, y = 5$$

$\therefore$ The point P is (1, 5).

Q is the point of intersection of the line

$$x + 2y = 10 \quad ...(1)$$
$$x + y = 6 \quad ...(2)$$

Subtracting (2) from (1),

$$y = 4, x = 2$$

The objective function is

$$Z = 16x + 20y$$

At F(0, 8), $Z = 16x + 20y = 0 + 160 = 160$

At P(1, 5), $Z = 16 + 100 = 116$

At Q(2, 4), $Z = 32 + 80 = 112$

At A(10, 0), $Z = 160 + 0 = 160$

This shows that minimum value of Z is ₹ 112.

Feasible region is unbounded. **1**

Consider the inequality $16x + 20y < 112$

or $4x + 5y = 28$

The line $4x + 5y = 28$ passes through (7, 0) and $\left(0, \frac{28}{5}\right)$.

Also, putting $x = 0$, $y = 0$ in $4x + 5y < 28$, we get, $0 < 28$ which is true.

$\therefore$ $8x + 10y = 33$ lies below the line $4x + 5y = 28$

Hence, there is no common point between feasible region and $4x + 5y < 33$.

$\therefore$ Mimimum value of $Z = 112$, $x = 2$, $y = 4$ *i.e.*, least value of mixture of ₹ 112 when 2 kg of food X and 4 kg of food Y are mixed. **1**

26. Total number of balls = 25

Number of balls marked 'X' = 10

Let X denote the event of getting a ball marked X.

Y = event of getting a ball Y

$$\therefore \quad P(X) = \frac{10}{25} = \frac{2}{5} = p$$

$$\therefore \quad P(Y) = 1 - \frac{2}{5} = \frac{3}{5} = q$$

Now 6 balls are drawn. **1**

(i) P(all the balls are marked X) = $\left(\frac{2}{5}\right)^6$

(ii) Event that not more than 2 will bear 'Y' mark)

$\Rightarrow$ $(6X, 0Y)$, $(5X, 1Y)$, $(4X, 2Y)$

$\therefore$ P(not more than 2 will bear the mark 'Y')

$$= \left(\frac{2}{5}\right)^6 + {}^6C_5\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^5 + {}^6C_4\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^4$$

$$= \left(\frac{2}{5}\right)^4\left(\frac{4}{25} + \frac{6 \times 3 \times 2}{25} + \frac{15 \times 9}{25}\right) \quad \textbf{1}$$

$$= \left(\frac{2}{5}\right)^4\left(\frac{4 + 36 + 135}{25}\right)$$

$= \left(\frac{2}{5}\right)^4 \left(\frac{175}{25}\right) = 7\left(\frac{2}{5}\right)^4$ **1**

(iii) Event that at least 1 ball will bear 'Y'

= [(5X, 1Y), (4X, 2Y), (3X, 3Y), (2X, 4Y), (X, 5Y), (0X, 6Y)} **1**

P(at least 1 ball will bear mark Y)

= 1 – P(no ball will bear mark Y)

= 1 – P(all balls bear mark Y)

$= 1 - \left(\frac{2}{5}\right)^6$ **1**

(iv)P(3 balls each are marked X and Y)

$$P(3) = {}^6C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^3$$

$$= \frac{6\times 5+4}{6}\times\frac{27}{125}\times\frac{8}{125}$$

$$= \frac{864}{3125}$$ **1**

SOLUTIONS

SAMPLE QUESTION PAPER - 9

Self Assessment______________________________

Time: 3 Hours **Maximum Marks: 100**

SECTION — A

1. Given, $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \times \vec{b} = 3$

$\therefore$ Projection of $\vec{b}$ on $\vec{a}$ $= \dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|}$

$$= \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} \quad \left[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}\right] \quad \tfrac{1}{2}$$

$$= \frac{3}{2} \quad \tfrac{1}{2}$$

$$\left[\because \vec{a}\cdot\vec{b} = 3 \text{ and } |\vec{a}| = 2\right]$$

2. Given, $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$

$$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$[\because |\hat{n}| = 1]$$

$$\Rightarrow \quad \cos\theta = \sin\theta$$

On dividing both sides by cos θ, we get

$$\tan\theta = 1$$

$$\Rightarrow \quad \tan\theta = \tan\frac{\pi}{4} \quad \left[\because 1 = \tan\frac{\pi}{4}\right]$$

$$\therefore \quad \theta = \frac{\pi}{4}$$

So, angle between $\vec{a}$ and $\vec{b}$ is $\dfrac{\pi}{4}$. **1**

3. (i) For a symmetric matrix A′ = A.

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$\therefore$ A′ = A ⇒ A is symmetric matrix. **1**

4. The rate of change of the circle w.r.t. time t is given to be 0.7 cm/sec *i.e.*,

$$\frac{dr}{dt} = 0.7 \text{ cm/sec.}$$

Now, circumference of the circle is C = 2π*r*.

$\therefore$ The rate of change of circumference w.r.t. t is given by

$$\frac{dC}{dt} = \left[\frac{d}{dr}(2\pi r).\frac{dr}{dt}\right] \quad \tfrac{1}{2}$$

$$= 2\pi.\frac{dr}{dt} = 2\pi \times 0.7$$

$$= 1.4\pi \text{ cm/sec.} \quad \tfrac{1}{2}$$

5. $\int (ax^2 + bx + c)\, dx$

$$= \int ax^2\, dx + \int bx\, dx + \int c\, dx$$

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C \quad \mathbf{1}$$

6. The equation of a plane through the line of intersection of planes

$x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is

$$(x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$$

$$x(1 + 2\lambda) + y(2 + \lambda) + z(3 - \lambda) - 4 + 5\lambda = 0 \quad ...(1)$$

This is perpendicular to the plane

$$5x + 3y + 6z + 8 = 0$$

$\therefore$ $5(1 + 2\lambda) + 3(2 + \lambda) + 6(3 - \lambda) = 0$

$$7\lambda + 29 = 0$$

$$\lambda = \frac{29}{7} \qquad \text{½}$$

Putting $\lambda = \dfrac{29}{7}$ in (1), we obtain the equation of the required plane as

$$-51x - 15y + 50z - 173 = 0$$

$$51x + 15y - 50z + 173 = 0. \qquad \text{½}$$

SECTION — B

7. To prove, $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$

LHS $= \cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18$

$$= \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$$

$$\left[\because \cot^{-1}x = \frac{1}{\tan^{-1}x}\right] \quad \mathbf{1}$$

$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\left(\frac{1}{8}\right)}\right) + \tan^{-1}\frac{1}{18}$$

$$\left[\because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)\right] \quad \mathbf{2}$$

$$= \tan^{-1}\left(\frac{15}{55}\right) + \tan^{-1}\frac{1}{18}$$

$$= \tan^{-1}\left(\frac{3}{11}\right) + \tan^{-1}\frac{1}{18}$$

$$= \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\left(\frac{3}{11}\right)\times\left(\frac{1}{18}\right)}\right) = \tan^{-1}\left(\frac{65}{195}\right) \quad \mathbf{1}$$

$$= \tan^{-1}\left(\frac{1}{3}\right) = \cot^{-1}3 = \text{RHS} \quad \mathbf{1}$$

Or

We have to prove

$$\cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right\} = \frac{\pi}{3}$$

$$\text{LHS} = \cos^{-1}(x) + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3-2x^2}}{2}\right\}$$

Let $\cos^{-1}x = \alpha \Rightarrow x = \cos\alpha$ 1

Then, LHS $= \alpha + \cos^{-1}$

$$\left[\cos\alpha\cdot\cos\frac{\pi}{3} + \frac{\sqrt{3}}{2}\sqrt{1-\cos^2\alpha}\right]$$

$$= \alpha + \cos^{-1}\left[\cos\frac{\pi}{3}\cos\alpha + \sin\frac{\pi}{3}\sin\alpha\right]$$

$$\left[\because \sin\alpha = \sqrt{1-\cos^2\alpha},\ \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right] \quad \mathbf{1}$$

$$= \alpha + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \alpha\right)\right]$$

[$\because$ cos (A – B) = cos A cos B + sin A sin B] **1**

$$= \alpha + \frac{\pi}{3} - \alpha = \frac{\pi}{3} = \text{RHS} \quad \mathbf{1}$$

8. Let $\quad B = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

and $\quad C = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$.

