ANSWER KEY- XII & DROPPER- JEE ADVANCED (PST-1)-DATE :03-01-2016

PAPER -1

CODE-1

PHYSICS															
QUS.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ANS.	D	B	B	BD	ABC	AB	AD	AB	ABCD	ABC	3	9	3	4	5
QUS.	16	17	18	19	20										
ANS.	5	4	5	9	7										
CHEMISTRY															
QUS.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
ANS.	C	BD	C	BC	ABC	ABC	ABCD	A	B	ABC	5	5	2	8	3
QUS.	36	37	38	39	40										
ANS.	3	6	2	9	9										
MATHEMATICS															
QUS.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
ANS.	D	BCD	ABC	B	D	AB	ACD	D	B	A	9	0	4	4	4
QUS.	56	57	58	59	60										
ANS.	8	5	9	0	3										

PAPER -2

CODE-1

PHYSICS															
QUS.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ANS.	AC	AB	ACD	ABD	AB	ACD	BD	BCD	BCD	AD	A	D	A	A	C
QUS.	16	17	18	19	20										
ANS.	D	B	C	A-p,B-q-C-r,s; D-t	A-p,r; B-q;C-q,s;D-q										
CHEMISTRY															
QUS.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
ANS.	ACD	BCD	ABC	AC	C	ACD	AC	ACD	A	ACD	D	D		A	C
QUS.	36	37	38	39	40										
ANS.	B	D	C	A-qrs ; B-pqrs ; C-pst ; D-rp	a- pqt; b-pq ; c-r ; d -pqst										
MATHEMATICS															
QUS.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
ANS.	C	ACD	B	B	A	ABC	ABCD	ABC	B	B	C	A	D	A	A
QUS.	56	57	58	59	60										
ANS.	A	B	C	A-r,B-p-C-s; D-t	A-s; B-p;C-q; D-t										

Note :-

Solutions can be *downloaded* from the website or will be displayed on the notice board !

ANSWER KEY- XII & DROPPER- JEE ADVANCED (PST-1)-DATE :03-01-2016

PAPER -1

CODE-2

PHYSICS															
QUS.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
ANS.	AB	AD	AB	ABCD	ABC	D	B	B	BD	ABC	5	5	4	5	9
QUS.	**16**	**17**	**18**	**19**	**20**										
ANS.	7	3	9	3	4										
CHEMISTRY															
QUS.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**
ANS.	ABC	ABCD	A	B	ABC	C	BD	C	BC	ABC	3	6	2	9	9
QUS.	**36**	**37**	**38**	**39**	**40**										
ANS.	5	5	2	8	3										
MATHEMATICS															
QUS.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
ANS.	AB	ACD	D	B	A	D	BCD	ACD	B	D	4	8	5	9	0
QUS.	**56**	**57**	**58**	**59**	**60**										
ANS.	3	9	0	4	4										

PAPER -2

CODE-2

PHYSICS															
QUS.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
ANS.	ACD	BD	BCD	BCD	AD	AC	AB	ACD	ABD	AB	A	D	A	D	C
QUS.	**16**	**17**	**18**	**19**	**20**										
ANS.	D	B	C	A-p,r; B-q;C-q,s;D-q	A-p; B-q; C-r,s; D-t										
CHEMISTRY															
QUS.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**
ANS.	ACD	AC	ACD	A	ACD	ACD	BCD	ABC	AC	C	C	B	D	C	D
QUS.	**36**	**37**	**38**	**39**	**40**										
ANS.	D	B	A	a- pqt; b-pq ; c-r ; d -pqst	A-qrs ; B-pqrs ; C-pst ; D-rp										
MATHEMATICS															
QUS.	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**
ANS.	ABCD	ABCD	ABC	B	B	C	ACD	B	B	AB	C	A	D	A	A
QUS.	**56**	**57**	**58**	**59**	**60**										
ANS.	A	B	C	A-s; B-p; C-q; D-t	A-r; B-p; C-s; D-t										

Note :-

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Hints & Solution

PAPER - 01

PART- I (PHYSICS)

1. A thin metallic spherical shell contains .....

Sol. Uniformly charged metallic sphere can be treated as a point charge for calculation of electric field at an external point.

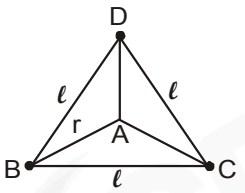
$\therefore$ Force on q is $\frac{k(Q+q_1)q}{d^2}$

$\therefore$ (D) is incorrect

2. Three equal masses m are rigidly .....

Sol. Moment of inertia about point B, $I_B = 2m\ell^2$

$$KE_B = \frac{1}{2}I_B.\omega^2 = \frac{1}{2}2m\ell^2.\omega^2 = m\ell^2\omega^2 \text{ J}$$



$\because$ $AB = AC = AD = r = \frac{\ell}{2\cos 30^\circ} = \frac{\ell}{\sqrt{3}}$

$\therefore$ Moment of inertia about point A,

$$I_A = 3\left[m\left(\frac{\ell}{\sqrt{3}}\right)^2\right] = m\ell^2$$

$$KE_A = \frac{1}{2}I_A.\omega^2 = \frac{1}{2}m\ell^2\omega^2 = \frac{m\ell^2\omega^2}{2} \text{ J}$$

$$\frac{KE_B}{KE_A} = \frac{2}{1}$$

3. The potential energy function .....

Sol. $U = -\int \vec{F}.\vec{dx} = -\int (4xy.dx + 2x^2dy)$

$= -2\int d(x^2y) = -2x^2y + C$

Also, we can differentiate the options and find F using

$F_x = -\frac{\delta U}{\delta x}$ and $F_y = -\frac{\delta U}{\delta y}$.

4. A uniform cylinder of mass *m* rests .....

Sol. When force F acts vertically downwards on the thread, the thread unwinds and the cylinder rotates clockwise. Hence, its surface tries to slip over the planks in backward direction. Hence, a friction comes into existence and acts on the cylinder in forward direction (rightward direction). It is the only horizontal force acting on the cylinder. Due to the friction, cylinder experiences a rightward translational acceleration. But the point at which the friction acts remains always at rest, therefore no energy is lost against the friction. Since the work is done by F and no energy is lost against the friction, therefore total kinetic energy of the cylinder at any instant is equal to work done by F. Hence, option (A) and (C) are wrong and option (D) is correct. If I is the moment of inertia of a body about its instantaneous axis of rotation, then total KE of the rotating body is equal to $I\omega^2/2$. Hence, option (B) is correct.

5. Which of the following statements .....

Sol. (A) Capacitance of a conductor depends on the size. If the plates are of different size their capacitance will be different and so their potential will be different according to the formula $V = \frac{Q}{C}$

So there can be potential difference between two adjacent conductors carrying the same magnitude of positive charge.
So the statement is correct

(B) $V_d = \left(\frac{e\tau}{m}\right)E = \left(\frac{e\tau}{m}\right)\frac{V}{L}$

$\therefore$ V_d is independent of diameter and will not change when diameter is changed.
(C) Since the bulbs are joined in series, so when one of the bulb is removed from the chain the resistance of the chain is decreased. Hence the current flowing through each bulb is increased. As heat produced is $\propto i^2$

Hence light is increased in the room.

(D) If a conductor is earthed then, its potential will become zero. If it is isolated, then only its net chrage will also become zero.

6. Three very large plates are given .....

Sol. Using the conservation of charge principle, distribute the charge on all the surfaces of the plates a, b, c, d, e, and f. Then use the concept, electric field is zero inside the plates.

7. One end of an ideal spring .....

Sol: Maximum work done by the spring = maximum loss in P.E. of the spring = $\frac{1}{2}kx^2$

The block will have maximum KE when the spring is in its undeformed state i.e., P.E =0 state

When spring does negative work the kinetic energy of the block will decrease.

While moving away from the wall the P E of the spring first decreases and then increases and therefore the K.E of the block first increases and then decreases.

8. A stone is projected from level ground .....

Sol. Let u_x and u_y be horizontal and vertical components of velocity respectively at t = 0. Then, $v_y = u_y - gt$

Hence, $v_y - t$ graph is straight line.

$$x = v_x t$$

Hence, x - t graph is straight line passing through origin.

The relation between y and t is

$y = u_y t - \frac{1}{2} gt^2$

Hence y-t graph is parabolic.

v_x = constant

Hence, v_x-t graph is a straight line.

9. Two small spheres of mass m_1 and .....

Sol. The velocity of centre of mass is always zero. At maximum deformation during head on collision, velocity of each sphere is equal to velocity of centre of mass and hence zero. Therefore at maximum deformation K.E. of system is also zero.

Velocity of separation after collision = e(velocity of approach before collision).

From centre of mass frame in a head-on collision, if $\vec{u}_1$ and $\vec{v}_1$ be velocity of a ball before and after collision $\vec{v}_1 = -e\vec{u}_1$. Since, v_{cm} = 0 from ground frame, ground frame and centre of mass frame carry same meaning.

10. Suppose a smooth tunnel is dug along.....

Sol. When the particle is dropped in the tunnel at its one end, it starts to execute simple harmonic motion with mean position as centre of tunnel. Hence, acceleration of the particle is directly proportional to its distance from centre of the tunnel. It means, option (D) is correct.

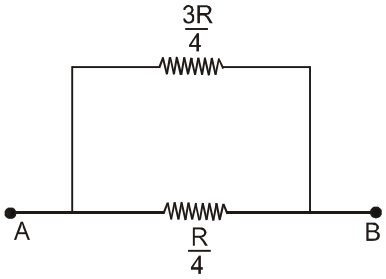
The particle has zero velocity at one end because it is dropped from rest at that end, therefore, ends of the tunnel are its extreme positions. Hence, at the other end, the velocity will become equal to zero. It means option (A) is wrong.

Velocity of a particle executing S.H.M. is maximum at its mean position. Hence, at centre of the tunnel, its velocity will be maximum possible. Hence, option (B) is wrong.

When particle moves from extreme position to centre of the tunnel, its velocity increases. It means KE increases or PE decreases. But initial potential energy of particle is negative. Hence, potential energy becomes more negative and it is least at mid-point of the tunnel because KE is maximum there. It means the gravitational PE can never be equal to zero. Hence, option (C) is also wrong.

11. Sixteen resistors each of resistance .....

Sol. 1



$$\Rightarrow R_{AB} = \frac{3R}{16} = \frac{3(16)}{16} = 3\Omega .$$

12. A solid conducting sphere of radius .....

Sol. On connec4ting, the entire amount of charge will shift to the outer sphere.

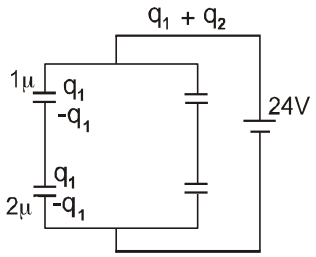
Heat generated

$$= U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R_1} - \frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$= \frac{(20\times10^{-6})^2 \times 9\times10^9}{2}\left[\frac{1}{0.10} - \frac{1}{0.20}\right] = 9\,J$$

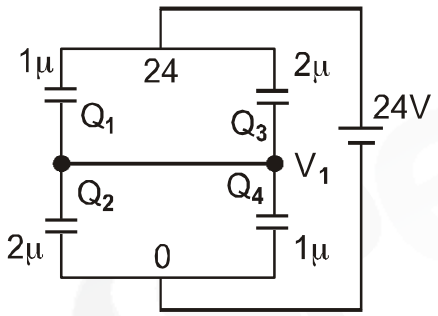
13. Four uncharged capacitors are .....

