MARKING SCHEME

SAMPLE PAPER

SECTION-A

1.	$\frac{1}{8}\left(5\vec{a}+3\vec{b}\right)$	1
2.	5 sq. units	1
3.	$\vec{r}.\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=29$	1
4.	-14	1
5.	m + n = 4	1
6.	$2x\frac{dy}{dx}-y=0$	1

SECTION-B

7. Sale matrix for A, B and C is $\begin{pmatrix}25 & 12 & 34\\ 22 & 15 & 28\\ 26 & 18 & 36\end{pmatrix}$ ½

Price matrix is $\begin{pmatrix}20\\ 15\\ 5\end{pmatrix}$ ½

$\therefore$ $\begin{pmatrix}25 & 12 & 34\\ 22 & 15 & 28\\ 26 & 18 & 36\end{pmatrix}\begin{pmatrix}20\\ 15\\ 5\end{pmatrix}=\begin{pmatrix}500+ & 180+ & 170\\ 440+ & 225+ & 140\\ 520+ & 270+ & 180\end{pmatrix}$ ½

$\therefore$ Amount raised by $=\begin{pmatrix}850\\ 805\\ 970\end{pmatrix}$ ½

School A = Rs 850, school B = Rs 805, school C = Rs 970

Values

- Helping the orphans 1
- Use of recycled paper 1

8. $A^2=\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}=\begin{pmatrix}1 & 12\\ -4 & 1\end{pmatrix}$ 1

$\therefore A^2 - 4A + 7I = \begin{pmatrix} 1 & 12 \\ -4 & 1 \end{pmatrix} + \begin{pmatrix} -8 & -12 \\ 4 & -8 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ 2

$A^2 = 4A - 7I \Rightarrow A^3 = 4A^2 - 7A = 4(4A-7I) - 7A$

$= 9A - 28I = \begin{pmatrix} 18 & 27 \\ -9 & 18 \end{pmatrix} + \begin{pmatrix} -28 & 0 \\ 0 & -28 \end{pmatrix}$

$= \begin{pmatrix} -10 & 27 \\ -9 & -10 \end{pmatrix}$ 1

OR

Write A = IA we get $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.A$ ½

$R_2 \longrightarrow R_2 - 2R_1 \Rightarrow$ $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \longrightarrow R_2 - 3R_3 \Rightarrow$ $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$\begin{matrix} R_1 \longrightarrow R_1 + R_2 \Rightarrow \\ R_3 \longrightarrow R_3 - 2R_2 \end{matrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} A$ 1

$\therefore A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix}$ ½

9. $\Delta = \begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix}$

$C_1 \longrightarrow C_1 - pC_2 - C_3$, $\Delta = \begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -p^2x - py - py - z & px+y & py+z \end{vmatrix}$ 1½

Expanding by R_3

$\Delta = (-p^2x - 2py - z)(xz - y^2)$ 1

Since x, y, z are in GP, $\therefore$ $y^2 = xz$ or $y^2 - xz = 0$ 1

$\therefore \quad \Delta = 0$ ½

10. $\int_{-1}^{1} |x.\cos\pi x|\, dx = 2\int_0^1 |x\cos\pi x| dx$ 1

$= 2\int_0^{1/2}(x\cos\pi x)\, dx + 2\int_{1/2}^{1} -(x\cos\pi x)dx$ 1

$= 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_0^{\frac{1}{2}} - 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{\frac{1}{2}}^{1}$ 1

$= 2\left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - 2\left[\frac{-1}{\pi^2} - \frac{1}{2\pi}\right] = \frac{2}{\pi}$ 1

11. $I = \int \frac{1+\sin 2x}{1+\cos 2x}.\, e^{2x}\, dx = \frac{1}{2}\int \frac{1+\sin t}{1+\cos}.\, e^t\, dt$ (where $2x=t$) ½

$= \frac{1}{2}\int\left(\frac{1}{2\cos^2 t/2} + \frac{2\sin t/2\, \cos t/2}{2\cos^2 t/2}\right) e^t\, dt$ 1