Then, the given matrix equaiton is A + B = C.

Now, $\quad A + B = C$

$\Rightarrow\ (A + B) + (-B) = C + (-B)$

$\Rightarrow\ A + (B + (-B)) = C + (-B)$

$\Rightarrow\ A + 0 = C - B$

$\Rightarrow\ A = C - B.$

$$\therefore \quad A = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 1 & -4 \end{bmatrix} \quad \mathbf{1}$$

$$= \begin{bmatrix} 3-2 & -6-3 \\ -3+1 & 8-4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix} \quad \mathbf{2}$$

9. $$\Delta = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{bmatrix},\ C_1 - C_3 \text{ and } C_2 - C_3$$

$$\Delta = \begin{bmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ b(c-a) & a(c-b) & ab \end{bmatrix} \quad \mathbf{1}$$

$$\Delta = (a-c)(b-c)\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -b & -a & ab \end{bmatrix} \quad \mathbf{1}$$

$$\Delta = (a-c)(b-c)[(-a)(-b)]$$

$$= (a-c)(b-c)(b-a) \quad \mathbf{1}$$

$$\Delta = (a-c)(b-c)(c-a). \quad \mathbf{1}$$

Or

Consider, $\begin{bmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{bmatrix}$

$$= \begin{bmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{bmatrix} \quad \mathbf{1}$$

[By performing $C_1 \to C_1 + (C_2 + C_3)$]

$$= 2(a+b+c)\begin{bmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{bmatrix}$$

[By taking $2(a+b+c)$ common from C_1] **1**

$$= 2(a+b+c)\begin{bmatrix} 1 & a & b \\ 0 & b+c+a & b \\ 0 & a & b+a+c \end{bmatrix}$$

[By performing $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$= 2(a+b+c)\begin{bmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & a+b+c \end{bmatrix} \quad \mathbf{1}$$

[By performing $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

Expanding along C_1, we get

$= 2(a+b+c)\,[(a+b+c)^2 - 0]$

$= 2(a+b+c)^3$. **1**

10. Let $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$= u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Now, $u = (\sin x)^x$ **1**

Taking log on both sides

$\log u = \log (\sin x)^x$

$= x \log (\sin x)$,

[$\because \log m^n = n \log m$]

Differentiating both sides w.r.t. x

$$\frac{1}{u}\frac{du}{dx} = 1.\log \sin x + x.\frac{1}{\sin x}\frac{d}{dx}(\sin x) \quad \mathbf{1}$$

[$(uv) = u'v + uv'$, $u = x'$, $v = \log (\sin x)$]

$$= \log \sin x + \frac{x\cos x}{\sin x}$$

$$\Rightarrow \frac{du}{dx} = u\,[\log \sin x + x \cot x]$$

$= (\sin x)^x\,[\log \sin x + x \cot x]$

$v = \sin^{-1}\sqrt{x}$

Put $v = \sin^{-1} t$, $t = \sqrt{x}$

$$\Rightarrow \frac{dv}{dt} = \frac{1}{\sqrt{1-t^2}}, \frac{dt}{dx} = \frac{1}{2}x^{1/2-1} = \frac{1}{2\sqrt{x}} \quad \mathbf{1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{dv}{dt}\times\frac{dt}{dx} = \frac{1}{\sqrt{1-t^2}}.\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^x\,[\log \sin x + x \cot x] + \frac{1}{2\sqrt{x-x^2}}$$

1

11. Let $P(n)$ be the given statement in the problem.

$$\therefore \quad P(n) : \frac{d}{dx}(x^n) = nx^{n-1} \quad ...(1)$$

To verify the statement for $n = 1$.

Put $n = 1$ in (1), we get

$$P(1) : \frac{d}{dx}(x^1) = nx^{1-1} = (1)x^0$$

$= (1)\,(1) = 1$ **1**

which is true as $\frac{d}{dx}(x) = 1$

We suppose $P(x)$ is true for $n = m$.

$$\Rightarrow P(m) : \frac{d}{dx}(x^m) = mx^{m-1} \quad ...(2) \quad \mathbf{1}$$

To establish the truth of $P(m+1)$, we prove

$$P(m+1) : \frac{d}{dx}(x^{m+1}) = (m+1)x^m$$

Since $x^{m+1} = x^1.x^m$

$$\therefore \frac{d}{dx}(x^{m+1}) = \frac{d}{dx}(x.x^m)$$

$$= x\frac{d}{dx}(x^m) + x^m.\frac{d}{dx}(x) \quad \mathbf{1}$$

$= x.mx^{m-1} + x^m.1$, [Using (2)]

$= mx^m + x^m = (m+1)x^m$

$= (m+1)x^{(m+1)-1}$

$\therefore$ $P(m+1)$ is true, if $P(m)$ is true. But $P(1)$ is true.

$\therefore$ By principle of induction, $P(n)$ is true for all $n \in N$. **1**

12. The given curves are

$x = y^2$ ...(1)

and $xy = k$ ...(2)

These meet where

$$y^2.y = k \Rightarrow y^3 = k \Rightarrow y = k^{1/3}$$

and $\quad x = y^2 = k^{2/3}$ **1**

i.e., at the point $P(k^{2/3}, k^{1/3})$

Differentiating (1) w.r.t. x, we get

$$1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$\therefore m_1$ = Slope of tangent to the curve (1) at

$$P = \left(\frac{dy}{dx}\right)_{at\ P} = \frac{1}{2k^{1/3}}$$ **1**

Similarly, differentiating (2) w.r.t. x, we get

$$x\frac{dy}{dx} + 1.y = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

$\therefore m_2$ = Slope of tangent to the curve (2) at P

$$= \left(\frac{dy}{dx}\right)_{at\ P} = \frac{k^{1/3}}{k^{2/3}}$$ **1**

For the curves to cut at right angle at P, we must have

$$m_1 m_2 = -1$$

$$\Rightarrow \frac{1}{2k^{1/3}} \times \left(\frac{k^{1/3}}{k^{2/3}}\right) = -1$$

$$\Rightarrow 2k^{2/3} = 1$$

$$\Rightarrow 2^3 k^2 = (1)^3$$

$$\Rightarrow 8k^2 = 1$$ **1**

Or

Given,

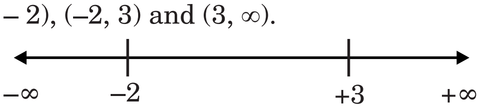
$$f(x) = 2x^3 - 3x^2 - 36x + 7$$

$$\Rightarrow f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6)$$ **1**

$$= 6(x - 3)(x + 2)$$

$\Rightarrow f'(x) = 0$ gives $x = 3, -2$.

The points $x = -2$ and $x = 3$ divides the real line into three disjoint intervals, namely, $(-\infty\ -2)$, $(-2, 3)$ and $(3, \infty)$.



1

In the intervals $(-\infty, -2)$ and $(3, \infty)$, $f'(x)$ is positive while in the interval $(-2, 3)$, $f'(x)$ is negative. Consequently, the function f' is strictly increasing in the intervals $(-\infty, -2)$ and $(3, \infty)$ while the function is strictly decreasing in the interval $(-2, 3)$. **2**

13. Let $\quad I = \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Write $2 \sin x + 3 \cos x$

$$= \lambda\frac{d}{dx}(3 \sin x + 4 \cos x) + \mu(3 \sin x + 4 \cos x)$$

1

$$= \lambda(3 \cos x - 4 \sin x) + \mu(3 \sin x + 4 \cos x)$$

$$\Rightarrow 2 = -4\lambda + 3\mu$$

$$3 = 3\lambda + 4\mu$$ **1**

Solving these, we get

$$\lambda = \frac{1}{25}, \mu = \frac{18}{25} \quad ...(1)$$

Hence,

$$I = \int \frac{\lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x)}{3\sin x + 4\cos x} dx$$

$$= \lambda\int\frac{3\cos x - 4\sin x)}{3\sin x + 4\cos x}dx + \mu\int dx + C$$ **1**

$$= \lambda\frac{1}{25}\log|3 \sin x + 4 \cos x| + \frac{18}{25}x + C,$$

Put $3 \sin x + 4 \cos x = 0$

$$\Rightarrow I = \frac{1}{25} + \frac{18}{25}x + C.$$ **1**

14. We have

$$2A + B + X = 0$$

$$X = -2A - B$$

$$\Rightarrow X = -2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ -1 & -5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