Sol. Initially,



$$q_1 = 24\left(\frac{2}{3}\mu C\right) = 16\mu C \;\&\; q_2 = 16\mu C$$

Finally, $(V_1 - 24)\times1 + (V_1 - 0)\times2 + (V_1 - 24)\times2 + (V_1 - 0)\times1 = 0$



$$V_1(1+2+2+1) = 24\times3 = 0$$

$$\Rightarrow V_1 = \frac{24\times3}{6} = 12V$$

$$Q_1 + Q_2 = (12-24)\times1 + (12-0)\times2$$
$$= -12 + 24 = 12\mu C$$

$$Q_3 + Q_4 = (12-24)\times2 + (12-0)\times1$$
$$= -24 + 12 = -12\mu C$$

Initial net charge on plates left of S = 0

Final net charge on plates left of

S = $Q_1 + Q_2 = 12\mu C$

Charge flowing through S = 12μC towards left.

14. A particle is projected at 60° to the .....

Sol. Velocity at the highest point = u cos 60° $= \frac{u}{2}$

Now, $k = \frac{1}{2}mu^2$

$$k' = \frac{1}{2}m\left(\frac{u}{2}\right)^2 = \frac{k}{4}$$

$\therefore n = 4$

15. A solid cylinder of height h and mass .....

Sol. If V_1 = volume submerged,

V_2 = V = volume of cylinder

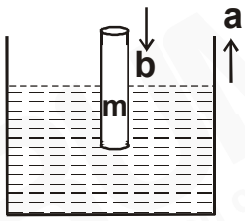
Density of Liquid = ρ_L,

Density of cylinder = ρ_m

From given condition $V_1\rho_L g = V_2\rho_m g$

$$\left(\frac{3h}{4}.A\right)\rho_L g = (hA)\rho_m g$$

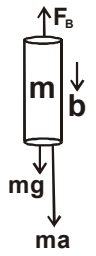
$$\frac{\rho_L}{\rho_m} = \frac{4}{3} \qquad .....(i)$$



For completely immersed cylinder, buoyant force

$$F_B = V\rho_L g = m g\left(\frac{\rho_L}{\rho_M}\right) = \frac{4\,m\,g}{3} \qquad .....(ii)$$

w.r.t. frame of vessel,



$$mg + ma - F_B = mb$$

$$g + a - \frac{4g}{3} = b \qquad .....(iii)$$

Since, relative acceleration of cylinder is = $\frac{1}{3}$ of acceleration of vessel.

$$\therefore\; b = \frac{a}{3}$$

From equation (iii)

$$g + a - \frac{4g}{3} = \frac{a}{3}$$

$$a = \frac{g}{2} = 5\ m/sec^2$$

16. A 10 H.P. motor pumps out water .....

Sol. Volume of water to raise

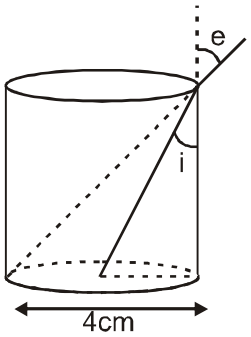
$= 22380\ \ell = 22380 \times 10^{-3} m^3$

$P = \frac{mgh}{t} = \frac{V\rho gh}{t} \quad \Rightarrow \quad t = \frac{V\rho gh}{P}$

$t = \frac{22380 \times 10^{-3} \times 10^3 \times 10 \times 10}{10 \times 746} = 5 min$

17. A glass beaker has diameter 4cm .....

Sol.



$\sqrt{\frac{5}{2}} \sin i = 1 \sin e$

$\Rightarrow \quad \sqrt{\frac{5}{2}} \frac{2cm}{\sqrt{4cm^2 + h^2}} = 1 \times \frac{4cm}{\sqrt{16cm^2 + h^2}}$

$\Rightarrow \quad h = 4\ cm$

18. A solid sphere of radius 'R' is .....

Sol. Assume 'ρ' and '$-\rho$' in the cavity then

$V_\rho = \frac{3}{2}\frac{K}{R}\left(\rho \cdot \frac{4}{3}\pi R^3\right) \Rightarrow V_{-\rho}$

$= \frac{K\left[-\rho \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]}{\frac{R}{2}}$

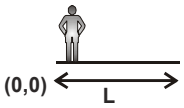
$V_C = V_\rho + V_{-\rho} = 2\pi K \rho R^2 - \frac{\pi K \rho R^2}{3}$

$= \frac{5\pi K \rho R^2}{3}$

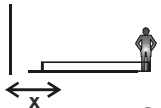
$V = \frac{5\rho R^2}{12 \in_0}$ **Ans.** $\frac{5\rho R^2}{12 \in_0} = 5$ Volts

19. A man of mass M stands at one end .....

Sol. $x_{com})i = \frac{M \times 0 + \frac{M}{3} \times \frac{L}{2}}{\left(M + \frac{M}{3}\right)}$



$x_{com})f = \frac{M(L + x) + \frac{M}{3}\left(\frac{L}{2} + x\right)}{\left(M + \frac{M}{3}\right)}$



$x_{com})i = x_{com})f$

$\frac{M}{3} \times \frac{L}{2} = ML + Mx + \frac{M}{3} \times \frac{L}{2} + \frac{M}{3}.x$

$\Rightarrow \quad x = -\frac{3L}{4}$

Distance moved by man w.r.t ground $= L - \frac{3L}{4} = \frac{L}{4}$.

20. A particle of mass 2m is projected .....

Sol. Velocity of particle after 1 s is

$v = 20\hat{i} + (20 - 10)\hat{j} = 20\hat{i} + 10\hat{j}$

Height of particle after 1 s,

$h_1 = 20 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 15\ m$

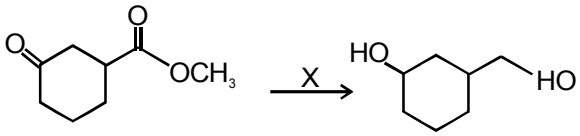
After explosion, one particle comes to rest. Therefore, by momentum conservation velocity of other particle will be $V' = 40\hat{i} + 20\hat{j}$

Height travelled by 2nd particle after explosion, $h_2 = \frac{(20)^2}{2 \times 10} = 20\ m$

Total height, $h = h_1 + h_2 = 15 + 20 = 35\ m$.

PART- II (CHEMISTRY)

21.



....

Sol. (A) $NaBH_4 | C_2H_5OH | H_3O^+$ – reduces alde hyde, ketone, acidhalid, alkylhalide but not to ester

(B) $N_2H_4 | OH^- |$ glycol$| \Delta$ – reduces aldehyde, ketone to corresponding alkane

(C) $LiAlH_4 | Et_2O | H_3O^+$ – reduces all the functional groups except alkene and alkyne

(D) DIBAL – H|THF|H_3O^+ – **reducess nitriles and ester to corresponding**

carbonyl

22. Au and Pt dissolves in aqua .....

Sol. $Au + 4H^+ + NO_3^- + 4Cl^- \longrightarrow [AuCl_4]^- + NO + 2H_2O$

$3Pt + 16H^+ + 4NO_3^- + 18Cl^- \longrightarrow 3[PtCl_6]^{2-} + 4NO + 8H_2O$

$X = H[AuCl_4]$; $Y = H_2[PtCl_6]$

23. an aqueous solution of aniline,.....

Sol. $C_6H_5NH_2 + H_2O \rightleftharpoons C_6H_5NH_3^+ + OH^-$

1	0	0
$1-\alpha$	α	α

($\because$ Dissociation occurs in presence of NaOH and thus dissociation of $C_6H_5NH_2$ will suppresh)

$$K_b = \frac{[C_6H_5NH_3^+][OH^-]}{[C_6H_5NH_2]} = \frac{10^{-8} \times [OH^-]}{[0.24]}$$

K_b for $C_6H_5NH_2$ =

$$\frac{K_w}{K_a \text{for} C_6H_5NH_3^+} = \frac{10^{-14}}{2.4 \times 10^{-5}}$$

$$[OH^-] = \frac{10^{-14} \times 0.24}{2.4 \times 10^{-5} \times 10^{-8}} = 0.01$$

$NaOH = 10^{-2}$ M

24. Consider the 1st order.....

Sol. $C_A = C_{AO}e^{-kt}$

$C_{AO}(1-\alpha) = C_{AO}e^{-kt}$

$\alpha = 1 - e^{-kt}$

on increasng temperature K increases hence α increases

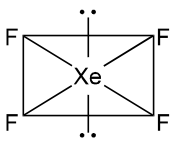
25. Xe when reacted with F_2 in .....

Sol. $Xe + 2F_2 \xrightarrow[7\,bar,\ Ni-tube]{873K,} XeF_4$

1 : 5

$6XeF_4 + 12H_2O \longrightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$

Shape square planar & geometry octahedral



hybridisation = sp^3d^2

26. The qualitative order of .....

Sol. Debroglie wavelength is given by

$$\lambda = \frac{h}{mv}$$

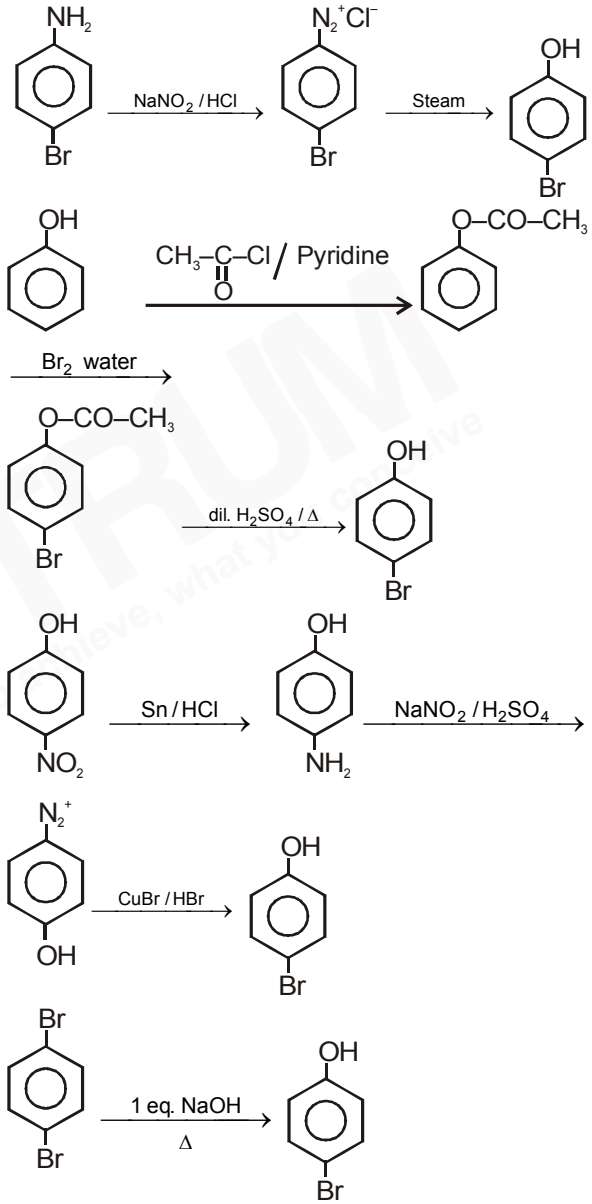
$$\lambda = \frac{h}{\sqrt{2m_e(KE)}}$$

$$\Rightarrow \quad \lambda = \left(\frac{150}{V}\right)^{\frac{1}{2}} \text{Å}$$

(on putting values of h, m_e and e)

27. Which of them yields .....

Sol.