$= \frac{1}{2}\int\left(\frac{1}{2}\sec^2 t/2 + \tan t/2\right) e^t\, dt$ 1

$\tan t/2 = f(t)$ then $f'(t) = \frac{1}{2}\sec^2 t/2$

Using $\int(f(t) + f'(t))\, e^t\, dt = f(t)\, e^t + C$, we get ½

$I = \frac{1}{2}\tan t/2.\, e^t + C = \frac{1}{2}\tan x.\, e^{2x} + C$ 1

OR

We have

$$\frac{x^4}{(x-1)(x^2+1)} = (x+1) + \frac{1}{x^3 - x^2 + x - 1}$$

$$= (x+1) + \frac{1}{(x-1)(x^2+1)} \quad \text{........ (1)}$$ 1

Now express $\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)}$ .... (2)

So,

$$1 = A(x^2 + 1) + (Bx + C)(x - 1)$$

$$= (A + B)x^2 + (C - B)x + A - C$$

Equating coefficients, $A + B = 0$, $C - B = 0$ and $A - C = 1$,

Which give $A = \frac{1}{2}$, $B = C = -\frac{1}{2}$. Substituting values of A, B, and C in (2), we get

$$\frac{1}{(x-1)(x^2+1)} = \frac{1}{2(x-1)} - \frac{1}{2}\frac{x}{(x^2+1)} - \frac{1}{2(x^2+1)} \quad \text{........ (3)} \qquad 1$$

Again, substituting (3) in (1), we have

$$\frac{x^4}{(x-1)(x^2+1)} = (x + 1) + \frac{1}{2(x-1)} - \frac{1}{2}\frac{x}{(x^2+1)} - \frac{1}{2(x^2+1)}$$

Therefore

$$\int \frac{x^4}{(x-1)(x^2+1)}dx = \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C \qquad 1+1$$

12. Let E : Die shows a number > 3

E : {H4, H5, H6] ½

and F : there is atleast one head.

∴ F : {HT, H1, H2, H3, H4, H5, H6} ½

$P(F) = 1 - \frac{1}{4} = \frac{3}{4}$ 1

$P(E \cap F) = \frac{3}{12} = \frac{1}{4}$ 1

∴ $P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$ 1

OR

p = $\frac{1}{2}$, q = $\frac{1}{2}$, let the coin be tossed n times

$\therefore$ P(r $\geq$ 1) > $\frac{90}{100}$ ½

or 1-P(r=0) > $\frac{90}{100}$ ½

P(r=0) < 1- $\frac{9}{10}$ = $\frac{1}{10}$ ½

${}^{n}C_0 \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^0 < \frac{1}{10} \Rightarrow \frac{1}{2^n} < \frac{1}{10}$ 1½

$\Rightarrow 2^n > 10, \therefore n = 4$ 1

13. $\left.\begin{array}{l} \vec{a} \times \vec{b} = \vec{c} \implies \vec{a} \perp \vec{b} \text{ and } \vec{b} \perp \vec{c} \\ \vec{a} \times \vec{c} = \vec{b} \implies \vec{a} \perp \vec{b} \text{ and } \vec{c} \perp \vec{b} \end{array}\right\} \Rightarrow \vec{a} \perp \vec{b} \perp \vec{c}$ 1

$|\vec{a} \times \vec{b}| = |\vec{c}|$ and $|\vec{a} \times \vec{c}| = |\vec{b}|$ 1

$\Rightarrow |\vec{a}|\,|\vec{b}| \sin\frac{\pi}{2} = |\vec{c}|$ and $|\vec{a}|\,|\vec{c}| \sin {}^{\pi}/_{2} = |\vec{b}|$

$\Rightarrow |\vec{a}|\,|\vec{b}| = |\vec{c}| \therefore |\vec{a}|\,|\vec{a}|\,|\vec{b}| = |\vec{b}| \Rightarrow |\vec{a}|^2 = 1 \Rightarrow |\vec{a}| = 1$ 1

