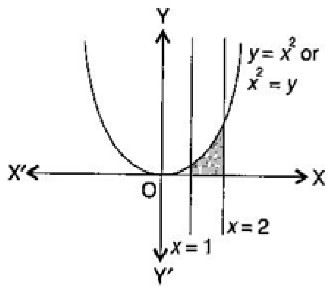
CLASS –XII MATHEMATICS NCERT SOLUTIONS
Applications of Integrals
Miscellaneous Exercise

Answers

1. (i) Equation of the curve (parabola) is

$y = x^2$ ....(i)

Required area bounded by curve (i), vertical line $x = 1, x = 2$ and $x -$ axis



$$= \left|\int_1^2 y\,dx\right| = \left|\int_1^2 x^2\,dx\right| = \left(\frac{x^3}{3}\right)_1^2$$

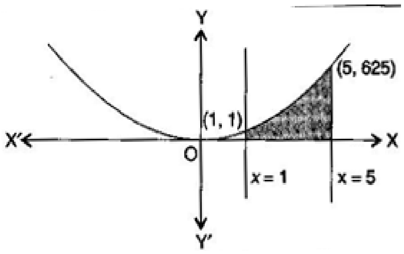
$$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \text{ sq. units}$$

(ii) Equation of the curve

$y = x^4$ .....(i)

It is clear that curve (i) passes through the origin because $x = 0$, from (i) $y = 0$.

Table of values for curve $y = x^4$ for $x = 1$ and $x = 5$ (given)



x	1	2	3	4	5
y	1	$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$

Required shaded area between the curve $y = x^4$, vertical lines $x = 1, x = 5$ and $x -$ axis

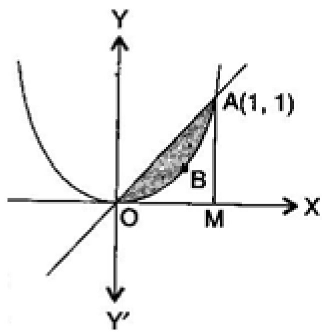
$$= \left|\int_1^5 y\,dx\right| = \left|\int_1^5 x^4\,dx\right| = \left(\frac{x^5}{5}\right)_1^5 = \frac{5^5}{5} - \frac{1^5}{5} = \frac{3125-1}{5} = \frac{3124}{5} = 624.8 \text{ sq. units}$$

2. Equation of one curve (straight line) is $y = x$ .....(i)

Equation of second curve (parabola) is $y = x^2$ .....(ii)

Solving eq. (i) and (ii), we get $x = 0$ or $x = 1$ and $y = 0$ or $y = 1$

$\therefore$ Points of intersection of line (i) and parabola (ii) are O (0, 0) and A (1, 1).



Now Area of triangle OAM

= Area bounded by line (i) and $x -$ axis

$$= \left|\int_0^1 y\,dx\right| = \left|\int_0^1 x\,dx\right| = \left(\frac{x^2}{2}\right)_0^1 = \frac{1}{2} - 0 = \frac{1}{2} \text{ sq. units}$$

Also Area OBAM = Area bounded by parabola (ii) and $x -$ axis

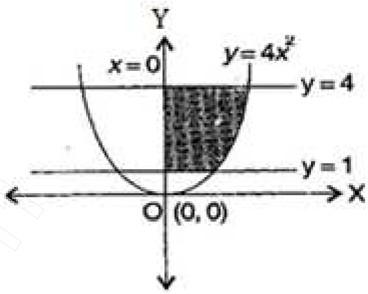
$$= \left|\int_0^1 y\,dx\right| = \left|\int_0^1 x^2\,dx\right| = \left(\frac{x^3}{3}\right)_0^1 = \frac{1}{3} - 0 = \frac{1}{3} \text{ sq. units}$$

$\therefore$ Required area OBA between line (i) and parabola (ii)

= Area of triangle OAM – Area of OBAM

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ sq. units Ans.

