# CBSE Sample Paper-06
# Mathematics
# Class – XII

Time allowed: 3 hours | Maximum Marks: 100

**General Instructions:**

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

## Section A

1. Prove that greatest integer function is neither one-one nor onto.
2. If A,B are symmetric matrices of the same order, prove that AB-BA is skew symmetric.
3. An operation * defined on Z+ is defined as a*b=a-b. Is * a binary operation? Justify.
4. If $A^{-1}=\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}, B^{-1}=\begin{bmatrix} 3 & 0 \\ -1 & 6 \end{bmatrix}$ find $(AB)^{-1}$.
5. Find the angle between the vectors $\vec{a}=3i+4j \ and \ \vec{b}=4i+3j$.
6. Without expanding prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix}=0$.

## Section B

7. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$
8. Find the vector and Cartesian equation of the plane which passes through the point (5,2,-4) and perpendicular to the line with direction ratios 2,3,-1.
9. The probability of solving a specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that (a) problem is solved (b) exactly one of them solves the problem.
10. Find the values of a and b such that the function defined by:

$$f(x)=\begin{cases} 5, x \le 2 \\ ax+b, 2 < x < 10 \\ 21, x \ge 10 \end{cases}$$ is continuous $\forall$ x.

11. If $F(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y)=F(x+y)$

12. Prove that $\frac{d}{dx}\left(\cos^{-1}\sqrt{\frac{1+x}{2}}\right)=\frac{-1}{2\sqrt{1-x^2}}$.

13. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by f(x)=2x-3 and $g:\mathbb{R}\to\mathbb{R}$ be defined by $g(x)=\frac{x+3}{2}$. Show that $f\circ g=I_R=g\circ f$.

14. Find the point on the curve $y=x^3-11x+5$ at which tangent is y=x-11.

15. The temperature T of a cooling object drops at a rate which is proportional to the difference T-S where S is the constant temperature of the surrounding medium. Thus, dT/dt=-c(T-S), where c>0. Solve the differential equation, given T(0)=40. Discuss two measures to prevent global warming.

16. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}$ *and* $\vec{b}\times\vec{c}=\vec{a}$; prove that $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular and $|\vec{b}|=1$ *and* $|\vec{c}|=|\vec{a}|$.

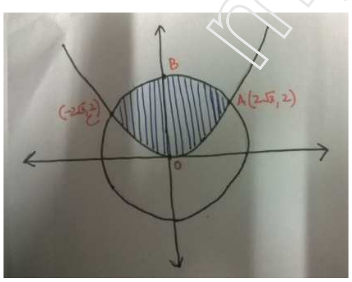
17. Integrate $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$.

18. Find the projection vector of 2i+3j-3k along 5j-k.

19. Check if the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ *and* $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ *are coplanar*.

**Section C**

20. Sketch the region common to the circle $x^2+y^2=16$ and the parabola $x^2=6y$. Also, find the area of the region using integration.




21. A factory has two machines A and B. Past record shows that the machine A produced 60% of the items of the output and machine B produced 40% of the items of the output. Further, 2% of the items produced by machine A and 1% of the items produced by machine B were defective. One

item is chosen at random. What is the probability that it was produced by machine B, given that it was defective?

22. Differentiate $\tan^{-1}\dfrac{2\sqrt{x}}{1-x}$ $w.r.t$ $\sin^{-1}\dfrac{2\sqrt{x}}{1+x}$ .

23. $\displaystyle\int_0^{\pi}\frac{xdx}{4\cos^2 x+9\sin^2 x}$

24. Prove that the semi vertical angle of the cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$.

25. The sum of three numbers is 6. If we multiply the third number by 3 and add second number to it we get 11. By adding first and third numbers we get double of the second number. Represent the following information mathematically and solve using matrices.

26. A factory manufactures two types of machines A and B. Each type is made of certain metal. The factory has only 480kgs of this metal available in a day. To manufacture machine A, 10 kgs of metal is required and 20kgsis required for B. Machine A and B require 15 and 10minutes to be painted. Painting department can use only 400 minutes in a day. The factory earns profit of 10,500 on machine A and 9000 on machine B. State as a linear programming problem and maximizes the profit.