JEE(Advanced) – 2016 TEST PAPER WITH SOLUTIONS
(HELD ON SUNDAY 22nd MAY, 2016)

PART-I : PHYSICS

SECTION–1 : (Maximum Marks : 18)

● This section contains **SIX** questions.

● Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

● For each question, darken the bubble corresponding to the correct option in the ORS.

● For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks   :   +3   If only the bubble corresponding to the correct option is darkened.

Zero Marks   :   0   If none of the bubbles is darkened.

Negative Marks :   –1   In all other cases.

1. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is

given by $E = \dfrac{3}{5} \dfrac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$

The measured masses of the neutron, $_1^1H$, $_7^{15}N$ and $_8^{15}O$ are 1.008665 u, 1.007825 u, 15.000109 u and

15.003065 u respectively. Given that the radii of both the $_7^{15}N$ and $_8^{15}O$ nuclei are same, 1u = 931.5 MeV/c^2

(c is the speed of light) and $\dfrac{e^2}{(4\pi\varepsilon_0)} = 1.44$ MeV fm. Assuming that the difference between the binding

energies of $_7^{15}N$ and $_8^{15}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is
(1fm = 10^{-15}m)

(A) 2.85 fm   (B) 3.03 fm   (C) 3.42 fm   (D) 3.80 fm

Ans. (C)

Sol. Electrostatic energy $= BE_N - BE_O$

$= \left[[7M_H + 8M_n - M_N] - [8M_H + 7M_n - M_O] \right] \times C^2$

$= [-M_H + M_n + M_O - M_N]C^2$

$= [-1.007825 + 1.008665 + 15.003065 - 15.000109] \times 931.5$

$= +3.5359$ MeV

$\Delta E = \dfrac{3}{5} \times \dfrac{1.44 \times 8 \times 7}{R} - \dfrac{3}{5} \times \dfrac{1.44 \times 7 \times 6}{R} = 3.5359$

$R = \dfrac{3 \times 1.44 \times 14}{5 \times 3.5359} = 3.42 fm$

PHYSICS

2. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\,K^{-1}$, the change in length of the wire PQ is

(A) 0.78 mm   (B) 0.90 mm   (C) 1.56 mm   (D) 2.34 mm

Ans. (A)

Sol.
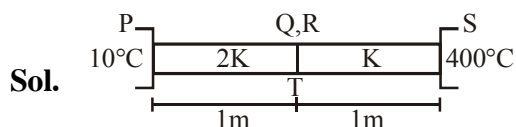
Heat flow from P to Q

$$\frac{dQ}{dt} = \frac{2KA(T-10)}{1}$$

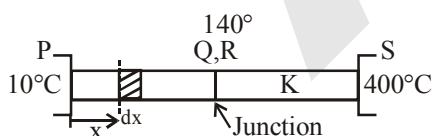Heat flow from Q to S

$$\frac{dQ}{dt} = \frac{KA(400-T)}{1}$$

At steady state heat flow is same in whole combination

$$\frac{2KA(T-10)}{1} = KA(400\text{-}T)$$

$2T - 20 = 400 - T$

$3T = 420$

$T = 140°$

Temp of junction is 140°C

Temp at a distance x from end P

is $T_x = (130x + 10°)$

Change in length dx is dy

$dy = \alpha dx(T_x - 10)$

$$\int_0^{\Delta y} dy = \int_0^1 \alpha dx(130x + 10 - 10)$$

$$\Delta y = \left[\frac{\alpha x^2}{2} \times 130 \right]_0^1$$

$$\Delta y = 1.2 \times 10^{-5} \times 65$$

$$\Delta y = 78.0 \times 10^{-5} \text{ m} = 0.78 \text{ mm}$$

3. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

(A) 64   (B) 90   (C) 108   (D) 120

Ans. (C)

Sol. Let the permissible level have activity of $A_{permissible}$

Thus, initially

$A_0 = 64\ A_{permissible}$ [Given]

Let number of days required be t.

$$\therefore \frac{A_0}{2^{t/t_{1/2}}} = A_{permissible}$$

$$\Rightarrow \frac{64 A_{permissible}}{2^{t/18}} = A_{permissible}$$

$$\therefore t = 108 \text{ days}$$

4. There are two vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively, are
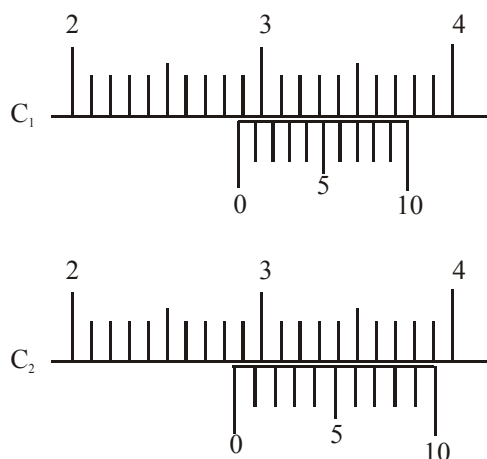
(A) 2.87 and 2.86   (B) 2.87 and 2.87   (C) 2.87 and 2.83   (D) 2.85 and 2.82

Ans. (C)

Sol. For caliper C_1

10 VSD = 9MSD

LC = 1MSD − 1VSD

LC = 0.01 cm

Measured value = Main scale reading + vernier scale reading

= (2.8 + 7 × 0.01) cm

= 2.87 cm

For Caliper C_2

10 VSD = 11 MSD

LC = 0.01 cm

Measured value = {2.8 + (10–7) × 0.01}cm

= 2.83 cm

5. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = \left(\dfrac{1}{32}\right) \times 10^5 \, Pa$ and V_f = 8 × 10^{-3} m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumes V_f. The amount of heat supplied to the system in the two step process is approximately

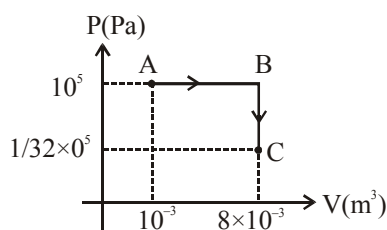(A) 112 J   (B) 294 J   (C) 588 J   (D) 813 J

Ans. (C)

Sol. In adiabatic process

P^3V^5 = constant

$\Rightarrow PV^{5/3}$ = constant

$\Rightarrow \gamma = \dfrac{5}{3}$ $\Rightarrow C_V = \dfrac{3}{2}R$ and $C_P = \dfrac{5}{2}R$

In another process

$\Delta Q = nC_P\Delta T + nC_V\Delta T$

$\quad = \dfrac{5}{2}nR(T_B - T_A) + \dfrac{3}{2}nR(T_C - T_B)$

$\Delta Q = \dfrac{5}{2}(P_B V_B - P_A V_A) + \dfrac{3}{2}(P_C V_C - P_B V_B)$

Putting values

$\Delta Q = 587.5\ J \approx 588J$

6. A small object is placed 50 cm to the left of thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure. If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are :
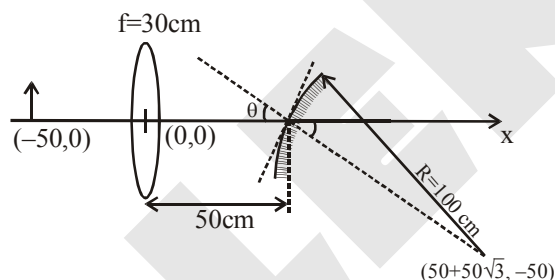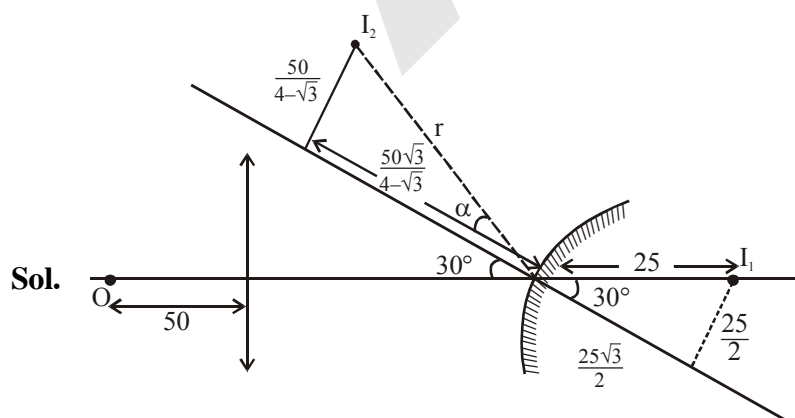

(A) $(25, 25\sqrt{3})$ 　　(B) $\left(\dfrac{125}{3}, \dfrac{25}{\sqrt{3}}\right)$ 　　(C) $(50 - 25\sqrt{3}, 25)$ 　　(D) $(0, 0)$

Ans. **(A)**

Sol.

PHYSICS

For lens $V = \dfrac{(-50)(30)}{-50+30} = 75$

For mirror $V = \dfrac{\left(\dfrac{25\sqrt{3}}{2}\right)(50)}{\dfrac{25\sqrt{3}}{2}-50} = \dfrac{-50\sqrt{3}}{4-\sqrt{3}}$

$m = -\dfrac{v}{u} = \dfrac{h_2}{h_1} \Rightarrow h_2 = -\left(\dfrac{\dfrac{-50\sqrt{3}}{4-\sqrt{3}}}{\dfrac{25\sqrt{3}}{2}}\right) \cdot \dfrac{25}{2}$

$h_2 = \dfrac{+50}{4-\sqrt{3}}$

The x coordinate of the images = $50 - v\cos 30 + h_2\cos 60 \approx 25$

The y coordinate of the images = $v\sin 30 + h_2\sin 60 \approx 25\sqrt{3}$

SECTION–2 : (Maximum Marks : 32)

● This section contains **EIGHT** questions.

● Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

● For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

● For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

● for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

7. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for z > 0) at a distance D = 3m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the sources d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is (are) true of the intensity pattern on the screen ?
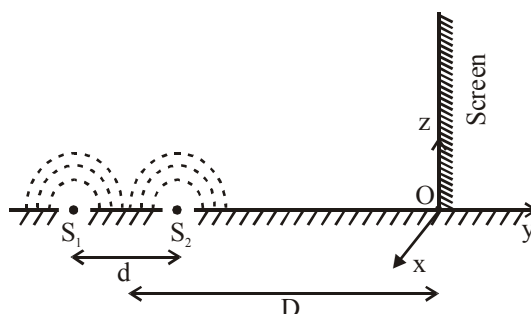
(A) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

(B) Semi circular bright and dark bands centered at point O

(C) The region very close to the point O will be dark

(D) Straight bright and dark bands parallel to the x-axis

Ans. (B, C)

Sol.
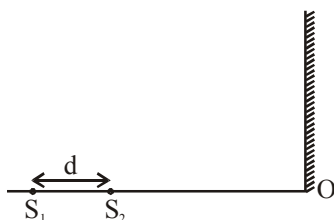
Path difference at point O = d = .6003 mm = 600300 nm

$$= \frac{2001}{2}(600\,\text{nm}) = 1000\lambda + \frac{\lambda}{2}$$

$\Rightarrow$ minima form at point O

Line S_1S_2 and screen are $\perp$ to each other so fringe pattern is circular (semi-circular because only half of screen is available)

8. In an experiment to determine the acceleration due to gravity g, the formula used for the time period

of a periodic motion is $T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm

and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true ?

(A) The error in the measurement of r is 10%

(B) The error in the measurement of T is 3.57 %

(C) The error in the measurement of T is 2%

(D) The error in the determined value of g is 11%

PHYSICS

Ans. **(A, B, D)**

Sol.

