CBSE Sample Paper-05
Mathematics
Class – XII

Time allowed: 3 hours | **ANSWERS** | Maximum Marks: 100

Section A

1. Solution: NO. Let 1∈A, 3(1)-1=2≠0

$Thus, (1,1) \notin R$

2. Solution:

$\vec{a} = 4i + 4j \ and \ \vec{b} = 4i - 2j$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{16-8}{(4\sqrt{2})2\sqrt{5}} = \frac{8}{8\sqrt{10}} \Rightarrow \theta = \cos^{-1}\left(\frac{8}{8\sqrt{10}}\right)$$

3. Solution: $\frac{\pi}{2}$

4. Solution: (1,12),(2,6),(3,4),(4,3),(6,2),(12,1)

5. Solution:

$$(A-A')' = A' - (A')' = A' - A = -(A-A')$$

Thus, A-A' is skew symmetric.

6. Solution: A scalar matrix is a diagonal matrix with equal diagonal elements.

$$\therefore z = 0, 2y = 6, x - y = 6$$
$$\Rightarrow x = 9, y = 3, z = 0$$

Section B

7. Solution:

$$3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$$

$$3(2\tan^{-1}x) - 4(2\tan^{-1}x) + 2(2\tan^{-1}x) = \frac{\pi}{3}$$

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$$2\tan^{-1}x = \frac{\pi}{3}$$

$$\tan^{-1}x = \frac{\pi}{6}$$

$$x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

8. Solution:

$$\vec{a} = i + j + k\ ,\ \vec{b} = i + 2j + 3k$$

$$\therefore \vec{a}+\vec{b} = 2i+3j+4k, \vec{a}-\vec{b} = -j-2k$$

$$Vector \perp to\ \vec{a}+\vec{b}, \vec{a}-\vec{b}\ is\ \left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)$$

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right) = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2i+4j-2k = \vec{c}$$

$$\therefore \left|\vec{c}\right| = 2\sqrt{6}$$

$$Unit\ vector = \frac{-2i+4j-2k}{2\sqrt{6}}$$

9. Solution:

Since A and B are independent events $P(A\cap B) = P(A)P(B)$

$$\therefore P(A\cup B) = P(A)+P(B)-P(A\cap B) = 0$$

$$\Rightarrow 0.6 = 0.2 + P(B) - .2P(B)$$

$$\Rightarrow P(B) = \frac{.4}{.8} = \frac{1}{2}$$

10. Solution:

$$Cost = y = y = \frac{3x(x+7)}{x+5} + 5 = \frac{3x^2+21}{x+5} + 5$$

$$\therefore MC = \frac{dy}{dx} = \frac{(x+5)(6x)-(3x^2+21).1}{(x+5)^2} = \frac{3x^2+30x+105}{(x+5)^2} = 3 + \frac{30}{(x+5)^2}$$

$$\frac{d}{dx}(MC) = \frac{-60}{(x+5)^2} < 0$$

Thus, marginal cost is a decreasing function of output(x).

11. Solution:

$$\frac{dy}{dx}+\frac{2y}{3}=\frac{x}{\sqrt{y}}$$

$$\sqrt{y}\frac{dy}{dx}+\frac{2y^{3/2}}{3}=x$$

$$Let\ z=y^{3/2}\Rightarrow\frac{dz}{dx}=\frac{3}{2}\sqrt{y}\frac{dy}{dx}$$

$$\Rightarrow\frac{2}{3}\frac{dz}{dx}+\frac{2}{3}z=x$$

$$\Rightarrow\frac{dz}{dx}+z=\frac{3}{2}x$$

$$\therefore P=1,Q=\frac{3}{2}x$$

$$\therefore I.F=e^{\int Pdx}=e^{x}$$

Solution is :

$$ze^{x}=\int\frac{3}{2}xe^{x}dx$$

$$\Rightarrow y^{3/2}e^{x}=\frac{3}{2}xe^{x}-e^{x}+c$$

$$\Rightarrow y^{3/2}=\frac{3}{2}(x-1)+Ce^{-x}$$

12. Solution:

$$I-A=\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}0&-\tan\alpha/2\\\tan\alpha/2&0\end{bmatrix}=\begin{bmatrix}1&\tan\alpha/2\\-\tan\alpha/2&1\end{bmatrix}$$

$$(I-A)\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}=\begin{bmatrix}1&\tan\alpha/2\\-\tan\alpha/2&1\end{bmatrix}\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha+(\tan\alpha/2)\sin\alpha&-\sin\alpha+(\tan\alpha/2)\cos\alpha\\\sin\alpha-(\tan\alpha/2)\cos\alpha&\cos\alpha+(\tan\alpha/2)\sin\alpha\end{bmatrix}$$