3. Equation of the curve (parabola) is $y = 4x^2$



$\Rightarrow \quad x^2 = \frac{y}{4}$ ....(i)

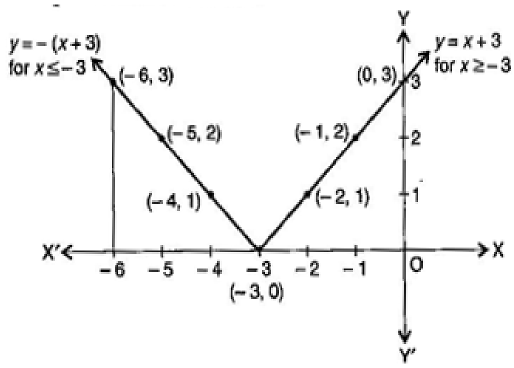
$\Rightarrow \quad x = \frac{\sqrt{y}}{2}$ ....(ii)

Here required shaded area of the region lying in first quadrant bounded by parabola (i), $x = 0$
and the horizontal lines $y = 1$ and $y = 4$ is

$$\left|\int_1^4 x\,dy\right| = \left|\int_1^4 \frac{\sqrt{y}}{2}\,dy\right| = \frac{1}{2}\left|\int_1^4 y^{\frac{1}{2}}\right|$$

$$= \frac{1}{2}\left|\frac{\left(y^{\frac{3}{2}}\right)_1^4}{\frac{3}{2}}\right| = \frac{1}{2}\cdot\frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{1}{3}\left(4\sqrt{4} - 1\right) = \frac{1}{3}(8-1) = \frac{7}{3} \text{ sq. units}$$ Ans.

4. Equation of the given curve is $y = |x+3|$ ....(i)



$\because \; y = |x+3| \geq 0$ for all real x

$\therefore$ Graph of curve is only above the $x-$axis i.e., in first and second quadrant only.

$\therefore \quad y = |x+3| = x+3$

If $x+3 \geq 0 \Rightarrow x \geq -3$ ....(ii)

And $\quad y = |x+3| = -(x+3)$

If $x+3 \leq 0 \Rightarrow x \leq -3$ ....(iii)

Table of values for $y = x+3$ for $x \geq -3$

x	–3	–2	–1	0
y	0	1	2	3

Table of values for $y = x+3$ for $x \leq -3$

x	–3	–4	–5	–6
y	0	1	2	3

Now, $\int_{-6}^{0} |x+3|\,dx = \int_{-6}^{-3} |x+3|\,dx + \int_{-3}^{0} |x+3|\,dx = \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{0} (x+3)\,dx$

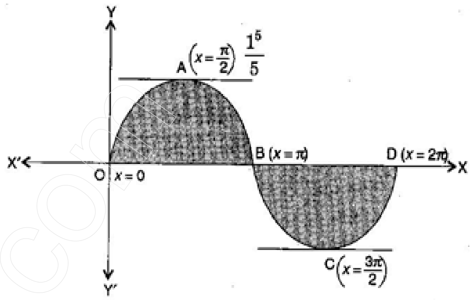
$$= \left(\frac{x^2}{2}+3x\right)_{-6}^{-3} + \left(\frac{x^2}{2}+3x\right)_{-3}^{0} = \left[\frac{9}{2}-9-(18-18)\right]+\left[0-\left(\frac{9}{2}-9\right)\right]$$

$$= \frac{9}{2}+9+0+0-\frac{9}{2}+9 = 18-\frac{18}{2}=18-9=9 \text{ sq. units}$$ Ans.

5. Equation of the curve is $y=\sin x$ ....(i)

$\because$ $y=\sin x \ge 0$ for $0\le x\le \pi$ i.e., graph is in first and second quadrant.

And $y=\sin x\le 0$ for $\pi \le x\le 2\pi$ i.e., graph is in third and fourth quadrant.



If tangent is parallel to $x-$axis, then $\frac{dy}{dx}=0$

$\Rightarrow$ $\cos x = 0$

$\Rightarrow$ $x=\frac{\pi}{2},\frac{3\pi}{2}$

Table of values for curve $y=\sin x$ between $x=0$ and $x=2\pi$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	-1	0

Now Required shaded area = Area OAB + Area BCD

$$= \left|\int_0^{\pi} y\,dx\right| + \left|\int_{\pi}^{2\pi} y\,dx\right|$$

$$= \left|\int_0^{\pi} \sin x\,dx\right| + \left|\int_{\pi}^{2\pi} \sin x\,dx\right| = \left|-(\cos x)_0^{\pi}\right| + \left|(\cos x)_{\pi}^{2\pi}\right|$$

