T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2016

Paper II – Code 0

PART I

1	2	3	4	5	6
C		**C**	**B**		**C**

7	8	9	10	11	12
	A, C	**B, C**	**A, B, D**		

13	14
	C, D

15	16	17	18

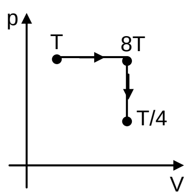
Section I

1. $A = \dfrac{A_0}{2^{t/\tau}} = 64\ A$

$2^{t/\tau} = 2^6$ $\quad t = 6 \times \tau$
$= 6 \times 18$
$= 108$ days

2.

3.



$\gamma = \dfrac{5}{3}$

$C_p = \dfrac{5R}{2}$

$C_V = \dfrac{3R}{2}$

$Q = nC_p(8T - T) + nC_V\left(\dfrac{T}{4} - 8T\right)$

$= n \times \dfrac{5R}{2} \times 7T + n \times \dfrac{3R}{2}\left(-\dfrac{31}{4}\right)T$

$= nRT\left[\dfrac{35}{2} - \dfrac{93}{8}\right] = p_1V_1 \times \dfrac{(140-93)}{8}$

$= 10^5 \times 10^{-3} \times \dfrac{47}{8} = 588$ J

4. C_1 $\quad$ 10 VSD = 9 MSD
1 VSD = 0.9 MSD
(L.C) = 1 MSD – 1 VSD
= 0.1 MSD = 0.1 mm
= 0.01 cm
Measured value
= MSR + VCD × L.C
= 2.8 + 7 × 0.01
= 2.87 cm
C_2 $\quad$ 10 VSD = 11 MSD
1 VSD = 1.1 MSD = 1.1 mm
= 0.11 cm
VCD 7
Measured value
= MSR + (8 MSD – 7 VSD)
= 2.8 + (8 × 0.1 – 7 × 0.11)
= 2.83 cm

5.

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

6. $\dfrac{2\lambda A(\theta-10)}{1} = \dfrac{\lambda A(400-\theta)}{1}$

Jn through $\theta = 140$

$\Delta\ell = \ell\alpha\theta$

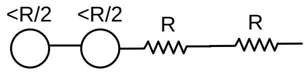
$= 1 \times 1.2 \times 10^{-5} \times (140 - 10)$

$= 156 \times 10^{-5}$ m = 1.56 mm

Section II

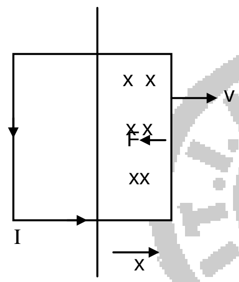
7.

8.



Maximum resistance all in series
So maximum voltage
Minimum resistors when all in parallel. Hence maximum current in this case.

9.



$\mathcal{E} = vBL$

$I = \dfrac{vBL}{R}$

$F = (-)ILB$

$= \dfrac{vBL}{R} \times LB$

$m\dfrac{dv}{dt} = (-)\dfrac{vB^2L^2}{R}$

$m\dfrac{dv}{dr}v = (-)\dfrac{vB^2L^2}{R}$

$dv = -\dfrac{B^2L^2}{mR}.dx$

$v - v_0 = -\dfrac{B^2L^2}{mR} \times x$

$v = v_0 - \dfrac{B^2L^2}{mR} \times x$

(C) correct

$I = \dfrac{vBL}{R}$, v is linear $\Rightarrow$ I is linear

ACW $\Rightarrow$ I +ve initially. I –ve when coming out

B correct

(D) wrong

Force is always retarding.

Hence (A) wrong.

