MODEL PAPER - I

SOLUTIONS AND MARKING SCHEME

SECTION A

Note : For 1 mark questions in Section A, full marks are given if answer is correct (i.e. the last step of the solution). Here, solution is given for your help.

Marks

1. We are given

$$\begin{bmatrix} 5x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -5 & 3 \end{bmatrix}$$

$\therefore$ $5x + y = 4$ and $-y = 1$

$\therefore$ $y = -1$ and $5x - 1 = 4$

or $5x = 5$

$\therefore$ $x = 1$ ...(1)

2. $6 * 4 =$ HCF of 6 and 4 = 2. ...(1)

3. $$\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \left|\sin^{-1} x\right|_0^{1/\sqrt{2}}$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1} 0$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$ ...(1)

4. Let $$I = \int \frac{\sec^2(\log x)}{x}\,dx$$

Let $\log x = t$

then $\frac{1}{x}dx = dt$

Marks

or $\quad dx = x\ dt$

$\therefore \quad I = \int \sec^2 t\ dt$

$= \tan t + c$

$= \tan (\log x) + c$ ...(1)

5. $\cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi - \frac{5\pi}{6}\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{5\pi}{6}\right)\right]$

$= \frac{5\pi}{6}$ ...(1)

6. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = \begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$

$= \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$

$= 0$ ...(1)

7. Here $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$

or $\quad 2x^2 - 8 = 0$

or $\quad x^2 - 4 = 0$

$x = \pm 2$ ...(1)

8. $\left(\hat{i} \times \hat{j}\right) \cdot \hat{k} + \left(\hat{j} \times \hat{k}\right) \cdot \hat{i} + \left(\hat{k} \times \hat{i}\right) \cdot \hat{j}$

$= \hat{k} \cdot \hat{k} + \hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{j}$

$= 1 + 1 + 1$

$= 3$ ...(1)

Marks

9. The d.c. of a line equally inclined to the coordinate axes are $\left(\frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}\right)$. ...(1)

10. $$(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$$

$\therefore$ $$|\vec{x}|^2 - |\vec{p}|^2 = 80$$

As $\vec{p}$ is a unit vector,

$$|\vec{p}| = 1$$

$\therefore$ $$|\vec{x}|^2 - 1 = 80$$

or $$|\vec{x}|^2 = 81$$

$\therefore$ $$|x| = 9$$ ...(1)

SECTION B

11. Let P be the perimeter and A be the area of the rectangle at any time t, then

$$P = 2(x + y) \text{ and } A = xy$$

It is given that $\frac{dx}{dt} = -5$ cm/minute

and $\frac{dy}{dt} = 4$ cm/minute ...(1)

(i) We have $P = 2(x + y)$

$\therefore$ $$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

Marks

$= 2\ (-5 + 4)$

$= -\ 2$ cm/minute ...(1½)

(ii) We have $A = xy$

$$\therefore \quad \frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$= [8 \times 4 + 6\ (-5)]$ ......($\because$ x = 8 and y = 6)

$= (32 - 30)$

$= 2$ cm^2/minute ...(1½)

OR

The given function is

$$f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$$

$$\therefore \quad f'(x) = \cos x - \sin x$$

$$= -\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)$$

$$= -\sqrt{2}\left(\sin x \cos\frac{\pi}{4} - \cos x \sin\frac{\pi}{4}\right)$$

$$= -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$ ...(1)

For strictly decreasing function,

$$f'(x) < 0$$

$$\therefore \quad -\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) < 0$$

or $$\sin\left(x - \frac{\pi}{4}\right) > 0$$

or $$0 < x - \frac{\pi}{4} < \pi$$

Marks

or $$\frac{\pi}{4} < x < \pi + \frac{\pi}{4}$$

or $$\frac{\pi}{4} < x < \frac{5\pi}{4}$$

Thus f(x) is a strictly decreasing function on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ ...(2)

