**MATHEMATICS SAMPLE PAPER**

**SOLUTIONS**

**SECTION–A**

1. Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{7\times 2 + 1\times 6 + (-4)\times 3}{\sqrt{2+6^2+3^2}} = \frac{14+6-12}{\sqrt{49}} = \frac{8}{7}$$

2. Since $\vec{a}, \vec{b}$ and $\vec{c}$ vectors are coplanar.

$\therefore \quad \left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$

$\Rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \qquad \Rightarrow \quad 1(-3+\lambda) - 3(6+0) + 1(2\lambda + 0) = 0$

$\Rightarrow \quad -3 + \lambda - 18 + 2\lambda = 0 \qquad \Rightarrow \quad 3\lambda - 21 = 0$

$\Rightarrow \quad \lambda = 7$

3. Let l, m, n, be direction cosine of given line.

$\therefore \quad \boldsymbol{l} = \cos 90^\circ = 0; \qquad m = \cos 60^\circ = \frac{1}{2}$ and n = cos $\theta$ and n = cos $\theta$

$\because \boldsymbol{l}^2 + m^2 + n^2 = 1 \qquad \Rightarrow \quad 0 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{3}{4} \qquad \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ $\qquad$ ($\because$ $\theta$ is acute angle)

$\Rightarrow \qquad \theta = \frac{\pi}{6}$

4. $a_{23} = \dfrac{|2-3|}{2} = \dfrac{|-1|}{2} = \dfrac{1}{2}$

5. Given family of curve is $\quad v = \dfrac{A}{r} + B$

Differentiating with respect to r, we get

$$\frac{dv}{dr} = \frac{-A}{r^2}$$

Again differentiating with respect to r, we get

$\dfrac{d^2v}{dr^2} = \dfrac{2A}{r^3} \qquad \Rightarrow \dfrac{d^2v}{dr^2} = \dfrac{2}{r}.\dfrac{A}{r^2}$

$\Rightarrow \quad \dfrac{d^2v}{dr^2} = \dfrac{2}{r}.\left(-\dfrac{dv}{dr}\right) \qquad \Rightarrow r\dfrac{d^2v}{dr^2} = -2\dfrac{dv}{dr}$

$\Rightarrow r\dfrac{d^2v}{dr^2} = +2\dfrac{dv}{dr} = 0$

6. Give differential equation is

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1 \qquad \Rightarrow \left(\frac{e^{-2\sqrt{x}}-y}{\sqrt{x}}\right).\frac{dx}{dy}=1$$

$$\Rightarrow \frac{dx}{dy}=\frac{\sqrt{x}}{e^{-2\sqrt{x}}-y} \qquad \Rightarrow \frac{dx}{dx}=\frac{e^{-2\sqrt{x}}-y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}} \qquad \Rightarrow \frac{dy}{dx}+\frac{1}{\sqrt{x}}.y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$IF=e^{\int\frac{1}{\sqrt{x}}dx}=e^{\int x^{\frac{-1}{2}}dx}=e^{2\sqrt{x}}$$

**SECTION-B**

7. $$A=\begin{vmatrix}2&0&1\\2&1&3\\1&-1&0\end{vmatrix}$$

$$\therefore A^2=A\times A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}4+0+1&0+0-1&2+0+0\\4+2+3&0+1-3&2+3+0\\2-2-+0&0-1+0&1-3+0\end{bmatrix}=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}$$

Now, $$A^2-5A+4I=\begin{vmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{vmatrix}-5\begin{vmatrix}2&0&1\\2&1&3\\1&-1&0\end{vmatrix}+4\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}-1&-1&-3\\-1&-3&-10\\-5&4&2\end{bmatrix}$$

Now given $A^2-5A+4I+X=0$

$$\Rightarrow \begin{bmatrix}-1&-1&-3\\-1&-3&-10\\-5&4&2\end{bmatrix}+X=0$$

$$\Rightarrow X=-\begin{bmatrix}-1&-1&-3\\-1&-3&-10\\-5&4&2\end{bmatrix} \qquad \Rightarrow X=\begin{bmatrix}1&1&3\\1&3&10\\5&-4&-2\end{bmatrix}$$

OR

Given $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$

$$A' = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$|A'| = 1(-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$$

Hence, $(A')^{-1}$ will exist.

$A_{11} = \begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix} = -1-8 = -9;$ $\quad A_{12} = -\begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix} = -(-2-6) = 8$

$A_{13} = -\begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = -8+3 = -5;$ $\quad A_{21} = -\begin{vmatrix} 0 & -2 \\ 4 & 1 \end{vmatrix} = -(0+8) = -8$

$A_{22} = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 1+6 = 7;$ $\quad A_{23} = -\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = -(4-0) = -4$

$A_{31} = \begin{vmatrix} 0 & -2 \\ -1 & 2 \end{vmatrix} = 0-2 = -2;$ $\quad A_{32} = -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(2-4) = 2$

$A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix} = -10 = -1$

Adj(A’) $= \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^7 = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

$$(A')^{-1}\frac{1}{1}\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

8. Here f(x) = $\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

Taking a common from $C_1$,we get

f(x) = $\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

applying $C_2 \to C_2 + C_{1,}$ we get

f(x) =a$\begin{vmatrix} 1 & 0 & 0 \\ x & a+x & -1 \\ x^2 & ax+x^2 & a \end{vmatrix}$

Expanding along $R_1$, we get

$f(x) = a[1(a^2 + ax + ax + x^2) - 0 + 0\}$

$\Rightarrow f(x) = a(a^2 + 2ax + x^2)$

$\Rightarrow f(x) = a(a+x)^2$

Now, $f(2x)-f(x)=a(a+2x)^2-a(a+x)^2$

$=a\{(a+2x)^2-(a+x)^2\}=a(a+2x+a+x)(a+2x-a-x)$

$=ax(2a+3x)$

9. Here

$$I = \int \frac{1}{\sin x + \sin 2x} dx$$

$$\Rightarrow I = \int \frac{1}{\sin x + 2\sin x \cos x} dx \quad \Rightarrow I = \int \frac{1}{\sin x(1 + 2\cos x)} dx$$

$$\Rightarrow I = \int \frac{\sin x}{\sin^2 x(1 + 2\cos x)} dx \quad \Rightarrow I = \int \frac{\sin x}{(1 - \cos^2 x)(1 + 2\cos x)} dx$$

