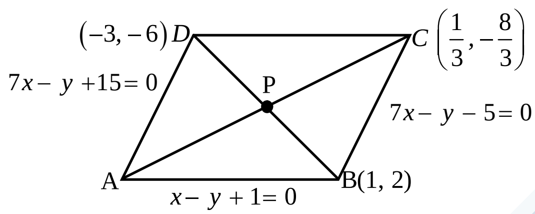
Joint Entrance Exam/JEE Mains 2016 Code – F

PART-A	CHEMISTRY

Coming Soon

PART-B	MATHEMATICS

31.(2)



32.(1) $a+d,\ a+4d,\ a+8d$

$(a+4d)^2=(a+d)(a+8d)$

$a^2+16d^2+8ad=a^2+9ad+8d^2$

$8d^2=ad$

$a=8d,\ d\neq 0$

$r=\dfrac{a+4d}{a+d}=\dfrac{12d}{9d}=\dfrac{4}{3}$

33.(4) $\left(2t^2,4t\right)\ (0,-6)$

$F(t)=4t^4+(4t+6)^2$

$=4\left(t^4+4t^2+9+12t\right)$

$=4\left(t^4+4t^2+12t+9\right)$

$F'(t)=4\left(4t^3+8t+12\right)=0$

$\Rightarrow\quad t^3+2t+3=0$

$t=-1$

$x^2+y^2-4x+8y+12=0$

34.(3) $\begin{vmatrix}1 & \lambda & -1\\ \lambda & -1 & -1\\ 1 & 1 & -\lambda\end{vmatrix}=0$

$(\lambda+1)-\lambda\left(-\lambda^2+1\right)-(\lambda+1)=0$

$(\lambda+1)\left(1+\lambda(\lambda-1)-1\right)=0$

$\lambda=-1\ or\qquad 0\ or\ 1$

35.(2) $f(x)+2f\left(\frac{1}{x}\right)=3x,\quad x\neq 0$

$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x}$

$$3f(x) = \frac{6}{x} - 3x$$

$$f(x) = \frac{2}{x} - x$$

$$f(x) = f(-x) \Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x$$

$$\frac{4}{x} = 2x$$

$$\Rightarrow \quad x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

36.(2) $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}} = \lim_{x \to 0^+} e^{\frac{1}{2x}\left(\tan^2 \sqrt{x}\right)} = e^{\frac{1}{2}}$

$$\log_e p = \frac{1}{2}$$

37.(3) $\operatorname{Re}\left((2 + 3i\sin\theta)(1 + 2i\sin\theta)\right) = 2 - 6\sin^2\theta = 0$

$$\Rightarrow \quad \sin^2\theta = \frac{1}{3}$$

38.(2) $\frac{2b^2}{a} = 8$

$$2b = ae$$

$$4b^2 = a^2e^2$$

$$4a^2\left(e^2 - 1\right) = a^2e^2$$

$$3e^2 = 4$$

$$e = \frac{2}{\sqrt{3}}$$

39.(1) Variance $= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{4 + 9 + a^2 + 121}{4} - \left(\frac{16 + a}{4}\right)^2$

$$= \frac{4\left(134 + a^2\right) - 256 - a^2 - 32a}{16}$$

$$3a^2 - 32a + 280 = 16 \cdot \left(\frac{7}{2}\right)^2 = 4 \times 49$$

$$3a^2 - 32a + 84 = 0$$

40. (1) $\int \frac{2x^{12} + 5x^9}{\left(x^5 + x^3 + 1\right)^3} dx$

$$\int \frac{2x^{12} + 5x^9}{x^{15}\left(1 + x^{-2} + x^{-5}\right)^3} dx$$

$$\int \frac{2x^{-3}+5x^{-6}}{\left(x^{-5}+x^{-2}+1\right)^3}dx$$

$$x^{-5}+x^{-2}+1=t$$

$$\left(+5x^{-6}+2x^{-3}\right)dx=-dt$$

$$-\int \frac{dt}{t^3}=-\left(\frac{t^{-2}}{-2}\right)+C=\frac{1}{2t^2}+C=\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$$

