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## **Senior School Certificate Examination**

**March — 2015**

### **Marking Scheme — Mathematics 65/1/B, 65/2/B, 65/3/B**

#### ***General Instructions :***

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question(s) on differential equations, constant of integration has to be written.
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
7. Separate Marking Scheme for all the three sets has been given.

**QUESTION PAPER CODE 65/1/B**

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

	Marks
1. $x = 2, y = 9$	(½ for correct x or y)
$\therefore x + y = 11$	$\frac{1}{2} m$
2.    order 3, or degree 1	$\frac{1}{2} m$
$\therefore \text{Degree} + \text{order} = 4$	$\frac{1}{2} m$
3. $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$ (Standard form)	$\frac{1}{2} m$
I.F. = $\log x$	$\frac{1}{2} m$
4. $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$	$\frac{1}{2} m$
$y = \pm \sqrt{40}$ or $\pm 2\sqrt{10}$	$\frac{1}{2} m$
5. $a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma$	$\frac{1}{2} m$
= $2 a^2$	$\frac{1}{2} m$
6.    using $\sin \theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a}  \cdot  \vec{b} }$	$\frac{1}{2} m$
$\Rightarrow \theta = 0^\circ$	$\frac{1}{2} m$

**SECTION - B**

7.  $[15000 \quad 15000] \begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800]$       2 m

$$\Rightarrow 300 + 150x = 1800 \quad 1 \text{ m}$$

$$\Rightarrow x = 10\% \quad 1 \text{ m}$$

yes : compassionate or any other relevant value 1 m

8.  $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \quad 1\frac{1}{2} \text{ m}$

and  $\tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}} \quad 1\frac{1}{2} \text{ m}$

$$\therefore \sin \left( \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \right) = \cos \left( \cos^{-1} \frac{1}{\sqrt{1+x^2}} \right)$$

$$\Rightarrow 1+x^2 + 2x + 1 = 1+x^2 \Rightarrow x = -\frac{1}{2} \quad 1 \text{ m}$$

OR

$$2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31}$$

$$= 2 \tan^{-1} \frac{3}{4} - \tan^{-1} \frac{17}{31} \quad 1 \text{ m}$$

$$= \tan^{-1} \frac{24}{7} - \tan^{-1} \frac{17}{31} \quad 1 \text{ m}$$

$$= \tan^{-1} 1 = \frac{\pi}{4} \quad 1+1 \text{ m}$$

9.  $C_1 \rightarrow C_1 + C_2 + C_3,$

$$(a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$R_2 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because a+b+c \neq 0) \quad 2 \text{ m}$$

$$\Rightarrow -a^2 - b^2 - c^2 + ab + bc + ca = 0 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow -\frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow a = b = c$$

10.  $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A \quad 1 \text{ m}$

$$R_2 \rightarrow R_2 - 2R_1,$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_2 \rightarrow R_2 - 3R_3$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_1 \rightarrow R_1 + R_2, \quad R_3 \rightarrow R_3 - 2R_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A \quad (2 \text{ marks for all operations})$$

$$\therefore A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} \quad 1 \text{ m}$$

$$11. \quad f(x) = x - |x - x^2| = |x - x(1-x)| = \begin{cases} 2x - x^2 & , \quad -1 \leq x < 0 \\ 0 & , \quad x = 0 \\ x^2 & , \quad 0 < x \leq 1 \end{cases} \quad 1 \text{ m}$$

$f(x)$  being a polynomial is continuous on  $[-1, 0] \cup [0, 1]$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2x - x^2) = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0$$

Also,  $f(0) = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x) \quad 1 \text{ m}$$

⇒ There is no point of discontinuity on  $[-1, 1]$  1 m

12.  $\frac{y}{x} = [\log x - \log(a + bx)]$  ½ m

$$\Rightarrow \frac{x \frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a + bx} \quad 1 \text{ m}$$

Differentiating again,

$$x \frac{d^2y}{dx^2} = \frac{a^2}{(a + b x)^2} \quad 1 \text{ m}$$

$$x^3 \cdot \frac{d^2y}{dx^2} = \left( \frac{ax}{a+bx} \right)^2 = \left( x \frac{dy}{dx} - y \right)^2 \text{ (using (i))}$$

$$13. \quad u = \sec^{-1} \left( \frac{1}{2x^2 - 1} \right) = 2 \cos^{-1} x \Rightarrow \frac{du}{dx} \frac{-2}{\sqrt{1-x^2}} \quad 1\frac{1}{2} \text{ m}$$

$$v = \sqrt{1-x^2} \Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \quad 1 \text{ m}$$

$$\frac{dv}{dx} \Big|_{x=\frac{1}{2}} = \frac{2}{x} = 4 \quad 1\frac{1}{2} \text{ m}$$