Let $\cos x = z \quad \Rightarrow - \sin x\, dx = dz$

$$\Rightarrow \quad I = \int \frac{-dz}{(1 - z^2)(1 + 2z)} \quad \Rightarrow \quad I = -\int \frac{dz}{(1 + z)(1 - z)(1 + 2z)}$$

Here , integrand is proper rational function. Therefore by the form of partial function , we can write

$$\frac{1}{(1 + z)(1 - z)(1 + 2z)} = \frac{A}{1 + z} + \frac{B}{1 - z} + \frac{C}{1 + 2z} \quad \text{.........(i)}$$

$$\Rightarrow \quad \frac{1}{(1 + z)(1 - z)(1 + 2z)} = \frac{A(1 - z)(1 + 2z) + B(1 + z)(1 + 2z) + C(1 + z)(1 - z)}{(1 + z)(1 - z)(1 - 2z)}$$

$1= A(1-z)( 1+2z) +B(1+z)(1+2z) +C(1+z)( 1 - z )$ .........(ii)

Putting the value of z=-1 in (ii) we get

$$\Rightarrow 1 = -2A + 0 + 0 \quad \Rightarrow \quad A = -\frac{1}{2}$$

Again, putting the value of z=1 in (ii),we get

$$\Rightarrow 1=0+B.2.(1+2)+0 \quad \Rightarrow \quad 1=6B \quad \Rightarrow \quad \mathbf{B=}\frac{1}{6}$$

**Similarly, putting the value of z=$-\frac{1}{2}$ in(ii) ,we get**

$$\Rightarrow 1 = 0 + 0 + C\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) \qquad \Rightarrow 1=\frac{3}{4}C \qquad \Rightarrow c=\frac{4}{3}$$

Putting the value of A,B,C in (i) we get

$$\frac{1}{(1 + z)(1 - z)(1 + 2z)} = -\frac{1}{2(1 + z)} + \frac{1}{6(1 - z)} + \frac{4}{3(1 + 2Z)}$$

$$I = -\int \left[ -\frac{1}{2(1 + z)} + \frac{1}{6(1 - z)} + \frac{4}{3(1 + 2Z)} \right] dz$$

$$I=\int\left[\frac{1}{2(1+z)}-\frac{1}{6(1-z)}-\frac{4}{3(1+2Z)}\right]dz$$

$$I=\frac{1}{2}\log|1+z|+\frac{1}{6}\log|1-z|-\frac{4}{3\times 2}\log|1+2z|+C$$

putting the value of z , we get

$$\Rightarrow \quad I=\frac{1}{2}\log|1+\cos x|+\frac{1}{6}\log|1-\cos x|-\frac{2}{3}\log|1+2\cos x|+C$$

OR

$$I=\int\frac{x^2-3x+1}{\sqrt{1-x^2}}dx=\int\frac{x^2-1+2-3x}{\sqrt{1-x^2}}dx$$

$$=\int\frac{-(1-x^{2)}}{\sqrt{1-x^2}}dx+2\int\frac{dx}{\sqrt{1-x^2}}-3\int\frac{xdx}{\sqrt{1-x^2}}$$

Let $=-\int\sqrt{1-x^2}dx+2\int\frac{dx}{\sqrt{1-x^2}}-3\int\frac{xdx}{\sqrt{1-x^2}}$

$$=-\frac{1}{2}x\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}x+2\sin^{-1}x+3\sqrt{1-x^2}+C$$

$$=\frac{3}{2}\sin^{-1}x-\frac{1}{2}x\sqrt{1-x^2}+3\sqrt{1-x^2}+C$$

10. Here $I=\int_{-\pi}^{\pi}(\cos\ ax-\sin bx)^2\ dx$

$$I=\int_{-\pi}^{\pi}\cos^2 ax+\sin^{2bx}-2\cos ax\sin bx)dx$$

$$I=\int_{-\pi}^{\pi}\cos^2 ax\,dx+\int_{-\pi}^{\pi}\sin^2 ax\,dx-0$$

[First two integranda are even function while third is odd function]

$$I=2\int_0^{\pi}2\cos^2 ax\,dx+\int_0^{\pi}2\sin^2 bx\,dx$$

$$I=\int_0^{\pi}(1+\cos 2ax)\,dx+\int_0^{\pi}(1-\cos 2bx)\,dx$$

$$I=\int_0^{\pi}dx+\int_0^{\pi}\cos 2ax\,dx+\int_0^{\pi}dx-\int_0^{\pi}\cos 2bx\,dx$$

$$I=2[x]_0^{\pi}+\left[\frac{\sin 2ax}{2a}\right]_0^{\pi}-\left[\frac{\sin 2bx}{2b}\right]_0^{\pi}$$

$$I=2\pi+\frac{\sin 2a\pi}{2a}-\frac{\sin 2b\pi}{2b}$$

11. Let E,F and A three events such that
E = selection of Bag A and F=selection of bag B
A= getting one red and one black ball of two

Here ,p(E)=P(getting 1 or 2 in a throw of die)= $\frac{2}{6}=\frac{1}{3}$

$\therefore\ p(F)=1-\frac{1}{3}=\frac{2}{3}$

Also, P(A/E)=P (getting one red and one black if bag A is selected)= $\frac{{}^6C_1\times{}^4C_1}{{}^{10}C_2}=\frac{24}{45}$

and P(A/F)=P(getting one red and one black if bag Black if bag B is selected)= $\frac{{}^3C_1\times 7C_1}{{}^{10}C_2}=\frac{21}{45}$

Now, by theorem of total probability,
p(A)=P(E).P(A/E)+P(F).P(A/F)