41.(3) $\ell(3)+m(-2)-(-4)=9$

$3\ell-2m=5$ ...... **(i)**

$2\ell-m-3=0$

$2\ell-m=3$ ...... **(ii)**

$4\ell-2m=6$ ...... **(iii)**

(iii) – (i)

$\ell=1$

$m=-1$ $\quad\quad \ell^2+m^2=2$

42.(2) $\cos x+\cos 4x+\cos 2x+\cos 3x=0$

$\Rightarrow \quad 2\cos\frac{5x}{2}\cos\frac{3x}{2}+2\cos\frac{5x}{2}\cdot\cos\frac{x}{2}=0$

$\Rightarrow \quad 2\cos\frac{5x}{2}\left[\cos\frac{3x}{2}+\cos\frac{x}{2}\right]=0$

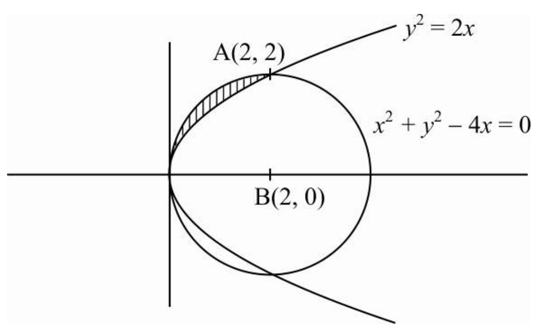
$\Rightarrow \quad 2\cos\frac{5x}{2}\cdot 2\cos x\cdot\cos\frac{x}{2}=0$

$\cos x=0 \quad\Rightarrow\quad x=\frac{\pi}{2},\frac{3\pi}{2}$

$\cos\frac{x}{2}=0 \quad\Rightarrow\quad x=\pi$

$\cos\frac{5x}{2}=0 \quad\Rightarrow\quad x=\frac{\pi}{5},\frac{3\pi}{5},\frac{7\pi}{5},\frac{9\pi}{5}$ $\quad\quad \left[\frac{5\pi}{5}=\pi \text{ is repeated}\right]$

43.(1) $x^2+y^2-4x\le 0$



$y^2\ge 2x$

$x^2+2x-4x=0$

$\Rightarrow \quad x^2-2x=0$

$\Rightarrow \quad x(x-2)=0$

$\Rightarrow \quad x=0,\ x=2$

$$\text{Area }=\int_0^2\left[\sqrt{4x-x^2}-\sqrt{2}\sqrt{x}\right]dx=\int_0^2\left[\sqrt{2^2-(x-2)^2}-\sqrt{2}\sqrt{x}\right]dx$$

$$= \left[\left| \frac{x-2}{2}\sqrt{4x-x^2} + \frac{4}{2}\sin^{-1}\frac{x-2}{2} - \sqrt{2}\times\frac{2}{3}x^{3/2}\right|_0^2\right] = \left[-\frac{2\sqrt{2}}{3}\times 2\sqrt{2} - \left\{-2\times\frac{\pi}{2}\right\}\right] = \left[\pi - \frac{8}{3}\right]$$

44.(3) $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right)$

$\Rightarrow \quad (\bar{a}\,.\,\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c} = \frac{\sqrt{3}}{2}\bar{b} + \frac{\sqrt{3}}{2}\bar{c}$

$\Rightarrow \quad \bar{a}\cdot\bar{c} = \frac{\sqrt{3}}{2}$ and $\bar{a}\cdot\bar{b} = -\frac{\sqrt{3}}{2}$