10. Error in measurement of r

$\dfrac{\Delta r}{r} = \dfrac{1\ \text{mm}}{10\ \text{m}} \times 100 = 10\%$

(A) correct

Average of T = $\dfrac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5}$

= 0.556 s = 0.56 s

Average of $|\Delta T|$ = $\dfrac{0.04 + 0 + 0.01 + 0.02 + 0.03}{5}$

$= \dfrac{0.1}{5} = 0.02$ s

% error $\dfrac{|\Delta T|}{T} = \dfrac{0.02}{0.56} \times 100 = 3.57\%$

(B) correct

(C) wrong

$\dfrac{\Delta g}{g} = \dfrac{2\Delta T}{T} + \dfrac{\Delta(R-r)}{R-r}$

$= 2 \times 3.57\% + \left(\dfrac{\Delta R + \Delta r}{R - r}\right) \times 100$

$= 7.14\% + \dfrac{2\ \text{mm}}{50\ \text{mm}} \times 100 = 11.14\%$

(D) correct

11. In case (i) there is loss of kinetic energy (perfectly inelastic collision). Hence amplitude will decrease. In the second case there is no loss of K.E, hence amplitude remains unchanged.

K.E = $\dfrac{p^2}{2M} = \dfrac{1}{2}kA^2$ (before m is attached)

K.E = $\dfrac{p^2}{2(M+m)} = \dfrac{1}{2}kA_1^2$ (after m is attached)

$\dfrac{A_1}{A} = \sqrt{\dfrac{M}{M+m}}$ (A) correct

(B) correct in both cases final

$T = 2\pi\sqrt{\dfrac{M+m}{R}}$

(C) Wrong. Total energy decreases only in the first case.

(D) Instantaneous speed at mean position decreases only in the first case.

D $\Rightarrow$ wrong.

12.

13.

14. A. $\lambda_e = \dfrac{h}{p} = \dfrac{h}{mv}$

$\lambda_{ph} = \dfrac{hc}{\lambda}$

As λ_{ph} increases λ decreases

B wrong

C. As ϕ increases λ_e decreases. So also about λ_{ph}

So (C) correct

D. $\lambda \propto \dfrac{1}{\sqrt{V}}$ V $\rightarrow$ 4 V then $\lambda \rightarrow \dfrac{\lambda}{2}$

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

Section III

15.

16.

17.

18.

PART II

19	20	21	22	23	24
D	**A**	**A**	**D**	**A**	**C**

25	26	27	28	29	30
B, C	**A, B**	**B, C, D**	**C**	**B, C, D**	**A, B, C**

31	32
A, C	**B, D**

33	34	35	36
B	**B**	**A**	**B**

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

Section I

19. Cell reaction is
$M^{4+} + H_2 \rightarrow M^{2+} + 2H^+$

$$E_{cell} = E^{\circ}_{cell} - \frac{0.059}{2}\log\frac{[M^{2+}][H^+]^2}{[M^{4+}]}$$

$$0.092 = 0.151 - \frac{0.059}{2}\log\frac{[M^{2+}]}{[M^{4+}]}$$

$$\frac{0.059}{2}\log\frac{[M^{2+}]}{[M^{4+}]} = 0.059$$

$$\frac{[M^{2+}]}{[M^{4+}]} = 10^2$$

x = 2

20. The order of acid strength is
I > II > III > IV

21. $[Ni(NH_3)_6]^{2+}$ octahedral
$[Pt(NH_3)_4]^{2+}$ square planar
$[Zn(NH_3)_4]^{2+}$ tetrahedral

22. As the concentration of CH_3OH(aq) increases, surface tension decreases slowly. Addition of an ionic compound increases the surface tension of aqueous solution. Addition of soap reduces the surface tension of water drastically.

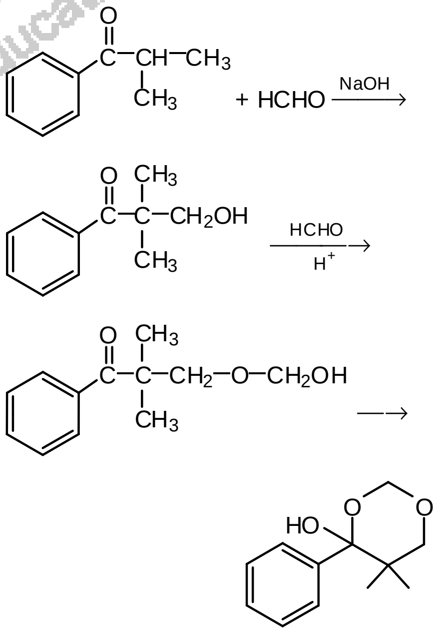
23. $2Ag^+ + S_2O_3^{2-} \rightarrow Ag_2SO_3 \downarrow$

$$Ag_2S_2O_3 + S_2O_3^{2-} \rightarrow \underset{\text{Soluble}}{[Ag(S_2O_3)_2]^{3-}}$$

$$[Ag(S_2O_3)_2]^{3-} + 3Ag^+ \rightarrow \underset{\text{white}}{2Ag_2S_2O_3} \downarrow$$

$$Ag_2S_2O_3 + H_2O \rightarrow \underset{\text{Black}}{Ag_2S} + H_2SO_4$$

24.



