

Sample Paper-04
Mathematics
Class – XII

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- d) Use of calculators is not permitted.

Section A

1. Give an example of a relation which is symmetric but not reflexive and transitive.
2. Find the direction cosines of x-axis.
3. What is the domain of $\sin^{-1} x$?
4. Find x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$.
5. Prove that $A-A'$ is skew symmetric.
6. Does inverse of $\begin{bmatrix} -2 & -1 \\ 4 & 2 \end{bmatrix}$ exist?

Section B

7. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$
8. If A, B, C have the co-ordinates (2,0,0) , (0,1,0) , (0,0,2) , then show that ABC is an isosceles triangle.
9. Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear?
10. The total revenue in rupees received from the sale of x units of a medicine is given by $R(x) = x^3 - e^x - 1/x$. Find the marginal revenue when $x=5$. List two precautions a responsible chemist should follow.

11. Find the differential equation of the system of circles touching x-axis at the origin
12. Using properties of determinants prove that
- $$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$
13. Find the equations of the tangent and the normal to the curve $x = \cos t, y = \sin t$ at $t = \frac{\pi}{4}$.
14. If $f(x) = \sqrt{x} (x > 0)$, $g(x) = x^2 - 1$, find if $f \circ g = g \circ f$.
15. Find $\frac{dy}{dx}$ if $y = (x \log x)^{\log(\log x)}$.
16. For any two vectors \vec{a} and \vec{b} , prove that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$.
17. Integrate $\int \sqrt{\frac{a-x}{a+x}} dx$.
18. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar.
- Also, find the equation of the plane containing these lines.
19. Find the vector and Cartesian equation of the planes passing through the intersection of the planes $\vec{r} \cdot (2\vec{i} + 6\vec{j}) + 12 = 0$ and $\vec{r} \cdot (3\vec{i} - \vec{j} + 4\vec{k}) = 0$ which are at unit distance from the origin.

Section C

20. Find the ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x=3$.
21. A man is known to speak the truth 3 out of 4 times. He throws a dice and reports that it is 4. Find the probability that is actually 4.
22. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ prove that $(1-x^2) \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - y = 0$
23. Integrate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x) dx$.
24. Determine the points on the curve $y = (1/4)x^2$ nearest to the point $(0,5)$.

25. The cost of 4 kg onions, 3kg wheat and 2 kg rice is 60. The cost of 2 kg onions, 4kg wheat and 6 kg rice is 90. The cost of 6 kg onions, 2kg wheat and 3 kg rice is 70. Find the per kg cost of each of the three commodities.
26. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of rs. 8000 on each piece of model A and 12000 on each piece of model B.