$\Rightarrow p(A)=\frac{1}{3}\times\frac{24}{45}+\frac{2}{3}\times\frac{21}{45}=\frac{8+14}{45}=\frac{22}{45}$

**OR**

Let number of head be random variable X in four tosses of a coin .X may have values 0,1,2,3 or 4 obviously repeated tosses of a coin are Bernoulli trials and thus X has binomial distribution with n=4 and p= probability of getting head in one toss= $\frac{1}{2}$

q=probability of getting tail (not head) in one toss= $1-\frac{1}{2}=\frac{1}{2}$

since, we know that P(X=r)= ${}^nC_rP^rq^{n-r},\quad r=0,1,2.........n$

therefore,

P(X=0)= ${}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{4-0}=1\times1\times\left(\frac{1}{2}\right)^4=\frac{1}{16}$

$P(X=1)={}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{4-1}=4\times\left(\frac{1}{2}\right)^1\times\left(\frac{1}{2}\right)^4=\frac{4}{16}=\frac{1}{4}$

$P(X=2)={}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2}=6\times\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^2=\frac{6}{16}=\frac{3}{8}$

$P(X=3)={}^4C_3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-3}=4\times\left(\frac{1}{2}\right)^3\times\left(\frac{1}{2}\right)^1=\frac{4}{16}=\frac{1}{4}$

$P(X=4)={}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{4-4}=1\times\left(\frac{1}{2}\right)^4\times\left(\frac{1}{2}\right)^0=\frac{1}{16}$

Now required probability distribution of X is

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 4P(x) | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{16}$ |

Required mean = $\mu=\sum x_ip_i$

$$=0\times\frac{1}{16}+1\times\frac{1}{4}+2\times\frac{3}{8}+3\times\frac{1}{4}+4\times\frac{1}{16}$$
$$=\frac{1}{4}+\frac{3}{4}+\frac{3}{4}+\frac{1}{4}=\frac{8}{4}=2$$

variance = $\sigma^2_x = \sum x_i p_i - \left(\sum x_i p_i\right)^2 = \sum X^2_i p_i - \mu^2$

$= \left(0\times\frac{1}{16}+1^2\times\frac{1}{4}+2^2\times\frac{3}{8}+3^2\times\frac{1}{4}+4^2\times\frac{1}{16}\right)-2^2$

$=\frac{1}{4}+\frac{3}{4}+\frac{9}{4}+1-4$

$=\frac{1}{4}+\frac{3}{4}+\frac{9}{4}-3$

$=\frac{1+6+9-12}{4}=\frac{4}{4}=1$

12. Here

Now

$\left(\vec{r}\times\hat{i}\right)\left(\vec{r}\times\hat{j}\right)+xy=\{(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i}\}.\{(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{j}\}+xy$

$=(-y\hat{k}+z\hat{j}).(x\hat{k}-z\hat{i})+xy=(0\hat{i}+z\hat{j}-y\hat{k})\cdot(-z\hat{i}+0\hat{j}+x\hat{k})+xy$

$=0+0-xy+xy=0$

13. Let P $(\alpha,\beta,\gamma)$ be the point of intersection of the given line (i) and plane (ii)

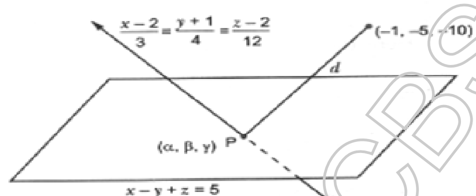
$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ ........(i)

and x – y + z = 5 ..........(ii)

since ,point P $(\alpha,\beta,\gamma)$ lies on line (i)( therefore it satisfy(i)

$\Rightarrow \frac{\alpha-2}{3}=\frac{\beta+1}{4}=\frac{\gamma-2}{12}=\lambda$

$\Rightarrow \alpha=3\lambda+2; \beta=4\lambda-1; \gamma=12\lambda+2$




Also point P $(\alpha,\beta,\gamma)$ lie on plane (ii)

$\Rightarrow \alpha-\beta+\gamma=5$ ......(iii)

putting the value of $\alpha,\beta,\gamma$ in (iii) we get

$\Rightarrow 3\lambda+2-4\lambda+1+12\lambda+2=5$

$\Rightarrow 11\lambda+5=5 \Rightarrow \lambda=0$

$\Rightarrow \alpha=2; \quad \beta=-1; \quad \gamma=2$

hence the coordinate of the point of intersect ion p is (-2,-1,2)

therefore ,required distance= d $=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$\sqrt{9+16+144}=\sqrt{169}=13\text{units}$

14. Here $\sin\left[\cot^{-1}(x+1)\right]=\cos(\tan^{-1}x)$

let $\cot^{-1}(x+1)=\theta$ $\Rightarrow \cot\theta=x+1$

$\Rightarrow \quad \operatorname{cosec}\theta=\sqrt{1+\cot^2\theta}=\sqrt{1+(x+1)^2}=\sqrt{x^2+2x+2}$

$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{x^2+2x+2}} \qquad \Rightarrow \theta = \sin^{-1}\left(\dfrac{1}{\sqrt{x^2+2x+2}}\right)$

$\Rightarrow \cot^{-1}(x+1) = \sin^{-1}\left(\dfrac{1}{\sqrt{x^2+2x+2}}\right)$

again $\tan^{-1}x = \alpha \Rightarrow \tan\alpha = x$

$\therefore \sec\alpha = \sqrt{1+\tan^2\alpha} = \sqrt{1+x^2}$

$\Rightarrow \cos\alpha = \dfrac{1}{\sqrt{1+x^2}} \qquad \Rightarrow \quad \alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

$\Rightarrow \quad \tan^{-1} = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

now equation (i) becomes

$\sin\left(\sin^{-1}\left(\dfrac{1}{\sqrt{x^2+2x+2}}\right)\right) = \cos\left(\cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)\right)$

$\dfrac{1}{\sqrt{x^2+2x+2}} = \dfrac{1}{\sqrt{1+x^2}} \Rightarrow \sqrt{x^2+2x+2} = \sqrt{1+x^2}$