Section II

25. (B) and (C) are only correct statements.

26. $CCl_4 + CH_3OH$ – positive deviation
$CS_2 + CH_3COCH_3$ – positive deviation
Benzene + Toluene – ideal solution
Phenol + Aniline – negative deviation

27. The number of nearest neighbours for an atom in the top most layer of ccp arrangement is 9
Packing efficiency for ccp = 74%
The number of octahedral and tetrahedral voids per atom are 1 and 2 respectively.
Edge length, $a = 2\sqrt{2}\,r$

28. $NaBH_4$ cannot reduce carboxyl and ester groups.

29. Reactions (B), (C) and (D) can give tert-butyl benzene.

30. Refining 'blister copper' is by
(a) Poling
(b) Electrolysis

31. $C_2^{2-} = KK, \sigma 2s^2, \sigma^*2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$

$O_2 = KK, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2,$
$\pi^* 2p_x^1 = \pi^* 2p_y^1$

B.O of O_2 = 2 B.O of $O_2^{2+} = 3$

N_2^+ has B.O = 2.5 N_2^- has B.O = 2.5
(one Bonding e^- less) (one e^- in ABMO)

He_2^+ molecular ion has $\frac{1}{2}$ bond order while 2 separate He atom have no bond order.

32. $2HNO_3 \xrightarrow[-H_2O]{P_2O_5} N_2O_5$
(A) P_4 reacts with HNO_3 to give H_3PO_4
(C) N_2O_5 has N–O–N bond
(D) $N_2O_5 + Na \rightarrow NaNO_3 + NO_2$ (brown gas)
(B) N_2O_5 is diamagnetic

