

Sample Paper-04 (unsolved)**Mathematics****Class – XII**

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- All questions are compulsory.
- The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- Use of calculators is not permitted.

Section A

- Compute $\text{adj}(A)$, where $A = \begin{bmatrix} 3 & -4 \\ 5 & 7 \end{bmatrix}$.
- Find a vector in the direction of $\vec{a} = 6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ whose magnitude is 5.
- Show that $f(x) = x^2 + x$ is an odd function.
- Solve for x , $\begin{vmatrix} x & 3 \\ 5 & 2x \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ 5 & 3 \end{vmatrix}$.
- Evaluate $\sin(\tan^{-1}(x))$, $|x| < 1$.
- Find the value of C s.t. $A - B + 2C = 0$, where $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & -1 & -3 \end{bmatrix}$, $B = A = \begin{bmatrix} -2 & 5 & -1 \\ 3 & 0 & 8 \end{bmatrix}$

Section B

- Express the following matrix as the sum of a symmetric and skew-symmetric matrix:

$$\begin{bmatrix} 7 & 1 & 0 \\ 1 & -5 & 6 \\ 0 & 8 & -4 \end{bmatrix}$$

8. Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$.
9. Given a function defined by $y=f(x)=\sqrt{4-x^2}$, $0 \leq x \leq 2$, $0 \leq y \leq 2$, find the inverse of f.
10. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs. 1000 is deposited with this bank. How much will it worth after 10 years? ($e^{0.5}=1.648$)
11. Solve $\tan^{-1}(2x) - \tan^{-1}(3x) = \frac{\pi}{4}$.
12. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$, which is perpendicular to the plane $x-y+z=0$.
13. A class consists of 10 boys and 8 girls. Three students are selected at random. What is the probability that selected group has (i) all boys (ii) at least one girl (iii) at most one girl? List two harmful consequences of female feticide.
14. Determine if the function f defined by :

$$f(x) = \begin{cases} x^2 \sin x & x \neq 0 \\ 0 & x = 0 \end{cases}$$
 is continuous.
15. Show that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.
16. Find the area of a parallelogram having adjacent sides a and b with $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} + 4\hat{k}$.
17. Integrate $\int \sin 3x \cos 5x dx$.
18. Find the angle between the planes $x+2y+3z-4=0$ and $2x+y-z=5$.
19. Find the value of the constant S s.t. the scalar product of the vector $i+j+k$ with the unit vector parallel to the sum of the vectors $2i+4j-5k$ and $5i+2j+3k$ is equal to 1.

Section C

20. Solve the following system of equations:

$$3x + y - z = 1$$

$$x - y + 2z = -4$$

$$19x + 5y - 4z = 1$$

21. Show that the semi vertical angle of the right circular cone of given total surface and maximum volume is $\sin^{-1}(1/3)$.
22. Kellogs is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 mg of iron. Knowing that bran contains 80 grams of protein and 40 mg of iron per kg and rice contains 100 grams of protein and 30 mg of iron per kg, find the minimum cost of producing this cereal if bran costs Rs. 5 per kg and rice costs Rs. 4 per kg.
23. Find the area bounded by $x^2 = 4y$ and the line $x = 4y - 2$.
24. A man is known to speak the truth 3 out of 4 times. He throws a dice and reports that it is a five. Find the probability that it is actually a 5.
25. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$, show that $(2y - 1) \frac{dy}{dx} = \frac{1}{x}$
26. Evaluate $\int \frac{1 - x^2}{x(1 - 2x)} dx$