$x^2+2x+2 = 1+x^2 \qquad \Rightarrow \quad 2x+2 = 1$

$\Rightarrow x = -\dfrac{1}{2}$

Or

Here

$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \dfrac{5\pi^2}{8}$

$\Rightarrow \quad (\tan^{-1}x)^2 + (\pi - \tan^{-1}x)^2 = \dfrac{5\pi^2}{8}$

$\Rightarrow \quad (\tan^{-1}x)^2 + (\tan^{-1}x)^2 + \dfrac{\pi^2}{4} - \pi\tan^{-1}x = \dfrac{5\pi^2}{8}$

$\Rightarrow \quad 2(\tan^{-1}x)^2 - \pi\tan^{-1}x + \dfrac{\pi^2}{4} - \dfrac{5\pi^2}{8} = 0$

$\Rightarrow \quad 2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \dfrac{3\pi^2}{8} = 8$

let $\tan^{-1}x = y$

$2y^2 - \pi y - \dfrac{3\pi^2}{8} = 0 \qquad \Rightarrow \quad 16y^2 - 8\pi y - 3\pi^2 = 0$

$16y^2 - 12\pi y + 4\pi y \ - 3\pi^2 = 0 \quad \Rightarrow 4y(4y-3\pi) + \pi(4y-3\pi) = 0$

$\Rightarrow (4y-3\pi)(4y+\pi) = 0 \quad \Rightarrow y = -\dfrac{\pi}{4} \text{ or } \quad y = \dfrac{3\pi}{4}$

$\Rightarrow \tan^{-1}x = -\dfrac{\pi}{4} \qquad [\because \dfrac{3\pi}{4} \text{ does not belongs to domain of } \tan^{-1}x \text{ i.e,} \left(-\dfrac{\pi}{2}, \dfrac{\pi}{4}\right)]$

$\Rightarrow x = \tan\left(-\dfrac{\pi}{4}\right) = -1$

15.

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\times\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{2+2\sqrt{1-x^4}}{1+x^2-1+x^2}\right) = \tan^{-1}\left(\frac{2+2\sqrt{1-x^4}}{2x^2}\right)$$

$$= \tan^{-1}\left(\frac{1+\sqrt{1-x^4}}{x^2}\right)$$

let $x^2 = \sin\theta$ $\Rightarrow \sin^{-1}(x^2) = \theta$

putting the value of $x^2$, we get

$$= \tan^{-1}\left\{\frac{1+\sqrt{1-\sin^2\theta}}{\sin\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{1+\cos\theta}{\sin\theta}\right\} = \tan^{-1}\left\{\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\cot\frac{\theta}{2}\right\} = \tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right\}$$

$$= \frac{\pi}{2}-\frac{\theta}{2} = \frac{\pi}{2}-\sin^{-1}x^2$$

$$\left(\begin{array}{l} \because 0 \le x^2 \le 1 \\ \Rightarrow \sin 0 < \sin\theta < \sin\frac{\pi}{2} \\ \Rightarrow 0 < \theta < \frac{\pi}{2} \Rightarrow 0 < \frac{\theta}{2} < \frac{\pi}{2} \\ \Rightarrow 0 < -\frac{\theta}{2} < -\frac{\pi}{4} \\ \Rightarrow \frac{\pi}{2} > \frac{\pi}{2}-\frac{\theta}{2} > \frac{\pi}{2}-\frac{\pi}{4} \\ \Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2}-\frac{\theta}{2}\right) > \frac{\pi}{4} \\ \left(\frac{\pi}{2}-\frac{\theta}{2}\right) \in \left(\frac{\pi}{4},\frac{\pi}{2}\right) \subset \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \end{array}\right)$$

differentiating both sides with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = -\frac{2x}{2\sqrt{1-x^4}} \Rightarrow \Rightarrow \frac{dy}{dx} = -\frac{x}{\sqrt{1-x^4}}$$

16. Given x= a cos $\theta$ +bsin $\theta$

$$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta + b\cos\theta$$

Also, $y = a\sin\theta - b\cos\theta$

$$\Rightarrow \frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\cos\theta + b\sin\theta}{-a\sin\theta + b\cos\theta}$$

$$\frac{dy}{dx} = -\frac{x}{y} \qquad \Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{y - x.\frac{dy}{dx}}{y^2}\right)$$

$$\Rightarrow y^2\frac{d^2y}{dx^2} = -y + x\frac{dy}{dx} \qquad \Rightarrow y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

17. Let ‘A’ be the area and ‘a’ be the side of an equilateral triangle.

$$A = \frac{\sqrt{3}}{4}a^2$$

Differentiating with respect to t we get

$\Rightarrow \quad \frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a\frac{da}{dt}$

$\Rightarrow \quad \frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times 2$ [Given $\frac{da}{dt}$ = 2cm/sec.]

$\Rightarrow \quad \frac{dA}{dt} = \sqrt{3}a$ $\quad \Rightarrow \left.\frac{da}{dt}\right]_{a=20cm} = 20\sqrt{3}\text{sq cm / s}$

18. Let $I = \int (x+3)\sqrt{3-4x-x^2}dx$

Let x + 3 = $A\frac{d}{dx}(3-4x-x^2)+B$

$\Rightarrow$ x + 3 = A (-4 – 2x) +B $\quad\Rightarrow$ x + 3 =- 4A =-2Ax + B

$\Rightarrow$ x + 3= (-4A + B) =- 2Ax [By comparing coefficients]

$\because$ -2A = 1 $\quad\Rightarrow$ A = $= \frac{1}{-2}$

Again, $\because$ -4A + B = 3

$\Rightarrow \quad -4\times\frac{1}{2}+B=3$ $\quad\Rightarrow$ 2 +B = 3 $\Rightarrow$ B = 1

Here, $x+3=-\frac{1}{2}(-2-4)+1$

$$I = \int\left\{-\frac{1}{2}(-2x-4)+1\right\}\sqrt{3-4x-x^2}dx$$

$$I = -\frac{1}{2}\int(-2x-4)\sqrt{3-4x-x^2}dx + \int\sqrt{3-4x-x^2}dx$$

$$I = -\frac{1}{2}I_1 + I_2$$ ...(i), where…………….