	T	Absolute error
1	0.52	−0.04
2	0.56	00
3	0.57	+0.01
4	0.54	−0.02
5	0.59	+0.03
	$T_{avg} = 0.556$ $= 0.56$	

Avg. absolute error $= \dfrac{.04 + 00 + .01 + .02 + .03}{5} = .02$

$\Rightarrow \dfrac{\Delta T}{T} \times 100\% = \dfrac{.02}{.56} \times 100\% \approx 3.57\%$ (B)

$\Rightarrow \dfrac{\Delta r}{r} \times 100\% = \dfrac{1}{10} \times 100 = 10\%$ (A)

also $\dfrac{\Delta g}{g} = \dfrac{\Delta R + \Delta r}{R - r} + \dfrac{2\Delta T}{T}$

$\dfrac{\Delta g}{g} \times 100\% = \dfrac{1+1}{50} \times 100\% + 2(3.57)\%$

$\approx 11\%$ (D)

9. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.
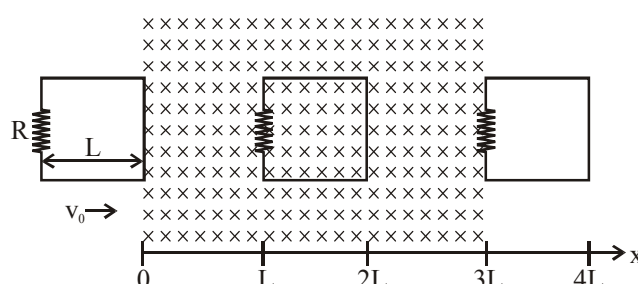
Which of the following schematic plot(s) is(are) correct ? (Ignore gravity)

(A)

(B)

(C)

(D)

Ans. (C, D)

Sol.
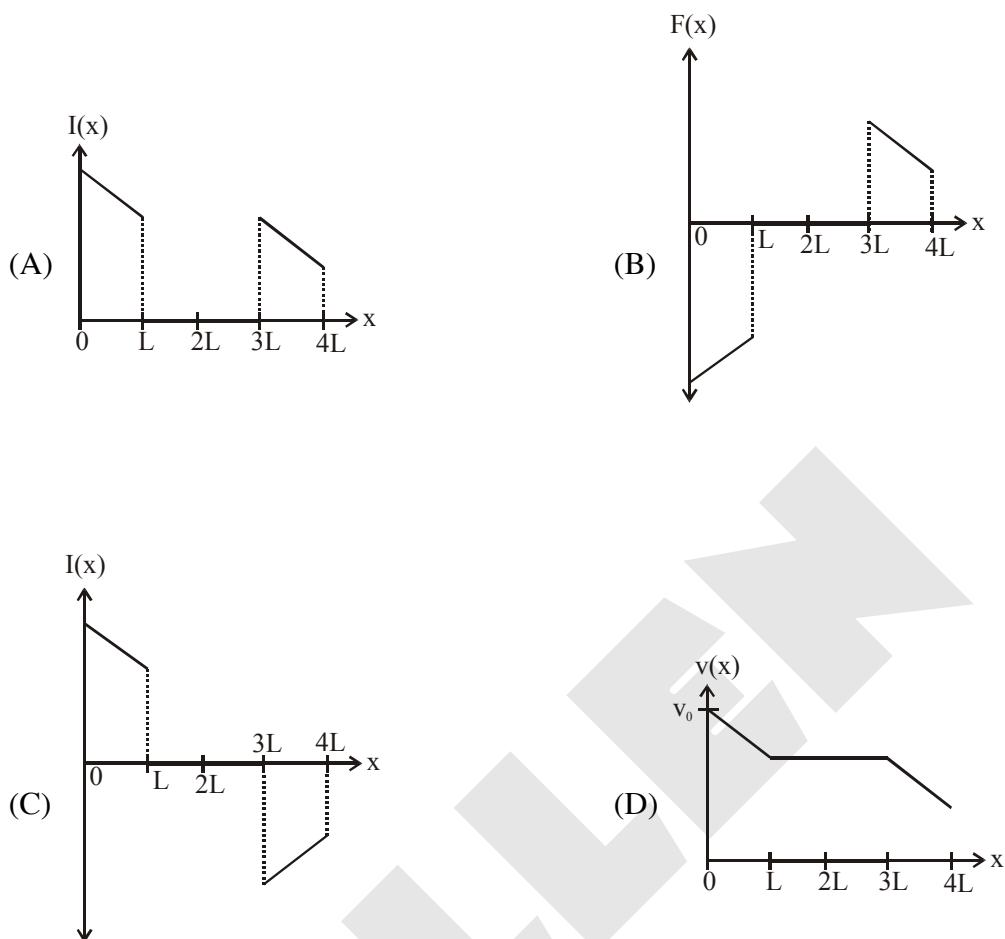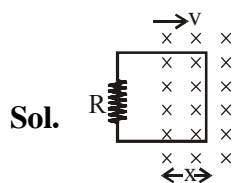
When loop was entering $(x < L)$

$\phi = BLx$

$e = -\dfrac{d\phi}{dt} = -BL\dfrac{dx}{dt}$

$|e| = BLV$

$i = \dfrac{e}{R} = \dfrac{BLV}{R}(ACW)$

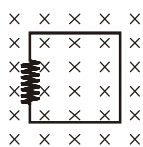$F = i\ell B$ (Left direction) $= \dfrac{B^2 L^2 V}{R}$ (in left direction)

PHYSICS

$$\Rightarrow a = \frac{F}{m} = -\frac{B^2L^2V}{mR} \quad a = V\frac{dV}{dx}$$

$$V\frac{dV}{dx} = -\frac{B^2L^2V}{mR} \quad \Rightarrow \int_{V_0}^{V} dV = -\frac{B^2L^2}{mR}\int_0^x dx$$

$$\Rightarrow V = V_0 - \frac{B^2L^2}{mR}x \quad \text{(straight line of negative slope for } x < L)$$

$$I = \frac{BL}{R}V \quad \Rightarrow \text{(I vs x will also be straight line of negative slope for } x < L)$$

$$L \le x \le 3L$$

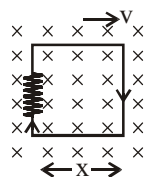
$$\frac{d\phi}{dt} = 0$$

$$e = 0 \quad i = 0$$

$$F = 0$$

$$x > 4L$$

$$e = B\ell v$$

Force also will be in left direction.

$$i = \frac{BLV}{R} \text{ (clockwise)} \quad a = -\frac{B^2L^2V}{mR} = V\frac{dV}{dx}$$

$$F = \frac{B^2L^2V}{R} \qquad \int_L^x -\frac{B^2L^2}{mR}dx = \int_{V_i}^{V_f} dV$$

$$\Rightarrow -\frac{B^2L^2}{mR}(x-L) = V_f - V_i$$

$$V_f = V_i - \frac{B^2L^2}{mR}(x-L) \quad \text{(straight line of negative slope)}$$

$$I = \frac{BLV}{R} \to \text{(Clockwise) (straight line of negative slope)}$$

10. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is(are) true ?
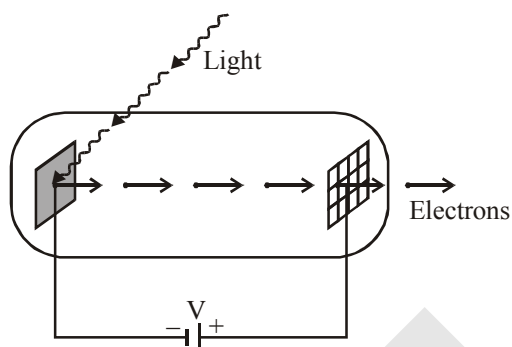
(A) For large potential difference $(V \gg \phi/e)$, λ_e is approximately halved if V is made four times

(B) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$

(C) λ_e is approximately halved, if d is doubled

(D) λ_e decreases with increase in ϕ and λ_{ph}

Ans. (A)

Sol. $K_{max} = \dfrac{hc}{\lambda_{Ph}} - \phi$

kinetic energy of e⁻ reaching the anode will be

$K = \dfrac{hc}{\lambda_{Ph}} - \phi + eV$

Now

$\lambda_e = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{\sqrt{2m\left(\dfrac{hc}{\lambda_{Ph}} - \phi + eV\right)}}$

If $eV \gg \phi$

$\lambda_e = \dfrac{h}{\sqrt{2m\left(\dfrac{hc}{\lambda_{Ph}} + eV\right)}}$

If $V_f = 4V_i$

$(\lambda_e)_f \simeq \dfrac{(\lambda_e)_i}{2}$

PHYSICS

11. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, right rod of length $\ell = \sqrt{24}a$ throught their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true ?
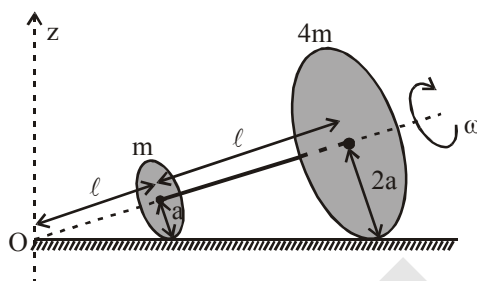
(A) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(B) The magnitude of the z-component of $\vec{L}$ is $55\ ma^2\omega$

(C) The magnitude of angular momentum of center of mass of the assembly about the point O is $81\ ma^2\omega$

(D) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5

Ans. (A,D)

Sol.
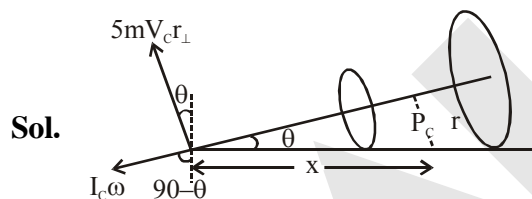
$$L_C = I_C\omega = \frac{1}{2}ma^2\omega + \frac{1}{2}\times 4m\times(2a)^2\omega = \frac{17ma^2\omega}{2}$$

$$V_C = \frac{m\times\omega a + 4m\times 2\omega a}{5m} = \frac{9\omega a}{5}$$

$$r_\perp = \frac{m\ell + 4m\times 2\ell}{5m} = \frac{9\ell}{5}$$

$$L\ of\ cm = \frac{5m\times 9\omega a}{5}\times\frac{9\ell}{5}$$

$$= 81\ m\omega a^2\times\frac{\sqrt{24}}{5}$$

$$L_z = 81 \ m\omega a^2 \ \frac{\sqrt{24}}{5} \times \cos\theta - I_C\omega \sin\theta$$

$$= 81 \ m\omega a^2 \times \frac{\sqrt{24}}{5} \times \frac{\sqrt{24}}{5} - \frac{17ma^2\omega}{10}$$

$$= \frac{m\omega a^2}{5}\left[\frac{81\times24}{5} - \frac{17}{2}\right]$$

$$= \frac{1134}{50}m\omega a^2$$

$$\omega'x = \omega r$$

$$\omega' = \frac{\omega r}{x} = \omega\sin\theta = \frac{\omega}{5}$$

12. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R/2$, which of the following statement(s) about any one of the galvanometers is(are) true ?