15. Let $I = \int \frac{5x}{(x+1)(x^2+9)} dx$

$$\frac{5x}{(x+1)(x^2+9)} dx = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

$$\therefore A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{9}{2}$$ **2**

$$\Rightarrow I = -\frac{1}{2}\int\frac{1}{x+1}dx + \frac{1}{2}\int\frac{x+9}{x^2+9}dx$$ **1**

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$$

1

16. Let $I = \int \frac{dx}{5\cos x - 12\sin x}$

$$= \int \frac{\sec^2\frac{x}{2}}{5 - 5\tan^2\frac{x}{2} - 24\tan\frac{x}{2}} dx$$ **1**

[Dividing numerator and denominator by $\cos^2 \frac{x}{2}$]

Let $\tan \frac{x}{2} = t$, so that $\frac{1}{2} \sec^2 \frac{x}{2}\, dx = dt$

$$\Rightarrow \quad I = \int \frac{2}{5 - 5t^2 - 24t} dt$$

$$= \frac{1}{5} \int \frac{2dt}{1 - \left(t^2 + \frac{24}{5} t\right)} \qquad \mathbf{1}$$

$$= \frac{2}{5} \int \frac{1}{\left(\frac{13}{5}\right)^2 - \left(t + \frac{12}{5} t\right)^2} dt$$

$$= \frac{2}{5} \times \frac{1}{2 \times \frac{13}{5}} \log \left| \frac{\frac{13}{5} + \left(t + \frac{12}{5}\right)}{\frac{13}{5} - \left(t + \frac{12}{5}\right)} \right| + C \qquad \mathbf{1}$$

$$= \frac{1}{13} \log \left| \frac{5 + \tan \frac{x}{2}}{5 - \tan \frac{x}{2}} \right| + C$$

$$= \frac{1}{13} \log \left| \frac{25 + \tan \frac{x}{2}}{1 - 5 \tan \frac{x}{2}} \right| + C \qquad \mathbf{1}$$

17. Given, $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 8$

$\vec{a}.(\vec{a} - \vec{b}) + \vec{b}.(\vec{a} - \vec{b}) = 8$

$\vec{a}.\vec{a} - \vec{a}.\vec{b} + \vec{b}.\vec{a} - \vec{b}.\vec{b}. = 8$ **1**

$\Rightarrow \ |\vec{a}|^2 - |\vec{b}|^2$

$[\because \ \vec{a}.\vec{b} = \vec{b}.\vec{a}]$

$\Rightarrow \ 64|\vec{b}|^2 - |\vec{b}|^2 = 8 \ [\because |\vec{a}| = 8|\vec{b}|]$ **1**

or $\quad 63|\vec{a}|^2 = 8 \ \because \ |\vec{b}|^2 = \frac{8}{63}$

But $\quad |\vec{a}| = 8\,|\vec{b}|$

$= 8 \times \sqrt{\frac{8}{63}} = \frac{8\sqrt{8}}{\sqrt{63}}$ **1**

Thus, $\quad |\vec{a}| = \frac{8\sqrt{8}}{\sqrt{63}}, \ |\vec{b}| = \sqrt{\frac{8}{63}}$ **1**

18. Let the direction ratios of the required line be a, b, c. Since, it is perpendicular to the two given lines.

Therefore, $a + 2b + 3c = 0$ ...(i) **1**

$-3a + 2b + 5c = 0$ ...(ii)

Solving (i) and (ii) by cross-multiplication, we get

$$\frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

$$\frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = k, \quad (say) \qquad \mathbf{1}$$

Thus, the required line passes through (– 1, 3, – 2) and has direction ratios proportional to (2, – 7, 4). So, its equation is **1**

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4} \qquad \mathbf{1}$$

Or

Equation of the plane through the point (3, 4, 2) is

$a(x - 3) + b(y - 4) + c(z - 2) = 0$ ...(i) **1**

Plane (i) passes through (2, – 2, – 1) and (7, 0, 6).

Therefore, $a(-1) + b(-6) + c(-3) = 0$ ...(ii)

and $\quad a(4) + b(-4) + c(4) = 0$

or $\quad a - b + c = 0$ ...(iii) **1**

Eliminating a, b, c from (i), (ii) and (iii), we get

$$\begin{vmatrix} x-3 & y-4 & z-2 \\ -1 & -6 & -3 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \ (x - 3)(-6 - 3) - (y - 4)(-1 + 3) +$
$(z - 2)(1 + 6) = 0$ **1**

$\Rightarrow \ -9x + 27 - 2y + 8 + 7z - 14 = 0$

$9x + 2y - 7z - 21 = 0$

is the required equation. **1**

19. Let

E_1 = the event that bag I is chosen

E_2 = the event that bag II is chosen

and A = the event that a white ball is drawn

$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}$ **1**

$$P\left(\frac{A}{E_1}\right) = \frac{3}{7}, P\left(\frac{A}{E_2}\right) = \frac{5}{11} \qquad \mathbf{1}$$

By Baye's theorem, the required probability is

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)} \quad \mathbf{1}$$

$$= \frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}} = \frac{\frac{3}{7}}{\frac{3}{7}+\frac{5}{11}} = \frac{33}{68} \quad \mathbf{1}$$

SECTION — C

20. We have $f(x) = x^2 + 2$ and $g(x) = \dfrac{x}{x-1}$; $x \neq 1$

Since, range f = domain g
and range g = domain f
$\therefore$ fog and gof exist.
For any $x \in$ R, we have $(fog)(x) = f[g(x)]$ **1**

$$= f\left[\frac{x}{x-1}\right] = \left(\frac{x}{x-1}\right)^2 + 2$$

$$= \frac{x^2+2(x-1)^2}{(x-1)^2} = \frac{x^2+2(x^2+1-2x)}{(x-1)^2}$$

$$= \frac{3x^2+2-4x}{(x-1)^2} \quad \mathbf{1}$$

$\therefore$ fog : R $\to$ R is defined by

$$(fog)(x) = \frac{3x^2-4x+2}{(x-1)^2}, \forall x \in R \quad ...(i) \quad \mathbf{1}$$

For any $x \in$ R, we have

$$(gof)(x) = g[f(x)]$$
$$= g[x^2 + 2]$$
$$= \frac{x^2+2}{x^2+2-1} = \frac{x^2+2}{x^2+1} \quad \mathbf{1}$$

$\therefore$ $gof : R \to R$ is defined by

$$(gof)(x) = \frac{x^2+2}{x^2+1} \forall x \in R$$

On putting $x = 2$ in eqn. (i), we get

$$(fog)(2) = \frac{3\times(2)^2-4(2)+2}{(2-1)^2}$$
$$= \frac{3\times4-8+2}{(1)^2} \quad \mathbf{1}$$
$$= 12 - 6 = 6$$

On putting $x = -3$ in Eq. (ii), we get

$$(gof)(-3) = \frac{(-3)^2+2}{(-3)^2+1}$$
$$= \frac{9+2}{9+1} = \frac{11}{10} \quad \mathbf{1}$$

Or

$f : R_+ \to [-9, \infty)$; $f(x) = 5x^2 + 6x - 9$; $f^{-1}(y)$

$$= \frac{\sqrt{54+5y}-3}{5} \quad \mathbf{1}$$

$fof^{-1}(y)$

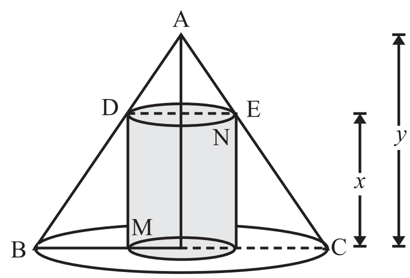
$$= 5\left\{\frac{\sqrt{54+5y}-3}{5}\right\}^2 + 6\left\{\frac{\sqrt{54+5y}-3}{5}\right\} - 9 = y^3 \quad \mathbf{2}$$

$$f^{-1}of\,(x) = \frac{\sqrt{54+5(5x^2+6x-9)}-3}{5} = x \quad \mathbf{2\tfrac{1}{2}}$$

Hence 'f' is invertible with $f^{-1}(y)$

$$= \frac{\sqrt{54+5y}-3}{5} \quad \mathbf{\tfrac{1}{2}}$$

21.