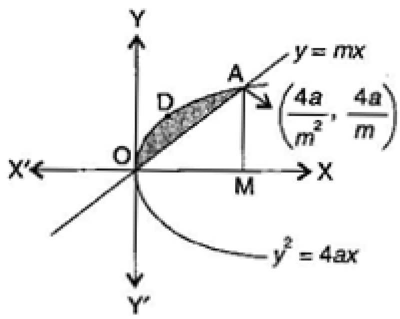
$$= \left|-(\cos\pi-\cos 0)\right| + \left|-(\cos 2\pi-\cos\pi)\right|$$

$$= \left|-1(-1-1)\right| + \left|-(1+1)\right| = 2+2 = 4 \text{ sq. units}$$ Ans.

6. Equation of parabola is $y^2=4ax$ ....(i)

The area enclosed between the parabola $y^2=4ax$ and line $y=mx$ is represented by shaded area OADO.

Here Points of intersection of curve (i) and line $y=mx$ are O (0, 0) and A$\left(\frac{4a}{m^2},\frac{4a}{m}\right)$.



Now Area ODAM = Area of parabola and $x-$axis

$$= \left|\int_0^{\frac{4a}{m^2}} 2\sqrt{a}.x^{\frac{1}{2}}\,dx\right| = 2\sqrt{a}\,\frac{\left(x^{\frac{3}{2}}\right)_0^{\frac{4a}{m^2}}}{\frac{3}{2}}$$

$$= \frac{4\sqrt{a}}{3}\left(\frac{4a}{m^2}\right)^{\frac{3}{2}} = \frac{4\sqrt{a}}{3}.\frac{4a}{m^2}\sqrt{\frac{4a}{m^2}} = \frac{32a^2}{3m^3} \qquad \text{..........(ii)}$$

Again Area of ΔOAM = Area between line $y = mx$ and $x-$axis

$$= \left|\int_0^{\frac{4a}{m^2}} mx\,dx\right| = m\left(\frac{x^2}{2}\right)_0^{\frac{4a}{m^2}} = \frac{m}{2}\left(\left(\frac{4a}{m^2}\right)^2 - 0\right) = \frac{m}{2}.\frac{16a^2}{m^4} = \frac{8a^2}{m^3} \qquad \text{..........(ii)}$$

$\therefore$ Requires shaded area = Area ODAM – Area of ΔOAM

$$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} = \frac{a^2}{m^3}\left(\frac{32}{3} - 8\right) = \frac{8a^2}{3m^3} \qquad \text{Ans.}$$

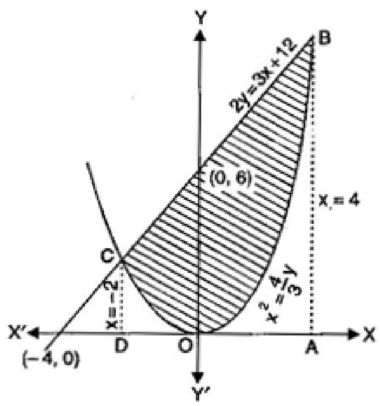
7. Equation of the parabola is

$4y = 3x^2$ ....(i)

$\Rightarrow \quad x^2 = \frac{4}{3}y$

Equation of the line is $2y = 3x + 12$ ......(ii)

In the graph, points of intersection are B (4, 12) and C$(-2,3)$.



Now, Area ABCD = $\left|\int_{-2}^{4}\left(\frac{3}{2}x + 6\right)dx\right|$

$= \left[\frac{3}{4}x^2 + 6x\right]_{-2}^{4} = (12+24)-(3-12)$

= 45 sq. units

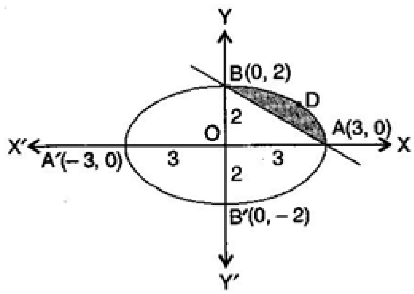
Again, Area CDO + Area OAB = $\left|\int_{-2}^{4}\left(\frac{3}{4}x^2\right)dx\right| = \left[\frac{3}{4}.\frac{x^3}{3}\right]_{-2}^{4} = \frac{1}{4}\left[64-(-8)\right]$ = 18 sq. units

$\therefore$ Required area = Area ABCD – (Area CDO + Area OAB)

= 45 – 18 = 27 sq. units Ans.