(A) The maximum voltage range is obtained when all the components are connected in series

(B) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

(C) The maximum current range is obtained when all the components are connected in parallel

(D) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

Ans. (B,C)

Sol.
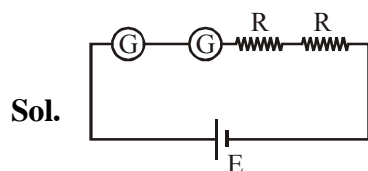
Range = $i_g(2R_C + 2R)$
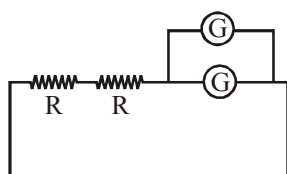
Range = $2i_g \times R_{eq}$

$$= 2i_g \left(2R + \frac{R_C}{2} \right)$$

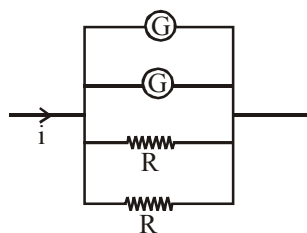$$= i_g \ (4R + R_C)$$

$$R_C < \frac{R}{2}$$

$2R_C < R \Rightarrow$ Range in $1 < 3i_g R$
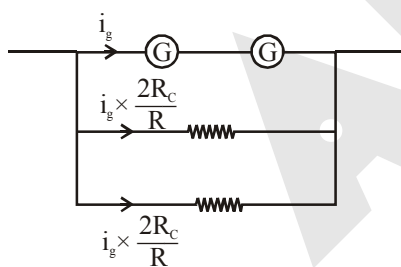
Range in $2 > 4i_g R$

So (B)

$$\text{Range} = 2i_g \ + \ 2i_g \frac{R_C}{R}$$

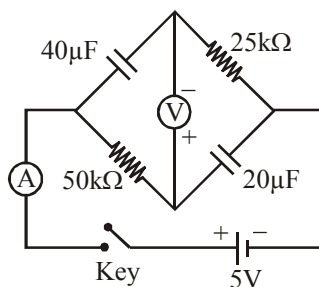$$= 2i_g \left(1 + \frac{R_C}{R} \right)$$

$$\text{Range} = i_g \ + \ 4i_g \frac{R_C}{R}$$

$$= 2i_g \left[\frac{1}{2} + \frac{2R_C}{R} \right]$$

$$= 2i_g \left(\frac{1}{2} + \frac{R_C}{R} + \frac{R_C}{R} \right)$$

So (C)

13. In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true?

(A) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time

(B) The voltmeter will display 0 V at time t = ln 2 seconds

(C) The current in the ammeter becomes 1/e of the initial value after 1 second

(D) The current in the ammeter becomes zero after a long time

Ans. (A,B,C,D)

Sol. (A) At t = 0, capacitor acts as short-circuit
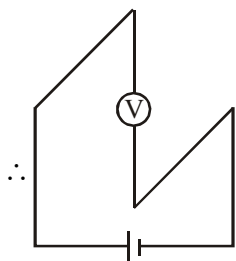
At t → ∞, capacitor acts as open circuit & no current flows through voltmeter.
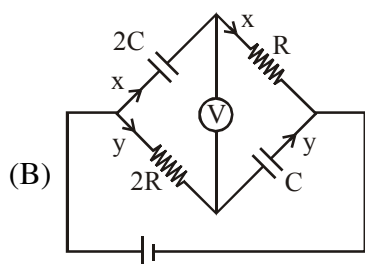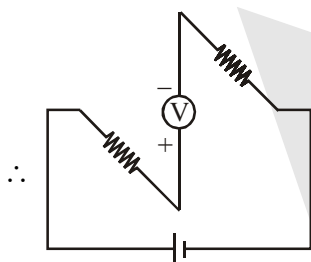
$$q_x = 2CV\left(1 - e^{-t/2CR}\right) \quad x = \frac{V}{R}e^{-t/2CR}$$

$$q_y = CV\left(1 - e^{-t/2CR}\right) \quad y = \frac{V}{2R}e^{-t/2CR}$$

$$\Delta V = -y2R + \frac{q_x}{2C}$$

$$= V\left[1 - 2e^{-t/2CR}\right] = 0$$

(C) $\tau = 1$ sec

So by $i = i_0 e^{-t/\tau}$ current at $t = 1$ sec is $= i_0/e$

(D) After long time no current flows since both capacitor & voltmeter does not allow.

14. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m ($<$M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M?

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\dfrac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

Ans. (A,B,D)

Sol. $T_i = 2\pi\sqrt{\dfrac{M}{K}}, T_f = 2\pi\sqrt{\dfrac{M+m}{K}}$

case (i) :

$M(A\omega) = (M + m)V$

$\therefore$ Velocity decreases at equilibrium position.

By energy conservation

$A_f = A_i\sqrt{\dfrac{M}{M+m}}$

case (ii) :

No energy loss, amplitude remains same

At equilibrium (x_0) velocity $= A\omega$.

In both cases ω decreases so velocity decreases in both cases

SECTION–3 : (Maximum Marks : 12)

● This section contains **TWO** paragraphs.

● Based on each paragraph, there are **TWO** questions.

● Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.

● For each question, darken the bubble corresponding to the correct option in the ORS.

● For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks   :   +3   If only the bubble corresponding to the correct answer is darkened.

*Zero Marks*   :   0   In all other cases.

PARAGRAPH 1

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{\upsilon}_{rot} \times \vec{\omega}\right) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega\hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.
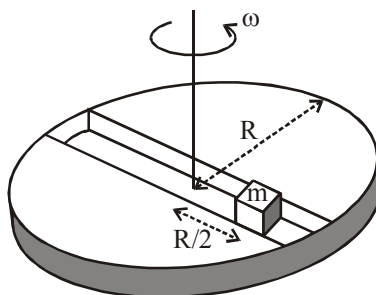
15. The distance r of the block at time t is :

(A) $\dfrac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$ (B) $\dfrac{R}{2}\cos 2\omega t$ (C) $\dfrac{R}{2}\cos \omega t$ (D) $\dfrac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

Ans. (D)

PHYSICS

Sol. Force on block along slot $= m\omega^2 r = ma = m\left(\dfrac{v\,dv}{dr}\right)$

$$\int_0^v v\,dv = \int_{R/2}^r \omega^2 r\,dr$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right) \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\Rightarrow \int_{R/4}^r \frac{dr}{\sqrt{r^2 - \dfrac{R^2}{4}}} = \int_0^t \omega\,dt$$

$$\ell n\left(\frac{r + \sqrt{r^2 - \dfrac{R^2}{4}}}{\dfrac{R}{2}}\right) - \ell n\left(\frac{R/2 + \sqrt{\dfrac{R^2}{4} - \dfrac{R^2}{4}}}{\dfrac{R}{2}}\right) = \omega t$$

$$\Rightarrow r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

$$\Rightarrow r = \frac{\dfrac{R^2}{4}e^{2\omega t} + \dfrac{R^2}{4}}{Re^{\omega t}} = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

16. The net reaction of the disc on the block is :
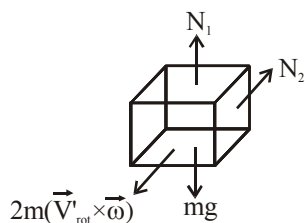
(A) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\hat{k}$ \qquad (B) $m\omega^2 R\sin\omega t\,\hat{j} - mg\hat{k}$

(C) $\dfrac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$ \qquad (D) $\dfrac{1}{2}m\omega^2 R\left(e^{2\omega t} - e^{-2\omega t}\right)\hat{j} + mg\hat{k}$

Ans. (C)

Sol.

$$\vec{N}_1 = mg\hat{k}$$

$$\vec{N}_2 = 2m\left(V'_{rot} \times \vec{\omega}\right)\hat{j}$$

$$= 2m\left[\frac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)\right]\omega\hat{j}$$

$$= \frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j}$$

Total reaction on block $= \vec{N}_1 + \vec{N}_2$

$$= \frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

PARAGRAPH 2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r <<h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collision between the balls and the interaction between them is negligible. (Ignore gravity)

PHYSICS
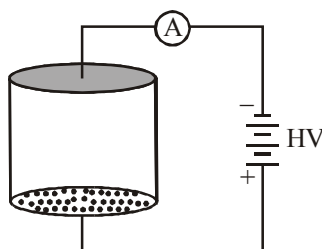
17. Which of the following statements is correct ?

(A) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(B) the balls will execute simple harmonic motion between the two plates

(C) The balls will bounce back to the bottom plate carrying the same charge they went up with

(D) The balls will stick to the top plate and remain there

Ans. **(A)**

Sol. Balls placed on +ve plate become positive charge and move upward due to electric field.

These balls on colliding with negative plate become negatively charged and move opposite to the direction of electric field.

18. The average current in the steady state registered by the ammeter in the circuit will be :

(A) Proportional to $V_0^{1/2}$ (B) Proportional to V_0^2

(C) Proportional to the potential V_0 (D) Zero

Ans. **(B)**

Sol.
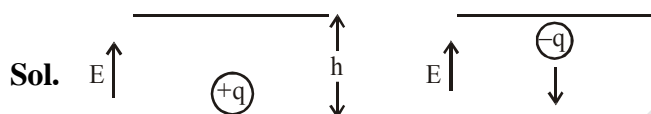
$$h = \frac{1}{2}at^2 \qquad [as \ u = 0]$$

$$\sqrt{\frac{2hm}{qE}} = time \Rightarrow time = \sqrt{\frac{2m}{q\Delta V}}$$

$$E = \frac{V_0}{h}$$

$$<current> = \frac{charge}{time} = \frac{q\sqrt{qV_0}}{\sqrt{2mh^2}}$$

$$q \propto V_0$$
$$< I > \propto V_0^2$$

JEE(Advanced) – 2016 TEST PAPER WITH SOLUTIONS

(HELD ON SUNDAY 22nd MAY, 2016)

# PART - II : CHEMISTRY

SECTION–1 : (Maximum Marks : 18)

● This section contains **SIX** questions.

● Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.

● For each question, darken the bubble corresponding to the correct option in the ORS.

● For each question, marks will be awarded in one of the following categories :

Full Marks   : +3   If only the bubble corresponding to the correct option is darkened.

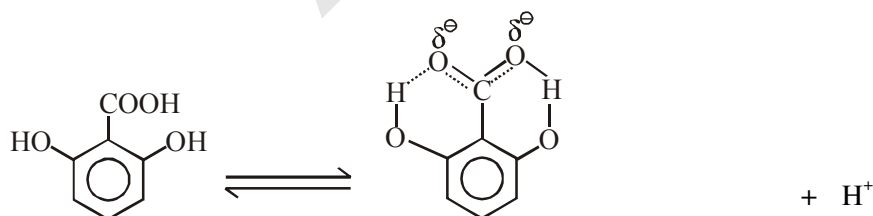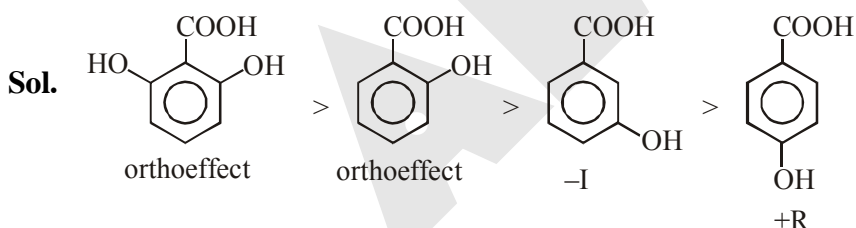*Zero Marks*   :   0   If none of the bubbles is darkened.

Negative Marks : −1   In all other cases.

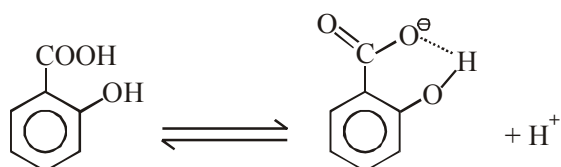**19.** The correct order of acidity for the following compounds is :
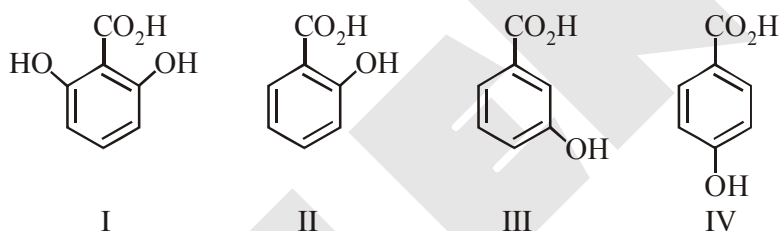
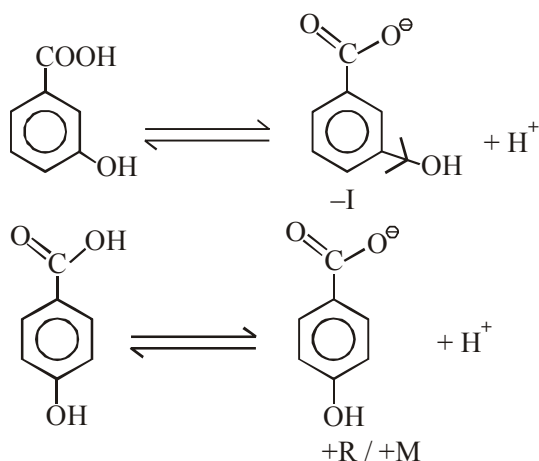(A) I > II > III > IV   (B) III > I > II > IV   (C) III > IV > II > I   (D) I > III > IV > II

Ans. (A)

Sol.

CHEMISTRY

$-I$

$+R / +M$

20. The geometries of the ammonia complexes of Ni^{2+}, Pt^{2+} and Zn^{2+}, respectively, are :

(A) octahedral, square planar and tetrahederal

(B) square planar, octahederal and tetrahederal

(C) tetrahederal, square planar and octahederal

(D) octahederal, tetrahederal and square planar

Ans. (A)

Sol.