14. Let  $I = \int_0^{\pi/2} \frac{5 \sin x + 3 \cos x}{\sin x + \cos x} dx \dots \dots \dots \text{(i)}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{5 \cos x + 3 \sin x}{\cos x + \sin x} dx \dots\dots \text{(ii)} \quad \left( \because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right) \quad 1\frac{1}{2} m$$

Adding (i) and (ii)  $1+1 \text{ m}$

$$2 I = 8 \int_0^{\pi/2} 1 \cdot dx = 4\pi$$

$$\Rightarrow I = 2\pi$$

OR

$$\text{put } \log x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt \quad 1 \text{ m}$$

$$= \int e^t \left( \log t + \frac{1}{t^2} \right) dt$$

$$= \int e^t \left[ \left( \log t - \frac{1}{t} \right) + \left( \frac{1}{t} + \frac{1}{t^2} \right) \right] dt$$

$$= e^t \left( \log t - \frac{1}{t} \right) + c$$

$$= x \left[ \log(\log x) - \frac{1}{\log x} \right] + c$$

15.  $I = \int \frac{x \cos x}{\cos x + x \sin x} dx$  1 m

$$\text{put } \cos x + x \sin x = t$$

$$\Rightarrow x \cos x \, dx = dt$$

$$= \int \frac{dt}{t} \quad 1 \text{ m}$$

$$= \log |\cos x + x \sin x| + c \quad 1 \text{ m}$$

16. 
$$\int \frac{x^4 dx}{(x-1)(x^2+1)} = \int \left[ (x+1) + \frac{1}{(x-1)(x^2+1)} \right] dx$$

(using partial fractions)

$$= \int (x+1) dx + \frac{1}{2} \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{x+1}{x^2+1} dx$$

$$= \frac{x^2}{2} + x + \frac{1}{2} \log|x-1| - \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + c$$

17. 
$$\begin{aligned} \overrightarrow{AB} &= -2\hat{i} - 5\hat{k} \\ \overrightarrow{AC} &= \hat{i} - 2\hat{j} - \hat{k} \end{aligned} \quad \left. \right\}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$$

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{165}$$

$$\hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|}$$

$$= \frac{(-10\hat{i} - 7\hat{j} + 4\hat{k})}{\sqrt{165}} \text{ or } \frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}}$$

18. 
$$\begin{aligned} \vec{a}_1 &= -\hat{i}, \quad \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k} \\ \vec{a}_2 &= -2\hat{j} + \hat{k}, \quad \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k} \end{aligned} \quad \left. \right\}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \frac{7}{12}$$

$$\text{S.D.} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = 2$$

OR

Foot of perpendicular are  $(0, b, c)$  &  $(a, 0, c)$  1 m  
 Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0 \quad \text{2 m}$$

$$\Rightarrow bcx + acy - abz = 0 \quad \text{1 m}$$

$$19. \quad p(x=2) = 9 \cdot P(x=3) \quad \text{1 m}$$

$$\Rightarrow {}^3C_2 p^2 q = 9 \cdot {}^3C_3 p^3 \cdot q^0 \quad \text{1 m}$$

$$\Rightarrow 3p^2(1-p) = 9p^3 \quad \text{1 m}$$

$$\Rightarrow p = \frac{1}{4} \quad \text{1 m}$$

OR

Let  $H_1$  be the event that red ball is drawn

$H_2$  be the event that black ball is drawn

$E$  be the event that both balls are red

$$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8} \quad \text{1 m}$$

$$P(E/H_1) = \frac{5_{C_2}}{10_{C_2}} = \frac{2}{9}, \quad P(E/H_2) = \frac{3_{C_2}}{10_{C_2}} = \frac{1}{15} \quad \text{1 m}$$

$$P(E) = P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2) \quad \text{1 m}$$

$$= \frac{3}{8} \cdot \frac{2}{9} + \frac{5}{8} \cdot \frac{1}{15} = \frac{1}{8} \quad \text{1 m}$$

20.