8. Equation of the ellipse is

$\frac{x^2}{9} + \frac{y^2}{4} = 1$ ....(i)



Here intersection of ellipse (i) with $x-$axis are

A (3, 0) and A'$(-3,0)$ and intersection of ellipse (i) with $y-$ axis are B (0, 2) and B'$(0,-2)$.

Also, the points of intersections of ellipse (i) and line $\frac{x}{3} + \frac{y}{2} = 1$ are A (3, 0) and B (0, 2).

$\therefore$ Area OADB = Area between ellipse (i) (arc AB of it) and $x-$axis

$$= \left|\int_0^3 \frac{2}{3}\sqrt{9-x^2}\ dx\right| = \left|\int_0^3 \frac{2}{3}\sqrt{3^2-x^2}\ dx\right| = \frac{2}{3}\left|\left[\frac{x}{2}\sqrt{3^2-x^2}+\frac{3^2}{2}\sin^{-1}\frac{x}{3}\right]\right|$$

$$= \frac{2}{3}\left[\frac{3}{2}\sqrt{9-9}+\frac{9}{2}\sin^{-1}1-\left(0+\frac{9}{2}\sin^{-1}0\right)\right]$$

$$= \frac{2}{3}\left[0+\frac{9}{2}.\frac{\pi}{2}-0\right] = \frac{2}{3}.\frac{9\pi}{4} = \frac{3\pi}{2} \text{ sq. units} \qquad \text{..........(ii)}$$

Again Area of triangle OAB = Area bounded by line AB and $x-$axis

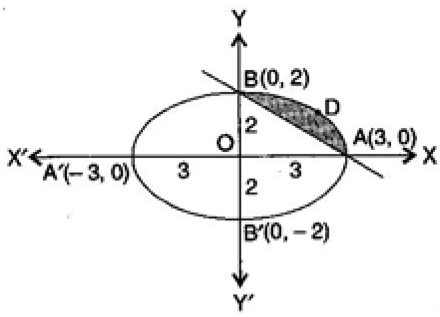
$$= \left|\int_0^3 \frac{2}{3}\sqrt{3-x}\ dx\right| = \frac{2}{3}\left|\left(3x-\frac{x^2}{2}\right)_0^3\right| = \frac{2}{3}\left\{\left(9-\frac{9}{2}\right)-0\right\} = \frac{2}{3}.\frac{9}{2}=3 \text{ sq. units} \quad \text{....(iii)}$$

Now Required shaded area = Area OADB – Area OAB

$$= \frac{3\pi}{2}-3 = 3\left(\frac{\pi}{2}-1\right)=\frac{3}{2}(\pi-2) \text{ sq. units} \qquad \text{Ans.}$$

9. Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ....(i)

Area between arc AB of the ellipse and $x-$axis



$$= \left|\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\ dx\right| = \frac{b}{a}\left|\int_0^a \sqrt{a^2-x^2}\ dx\right|$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{b}{a}\left[0+\frac{a^2}{2}\sin^{-1}1-(0+0)\right]$$

$$= \frac{b}{a}.\frac{a^2}{2}.\frac{\pi}{2}=\frac{\pi ab}{4} \qquad \text{...........(ii)}$$

Also Area between chord AB and $x-$axis

$$= \left|\int_0^a \frac{b}{a}(a-x)\ dx\right| = \frac{b}{a}\left|\int_0^a (a-x)\ dx\right| = \frac{b}{a}\left[ax-\frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left(a^2-\frac{a^2}{2}\right) = \frac{b}{a}.\frac{a^2}{2}=\frac{1}{2}ab$$

Now Required area

= Area between arc AB of the ellipse and $x-$axis – Area between chord AB and $x-$axis

$$= \frac{\pi ab}{4}=\frac{ab}{2}=\frac{ab}{4}(\pi-2) \text{ sq. units} \qquad \text{Ans.}$$

10. Equation of parabola is $x^2=y$ ....(i)

Equation of line is $y=x+2$ ....(ii)

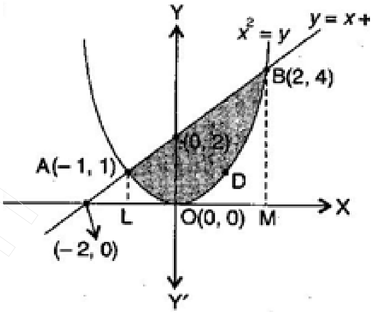
Here the two points of intersections of parabola (i) and line (ii) are $A(-1,1)$ and B (2, 4).