As sin x and cos x are well defined in [0, 2π],

f (x) = sin x + cos x is an increasing function in the complement of interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

i.e., in $$\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$$ ...(1)

12. We are given

$$(\cos x)^y = (\sin y)^x$$

Taking log of both sides, we get

$$y \log \cos x = x \log \sin y$$ ...(½)

Differentiating w.r.t. x, we get

$$y \cdot \frac{1}{\cos x} \cdot (-\sin x) + \log \cos x \cdot \frac{dy}{dx}$$
$$= x \cdot \frac{1}{\sin y} \cdot (\cos y)\frac{dy}{dx} + \log \sin y \cdot 1$$ ...(2)

or $$-y \tan x + \log \cos x \frac{dy}{dx} = x \cot y \frac{dy}{dx} + \log \sin y$$

$$\Rightarrow \quad \frac{dy}{dx}(\log \cos x - x \cot y) = \log \sin y + y \tan x$$ ...(1)

$$\therefore \quad \frac{dy}{dx} = \frac{\log \sin y + y \tan x}{\log \cos x - x \cot y}$$ ...(½)

Marks

13. For showing f is one-one ...(1½)

For showing f is onto ...(1)

As f is one-one and onto f is invertible ...(½)

For finding $f^{-1}(x) = \dfrac{4x}{4-3x}$ ...(1)

14. Let $I = \displaystyle\int \frac{dx}{\sqrt{5-4x-2x^2}}$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - 2x - x^2}}$$ ...(½)

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - \left(2x + x^2 + 1 - 1\right)}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{7}{2} - (x+1)^2}}$$ ...(1½)

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{7}{2}\right)^2 - (x+1)^2}}$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x+1}{\frac{\sqrt{7}}{\sqrt{2}}}\right) + c$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{\sqrt{2}(x+1)}{\sqrt{7}}\right) + c$$ ...(2)

Marks

OR

Let $I = \int x \sin^{-1} x \, dx$

$= \int \sin^{-1} x . x \, dx$

$= \sin^{-1} x \cdot \frac{x^2}{2} - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$ ...(1)

$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx$

$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$

$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \frac{dx}{\sqrt{1-x^2}}$ ...(1)

$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \cdot \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right] - \frac{1}{2} \sin^{-1} x + c$

...(1)

$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + c$

$= \frac{x^2 \sin^{-1} x}{2} - \frac{1}{4} \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + c$ ...(1)

15. We have

$$y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$$

Marks

$$\Rightarrow \qquad y\sqrt{1-x^2} = \sin^{-1} x$$

Differentiating w.r.t. x, we get

$$y.\frac{(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

or $$-xy + (1-x^2)\frac{dy}{dx} = 1$$ ...(1½)

Differentiating again,

$$-x\frac{dy}{dx} - y + (1-x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(-2x) = 0$$

or $$(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$$ ...(2½)

which is the required result.

16. (i) P (both the students A and B pass the examination)

$= P(A \cap B)$ ...(½)

$= P(A)\ P(B)$ ...(½)

$= \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$ ...(½)

(ii) P (atleast one of the students A and B passes the examination)

= 1 – P (none of the students pass) ...(½)

$= 1 - \frac{1}{5} \times \frac{2}{5}$ ...(½)

$= 1 - \frac{2}{25}$ ...(½)

$= \frac{23}{25}$ ...(½)

Marks

When appearing in an examination, a student should have no intention of copying or cheating as it inculcates habit of dishonesty which leads to corruption and many other ills. ...(1)

17. LHS $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$R_1 \to R_1 + R_2$

$R_2 \to R_2 + R_3$

$$\begin{vmatrix} a+b & a+b & -(a+b) \\ -(b+c) & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix} \quad ...(2)$$

$$= (a+b)(b+c)\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix} \quad ...(½)$$

$C_1 \to C_1 + C_3$

$$= (a+b)(b+c)\begin{vmatrix} 0 & 1 & -1 \\ 0 & 1 & 1 \\ c+a & -a & a+b+c \end{vmatrix} \quad ...(1)$$