$\Rightarrow$ Angle between $\bar{a}$ & $\bar{c} = 30^\circ$

$\bar{a}$ & $\bar{b} = 150^\circ = \frac{5\pi}{6}$

45.(2) $4x + 2\pi r = 2 \quad \Rightarrow \quad 2x + \pi r = 1$

$\Rightarrow \quad r = \frac{1-2x}{\pi}$

$f(x) = x^2 + \pi r^2$

$= x^2 + \pi \times \frac{[1-2x]^2}{\pi^2}$

$f(x) = x^2 + \frac{(1-2x)^2}{\pi}$

$f'(x) = 2x - \frac{2(1-2x)\times(2)}{\pi} = 0$

$\Rightarrow \quad x = \frac{2(1-2x)}{\pi}$

$\Rightarrow \quad \pi x = 2 - 4x$

$\Rightarrow \quad \pi x = 2 - 4\left[\frac{1-\pi r}{2}\right]$

$\pi x = 2 - 2(1-\pi r)$

$\pi x = 2 - 2 + 2\pi r$

$\pi x = 2\pi r$.

46.(1) Equation of line: $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = \lambda$

Any point is $(\lambda+1, \lambda-5, \lambda+9)$

It lies on plane

$\Rightarrow \quad (\lambda+1) - (\lambda-5) + (\lambda+9) = 5$

$\Rightarrow \quad \lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

$\Rightarrow \quad \lambda + 10 = 0$

$\Rightarrow \quad \lambda = -10$

$\therefore$ Point is $(-9, -15, -1)$, another is $(1, -5, 9)$

Distance = $\sqrt{100+100+100} = 10\sqrt{3}$

47.(3) $y(1+xy)dx = xdy$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} + y^2 \quad \Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = y^2$

Bernaulli's DE

$n = 2$

I.F $= \int_e (1-2)\left(-\frac{1}{x}\right)dx = \int_e \frac{1}{x}dx = x$, Solution $y^{1-2}x = \int(1-2)\cdot x\cdot 1\cdot dx$

$\Rightarrow \quad \frac{x}{y} = -\frac{x^2}{2} + C$

Given $\quad f(1) = -1$

$\Rightarrow \quad \frac{1}{-1} = -\frac{1}{2} + C \quad \Rightarrow \quad C = -\frac{1}{2}$

$\therefore \quad$ equation $\frac{x}{y} = -\frac{x^2}{2} - \frac{1}{2}$

When $x = -\frac{1}{2}$, we have $-\frac{1}{2y} = -\frac{1}{4\times 2} - \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{y} = -\frac{5}{4} \quad \Rightarrow \quad y = \frac{4}{5}$

48.(3) $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$

Assuming all dissimilar terms

${}^{n+2}C_2 = 28$

$n = 6$

Sum of all coefficients $= 3^6 = 729$

49.(1) $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right); \quad x \in \left(0, \frac{\pi}{2}\right)$

$$f'(x) = \frac{1}{1+\frac{1+\sin x}{1-\sin x}} \times \frac{1}{2\sqrt{\frac{1+\sin x}{1-\sin x}}} \times \left\{\frac{(1-\sin x)(\cos x) - (1-\sin x)(-\cos x)}{(1-\sin x)^2}\right\}$$

At $x = \frac{\pi}{6}$

$$f'\left(\frac{\pi}{6}\right) = \frac{1}{1+\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} \times \frac{1}{2\sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}}} \times \left\{\frac{2\times\sqrt{3}/2}{\left(\frac{1}{2}\right)^2}\right\} = \frac{1}{1+3} \times \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\frac{1}{4}} = \frac{1}{4} \times \frac{1}{2\sqrt{3}} \times 4 \times \sqrt{3} = \frac{1}{2}$$

Slope of normal $= -2$

Point at $x = \frac{\pi}{6} \quad f\left(\frac{\pi}{6}\right) = \tan^{-1}\sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore \quad$ equation $y - \frac{\pi}{3} = (-2)\left(x - \frac{\pi}{6}\right) \quad \Rightarrow \quad y - \frac{\pi}{3} = -2x + \frac{\pi}{3} \quad \Rightarrow \quad y + 2x = \frac{2\pi}{3}$