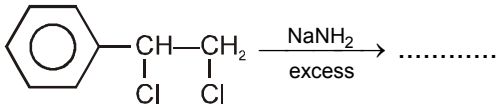
28. Mole fraction of CH_3OH.....

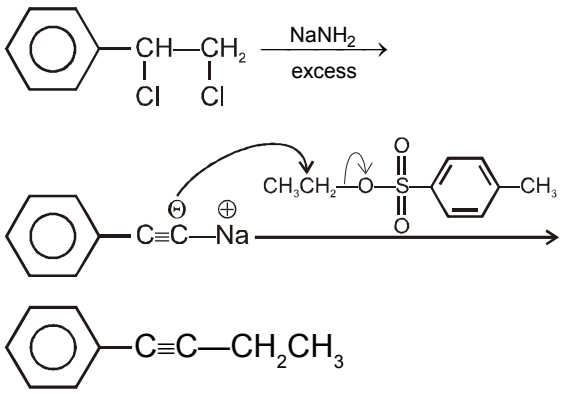
Sol.

1 mole mixture → $CH_3OH = 0.5$ mol $= 1/2 \times 32 = 16$ gm

1 mole mixture → $C_2H_5OH = 0.5$ mol $\equiv 1/2 \times 46 = 23$ gm

$$\text{mass \% } CH_3OH = \frac{16}{39} \times 100 = 41\%$$

29. 

Sol. 

1-phenylbut-1-yne

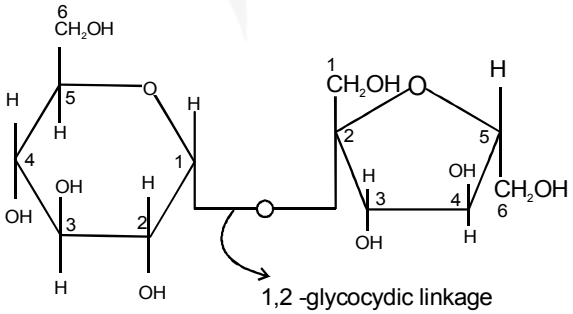
30. Which of the following.....

Sol Sucrose on hydrolysis gives equimolar mixture of D-(+)-glucose and D-(-) fructose.

Sucrose $\xrightarrow{H_2O}$ **α–D – glucose + βD–fructose**

$\alpha = +60^0$ $\alpha = 52^0$ $\alpha = -92^0$

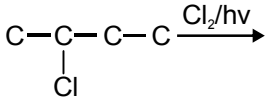
These two monosaccharides are held together by a glycosidic linkage between C1 of α–glucose and C2 of β–fructose. It has 1,2–glycosidic linkage. Since the reducing groups of glucose and fructose are involved in glycosidic bond formation, sucrose is a non reducing sugar.

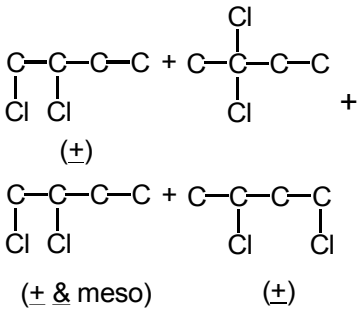


31. Rate constant for reaction.....

Sol. Order is zero $\therefore t_{0.5} = \frac{a_0}{2k} = \frac{6.93}{2 \times 0.693}$

34. find total dichloro.....

Sol. 



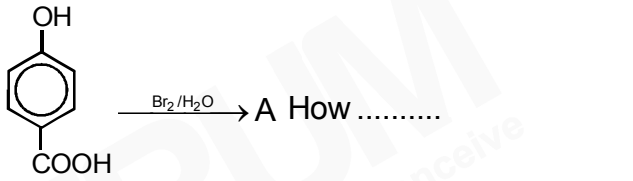
35. Specific gravity of a solution .....

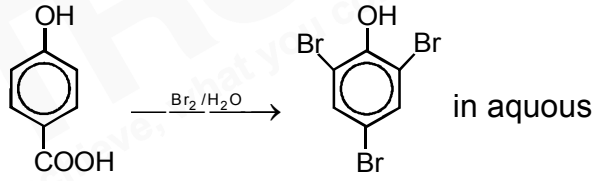
Sol. Let V ml water is added

$\frac{100 \times 1.8 + V}{100 + V} = 1.2$

$100 \times 0.6 = 0.2\ V$

$V = 300$ ml

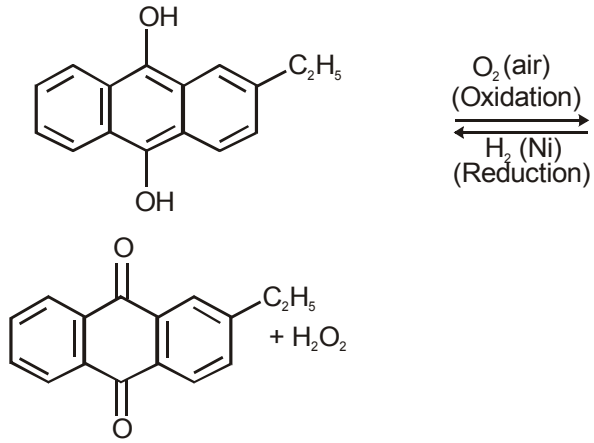
36. 

Sol. 
in aquous solution COOH group is replaced by Br^+ electrophile

37. How many of the following.....

Sol. 1,2,4,6,7 and 8 are stronger acid than phenol

38. When 2-ethyl anthroquinnol.....

Sol. 

2- Ethyl anthraquinol
2-Ehtylanthraquinone

39. In He^+ ion e^- is present.....

Sol. Binding energy = $13.6 \frac{Z^2}{n^2}$

$1.36 \times \frac{4}{n^2} = 6.04$

$n^2 = \frac{54.4}{6.04}$

$n = 3$

hence state is 3 and it has total 9 orbital in 3s 3p 3d and all have same energy

40. K_a of a substituted .....

Sol. At equivalence point it is the salt of WASB and conc. of salt is

$\frac{20 \times 0.2}{20 + 20} = 0.1M$

$pH = \frac{1}{2}(14 + pk_a + \log C)$

$= \frac{1}{2}(14 + 5 - 1) = 9$

PART- III (MATHS)

41. Let R be the set of real numbers and .....

[Sol. Put $x = x + h$

$y = x$

$\therefore \quad |f(x + h) - f(x)| \le |h|^3$

$\Rightarrow \quad \lim_{h \to 0} \left| \frac{f(x+h) - f(x)}{h} \right| \le \lim_{h \to 0} h^2$

$\Rightarrow \quad |f'(x)| \le 0 \Rightarrow |f'(x)| = 0$

$\Rightarrow f(x) = \text{constant}$

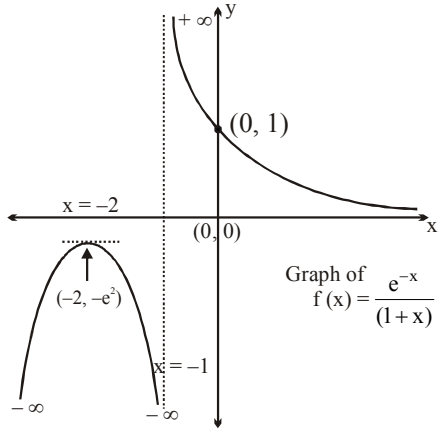
$\therefore \quad f(x) = 100$

Hence, $f(20) = 100$ Ans.]

42. For the equation $\frac{e^{-x}}{1+x} = p$, ...

[Sol. Let $f(x) = \frac{e^{-x}}{1+x}$

$\Rightarrow \quad f'(x) = \frac{-e^{-x}(x+2)}{(1+x)^2}$



Now, verify alternatives.]

43. Let f be a differentiable function on R .....

[Sol. We have, $\int_0^x f(t)\,dt + x\int_0^x f(t)\,dt - \int_0^x t \cdot f(t)\,dt$

$= \left(-1 + e^{-x}\right)$

Differentiate both sides with respect to x, we get

$f(x) + \int_0^x f(t)\,dt = -e^{-x} \qquad \ldots.(i)$

put $x = 0$, we get $f(0) = -1$

Again, differentiate both sides of equation (i) with respect to x, we get

$e^x\left(f'(x) + f(x)\right) = 1$

$\therefore$ On integrating both sides with respect to x, we get

$e^x \cdot f(x) = x + c$

As, $f(0) = -1 \Rightarrow -1 = 0 + c$

$\Rightarrow \quad c = -1.$

$\therefore \; f(x) = (x - 1)e^{-x}.$

Now verify alternatives.]

44. Given two independent events .....

[Sol. $P(A) + P(B) - 2P(A)P(B) = \frac{26}{49}$ .....(1)

and $P(A) + P(B) - P(A)P(B) = \frac{34}{49}$ .....(2)

$\therefore \quad P(A) = \frac{4}{7}, \; P(B) = \frac{2}{7}$

or $\quad P(A) = \frac{2}{7}, \quad P(B) = \frac{4}{7}.$]

45. The solution of differential equation .....

[Sol. $\left(\frac{y+1}{y}\right)dy + e^x(\cos^2 x - \sin 2x)\,dx = 0$

$\therefore$ On integrating, we get

$y + \ln y + e^x \cos^2 x = c.$

As, $y(0) = 1 \Rightarrow c = 2.$]

46. Let $L = \lim\limits_{x\to 0} \dfrac{3px + (p-2)\sin x}{(\sin^{-1} x)^3}$ .....

[Sol. $L = \lim\limits_{x\to 0} \dfrac{3px + (p-2)x - \frac{1}{6}(p-2)x^3 + ...}{x^3}$

So, $p = \dfrac{1}{2}$ and $L = \dfrac{1}{4}$.]

47. Let $f(x) = \text{sgn}(\sin^2 x + 2\sin x - 3)$, $x \in R$ .....

[Sol. $m > 1$ or $m < -3$]

48. If 4 elements are randomly selected .....

[Sol. 4 randomly selected elements can be

$-2, 0, 2, 0$ (1 is not taken)

$\Rightarrow$ 4 are invertible

$-2, 0, 1, 0$ (2 is not taken)

$\Rightarrow$ 4 are invertible

$0, 1, 2, 0$ (-2 is not taken)

$\Rightarrow$ 4 are invertible

$-2, 0, 1, 2$ (one 0's is not taken)

$\Rightarrow$ 24 are invertible

Total (2×2 matrices) $= 3\left(\dfrac{4!}{2!}\right) + 4!$

$\therefore$ $n(S) = 36 + 24 = 60$

$\therefore P(A) = \dfrac{36}{60} = \dfrac{3}{5}$ **Ans.**]

49. $\displaystyle\int_0^{\pi/2} \frac{\sin^3 x}{(\cos^4 x + 3\cos^2 x + 1)\tan^{-1}(\sec x + \cos x)}\,dx$ .....