$$=\begin{bmatrix}(2\cos^{2}\alpha/2-1)+(\tan\alpha/2)(2\sin\alpha/2\cos\alpha/2)&-(2\sin\alpha/2\cos\alpha/2)+(\tan\alpha/2)(2\cos^{2}\alpha/2-1)\\(2\sin\alpha/2\cos\alpha/2)-(\tan\alpha/2)(2\cos^{2}\alpha/2-1)&(2\cos^{2}\alpha/2-1)+(\tan\alpha/2)(2\sin\alpha/2\cos\alpha/2)\end{bmatrix}$$

$$=\begin{bmatrix}1&-\tan\alpha/2\\\tan\alpha/2&1\end{bmatrix}=I+A$$

13. Solution:

$$When\ t=\frac{\pi}{4},x=\frac{1}{\sqrt{2}},y=\frac{1}{\sqrt{2}}$$

$$\therefore \text{point of contact} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$y = \sin t \Rightarrow \frac{dy}{dt} = \cos t, x = \cos t \Rightarrow \frac{dx}{dt} = -\sin t$$
$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t} = -\cot t$$
$$\therefore \text{slope at } t = \frac{\pi}{4} = -\cot\frac{\pi}{4} = -1$$
$$\therefore \text{equation of tangent} = (y - \frac{1}{\sqrt{2}}) = -1(x - \frac{1}{\sqrt{2}}) \Rightarrow x + y - \sqrt{2} = 0$$
$$\text{Slope of normal} = m, m(-1) = -1 \Rightarrow m = 1$$
$$\therefore \text{equation of normal} = (y - \frac{1}{\sqrt{2}}) = 1(x - \frac{1}{\sqrt{2}}) \Rightarrow x - y = 0$$

14. Solution:

Reflexive:

Let a$\in \mathbb{Z}$

a-a=0, 7 divides 0.

$\therefore (a,a) \in R \forall a \in \mathbb{Z}$

Hence, R is reflexive.

Symmetric:

$$\text{Suppose } (a,b) \in R \Rightarrow 7/a - b \Rightarrow a - b = 7k \text{ f.s. } k \in \mathbb{Z}$$
$$\Rightarrow b - a = -7k = 7k' \text{ f.s. } k' \in \mathbb{Z}$$
$$\therefore (b,a) \in \mathbb{Z}$$

Transitive:

$$\text{Suppose } (a,b),(b,c) \in R$$
$$\Rightarrow b - a = 7m \text{ f.s. } m \in \mathbb{Z}, c - b = 7n \text{ f.s. } n \in \mathbb{Z}$$
$$\therefore c - a = c - b + b - a = 7(n - m), \text{where } n - m \in \mathbb{Z}$$
$$\therefore (a,c) \in R$$

Hence the above relation is an equivalence relation.

15. Solution:

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$$y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x}+\ldots}}$$
$$\Rightarrow y=\sqrt{\log x+y}$$
$$\Rightarrow y^2=\log x+y$$

Differentiating both sides w.r.t x, we get

$$2y\frac{dy}{dx}=\frac{1}{x}+\frac{dy}{dx}\Rightarrow(2y-1)\frac{dy}{dx}=\frac{1}{x}$$

16. Solution:

$$\left|\vec{a}+\vec{b}\right|^2=\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)$$
$$=\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}$$
$$=\left|\vec{a}\right|^2+2\vec{a}.\vec{b}+\left|\vec{b}\right|^2\le\left|\vec{a}\right|^2+2\left|\vec{a}.\vec{b}\right|+\left|\vec{b}\right|^2\le\left|\vec{a}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|+\left|\vec{b}\right|^2$$
$$=\left(\left|\vec{a}\right|+\left|\vec{b}\right|\right)^2$$
$$\therefore\left|\vec{a}+\vec{b}\right|\le\left|\vec{a}\right|+\left|\vec{b}\right|$$

17. Solution:

$$I=\int\frac{e^x}{e^{2x}-4}dx$$
$$Let\ e^x=t\Rightarrow e^xdx=dt$$
$$I=\int\frac{1}{t^2-4}dt=\int\frac{1}{(t-2)(t+2)}dt$$
$$=\frac{1}{4}\int\left(\frac{1}{t-2}-\frac{1}{t+2}\right)dt$$
$$=\frac{1}{4}(\log|t-2|-\log|t+2|)+c$$
$$=\frac{1}{4}\log\left|\frac{t-2}{t+2}\right|+c$$
$$=\frac{1}{4}\log\left|\frac{e^x-2}{e^x+2}\right|+c$$

1.We should not judge people by their religion.
2. Women should be empowered by educating them.