50.(1) $\quad g(x) = f(f(x))$

$\Rightarrow \quad g'(x) = f'(f(x))\, f'(x)$

$\Rightarrow \quad g'(0) = f'(f(0))\, f'(0)$

For $x \to 0,\ \log 2 > \sin x$

$\therefore \quad f(x) = \log 2 - \sin x \quad \therefore \quad f'(x) = -\cos x \quad \Rightarrow \quad f'(0) = -1$

Also, $x \to \log 2,\ \log 2 > \sin x \quad \therefore \quad f(x) = \log 2 - \sin x$

$\therefore \quad f'(x) = -\cos x \quad \Rightarrow \quad f'(\log 2) = -\cos(\log 2)$

$\therefore \quad g'(0) = (-\cos(\log 2))(-1) = \cos(\log 2)$

51.(3) $P(E_1) = \dfrac{1}{6}$

$P(E_2) = \dfrac{1}{6}$

$P(E_3) = \dfrac{2+4+6+4+2}{36} = \dfrac{1}{2}$

$P(E_2 \cap E_3) = \dfrac{1}{6} \times \dfrac{1}{2} = P(E_2) \times P(E_3)$

$P(E_1 \cap E_3) = \dfrac{1}{6} \times \dfrac{1}{2} = P(E_1) \times P(E_3)$

$P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1)P(E_2)P(E_3)$

52.(1) $A(adj\ A) = |A| I_n = AA^T$ [Given]

$|A| = 10a + 3b$

$A^T = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$$AA^T \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} = \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix} = \begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix}$$

$\Rightarrow \quad 15a - 2b = 0 \quad \Rightarrow \quad a = \dfrac{2b}{15}\ \&\ 10a + 3b = 13 \quad \Rightarrow \quad a = \dfrac{13-3b}{10}$

$\Rightarrow \quad \dfrac{2b}{15} = \dfrac{13-3b}{10} \quad \Rightarrow \quad 4b = 39 - 9b \quad \Rightarrow \quad 13b = 39 \quad \Rightarrow \quad b = 3$

$\Rightarrow \quad a = \dfrac{2}{15} \times 3 = \dfrac{6}{15} = \dfrac{2}{5} \quad \Rightarrow \quad 5a = 2$

$\therefore \quad 5a + b = 2 + 3 = 5$

53.(2) $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$= [(p \vee q) \wedge (\sim q \vee q)] \vee (\sim p \wedge q)$

$= [(p \vee q) \wedge t] \vee (\sim p \wedge q)$

$= (p \vee q) \vee (\sim p \wedge q)$

$= [(p \vee q \vee \sim p) \wedge (p \vee q \vee q)]$

$= (t \vee q) \wedge (p \vee q) = t \wedge (p \vee q) = p \vee q$

54.(4) $\left(x^2-5x+5\right)^{x^2+4x-60}=1$

Case – I

$x^2+4x-60=0$

$x=-10$

$x=6$

Case – II

$x^2-5x+5=1$

$x^2-5x+4=0$

$x=1$

$x=4$

Case – III

$x^2-5x+5=-1$

$x^2-5x+6=0$

$x=2$ or 3

For $x=2$

$x^2+4x-60=-48$

For $x=3$

$x^2+4x-60=-39$

$\therefore \quad x=2$

Sum of all real value $=3$

55.(3) $x^2+y^2-8x-8y-4=0$

$C\equiv(4,4) \qquad r=6$

Let centre be (x_1, y_1)