[Sol. $\displaystyle\int_0^{\pi/2} \frac{\sin^3 x}{(\cos^4 x + 3\cos^2 x + 1)\tan^{-1}(\sec x + \cos x)}\,dx$

Put $\sec x + \cos x = t \Rightarrow (\sec x \tan x - \sin x)$

$dx = dt \Rightarrow \sin x \dfrac{\sin^2 x}{\cos^2 x}\,dx = dt$

$\Rightarrow \sin^3 x\,dx = \cos^2 x\,dt$

$I = \displaystyle\int_2^{\infty} \frac{dt}{(\cos^2 x + \sec^2 x + 3)\tan^{-1} t}$

$= \displaystyle\int_2^{\infty} \frac{dt}{(t^2+1)\tan^{-1} t}$; Put $\tan^{-1} t = z$

$I = \ln z\,\Big]_{\tan^{-1} 2}^{\frac{\pi}{2}} = \ln \dfrac{\pi}{2} - \ln(\tan^{-1} 2)$. **Ans.**]

50. Let $\alpha > -1$ and $\beta > -1$, then the .....

[Sol. $\lim\limits_{n\to\infty} \dfrac{n^\beta}{n^\alpha} \dfrac{\sum\limits_{r=1}^{n} r^\alpha}{\sum\limits_{r=1}^{n} r^\beta} = \dfrac{\int_0^1 x^\alpha\,dx}{\int_0^1 x^\beta\,dx} = \dfrac{\beta+1}{\alpha+1}$. **Ans.**]

51. Graph of a function $y = f(x)$ is shown .....

[Sol. $I_1 = \lim\limits_{x\to 0^+}[f(x) - 2] = -1$ $\{\because f(0^+) < 2\}$

$I_2 = \lim\limits_{x\to 0^-}[f(x^2) + f(x)] = [f(0^+) + f(0^-)] = 5$

$I_3 = \lim\limits_{x\to 0^+}[\sin^{-1}(\sin(f[x])) + f(0)]$

$= [\sin^{-1}\sin(f(0)) + f(0)] =$

$[\sin^{-1}(\sin 3) + 3] = [\pi - 3 + 3] = 3$

$I_4 = \lim\limits_{x\to 0^-}(\sin^{-1}(\sin(f(x^2)))) + \tan^{-1}(\tan(f(x)))$

$= \sin^{-1}(\sin(f(0^+))) + \tan^{-1}(\tan(f(0^-)))$

$= \sin^{-1}(\sin(2)) + \tan^{-1}(\tan 4)$

$= \pi - 2 + 4 - \pi = 2$

$\therefore I_1 + I_2 + I_3 + I_4) = -1 + 5 + 3 + 2 = 9.$

52. Let $I_1 = \displaystyle\int_{\frac{1}{2}}^{2} \frac{x^{2012} - 1}{x^{2014} + 1}\,dx$ .....

[Sol. Put $x = \dfrac{1}{y}$ in I_1, we get $I_1 = -I_1$

$\Rightarrow I_1 = 0.$

Let $I_2 = \displaystyle\int_2^4 \frac{\ln 2 \cdot (\ln x - 1)}{(\ln x)^2}\,dx$; Put $\ln x = t$

$\Rightarrow$ $x = e^t$

So, $I_2 = (\ln 2)\displaystyle\int_{\ln 2}^{\ln 4} e^t \frac{(t-1)}{t^2}\,dt$

$= (\ln 2)\left(\dfrac{e^t}{t}\right)_{\ln 2}^{\ln 4} = 0.$]

53. A set of 3 numbers is chosen from .....

[Sol. $1 \cdot 3 \cdot 5, ..., (2n+1) \rightarrow (n+1)$

$2, 4, 6, ..., (2n) \rightarrow n$

$n(S) = {}^{(2n+1)}C_3$

n (A) = $^{n+1}C_2 + {}^nC_2 = \frac{(n+1)n}{2} + \frac{(n-1)n}{2}$

$= \frac{n}{2}[(n+1)(n-1)] = n^2$

$P(A) = \frac{n^2\ 6}{(2n+1)\cdot 2n(2n-6)}$

$\frac{4}{21} = \frac{3n}{4n^2-1}$

$\Rightarrow \quad 16n^2 - 4 = 63\,n$

$\Rightarrow \quad 16n^2 - 63\,n - 4 = 0$

$16n^2 - 64\,n + n - 4 = 0$

$\Rightarrow \quad 16n(n-4) + (n-4) = 0$

$\Rightarrow \quad (n-4)(16n+1) = 0$

$n = 4$ or $n = \frac{-1}{16}$ (rejected)]

54. If equation of tangent to the graph .....

[Sol. $f(2) = 5, \quad f'(2) = 4, \quad f''(2) = 0$

$\lim_{x\to 2} \frac{f(x^2-2) - f(f(x)-3)}{(x-2)^2} \quad \left(\frac{0}{0}\text{form}\right)$

$\lim_{x\to 2} \frac{f'(x^2-2)\cdot 2x - f'(f(x)-3)f'(x)}{2(x-2)}$

$\lim_{x\to 2} \frac{2f'(x^2-2)+2xf''(x^2-2)\cdot 2x - \left(f'(f(x)-3)\,f''(x)+f'(x)f''(f(x)-3)f'(x)\right)}{2}$

$\frac{2f'(2)+0-(0+0)}{2} = f'(2) = 4$

Note that $x = 2$ is a point of inflection of $y = f(x)$.]

55. An urn contains 10 balls, 4 red and 6 blue ...

[Sol.

$I(10) \begin{cases} 4R \\ 6B \end{cases} \xrightarrow{1\text{ ball}}$

$II(16+n) \begin{cases} 16R \\ nB \end{cases} \xrightarrow{1\text{ ball}}$

P(ball drawn is of same colour)

$= \frac{4}{10}\cdot\frac{16}{16+n} + \frac{6}{10}\cdot\frac{n}{16+n}$

$\frac{2}{5}\cdot\frac{16}{16+n} + \frac{3}{5}\cdot\frac{n}{16+n} = 0.44$

$\frac{[32+3n]}{5(16+n)} = \frac{44}{100} = \frac{11}{25} \Rightarrow 11\cdot 5(16+n)$

$= 25(32+3n)$

$\Rightarrow 55(16+n) = 800 + 75n$

$\Rightarrow 55\cdot 16 + 55n = 800$

$\Rightarrow 55\cdot 16 - 800 = 20n$

$\Rightarrow 880 - 800 = 20n \Rightarrow 80 = 20n$

$\Rightarrow n = 4$. **Ans.**]

56. The function f (x) .....

[Sol. Using expansions of sin 3x, sin 5x and sin x, we get C = f (0) =

$\lim_{x\to 0} \frac{\left[\frac{(3x)}{1!} - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - .....\right] + A\left[5x - \frac{(5x)^3}{3!} + \frac{(5x)^5}{5!} - .....\right] + B\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - .....\right]}{x^5}$

for limit to exist

coefficient of x = 0 and coefficient of $x^3 = 0$

$3 + 5A + B = 0$ and $\frac{9}{2} + \frac{125A}{6} + \frac{B}{6} = 0$

solving $A = -\frac{1}{5}; \quad B = -2$

$\therefore \quad f(0) = \text{coefficient of } x^5 = -\frac{16}{5}$

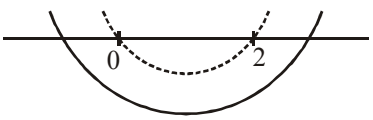
hence $A = -\frac{1}{5}$; $B = -2$ and $C = -\frac{16}{5}$;

hence $\frac{4AB+C}{A} = 8$ Ans.]

57. If $\alpha + \frac{1}{\alpha}$ and $2 - \beta - \frac{1}{\beta}$ $(\alpha, \beta > 0)$ .....

[Sol. $\alpha + \frac{1}{\alpha} \geq 2$

$\beta + \frac{1}{\beta} \Rightarrow -\beta - \frac{1}{\beta} \leq -2 \Rightarrow 2 - \beta - \frac{1}{\beta} \leq 0$



$\therefore$ one root is greater than or equal to 2 and other root is smaller than or equal to 0.

$x^2 - 2(a+1)x + a - 3 = 0$

(i) $D \geq 0$

$4(a+1)^2 - 4(a-3) \geq 0 \Rightarrow a^2 + 2a + 1 - a + 3 \geq 0 \Rightarrow a^2 + a + 4 \geq 0 \Rightarrow a \in R$

(ii) $f(0) \le 0 \Rightarrow a - 3 \le 0 \Rightarrow a \le 3$

(iii) $f(2) \le 0 \Rightarrow 4 - 4(a+1) + a - 3 \le 0$

$\Rightarrow -3a - 3 \le 0 \Rightarrow a \ge -1$

$\therefore a \in [-1, 3]$

Required sum = $-1 + 0 + 1 + 2 + 3 = 5$.

58. Let p_1 and p_2 be two values .....

[Sol. $f'(0^-) = \lim\limits_{h\to 0} \dfrac{f(-h) - f(0)}{-h}$

$= \lim\limits_{h\to 0} \dfrac{\dfrac{-h\, ln(1-h^2)}{\sec h - \cos h} - 0}{-h}$

$= \lim\limits_{h\to 0} \dfrac{ln(1-h^2)}{\dfrac{(1-\cos^2 h)}{\cos h}}$ **[ACC] [MPP]**

$= \lim\limits_{h\to 0} \dfrac{\dfrac{\ln(1-h^2)}{-h^2} \times -h^2}{\left(\dfrac{1-\cos h}{h^2}\right) h^2 (1+\cos h)}$

$= \lim\limits_{h\to 0} \dfrac{-h^2}{\dfrac{1}{2} \times 2h^2} = -1$ **New for 13th 2010**

$f'(0^+) = \lim\limits_{h\to 0} \dfrac{f(h) - f(0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{(p^2 - 3p + 1)h + h^2 - 0}{h}$

$= p^2 - 3p + 1$ **[13th, 05-09-2010, P-2]**

Using $f'(0^+) = f'(0^-)$ gives $p^2 - 3p - 1 = -1$ or $p^2 - 3p = 0$

$\Rightarrow$ $p = 0$ or 3 $\quad (p_1^2 + p_2^2) = 9$ **Ans.**]

59. Let f be a differentiable function on R .....

[Sol. $e^{-x} \int\limits_0^x f(t)\, dt = a \int\limits_0^x e^{-t} f(t)\, dt$

Differentiate both sides with respect to x, we get

$e^{-x} \cdot f(x) - \left(\int\limits_0^x f(t)\, dt \right) e^{-x} = a e^{-x} f(x)$

$\therefore f(x)(1 - a) = \int\limits_0^x f(t)\, dt$

Differentiate both sides with respect to x, we get

$(1 - a) f'(x) = f(x) \quad \therefore \dfrac{f'(x)}{f(x)} = \dfrac{1}{1-a}$

$\Rightarrow \quad ln\,(f(x)) = \left(\dfrac{1}{1-a}\right) x + C$

$\Rightarrow \quad f(x) = k \cdot e^{\frac{x}{1-a}}$

Also $f(0) = 0$

$\Rightarrow k = 0 \qquad \Rightarrow \quad f(x) = 0.$

So, $\left(f(e) + f(e^2)\right) = 0$ **Ans.**]

60. Let $f(x) = x \cos x$, $x \in \left[\dfrac{3\pi}{2}, 2\pi\right]$ and .....

[Sol. $f(x) = x \cos x, \quad x \in \left[\dfrac{3\pi}{2}, 2\pi\right]$

$\therefore \quad f'(x) = (-x \sin x + \cos x) > 0$,

$x \in \left[\dfrac{3\pi}{2}, 2\pi\right]$

$\Rightarrow$ f(x) is increasing function in $\left[\dfrac{3\pi}{2}, 2\pi\right]$.