Metal ion	Complex with NH_3	Geometry
Ni^{2+}	$[Ni(NH_3)_6]^{2+}$	Octahedral
Pt^{2+}	$[Pt(NH_3)_4]^{2+}$	Square planar
Zn^{2+}	$[Zn(NH_3)_4]^{2+}$	Tetrahedral

So, option (A) is correct.

21. For the following electrochemical cell at 298K,

$Pt(s) \mid H_2(g, 1bar) \mid H^+ (aq, 1M) \parallel M^{4+}(aq.), M^{2+}(aq.) \mid Pt(s)$

$E_{cell} = 0.092$ V when $\dfrac{[M^{2+}(aq.)]}{[M^{4+}(aq.)]} = 10^x$

Given : $E^0_{M^{4+}/M^{2+}} = 0.151V$; $2.303 \dfrac{RT}{F} = 0.059$

The value of x is -

(A) –2   (B) –1   (C) 1   (D) 2

Ans. (D)

Sol. At anode : $H_2(g) \rightleftharpoons 2H^+ (aq) + 2e^-$

At cathode : $M^{4+}(aq) + 2e^- \rightleftharpoons M^{2+} (aq)$

Net cell reaction : $H_2(g) + M^{4+} (aq) \rightleftharpoons 2H^+ (aq) + M^{2+} (aq)$
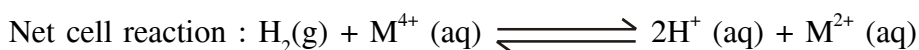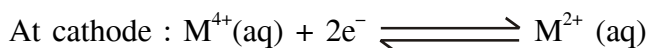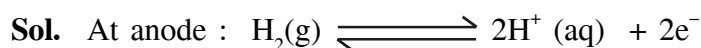
Now, $E_{cell} = \left(E^{\circ}_{M^{4+}/M^{2+}} - E^{\circ}_{H^+/H_2} \right) - \dfrac{0.059}{n} . \log \dfrac{\left[H^+ \right]^2 \left[M^{2+} \right]}{P_{H_2} . \left[M^{4+} \right]}$

or, $0.092 = (0.151 - 0) - \dfrac{0.059}{2}.\log\dfrac{1^2 \times [M^{2+}]}{1 \times [M^{4+}]}$

$\therefore \quad \dfrac{[M^{2+}]}{[M^{4+}]} = 10^2 \quad \Rightarrow x = 2$

22. The major product of the following reaction sequence is :

$\xrightarrow[\text{(ii) HCHO / H}^+ \text{ (catalytic amount)}]{\text{(i) HCHO (excess) / NaOH, heat}}$
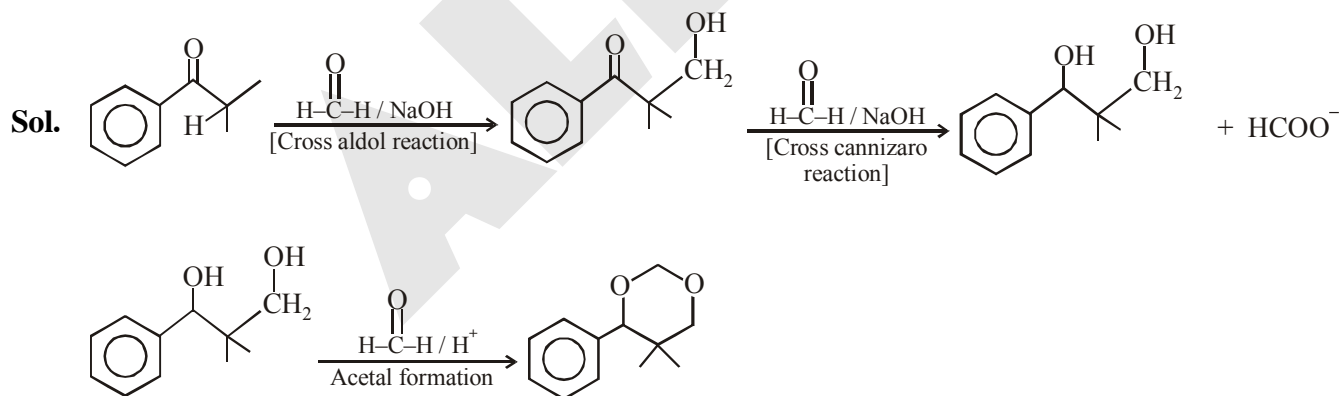
(A)

(B)

(C)

(D)

Ans. (A)

Sol.

$\xrightarrow[\text{[Cross aldol reaction]}]{\text{H–C–H / NaOH}}$ $\xrightarrow[\text{[Cross cannizaro reaction]}]{\text{H–C–H / NaOH}}$ $+ \; HCOO^-$

 $\xrightarrow[\text{Acetal formation}]{\text{H–C–H / H}^+}$
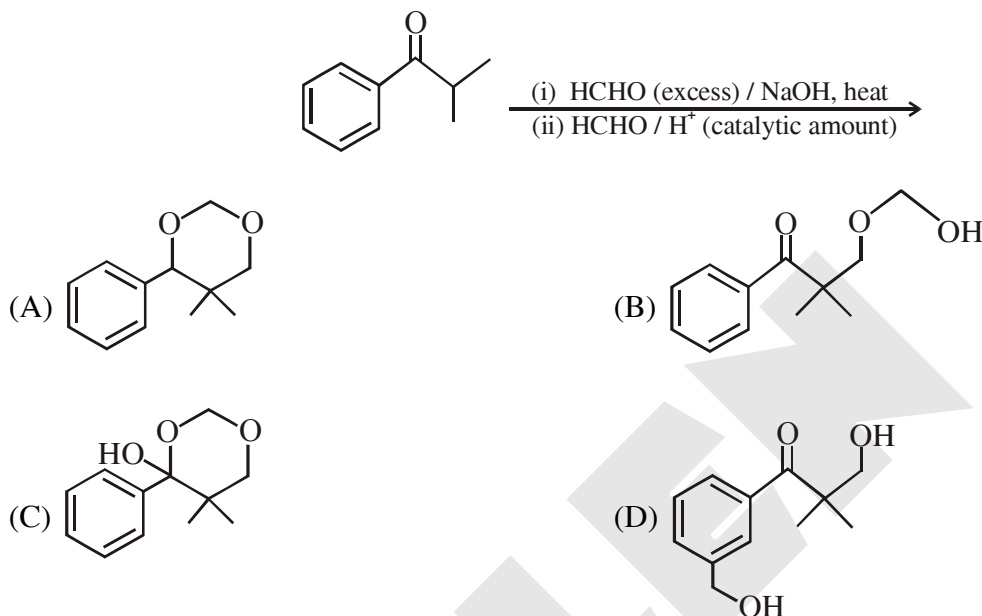
23. In the following reaction sequence in aqueous soluiton, the species **X**, **Y** and **Z** respectively, are -

$S_2O_3^{2-} \xrightarrow{Ag^+} \underset{\substack{\text{clear} \\ \text{solution}}}{X} \xrightarrow{Ag^+} \underset{\substack{\text{white} \\ \text{precipitate}}}{Y} \xrightarrow{\text{With time}} \underset{\substack{\text{black} \\ \text{precipitate}}}{Z}$

(A) $[Ag(S_2O_3)_2]^{3-}$, $Ag_2S_2O_3$, Ag_2S

(B) $[Ag(S_2O_3)_3]^{5-}$, Ag_2SO_3, Ag_2S

(C) $[Ag(SO_3)_2]^{3-}$, $Ag_2S_2O_3$, Ag

(D) $[Ag(SO_3)_3]^{3-}$, Ag_2SO_4, Ag

Ans. (A)

CHEMISTRY

Sol. $S_2O_3^{2-} \xrightarrow{Ag^+} [Ag(S_2O_3)_2]^{3-} \xrightarrow{Ag^+} Ag_2S_2O_3\downarrow$
$\qquad\qquad\qquad\quad$ (X) $\qquad\qquad\qquad$ (Y)
$\qquad\qquad\quad$ clear solution $\qquad$ white precipitate

$\qquad\qquad\qquad\qquad\qquad\qquad$ $\downarrow$ with time

$\qquad\qquad\qquad\qquad\qquad$ $Ag_2S\downarrow$
$\qquad\qquad\qquad\qquad\qquad$ (Z)
$\qquad\qquad\qquad\qquad$ black precipitate
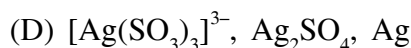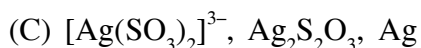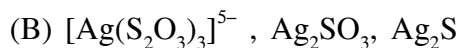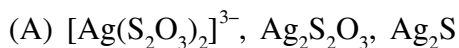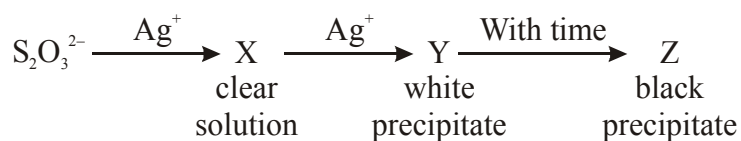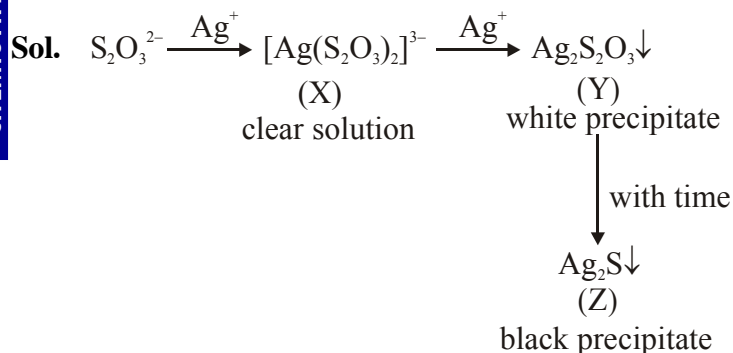
So, X, Y and Z are $[Ag(S_2O_3)_2]^{3-}$, $Ag_2S_2O_3$ and Ag_2S respectively.

24. The qualitative sketches I , II and III given below show the variation of surface tension with molar concentration of three different aqueous solutions of KCl, CH_3OH and $CH_3(CH_2)_{11}OSO_3^-$ Na^+ at room temperature. The correct assignment of the sketches is -
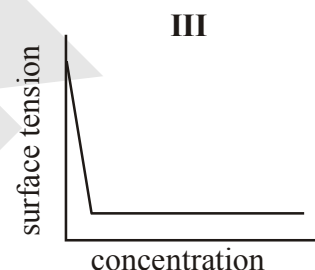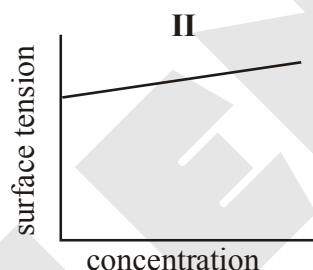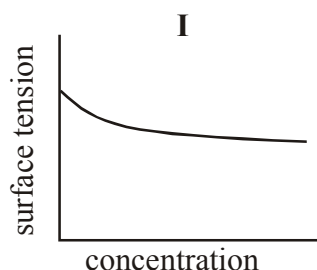
(A) **I :** KCl $\qquad\qquad$ **II :** CH_3OH $\qquad\qquad$ **III :** $CH_3(CH_2)_{11}OSO_3^-$ Na^+

(B) **I :** $CH_3(CH_2)_{11}OSO_3^-$ Na^+ $\quad$ **II :** CH_3OH $\qquad\qquad$ **III :** KCl

(C) **I :** KCl $\qquad\qquad$ **II :** $CH_3(CH_2)_{11}OSO_3^-$ Na^+ $\quad$ **III :** CH_3OH

(D) **I :** CH_3OH $\qquad\qquad$ **II :** KCl $\qquad\qquad$ **III :** $CH_3(CH_2)_{11}OSO_3^-$ Na^+

Ans. (D)

Sol. Water has large surface tension due to very strong interaction. Generally adding organic derivatives to water decreases its surface tension due to hydrophobic interaction.

In case III, hydrophobic interaction is stronger than case I causing surface tension to decrease more rapidly.