$= 2(a + b)(b + c)(c + a)$ ...(½)

18. The given differential equation is

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

or $$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$

Let $$y = zx$$

$\therefore$ $$\frac{dy}{dx} = z + x\frac{dz}{dx} \quad ...(1)$$

Marks

$\therefore \quad z + x\dfrac{dz}{dx} = z - \tan z$

or $\quad x\dfrac{dz}{dx} = -\tan z$ ...(1)

or $\quad \int \cot z \; dz + \int \dfrac{dx}{x} = 0$ ...(½)

$\therefore \quad \log \sin z + \log x = \log c$

or $\quad \log (x \sin z) = \log c$ ...(1)

or $\quad x \sin\left(\dfrac{y}{x}\right) = c$ ...(½)

which is the required solution.

19. The given differential equation is

$$\cos^2 x \dfrac{dy}{dx} + y = \tan x$$

or $\quad \dfrac{dy}{dx} + \sec^2 x.\, y = \tan x.\, \sec^2 x$

It is a linear differential equation

Integrating factor $= e^{\int \sec^2 x \; dx} = e^{\tan x}$ ...(1)

$\therefore$ Solution of the differential equation is

$$y.e^{\tan x} = \int e^{\tan x}.\tan x \sec^2 x \, dx + c$$ ...(½)

Now, we find $\quad I_1 = \int e^{\tan x} \cdot \tan x \sec^2 x \, dx$

Let $\quad \tan x = t, \sec^2 x \; dx = dt$

Marks

$\therefore \quad I_1 = \int te^t \, dt$

$= t.e^t - \int e^t dt$

$= t \,.\, e^t - e^t$

$= (t - 1)e^t = (\tan x - 1)\, e^{\tan x}$ ...(2)

$\therefore$ From (i), solution is

$$y \,.\, e^{\tan x} = (\tan x - 1)\, e^{\tan x} + c$$

or $y = (\tan x - 1) + ce^{-\tan x}$ ...(½)

20. Equations of the two lines are :

$$\overrightarrow{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k}$$

or $\overrightarrow{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ ...(i)

and $\overrightarrow{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ ...(ii)

Here $\overrightarrow{a_1} = \hat{i} + 2\hat{j} + \hat{k}$ and $\overrightarrow{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

and $\overrightarrow{b_1} = \hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$ ...(1)

$\therefore \quad \overrightarrow{a_2} - \overrightarrow{a_1} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right)$

$= \hat{i} - 3\hat{j} - 2\hat{k}$ ...(½)

and $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$= \hat{i}(-3) - \hat{j}(0) + \hat{k}(3)$

$= -3\hat{i} + 3\hat{k}$ ...(1)

Marks

$\therefore \quad \left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{9+9} = 3\sqrt{2}$

$\therefore \quad$ S.D. between the lines $= \dfrac{\left|\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)\right|}{\left|\vec{b_1} \times \vec{b_2}\right|}$ ...(½)

$$= \frac{\left|\left(\hat{i} - 3\hat{j} - 2\hat{k}\right)\cdot\left(-3\hat{i} + 3\hat{k}\right)\right|}{3\sqrt{2}}$$

$$= \frac{|-3-6|}{3\sqrt{2}}$$

$$= \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ units}$$ ...(1)

21. $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right]$$ ...(1)

...$\left[\because x \in \left(0, \frac{\pi}{4}\right)\right]$

$$= \cot^{-1}\left[\frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) - \left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}\right]$$ ...(1)

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$$ ...(1)

$$= \cot^{-1}\left[\cot\left(\frac{x}{2}\right)\right]$$

$$= \frac{x}{2}$$ ...(1)