Also, $f\left(\dfrac{3\pi}{2}\right) = 0$; $f(2\pi) = 2\pi$

So, $\int\limits_{\frac{3\pi}{2}}^{2\pi} f(x)\, dx + \int\limits_0^{2\pi} g(x)\, dx = 4\pi^2$ ..... (i)

Now, $\int\limits_{\frac{3\pi}{2}}^{2\pi} \underset{I}{x} \cdot \underset{\substack{II \\ (I.B.P)}}{(\cos x)}\, dx = 1 + \dfrac{3\pi}{2}$

$\Rightarrow \quad \int\limits_0^{2\pi} g(x)\, dx = \left(4\pi^2 - \dfrac{3\pi}{2} - 1\right)$]

PAPER - 02

PART- I (PHYSICS)

1. Velocity-time graph for a car moving .....

Sol. (A),(D) Velocity never goest to negative that means car never turns back hence displacement = Distance and D is also incorrect.

(B) Acceleration of car = $\frac{dv}{dt}$ = slope of curve it is zero at the top of the semicircle.

(C) Mean speed $= \frac{AUC}{time} = \frac{\pi ab/2}{2} = \frac{\pi}{4}$.

$a = 1\,sec$, $b = 1\,m/sec$ both are the radius.

2. From figure, select the correct alternative .....

Sol. In escaping condition. K.E. of particle is equal to negative of total potential energy i.e., T.E. is zero

Now total P.E. of particle of mass (m) is

$$\left(-\frac{GM_em}{\frac{r}{2}}\right)+\left(\frac{GM_mm}{\frac{r}{2}}\right)$$

Escaping K.E. of particle $= \frac{1}{2}mv_e^2$

In escaping condition

$$\frac{2GM_em}{r}+\frac{2GM_mm}{r}=\frac{1}{2}mv_e^2$$

$$v_e=\sqrt{\frac{4G}{r}(M_e+M_m)}$$

Choices a and b are right from above explanation.

3. A parallel plate capacitor with plate .....

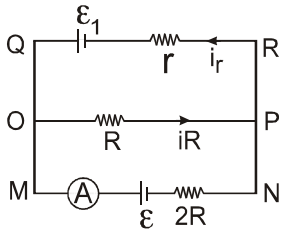
Sol. Due to induced charges on dielectric electric field reduces to $\frac{E_0}{K}$ and hence potential difference decreases in same proportion. This results in increase in capacitances by K times $\theta' = \theta\left(1-\frac{1}{K}\right)$.

4. Identify the correct options according .....

Sol. Since reading in Ammeter is zero, so the current (i) will only flow in the upper circuit.

where $I = \frac{\varepsilon_1}{R+r} = i_r = i_R$,

$\therefore$ (B) is correct.



Now

Here, $V_Q - V_R = V_O - V_P = V_M - V_N = \varepsilon$

also $V_Q - V_R = \varepsilon_1 - r\,i_r = \varepsilon_1 - r\frac{\varepsilon_1}{R+r} = \frac{\varepsilon_1 R}{R+r} = \varepsilon$

$\therefore \varepsilon_1 = \frac{\varepsilon(R+r)}{R}$

$\therefore$ (A) is correct and (C) is incorrect

$\because$ no current is flowing through 2R

$\therefore$ potential difference across 2R = 0

5. The two blocks A and B of equal mass .....

Sol. $a_A = (g\sin\theta - \mu_1 g\cos\theta)$

; $a_B = (g\sin\theta - \mu_2 g\cos\theta)$

If $\mu_1 > \mu_2$ then $a_A < a_B$ so both blocks remains in contact and their common acceleration is $a_A = a_B = \frac{2mg\sin\theta - (\mu_1+\mu_2)mg\cos\theta}{2m}$

$$= \frac{2g\sin\theta - (\mu_1+\mu_2)g\cos\theta}{2}$$

Hence (A) is correct and (C) is wrong.

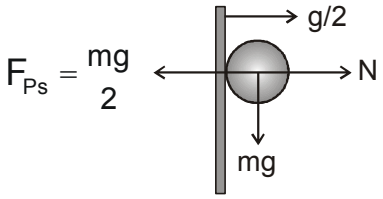
If $\mu_1 < \mu_2$ then $a_A > a_B$. Both blocks will slide with different acceleration and they will loose the contact and (D) is also wrong.

7. A uniform solid sphere rolls down .....

Sol. To prevent slipping $\alpha.r = a_C$ and static friction will act.

[a_c = linear acceleration of C.O.M. of sphere] Torque about COM

$\tau = f.r$



$F_{Ps} = \frac{mg}{2}$

$\tau = f.r = I\alpha = \frac{2}{5}.mr^2.\alpha$

$\alpha = \frac{5f}{2mr}$ .....(i)

$a_C = \frac{mg-f}{m} \Rightarrow a_C = \frac{mg-\frac{\mu mg}{2}}{m}$

$\Rightarrow a_C = g\frac{(2-\mu)}{2}$ .....(ii)

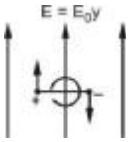
From (i) and (i) $\frac{mg-f}{m} = \frac{5f}{2m} \Rightarrow f = \frac{2mg}{7}$

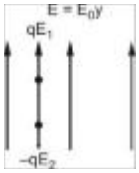
$\because \quad f \le \mu N \Rightarrow \frac{2mg}{7} \le \mu \frac{mg}{2}$

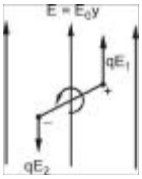
or $\mu \ge \frac{4}{7}$

8. An electric dipole is placed in electric .....

Sol. Following 3 diagrams explain why (B), (C), (D) are correct and (A) is incorrect







In figure 1, $F = 0, \tau \ne 0$

In figure 2, $F_{net} \ne 0$ but $\tau = 0$

In figure 3, $F_{net} \ne 0$ & $\tau_{net} \ne 0$

9. Which of the following statements .....

Sol. Since focal length of lens depends on refractive index of lens with respect to medium

as $\frac{1}{f} = \left(\frac{n_{lens}}{n_{surrounding}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

so f will increase.

Focal length of mirror does not depends on medium.

11. The particle comes to rest in a time .....

Sol. (A) Given $a = -kv^{1/2}$ or $\frac{dv}{dt} = -kv^{1/2}$

Thus $v^{\frac{1}{2}}dv = -k\,dt$

Integrating, we have

$\int v^{-1/2}dv = -k\int dt$ or

$2v^{1/2} = -kt + C$ (i)

where C is the constant of integration.

Given that at $t = 0$, $v = u$.

Using this in (i), we get $2u^{1/2} = C$

Using this value of C in (i), $2(v^{1/2} - u^{1/2}) = -kt$

(ii) Let τ be the time taken by the particle to come to rest. Then, v = 0 at $t = \tau$. Using this in (ii), we get

$2(0 - u^{1/2}) = -k\tau$ or $\tau = \frac{2u^{1/2}}{k}$ (iii)

12. The distance covered by the particle .....

Sol. (D)

To find the distance 's' covered in this time, we use Eq. (i) to get $v^{1/2} = u^{1/2} - \frac{kt}{2}$

Squaring, we have $v = u - ktu^{1/2} + \frac{k^2t^2}{4}$

But $v = \frac{ds}{dt}$

Therefore, $\frac{ds}{dt} = u - ktu^{1/2} + \frac{k^2t^2}{4}$

Integrating from t = 0 to $t = \tau$, we have

$s = \left|ut - \frac{ku^{1/2}t^2}{2} + \frac{k^2t^3}{12}\right|_0^{\tau}$

Substituting the value of t from (iii) in (iv), we get $s = u\tau - \frac{1}{2}ku^{1/2}\tau^2 + \frac{1}{12}k^2\tau^3$ or

$s = \frac{2u^{3/2}}{3k}$.

13. The distance between block and .....

14. The least tension in string .....

Sol **(13 to 14).** Since $u = \sqrt{2gL}$, the highest point to which the block shall reach is $\frac{u^2}{2g}$ = L distance above its initial position. Hence at highest point, the thread has rotated by $\frac{\pi}{3}$ = 60° and the block is at same horizontal level as centre of cylinder as shown. $x = \frac{R}{\sin 30} = 2R$

$T = mg\cos 60° = \frac{mg}{2}$

15. The velocity of efflux .....

Sol. Since area of a hole is very small in comparison to base area A of the cylinder.

Therefore, velocity of liquid inside the cylinder is negligible.

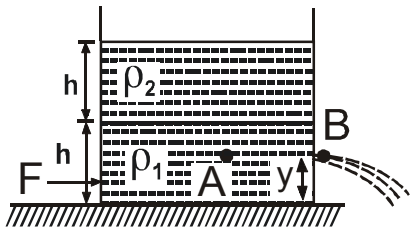
Let velocity of efflux be v and atmospheric pressure P_0.

Consider two points A (inside the cylinder)

and B (just outside the hole) in the same horizontal line as shown in figure.

Pressure at A,

$P_A = P_0 + h\rho_2 g + (h-y)\rho_1 g$



Pressure at B, $P_B = P_0$

According to Bernoulli's theorem,

Pressure energy at A = Pressure energy at B + Kinetic energy at B.

$\therefore P_A = P_B + \frac{1}{2}\rho_1 v^2$

$\therefore \quad v = 4\ ms^{-1}$

16. Minimum and maximum values .....

Sol. Total mass of the liquid in the cylinder is

$m = Ah\rho_1 + Ah\rho_2 = 450$ Kg

Limiting friction = $\mu mg = 45$ N.

F < limiting friction, therefore, minimum force required is zero.

Consider free body diagram for maximum vertical forces, N = mg

Now considering horizontal forces,

$F_{max} = F + \mu N$ or $F_{max} = 52.2$ N.

17. The magnitude of the force is .....

Sol. For maximum force,

$\frac{dF}{dt} = 0 \Rightarrow a - 2bt = 0 \Rightarrow t_1 = \frac{a}{2b}$

$F_{max} = at_1 - b^2{}_1 = a \times \frac{a}{2b} - b \times \left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$

18. The maximum velocity v_{max} attained .....

Sol. Since force is always positive, velocity will continuously increase.