Area ALODBM = Area bounded by parabola (i) and $x-$axis

$$= \left|\int_{-1}^{2} x^2\, dx\right| = \left(\frac{x^3}{3}\right)_{-1}^{2} = \frac{8}{3}-\left(\frac{-1}{3}\right) = \frac{8}{3}+\frac{1}{3} = \frac{9}{3} = 3 \text{ sq. units}$$

Also Area of trapezium ALMB =

Area bounded by line (ii) and $x-$axis



$$= \left|\int_{-1}^{2} (x+2)\, dx\right| = \left(\frac{x^2}{2}+2x\right)_{-1}^{2} = 2+4-\left(\frac{1}{2}-2\right) = 6-\frac{1}{2}+2 = \frac{15}{2} \text{ sq.}$$

units

Now Required area = Area of trapezium ALMB – Area ALODBM

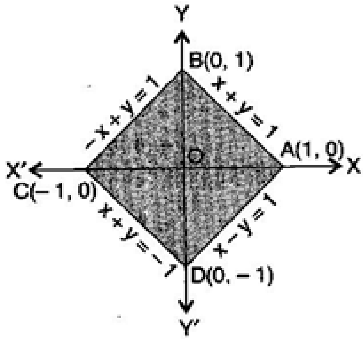
$$= \frac{15}{2}-3=\frac{9}{2} \text{ sq. units}$$

Ans.

11. Equation of the curve (graph) is

$|x|+|y|=1$ ....(i)

The area bounded by the curve (i) is represented by the shaded region ABCD.



The curve intersects the axes at points A (1, 0), B (0, 1), $C(-1,0)$ and $D(0,-1)$.

It is observed clearly that given curve is symmetrical about $x-$axis and $y-$axis.

$\therefore$ Area bounded by the curve

= Area of square ABCD

= 4 x Δ OAB

$$= 4\left|\int_0^1 (1-x)\, dx\right| = 4\left(x-\frac{x^2}{2}\right)_0^1 = 4\left[\left(1-\frac{1}{2}\right)-0\right] = 4\times\frac{1}{2} = 2 \text{ sq. units}$$ Ans.

12. The area bounded by the curves $\{(x,y): y \geq x^2 \text{ and } y=|x|\}$ is represented by the shaded region.

It is clearly observed that the required area is symmetrical about $y-$axis.

$\therefore$ Required area

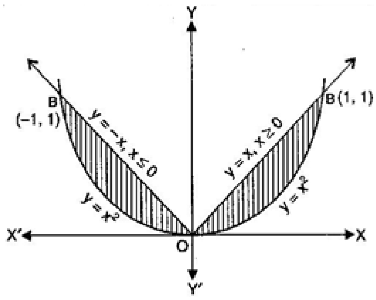
= Area between parabola $y=x^2$ and $x-$axis between limits $x=0$ and $x=1$

$$= \int_0^1 y\, dx = \int_0^1 x^2\, dx = \left(\frac{x^3}{3}\right)_0^1 = \frac{1}{3} \qquad \text{...........(i)}$$

And Area of ray $y=x$ and $x-$axis,

$= \int_0^1 y\ dx = \int_0^1 x\ dx = \left(\frac{x^2}{2}\right)_0^1 = \frac{1}{2}$

..........(ii)



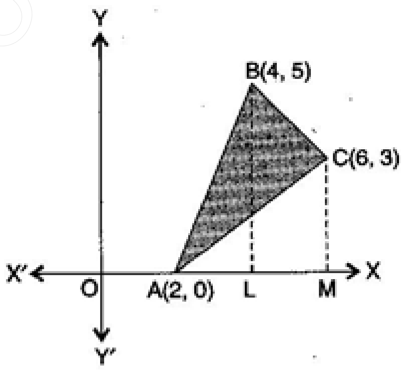
$\therefore$ Required shaded area in first quadrant

= Area between ray $y = x$ for $x \geq 0$ and $x -$ axis –

Area between parabola $y = x^2$ and $x -$ axis in first quadrant

= Area given by eq. (ii) – Area given by eq. (i) = $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ sq. units Ans.