Marks

OR

The given equation is

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x) \quad \text{...(1½)}$$

$$\Rightarrow \quad \frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x \quad \text{...(1)}$$

$$\Rightarrow \quad \cos x = \operatorname{cosec} x \,.\, \sin^2 x$$

$$\Rightarrow \quad \cos x = \sin x$$

$$\therefore \quad x = \frac{\pi}{4} \quad \text{...(1½)}$$

22. Unit vector along the sum of vectors

$$\overrightarrow{a} = 2\hat{i} + 4\hat{j} - 5\hat{k} \text{ and } \overrightarrow{b} = \lambda\hat{i} + 2\hat{j} + 3\hat{k} \text{ is}$$

$$\frac{\overrightarrow{a}+\overrightarrow{b}}{\left|\overrightarrow{a}+\overrightarrow{b}\right|} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 6^2 + (-2)^2}}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \quad \text{...(1½)}$$

We are given that dot product of above unit vector with the vector $\hat{i} + \hat{j} + \hat{k}$ is 1.

$$\therefore \quad \frac{(2+\lambda)}{\sqrt{\lambda^2+4\lambda+44}} \cdot 1 + \frac{6}{\sqrt{\lambda^2+4\lambda+44}} - \frac{2}{\sqrt{\lambda^2+4\lambda+44}} = 1 \quad \text{...(1)}$$

or $\quad 2 + \lambda + 6 - 2 = \sqrt{\lambda^2 + 4\lambda + 44}$

or $\quad (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$

or $\quad \lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$

Marks

or $\quad 8\lambda = 8$

or $\quad \lambda = 1$ ...(1½)

OR

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar $\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$ ...(½)

$\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar if

$[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 0$ (½)

For showing

$$[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}] \quad (3)$$

SECTION C

23. Equation of the plane through the points A (3, – 1, 2), B (5, 2, 4) and C (–1, –1, 6) is

$$\begin{vmatrix} x - 3 & y + 1 & z - 2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0 \quad ...(2\tfrac{1}{2})$$

i.e. $\quad 3x - 4y + 3z = 19$ ...(1½)

Distance of point (6, 5, 9) from plane $3x - 4y + 3z = 19$

$$= \frac{|18 - 20 + 27 - 19|}{\sqrt{9 + 16 + 9}}$$

$$= \frac{6}{\sqrt{34}} \text{ units} \quad ...(2)$$

24. The given parabola is $y^2 = x$ ....(i)

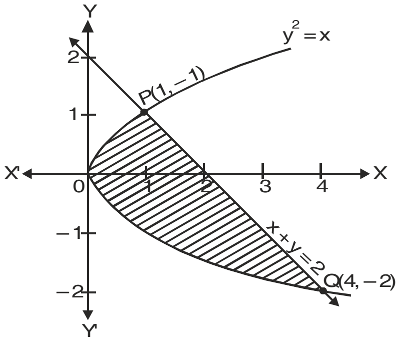
It represents a parabola with vertex at O (0, 0)

Marks

The given line is

$$x + y = 2$$

or $$x = 2 - y \qquad \text{....(ii)}$$



...(1)

Solving (i) and (ii), we get the point of intersection P (1, 1) and Q (4, − 2) ...(1)

Required area = Area of the shaded region

$$= \int_{-2}^{1} \left[(2 - y) - y^2\right] dy \qquad \text{...(2)}$$

$$= \left(2y - \frac{y^2}{2} - \frac{y^3}{3}\right)_{-2}^{1} \qquad \text{...(1)}$$

$$= \left[\left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)\right]$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3} + 4 + 2 - \frac{8}{3}\right)$$

$$= \frac{12 - 3 - 2 + 24 + 12 - 16}{6}$$

$$= \frac{27}{6}$$

$$= \frac{9}{2} \text{ sq. units} \qquad \text{...(1)}$$

Marks

25. Let $I = \int_0^{\pi} \frac{x\ dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

or $I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2 \cos^2(\pi - x) + b^2 \sin^2(\pi - x)}$

or $I = \int_0^{\pi} \frac{(\pi - x)dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ ....(ii)