Section III

33. $X_{2(g)} \rightleftharpoons 2X_{(g)}$ Total

$1-\frac{\beta}{2}$ $\frac{\beta}{2}$ $1+\frac{\beta}{2}$

$p_{x_2} = \frac{(2-\beta)2}{2+\beta}$

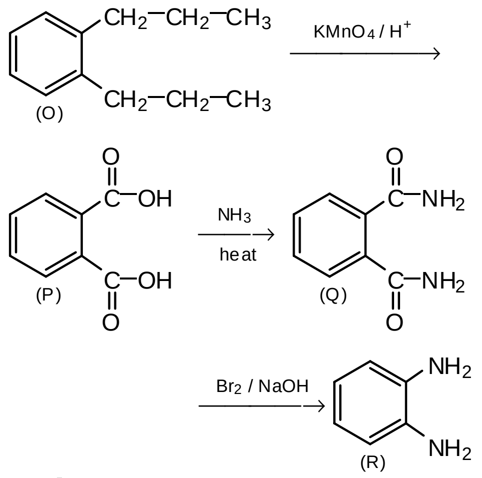
$p_x = \frac{\beta \times 4}{2+\beta}$

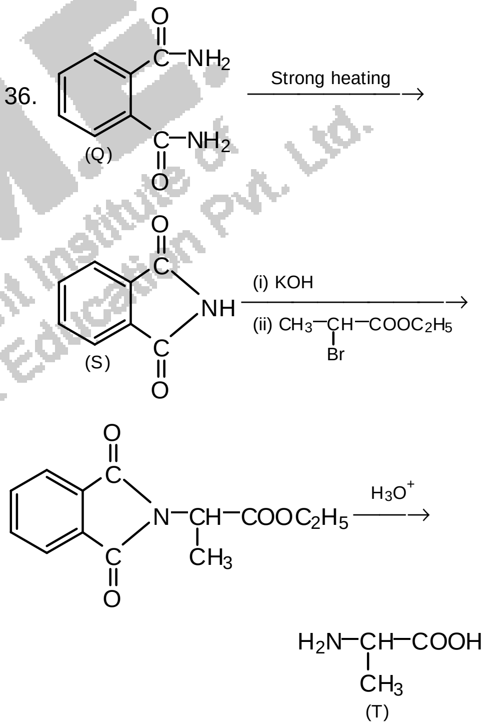
$K_p = \frac{p_x^2}{p_{x_2}}$

$= \frac{16\beta^2}{(2+\beta)^2} \times \frac{2+\beta}{(2-\beta)2}$

$= \frac{8\beta^2}{4-\beta^2}$

34. Since it is a thermal decomposition, the reaction initiates only on heating. Thus it is non-spontaneous at the start.

35.


36.


T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

PART III

37	38	39	40	41	42
C	**B**	**B**	**C**	**B**	**A**

43	44	45	46	47	48
A, D	**A, C, D**	**A, B**	**B, C**	**B, C**	**B, C, D**

49	50
B, C	**A, C, D**

51	52	53	54
B	**C**	**A**	**C**

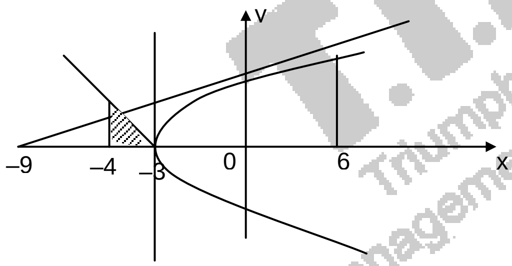
T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

Section I

37. Image of (3, 1, 7) with respect to x – y + z = 3 is (–1, 5, 3) and (0, 0, 0) is
Equation of plane passing through (–1, 5, 3), (0, 0, 0) and counting the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$
is x – 4y + 7z = 0

38. $y \ge \sqrt{|x+3|}$

$y^2 = x + 3, x > -3$



$5y \le x + 9$

$y \le \frac{3}{9}+\frac{9}{5}$

$5y = x + 9$

$x - 5y + 9 = 0$

$x \le 6$

Area of the trapezium

$= \frac{1}{2}(10)\times 4)$

$= 20$

$$\int_{-4}^{-3}\sqrt{-3-x}\ dx + \int_{-3}^{6}\left[\frac{2(x+3)^{\frac{3}{2}}}{3}\right]_{-3}^{6}$$

x = –t

dx = –dt

$= \int_{-4}^{-3}\sqrt{-3+t}\,(-dt)$

$= \int_{3}^{4}\sqrt{t-3}\ dt$ $\qquad \frac{2}{3}\times\left(3^2\right)^{\frac{3}{2}}$

$= \frac{2\times 27}{3} = 18$

$\left[\frac{2}{3}(t-3)^{\frac{3}{2}}\right]_3^4$

$= \frac{2}{3}$

Area = $20 - \left\{18+\frac{2}{3}\right\}$

$= 2 - \frac{2}{3} = \frac{4}{3}$

39. b_1, b_2, b_3 …….. b_{101} are G.P with C R = 2
a_1, a_2 ……….a_{101} are A.P
$a_1 = b_1 \quad a_{51} = b_{51}$

$\frac{b_2}{b_1} = 2 \quad \Rightarrow b_2 = 2b_1$

$\frac{b_3}{b_2} = 2 \quad b_3 = 2b_2 = 4b_1$

$\therefore$ t = (1 + 2 + 4+ …..+ 102)b_1 $\qquad \frac{n(n+1)}{2}$

= 1 + 2 (1 + 2 …… + 51)b_1

$= 1 + 2\ \frac{51\times 52}{2}b_1$

= 1 + 2652

S = $a_1 + a_2$ + ….. + a_{51}

= b_1 ……. + b_{51}

< $b_1 + b_2$ ….. + b_5

$\therefore$ S > t and $a_{101} < b_{01}$

40. $$\frac{2}{2\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$

$$=\frac{2}{2\cos\frac{\pi}{6}-\cos(k+1)\frac{\pi}{3}}$$

$$=\frac{-4}{\sqrt{3}-2\cos(k+1)\frac{\pi}{3}}$$

$$\sum_{k=1}^{13}\frac{4}{\sqrt{3}-2\cos(k+1)\frac{\pi}{3}} \qquad \because \sum\cos(k+1)\frac{\pi}{3}=\frac{-1}{2}$$