13. Vertices of the given triangle are A (2, 0), B (4, 5) and C (6, 3).



Equation of side AB is $y - 0 = \frac{5-0}{4-2}(x-2)$

$= y = \frac{5}{2}(x-2)$

Equation of side BC is $y - 5 = \frac{3-5}{6-4}(x-4)$

$= y = 9 - x$

Equation of side AC is $y - 0 = \frac{3-0}{6-2}(x-2)$

$= y = \frac{3}{4}(x-2)$

Now, Required shaded area = Area ΔALB + Area of trapezium BLMC – Area ΔAMC

$= \left|\int_2^4 \frac{5}{2}(x-2)\ dx\right| + \left|\int_4^6 (9-x)\ dx\right| - \left|\int_2^6 \frac{3}{4}(x-2)\ dx\right|$

$= \frac{5}{2}\left(\frac{x^2}{2} - 2x\right)_2^4 + \left|\left(9x - \frac{x^2}{2}\right)_4^6\right| - \frac{3}{4}\left(\frac{x^2}{2} - 2x\right)_2^6$

$= \left[\frac{5}{2}(8-8) - (2-4)\right] + |54 - 18 - (36-8)| - \left[\frac{3}{4}\{18 - 12 - (2-4)\}\right]$

$= \frac{5}{2}(0+2) + |36 - 36 + 8| - \frac{3}{4}(6+2)$

$= \frac{5}{2} \times 2 + 8 - \frac{3}{4} \times 8$

= 5 + 8 – 6 = 7 sq. units Ans.

14. Equation of one line l_1 is

$2x + y = 4$,

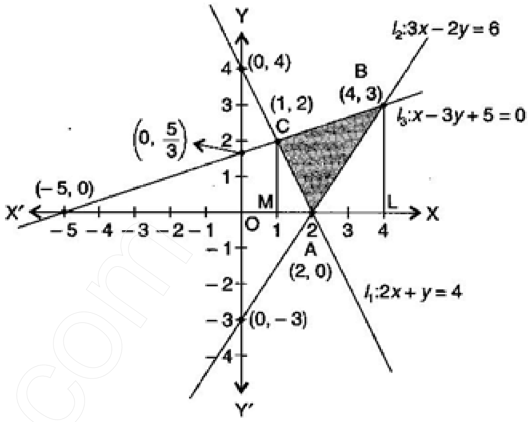
Equation of second line l_2 is

$$3x - 2y = 6$$

And Equation of third line l_3 is

$$x - 3y + 5 = 0.$$

Here, vertices of triangle ABC are A (2, 0), B (4, 3) and C (1, 2).



Now, Required area of triangle

= Area of trapezium CLMB

– Area ΔACM – Area ΔABL

$$= \left|\int_1^4 \frac{1}{3}(x+5)\,dx\right| - \left|\int_1^2 (4-2x)\,dx\right| - \left|\int_2^4 \frac{3}{2}(x-2)\,dx\right|$$

=

$$\frac{1}{3}\left|\left(\frac{x^2}{2}+5x\right)_1^4\right| - \left|\left(4x-\frac{2x^2}{2}\right)_1^2\right| - \frac{3}{2}\left|\left(\frac{x^2}{2}-2x\right)_2^4\right|$$

$$= \frac{1}{3}\left[8+20-\left(\frac{1}{2}+5\right)\right] - \{(8-4)-(4-1)\} - \frac{3}{2}\left|(8-8)-(2-4)\right|$$

$$= \frac{1}{3}\left(28-\frac{11}{2}\right) - (4-3) - \frac{3}{2}\times 2$$

$$= \frac{1}{3}\times\frac{45}{2} - 1 - 3 = \frac{15}{2} - 1 - 3 = \frac{7}{2} \text{ sq. units}$$ Ans.

