Sample Paper-02
Mathematics
Class – XII

Time allowed: 3 hours Maximum Marks: 100

General Instructions:

a) All questions are compulsory.

b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.

c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

d) Use of calculators is not permitted.

Section A

1. Give example of a function which is neither one-one nor onto .
2. Calculate the direction cosines of the vector $\vec{a} = 3i - 2j + 5k$.
3. Let L be the set of all lines in a plane and R be the relation in L defined as R={(L_1,L_2):L_1 is parallel to L_2}. Is L reflexive?
4. Suppose X is a 2x3 matrix, Z is a 5x3 matrix. Find the order of Y such that both XY and YZ are well defined.
5. Find the area of the triangle with vertices at the points (1,0),(6,0),(4,3).
6. Find x such that $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix}$

Section B

7. Evaluate $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right)$
8. Show that the points A,B and C with position vectors $\vec{a} = 3i - 4j - 4k, \vec{b} = 2i - j + k, \vec{c} = i - 3j - 5k$ form the vertices of a right angled triangle.
9. The probability of solving a specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that (a) problem is solved (b) exactly one of them solves the problem.
10. Find all points of discontinuity of the function f where f is defined by:

$$f(x) = \begin{cases} x^3 - x + 1, x \le -3 \\ -2x, -3 < x < 3 \\ 3x + 2, x \ge 3 \end{cases}$$

11. Solve the differential equation $x\frac{dy}{dx} - y + x\operatorname{cos}ec\left(\frac{y}{x}\right) = 0,\ y(1) = 0$

12. Using properties of determinants prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

13. A poisonous substance is dropped in a lake next to a village. The waves move is circles at a speed of 2cm per second. At the instant when radius of the circular wave is 14cm, evaluate how fast the enclosed area is increasing. Discuss two harmful consequences of polluting water bodies.

14. Show that if $f : A \to B$ and $g : B \to C$ are onto, then $g \circ f : A \to C$ is onto.

15. Verify Rolle's theorem for $f(x)=x^2+2x-8$, $x\in[-4,2]$.

16. Show that $\left(\left|\vec{a}\right|\vec{b}+\left|\vec{b}\right|\vec{a}\right).\left(\left|\vec{a}\right|\vec{b}-\left|\vec{b}\right|\vec{a}\right)=0$

17. Integrate $\int \frac{dx}{x(x^4-1)}$.

18. Find the distance between the lines l_1 and l_2 given by :

$\vec{r}=(i+3j-2k)+\lambda(2i-3j+k)$

$\vec{r}=(2i+4j-k)+\mu(2i-3j+k)$

19. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2i+2j-3k)=7, \vec{r}.(2i+5j+3k)=9$ and the point $(2,1,3)$

Section C

20. A factory can hire two tailors A and B in order to stich pants and shirts. Tailor A can stich 6 shirts and 4 pants in a day. Tailor B can stich 10 shirts and 4 pants in a day. Tailor A charges 15 per day and tailor B charges 20 per day. The factory has to produce minimum 60 shirts and 32 pants. State as a linear programming problem and minimize the labour cost.

21. Find the area of the region included between the two parabolas $y^2=4ax$ and $x^2=4ay$, $a>0$.

22. Bag X contains 2 white and 3 red balls. Bag Y contains 5 white and 4 red balls. Bag Z contains 2 white and 3 red balls. A ball is drawn at random from one of the bags and it is found to be red. What is the probability that it is drawn from bag Y?

23. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

24. Solve the following system of equations using matrix method

$\frac{3}{x}-\frac{2}{y}+\frac{3}{z}=8$

$\frac{2}{x}+\frac{1}{y}-\frac{1}{z}=1$

$\frac{4}{x}-\frac{3}{y}+\frac{3}{z}=4$

25. If x=a(cost+tsint), y=a(sint-tcost), find $\frac{d^2y}{dx^2}$

26. $\int_0^{\pi}\frac{xdx}{4\cos^2 x+9\sin^2 x}$