Adding (i) and (ii), we get

$2I = \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ ....(iii) ...(1)

or $2I = \pi . 2 \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

Using property $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$, If $f(2a - x) = f(x)$ ...(1)

or $I = \pi \int_0^{\pi/2} \frac{\sec^2 x\ dx}{a^2 + b^2 \tan^2 x}$ ...(1)

Let $\tan x = t$ then $\sec^2 x\ dx = dt$

When $x = 0$, $t = 0$ and when $x \to \frac{\pi}{2}, t \to \infty$

$\therefore \quad I = \pi \int_0^{\infty} \frac{dt}{a^2 + b^2 t^2}$ ...(1)

$= \frac{\pi}{b^2} \int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$

Marks

$$= \frac{\pi}{b^2} \cdot \frac{1}{a/b}\left[\tan^{-1}\frac{t}{a/b}\right]_0^{\infty}$$ ...(1)

$$= \frac{\pi}{ab}\left[\tan^{-1}\frac{bt}{a}\right]_0^{\infty}$$

$$= \frac{\pi}{ab}\left[\frac{\pi}{2}\right]$$

$$= \frac{\pi^2}{2ab}$$ ...(1)

26. Let the amount of prize for three values honesty, regularity and discipline be represented by x, y and z respectively. Then

$$5x + 4y + 3z = 11000$$

$$4x + 3y + 5z = 10700$$

$$x + y + z = 2700$$ ...(1½)

AX = B, where

$$A = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11000 \\ 10700 \\ 2700 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{vmatrix} = -3 \neq 0$$ ...(½)

So, A^{-1} exists.

$$\text{Now} \quad \text{adj } A = \begin{bmatrix} -2 & -1 & 11 \\ 1 & 2 & -13 \\ 1 & -1 & -1 \end{bmatrix}$$ ...(1)

Marks

$$A^{-1} = \frac{\text{adj } A}{|A|} = -\frac{1}{3}\begin{bmatrix} -2 & -1 & 11 \\ 1 & 2 & -13 \\ 1 & -1 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

So, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -2 & -1 & 11 \\ 1 & 2 & -13 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 11000 \\ 10700 \\ 2700 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -3000 \\ -2700 \\ -2400 \end{bmatrix} = \begin{bmatrix} 1000 \\ 900 \\ 800 \end{bmatrix}$$

So, $x = 1000$, $y = 900$, $z = 800$

i.e., The amount of prize for the values honesty, regularity and discipline are Rs. 1000, Rs. 900 and Rs. 800 respectively. ...(1)

(ii) I prefer honesty because corruption is the root cause of all problems for the citizens of the country. Honest persons are always disciplined and regular in approach. ...(2)

OR

26. By using elementary row transformations, we can write

$$A = IA$$

i.e., $$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(1)

Applying $R_1 \to R_2 - R_2$, we get

$$\begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ ...(1)

Marks

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad ...(1)$$

Applying $R_1 \to R_1 + R_3$, we get

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad ...(½)$$

Applying $R_2 \to R_2 - 2R_3$, we get

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \qquad ...(½)$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \qquad ...(½)$$

Applying $R_3 \to R_3 - 4R_2$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & 12 & 9 \end{bmatrix} A \qquad ...(½)$$

$$A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \qquad ...(1)$$

Marks

27. Let the events be

E_1 : Bag I is selected

E_2 : Bag II is selected

E_3 : Bag III is selected

and A : a black and a red ball are drawn ...(1)

$\therefore$ $P(E_1) = P(E_2) = P(E_3) = \dfrac{1}{3}$ ...(1)

$$P(A/E_1) = \frac{1 \times 3}{{}^6C_2} = \frac{3}{15} = \frac{1}{5}$$

$$P(A/E_2) = \frac{2 \times 1}{{}^7C_2} = \frac{2}{21}$$

$$P(A/E_3) = \frac{4 \times 3}{{}^{12}C_2} = \frac{4 \times 3}{66} = \frac{2}{11}$$ ...(1½)

$$\therefore \quad P(E_1/A) = \frac{P(A/E_1).P(E_1)}{P(A/E_1)\ P/(E_1) + P(A/E_2)\ P(E_2) + P(A/E_3).P(E_3)}$$ ...(1)