……………

Now $\quad I_1\int(-2x-4)\sqrt{3-4x-x^2}dx$

Let $\quad$ 3-4x – x² = z $\quad\Rightarrow\quad$ (-2x – 4) dx = dz

$I_1\int\sqrt{z}dz = \frac{2}{3}(z)^{\frac{2}{3}} + C_1 \quad\Rightarrow\quad I_2 = \frac{2}{3}(3-4x-x^2)^{\frac{3}{2}+}C_1$

Again $\quad I_2\int\sqrt{3-4x-x^2}dx$

$\Rightarrow \quad I_2 = \int\sqrt{-(x^2+4x-3)}dx \quad\Rightarrow\quad I_2 = \int\sqrt{-\{x+2)^2-7\}}dx$

$$I_2 = \int\sqrt{(\sqrt{7})^2-(x+2)^2}dx$$

$$I_2 = \frac{1}{2}(x+2)\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+C_2$$

Putting the value of $I_1$ and $I_2$ in (i), we get

$\because$ $$I=-\frac{1}{2}\times\frac{2}{3}(3-4x-x^2)^{\frac{3}{2}}-\frac{C_1}{C_2}(x+2)\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+C_2$$

$\Rightarrow$ $$I=-\frac{1}{3}(3-4x-x^2)^{\frac{3}{2}}+\frac{1}{2}(x+2)\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+C_2$$

$$=\sqrt{3-4x-x^2}\left[-\frac{1}{3}(3-4x-x^2)+\frac{1}{2}(x+2)\right]+\frac{7}{2}\sin^{-1}\frac{x+2}{7}+C$$

$$=\frac{x}{6}\sqrt{3-4x-x^2}(2x+11)+\frac{7}{2}\sin^{-1}\frac{x+2}{7}+C,\ \text{where } C=C_2-\frac{C_1}{2}$$

19. The number of handmade fans, mats and plates sold by three school A, B and can be represented by $3\times3$ matrix as

$$X=\begin{array}{c}A\\B\\C\end{array}\begin{bmatrix}40&50&20\\25&40&30\\35&50&40\end{bmatrix}$$

And their selling price can be represented by $3\times1$ matrix as

$$Y=\begin{bmatrix}25\\100\\50\end{bmatrix}\begin{array}{l}\to\text{Handmade fans}\\\to\quad\text{Mats}\\\to\quad\text{Plates}\end{array}$$

Now, the total funds collected by each school is given by the matrix multiplication as

$$XY=\begin{array}{c}A\\B\\C\end{array}\begin{bmatrix}40&50&20\\25&40&30\\35&50&40\end{bmatrix}\begin{bmatrix}25\\100\\50\end{bmatrix}$$

$\Rightarrow$ $$XY=\begin{bmatrix}40\times25+50\times100+20\times50\\25\times25+40\times100+30\times50\\35\times25+50\times100+40\times50\end{bmatrix}\Rightarrow XY=\begin{bmatrix}7000\\6125\\7875\end{bmatrix}$$

Hence, total funds collected by school A = Rs.7000
Total funds collected by school B = Rs.6125
Total funds collected by school C = Rs.7875
Total funds collected for the purpose = Rs.(7000+6125+7875)
= Rs. 21000
Value: Students are motivated for social service.

**SECTION-C**

20 Reflexivity: By commutative law under addition and multiplication

B + a = a +b $\quad\forall a,b\in N$
Ab=ba $\quad\forall a,b\in N$
Ab(b+a)=ba(a+b) $\quad\forall a,b\in N$
(a,b)R(a,b) Hence, R is reflexive

Symmetry: Let(a,b)R(c,d)
(a,b)R(c,d) $\Rightarrow$ ad(b + c) = bc (a + d)
$\Rightarrow$ bc(a + d) = ab(b + c)

$\Rightarrow \quad cb(d + a)\ da(c + b)$

[By commutative law under addition and multiplication]

$\Rightarrow \quad (c + d)\ R(a, b)$

Hence, R is symmetric.

Transitivity: Let(a, b) R (c, d) and (c, d) R (e, f)

Now, (a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow \quad ad( b + c) = bc(a + d)$ and $cf(d + e)\ de( c + f)$

$\frac{b+c}{bc} = \frac{a+d}{ad}$ and $\frac{d+e}{de} = \frac{c+f}{cf}$

$\Rightarrow \quad \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

Adding both, we get

$\Rightarrow \quad \frac{1}{c} + \frac{1}{v} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

$\Rightarrow \quad \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \quad \Rightarrow \quad \frac{e+b}{be} = \frac{f+a}{af}$

$\Rightarrow \quad af(b+e) = be(a +f) \quad \Rightarrow \quad (a, b)\ R\ (e, f) \quad [c,d \neq 0]$

Hence, r is transitive.

In this way, r is reflexive symmetric and transitive

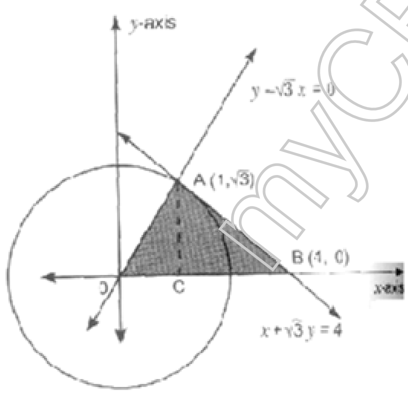
Therefore, r is an equivalence relation.