Due to K^+Cl^- (inorganic electrolyte) intermolecular forces increases, surface tension increases.
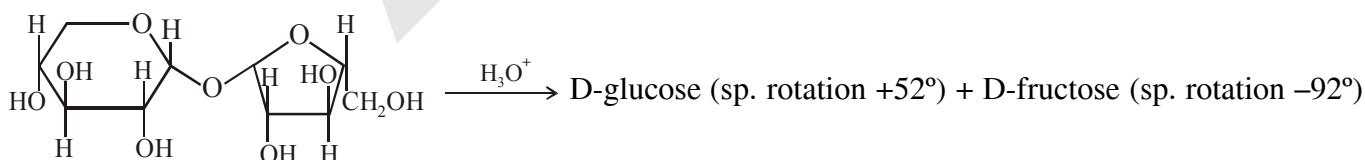
SECTION–2 : (Maximum Marks : 32)

● This section contains **EIGHT** questions.

● Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

● For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

● For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

● for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

25. For 'invert sugar', the correct statement(s) is (are)

(Given : specific rotations of (+)-sucrose, (+)-maltose, L-(–)-glucose and L-(+)-fructose in aqueous solution are + 66°, +140°, –52° and + 92°, respectively)

(A) 'invert sugar' is prepared by acid catalyzed hydrolysis of maltose

(B) 'invert sugar' is an equimolar mixture of D-(+) glucose and D-(–)-fructose

(C) specific rotation of 'invert surgar' is –20°

(D) on reaction with Br_2 water, 'invert sugar' forms saccharic acid as one of the products

Ans. (B,C)

Sol. Invert sugar is equailmolar mixture of D-glucose and D-fructose which is obtained by hydrolysis of sucrose

Specific rotation of mixture is half of sum of sp. rotation of both components $\dfrac{+52° + (-92°)}{2} = -20°$

Sucrose sp. rotation + 66°

$\xrightarrow{H_3O^+}$ D-glucose (sp. rotation +52°) + D-fructose (sp. rotation –92°)

D-glucose $\xrightarrow{Br_2, H_2O}$ D-gluconic acid (not saccharic acid)
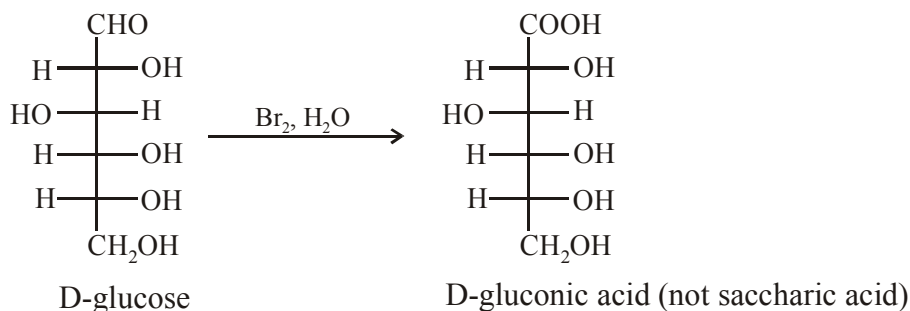
D-fructose $\xrightarrow{Br_2, H_2O}$ No reaction

CHEMISTRY

26. Among the following reaction(s) which gives (give) tert-butyl benzene as the major product is(are)

(A) $\xrightarrow{NaOC_2H_5}$

(B) $\xrightarrow{AlCl_3}$

(C) $\xrightarrow{H_2SO_4}$

(D) $\xrightarrow{BF_3.OEt_2}$

Ans. **(B,C,D)**

Sol.
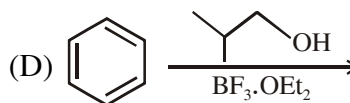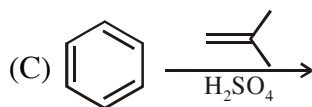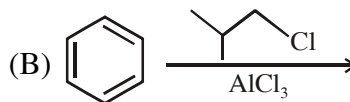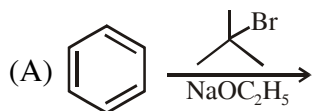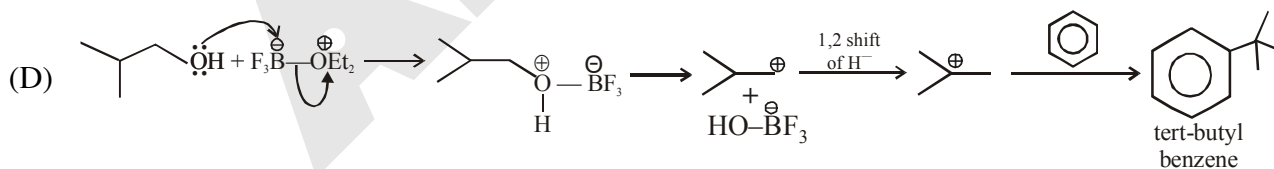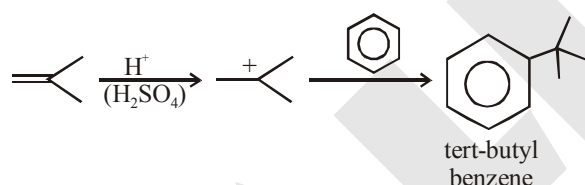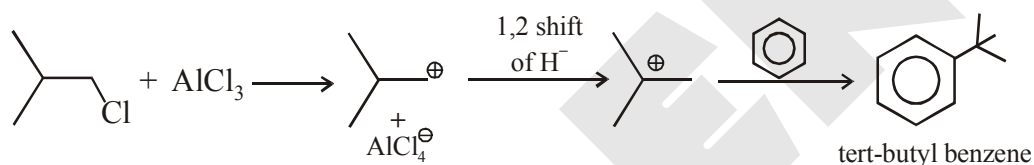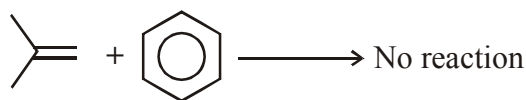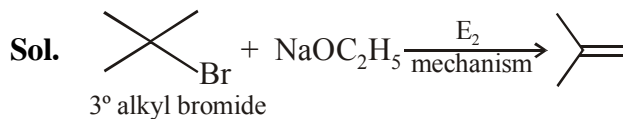
27. Extraction of copper from copper pyrite ($CuFeS_2$) involves

(A) crushing followed by concentration of the ore by froth-flotation

(B) removal of iron as slag

(C) self-reduction step to produce 'blister copper' follwoing evolution of SO_2

(D) refining of 'blister copper' by carbon reduction

Ans. **(A,B,C)**

Sol.   Copper pyrite [$CuFeS_2$]

$\downarrow$ Crushing into fine powder

Concentrated by froth floatation process

$\downarrow$

Roasting take place in reverberatory furnace

$\begin{cases} 2CuFeS_2 + O_2 \xrightarrow[\text{(air)}]{\Delta} Cu_2S + 2FeS + SO_2 \\ FeS + O_2 \xrightarrow{\Delta} FeO + SO_2 \end{cases}$

$\downarrow$ Smelting $\begin{bmatrix} \text{Iron is removed as slag of } FeSiO_3 \\ FeO + SiO_2(\text{flux}) \xrightarrow{\Delta} FeSiO_3(\ell)(\text{slag}) \end{bmatrix}$

Copper matte (Cu_2S + FeS)

$\downarrow$ Self reduction $\begin{bmatrix} \text{Self reduction occurs in Bessemer converter} \\ 2Cu_2S + 3O_2 \longrightarrow 2Cu_2S + 2SO_2(\uparrow) \\ Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2(\uparrow) \end{bmatrix}$
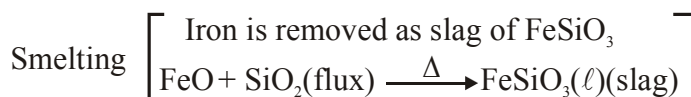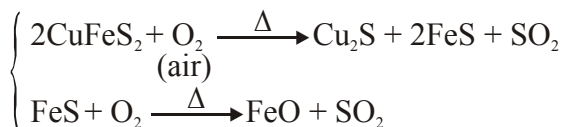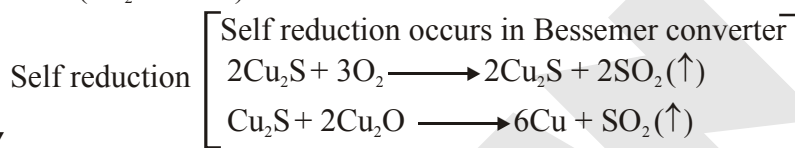
(Blister copper)

$\downarrow$

Refining of blister copper is done by poling followed by electrorefining but not by carbon reduction method.

28. The **CORRECT** statement(s) for cubic close packed (ccp) three dimensional structure is (are)

(A) The number of the nearest neighbours of an atom present in the topmost layer is 12

(B) The efficiency of atom packing is 74%

(C) The number of octahedral and tetrahedral voids per atom are 1 and 2, respectively

(D) The unit cell edge length is $2\sqrt{2}$ times the radius of the atom

Ans. (B,C,D)

Sol. CCP is ABC ABC ..... type packing

(A) In topmost layer, each atom is in contact with 6 atoms in same layer and 3 atoms below this layer.

(B) Packing fraction = $\dfrac{4 \times \dfrac{4}{3}\pi r^3}{\left(\dfrac{4r}{\sqrt{2}}\right)^3} = (0.74)$

(C) Each FCC unit has effective no of atoms = 4

Octahedral void = 4

Tetrahedral void = 8

(D) $4r = a\sqrt{2}$

CHEMISTRY

29. Reagent(s) which can be used to bring about the following transformation is(are)
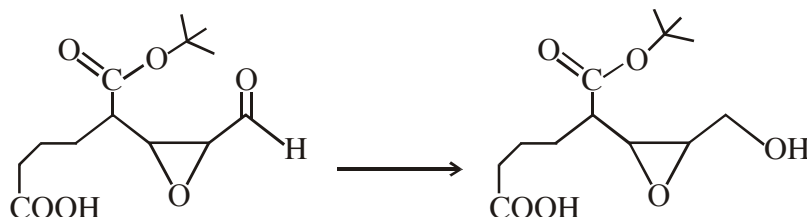
(A) LiAlH$_4$ in (C$_2$H$_5$)$_2$O   (B) BH$_3$ in THF

(C) NaBH$_4$ in C$_2$H$_5$OH (D) Raney Ni / H$_2$ in THF

Ans. (C)

Sol.
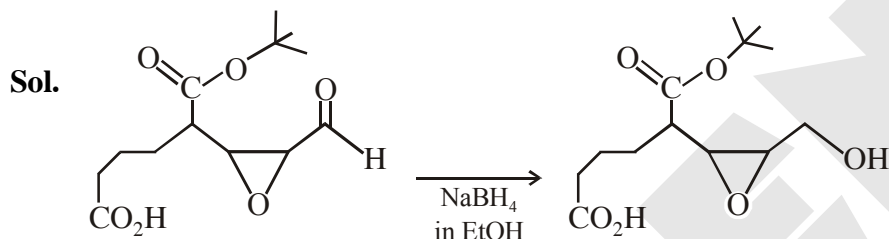
LiAlH$_4$ in (C$_2$H$_5$)$_2$O ; BH$_3$ in (THF) ; Raney Ni (H$_2$) either can reduce all functional group or can reduce some of the functional group of the compound given above in reactant side.

But NaBH$_4$ is example of selective reducing agent. It can not reduce –C–O– (ester group) , –C–OH (carboxylic acid group) , (epoxide group) , but reduces –CH=O (aldehyde group) into –CH$_2$OH (1° alcohol)

30. Mixture(s) showing positive deviation from Raoult's law at 35°C is (are)

(A) carbon tetrachloride + methanol   (B) carbon disulphide + acetone

(C) benzene + toluene   (D) phenol + aniline

Ans. (A, B)

Sol. (A) H-bonding of methanol breaks when CCl$_4$ is added so bonds become weaker, resulting positive deviation.

(B) Mixing of polar and non-polar liquids will produce a solution of weaker interaction, resulting positive deviation

(C) Ideal solution

(D) –ve deviation because stronger H-bond is formed.

