CBSE Sample Paper-05
Mathematics
Class – XII

Time allowed: 3 hours | Maximum Marks: **100**

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. Is R defined on the set A={1,2,3,.....14,15} defined as R={(x,y):3x-y=0} reflexive?

2. Find the angle between the vectors $\vec{a} = 4i + 4j$ and $\vec{b} = 4i - 2j$.

3. Evaluate $\sin^{-1}\left(\frac{1}{6}\right) + \cos^{-1}\left(\frac{1}{6}\right)$?

4. If a matrix has 12 elements, what are the possible orders it can have?

5. Prove that A-A' is skew symmetric.

6. Find the values of x,y,z s.t $\begin{bmatrix} x-y & 0 & 0 \\ z & 6 & 0 \\ 0 & 0 & 2y \end{bmatrix}$ is a scalar matrix?

Section B

7. Solve: $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

8. Find a unit vector perpendicular to each of the vectors $\left(\vec{a} - \vec{b}\right)$ and $\left(\vec{a} + \vec{b}\right)$ where $\vec{a} = i + j + k$ and $\vec{b} = i + 2j + 3k$.

9. If A and B are independent events such that P(AUB)=0.6, P(A)=0.2. Find P(B)

10. The relation between the total cost y and the total output x is given by

$y = \frac{3x(x+7)}{x+5} + 5$. Prove that the marginal cost continuously falls as output increases.

11. Solve $\frac{dy}{dx} + \frac{2y}{3} = \frac{x}{\sqrt{y}}$

12. If $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and I is the identity matrix of order2 , show that

$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

13. Find the equations of the tangent and the normal to the curve $x = \cos t, y = \sin t$ *at* $t = \frac{\pi}{4}$.

14. Show that the relation R in the set $\mathbb{Z}$ of integers given by R={(a,b):7 divides a-b} is an equivalence relation.

15. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots}}}$ *prove that* $(2y-1)\frac{dy}{dx} = \frac{1}{x}$.

16. For any two vectors $\vec{a}$ *and* $\vec{b}$, *prove that* $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$.

17. Integrate $\int \frac{e^x}{e^{2x} - 4} dx$. Write ant points for promoting national integration.

18 Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2i + 2j - 3k) = 7, \vec{r}.(2i + 5j + 3k) = 9$ *and the point* $(2,1,3)$.

19. Find the equation of the plane passing through the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7,-7).

Show that the line $x = \frac{7-y}{3} = \frac{z+7}{2}$ lies on the plane.

Section C

20. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let ¾ be the probability that he knows the answer and ¼ be the probability that he guesses. Assume that a student who guesses the answer would answer correctly with probability 1/4. What is the probability that a student knows the answer, given that he has answered it correctly.

21. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $w.r.t.\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

22. Integrate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x)dx.$

23. Prove that the volume of the largest cone that can be inscribed in a sphere of radius a is 8/27 of the volume of the sphere.

24. Solve the following system of equations using matrix method

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

25. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per kg of vitamin A and 2 unit per kg of vitamin C. Food 1 costs Rs.50 per kg and Food 2 costs Rs.70 per kg. Using linear programming , find the minimum cost of such a mixture.

26. Draw a rough sketch of the region $\{(x, y) : y^2 < 4x\,,\, 4x^2 + 4y^2 \le 9\}$ and find the area enclosed.