18 Solution:

$$\overrightarrow{n_1}=2i+2j-3k,d_1=7$$
$$\overrightarrow{n_2}=2i+5j+3k,d_2=9$$

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Equation of plane:

$$\vec{r}.(\vec{n_1}+\lambda\vec{n_2})=d_1+\lambda d_2$$
$$\vec{r}.(2i+2j-3k+\lambda(2i+5j+3k))=7+9\lambda$$
$$Let\ \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$$
$$\therefore x(2+2\lambda)+y(2+5\lambda)+z(-3+3\lambda)=7+9\lambda$$
$$Putting\ (x,y,z)=(2,1,3)\ we\ get\ \lambda=\frac{10}{9}$$
$$Substituting\ the\ value\ of\ \lambda\ we\ get,\vec{r}.(38i+68j+3k)=153$$

19. Solution:

Equation of any plane containing the given line is:

A(x+1)+B(y-3)+C(z+2)=0, where -3A+2B+C=0

Since , plane passes through (0,7,-7) we have,

A(1)+B(4)+C(-5)=0, i.eA+4B-5C=0

Solving the above equations for A, B,C we have

$$\frac{A}{-10-4}=\frac{B}{1-15}=\frac{C}{-12-2}\Rightarrow\frac{A}{1}=\frac{B}{1}=\frac{C}{1}=k$$

Thus, substituting the value of A, B,C we get the equation of the plane as x+y+z=0

Now, the line $x=\frac{7-y}{3}=\frac{z+7}{2}$ or $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ lies on the plane if it is parallel to the plane and any point on it lies on the plane.

Since, 1(1)+-3(1)+2(1)=0 $\Rightarrow$ line is parallel to the plane.

Again as (0,7,-7) lies on the plane, we conclude the line lies on the plane.

Section C

20. Solution:

Let E be the event that the student answered correctly.

Let E_1denote the event that the student knows the answer. Let E_2denote the event that the student guesses the answer.

$P(E_1/E)=?$

$P(E_1)=3/4$, $P(E_2)=1/4$

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$P(E/E_1)=1$, $P(E/E_2)=1/4$

Thus, by Baye's Theorem

$$P(E_1/E)=\frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$$
$$=\frac{(3/4)(1)}{(3/4)1+(1/4)(1/4)}=\frac{12}{13}$$

21. Solution:

$$Let\ y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), z=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$
$$Put\text{ in }g\ x=\tan\theta,$$
$$y=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$
$$=\tan^{-1}\left(\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}\right)=\tan^{-1}(\tan\theta/2)=\vartheta/2$$
$$z=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}(\sin 2\theta)=2\theta$$
$$\therefore \theta=z/2$$
$$\therefore y=\frac{1}{2}\left(\frac{z}{2}\right)=\frac{z}{4}$$
$$\frac{dy}{dz}=\frac{1}{4}$$

22. Solution:

$$I=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos 2x\log(\sin x)dx.$$
$$=\log(\sin x).\frac{\sin 2x}{2}\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1}{\sin x}\cos x.\frac{\sin 2x}{2}dx$$
$$=0-\log\left(\frac{1}{\sqrt{2}}\right)\frac{1}{2}-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1}{\cancel{\sin x}}\cos x.\frac{\cancel{2}\cancel{\sin x}\cos x}{\cancel{2}}dx$$
$$=-\frac{1}{2}\log\left(\frac{1}{\sqrt{2}}\right)-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1+\cos 2x}{2}dx=-\frac{1}{2}\log\left(\frac{1}{\sqrt{2}}\right)-\frac{\pi}{8}+\frac{1}{4}$$

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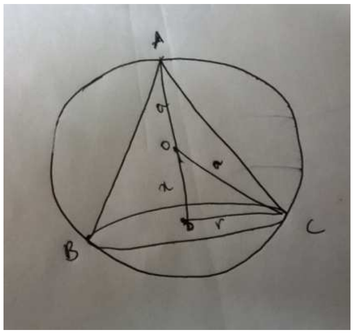
23. Solution:

Consider a sphere of radius a with center O s.t. OD=x and DC=r.



Let h be the height of the cone.