31. The nitrogen containing compound produced in the reaction of HNO_3 with P_4O_{10}

(A) can also be prepared by reaction of P_4 and HNO_3

(B) is diamagnetic

(C) contains one N-N bond

(D) reacts with Na metal producing a brown gas

Ans. (B,D)

Sol. $P_4O_{10} + 4HNO_3 \xrightarrow{\text{dehydration of } HNO_3} 4(HPO_3) + 2N_2O_5$

$$\text{(required product)}$$

(A) $P_4 + 20\,HNO_3 \rightarrow 4H_3PO_4 + 20NO_2\uparrow + 4H_2O$
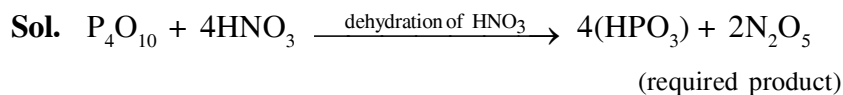
(B) N_2O_5 is diamagnetic in nature

(C) $N_2O_5 \rightarrow$
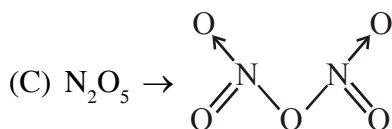
N_2O_5 contains one N–O–N bond not N–N bond.

(D) $Na + N_2O_5 \rightarrow NaNO_3 + NO_2\uparrow$

$$\text{(Brown gas)}$$
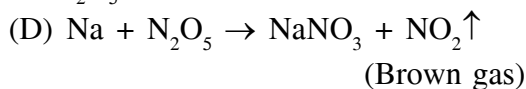
32. According to Molecular Orbital Theory,

(A) C_2^{2-} is expected to be diamagnetic

(B) O_2^{2+} is expected to have a longer bond length than O_2

(C) N_2^+ and N_2^- have the same bond order

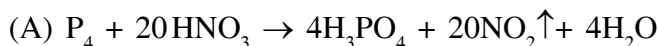(D) He_2^+ has the same energy as two isolated He atoms

Ans. (A,C)

Sol (A) The molecular orbital energy configuration of C_2^{2-} is

$$\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^2$$

In the MO of C_2^{2-} there is no unpaired electron hence it is diamagnatic

(B) Bond order of O_2^{2+} is 3 and O_2 is 2 therefore bond length of O_2 is greater than O_2^{2+}

(C) The molecular orbital energy configuration of N_2^+ is

$$\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^1$$

Bond order of $N_2^+ = \dfrac{1}{2}(9-4)$

$$= 2.5$$

The molecular orbital energy configuration of N_2^- is

$$\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2 = \pi_{2p_y}^2, \sigma_{2p_z}^2, \pi_{2p_x}^{*1} = \pi_{2p_y}^*$$

Bond order of $N_2^- = \dfrac{1}{2}(10-5)$

$$= 2.5$$

(D) He_2^+ has less energy as compare to two isolated He atoms

CHEMISTRY

SECTION–3 : (Maximum Marks : 12)

● This section contains **TWO** paragraphs.
● Based on each paragraph, there are **TWO** questions.
● Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
● For each question, darken the bubble corresponding to the correct option in the ORS.
● For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks   : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks   :   0 In all other cases.

PARAGRAPH 1

Thermal decomposition of gaseous X_2 to gaseous X at 298 K takes place according to the following equation :

$$X_2(g) \rightleftharpoons 2X(g)$$
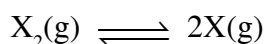
The standard reaction Gibbs energy, $\Delta_r G°$, of this reaction is positive. At the start of the reaction, there is one mole of X_2 and no X. As the reaction proceeds, the number of moles of X formed is given by β. Thus, $\beta_{equilibrium}$ is the number of moles of X formed at equilibrium. The reaction is carried out at a constant total pressure of 2 bar. Consider the gases to behave ideally.

(Given : R = 0.083 L bar K^{-1} mol^{-1})

33. The equilibrium constant K_P for this reaction at 298 K, in terms of $\beta_{equilibrium}$, is

(A) $\dfrac{8\beta_{equilibrium}^2}{2-\beta_{equilibrium}}$ (B) $\dfrac{8\beta_{equilibrium}^2}{4-\beta_{equilibrium}^2}$ (C) $\dfrac{4\beta_{equilibrium}^2}{2-\beta_{equilibrium}}$ (D) $\dfrac{4\beta_{equilibrium}^2}{4-\beta_{equilibrium}^2}$

Ans. (B)

Sol. $X_2(g) \rightleftharpoons 2X(g)$

$$1-\frac{\beta_{eq.}}{2} \qquad\qquad \beta_{eq.}$$
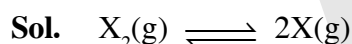
$$K_P = \frac{P_X^2}{P_{X_2}} = \frac{\left(\dfrac{\dfrac{\beta_{eq.}}{1+\dfrac{\beta_{eq}}{2}}P_T\right)^2}{\left(\dfrac{1-\dfrac{\beta_{eq.}}{2}}{1+\dfrac{\beta_{eq}}{2}}P_T\right)}$$

$$K_P = \frac{\beta_{eq.}^2}{1-\dfrac{\beta_{eq.}^2}{4}}P_T = \frac{2\beta_{eq}^2}{1-\dfrac{\beta_{eq.}}{4}}$$

$$= \frac{8\beta_{eq.}^2}{4-\beta_{eq.}^2}$$

34. The **INCORRECT** statement among the following , for this reaction, is

(A) Decrease in the total pressure will result in formation of more moles of gaseous X

(B) At the start of the reaction, dissociation of gaseous X_2 takes place spontaneously

(C) $\beta_{equilibrium}$ = 0.7

(D) K_C < 1

Ans. (C)

Sol. (A) On decreasing P_T $\left[Q = \dfrac{n_{x^2} P_T}{n_{x_2} n_T} \right]$ Q will be less than Kp reaction will move in forward direction

(B) At the start of the reaction $\Delta G = \Delta G^0 + RT \ln Q$

t = 0 , Q = 0 $\Rightarrow \Delta_{rxn} G = -ve$ (spontaneous)

(C) if $\beta_{eq} = 0.7$

$K_p = \dfrac{8 \times 0.49}{4 - 0.49} = \dfrac{3.92}{3.51}$

$K_p > 1$

Since it is given that

$\Delta G^0 > 0 \Rightarrow K_p < 1$

∴ This is incorrect

(D) $K_p = K_C \times (RT)^{\Delta n_g}$

$K_C = \dfrac{K_P}{(R \times 298)^1}$

$K_C < 1$

PARAGRAPH 2

Treatment of compound **O** with $KMnO_4$ / H^+ gave **P**, which on heating with ammonia gave **Q**. The compound **Q** on treatment with Br_2 / NaOH produced **R**. On strong heating, **Q** gave **S**, which on further treatmenet with ethyl 2-bromopropanoate in the presence of KOH following by acidification, gave a compound **T**.

(O)

35. The compound **R** is :

(A) benzene ring with NH$_2$ and NH$_2$ substituents (ortho-diaminobenzene)   (B) benzene ring with two C(=O)Br groups   (C) benzene ring with two C(=O)NHBr groups   (D) benzene ring with imide N-Br
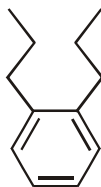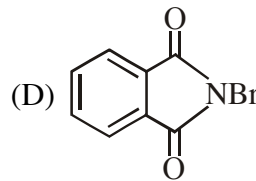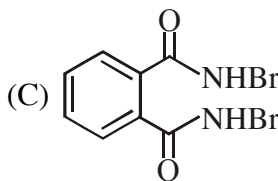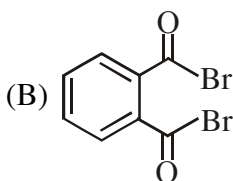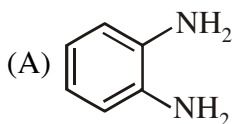
Ans. (A)

36. The compound **T** is :

(A) Glycine   (B) Alanine   (C) Valine   (D) Serine

Ans. (B)

Sol.

Solution Q.35 & 36.
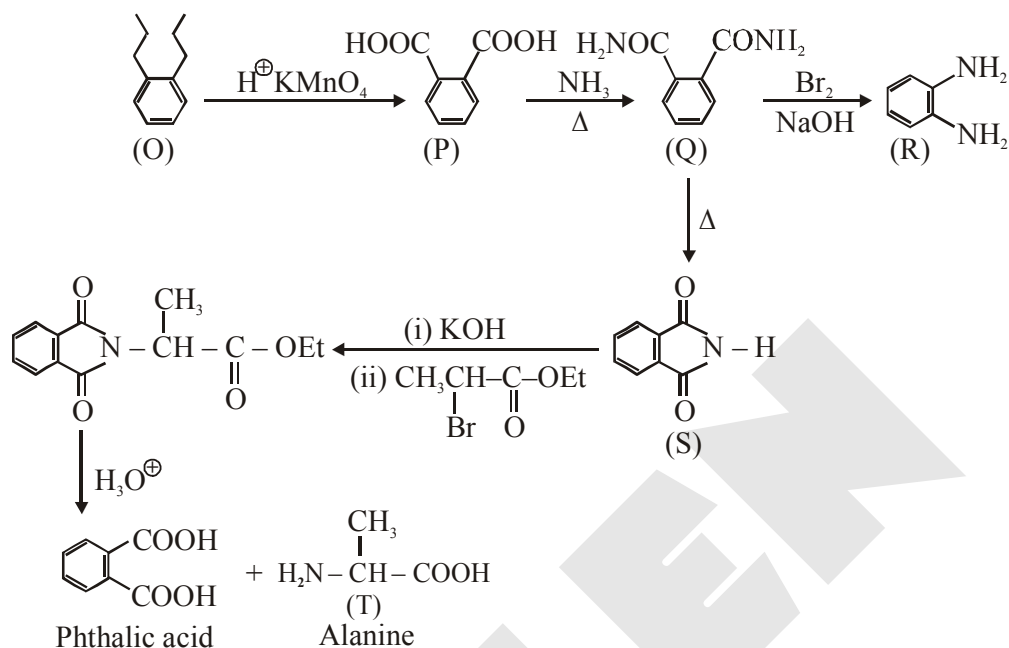
Q to R is Hoffmann's bromamide degradation reaction

S to T is Gabriel's phthalimide sysnthesis

JEE(Advanced) – 2016 TEST PAPER WITH SOLUTIONS
(HELD ON SUNDAY 22nd MAY, 2016)

PART - III : MATHEMATICS

SECTION–1 : (Maximum Marks : 18)
- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u> :
  Full Marks   : +3 If only the bubble corresponding to the correct option is darkened.
  Zero Marks   :   0 If none of the bubbles is darkened.
  Negative Marks : −1 In all other cases.

37. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that

$P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals

(A) 52   (B) 103   (C) 201   (D) 205

Ans. (B)

Sol. $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \Rightarrow P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 16+32 & 8 & 1 \end{bmatrix}$

so, $P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 16+32+48 & 12 & 1 \end{bmatrix}$ (from the symmetry)

$P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ \dfrac{16.50.51}{2} & 200 & 1 \end{bmatrix}$

As, $P^{50} - Q = I \Rightarrow q_{31} = \dfrac{16.50.51}{2}$

$q_{32} = 200$ and $q_{21} = 200$

$\therefore \dfrac{q_{31} + q_{32}}{q_{21}} = \dfrac{16.50.51}{2.200} + 1 = 102 + 1 = 103$

38. Area of the region $\left\{(x, y) \in \mathbb{R}^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\right\}$ is equal to -

(A) $\dfrac{1}{6}$ 　　　　(B) $\dfrac{4}{3}$ 　　　　(C) $\dfrac{3}{2}$ 　　　　(D) $\dfrac{5}{3}$

Ans. (C)

Sol.
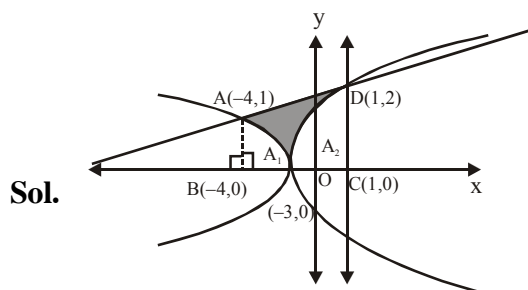
Clearly required area = area (trapezium ABCD) − (A$_1$ + A$_2$)　　.....(i)

area (trapezium ABCD) $= \dfrac{1}{2}(1+2)(5) = \dfrac{15}{2}$

$A_1 = \displaystyle\int_{-4}^{-3} \sqrt{-(x+3)}\, dx$

$= \dfrac{2}{3}$

and $A_2 = \displaystyle\int_{-3}^{1} (x+3)^{1/2}\, dx = \dfrac{16}{3}$

$\therefore$ From equation (1), we get required area $= \dfrac{15}{2} - \left(\dfrac{2}{3} + \dfrac{16}{3}\right) = \dfrac{3}{2}$

39. The value of $\displaystyle\sum_{k=1}^{13} \dfrac{1}{\sin\left(\dfrac{\pi}{4} + \dfrac{(k-1)\pi}{6}\right)\sin\left(\dfrac{\pi}{4} + \dfrac{k\pi}{6}\right)}$ is equal to

(A) $3 - \sqrt{3}$ 　　(B) $2(3 - \sqrt{3})$ 　　(C) $2(\sqrt{3} - 1)$ 　　(D) $2(2 + \sqrt{3})$

Ans. (C)

Sol. We have,

$= 2.\displaystyle\sum_{k=1}^{13} \dfrac{\sin\left(\left(\dfrac{k\pi}{6} + \dfrac{\pi}{4}\right) - \left((k-1)\dfrac{\pi}{6} + \dfrac{\pi}{4}\right)\right)}{\sin\left(\dfrac{\pi}{4} + (k-1)\dfrac{\pi}{6}\right).\sin\left(\dfrac{\pi}{4} + \dfrac{k\pi}{6}\right)} = 2\displaystyle\sum_{k=1}^{13}\left(\cot\left((k-1)\dfrac{\pi}{6} + \dfrac{\pi}{4}\right) - \cot\left(\dfrac{k\pi}{6} + \dfrac{\pi}{4}\right)\right)$

$= 2\left[\cot\dfrac{\pi}{4} - \cot\left(\dfrac{13\pi}{6} + \dfrac{\pi}{4}\right)\right] = 2\left(1 - \cot\left(\dfrac{5\pi}{12}\right)\right) = 2\left(1 - (2 - \sqrt{3})\right) = 2(\sqrt{3} - 1)$

40. Let $b_i > 1$ for $i = 1, 2, ..., 101$. Suppose $\log_e b_1, \log_e b_2,.....,\log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2,......, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a_{51}$ then

(A) $s > t$ and $a_{101} > b_{101}$

(B) $s > t$ and $a_{101} < b_{101}$

(C) $s < t$ and $a_{101} > b_{101}$

(D) $s < t$ and $a_{101} < b_{101}$

Ans. (B)

Sol. If $\log_e b_1, \log_e b_2......\log_e b_{101} \to$ AP ; D $= \log_e 2$

$\Rightarrow b_1\, b_2\, b_3....b_{101} \to$ GP ; r $= 2$

$\therefore\quad b_1, 2b_1,\, 2^2 b_1,........., 2^{100} b_1......$ GP

$a_1\, a_2\, a_3\, ...a_{101}\, ...$ AP

Given, $a_1 = b_1$ & $a_{51} = b_{51}$

$\Rightarrow\quad a_1 + 50D = 2^{50} b_1$

$\therefore\quad a_1 + 50D = 2^{50} a_1$ (As $b_1 = a_1$)

Now, $t = b_1(2^{51} - 1)$; $s = \dfrac{51}{2}(2a_1 + 50D)$

$t = a_1 . 2^{51} - a_1 \Rightarrow t < a_1 . 2^{51}$ ....(i) ; $s = \dfrac{51}{2}(a_1 + a_1 + 50D)$

$$s = \dfrac{51}{2}(a_1 + 2^{50} a_1)$$

$$s = \dfrac{51a_1}{2} + \dfrac{51}{2} . 2^{50} a_1$$

$\Rightarrow s > a_1 . 2^{51}$ ....(ii)

clearly $s > t$ (from equation (i) and (ii))

Also $a_{101} = a_1 + 100D$; $b_{101} = b_1 . 2^{100}$

$\therefore\quad a_{101} = a_1 + 100\left(\dfrac{2^{50} a_1 - a_1}{50}\right)$; $b_{101} = 2^{100} a_1$ ....(iii)

$a_{101} = a_1 + 2^{51} a_1 - 2a_1 \Rightarrow a_{101} = 2^{51} a_1 - a_1 \Rightarrow a_{101} < 2^{51} a_1$ ....(iv)

clearly $b_{101} > a_{101}$ (from equation (iii) and (iv))

41. The value of $\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \dfrac{x^2 \cos x}{1+e^x}\,dx$ is equal to

(A) $\dfrac{\pi^2}{4}-2$ 　　　(B) $\dfrac{\pi^2}{4}+2$ 　　　(C) $\pi^2-e^{\frac{\pi}{2}}$ 　　　(D) $\pi^2+e^{\frac{\pi}{2}}$

Ans. (A)

Sol. Let $I = \int\limits_{-\pi/2}^{\pi/2} \dfrac{x^2 \cos x}{1+e^x}\,dx = \int\limits_{0}^{\pi/2}\left(\dfrac{1}{1+e^x}+\dfrac{1}{1+e^{-x}}\right)x^2 \cos x\,dx$

$= \int\limits_{0}^{\pi/2} x^2 \cos x\,dx = \underbrace{\left(x^2 \sin x\right)_{0}^{\pi/2}}_{\text{(I)}\ \ \text{(II)}} - 2\int\limits_{0}^{\pi/2} \underbrace{x.\sin x\,dx}_{\text{(I)}\ \ \text{(II)}}$

$= \dfrac{\pi^2}{4} - 2\left[-\left(x.\cos x\right)_{0}^{\pi/2} + \int\limits_{0}^{\pi/2} 1.\cos x\right] = \dfrac{\pi^2}{4} - 2[0+1] = \left(\dfrac{\pi^2}{4}-2\right)$

42. Let P be the image of the point (3, 1, 7) with respect to the plane x − y + z = 3. Then the equation of the plane passing through P and containing the straight line $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{1}$ is

(A) x + y − 3z = 0 　　　　　　(B) 3x + z = 0

(C) x − 4y + 7z = 0 　　　　　(D) 2x − y = 0

Ans. (C)

Sol. Line AP : $\dfrac{x-3}{1}=\dfrac{y-1}{-1}=\dfrac{z-7}{1}=\lambda$

$\Rightarrow$ F(3 + λ, 1 − λ, λ + 7) lies in the plane

$\therefore$ 3 + λ − (1 − λ) + λ + 7 = 3

3λ = −6 ⇒ λ = −2

$\Rightarrow$ F(1,3,5)

$\Rightarrow$ P(−1,5,3)
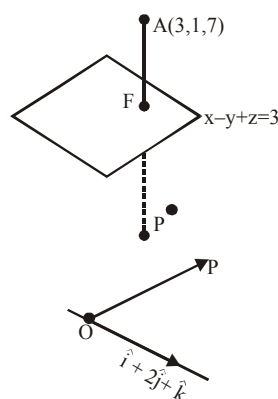
so required plane is $\begin{vmatrix} x-0 & y-0 & z-0 \\ 1 & 2 & 1 \\ -1 & 5 & 3 \end{vmatrix} = 0$

$\therefore$ x − 4y + 7z = 0

SECTION–2 : (Maximum Marks : 32)

● This section contains **EIGHT** questions.

● Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.

● For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

● For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

● for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in −2 marks, as a wrong option is also darkened.

43. Let a, b $\in$ $\mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$. Then f is -

(A) differentiable at x = 0 if a = 0 and b = 1

(B) differentiable at x = 1 if a = 1 and b = 0

(C) **NOT** differentiable at x = 0 if a = 1 and b = 0

(D) **NOT** differentiable at x = 1 if a = 1 and b = 1

Ans. (A,B)

Sol. If $x^3 - x \geq 0 \Rightarrow \cos|x^3 - x| = \cos(x^3 - x)$

$x^3 - x < 0 \Rightarrow \cos|x^3 - x| = \cos(x^3 - x)$

Similarly $b|x|\sin|x^3 + x| = bx\sin(x^3 + x)$ for all $x \in R$

$\therefore \quad f(x) = a\cos(x^3 - x) + bx\sin(x^3 + x)$

which is composition and sum of differentiable functions

therefore always continuous and differentiable.

44. Let $f(x) = \lim\limits_{n \to \infty} \left(\dfrac{n^n(x+n)\left(x + \dfrac{n}{2}\right).....\left(x + \dfrac{n}{n}\right)}{n!(x^2 + n^2)\left(x^2 + \dfrac{n^2}{4}\right).....\left(x^2 + \dfrac{n^2}{n^2}\right)} \right)^{x/n}$, for all $x > 0$. Then

(A) $f\left(\dfrac{1}{2}\right) \geq f(1)$　　　(B) $f\left(\dfrac{1}{3}\right) \leq f\left(\dfrac{2}{3}\right)$　　　(C) $f'(2) \leq 0$　　　(D) $\dfrac{f'(3)}{f(3)} \geq \dfrac{f'(2)}{f(2)}$

Ans. (B,C)

MATHEMATICS

Sol. $\ln f(x) = \lim\limits_{n \to \infty} \dfrac{x}{n} \ln \left[\dfrac{\prod\limits_{r=1}^{n}\left(x + \dfrac{1}{r/n}\right)}{\prod\limits_{r=1}^{n}\left(x^2 + \dfrac{1}{(r/n)^2}\right) \prod\limits_{r=1}^{n}(r/n)} \cdot \dfrac{1}{\prod\limits_{r=1}^{n}(r/n)} \right]$

$= x \lim\limits_{n \to \infty} \dfrac{1}{n} \sum\limits_{r=1}^{n} \ln \left(\dfrac{x\dfrac{r}{n} + 1}{\left(x\dfrac{r}{n}\right)^2 + 1} \right)$

$= x \int\limits_{0}^{1} \ln \left(\dfrac{1 + tx}{1 + t^2 x^2} \right) dt$ 　　　put tx = z

$\ln f(x) = \int\limits_{0}^{x} \ln \left(\dfrac{1 + z}{1 + z^2} \right) dz$

$\Rightarrow \quad \dfrac{f'(x)}{f(x)} = \ln \left(\dfrac{1 + x}{1 + x^2} \right)$

sign scheme of $f'(x)$ 　$\dfrac{+ \qquad -}{\qquad 1 \qquad}$ 　also $f'(1) = 0$

$\Rightarrow \quad f\left(\dfrac{1}{2}\right) < f(1), \ f\left(\dfrac{1}{3}\right) < f\left(\dfrac{2}{3}\right), \ f'(2) < 0$

Also $\dfrac{f'(3)}{f(3)} - \dfrac{f'(2)}{f(2)} = \ln\left(\dfrac{4}{10}\right) - \ln\left(\dfrac{3}{5}\right)$

$= \ln\left(\dfrac{4}{6}\right) < 0 \Rightarrow \dfrac{f'(3)}{f(3)} < \dfrac{f'(2)}{f(2)}$

45. Let $f : \mathbb{R} \to (0, \infty)$ and $g : \mathbb{R} \to \mathbb{R}$ be twice differentiable function such that f'' and g'' ar continuous

functions on $\mathbb{R}$. Suppose $f'(2) = g(2) = 0$, $f''(2) \neq 0$ and g'(2) $\neq$ 0. If $\lim\limits_{x \to 2} \dfrac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

(A) f has a local minimum at x = 2 　　　　(B) f has a local maximum at x = 2

(C) $f''(2) > f(2)$ 　　　　(D) $f(x) - f''(x) = 0$ for at least one x $\in \mathbb{R}$

Ans. (A,D)

Sol. Using L'Hôpital's Rule

$\lim\limits_{x \to 2} \dfrac{f'(x)g(x) + f(x)g'(x)}{f''(x)g'(x) + f'(x)g''(x)} = 1$

$\Rightarrow \quad \dfrac{f(2)g'(2)}{f''(2)g'(2)} = 1 \quad \Rightarrow \quad f'(2) = f(2) > 0$

option (D) is right and option (C) is wrong

also $f'(2) = 0$ and $f''(2) > 0$ 　　$\therefore$ 　x = 2 is local minima.

46. Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in $\mathbb{R}^2$ and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u} \times \vec{v}| = 1$ and $\hat{w}.(\hat{u} \times \vec{v}) = 1$. Which of the following statement(s) is(are) correct?
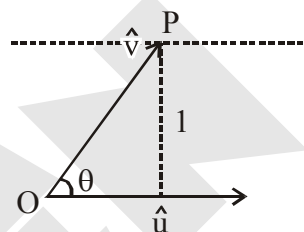
(A) There is exactly one choice for such $\vec{v}$

(B) There are infinitely many choice for such $\vec{v}$

(C) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

(D) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

Ans. (B,C)

Sol. $|\hat{w}| |\hat{u} \times \hat{v}| \cos\phi = 1 \Rightarrow \phi = 0$

$\Rightarrow \hat{u} \times \vec{v} = \hat{w}$ also $|\vec{v}| \sin\theta = 1$

$\Rightarrow$ there may be infinite vectors $\vec{v} = \overrightarrow{OP}$

such that P is always 1 unit dist. from $\hat{u}$

For option (C) : $\hat{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & 0 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$\hat{w} = (u_2 v_3)\hat{i} - (u_1 v_3)\hat{j} + (u_1 v_2 - u_2 v_1)\hat{k}$

$u_2 v_3 = \frac{1}{\sqrt{6}}, -u_1 v_3 = \frac{1}{\sqrt{6}} \Rightarrow |u_1| = |u_2|$

for option (D) : $\hat{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & 0 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$\hat{w} = (-v_2 u_3)\hat{i} - (u_1 v_3 - u_3 v_1)\hat{j} + (u_1 v_2)\hat{k}$

$-v_2 u_3 = \frac{1}{\sqrt{6}}, u_1 v_2 = \frac{2}{\sqrt{6}}$

$\Rightarrow 2|u_3| = |u_1|$ So (D) is wrong

47. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then-

(A) $SP = 2\sqrt{5}$

(B) $SQ:QP = (\sqrt{5}+1):2$

(C) the x-intercept of the normal to the parabola at P is 6

(D) the slope of the tangent to the circle at Q is $\frac{1}{2}$

Ans. (A,C,D)

MATHEMATICS

Sol..
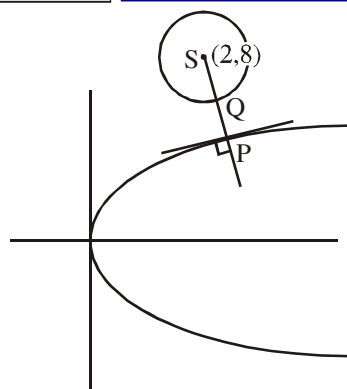
$y^2 = 4x$

point P lies on normal to parabola passing through centre of circle

$y + tx = 2t + t^3$ ....(i)

$8 + 2t = 2t + t^3$

$t = 2$

$P(4, 4)$

$SP = \sqrt{(4-2)^2 + (4-8)^2}$

$SP = 2\sqrt{5}$

$SQ = 2$

$\Rightarrow PQ = 2\sqrt{5} - 2$

$\dfrac{SQ}{QP} = \dfrac{1}{\sqrt{5}-1} = \dfrac{\sqrt{5}+1}{4}$

To find x intercept

put $y = 0$ in (i)

$\Rightarrow x = 2 + t^2$

$x = 6$

$\because$ Slope of common normal $= -t = -2$

$\therefore$ Slope of tangent $= \dfrac{1}{2}$

48. Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Suppose $S = \left\{ z \in \mathbb{C} : z = \dfrac{1}{a + ibt}, t \in \mathbb{R}, t \neq 0 \right\}$, where $i = \sqrt{-1}$.

If $z = x + iy$ and $z \in S$, then (x,y) lies on

(A) the circle with radius $\dfrac{1}{2a}$ and centre $\left(\dfrac{1}{2a}, 0 \right)$ for $a > 0$, $b \neq 0$

(B) the circle with radius $-\dfrac{1}{2a}$ and centre $\left(-\dfrac{1}{2a}, 0 \right)$ for $a < 0$, $b \neq 0$

(C) the x-axis for $a \neq 0$, $b = 0$

(D) the y-axis for $a = 0$, $b \neq 0$

Ans. (A,C,D)

Sol.. $x + iy = \dfrac{1}{a + ibt}$

$x + iy = \dfrac{a - ibt}{a^2 + b^2 t^2}$

Let $a \neq 0$ & $b \neq 0$

$x = \dfrac{a}{a^2 + b^2 t^2}$ ....(1)

$y = \dfrac{-bt}{a^2 + b^2 t^2}$ ....(2)

$\dfrac{y}{x} = \dfrac{-bt}{a} \Rightarrow t = -\dfrac{ay}{bx}$

put in (1)

$x \left\{ a^2 + b^2 . \dfrac{a^2 y^2}{b^2 x^2} \right\} = a$

$a^2(x^2 + y^2) = ax$

$x^2 + y^2 - \dfrac{1}{a} x = 0$

$\left(x - \dfrac{1}{2a} \right)^2 + y^2 = \dfrac{1}{4a^2}$

$\Rightarrow$ option (A) is correct

for $a \neq 0$, $b = 0$

$x + iy = \dfrac{1}{a}$

$x = \dfrac{1}{a}$, $y = 0 \Rightarrow$ z lies on x-axis $\Rightarrow$ option (C) is correct

for $a = 0$, $b \neq 0$

$x + iy = \dfrac{1}{ibt}$

$y = -\dfrac{1}{bt} i$, $x = 0$

$\Rightarrow$ z lies on y-axis. $\Rightarrow$ option (D) is correct

49. Let $a, \lambda, m \in \mathbb{R}$. Consider the system of linear equations

$ax + 2y = \lambda$

$3x - 2y = \mu$

Which of the following statement(s) is(are) correct ?

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

Ans. (B,C,D)

Sol. $ax + 2y = \lambda$

$3x - 2y = \mu$

for $a = -3$ above lies will be parallel or coincident

parallel for $\lambda + \mu \neq 0$ and coincident if $\lambda + \mu = 0$

and if $a \neq -3$ lies are intersecting $\Rightarrow$ unique solution.

50. Let $f : \left[-\dfrac{1}{2}, 2\right] \to \mathbb{R}$ and $g : \left[-\dfrac{1}{2}, 2\right] \to \mathbb{R}$ be function defined by $f(x) = [x^2 - 3]$ and

$g(x) = |x| \, f(x) + |4x - 7| \, f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

(A) f is discontinuous exactly at three points in $\left[-\dfrac{1}{2}, 2\right]$

(B) f is discontinuous exactly at four points in $\left[-\dfrac{1}{2}, 2\right]$

(C) g is NOT differentiable exactly at four points in $\left(-\dfrac{1}{2}, 2\right)$

(D) g is NOT differentiable exactly at five points in $\left(-\dfrac{1}{2}, 2\right)$

Ans. (B,C)

Sol. $f(x) = [x^2] - 3$

$g(x) = (|x| + |4x - 7|)([x^2] - 3)$

$\because$ f is discontinuous at $x = 1, \sqrt{2}, \sqrt{3}, 2$ in $\left[-\dfrac{1}{2}, 2\right]$

and $|x| + |4x - 7| \neq 0$ at $x = 1, \sqrt{2}, \sqrt{3}, 2$

$\Rightarrow$ g(x) is discontinuous at $x = 1, \sqrt{2}, \sqrt{3}$ in $\left(-\dfrac{1}{2}, 2\right)$

In $(0 - \delta, 0 + \delta)$

$g(x) = (|x| + |4x - 7|) . (-3)$

$\Rightarrow$ 'g' is non derivable at $x = 0$.

In $\left(\dfrac{7}{4} - \delta, \dfrac{7}{4} + \delta\right)$

$g(x) = 0$ as $f(x) = 0$

$\Rightarrow$ Derivable at $x = \dfrac{7}{4}$

$\therefore$ 'g' is non-derivable at $0, 1, \sqrt{2}, \dfrac{7}{4}$

SECTION–3 : (Maximum Marks : 12)

● This section contains **TWO** paragraphs.

● Based on each paragraph, there are **TWO** questions.

● Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.

● For each question, darken the bubble corresponding to the correct option in the ORS.

● For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks   :   +3   If only the bubble corresponding to the correct answer is darkened.

Zero Marks   :   0   In all other cases.

PARAGRAPH 1

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\dfrac{1}{2}$, $\dfrac{1}{6}$ and $\dfrac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games

51. $P(X > Y)$ is-

(A) $\dfrac{1}{4}$ (B) $\dfrac{5}{12}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{7}{12}$

Ans. (B)

Sol.. $P(X > Y) = P(T_1 \text{ win}) P(T_1 \text{ win}) + P(T_1 \text{ win}) P(\text{match draw}) + P(\text{match draw}).P(T_1 \text{ win})$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{2} = \frac{5}{12}$$

52. $P(X = Y)$ is-

(A) $\dfrac{11}{36}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{13}{36}$ (D) $\dfrac{1}{2}$

Ans. (C)

Sol. $P(X = Y) = P(\text{match draw}) P(\text{match Draw}) + P(T_1 \text{ win}) P(T_2 \text{ win}) + P(T_2 \text{ win}) P(T_1 \text{ win})$

$$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{13}{36}$$
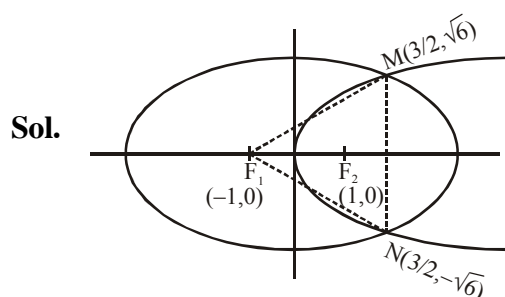
PARAGRAPH 2

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

53. The orthocentre of the triangle F_1MN is-

(A) $\left(-\dfrac{9}{10}, 0\right)$ (B) $\left(\dfrac{2}{3}, 0\right)$ (C) $\left(\dfrac{9}{10}, 0\right)$ (D) $\left(\dfrac{2}{3}, \sqrt{6}\right)$

Ans. (A)

Sol.

Orthocentre lies on x-axis

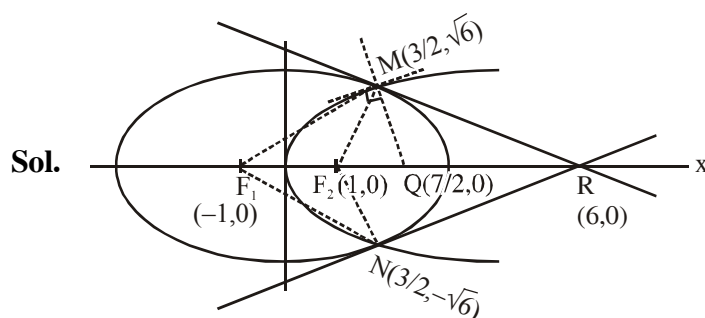Equation of altitude through M : $y - \sqrt{6} = \dfrac{5}{2\sqrt{6}}\left(x - \dfrac{3}{2}\right)$

Equation of altitude through F_1 : $y = 0$

solving, we get orthocentre $\left(-\dfrac{9}{10}, 0\right)$

54. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is-

(A) 3 : 4   (B) 4 : 5   (C) 5 : 8   (D) 2 : 3

Ans. (C)

Sol.

Normal to parabola at M : $y - \sqrt{6} = -\dfrac{\sqrt{6}}{2.1}\left(x - \dfrac{3}{2}\right)$

Solving it with y = 0, we get $Q \equiv \left(\dfrac{7}{2}, 0 \right)$

Tangent to ellipse at M : $\dfrac{x.\dfrac{3}{2}}{9} + \dfrac{y(\sqrt{6})}{8} = 1$

Solving it with y = 0, we get R ≡ (6, 0)

∴   Area of triangle MQR $= \dfrac{1}{2}.\left(6 - \dfrac{7}{2} \right).\sqrt{6} = \dfrac{5\sqrt{6}}{4}$

Area of quadrilateral $MF_1NF_2 = 2.\dfrac{1}{2}.\left(1 - (-1) \right).\sqrt{6} = 2\sqrt{6}$

Required ratio = 5 : 8