15. Equation of parabola is $y^2 = 4x$ ……(i)

And equation of circle is $4x^2 + 4y^2 = 9$ ……(ii)

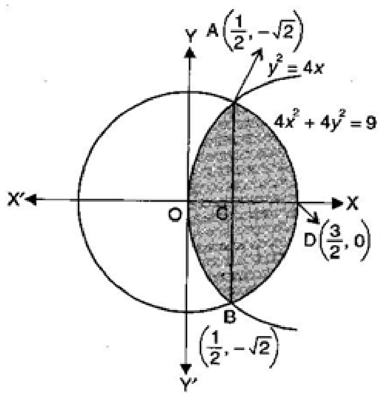
Here, the two points of intersection of parabola (i) and circle (ii) are $A\left(\frac{1}{2}, \sqrt{2}\right)$ and $B\left(\frac{1}{2}, -\sqrt{2}\right)$

Required shaded area OADBO (Area of the circle which is interior to the parabola)

= 2 x Area OADO = 2 [Area OAC + Area CAD]

$$= 2\left[\left|\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \left|\int_{\frac{1}{2}}^{\frac{3}{2}} \sqrt{\frac{9}{4}-x^2}\,dx\right|\right|\right]$$

$$= \left[\left\{2.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right\}_0^{\frac{1}{2}} + \left\{\frac{x\sqrt{\frac{9}{4}-x^2}}{2} + \frac{\frac{9}{4}}{2}\sin^{-1}\frac{x}{\frac{3}{2}}\right\}_{\frac{1}{2}}^{\frac{3}{2}}\right]$$



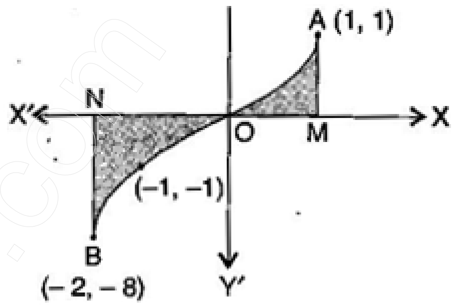
$$= 2\left[\frac{4}{3}\times\frac{1}{2\sqrt{2}}+\frac{9}{8}\sin^{-1}1-\frac{\frac{1}{2}\sqrt{2}}{2}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$

$$= 2\left[\frac{\sqrt{2}}{3}+\frac{9}{8}.\frac{\pi}{2}-\frac{\sqrt{2}}{4}-\frac{9}{8}\sin^{-1}\frac{1}{3}\right] = \left(\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}+\frac{\sqrt{2}}{6}\right) \text{ sq. units} \qquad \text{Ans.}$$

16. Equation of the curve is $y = x^3$ **Y**

To find: Area OBN ($y = x^3$ for $-2 \le x \le 0$) and Area OAM ($y = x^3$ for $0 \le x \le 1$)

$\therefore$ Required area = Area OBN + Area OAM



$$= \left|\int_{-2}^{0} x^3\, dx\right| + \left|\int_{0}^{1} x^3\, dx\right|$$

$$= \left|\left(\frac{x^4}{4}\right)_{-2}^{0}\right| + \left|\left(\frac{x^4}{4}\right)_{0}^{1}\right|$$

$$= \left|0-\frac{16}{4}\right| + \left|\frac{1}{4}-0\right| = 4+\frac{1}{4} = \frac{17}{4} \text{ sq. units}$$

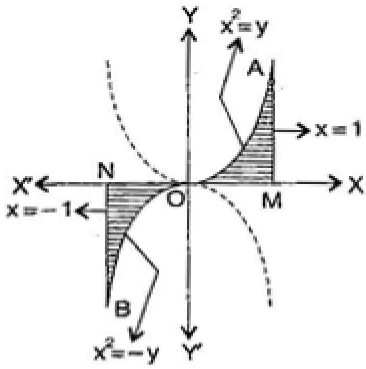
Therefore, option (D) is correct,

17. Equation of the curve is

$y = x|x| = x(x) = x^2$ if $x \ge 0$ ....(i)

And $y = x|x| = x(-x) = -x^2$ if $x \le 0$ ....(ii)

Required area = Area ONBO + Area OAMO



$$= \left|\int_{-1}^{0} -x^2\, dx\right| + \left|\int_{0}^{1} x^2\, dx\right|$$

$$= \left|\left(\frac{-x^3}{3}\right)_{-1}^{0}\right| + \left|\left(\frac{x^3}{3}\right)_{0}^{1}\right|$$

$$= 0-\left(\frac{-1}{3}\right)+\frac{1}{3}-0 = \frac{2}{3} \text{ sq. units}$$

Therefore, option (C) is correct.