$\Rightarrow 1.\,|\vec{b}| = |\vec{c}| \Rightarrow |\vec{b}| = |\vec{c}|$

14. DR's of line (L_1) joining (4, 3, 2) and (1, -1, 0) are <3, 4, 2> ½

DR's of line (L_2) joining (1, 2, -1) and (2, 1, 1) are <1, -1, 2> ½

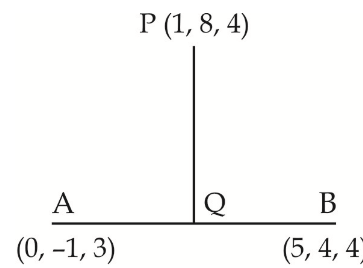
A vector $\perp$ to L_1 and L_2 is $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 10\hat{\imath}-4\hat{\jmath}-7\hat{k}$ 1½

$\therefore$ Equation of the line passing through (1, -1, 1) and $\perp$ to L_1 and L_2 is

$\vec{r} = (\hat{\imath}-\hat{\jmath}+\hat{k}) + \lambda\,(10\hat{\imath}-4\hat{\jmath}-7\hat{k})$ 1½

OR

Equation of line AB is



$\vec{r} = (-\hat{j}+3\hat{k}) + \lambda\ (5\hat{i}+5\hat{j}+\hat{k})$ 1

$\therefore$ Point Q is $(5\lambda, -1+5\lambda, 3+\lambda)$ ½

$\overrightarrow{PQ} = (5\lambda-1)\ \hat{\imath} + (5\lambda-9)\ \hat{\jmath} + (\lambda-1)\ \hat{k}$ ½

PQ $\perp$ AB $\Longrightarrow 5(5\lambda-1) + 5\ (5\lambda-9) + 1\ (\lambda-1) = 0$

$51\lambda = 51 \Longrightarrow \lambda=1$ ½

$\Longrightarrow$ foot of perpendicular (Q) is (5, 4, 4) ½

Length of perpendicular PQ = $\sqrt{4^2 + (-4)^2 + 0^2} = 4\sqrt{2}$ units 1

15. $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}\ x = -\frac{\pi}{2}$

$\Longrightarrow \sin^{-1} 6x = \left(\frac{-\pi}{2} - \sin^{-1} 6\sqrt{3x}\right)$ ½

$\Longrightarrow 6x = \sin\left[-\frac{\pi}{2} - \sin^{-1}6\sqrt{3}x\right] = -\sin\left[\frac{\pi}{2} + \sin^{-1} 6\sqrt{3}x\right]$ ½

$= -\cos\left[\sin^{-1} 6\sqrt{3}x\right] = -\sqrt{1-108x^2}$ 1

$\Longrightarrow 36x^2 = 1-108\ x^2 \Longrightarrow 144\ x^2 = 1$

$\Longrightarrow \quad x = \pm\frac{1}{12}$

since $x = \frac{1}{12}$ does not satisfy the given equation

$\therefore x = -\frac{1}{12}$ 1

OR

LHS = $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1

$= \tan^{-1}\left(\frac{2.\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\frac{17}{31}$ 1

$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\frac{17}{31}$ 1

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}.\frac{17}{31}}\right) = \tan^{-1}(1) = {}^{\pi}/_{4}$ 1

16. $x = \sin t$ and $y = \sin kt$

$\frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = k\cos t\, kt$

$\Longrightarrow \frac{dy}{dx} = k\frac{\cos kt}{\cos t}$ 1

or $\cos t.\frac{dy}{dx} = k.\cos kt$

$\cos^2 t\left(\frac{dy}{dx}\right)^2 = k^2\cos^2 kt$

$\cos^2 t\left(\frac{dy}{dx}\right)^2 = k^2\cos^2 kt$ ½

$(1-x^2)\left(\frac{dy}{dx}\right)^2 = k^2(1-y^2)$ 1

Differentiating w.r.t.x

$(1-x^2)\,2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = -2k^2y\frac{dy}{dx}$ 1