So, velocity is maximum when

F = 0 i.e. at t = a/b

Now, Change in momentum = Impulse = mv – 0 = mv

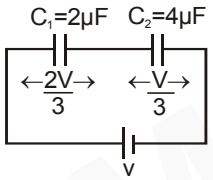
$= \int_0^{\frac{a}{b}} (at - bt^2)dt = \left[a\frac{t^2}{2} - b\frac{t^3}{3}\right]_0^{\frac{a}{b}} = \frac{a^3}{2b^2} - \frac{a^3}{3b^2} = \frac{a^3}{6b^2}$

$\therefore \qquad v_{max} = \frac{a^3}{6mb^2}$

20. In the given figure, the separation .....

Sol. U_i for $C_1 = C_1 \times \frac{4}{9}V^2 \times \frac{1}{2} = \frac{4V^2}{9}$

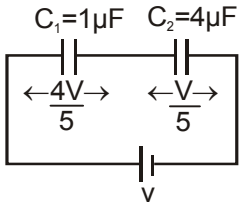
U_i for $C_2 = 4 \times \frac{V^2}{9} \times \frac{1}{2} = \frac{2V^2}{9}$



When separation of plates of C_1 get doubled, its capacity becomes half :

U_f for $C_1 = 1 \times \frac{16V^2}{25} \times \frac{1}{2} = \frac{8V^2}{25}$

U_f for $C_2 = 4 \times \frac{V^2}{25} \times \frac{1}{2} = \frac{2V^2}{25}$



PART- II (CHEMISTRY)

21. Which of the following is/are.....

Sol. thermodynamics stabilty P(black) > p(Red > P(W)

$(\Delta_f H^\circ)_{P(w)} = 0$

$P(w) \rightarrow P(R)/P(B)$

process is exothermic also entropy decreases due to polymerisation

22. Identify the correct .....

Sol. $K = Ae - Ae^{-Ea/RT}$

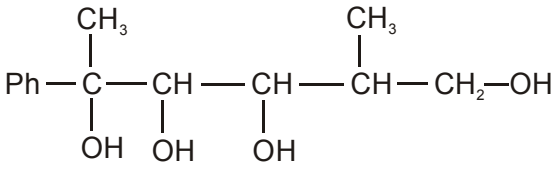
$E_a \uparrow, K \downarrow$

* Concentration of reactant, product and catalyst may appear in rate law

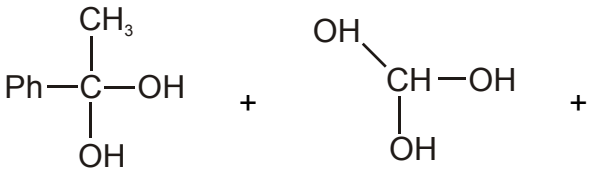
* molecularity includes only no. of reactant molecules involved in R.D.S forming transition state

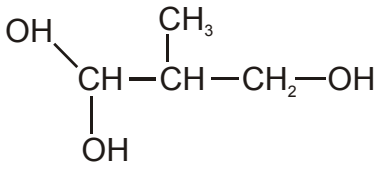
23. The product(s) which is/are .....

Sol.



$\xrightarrow{HIO_4}$



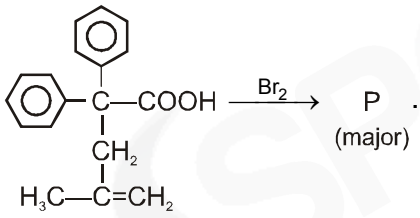


Hence products are

24. Which of the .....

Sol. all type of work depends upon path hence work is path function. Entropy, Enthalpy, internal energy, free energy are state function

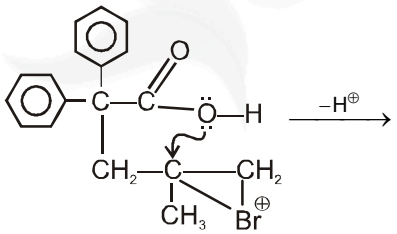
25.

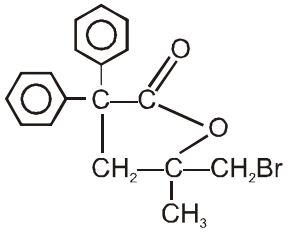


.....

(major)

Sol.





+ HBr

26. $2CaSO_4(s) \rightleftharpoons 2CaO(s)$ .....

Sol. (A) As reaction is endothermic therefore it will go in the forward direction hence moles of CaO will increase.

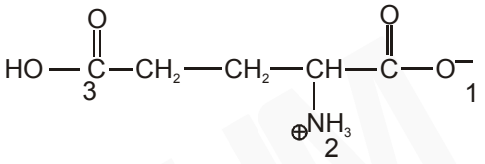
(C) With the increase or decrease of volume particle pressure of the gases will remain same.

(D) Due to the addition of inest gas at constant pressure reaction will proceede in the direct in which more number of gaseous moles are formed.

27. For the cationic form of .....

Sol. 1 $\rightarrow$ is strongest acid as $\overset{\oplus}{N}H_3$ group with – I effect is close to 1 and also COOH is stronger acid than $-\overset{\oplus}{N}H_3$ group

Z witter ion form of the amino acid is



hence isoelectric point = $\frac{pk_{a_1} + pk_{a_2}}{2}$

$= \frac{2.2 + 4.2}{2}$

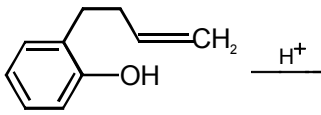
$= 3.2$

28. When I_2 is added to .....

Sol. If excess of chlorine water is added, I_2 is oxidised to iodic acid (colourless)

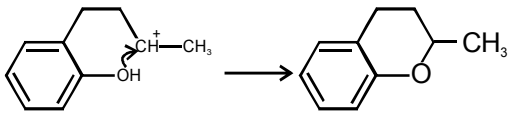
$I_3^- + 8Cl_2 \uparrow + 9H_2O \longrightarrow 3IO_3^- + 16Cl^- + 18\ H^+$

29.



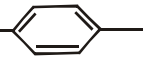
.....

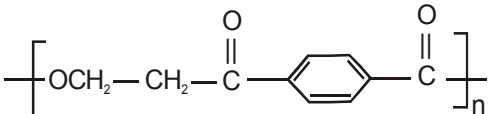
Sol.



30. which of the following .....

Sol. It is condensation co-polymer and polyester

$nHOH_2C - CH_2OH + nHOOC$  $COOH \rightarrow$



Terylene or dacron

Ethylene glycol (Ethane-1,2–diol)

Terephthalic acid (Benzene- 1,4 - di) carboxylic acid

Paragraph for Question Nos. 31 & 32

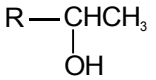
32. Which of the following .....

Sol. D.B.E of (A) = $10 - \frac{12}{2} + 1 = 5$

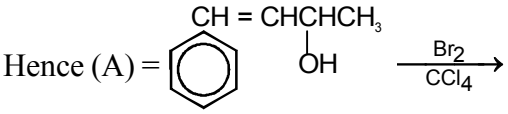
Since (A) declolories bromine water and adds only molecules of bromine, (A) should have a double bond

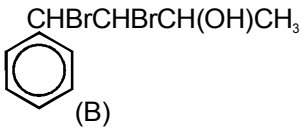
$\because$ B.D.E of A = 5; it should also have a benzene ring

(A) gives positive iodo form test and should have structure of the type

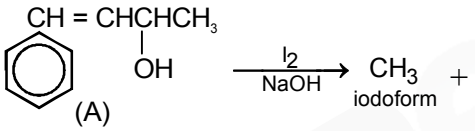


(R = alkyl/aryl group)

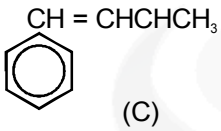




(B)



(A)



(C)

Paragraph for Question Nos. 33 & 34

34. Identify the correct.....

Sol. Rb gives red colour in flame

$Rb + O_2 \rightarrow RbO_2$

$RbO_2 + H_2O \rightarrow RbOH + H_2O_2 + O_2(g)$

On a large scale it is prepared mainly by Ostwald's process.

This method is based upon catalytic oxidation of NH_3 by atmoshperic oxygen.

$$4\ NH_3\ (g) + 5\ O_2\ (g)\ (\text{from air}) \xrightarrow[500\ K,\ 9\ bar]{Pt/Rh\ \text{gauge catalyst}} 4\ NO\ (g) + 6\ H_2O\ (g)$$

Nitric oxide thus formed combines with oxygen giving NO_2.

$2\ NO\ (g) + O_2\ (g) \rightleftharpoons 2\ NO_2\ (g)$

Nitrogen dioxide so formed, dissolves in water to give HNO_3.

$3\ NO_2\ (g) + H_2O\ (\ell) \longrightarrow 2\ HNO_3\ (aq) + NO\ (g)$

NO thus formed is recycled and the aqueous HNO_3 can be concentrated by distillation upto ~ 68% by mass. Further concentration to 98% can be achieved by dehydration with concentrated H_2SO_4.

Nitrous acid and nitrites are good oxidizing agents and convert iodides to iodine, ferrous salts to ferric, stannous to stannic and sulphites to sulphates eg.

Paragraph for Questions 35 and 36

36. For the process which is true**.....**

Sol. $q = 0 \quad \Rightarrow \quad \Delta U = w$

$nC_V\ (T_2 - T_1) = -P_{ext.}\ (V_2 - V_1)$

$$C_V\ (T_2 - 300) = -1\left(\frac{R \times T_2}{1.2} - \frac{R \times 300}{6}\right)$$

$$\Rightarrow \frac{5}{2}(T_2 - 300) = \frac{300}{6} - \frac{T_2}{1.2}$$

$$5T_2 - 1500 = 100 - \frac{5T_2}{3}$$

$$\frac{20T_2}{3} = 1600 \quad \therefore\ T_2 = 240\ K$$

$\Delta U = nC_V\Delta T$

$$= 10 \times \frac{5R}{2}\ (240 - 300)$$

$= -1500\ R$

$\Delta H = \gamma\ \Delta\ U$

$$= \frac{7}{5} \times (-1500R) = -2100\ R$$

$$\Delta S_{sys.} = nC_V \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1}$$

$$= 10 \times \frac{5R}{2} \ln\left(\frac{240}{300}\right) + 10\ R \ln\left(\frac{T_2/P_2}{T_1/P_1}\right)$$

$$= 25\ R \ln 0.8 + 10R \ln\left(\frac{240/1.2}{300/6}\right)$$

$= 25\ R \ln 0.8 + 10\ R \ln 4 = 9R.$

$\therefore$ **2109 Ans**

Paragraph for Questions 37 and 38

38. Aprroximate CrO_4^{2-} .....

Sol As solubility of Ag_2CrO_4 in 0.1 M $AgNO_3$ is 10^{-10} M hence $k_{sp} = (0.1)^2 \times 10^{-10} = 10^{-12}$

$$2AgNO_3 + K_2CrO_4 \rightleftharpoons Ag_2CrO_4 + KNO_3$$

m moles at T = 0

40×0.1	60×0.05	0	0
=4	=3	0	0

Let us first see whether precipitation occurs or not on mixing.