$$\frac{4}{\sqrt{3}-2\left(\frac{-1}{2}\right)}=\frac{4}{\sqrt{3}+1}=\frac{4\left(\sqrt{3}-1\right)}{2}$$

$$=2\sqrt{3}-2$$

41. $$|P|=\begin{vmatrix}1&0&0\\4&1&0\\16&4&1\end{vmatrix}=1$$

$P^{50} = I + R$

$$P^2=\begin{pmatrix}1&0&0\\4&1&0\\16&4&1\end{pmatrix}\begin{pmatrix}1&0&0\\4&1&0\\16&4&1\end{pmatrix}$$

$$=\begin{pmatrix}1&0&0\\8&1&0\\48&8&1\end{pmatrix}$$

$$P^3=\begin{pmatrix}1&0&0\\8&1&0\\48&8&1\end{pmatrix}\begin{pmatrix}1&0&0\\4&1&0\\16&4&1\end{pmatrix}$$

$$=\begin{pmatrix}1&0&0\\12&1&0\\96&12&1\end{pmatrix}$$

$$P^4=\begin{pmatrix}1&0&0\\12&1&0\\96&12&1\end{pmatrix}\begin{pmatrix}1&0&0\\4&1&0\\16&4&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\16&1&0\\160&16&1\end{pmatrix}$$

16, 48, 96, 160, 240, ……

32, 48, 64, 80 $\qquad$ $An^2 + Bn + C$

16, 16, 16

$A + B + C = 16$

$4A + 2B + C = 48$

$9A + 3B + C = 96$

$3A + B = 32$

$\underline{5A + B = 48}$

$2A = 16$

$A = 8$

$B = 32 - 24 = 8$

$C = 16 - 8 - 8 = 0$

Hence, the 3rd row 1st column element of P^{50}

$= 8n^2 + 8n$ where $n = 50$

$= 8 \times 2500 + 400$

$= 20400$

3rd row 2nd column element of P^{50}

$= 4 + 49 \times 4$

$= 200$

$$\frac{q_{31}+q_{32}}{q_{21}}=\frac{20400+200}{4\times 50}$$

$$=\frac{20600}{200}=103$$

42. $$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2\cos x}{1+e^x}\,dx$$

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2\cos x}{1+e^{-x}}\,dx$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2\cos x\ e^x}{1+e^x}\,dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\left(1+e^x\right)x^2\cos x}{1+e^x}\,dx$$

$$=2\int_0^{\frac{\pi}{2}}x^2\cos x\ dx$$

$$I=\left(x^2\sin x\right)_0^{\frac{\pi}{2}}-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}2x\sin x\ dx$$

$$=\frac{\pi^2}{4}-2(-x\ \cos x)+\int\cos x\ dx$$

$$=\frac{\pi^2}{4}-2$$

43. $f'(2) = 0 = g(2)$

$f''(2) \neq 0$

$g'(2) \neq 0$

$$\lim_{x\to 2}\frac{f(x)\,g(x)}{f'(x)\ g'(x)}\to\left(\frac{0}{0}\ \text{ form}\right)$$

$$=\lim_{x\to 2}\frac{f(x)\,g'(x)+f'(x)\ g(x)}{f'(x)\ g''(x)+f''(x)\ g'(x)}$$

$$=\frac{f(2)\times g'(2)+f'(2)\ g(2)}{f'(2)\ g''(2)+f''(2)\ g'(2)}$$

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

$$= \frac{f(2)\ g'(2)}{f'(2)\ g''(2) + f''(2)\ g'(2)} = 1$$

$$= \frac{f(2)\ g'(2)}{f'(2)\ g''(2) + g'(2)} = 1$$

$\Rightarrow f''(2) = f(2)$

Since the range of f is $(0, \infty)$ $\quad 2 = 4 - 8+$

$f(2) > 0$

$\Rightarrow f''(2) > 0$

$\Rightarrow$ f is minimum at $x = 2 \rightarrow$ (A) is true

Since $f''(2) - f(2) = 0$

$\Rightarrow$ (D) is true

44. $y^2 = 4x$

$P(t^2, 2t)$

Normal at $P \rightarrow y + xt = 2t + t^3$

It passes through the centre S(2, 8) of the circle

$8 + 2t = 2t + t^3$

$t = 2$

P(4, 4)