21. Given circle is $x^2 + y^2 = 4$

$\Rightarrow \quad 2x + 2y\frac{dy}{dx} = 0 \qquad$ [By differentiating]

$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$

Now, slope of tangent at $(1, \sqrt{3}) = \left.\frac{dy}{dx}\right]_{(1,\sqrt{3})} = -\frac{1}{\sqrt{3}}.$




$\therefore \quad$ Slope of normal at $(1, \sqrt{3}) = \sqrt{3}$

Therefore, equation of tangent is

$$\frac{y - \sqrt{3}}{x - 1} = -\frac{1}{\sqrt{3}}$$

$\Rightarrow \quad x + \sqrt{3}y = 4$

Again, equation of normal is

$$\frac{y-\sqrt{3}}{x-1}=\sqrt{3} \quad \Rightarrow \quad y-\sqrt{3}x=0$$

To draw the graph of the triangle formed by the lines x-axis, (i) and (ii), we find the intersecting of these three lines which give vertices of required triangle. Let O, A, B be the intersecting of these lines.

Obviously, the coordinate of O, A, B are (0, 0), $\left(1,\sqrt{3}\right)$ and (4, 0) respectively.

Required area = area of triangle OAB

= area of region OAC + area of region CAB

$=\int_0^1 y\,dx+\int_1^4 y\,dx$ [Where in 1st integrand $y=\sqrt{3x}$ and in 2nd $y=\frac{4-x}{\sqrt{3}}$]

$$=\int_0^1 \sqrt{3}x dx+\int_0^1 \frac{4-x}{\sqrt{3}}dx=\sqrt{3}\left[\frac{x^2}{2}\right]_0^1-\frac{1}{\sqrt{3}}\left[\frac{(4-x)^2}{2}\right]_1^4$$

$$=\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{3}}\left[0-\frac{9}{2}\right]$$

$$=\frac{\sqrt{3}}{2}+\frac{9}{2\sqrt{3}}=\frac{12}{2\sqrt{3}}=2\sqrt{3}\text{ sq units.}$$

Or

$$\int_1^3 (e^{2-3x}+x^2+1)dx=e^2\int_1^3 e^{-3x}dx+\int_1^3 (x^2+1)dx$$

$$=e^2.I_1+I_2 \qquad \text{.... (i), } I_2=\int_1^3 e^{-3x}dx; I_2=\int_1^3 (x^2+1)dx$$

We have, $\int_a^b f(x)dx=\lim_{h\to 0} h\{(a+b)+f(a+2h)+\ldots+f(a+nh)\}$

For $I_1$

$f(x)=e^{-3x}$, a = 1, b = 3

$h=\frac{b-a}{n}\Rightarrow h=\frac{2}{n}\Rightarrow nh=2$

Now, $\int_1^3 e^{-3x}dx=\lim_{h\to 0} h\{f(1+h)+f(1+2h)+\ldots f(1+nh)\}$

$$=\lim_{h\to 0} h\left\{e^{-3(1+h)}+e^{-3(1+2h)}+\ldots+e^{-3(1+nh)}\right\}$$

$$=\lim_{h\to 0} h\left\{e^{-3}.e^{-3h}+e^{-3}.e^{-6(1+2h)}+\ldots+e^{-3nh}\right\}$$

$$=\lim_{h\to 0} h\left\{e^{-3h}+(e^{-3h})^2+\ldots+(e^{-3h})^n\right\}$$

$$=e^{-3}.\lim_{h\to 0} h\left\{\frac{e^{-3h}(-1-(e^{-3h})^n}{1-e^{-3h}}\right\}$$

$$=e^{-3}.\lim_{h\to 0} h\left\{\frac{e^{-3h}(1-e^{-3nh})}{1-e^{-3h}}\right\} \qquad \text{[Applying formula for sum of GP]}$$

$$= e^{-3}.\lim_{h\to 0}\left\{\frac{e^{-3h}((1-e^{-6})}{1-e^{-3h}}\right\} = e^{-3}(1-e^{-6}).e^{0}.\frac{1}{\lim\limits_{3h\to 0}\frac{e^{-3h}-1}{-3h}\times 3}$$

$$= \frac{e^{-3}(1-e^{-6})}{3}$$

For $I_2$ F(x) = x² + 1, a = 1, b = 3 $\Rightarrow$ $h = \frac{b-a}{n}$ $\Rightarrow$ $h = \frac{2}{n} \Rightarrow b$

nh=2

Now, $\int_1^3 (x^2+1)dx = \lim_{h\to 0} h\{f(1+h)+f(1+2h)+....+f(1+nh)\}$

$$= \lim_{h\to 0} h\left[\left\{(1+h)^2+1\right\}+1\left\{(1+2h)^2+1\right\}+...+\left\{(1+nh)^2+1\right\}\right]$$

$$= \lim_{h\to 0} h\left[n+(1+h^2+2h)+(1+4h^2+4h)+(1+9h^2+6h)+...+(1+n^2h^2+2nh)\right]$$

$$= \lim_{h\to 0} h\left[n+n+h^2(1^2+2^2+3^2+...n^2)+2h(1+2+3...+n)\right]$$

$$= \lim_{h\to 0} h\left[2n+\frac{h^2 n(n+1)(2+1)}{6}+\frac{2h\,n(n+1)}{2}\right]$$

$$= \lim_{h\to 0}\left[2nh++\frac{h\,nh(nh+h)(2nh+h)}{6}+nh(nh+h)\right]$$

$$= \lim_{h\to 0}\left[4+\frac{2(2+h)(2\times 2+h)}{6}+nh(nh+h)\right] = 4+\frac{2(2+0)(4+0)}{6}+2(2+0)$$

$$= 4+\frac{16}{6}+4 = 8+\frac{8}{3} = \frac{32}{3}$$

Putting the value of $I_1$ and $I_2$ in (i), we ge,

$$I = \frac{e^2.e^{-3}(1-e^{-6})}{3}+\frac{32}{3} = \frac{e^{-1}(1-e^{-6})}{3}+\frac{32}{3} = \frac{32+(e^{-1}-e^{-7})}{3}$$

22. The given differential equation can be written as

$$\frac{dx}{dy}+\frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

Now (i) is linear differential equation of the form $\frac{dx}{dy}+P_1x = Q_1$,

where , $p_1 = \frac{1}{1+y^2}$ and $Q_1 = \frac{\tan^{-1}y}{1+y^2}$

Therefore, I.F = $e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

Thus , the solution of the given differential equation is

$$xe^{\tan^{-1}y} = \int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy + C$$