$$\angle BAM = \alpha$$
$$DN = r$$
$$MN = x$$
$$AN = AM - NM \quad \mathbf{1}$$
$$= h - x$$

In ΔADN, $\tan\alpha = \dfrac{r}{h-x}$

Volume of cylinder (V)

$$V = \pi r^2 x = \pi(h-x)^2 \tan^2 \pi x$$
$$= \pi \tan^2 \alpha(x^3 - 2hx^2 + h^2x) \quad \mathbf{1}$$
$$\frac{dV}{dx} = \pi \tan^2 \alpha(3x^2 - 4hx + h^2)$$
$$\frac{d^2V}{dx^2} = \pi \tan^2 \alpha(6x^2 - 4h) \quad \mathbf{1}$$
$$\frac{dV}{dx} = 0$$
$$\Rightarrow 3x^2 - 4hx + h^2 = 0$$
$$(x-h)(3x-h) = 0 \quad \mathbf{1}$$
$$x = h$$

or $x = \dfrac{h}{3}$

$x = h$, not possible

$\therefore$ $x = \dfrac{h}{3}$

When $\quad x = \frac{h}{3}$. **1**

$$\frac{d^2V}{dx^2} = \pi \tan^2 \alpha(2h - 4h)$$

$$= -2\pi h \tan^2 \alpha < 0$$

$\therefore$ V is max, when $x = \frac{h}{3}$. **1**

22. Firstly, we draw a square formed by the lines $x = 0$, $x = 4$, $y = 4$ and $y = 0$ and after that, we draw two parabolas which intersect each other on the square such that the whole region divided into three parts. Now, we find separately area of each part and see area of each part is equal.

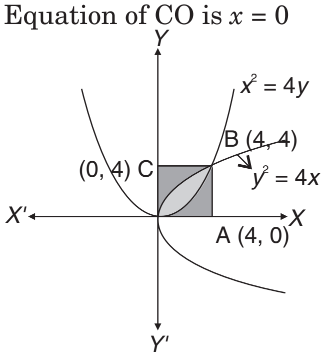
Let ABCD be the square whose sides are represented by following equations :

Equation of OA is $y = 0$

Equation of AB is $x = 4$

Equation of BC is $y = 4$

Equation of CO is $x = 0$



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On solving equations $y^2 = 4x$ and $x^2 = 4y$ simultaneously, we get A(0, 0) and B(4, 4) as their points of intersection.

Now, area bounded by these curves is given by

$$= \int_0^4 \left[\left(y = 2\sqrt{x}\right) - \left(y = \frac{x^2}{4}\right)\right]dx$$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx = \left[2\cdot\frac{2}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]^4 = \frac{4}{3}\cdot(4)^{3/2} - \frac{64}{12}$$

$$= \frac{4}{3}\cdot(2^2)^{3/2} - \frac{64}{12} = \frac{4}{3}\cdot(2)^3 - \frac{64}{12}$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq units}$$

Hence, area bounded by curves $y^2 = 4x$ and $x^2 = 4y$ is $\frac{16}{3}$ sq. units ...(i) **1**

Now, area bounded by curve $2x^2 = 4y$ and the lines $x = 0$, $x = 4$ and x-axis

$$= \int_0^4 \left(y = \frac{x^2}{4}\right)dx = \int_0^4 \frac{x^2}{4}dx$$

$$= \left[\frac{x^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq. units} \qquad \text{...(ii) } \mathbf{1\tfrac{1}{2}}$$

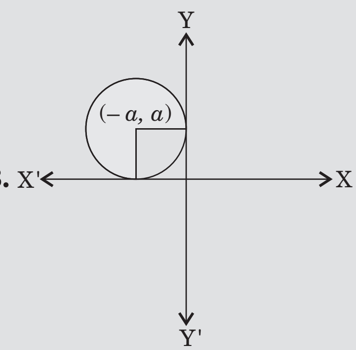
Similarly, the area bounded by curve $y^2 = 4x$ and the line $y = 0$, $y = 4$ and y-axis.

$$= \int_0^4 \left(x = \frac{y^2}{4}\right)dy = \int_0^4 \left(\frac{y^2}{4}\right)dy$$

$$= \left[\frac{y^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq. units} \qquad \text{...(iii) } \mathbf{1\tfrac{1}{2}}$$

From Eqn. (i), (ii) and (iii), it is clear that area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ divides the area of square in three equal parts. **Hence Proved. 1**

23.



1

Equation of family of circles is,

$(x - a)^2 + (y - a)^2 = a^2$ **½**

or $x^2 + y^2 + 2ax - 2ay + a^2 = 0$ **½**

Differentiating, we get

$$2x + 2y\frac{dy}{dx} + 2a - 2a\frac{dy}{dx} = 0 \qquad \mathbf{1}$$

$$x + y\frac{dy}{dx} = a\left(\frac{dy}{dx} - 1\right) \qquad \mathbf{1}$$

or $\quad a = \frac{x + yy'}{y' - 1}$ where $y' = \frac{dy}{dx}$

Substituing the value of (a) in (i)

we get, $(xy' - x + x + yy')^2 + (yy' - y - x - yy')^2$

$= (x + yy')^2$ **1**

or $(x + y)^2[(y')^2 + 1] = (x + yy')^2$ **1**

[CBSE Marking Scheme 2011]

24. Equation of is any plane passing through $(-1, -1, 2)$

$a(x + 1) + b(y + 1) + c(z - 2) = 0$ ...(1)

If plane is $\perp$ to each one of the plane

$2x + 3y - 3z - 2 = 0$

and, $5x - 4y + z - 6 = 0$

$2a + 3b - 3c = 0$ ...(2)

$5a - 4b + c = 0$ ...(3) **1**

On solving (2) and (3), we get

$a = -9,$

$b = -17,$

$c = -23$ **1**

$\therefore$ $9(x + 1) + 17(y + 1) + 23(z - 2) = 0$

$9x + 17y + 23z + 9 + 17 - 46 = 0$

$9x + 17y + 23z - 20 = 0$ **1**

The equation of the plane passing through $(-1, -1, 2)$ is

$a(x + 1) + b(y + 1) + c(z - 2) = 0$, where a, b, c are direction ratios of normal to the plane

(i) is $\perp$ to $2x + 3y - 3z = 2$ and $5x - 4y + z = 6$

$\therefore$ $2a + 3b - 3c = 0$ **1**

and $5a - 4b + c = 0$

$\therefore$ $\frac{a}{-9} = \frac{b}{-17} = \frac{c}{-23}$

The direction ratios of normal to the plane are 9, 17, 23

$\Rightarrow$ Equation of plane is **1**

$9(x + 1) + 17(y + 1) + 23(z - 2) = 0$

$\Rightarrow$ $9x + 17y + 23z = 0$ **1**

Or

Let the coordinates of points A, B and C be $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ respectively. **½**

$\Rightarrow$ Equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and **1**

$\frac{a+0+0}{3} = 1, \; \frac{0+b+0}{3} = -2, \; \frac{0+0+c}{3} = 3$ **1½**

$\therefore$ a = 3, b = –6 and c = 9

$\Rightarrow$ Equation of plane is $\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$ **1**

or $6x - 3y + 2z - 18 = 0$ **1**

which in vector form is

$\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 18$

The cartesian form is $6x - 3y + 2z - 18 = 0$ and the vector form of equation is

$\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 18.$ **1**

25. Let x kg and y kg of fertilizers of type I and II meet the requirement. According to hypothesis, the L.P.P. is

Minimize $Z = 2x + 3y$

Subject to the constraints,

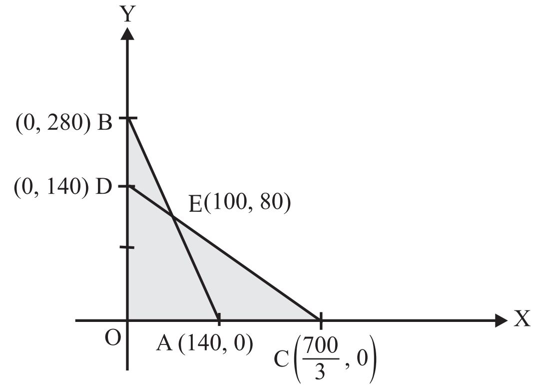
$x \times \frac{10}{100} + y \times \frac{5}{100} \geq 14$ **1**

$\Rightarrow$ $10x + 5y \geq 1400$

$x \times \frac{10}{100} + y \times \frac{10}{100} \geq 14$ **1**

$\Rightarrow$ $6x + 10y \geq 1400$

$x \geq 0, y \Rightarrow 0$ **1**



Now, draw the lines

AB : $10x + 5y = 1400$

CD : $6x + 10y = 1400$ **1**

Lines AB and CD meet at E(100, 80).