$SP^2 = (4 - 2)^2 + (8 - 4)^2 = 4 + 16 = 2D$

$SP = 2\sqrt{5}$ $\Rightarrow$ (A) is true

Q is the point (3, 6) $\Rightarrow$ (B) is false

Normal at P(4, 4)

$y + 2x = 4 + 8 = 12$

$\frac{x}{6} + \frac{y}{12} = 1$ $\Rightarrow$ (C) true

Circle is $(x - 2)^2 + (y - 8)^2 = 4$

$2(x - 2) + 2(y - 8)y' = 0$

$$y' = -\frac{(x-2)}{(y-8)}$$

$$= -\left(\frac{3-2}{6-8}\right) = \frac{1}{2}$$

45. $f(x) = a\cos|x^3 - x| + b|x|\sin(|x^3 + x|)$

$x^3 - x = x(x^2 - 1)$

$= (x + 1)\,x\,(x - 1)$

$x^3 + x = (x^2 + 2)x$

$$f(x) = \begin{cases} a\cos(x - x^3) - bx[-\sin(x^3 + x)], & -\infty < x < -1 \\ a\cos(x^3 - x) - bx[-\sin(x^3 + x)] & -1 < x < 0 \\ a\cos(x - x^3) + bx\sin(x^3 + x), & 0 < x < 1 \\ a\cos(x^3 - x) + bx\sin(x^3 + x) & x > 1 \end{cases}$$

$f(0^-) = a = f(0^+)$

$f(1^-) = a + b\sin 2 = f(1^+)$

f(x) is continuous at x = 0, 1

$f'(x) = [-a\sin(x^3 - x)]\,[3x^2 - 1]$

$+b\{\sin(x^3 + x)$

$+3(x^2 + 1)x\cos(x^3 + x)\}$

$-1 < x < 0$

$= [-a\sin(x - x^3)]\,[1 - 3x^2]$

$+b\{\sin(x^3 + x) + x(3x^2 + 1)\cos(x^3 + x)\}$

$0 < x < 1$

$= [-a\sin(x^3 - x)]\ [3x^2 - 1]$

$+ b\{\sin(x^3 + x) + x(3x^2 + 1)\cos(x^3 + x)\}\ x > 1$

$f'(0^-) = 0$

$f'(0^+) = 0$

$f'(0^+) = 0$

$f'(1^-) = b\{\sin 2 + 4\cos 2\}$

$f'(1^+) = b\{\sin 2 + 4\cos 2\}$

46. $f(x) = [x^2 - 3]$

When $x \in \left[-\frac{1}{2}, 2\right]$

$x^2 - 3$ varies from -2.75 to 1

f(x) varies -2.75 to -2 (break point)

-2 to -1 (break point)

-1 to 0 (break point)

0 to 1 (break point)

f(x) is discontinuous at 4 points

option (B)

$g(x) = [x^2 - 3]\ \{|x| + |4x - 7|\}$

In $\left(\frac{-1}{2}, 2\right)$, $\left[x^2 - 3\right]$ discontinuous at 3 points

and $|x| + |4x - 7|$ is discontinuous

at $x = 0, \frac{7}{4}$

But as x = 0 is already included

$\Rightarrow$ total 4 discontinuities

$\therefore$ (B), (C) true

47 $\log f(x) = x\int_0^1 \log(1 + tx)\,dt$

$\therefore$ f(x) is increasing from (0, 1)

and decreasing from $(1, \infty)$

$\therefore$ (B) and (C) true

48. $ax + 2y = \lambda$

$3x - 2y = \mu$

$$\begin{vmatrix} a & 2 \\ 3 & -2 \end{vmatrix} = -2a - 6$$

$= -2(a + 3)$

(B) is true

When $a = -3$ $\begin{vmatrix} a & \lambda \\ 3 & \mu \end{vmatrix} = a\mu - 3\lambda$

$= -3\mu - 3\lambda$

$= -3(\lambda + \mu)$

$$\begin{vmatrix} 2 & \lambda \\ -2 & \mu \end{vmatrix} = 2\mu + 2\lambda$$

$\Rightarrow$ (C) is true

(D) is true

Section II

49. $|u|\ |v| \sin\theta = 1$

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.