Let $I = \int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy$

substituting $\tan^{-1}y = t$ so that $\left(\frac{1}{1+y^2}\right)dy = dt$, we get

$I=\int t\,e^t dt = t\,e^t - \int 1.e^t dt = t\,e^t - e^t \equiv e^t(t-1)$

$I = e^{\tan^{-1}y}(\tan^{-1}y-1)$

substituting the valiue of I in the equation (ii) , we get

$x.e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y-1)+C$ or $x=(\tan^{-1}y-1)+C\,e^{-\tan^{-1}y}$

which is the gernal solution of the given differential equation

OR

Given differential equation is

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2} \quad .....(i)$$

Let y= $\nu$x $\Rightarrow$ $\frac{dy}{dx}=\nu+x\frac{d\nu}{dx}$

Now (i) becomes

$$\nu+x\frac{d\nu}{dx}=\frac{\nu x^2}{x^2+\nu^2x^2} \Rightarrow \nu+x\frac{d\nu}{dx}=\frac{\nu x^2}{x^2(1+\nu^2)}$$

$$\Rightarrow \nu+x\frac{d\nu}{dx}=\frac{\nu}{1+x^2} \Rightarrow x\frac{d\nu}{dx}=\frac{\nu}{1+\nu^2}-\nu$$

$$\Rightarrow x\frac{d\nu}{dx}=\frac{\nu-\nu-\nu^3}{1+\nu^2} \Rightarrow x\frac{d\nu}{dx}=\frac{-\nu^3}{1+\nu^2}$$

$$\Rightarrow -\frac{dx}{x}=\frac{1+\nu^2}{\nu^3}d\nu$$

Integrating both sides, we get

$$\Rightarrow \int\frac{1+\nu^2}{\nu^3}d\nu=-\int\frac{dx}{x} \Rightarrow \int\frac{d\nu}{\nu^3}+\int\frac{d\nu}{\nu}=-\int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2\nu^2}+\log|\nu|=-\log|x|+C$$

putting the value of $\nu=\frac{y}{x}$ ,we get

$$\Rightarrow -\frac{x^2}{2y^2}+\log\left|\frac{y}{x}\right|+\log|x|=C \Rightarrow -\frac{x^2}{2y^2}+\log|y|-\log|x|+\log|x|=C$$

$$\Rightarrow -\frac{x^2}{2y^2}+\log|y|=C$$

put y=1 and x=0 in(ii) 0+log$|1|$=C $\Rightarrow$ C = 0

Therefore required particular solution is $-\frac{x^2}{2y^2}+\log|y|=0$

23. Let the given lines

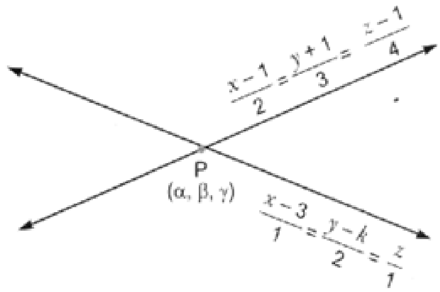
$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4} \quad ......(i)$$

and $\frac{x-3}{1}=\frac{y-k}{3}=\frac{z}{1}$ ......(ii)intersect at P $(\alpha,\beta,\gamma)$

$\because$ P lie in (i)

$$\Rightarrow \frac{\alpha-1}{2}=\frac{\beta+1}{3}=\frac{\gamma-1}{2}=\lambda\text{ (say)}$$

$\Rightarrow\ \alpha = 2\lambda+1, \beta = 3\lambda-1, \gamma = 4\lambda+1$




again, $\because$ P lie on (ii)also

$\Rightarrow \frac{\alpha-3}{1} = \frac{\beta-k}{2} = \gamma$

$\Rightarrow \quad \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1}$

$\Rightarrow \quad \frac{2\lambda-2}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1}$

I II III

from I and II

$\Rightarrow \quad 2\lambda-2 = \frac{3\lambda-1-k}{2} \quad \Rightarrow \quad 4\lambda-4 = 3\lambda-1-k$

$\Rightarrow \quad k=3\lambda-1-4\lambda+4 \quad \Rightarrow k=-\lambda+3$

$\Rightarrow \quad k = \frac{3}{2}+3 = \frac{9}{2}$

Now, we know that equation of plane containing lines.

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Therefore , required equation is $\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow$ (x-1)(3-8)-(y+1)(2-4)+(z-1)(4-3)=0

$\Rightarrow$ -5(x-1)+2(y+1)+(z-1)=0

$\Rightarrow$ -5x+2y+z+6=0 $\quad \Rightarrow$ 5x-2y-z-6=0

24. we know that "if A and B are two independent events" then

$P(A \cap B) = P(A).P(B)$

Also, since A and B are two independent events $\bar{A}$,B and A, $\bar{B}$ are also independent events .

$\therefore$ $P(\bar{A} \cap B) = P(\bar{A}).P(B)$

$P(A \cap \bar{B}) = P(\bar{A}).P(\bar{B})$

Now, let P(A) = x and P(B) = y

$\Rightarrow P(\bar{A}) = 1 - x$ and $P(\bar{B}) = 1 - y$

Given $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$

$\Rightarrow P(\overline{A}).P(B) = \frac{2}{15}$ and $P(A).P(\overline{B}) = \frac{1}{6}$

$\Rightarrow (1-x).y = \frac{2}{15}$ and $x.(1-y) = \frac{1}{6}$

$\Rightarrow y - xy = \frac{2}{15}$ .....(i) and $x - xy = \frac{1}{6}$ ......(ii)

From(i) $y.(1-x) = \frac{2}{15}$ $\Rightarrow$ $y = \frac{2}{15(1-x)}$

Putting the value of y in (ii) ,we get

$x - x \times \frac{2}{15(1-x)} = \frac{1}{6}$ $\Rightarrow$ $\frac{15x - 15x^2 - 2x}{15 - 15x} = \frac{1}{6}$