The feasible region has been shaded. The unbounded region has vertices C, E and B.

At C$\left(\frac{700}{3}, 0\right)$, the cost is

$Z = 2 \times \frac{700}{3} + 3 \times 0 = \frac{1400}{3}$ **1**

At E (100, 80), the cost is

$Z = 2 \times 100 + 3 \times 80 = 440$

At B(0, 280), the cost is

$Z = 2 \times 0 + 3 \times 280 = 840$

Clearly, 440 is the minimum of the three.

Thus, 100 kg of type I and 80 kg of type II fertilisers should be used to meet the desired requirement and minimum cost. **1**

Value : Fertilizers should not be used to increase yield and minimise cost as they are harmful for our health.

26. Consider the following events

E = A hits the target.

F = B hits the target

and G = C hits the target

We have, $P(E) = \frac{4}{5}$, $P(F) = \frac{3}{4}$ and $P(G) = \frac{2}{3}$

(i) We have,

Required probability $= P(E \cap F \cap G)$

Required probability $= P(E)\,P(F)\,P(G)$

{$\because$ A,B, C are independent events}

$\Rightarrow$ Required probability $= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$ **1**

(ii) We have,

Required probability $= P(\overline{E} \cap F \cap G)$

$\therefore$ Required probability $= P(\overline{E})\, P(F)\, P(G)$

[$\because$ A, B, C are independent events]

Required probability $= \left(1 - \frac{4}{5}\right) \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{10}$ **1**

(iii) We have,

$\Rightarrow$ Required probability $= P(E \cap F \cap \overline{G})$

$\cup (\overline{E} \cap F \cap G) \cup (E \cap \overline{F} \cap G)$ **1**

$\Rightarrow$ Required probability $= P(E)\, P(F)\, P(\overline{G})$

$P(\overline{E})\, P(F)\, P(G) + P(E)\, P(\overline{F})\, P(G)$ **1**

$\Rightarrow$ Required probability

$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{1}{5} \times \frac{3}{4} \times \frac{2}{5} \times \frac{4}{5} + \frac{1}{4} \times \frac{2}{3} = \frac{13}{30}$ **1**

(iv) Required probability $= P(\overline{E} \cap \overline{F} \cap \overline{G})$

$= P(\overline{E})\, P(\overline{F})\, P(\overline{G})$

$= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$ **1**

■■■

S O L U T I O N S

SAMPLE
QUESTION PAPER - 10

Self Assessment

Time: 3 Hours **Maximum Marks: 100**

SECTION — A

1. Given, $|\vec{a}| = l, |\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = \sqrt{3}$

$\Rightarrow |\vec{a}||\vec{b}| \sin\theta = \sqrt{3}$

$\left[\because \vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta \cdot \hat{n} \text{ and } |\hat{n}| = 1\right]$

$\Rightarrow 1 \times 2 \times \sin\theta = \sqrt{3}$

$\left[\because |\vec{a}| = 1 \text{ and } |\vec{b}| = 2\right]$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$

Hence, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. **1**

2. Given,

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix}\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing the corresponding elements, we get

$8 + y = 0$ and $2x + 1 = 5$

$\Rightarrow \quad y = -8$ and $x = \frac{5-1}{2} = 2$

$\therefore \quad x - y = 2 - (-8) = 10$

3. Ellipse is

$16x^2 + 9y^2 = 400$ ...(1)

Differentiating (1) w.r.t. x, we get

$32x + 18y\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{16}{9}\cdot\frac{x}{y}$ ...(2)

By hypothesis,

$\frac{dy}{dt} = -\frac{dx}{dt}$

$\Rightarrow \quad \frac{dy}{dx} = -1$ ...(3)

$\therefore \quad \frac{16}{9}\frac{x}{y} = 1,$ [By (2) and (3)]

$\Rightarrow \quad x = \frac{9}{16}y,$ ...(4) ½

$\therefore$ From (1), we get

$16\left(\frac{9}{16}y\right)^2 + 9y^2 = 400$

$\Rightarrow \quad \left(\frac{81}{16} + 9\right)y^2 = 400$

$\Rightarrow \quad y^2 = \frac{400 \times 16}{225} = \frac{16 \times 16}{9}$

$= \left(\frac{16}{3}\right)^2$

$\Rightarrow \qquad y = \pm\dfrac{16}{3}$

From (4), we get

$x = \pm 3$

$\therefore$ At the points $\left(3, \dfrac{16}{3}\right)$ and $\left(-3, -\dfrac{16}{3}\right)$, ellipse behaves as desired. **½**

4. Direction ratio's are 0 – 1, 1 – 0, 1 – 0

i.e., – 1, 1, 1

$\because \quad \sqrt{(-1)^2+1^2+1^2} = \sqrt{3}.$

$\therefore$ Direction cosines are $\dfrac{-1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}$. **1**

5. $y = e^x + 1$

Differentiating w.r.t. x,

$y' = e^x + 0 \qquad ...(1)$

Again differentiating

$y'' = e^x \qquad ...(2)$

Subtracting (1) and (2),

$y'' - y' = e^x - e^x = 0$

$\Rightarrow$ Differential equation is $y'' - y' = 0$. **1**

6. We have, $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}-2p\hat{j}+3\hat{k}$ are two parallel vectors, so their direction ratios will be proportional.

$$\therefore \quad \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$$

$$\Rightarrow \quad \frac{2}{-2p} = \frac{3}{1}$$

$$\Rightarrow \quad -6p = 2$$

$$\Rightarrow \quad p = \frac{2}{-6}$$

$$p = -\frac{1}{3}$$

SECTION — B

7. We have,

$$x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

and $$x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$(x + y) + (x - y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad \textbf{1}$$

$$2x = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} \qquad \textbf{1}$$

$$\Rightarrow \quad 2x = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\Rightarrow \quad x = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} \qquad \textbf{1}$$

and, $$(x + y) - (x - y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$\Rightarrow \quad 2y = \begin{bmatrix} 7-3 & 0+0 \\ 2+0 & 5-3 \end{bmatrix} \qquad \textbf{1}$$

$$\Rightarrow \quad 2y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow \quad y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Thus, $$x = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

and $$y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \qquad \textbf{1}$$

8. Consider $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$$= \begin{vmatrix} 2(b+c) & 2(c+a) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \qquad \textbf{1}$$

[By performing $R_1 \to R_1 + (R_2 + R_3)$]

$$= 2\begin{vmatrix} b+c & c+a & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

[By taking 2 common from R_1]

$$= 2\begin{vmatrix} b+c & c+a & a+b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

[By performing $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$= \begin{vmatrix} 0 & c & b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix} \qquad \textbf{1}$$

[By performing $R_1 \to R_1 + (R_2 + R_3)$]

Performing $C_2 \to bC_2$ and $C_3 \to cC_3$ and dividing the determinant by bc, we get

$$= \frac{2}{bc}\begin{vmatrix} 0 & bc & bc \\ -c & 0 & -ac \\ -b & -ab & 0 \end{vmatrix}$$

$$= \frac{2}{bc}\begin{vmatrix} 0 & 0 & bc \\ -c & ab & -ac \\ -b & -ab & 0 \end{vmatrix} \quad \mathbf{1}$$

[By performing $C_2 \to C_2 - C_3$]

$$= \frac{2}{bc}[bc\,(abc + abc)]$$

$$= 4abc = \text{R.H.S.} \quad \mathbf{1}$$

Or

$$\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$$

$$= \begin{vmatrix} A & x & zy \\ B & y & zx \\ C & z & xy \end{vmatrix} \quad \mathbf{1}$$