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

4 m

$\forall a \in \{0, 1, 2, 3, 4, 5, 6\}$

$$a * 0 = a = 0 * a \Rightarrow 0 \text{ is identity}$$

1 m

$$\forall a \in \{1, 2, 3, 4, 5, 6\}$$

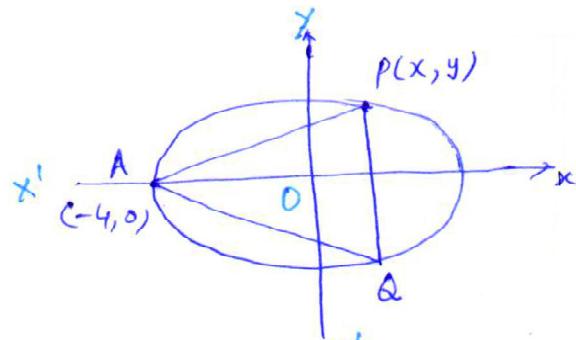
$$a * b = 0 = b * a$$

$$\Rightarrow a * (7 - a) = 0 = (7 - a) * a$$

$\Rightarrow$  (7 - a) is inverse of a

1 m

$$21. \quad A = y(x + 4)$$



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

1 m

$$= \frac{9}{16} (4 - x) (4 + x)^3$$

1 m

$$\frac{dz}{dx} = \frac{9}{16} (4+x)^2 (8-4x)$$

1 m

$$\frac{dz}{dx} = 0 \Rightarrow x = 2$$

1 m

$$\frac{d^2z}{dx^2} = -\frac{9}{4}(4+x)^2 + \frac{9}{8}(4+x)(8-4x)$$

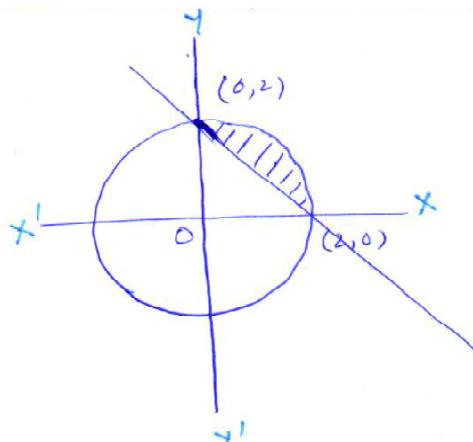
$$\left. \frac{d^2z}{dx^2} \right|_{x=2} < 0$$

1 m

∴ Maximum value of A =  $9\sqrt{3}$  sq. units

1 m

22.



1 m

Required Area

$$= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx$$

2 m

$$= \left[ \frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 2x - \frac{x^2}{2} \right]_0^2$$

1+1 m

$$= (\pi - 2) \text{ sq. units}$$

1 m

23.  $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

put  $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$  1 m

$$\Rightarrow \frac{1+v^2}{v^3} = -\frac{dx}{x} \quad 1 \text{ m}$$

Integrating both sides

$$-\frac{1}{2v^2} + \log v = -\log x + c \quad 1 \text{ m}$$

$$\Rightarrow -\frac{x^2}{2y^2} + \log y = c \quad 1 \text{ m}$$

when  $x = 1, y = 1 \Rightarrow c = -\frac{1}{2}$  1 m

$$\Rightarrow \log y = \frac{x^2 - y^2}{2y^2} \quad \frac{1}{2} \text{ m}$$

when  $x = x_0, y = e \Rightarrow x_0 = \sqrt{3}e \quad 1\frac{1}{2} \text{ m}$

OR

$$I.F = e^{\int \tan x dx} = e^{\log \sec x} = \sec x \quad 1 \text{ m}$$

$$\therefore \frac{d}{dx} (y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x \quad 1 \text{ m}$$

$$\Rightarrow y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x dx + c \quad 2 \text{ m}$$

$$\Rightarrow y = x^3 + c \cos x$$

$$\text{when } x = \frac{\pi}{3}, y = 0; \text{ we get } c = \frac{-2\pi^3}{27} \quad 1 \text{ m}$$

$$\therefore y = x^3 - \frac{2\pi^3}{27} \cos x \quad 1 \text{ m}$$

24. Equation of line is  $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$  1 m

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0 \quad 1 \text{ m}$$

$\Rightarrow 2x + y + z - 7 = 0 \dots \dots \dots \text{(i)}$  1 m

general point on given line  $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$  lies on (i) 1 m

$$\therefore 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\frac{2}{3} \quad 1 \text{ m}$$

$$\therefore \text{Point of intersection } \left(\frac{5}{3}, 6, -\frac{7}{3}\right) \quad 1 \text{ m}$$

25. Let  $H_1$  : be the event 1, 2 appears

$H_2$  : be the event 3, 4, 5, 6 appears 1 m

$E_3$  : be the event that head appears

$$P(H_1) = \frac{2}{6} = \frac{1}{3}, \quad P(H_2) = \frac{4}{6} = \frac{2}{3} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{3}{8} \quad P(E/H_2) = \frac{1}{2} \quad 1 \text{ m}$$

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)} \quad 1 \text{ m}$$

$$= \frac{8}{11} \quad 2 \text{ m}$$

OR

Let  $H_1$  : be the event that 4 occurs

$H_2$  : be the event that 4 does not occurs 1 m

$E$  : be the event that man reports 4 occurs  
on a throw of dice

$$P(H_1) = \frac{1}{6}, \quad P(H_2) = \frac{5}{6} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{3}{5} \quad P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5} \quad 1 \text{ m}$$