$1 \times |v| \sin\theta = 1$ ——— (1)

$w (u \times v) = 1$

$[w\ u\ v] = 1$

$\Rightarrow |w|\ |v|\ |u| = 1$

$\Rightarrow |v| = 1$

Sub in (1)

$\sin\theta = 90$

$\therefore$ u and v are $\perp^r$ and $|u| = |v|-= 1$

$\therefore$ (A) incorrect

$\therefore$ B is true

If $|u_1| = |u_2|$ then $|u| = 1$

$(u_1, u2_{, 0)}$

$\Rightarrow \sqrt{u_1^2 + u_2^2} = 1$ only if $|u_1| = u_2|$ $\therefore$ (C) correct

$(u_1, 0, u_3)$

$\Rightarrow \sqrt{u_1^2 + u_3^3} \neq$ if $2|u_1| = |u_2|$

$\therefore$ D incorrect

50. $x + iy = \dfrac{1}{a+ibt}$

$= \dfrac{a-ibt}{a^2+b^2t^2}$

$\Rightarrow x = \dfrac{a}{a^2+b^2t^2}$ and $y = \dfrac{-bt}{a^2+b^2t^2}$

Consider option (A)

Given circle is

$$\left(x - \frac{1}{2a}\right)^2 + y^2 = \frac{1}{4a^2}$$

i.e., $a^2x^2 + a^2y^2 - x = 0$ ——— (1)

$x = \dfrac{a}{a^2+b^2t^2}$ and $y = \dfrac{-bt}{a^2+b^2t}$ satisfy (1)

$\therefore$ (x, y) lies on (1)

(A) correct

But x and y do not satisfy are circle given in option (B)

(B) incorrect

Now, for $b = 0$ and $a \neq 0$,

$x + iy = \dfrac{1}{a}$

$\Rightarrow x = \dfrac{1}{a}$ and $y = 0$

$\Rightarrow$ lies on x – axis

(C) correct

For $a = 0$ and $b \neq 0$,

$x + iy = -\dfrac{i}{bt}$

$\Rightarrow x = 0$ and $y = -\dfrac{1}{bt} \Rightarrow$ lies on y-axis

(D) correct

Section III

Possibilities are T_1T_1, T_1T_2, T_2T_2, T_1D, T_2D, DT_1, DT_2 with (x, y) values (6, 0) (3, 3), (3, 3), (0, 6), (4, 1), (1, 4), (4, 1), (1, 4), (D, D) (2, 2)

51. $P(x > 4) = P(\ (6, 0), (4, 1), (4, 1))$

i.e. $P(T_1T_1, T_1D, DT_1)$

$= \dfrac{1}{2}\times\dfrac{1}{2} + \dfrac{1}{2}\times\dfrac{1}{6} + \dfrac{1}{6}\times\dfrac{1}{2}$

$= \dfrac{5}{12}$

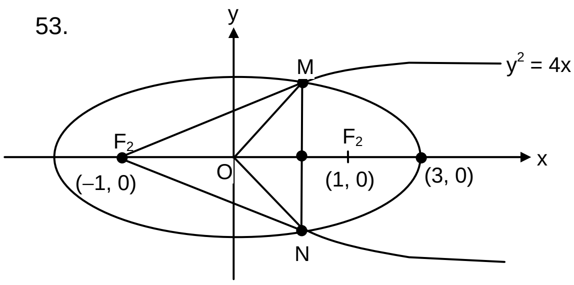
52. $P(x = 4) = P((3, 3), (3, 3)\ (2, 2))$

i.e. $P(T_1T_2, T_2T_1)$

$= \dfrac{1}{2}\times\dfrac{1}{3} + \dfrac{1}{3}\times\dfrac{1}{2} + \dfrac{1}{6}\times\dfrac{1}{6} = \dfrac{1}{6} + \dfrac{1}{6} + \dfrac{1}{36}$

$\dfrac{13}{36}$

53.