Radius = $|y_1|$ $\qquad C_1C_2 = r_1 + r_2$

$\Rightarrow \quad \sqrt{(x_1-4)^2+(y_1-4)^2} = 6+|y_1|$

$\Rightarrow \quad (x_1-4)^2+(y_1-4)^2 = 36+y_1^2+12|y_1|$

$\Rightarrow \quad x_1^2-8x_1-8y_1-4=12|y_1|$

$y_1>0 \quad \Rightarrow \quad x_1^2-8x_1-8y_1-4=12y_1$

$\Rightarrow \quad x_1^2-8x_1-4=20\,y_1$

$\Rightarrow \quad (x_1-4)^2-20=20y_1$

$\Rightarrow \quad (x_1-4)^2=20(y_1+1)$ Parabola

$y_1<0 \quad \Rightarrow \quad x_1^2-8x_1-8y_1-4=-12y_1$

$\Rightarrow \quad x_1^2-8x_1-4=-4y_1$

$\Rightarrow \quad (x_1-4)^2=20-4y_1 \Rightarrow (x_1-4)^2=-4(y-5)$ parabola

56.(3) $A \quad \dfrac{\lfloor 4}{\lfloor 2}=12$

$L \quad \lfloor 4 = 24$

$M \quad \dfrac{\lfloor 4}{\lfloor 2}=12$

$SA \qquad \frac{\lfloor 3}{\lfloor 2} = 3$

$SL \qquad \lfloor 3 = 6$

Total 57

Next word is SMALL.

57.(1) $\ell = \left(\frac{(n+1)}{n} \cdot \frac{n+2}{n} \cdot \frac{n+3}{n} \cdot .... \frac{n+2n}{n}\right)^{1/n}$

$$\log \ell = \frac{1}{n}\left[\log\left(\frac{n+1}{n}\right) + \log\left(\frac{n+2}{n}\right) + ... + \log\left(\frac{n+2n}{n}\right)\right]$$

$$\log \ell = \int_0^2 \log(1+x)\,dx \qquad\qquad 1 + x = t$$

$$\log \ell = \int_1^3 \log t \quad dt$$

$$\log \ell = t \log t - \int \frac{1}{t} \cdot t \ dt$$

$$\log \ell = t(\log t - 1)$$

$$\log \ell = 3(\log 3 - 1) - 1(\log 1 - 1)$$

$$= 3 \log 3 - 2$$

$$= \log 27 - \log e^2$$

$$= \log \ell = \log \frac{27}{e^2}$$

$$\ell = \frac{27}{e^2}$$

58.(1) $S_n = \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2$

$$S_n = \frac{1}{25}\left[8^2 + 12^2 + 16^2 + 20^2 + ...\right]$$

$$S_n = \sum_{n=1}^{10} \frac{1}{25}\left[(4n+4)^2\right]$$

$$= \sum_{n=1}^{10} \frac{16}{25}\left[n + 1^2\right]$$

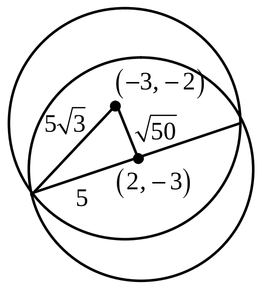
$$= \sum_{n=1}^{10} \frac{16}{25}\left[n^2 + 2n + 1\right] \quad 35.11$$

$$= \frac{16}{25}\left[\frac{10.11.\ 21}{6} + \frac{10.11}{2} + 10\right]$$

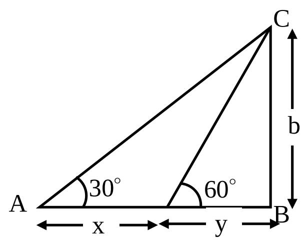
$$= \frac{16}{25}[385 + 110 + 10]$$

$$= \frac{16}{25} \times 505 \quad \equiv \frac{16}{5} \times 101 \qquad \Rightarrow \ m = 101$$

59.(1)



60.(3)



Let speed be " v "

$$\tan 60^\circ = \frac{b}{y} = \sqrt{3}$$

$b = \sqrt{3}y$ …(i)

$\tan 30^\circ = \dfrac{b}{x+y}$

$\dfrac{1}{\sqrt{3}} = \dfrac{b}{x+y}$

$x + y = b\sqrt{3}$ …(ii)

$10v + vt_1 = \sqrt{3}\ y\ .\ \sqrt{3}$

$10v + vt_1 = 3y$

$10v + vt_1 = 3vt_1$

$10 + t_1 = 3t_1$

$2t_1 = 10$

$t_1 = 5.$

PART-C	PHYSICS

Coming Soon