$\Longrightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$ ½

17. let $u = y^x$, $v = x^y$, $w = x^x$

(i) $\log u = x\log y \Longrightarrow \frac{du}{dx} = y^x\left[\log y + \frac{x}{y}\frac{dy}{dx}\right]$ 1

(ii) $\log v = y\log x \Longrightarrow \frac{dv}{dx} = x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$ ½

(iii) $\log w = x \log x \Rightarrow \frac{dw}{dx} = x^x, (1+\log x)$ ½

$\Rightarrow y^x \left[\log y + \frac{x}{y}\frac{dy}{dx}\right] + x^y \left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + x^x (1+\log x) = 0$ 1

$\Rightarrow \frac{dy}{dx} = -\frac{x^x(1+\log x) + y\, x^{y-1} + y^x \log y}{x . y^{x-1} + \log x}$ 1

18. $f(x) = x^3 + bx^2 + ax + 5$ on $[1, 3]$

$f'(x) = 3x^2 + 2bx + a$

$f'(c) = 0 \Rightarrow 3\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + a = 0$ - - - - - - (i) 1

$f(1) = f(3) \Rightarrow b + a + 6 = 32 + 9b + 3a$

or $a + 4b = -13$ - - - - - - - - - - (ii) 1

Solving (i) and (ii) to get a=11, b= -6 1

19. Let $3x + 1 = A(-2x - 2) + B \Rightarrow A = -3/2, B = -2$ 1

$I = \int \frac{-\frac{3}{2}(-2x-2)}{\sqrt{5-2x-x^2}} dx - 2\int \frac{1}{\sqrt{\left(\sqrt{6}\right)^2 - (x+1)^2}} dx$ 1+1

$= -3\sqrt{5-2x-x^2} - 2.\ \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C$ 1

SECTION–C

20. (i) for all a, b ∈ A, $(a, b)\ R\ (a, b)$, as $a + b = b + a$

∴ R is reflexive 1

(ii) for a, b, c, d ∈ A, let $(a, b)\ R\ (c, d)$

∴ $a + d = b + c \Rightarrow c + b = d + a \Rightarrow (c, d)\ R\ (a, b)$

∴ R is symmetric 1

(iii) for a, b, c, d, e, f, ∈ A, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\therefore a + d = b + c$ and $c + f = d + e$

$\Rightarrow a + d + c + f = b + c + d + e$ or $a + f = b + e$

$\Rightarrow (a, b)\ R\ (e, f)$ $\therefore$ R is Transitive 2

Hence R is an equivalence relation and equivalence class [(2, 5)] is ½

{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)} 1½

OR

Let $y \in S$, then $y=4x^2+12x+15$, for some $x \in N$

$\Rightarrow y = (2x + 3)^2 + 6 \Rightarrow x = \frac{(\sqrt{y-6})-3}{2}$, as $y > 6$ 1

Let $g : S \longrightarrow N$ is defined by $g(y) = \frac{(\sqrt{y-6})-3}{2}$ 1

$\therefore gof(x) = g(4x^2+12x+15) = g((2x+3)^2+6) = \frac{\sqrt{(2x+3)^2}-3}{2} = x$ 1

and $fog(y) = f\left(\frac{(\sqrt{y-6})-3}{2}\right) = \left[\frac{2\{(\sqrt{y-6})-3\}}{2} + 3\right]^2 +6 = y$ 1

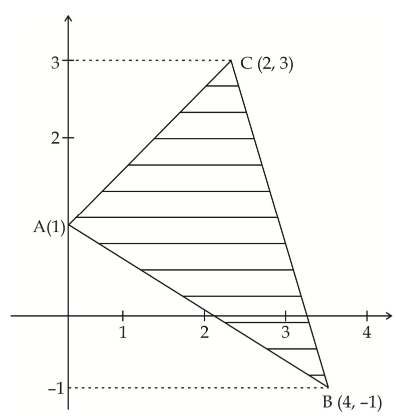
Hence $fog(y) = I_S$ and $gof(x) = I_N$

$\Rightarrow f$ is invertible and $f^{-1} = g$ 1

21. Let the lines be, AB: $x+2y = 2$, BC: $2x+y = 7$, AC = $y-x = 1$ 1

$\therefore$ Points of intersection are

A(0,1), B(4,-1) and C(2, 3) 1½



$A = \frac{1}{2}\int_{-1}^{3}(7-y)\,dy - \int_{-1}^{1}(2-2y)\,dy - \int_{1}^{3}(y-1)\,dy$ 1½