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{2}{21} + \frac{1}{3} \times \frac{2}{11}}$$ ...(½)

$$= \frac{\frac{1}{15}}{\frac{1}{15} + \frac{2}{63} + \frac{2}{33}}$$

$$= \frac{\frac{1}{15}}{\frac{551}{3465}}$$

Marks

$$= \frac{1}{15} \times \frac{3465}{551} = \frac{231}{551} \qquad ...(1)$$

28. Let us suppose that the dealer buys x fans and y sewing machines,

Thus L.P. problem is

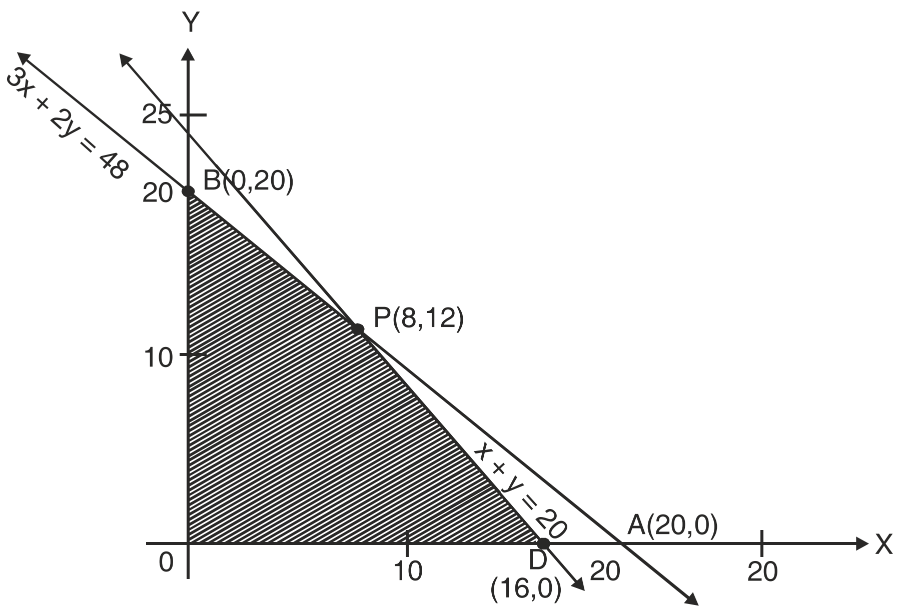
Maximise $\quad Z = 22x + 18y \quad$ ...(½)

subject to constraints,

$$x + y \leq 20$$

$$360x + 240y \leq 5760 \text{ or } 3x + 2y \leq 48$$

$$x \geq 0, \ y \geq 0 \qquad ...(1½)$$



For correct graph ...(1½)

The feasible region ODPB of the L.P.P. is the shaded region which has the corners O (0, 0), D (16, 0), P (8, 12) and B (0, 20)

The values of the objective function Z at O, D, P and B are :

Marks

	At O,	$Z = 22 \times 0 + 18 \times 0 = 0$
	At D,	$Z = 22 \times 16 + 18 \times 0 = 352$
	At P,	$Z = 22 \times 8 + 18 \times 12 = 392 \rightarrow$ Maximum
and	At B,	$Z = 22 \times 0 + 18 \times 20 = 360$

Thus Z is maximum at $x = 8$ and $y = 12$ and the maximum value of $z =$ Rs 392.