$R_1 \to xR_1$, $R_2 \to yR_2$, $R_3 \to zR_3$

$$= \frac{1}{xyz}\begin{vmatrix} Ax & x^2 & xyz \\ By & y^2 & xyz \\ CZ & z^2 & xyz \end{vmatrix} \quad \mathbf{1}$$

$$= \frac{xyz}{xyz}\begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$$

$$= \Delta \quad \mathbf{1}$$

$\therefore \quad \Delta_1 = \Delta$ **Hence, proved.** **1**

9. We have $y = e^{a\cos^{-1}x}$

$$\therefore \quad \frac{dy}{dx} = e^{a\cos^{-1}}\frac{-a}{\sqrt{1-x^2}} \quad \mathbf{1}$$

$$\Rightarrow \sqrt{1-x^2}\,y_2 + y_1.\frac{1}{2}.\frac{1}{\sqrt{1-x^2}}(-2x) \quad \mathbf{1}$$

$$= \frac{a^2e^{a\cos^{-1}x}}{\sqrt{1-x^2}}$$

$\Rightarrow \quad (1 - x^2)y_2 + xy_1 = a^2y$

$\Rightarrow \quad (1 - x^2)y_2 + xy_1 - a^2y = 0.$ **1**

$\Rightarrow \quad (1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0.$ **Proved** **1**

10. The given equation may be written as

$$x\sqrt{1+y} = -y\sqrt{1+x}$$

Squaring both sides, we get

$$x^2(1 + y) = y^2(1 + x) \quad \mathbf{1}$$

$\Rightarrow \quad x^2 - y^2 = y^2x - x^2y$

$\Rightarrow \quad (x + y)(x - y) = -xy(x - y)$

$\Rightarrow \quad x + y = -xy, \quad [\because x - y \neq 0]$

$\Rightarrow \quad x = -y - xy$ **1**

$\Rightarrow \quad y(1 + x) = -x$

$$\therefore \quad y = -\frac{x}{1+x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{(1+x).1 - x(0+1)}{(1+x)^2}\right] \quad \mathbf{1}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{(1+x)^2} \quad \mathbf{1}$$

11. $x = \sin 3t,\ y = \cos 2t$

$$\frac{dx}{dt} = 3\cos 3t$$

$$\frac{dy}{dt} = -2\sin 2t$$

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} \quad \mathbf{1}$$

$$= \frac{-2\sin 2t}{3\cos 3t}$$

$$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \frac{-2\sin 2.\frac{\pi}{4}}{3\cos 3.\frac{\pi}{4}}$$

$$= \frac{-2\sin\frac{\pi}{4}}{3\cos 3\frac{\pi}{4}}$$

$$= \frac{-2\times 1}{-3\times\frac{1}{\sqrt{2}}}$$

$$= \frac{2\sqrt{2}}{3} \quad \mathbf{1}$$

$$(y - \cos 2t) = \frac{2\sqrt{2}}{3}(x - \sin 3t)$$

$$y - \cos 2.\frac{\pi}{4} = \frac{2\sqrt{2}}{3}\left(x - \sin\frac{3\pi}{4}\right)$$

$$y = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right) \quad \mathbf{1}$$

$$y = \frac{2\sqrt{2}}{3}\left(\frac{\sqrt{2}x - 1}{\sqrt{2}}\right)$$

$$3\sqrt{2}\,y = 4x - 2\sqrt{2} \quad \mathbf{1}$$

$$4x - 3\sqrt{2}\,y - 2\sqrt{2} = 0$$

Or

Let at point (x, y), tangent is parallel to x-axis.

$y = 2x^3 - 15x^2 + 36x - 21$ ...(i)

$\therefore$ Slope of tangent

$= \left(\frac{dy}{dx}\right)_{(x,y)}$ **1**

$= 6x^2 - 30x + 36$

$\because$ tangent is parallel to x-axis

$\therefore \quad m_{\text{tangent}} = m_{x\text{-axis}}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x,y)} = 0$

$\Rightarrow \quad 6x^2 - 30x + 36 = 0$

$\Rightarrow \quad 6(x^2 - 5x + 6) = 0$

$\Rightarrow \quad (x-2)(x-3) = 0$

$\Rightarrow \quad x = 2, x = 3$ **1**

Putting these values of x in equation (i),

Putting $x = 2$,

$y = 2(2)^3 - 15(2)^2 + 36(2) - 21$
$= 16 - 60 + 72 - 21$
$= 7$

Putting $x = 3$,

$y = 2(3)^3 - 15(3)^2 + 36(3) - 21$
$= 54 - 135 + 108 - 21$ **1**
$= 16$

$\therefore$ At points (2, 7) and (3, 16), the tangents are parallel to x-axis.

Equation of tangent at (2, 7) is

$y - y_1 = m(x - x_1)$

$\Rightarrow \quad (y - 7) = 0\,(x - 2)$

$\Rightarrow \quad y = 7$

Equation of tangent at (3, 16) is

$y - y_1 = m(x - x_1)$

$y - 16 = 0\,(x - 3)$ **1**

$\Rightarrow \quad y = 16$

12. $I = \int \sin x\sqrt{1 + (2\cos^2 x - 1)}\,dx$

$= \int \sin x\sqrt{2}\cos x\,dx$ **1**

$= \sqrt{2}\int \sin x \cos x\,dx$

Let $\sin x = t \Rightarrow \cos x\,dx = dt$ **1**

$I = \sqrt{2}\int t\,dt + C = \sqrt{2}.\frac{t^2}{2} + C$ **1**

$= \frac{t^2}{2} + C = \frac{\sin^2 x}{2} + C$ **1**

Or

$I = \int \frac{x^2 + x + 1}{(x+2)(x+1)^2}dx$

Let $\int \frac{x^2 + x + 1}{(x+2)(x+1)^2}dx =$

$\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$ **1**

Comparing the numerators on both sides, we have

$(x^2 + x + 1) = A(x + 1)(x + 2) + B(x + 2) + C(x + 1)^2$ ...(1)

Putting $x = -1$ in (1), we get

$1 - 1 + 1 = A(0)(1) + B(1) + C(0)^2$

$\Rightarrow \quad B = 1$ ...(2) **1**

Putting $x = -2$ in (1), we get

$4 - 2 + 1 = A(-1)(0) + B(0) + C(-1)^2$

$\Rightarrow \quad 3 = C$

Comparing the coefficients of x^2 on both sides of (1)

$1 = A + C \Rightarrow A = 1 - C = 1 - 3$ **1**

$A = -2$

Thus, $I = \int \frac{-2}{x+1}dx + \int \frac{dx}{(x+1)^2} + \int \frac{-3}{x+2}dx$

$= -2\log|x + 1| - \frac{1}{x+1} + 3\log|x + 2| + C.$ **1**

13. LHS $= \cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18$

$= \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$ **½**

$= \tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}} + \tan^{-1}\frac{1}{18}$ **1**

$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18}$ **½**

$= \tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\cdot\frac{1}{18}} = \tan^{-1}\frac{65}{195}$

$= \tan^{-1}\frac{1}{3}$ **1½**

$= \cot^{-1}3 =$ RHS **½**

Or

$I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

$= \int \frac{\frac{5}{2}(2x+4) - 7}{\sqrt{x^2+4x+10}}dx$ **1**

$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{1}{\sqrt{(x+2)^2(\sqrt{6})^2}}dx$ **1**

$= 5\sqrt{x^2+4x+10}$

$-7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + C$ **2**

[CBSE Marking Scheme 2011]

14. $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = (\sin^{-1} x + \cos^{-1} x)^2 - 2\sin^{-1} x \cos^{-1} x$ **½**

$$= \left(\frac{\pi}{2}\right)^2 - 2\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right) \quad \textbf{½}$$

$$= \frac{\pi^2}{4} - \pi\sin^{-1} x + 2(\sin^{-1} x)^2$$

$$= 2\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x + \frac{\pi^2}{8}\right]$$

$$= 2\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] \quad \textbf{1}$$

$\therefore$ least value $= 2\left[\frac{\pi^2}{16}\right] = \frac{\pi^2}{8}$ **1**

and greatest value $= 2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$

$$= \frac{5\pi^2}{4} \quad \textbf{1}$$

15. Given, $x(1 + y^2)\, dx - y(1 + x^2)\, dy = 0$

$\Rightarrow \quad x(1 + y^2)\, dx = y(1 + x^2)\, dy$

$$\frac{x}{1+x^2}\, dx = \frac{y}{1+y^2}\, dy \quad \textbf{1}$$

Integrating both sides,

$$\int \frac{x}{1+x^2}\, dx = \int \frac{y}{1+y^2}\, dy$$

Let $1 + x^2 = t$ and $1 + y^2 = u$

$dx\,(2x) = dt,\ 2y\, dy = du$

$\therefore$ Given equation becomes

$$\frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\int \frac{du}{u} \quad \textbf{1}$$