$= \frac{1}{2}\left(7y - \frac{y^2}{2}\right)_{-1}^{3} - (2y-y^2)_{-1}^{1} - \left(\frac{y^2}{2} - y\right)_{1}^{3}$ 1½

= 12 – 4 – 2 = 6sq.Unit. ½

22. Given differential equation is homogenous.

$\therefore$ Putting y = vx to get $\frac{dy}{dx}$ = v + $x\frac{dv}{dx}$ ½

$\frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - xe^{y/x}}{x\sin\left(\frac{y}{x}\right)} \Rightarrow v + x\frac{dv}{dx} = \frac{v\, sinv - e^{v}}{sin\, v}$ 1

$\therefore v + x\frac{dv}{dx} = v - \frac{e^{v}}{sinv}$ or $x\frac{dv}{dx} = -\frac{e^{v}}{sinv}$

$\therefore \int sinv\, e^{-v}\, dv = -\int \frac{dx}{x}\ or\ I_1 = -logx + c_1$ - - - - - - - - (i) 1

$I_1 = sinv.e^{-v} + \int \cos v\, e^{-v} dv$

$= -sinv.e^{-v} - cosv\, e^{-v} - \int \sin v.\, e^{-v}\ dv$

$I_1 = -\frac{1}{2}(\sin v + cosv)\, e^{-v}$ 1

Putting (i), $\frac{1}{2}(sinv + cosv)\, e^{-v} = \log x + C_2$

$\Rightarrow \left[\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right] e^{\frac{-y}{x}} = \log x^2 + C$ 1

$x = 1, y = 0 \Rightarrow c = 1$ 1

Hence, Solution is $\left[\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right] e^{\frac{-y}{x}} = \log x^2 + 1$ ½

OR

$(x-a)^2 + (y-b)^2 = r^2$ ....(i)

$\Rightarrow 2(x-a) + 2(y-b)\frac{dy}{dx} = 0$ ....(ii) ½

$\Rightarrow 1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ ....(iii) ½

$\therefore (y-b) = -\frac{(1+y_1^2)}{y^2}$ 1½

From (ii), $(x-a) = \frac{y_1(1+y_1^2)}{y_2}$ 1½

Putting these values in (i)

$$\frac{y_1^2(1+y_1^2)^2}{y_2^2}+\frac{(1+y_1^2)^2}{y_2^2}=r^2$$ 1

or $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$$ 1

23. Here $\vec{a}_1=-3\hat{i}+\hat{j}+5\hat{k}\,,\vec{b}_1=3\hat{i}+\hat{j}+5\hat{k}$

$\vec{a}_2=-\hat{i}+2\hat{j}+5\hat{k}\,,\vec{b}_2=-\hat{i}+2\hat{j}+5\hat{k}$ ½

$$(\vec{a}_2-\vec{a}_1).(\vec{b}_1\text{x}\,\vec{b}_2)=\begin{vmatrix}2 & 1 & 0\\ -3 & 1 & 5\\ -1 & 2 & 5\end{vmatrix}=2(-5)-1(-15+5)$$ 1½

= –10 + 10 = 0

∴ lines are co–planer. ½

Perpendicular vector $(\vec{\text{n}})$ to the plane = $\overrightarrow{\text{b}_1}\text{x}\,\overrightarrow{\text{b}_2}$

$$\begin{vmatrix}i & \text{j} & \hat{\text{k}}\\ -3 & 1 & 5\\ -1 & 2 & 5\end{vmatrix}=-5\hat{i}+10\hat{j}-5\hat{\text{k}}$$ 2

or $\hat{i}-2\hat{\text{j}}+\hat{\text{k}}$ 2

∴Eqn. of plane is $\vec{\text{r}}.(\hat{i}-2\hat{\text{j}}+\hat{\text{k}})=(\hat{i}-2\hat{j}+\hat{k}).(-3\hat{i}+\hat{j}+5\hat{k})=0$ 1½

or x – 2y + z = 0

24. Let E_1: Student resides in the hostel

E_2: Student resides outside the hostel

$\text{P}(\text{E}_1)=\frac{40}{100}=\frac{2}{5},\text{P}(\text{E}_2)=\frac{3}{5}$ ½+½