Then, h=OA+x=a+x

Also, $for\ \triangle ODC, x^2 + r^2 = a^2$

Let V be the volume of the cone.

$$\therefore V = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow V(x) = \frac{1}{3}\pi(a^2 - x^2)(a+x)$$

$$\therefore V'(x) = \frac{1}{3}\pi\left[(a^2 - x^2) + (a+x)(-2x)\right] = \frac{\pi}{3}(a+x)(a-3x)$$

$$Also, V''(x) = \frac{\pi}{3}\left[(a+x)(-3x) + (a-3x)1\right]$$

$$V'(x) = 0 \Rightarrow x = -a, \frac{a}{3}$$

$$Neglecting\ x = -a,$$

$$V''\left(\frac{a}{3}\right) = \frac{-4\pi a}{3} < 0$$

$$\therefore volume\ is\ \max imum\ when\ x = a/3.$$

$$\therefore h = a + \frac{a}{3} = \frac{4a}{3}$$

$$r^2 = a^2 - \frac{a^2}{9} = \frac{8a^2}{9}$$

$$\therefore Volume = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{8a^2}{9}\right)\left(\frac{4a}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi a^3\right)$$

24. Solution:

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Let $\frac{1}{x}=u,\frac{1}{y}=v,\frac{1}{z}=w$

$\therefore$ the system of equations becomes,

$$\begin{aligned}&2u+3v+10w=4\\&4u-6v+5w=1\\&6u+9v-20w=2\end{aligned}$$

$$Let\ A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix},b=\begin{bmatrix}4\\1\\2\end{bmatrix}$$

$$|A|=1200\neq 0, A^{-1}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}, U=A^{-1}b=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\1\\2\end{bmatrix}=\begin{bmatrix}1/2\\1/3\\1/5\end{bmatrix}$$

$\therefore u=1/2\Rightarrow x=2, v=1/3\Rightarrow y=3, w=1/5\Rightarrow z=5$

25. Solution:

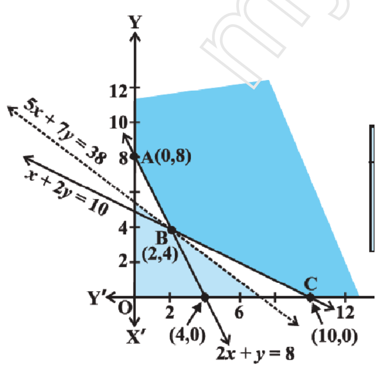
Suppose the mixture contains x kg of food 1 and y kgs of food2.

Then, Cost Z=50x + 70y

The mathematical formulation of the problem is as follows:

Min Z= 50x+70y

s.t
$$\begin{aligned}&2x+y\geq 8(requirement\ of\ VitA)\\&x+2y\geq 10(requirement\ of\ VitC)\\&x\geq 0, y\geq 0\end{aligned}$$



We graph the above inequalities. The feasible region as shown in the figure is unbounded. The corner points areA,B and C. The co-ordinates of the corner points are (0,8), (2,4),(10,0).

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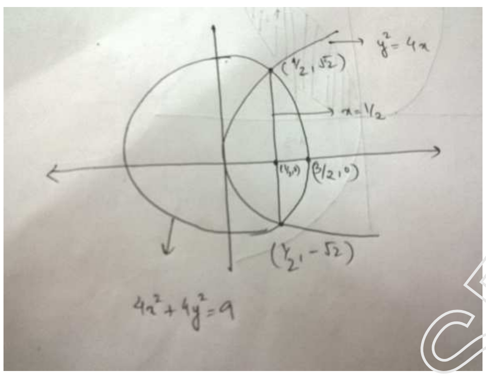
Corner Point	Z=50x +70y
(0,8)	560
(2,4)	**380**
(10,0)	500

Thus cost is minimized by mixing 2 units of food 1 and 4 units of food 2 and minimum cost is 380.

26. Solution:

The point of intersection of the curves $y^2=4x$, $4x^2+4y^2=9$:

$$4x^2+4(4x)=9 \Rightarrow 4x^2-16x-9=0 \Rightarrow (2x-1)(2x+9)=0$$
$$\Rightarrow x=1/2,-9/2$$
$$but\ x=-9/2\ is\ not\ a\ possible\ solution(\because y^2=-18, not\ possible)$$
$$\therefore x=1/2 \Rightarrow y=\pm\sqrt{2}$$



The shaded area is the required area.

$$Area=2\left[\int_0^{1/2} 2\sqrt{x}dx+\int_{1/2}^{3/2}\sqrt{\left(\frac{3}{2}\right)^2-x^2}dx\right]$$

$$=2\left[\left.\frac{2x^{3/2}}{3/2}\right|_0^{1/2}+\left.\left(\frac{x}{2}\sqrt{\left(\frac{9}{4}\right)-x^2}+\frac{9}{8}\sin^{-1}\left(\frac{x}{3/2}\right)\right)\right|_{1/2}^{3/2}\right]=\frac{9}{8}\pi-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)+\frac{5}{6\sqrt{2}}$$