$[Ag^+]^2\ [CrO_4^{2-}] = K_{sp} = 10^{-12}$

Also after mixing

$$[Ag^+]^2[CrO_4^{2-}] = \left[\frac{4}{100}\right]^2\left[\frac{3}{100}\right] = 4.8 \times 10^{-5} > K_{sp}$$

Thus, precipitation will take place

$2AgNO_3 + K_2CrO_4 \rightleftharpoons Ag_2CrO_4 + 2KNO_3$

m mole before mixing			
4	3	0	0
m mole after mixing			
0	(3— 2)	2	4
x	1	2	4

$$[CrO_4^{2-}] = \frac{1}{100} = 0.01\ M$$

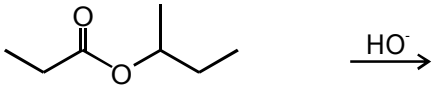
$K_{sp} = [Ag^+]^2 \times [CrO_4^{2-}]$

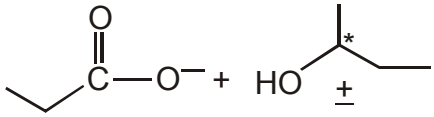
$10^{-12} = (x)^2 \times 0.01$

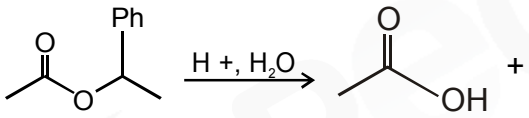
$x = 10^{-5}$

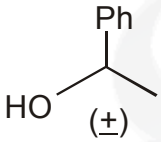
39. COLUMN -I .....

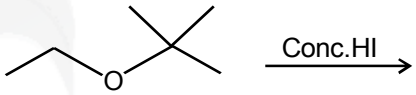
Sol. Reaction proceeds via nucleophilic aromatic substitution or addition elemination mechanism

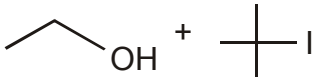
(A) 

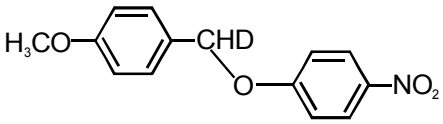


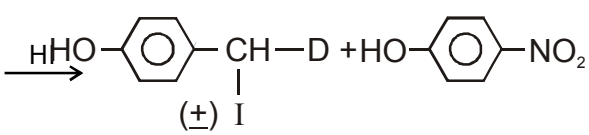
(B) 



(C) S_N1 



(D) 



+ CH_3OH

40. Match the following

Sol. $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + NO_2 + O_2$

$Cu + Conc\ HNO_3 \longrightarrow Cu(NO_3)_2 + NO_2 + H_2O$

$HNO_3 + P_4O_{10} \longrightarrow N_2O_5 + HPO_3$

$AgNO_3 \xrightarrow{\Delta} Ag + NO_2 + O_2$

PART- III (MATHS)

41. The number of ways in which 18 .....

[Sol. Giving one to each we have 13 coins to be distributed in 5 children.

O O O O O O O O O O O O O O O Ø Ø Ø Ø

using beggar $^{17}C_4$ ways

Now, give 8 more to A or B or C or D or E we have number of ways to be rejected

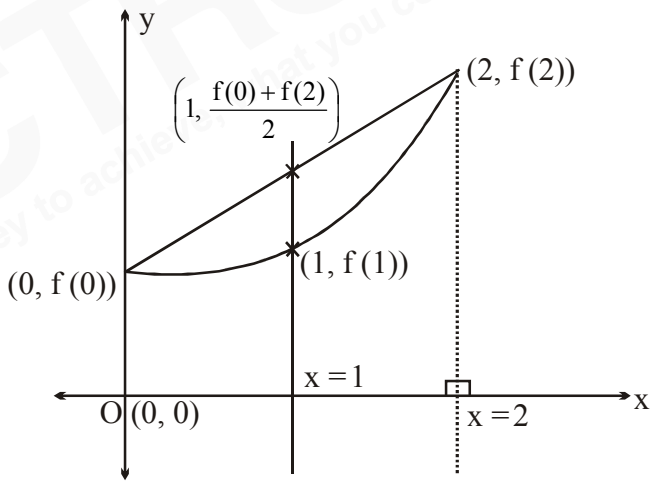
O O O O O Ø Ø Ø Ø = $5 \cdot {}^9C_4$

Required ways = $^{17}C_4 - 5 \cdot {}^9C_4$

= 2380 – 630 = 1750 ways.]

42. Let f (x) be a twice differentiable .....

[Sol.



Now, verify alternatives.]

43. The value of .....

[Sol. $\frac{x}{\sin^{-1}x} < 1 \Rightarrow \left[\frac{x}{\sin^{-1}x}\right] = 0$

$\Rightarrow \quad \text{sgn}\left(1 - \left[\frac{x}{\sin^{-1}x}\right]\right) = 1$

$$\lim_{x\to 0} \frac{\sin^3 x - x^3}{x.\tan^2 x.\sin(\pi\cos x)}$$

$$= \lim_{x\to 0} \frac{(\sin x - x)\,(\sin^2 x + x\sin x + x^2)}{x \cdot \frac{\tan^2 x}{x^2} \cdot x^2\ \sin(\pi - \pi\cos x)}$$

$$= \lim_{x\to 0} \frac{(\sin x - x)\left(\frac{\sin^2 x}{x^2} + \frac{x\sin x}{x^2} + \frac{x^2}{x^2}\right)}{x\left(\frac{\sin(\pi - \pi\cos x)}{\pi - \pi\cos x}\right)\frac{\pi(1-\cos x)}{x^2}\cdot x^2}$$

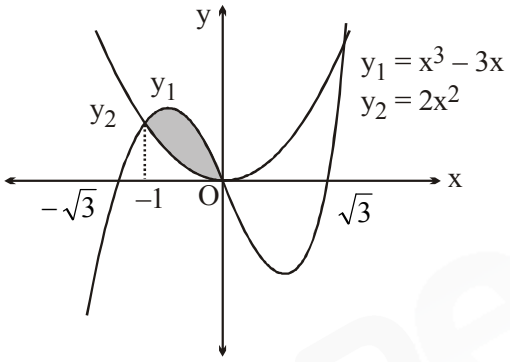
$= -\frac{1}{6}\times\frac{3}{\pi}\times 2 = \frac{-1}{\pi}$ **Ans.**]

44. The area bounded by the curves .....

[Sol. Solving y_1 and y_2, we get $x = 0, -1, 3$

Required area $= \int_{-1}^{0} (y_1 - y_2)\,dx$

$= \int_{-1}^{0} (x^3 - 3x - 2x^2)\,dx$



$= \left(\frac{x^4}{4} - \frac{3x^2}{2} - \frac{2x^3}{3}\right)_{-1}^{0} = 0 - \left(\frac{1}{4} - \frac{3}{2} + \frac{2}{3}\right)$

$= -\left(\frac{3-18+8}{12}\right) = \frac{7}{12}$. **Ans.**]

45. Let $L = \lim_{x\to 1} \frac{\sin(6\cos^{-1} x)}{\sqrt{1-x^2}}$ .....

[Sol. $L = 6; M = 18$]

46. If α is a real number for which .....

[Sol. $f(x) = \ln\left(3\cos^{-1}\left(\frac{3x}{7}\right) - \pi\right)$

$3\cos^{-1}\left(\frac{3x}{7}\right) - \pi > 0 \Rightarrow \cos^{-1}\left(\frac{3x}{7}\right) > \frac{\pi}{3}$

$\Rightarrow \frac{3x}{7} < \frac{1}{2} \Rightarrow x < \frac{7}{6}$

and $-1 \le \frac{3x}{7} \le 1 \Rightarrow \frac{-7}{3} \le x \le \frac{7}{3}$

$\therefore x \in \left[\frac{-7}{3}, \frac{7}{6}\right)$.

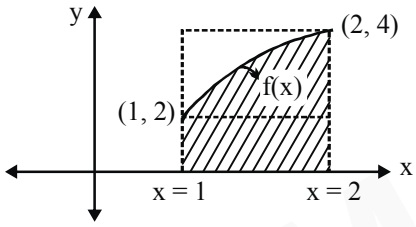
47. A pair of dice is rolled. Two events E .....

[Sol. $E = \{(1,1)(2,2),(3,3),(4,4),(5,5),(6,6)\}$
$F = \{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$

$\therefore P(E) = \frac{1}{6}$, $P(F) = \frac{1}{6}$ and $P(E \cap F) = 0$.]

48. Let f be a strictly increasing .....

[Sol.



Now, verify alternatives.]

49. The value of definite .....

[Sol. Let $I = \int_{1/3}^{2/3} \frac{\ln(x)}{\ln x + \ln(1-x)}\,dx$ .....(1)

Also, $I = \int_{1/3}^{2/3} \frac{\ln(1-x)}{\ln(1-x) + \ln x}\,dx$

(using king property) .....(2)

$\therefore$ On adding (1) and (2), we get

$2I = \int_{1/3}^{2/3} 1\cdot dx \Rightarrow I = \frac{1}{6}$. **Ans.**]

50. Let f(x) be a continuous and .....

[Sol. Given $\int_{-2T}^{a+5T} f(x)dx = 19$

$\Rightarrow \int_{-2T}^{5T} f(x)dx + \int_{5T}^{a+5T} f(x)dx$

$= 19 \Rightarrow 7\int_0^T f(x)\,dx + \int_0^a f(x)dx = 19$

$\Rightarrow \int_0^a f(x)\,dx = 19 - 7\times 2 = 19 - 14 = 5$. **Ans.**]

51. The number of solution(s) of the .....

52. If $f(x) + g(x) = \frac{\pi}{8}$ then x equals .....

Sol Putting $x = \tan\theta$ $\because x \in R - \{0\}$

$\Rightarrow \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\}$

$f(x) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

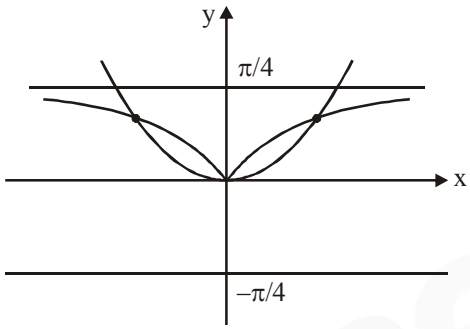
$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

$g(x) = \text{cosec}^{-1}\left(\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}\right)$

$= \text{cosec}^{-1}(\text{cosec}\,\theta) = \theta = \tan^{-1}x$

(i) $f(x) - g(x) = -\frac{1}{2}\tan^{-1}x$

$x^2 = |f(x) - g(x)|$



clearly number of solutions are 2.

(ii) $f(x) + g(x) = \frac{\pi}{8}$

$\frac{3}{2}\tan^{-1}x = \frac{\pi}{8} \Rightarrow \tan^{-1}x = \frac{\pi}{12} \Rightarrow x = 2-\sqrt{3}$

53. The value of (a + b) is equal to .....

54. Number of integral values of λ so that .....

[Sol. $f(x) = e^{\lim\limits_{n\to\infty}\left(\frac{\sin x\left(e^{1/n}-1\right)}{1/n}\right)} = e^{\sin x}$

$\therefore a = \frac{2}{11} \times [0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10] = 10$

$b = \lim\limits_{x\to0}\left(\frac{x^2}{\left[\frac{\tan x}{x}\right]-\cos x}\right)$

$= \lim\limits_{x\to0}\left(\frac{x^2}{1-\cos x}\right) = 2$

(i) $\therefore$ (a + b) = 12

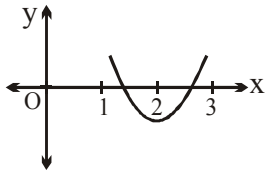
(ii) $bx^2 - b^2x + \lambda = 0$

$2x^2 - 4x + \lambda = 0$

$f(1) \cdot f(2) < 0$

$(\lambda - 2)(\lambda) < 0$

$\Rightarrow \lambda \in (0, 2)$ and



$f(2) \cdot f(3) < 0$

$(\lambda)(6 + \lambda) < 0 \Rightarrow \lambda \in (-6, 0)$

Hence, no possible values of λ.