Equation of the parabola is $y^2 = 4c$

$\therefore \dfrac{x^2}{9} + \dfrac{y^2}{8} = 1 \Rightarrow \dfrac{x^2}{9} + \dfrac{4x}{8} = 1 \Rightarrow \dfrac{x^2}{9} + \dfrac{x}{2} = 1$

$\Rightarrow 2x^2 + 9x - 18 = 0$

$\Rightarrow 2x^2 + 12x - 3x - 18 = 0$

$\Rightarrow 2x(x + 6) - 3(x + 6) = 0 \Rightarrow (x + 6)(2x - 3) = 0$

$\Rightarrow x = -6$ or $\dfrac{3}{2}$

Since $x > 0$, take $x = \dfrac{3}{2}$

$\Rightarrow y = +\sqrt{6}$

$\therefore$ M is $\left(\dfrac{3}{2}, \sqrt{6}\right)$ and N is $\left(\dfrac{3}{2}, -\sqrt{6}\right)$

Slope of $F_2N = \dfrac{-2\sqrt{6}}{5}$

$\therefore$ Slope of the altitude through M is $\dfrac{5}{2\sqrt{6}}$

$\therefore$ Equation of this altitude is

$y - \sqrt{6} = \dfrac{5}{2\sqrt{6}}\left(x - \dfrac{3}{2}\right)$ ——— (1)

Also x – axis (y = 0) is an altitude

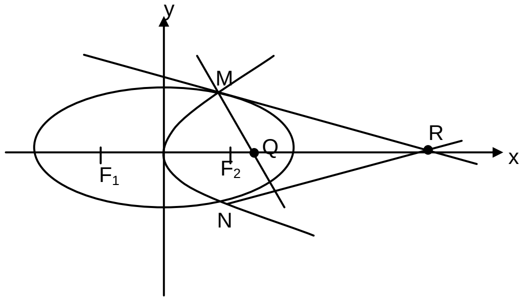
Put y = 0 in (1)

$\therefore -\sqrt{6} = \dfrac{5}{2\sqrt{6}}\left(x - \dfrac{3}{2}\right)$

$\Rightarrow x = \dfrac{-9}{10}$

$\therefore$ orthocenter is $\left(\dfrac{-9}{10}, 0\right)$

54.



Slope of the tangent at $M\left(\frac{3}{2}, \sqrt{6}\right)$ to the ellipse

$$= \frac{dy}{dx} = \frac{-4}{3\sqrt{6}}$$

$\therefore$ Equation of the tangent at M is

$$y - \sqrt{6} = \frac{-4}{3\sqrt{6}}\left(x - \frac{3}{2}\right)$$

Put y = 0. Then x = 6

$\therefore$ R is (6, 0)

Slope of tangent at $M\left(\frac{3}{2}, \sqrt{6}\right)$ to the parabola

$$\frac{dy}{dx} = \frac{2}{\sqrt{6}}$$

Equation of the normal at M is

$$y - \sqrt{6} = \frac{-\sqrt{6}}{2}\left(x - \frac{3}{2}\right)$$

Put y = 0. Then $x = \frac{7}{2}$

$\therefore$ Q is $\left(\frac{7}{2}, 0\right)$

Area of $\Delta MQR = \frac{5\sqrt{6}}{4}$

Area of quadrilateral MF_1NF_2

$= 2 \times$ Area of ΔF_1MF_2

$= 2\sqrt{6}$

$\therefore$ Ratio is $\frac{5\sqrt{6}}{4} : 2\sqrt{6} = 5 : 8$

***This key had been prepared by our academic team. However, in questions where multiple interpretations are possible, there may be divergence from the official answer key published / to be published by the examination authorities and no claim shall lie against T.I.M.E. Pvt. Ltd. in the event of any such mismatch between official key and T.I.M.E.s key.**

T.I.M.E. Triumphant Institute of Management Education Pvt. Ltd.