$\Rightarrow 6(-15x^2+13x)=15-15x$ $\Rightarrow$ $-90x^2+78x=15-15x$

$\Rightarrow -90x^2 + 93x - 15 = 0$ $\Rightarrow$ $30x^2 - 31x + 5=0$

$\Rightarrow 30x^2 - 25x - 6x + 5=0$ $\Rightarrow$ $5x(6x-5) - 1(6x-5)=0$

$\Rightarrow (6x-5)(5x-1)=0$ $\Rightarrow$ $x = \frac{5}{6}$ or $x = \frac{1}{5}$

Now, $x = \frac{5}{6}$ $\Rightarrow$ $y = \frac{2}{15\left(1-\frac{5}{6}\right)} = \frac{12}{15} = \frac{4}{5}$

and $x = \frac{1}{5}$ $\Rightarrow$ $y = \frac{2}{15\left(1-\frac{1}{5}\right)} = \frac{1}{6}$

Hence $P(A) = \frac{5}{6}$ and $P(B) = \frac{4}{5}$

or $P(A) = \frac{1}{5}$ $P(B) = \frac{1}{6}$

25. Given $f(x) = \sin x - \cos x$ $\Rightarrow f'(x)=\cos x+\sin x$

for critical points

$f'(x)=0$ $\Rightarrow$ $\cos x+\sin x=0$

$\Rightarrow \sin x = -\cos x$ $\Rightarrow$ $\tan x=-1$

$\Rightarrow \tan x = \tan\frac{3\pi}{4}$ $\Rightarrow$ $x = n\pi + \frac{3\pi}{4}$ $, n \in Z$

$\Rightarrow$ $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ [other value does not belong to $(0, 2\pi)$]

$\Rightarrow$ Now $f''(x) = -\sin x + \cos x$

$$f''(x)_{x=\frac{3\pi}{4}} = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2} < 0$$

i.e., $f(x)$ is maximum at $x = \frac{3\pi}{4}$

$\Rightarrow$ Local maximum value of $f(x) = f\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4} - \cos\frac{3\pi}{4}$

$$-\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)=\frac{2}{\sqrt{2}}=\sqrt{2}$$

Again $f''(x)_{x=\frac{7\pi}{4}} = -\sin\frac{7\pi}{4} + \cos\frac{7\pi}{4} = -\left(-\frac{1}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}>0$

i.e., f(x) is minimum at x= $\frac{7\pi}{4}$

$\Rightarrow$ Local minimum value of f(x) =f( $\frac{7\pi}{4}$ )=sin $\frac{7\pi}{4}$ - cos $\frac{7\pi}{4}$ $=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=-\frac{2}{\sqrt{2}}=-\sqrt{2}$

Therefore, local maximum and local minimum values are $\sqrt{2}$ and - $\sqrt{2}$ respectively.

26. Given constraints are

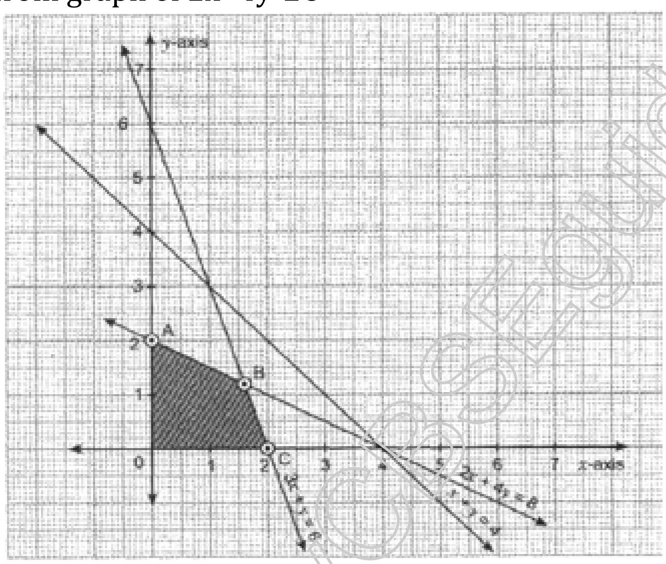
$2x+4y \le 8$ ........(i)

$3x+y\le 4$ .........(ii)

$x+y\le 4$ ........(iii)

$x\ge 0, y\ge 0$ ........(iv)

from graph of $2x+4y \le 8$




we draw the graph of 2x+ 4y = 8 as

| | | |
|---|---|---|
| X | 0 | 4 |
| y | 2 | 0 |

$\because$ $2\times 0+4\times 0\le 8$

$\Rightarrow$ (0,0) origin satisfy the constraints.

Hence , fesible region lie origin side of line 2x + 4y = 8

For graph $3x+y\le 4$

we draw the graph of line 3x+ y =6

| | | |
|---|---|---|
| X | 0 | 2 |
| y | 6 | 0 |

$\because$ $3\times 0+0\le 6$

$\Rightarrow$ origin (0,0) satisfy $3x+ y \le 6$
hence , fesible region lie origin side of line 3x + y =6
for graph of $x+ y \le 4$
we draw the graph of line x+ y=4

| | | |
|---|---|---|
| X | 0 | 4 |
| y | 4 | 0 |

$\because \quad 0+0 \le 4 \Rightarrow$ origin (0,0) satisfy $x + y \le 4$
hence feasible region lie origin side of line x+4= 4
also $\quad x \ge 0, y \ge 0$ says feasible region is in ist quadrant.
therefore, OABC is required feasible region.
Having corner point O(0,0) , (0,2) , B $(\frac{8}{5},\frac{6}{5})$ , C(2,0)

here feasible region is bounded.
NOW the value of objective function Z = 2x + 5y is obtained as

| Corner point | Z=2x+5y |
|---|---|
| O(0,0) | 0 |
| (0,2) | $2\times0\times5\times2$ =10 |
| B $(\frac{8}{5},\frac{6}{5})$ | $2\times\frac{8}{5}\times5\times\frac{6}{5}$ =9.2 |
| C(2,0) | $2\times2\times5\times0$ =4 |

Hence, maximum value of Z is 10 at x =0, y =2