55. For $\lambda = 1$, if $f(3x^2 - 2x + 1) < f$ .....

56. If f (x) is increasing for all $x \in R$ then .....

[Sol. $f(x) = \cos 2x + 2x\lambda^2 + (2\lambda + 1)(\lambda - 1)x^2$

(i) For $\lambda = 1$

$f(x) = \cos 2x + 2x$

$f'(x) = -2\sin 2x + 2 = 2(1 - \sin 2x)$

$f'(x) \geq 0 \Rightarrow f(x)$ is increasing $\forall x \in R$

$f(3x^2 - 2x + 1) < f(x^2 - 2x + 9)$

$\Rightarrow 3x^2 - 2x + 1 < x^2 - 2x + 9$

$\Rightarrow 2x^2 - 8 < 0 \quad \Rightarrow x \in (-2, 2)$

(ii) $f'(x) = -2\sin 2x + 2\lambda^2 + (2\lambda + 1)(\lambda - 1) \cdot 2x$

for f (x) increasing $\forall x \in R$, coefficient of x must be zero.

$\therefore \lambda = \frac{-1}{2}$ or 1

For $\lambda = \frac{-1}{2}$, $f'(x) = \frac{1}{2} - 2\sin 2x$ (rejected)

For $\lambda = 1$, $f'(x) = 2 - 2\sin 2x$

$= 2(1 - \sin 2x) \geq 0 \quad \forall x \in R$

57. The number of solutions(s) of the .....

58. If area enclosed by $y = f_1(x)$, $y = e^x$ and .....

[Sol. $\frac{ydx + dy}{e^x y^2} = -dy \Rightarrow \frac{e^{-x}}{y}dx + e^{-x}\frac{dy}{y^2} = -dy$

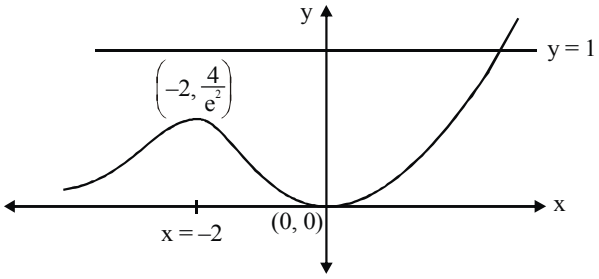
$\Rightarrow -d\left(\frac{e^{-x}}{y}\right) = -dy$

$\therefore \quad \frac{e^{-x}}{y} = y + \lambda$

(i) $f_1(x) \cdot f_2(x) + x^2 = 0$

$e^{\frac{-x}{2}} \cdot \left(-e^{\frac{-x}{2}}\right) + x^2 = 0 \Rightarrow -e^{-x} + x^2 = 0$

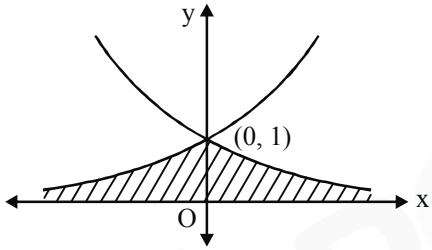
$\Rightarrow x^2 = e^{-x} \Rightarrow e^x \cdot x^2 = 1$



Graph of $f(x) = x^2e^x$

Clearly, there are three solutions.

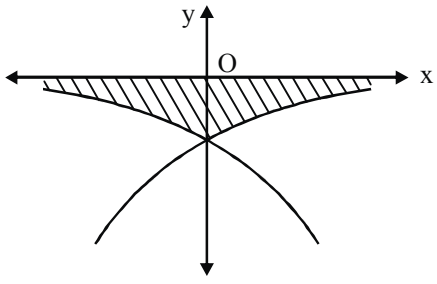
(ii) Area enclosed by $y = f_1(x)$, $y = e^x$ and the x-axis



$A_1 = \int_{-\infty}^{0} e^x dx + \int_{0}^{\infty} e^{\frac{-x}{2}} dx$

$= \left(e^x\right)_{-\infty}^{0} + \left(-2\left(e^{\frac{-x}{2}}\right)_0^{\infty}\right) = 1 - 0 - 2(0 - 1) = 3$

Area enclosed by $y = f_2(x)$; $y = -e^x$ and the x-axis.



$A_2 = \left|\int_{-\infty}^{0} (-e^x) dx + \int_{0}^{\infty} \left(-e^{\frac{-x}{2}}\right) dx\right| = A_1 = 3$

$\therefore \quad A_1 + A_2 = 6.$]

59. **Column-I** **Column-II**

(A) If

$$\lim_{n\to\infty} \frac{(1^3 + 2^3 + 3^3 + \ldots\ldots + n^3)(1^5 + 2^5 + 3^5 + \ldots\ldots + n^5)}{(1^9 + 2^9 + 3^9 + \ldots\ldots + n^9)}$$

[Sol.

(A) $$\lim_{n\to\infty} \frac{(1^3 + 2^3 + 3^3 + \ldots\ldots + n^3)(1^5 + 2^5 + \ldots\ldots + n^5)}{(1^9 + 2^9 + \ldots\ldots + n^9)}$$

$$= \lim_{n\to\infty} \frac{\left(\sum_{r=1}^{n} r^3\right)\left(\sum_{r=1}^{n} r^5\right)}{\sum_{r=1}^{n} r^9}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}\left(\sum_{r=1}^{n}\left(\frac{r}{n}\right)^3\right)\frac{1}{n}\left(\sum_{r=1}^{n}\left(\frac{r}{n}\right)^5\right)}{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^9}$$

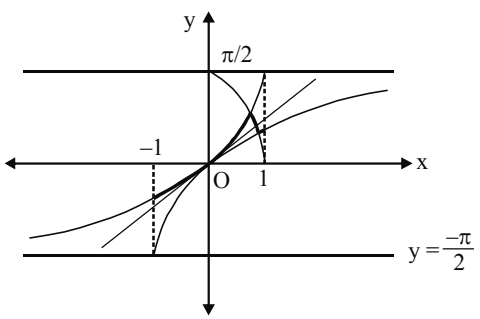
$$= \frac{\left(\int_0^1 x^3 dx\right)\left(\int_0^1 x^5 dx\right)}{\int_0^1 x^9 dx} = \frac{\frac{1}{4} \times \frac{1}{6}}{\frac{1}{10}} = \frac{5}{12}$$

$\Rightarrow |p - q|_{least} = 07$

(B) Number of points where f (x) is non-derivable is 2 i.e. $x = \frac{1}{\sqrt{2}}, \sqrt{\frac{\sqrt{5}-1}{2}}$

Here, $f(x) = \begin{cases} \tan^{-1} x; & -1 \le x < 0 \\ \sin^{-1} x; & 0 \le x \le \frac{1}{\sqrt{2}} \\ \cos^{-1} x; & \frac{1}{\sqrt{2}} < x < \sqrt{\frac{\sqrt{5}-1}{2}} \\ \tan^{-1} x; & \sqrt{\frac{\sqrt{5}-1}{2}} \le x \le 1 \end{cases}$

Note that f (x) is derivable at x = 0.



(C) $f(x) = \sqrt{\log_2\left(\frac{10x-4}{4-x^2}\right)-1}$

$\log_2\left(\frac{10x-4}{4-x^2}\right) - 1 \geq 0 \Rightarrow \frac{10x-4}{4-x^2} \geq 2$

$\Rightarrow \frac{10x-4-8+2x^2}{4-x^2} \geq 0 \Rightarrow \frac{x^2+5x-6}{4-x^2} \geq 0$

$\Rightarrow \frac{(x+6)(x-1)}{(x+2)(x-2)} \leq 0 \Rightarrow x \in [-6,-2) \cup [1,2)$

$\therefore$ Number of integers in the domain = 5

Required probability = $\frac{5}{13} \equiv \frac{a}{b}$

$\Rightarrow$ (b – a) = 13 – 5 = 8

(D) $f(x) = c\,x\,e^{-x} - \frac{x^2}{2} + x$,

we must have, $f'(x) \leq 0 \ \forall \ x \leq 0$

$\Rightarrow$ $(1-x)\ (c\,e^{-x}+1) \leq 0 \ \forall \ x \leq 0$

$\Rightarrow$ $c \leq -e^x \ \forall \ x \leq 0$

so, $c \in (-\infty, -1]$

$\Rightarrow$ Least value of $c^2 = 1$]

60. Column – I Column – II

(A) If $\lim\limits_{x\to\frac{\pi}{2}} \frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$.....

[Sol.

(A) $\lim\limits_{x\to\frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{(\pi-2x)^3}$

$x = \frac{\pi}{2} + h \quad \Rightarrow \quad \frac{x}{2} = \frac{\pi}{4} + \frac{h}{2}$

$= \lim\limits_{h\to 0} \frac{-\tan\frac{h}{2}(1-\cos x)}{-8h^3}$

$= \lim\limits_{h\to 0} \frac{\left(\tan\frac{h}{2}\right)(1-\cos x)}{\frac{h}{2}\times 16\ \ h^2} = \frac{1}{32}$

$\Rightarrow$ k = 32 **Ans.**

(B) Domain is $\frac{\pi^2}{16} - x^2 \geq 0$

Hence, $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

Since, f(x) is even

$\therefore$ In $\left[0, \frac{\pi}{4}\right]$ range of f(x) is $\in \left[0, \frac{3}{\sqrt{2}}\right]$

$\Rightarrow$ a = 0, b = $\frac{3}{\sqrt{2}}$

Hence $2(a^2 + b^2) = 2\left(0 + \frac{9}{2}\right) = 9$ **Ans.**

(C) 2 cos x(cosec x – 2) = (cosec x – 2)

(2 cos x – 1) (cosec x – 2) = 0

$\therefore$ cos x = $\frac{1}{2}$ or sin x = $\frac{1}{2}$

$\therefore$ sum = $3\pi = \frac{12\pi}{4} \equiv \frac{k\pi}{4}$

$\Rightarrow$ k = 12 **Ans.**]

(D) Given, $(e-1)e^{xy} + x^2 = e^{x^2+y^2}$

Differentiate both sides with respect to x,

we get

$(e-1)\cdot e^{xy}\cdot\left(x\cdot\frac{dy}{dx}+y\right) + 2x$

$= e^{x^2+y^2}\cdot\left(2x+2y\cdot\frac{dy}{dx}\right)$

put x = – 1, y = 0;

we get $(e-1)\cdot\left(\frac{dy}{dx}\right) + 2 = e\,(2+0)$

$\Rightarrow \left.\frac{dy}{dx}\right|_{(1,0)} = 2$

* * *