A: Getting A grade in the examination

$\text{P}\left(\frac{\text{A}}{\text{E}_1}\right)=\frac{50}{100}=\frac{1}{2}$ $\quad$ $\text{P}\left(\frac{\text{A}}{\text{E}_2}\right)=\frac{30}{100}=\frac{3}{10}$ 1+1

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P(\frac{A}{E_1})}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P(\frac{A}{E_2})}$$ 1

$$= \frac{\frac{2}{5}\cdot\frac{1}{2}}{\frac{2}{5}\cdot\frac{1}{2}+\frac{3}{5}\cdot\frac{3}{10}} = \frac{10}{19}$$ 1+1

25. Let the distance travelled @ 50 km/h be x km.

and that @ 80 km/h be y km.

∴ LPP is

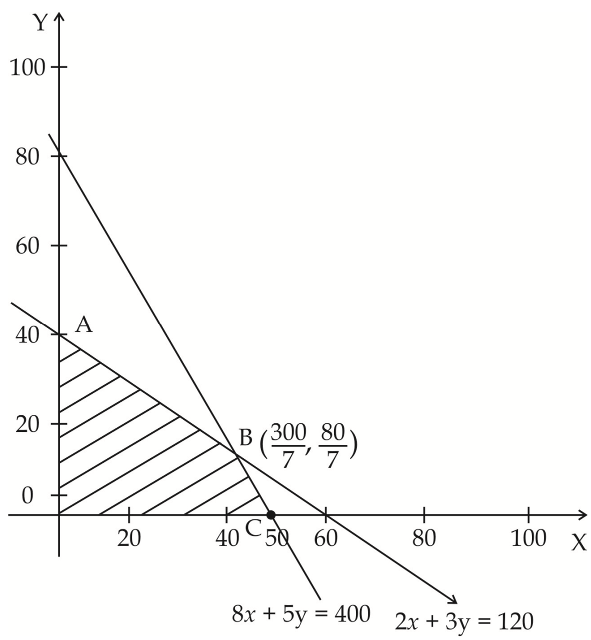
Maximize D = x + y ½

St. $2x + 3y \leq 120$

$\frac{x}{50} + \frac{y}{80} \leq 1$ or $8x + 5y \leq 400$ 2

$x \geq 0$, $y \geq 0$

2



Vertices are.

$(0, 40), \left(\frac{300}{7}, \frac{80}{7}\right), (50,0)$

Max. D is at $\left(\frac{300}{7}, \frac{80}{7}\right)$

Max. D = $\frac{380}{7} = 54\frac{2}{7}$ km. 1½

26. Let P(x, y) be the position of the jet and the soldier is placed at A(3, 2)

$\Rightarrow$ AP = $\sqrt{(x-3)^2 + (y-2)^2}$ ....(i) ½

As y = $x^2 + 2 \Rightarrow y - 2 = x^2$ ....(ii) $\Rightarrow$ AP^2 = $(x-3)^2 + x^4$ = z (say) ½

$\frac{dz}{dx} = 2(x-3) + 4x^3$ and $\frac{d^2z}{dx^2} = 12x^2 + 2$ 2

$\frac{dz}{dx} = 0 \Rightarrow x = 1$ and $\frac{d^2z}{dx^2}$ (at $x = 1$) > 0 1+1

$\therefore$ z is minimum when $x = 1$, when $x = 1$, y = 1+2 = 3

$\therefore$ minimum distance = $\sqrt{(3-1)^2 + 1^2} = \sqrt{5}$ 1