$\log(t) = \log(u) + C$

$\log(1 + x^2) = \log(1 + y^2) + C$

Given $y = 0$ at $x = 1$

$\log(1 + 1) = \log 1 + C$ **1**

$\log 2 = C$

$\therefore$ Required equation becomes

$\log(1 + x^2) = \log(1 + y^2) + \log 2.$ **1**

16. The green matrix equation can be written as

$$\begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix} \quad \textbf{1}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix} \quad \textbf{2}$$

$\Rightarrow$ $2x + 3 = 9,\ 2z - 3 = 15,\ 2y = 12$

and $2t + 6 = 18$

$\Rightarrow$ $x = 3, z = 9,$ y $= 6$ and $t = 6.$

17. We know that

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}| \cos\theta$

Here $\vec{a}\cdot\vec{b} = \frac{1}{2}, |\vec{a}| = |\vec{b}|$

and $\alpha = 60^\circ$ **1**

$\therefore$ $\frac{1}{2} = |\vec{a}||\vec{b}| \cos 60^\circ$

$\Rightarrow$ $\frac{1}{2} = |\vec{a}|^2 \left(\frac{1}{2}\right)$ **1**

$\therefore$ $|\vec{a}|^2 = 1$

[$\because |\vec{a}| = |\vec{b}|$, given]

$\Rightarrow$ $|\vec{a}| = 1$

$|\vec{b}| = |\vec{a}| = 1$ **1**

Thus, $|\vec{a}| = 1, |\vec{b}| = 1$ **1**

18. Given, $n = 6$ (No. of trials)

x be a binomial variate

It satisfies the relation,

$9P(x = 4) = P(x = 2)$

For an equation,

$P(x = 4) = MC_r (p)^r (q)^{n-r}$ **1**

p = probability of succese.

$q = 1 - p$

$P(x = 4) = 6C^4 (p)^4 (1 - p)^2$ ...(i) **1**

Again, $P(x = 2) = MC_r (p)^r (q)^{n-r}$

$P(x = 2) = 6C_2 (p)^2 (q)^2$ ...(ii) **1**

By equations (i) & (ii),

$9[6C_4 p^4 (1 - p)^2] = [6C_2 p^2 q^4]$

$9p^2 = (1 - p)^2$

$\Rightarrow$ $3p = 1 - p$

$p = \frac{1}{4}$ **1**

19. Equation of the given lines can be written in standard form as

$$l_1 : \frac{x-1}{-3} = \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2}$$

and $$l_2 : \frac{x-1}{-\frac{3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \quad \textbf{1}$$

Direction ratios of these lines are $-3, \frac{p}{7}, 2$ and $-\frac{3p}{7}, 1, -5,$ respectively. **1**

We know that, two lines of direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular to each other, if

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore\ (-3)\left(\frac{-3p}{7}\right)+\left(\frac{p}{7}\right)(1)+(2)(-5) = 0$$

$$\Rightarrow \quad \frac{9p}{7}+\frac{p}{7} - 10 = 0$$

$$\Rightarrow \quad \frac{10p}{7} = 10 \Rightarrow p = 7 \qquad \mathbf{1}$$

Thus, the value of p is 7.

Also, we know that, the equation of a line which passes through the point (x_1, y_1, z_1) with direction ratios a, b, c is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Hence, required line is parallel to line 4.

So, $\quad a = -3, b = \frac{7}{7} = 1$ and $c = 2$

Now, equation of line passing through the point (3, 2, – 4) and having direction ratios (– 3, 1, 2) is

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$

$$\frac{3-x}{3} = \frac{y-2}{1} = \frac{z+4}{2} \qquad \mathbf{1}$$

SECTION — C

20. $f : A \to B'$ where $A = R - \{3\}$, $B = R - \{1\}$, f is defined by

$$f(x) = \frac{x-2}{x-3}$$

(a) $$f(x_1) = \frac{x_1-2}{x_1-3}, f(x_2) = \frac{x_2-2}{x_2-3} \qquad \mathbf{1}$$

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \qquad \mathbf{1}$$

or $\quad (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

or $\quad x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

i.e., $\quad -3x_1 - 2x_2 = -2x_1 - 3x_2$

or $\quad -x_1 = -x_2 \qquad \mathbf{1}$

$\Rightarrow \quad x_1 = x_2 \qquad \mathbf{1}$

$\therefore\ f$ is one-one.

(b) Let $$y = \frac{x-2}{x-3}, xy - 3y = x - 2$$

or $$x(y-1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1} \qquad \mathbf{1}$$

$\Rightarrow$ For every value of y except $y = 1$, there is a pre-image.

$$x = \frac{3y-2}{y-1} \qquad \mathbf{1}$$

$\Rightarrow f$ is onto.

Thus, f is one-one and onto.

21. Given, Region is $\{(x, y) : y^2 \geq 6x, x^2 + y^2 \leq 16\}$

Above region has two equation

$$y^2 = 6x \qquad \text{...(i)}$$

Which is a parabola with vertex (0, 0) and axist along X-axis

and $$x^2 + y^2 = 16 \qquad \text{...(ii)}$$

Which is a circle with centre (0, 0) and radius $r = 4$.

On putting $y^2 = 6x$ from Eq. (i) in Eq. (ii) we get

$$x^2 + 6x - 16 = 0$$

$$\Rightarrow x^2 + 8x - 2x - 16 = 0$$

$$\Rightarrow x(x+8) - 2(x+8) = 0$$

$$\Rightarrow (x-2)(x+8) = 0$$

$$\Rightarrow x = -8 \text{ or } 2$$

When $x = -8$, then $y^2 = 6(-8) = -48$ which is not possible. $\qquad \mathbf{1}$

So, $\quad x = -8$ is rejected

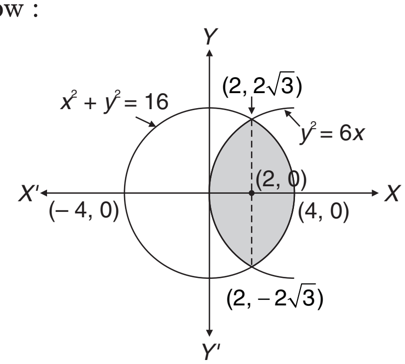
and when $x = 2$, then $y^2 = 6x = 6(2) = 12$

$$\Rightarrow \quad y = \pm\sqrt{12}$$

$$\Rightarrow \quad y = \pm 2\sqrt{3}$$

So, points of intersection are $(2, 2\sqrt{3})$ and $(2, -2\sqrt{3})$. $\qquad \mathbf{1}$

Now, draw the graph of given curve as given below :



$\therefore$ Required area

$$= 2\left[\int_0^2 y(\text{parabola}) + \int_2^4 y(\text{circle})\right]dx$$

$$= 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16-x^2}\,dx\right] \qquad \mathbf{1}$$

$$= 2\sqrt{6}\int_0^2 \sqrt{x}\,dx + 2\int_2^4 \sqrt{16-x^2}\,dx$$

$$= 2\sqrt{6}\left[x^{3/2}.\frac{2}{3}\right]_0^2 + 2\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$$ **1**

$$\left[\because \int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$= \frac{4}{3}\sqrt{6}\left[2^{3/2}\right] + 2\left[0 + \frac{16}{2}\sin^{-1}1 - \sqrt{16-4} - \frac{16}{2}\sin^{-1}\frac{2}{4}\right]$$ **1**

$$= \frac{4}{3}\sqrt{6}\cdot 2\sqrt{2} + 2\left[8\sin^{-1}\left(\sin\frac{\pi}{2}\right) - \sqrt{12} - 8\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right]$$

$$\left[\because 1 = \sin\frac{\pi}{2} \text{ and } \frac{1}{2} = \sin\frac{\pi}{6}\right]$$

$$= \frac{8}{3}\sqrt{3}\times\sqrt{2}\times\sqrt{2} + 2\left[4\pi - 2\sqrt{3} - 8\times\frac{\pi}{6}\right]$$