Hence the dealer should purchase 8 fans and 12 sewing machines to obtain maximum profit. ...(½)

Values promoted are the maximum utility of money and space of storage. ...(2)

29. Let ABC be a right angled triangle with base $BC = x$ and hypotenuse $AB = y$

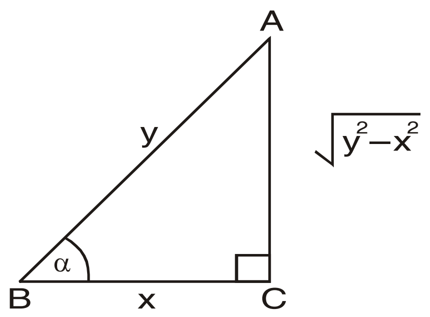
such that

$$x + y = k \text{ where } k \text{ is a constant} \quad \text{...(½)}$$

Let α be the angle between the base and the hypotenuse.

The area of triangle, $A = \frac{1}{2} BC \times AC$

$$= \frac{1}{2} x \sqrt{y^2 - x^2}$$



...(1½)

Marks

$$\therefore \qquad A^2 = \frac{x^2}{4}(y^2 - x^2)$$

$$= \frac{x^2}{4}\left[(k - x)^2 - x^2\right]$$

or $$A^2 = \frac{x^2}{4}\left[k^2 - 2kx\right] = \frac{k^2x^2 - 2kx^3}{4} \quad ..(i)$$

Differentiating w.r.t. x we get

$$2A\frac{dA}{dx} = \frac{2k^2x - 6kx^2}{4} \quad ....(ii)$$

or $$\frac{dA}{dx} = \frac{k^2x - 3kx^2}{4A} \quad ...(1)$$

For maximum or minimum,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \qquad \frac{k^2x - 3kx^2}{4} = 0$$

$$\Rightarrow \qquad x = \frac{k}{3} \quad ...(1)$$

Differentiating (ii) w.r.t.x. we get

$$2\left(\frac{dA}{dx}\right)^2 + 2A\frac{d^2A}{dx^2} = \frac{2k^2 - 12kx}{4}$$

Putting, $\frac{dA}{dx} = 0$ and $x = \frac{k}{3}$, we get

$$\frac{d^2A}{dx^2} = \frac{-k^2}{4A} < 0$$

Marks

$\therefore$ A is maximum when $x = \frac{k}{3}$ ...(1)

Now $x = \frac{k}{3} \Rightarrow y = k - \frac{k}{3} = \frac{2k}{3}$

$\therefore$ $\cos\alpha = \frac{x}{y} \Rightarrow \cos\alpha = \frac{k/3}{2k/3} = \frac{1}{2}$

$$\alpha = \frac{\pi}{3}$$ ...(1)

OR

Let the length of the tank be x metres and breadth by y metres

$\therefore$ Depth of the tank = 2 metre

$\therefore$ Volume $= x \times y \times 2 = 8$

$$xy = 4$$

or $y = \frac{4}{x}$ ...(1)

Area of base = xy sq m

Area of 4 walls $= 2\,[2x + 2y] = 4\,(x + y)$

$\therefore$ Cost $C\,(x, y) = 70\,(xy) + 45\,(4x + 4y)$

or $C\,(x, y) = 70 \times 4 + 180\,(x + y)$ ...(1)

$\therefore$ $C(x) = 280 + 180\left(x + \frac{4}{x}\right)$ ...(½)

Now $\frac{dC}{dx} = 180\left(1 - \frac{4}{x^2}\right)$ ...(1)

For maximum or minimum, $\frac{dC}{dx} = 0$

$\therefore$ $180\left(1 - \frac{4}{x^2}\right) = 0$

Marks

or $x^2 = 4$

or $x = 2$ ...(½)

and $$\frac{d^2C}{dx^2} = 180\left(\frac{8}{x^3}\right) > 0$$

$$\left.\frac{d^2C}{dx^2}\right|_{x=2} = 180\left(\frac{8}{8}\right) > 0 \qquad \text{...(1)}$$

$\therefore$ C is minimum at $x = 2$

Least Cost = Rs [(280 + 180 (2 + 2)]

= Rs [(280 + 720] = Rs 1000 ...(1)