18. Equation of the circle is $x^2 + y^2 = 16$ .....(i)

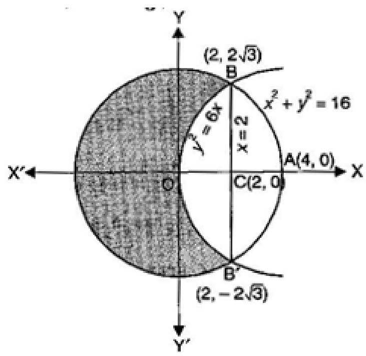
This circle is symmetrical about $x-$ axis and $y-$ axis.

Here two points of intersection are $B\left(2, 2\sqrt{3}\right)$ and $B'\left(2, -2\sqrt{3}\right)$.

Required area = Area of circle – Area of circle interior to the parabola

= $\pi r^2 -$ Area OBAB'O = $16\pi -$ 2 x Area OBACO $[\because r = 4]$

= $16\pi - 2$[Area OBCO + Area BACB]



$$= 16\pi - 2\left[\left|\int_0^2 \sqrt{6x}\ dx\right| + \left|\int_2^4 \sqrt{16-x^2}\ dx\right|\right]$$

$$= 16\pi - 2\left[\sqrt{6}.\left(\frac{x^{3/2}}{3/2}\right)_0^2 + \left(\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{\pi}{4}\right)\right]_2^4$$

$$= 16\pi - 2\left[\frac{2}{3}\sqrt{6}\left(2\sqrt{2}\right) + 8\sin^{-1}1 - \sqrt{12} - 8\sin^{-1}\frac{1}{2}\right]$$

$$= 16\pi - 2\left[\frac{8}{\sqrt{3}} + 8.\frac{\pi}{2} - 2\sqrt{3} - 8.\frac{\pi}{6}\right]$$

$$= 16\pi - 2\left[\frac{8}{\sqrt{3}} - 2\sqrt{3} + 8\pi\left(\frac{1}{2} - \frac{1}{6}\right)\right]$$

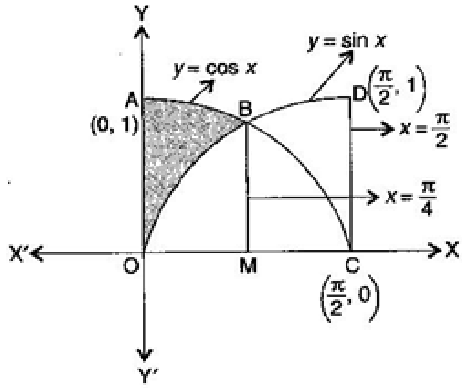
$$= 16\pi - 2\left[\frac{8-6}{\sqrt{3}} + 8\pi\left(\frac{3-1}{6}\right)\right] = 16\pi - 2\left[\frac{2}{\sqrt{3}} + \frac{8\pi}{3}\right]$$

$$= 16\pi - \frac{4}{\sqrt{3}} - \frac{16\pi}{3} = 16\pi\left(1 - \frac{1}{3}\right) - \frac{4}{\sqrt{3}} = \frac{32\pi}{3} - \frac{4}{\sqrt{3}}$$

$$= \frac{32\pi}{3} - \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4}{3}\left(8\pi - \sqrt{3}\right) \text{ sq. units}$$ Ans.

19. Here both graphs intersect at the point $B\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$.

Required shaded area = Area OABC – Area OBC
= Area OABC – (Area OBM + Area BCM)



$$= \left|\int_0^{\pi/2} \cos x\ dx\right| - \left(\left|\int_0^{\pi/4} \sin x\ dx\right| + \left|\int_{\pi/4}^{\pi/2} \cos x\ dx\right|\right)$$

$$= \left|(\sin x)_0^{\pi/2}\right| - \left(\left|(-\cos x)_0^{\pi/4}\right| + (\sin x)_{\pi/4}^{\pi/2}\right)$$

$$= \left(\sin\frac{\pi}{2} - \sin 0^\circ\right) - \left(\left|-\cos\frac{\pi}{4} + \cos 0^\circ\right| + \left|\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right|\right)$$

$$= 1 - 0 - \left(\frac{-1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}}\right) = 1 + \frac{1}{\sqrt{2}} - 1 - 1 + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} - 1 = \left(\sqrt{2} - 1\right) \text{ sq. units}$$

Therefore, option (B) is correct.