$$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{16\pi}{6}$$

$$= \frac{16\sqrt{3}}{3} + \frac{32\pi}{6} - 4\sqrt{3}$$

$$= \frac{16\sqrt{3}}{3} + \frac{16\pi}{3} - 4\sqrt{3}$$

$$= \frac{4\sqrt{3}}{3} + \frac{16\pi}{3} \text{ sq. units}$$

22. Let $I = \int_0^1 (\tan^{-1} x)^2.x\,dx$

$$I = \left[(\tan^{-1}x)^2.\frac{x^2}{2}\right]_0^1 - \int_0^1 2\tan^{-1}x.\frac{1}{1+x^2}\frac{x^2}{2}\,dx$$ **1**

$$I = \frac{\pi^2}{32} - \int_0^1 \tan^{-1}x.\frac{x^2}{1+x^2}\,dx$$ **½**

$[x = \tan\theta \Rightarrow dx = \sec^2\theta\,d\theta]$ **½**

$$I = \frac{\pi^2}{32} - \int_0^{\pi/4}\theta.\tan^2\theta\,d\theta$$

$$I = \frac{\pi^2}{32} - \int_0^{\pi/4}\theta.\sec^2\theta\,d\theta + \int_0^{\pi/4}\theta.d\theta$$ **1**

$$I = \frac{\pi^2}{32} - [\theta\tan\theta]_0^{\pi/4}\int_0^{\pi/4}\tan\theta\,d\theta + \left[\frac{\theta^2}{2}\right]_0^{\pi/4}$$ **1**

$$I = \frac{\pi^2}{32} - \frac{\pi}{4} + [\log\ \sec\theta]_0^{\pi/4} + \frac{\pi^2}{32}$$ **1**

$$I = \frac{2\pi^2}{32} - \frac{\pi}{4} + \frac{1}{2}\log 2$$

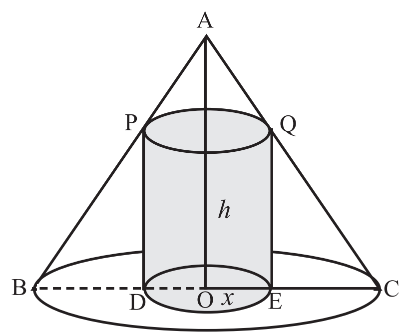
or $\dfrac{\pi^2 - 4\pi}{16} + \dfrac{1}{2}\log 2$ **1**

23. Let r = OC be the radius of the cone and h = OA be its height. Let a cylinder with radius of base circle OE = x be inscribed in the given cone. The height QE of the cylinder is given

$$\frac{QE}{QA} = \frac{EC}{OC} \quad (\text{since } \Delta QEC \sim \Delta AOC)$$

$$\Rightarrow \quad \frac{QE}{h} = \frac{r-x}{r}$$

$$QE = \frac{h(r-x)}{r}$$ **1**



1

Let S = S(x) be the curved surface of the cylinder, then

$$S_{(x)} = \frac{2\pi h}{r}(rh - x^2)$$

$$\Rightarrow \quad S'_{(x)} = \frac{2\pi h}{r}(r - 2x)$$ **1**

$$S''_{(x)} = \frac{4\pi h}{r}$$

Now $S'_{(x)} = 0$ give $x = \dfrac{r}{2}$ **1**

Since S''(x) < 0 for all x, and in particular at $x = \dfrac{r}{2}$ it follows that $x = \dfrac{r}{2}$ is a point of maxima of S. **1**

Hence, the radius of cylinder of greatest curved surface are a which can be inscribed in a given cone is half of that of the cone. **1**

24. Let us define the events as

E_1 : Motorcycle manufactured in plant I

E_2 : Motorcycle manufactured in plant II

A : Motorcycle of standard quality **1**

Now, $P(E_1) = \frac{70}{100} = \frac{7}{10}$

$P(A/E_1) = \frac{80}{100} = \frac{8}{10}$

$P(E_2) = \frac{30}{100} = \frac{3}{10}$

$P(A/E_2) = \frac{90}{100} = \frac{9}{10}$ **2**

The probability that the plant II manufactured a standard quality of motorcycle,

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$ **1**

$$= \frac{\frac{3}{10}\times\frac{9}{10}}{\frac{7}{10}\times\frac{8}{10}+\frac{3}{10}\times\frac{9}{10}}$$ **1**

$$= \frac{27}{56+27} = \frac{27}{83}$$ **1**

25. The two planes are

$4x + 8y + z - 8 = 0$ ...(1)

$y + z - 4 = 0$ ...(2)

The direction ratio of normal of the planes (1) are 4, 8, 1.

The direction ratio of normal of the planes (2) are 0, 1, 1.

These planes are not parallel. **1**

Since, $\frac{4}{0} \neq \frac{8}{1} \neq \frac{1}{1}$

The planes are not perpendicular.

Since $a_1a_2 + b_1b_2 + c_1c_2 = 4.0 + 8.1 + 1.1 \neq 0$ **1**

$\therefore$ Planes are not perpendicular.

The angle θ between them is given by

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$$ **1**

$$= \frac{4.0+8.1+1.1}{\sqrt{4^2+8^2+1^2}\sqrt{0+1^2+1^2}}$$ **1**

$$= \frac{8+1}{\sqrt{16+64+1}\sqrt{1+1}}$$ **1**

$$= \frac{9}{\sqrt{81}\sqrt{2}} = \frac{9}{9\sqrt{2}} = \frac{1}{\sqrt{2}}$$ **1**

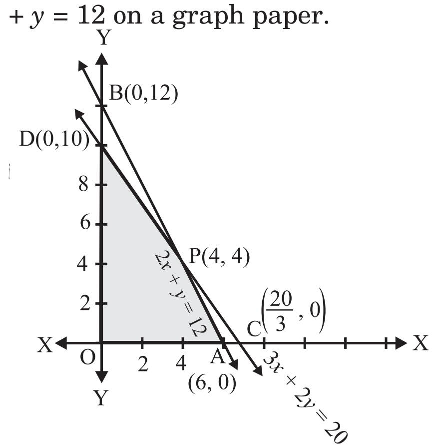
26. Let the manufacturer produces x pedestal lamps and y wooden shades; then the time taken by x pedestal lamps and y wooden shades on grinding/cutting machines = $(2x + y)$ hours and time taken by x pedestal lamps and y shades on the sprayer = $(3x + 2y)$ hours.

Since grinding/cutting machine is available for at the most 12 hours, $2x + y \leq 12$ and sprayer is available for at the most 20 hours. We have,

$$3x + 2y \leq 20$$

Profit from the sale of x lamps and y shades $Z = 5x + 3y$. **1**

So, our problem is to maximize $Z = 5x + 3y$ subjects to constraints $3x + 2y \leq 20$, $2x + y \leq 12$, $x, y \geq 0$. Plot the equations $3x + 2y = 20$, $2x + y = 12$ on a graph paper. **1**



1

(i) The line $2x + y = 12$ passes through A(6, 0) and B(0, 12)

Putting $x = 0$, $y = 0$ in $2x + y \leq 12$, we get $0 \leq 12$, which is true.

$\Rightarrow 2x + y \leq 12$ lies on and below AB.

(ii) The line $3x + 2y = 20$ passes through C $\left(\frac{20}{3}, 0\right)$ and D(0, 10). **1**

Putting $x = 0$, $y = 0$ in $3x + 2y \leq 20$, we get $0 \leq 20$, which is true.

$\Rightarrow$ The region $3x + 2y \leq 20$ lies on and below CD.

(iii) $x \geq 0$, is the region on and to the right of y-axis.

(iv) $y \geq 0$, is the region on and above x-axis

The shaded area OAPD is the feasible region where P is intersection of AB and CD.

Now, AB : $2x + y = 12$ ...(1)

CD : $3x + 2y = 20$ ...(2) **1**

Multiplying (1) by 2 and subtract (2) from it

$x = 24 - 20 = 4$

From (1), $y = 12 - 2x = 12 - 8 = 4$

$\therefore$ Point P is (4, 4).

Now, $Z = 5x + 3y$

At A(6, 0) $Z = 30$

At P(4, 4) $Z = 20 + 12 = 32$

At D(0, 10) $Z = 0 + 30$

Thus, the manufacturer should produce 4 lamps at 4 shades to get a maximum profit of ₹ 32. **1**