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)} \quad 1 \text{ m}$$

$$= \frac{3}{13} \quad 2 \text{ m}$$

26. Let us consider the man invested on x electronic and y manually operated machines

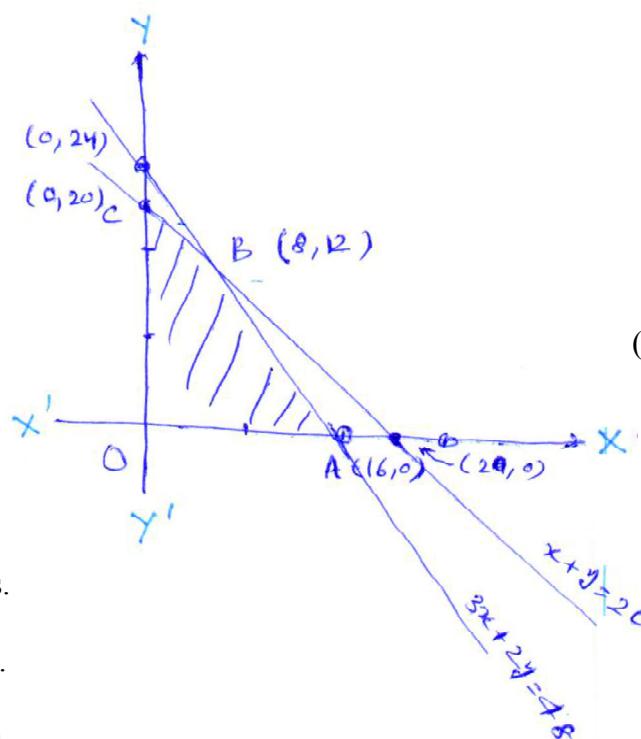
$$\text{Maximise } P = 220x + 180y \dots \text{(i)} \quad 1 \text{ m}$$

subject to

$$x + y \leq 20$$

$$3600x + 2400y \leq 57600 \Rightarrow 3x + 2y \leq 48 \quad 1\frac{1}{2} \text{ m}$$

$$x, y \geq 0$$



$$P|_{A(16,0)} = 3520 \text{ Rs.}$$

$$P|_{B(8,12)} = 3920 \text{ Rs.}$$

$$P|_{C(0,20)} = 3600 \text{ Rs.}$$

Maximum profit is Rs. 3920 at x = 8, y = 12

(1 mark for plotting each line) = 2 m

( $\frac{1}{2}$  to find the vertices of feasible region)

1 m

**QUESTION PAPER CODE 65/2/B**

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

Marks

1.  $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$   $\frac{1}{2}$  m

$$y = \pm \sqrt{40} \text{ or } \pm 2\sqrt{10}  $\frac{1}{2}$  m$$

2. 
$$\begin{aligned} a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma \\ = 2 a^2 \end{aligned}  $\frac{1}{2}$  m$$

3. using  $\sin \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$   $\frac{1}{2}$  m

$$\Rightarrow \theta = 0^\circ  $\frac{1}{2}$  m$$

4.  $x = 2, y = 9$  ( $\frac{1}{2}$  for correct x or y)

$$\therefore x + y = 11  $\frac{1}{2}$  m$$

5. order 3, or degree 1  $\frac{1}{2}$  m

$$\therefore \text{Degree} + \text{order} = 4  $\frac{1}{2}$  m$$

6. 
$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x} \quad (\text{Standard form})  $\frac{1}{2}$  m$$

$$\text{I.F.} = \log x  $\frac{1}{2}$  m$$

**SECTION - B**

7. 
$$\frac{y}{x} = [\log x - \log(a + b x)]  $\frac{1}{2}$  m$$

$$\Rightarrow \frac{x \frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a + b x} 1 m$$

Differentiating again,

$$x \frac{d^2y}{dx^2} = \frac{a^2}{(a + b x)^2} \quad 1 \text{ m}$$

$$x^3 \cdot \frac{d^2y}{dx^2} = \left( \frac{ax}{a+bx} \right)^2 = \left( x \frac{dy}{dx} - y \right)^2 \text{ (using (i))}$$

$$8. \quad u = \sec^{-1} \left( \frac{1}{2x^2 - 1} \right) = 2 \cos^{-1} x \Rightarrow \frac{du}{dx} \frac{-2}{\sqrt{1-x^2}} \quad 1\frac{1}{2} \text{ m}$$

$$v = \sqrt{1-x^2} \Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \quad 1 \text{ m}$$

$$\frac{dv}{dx} \Big|_{x=\frac{1}{2}} = \frac{2}{x} = 4 \quad 1\frac{1}{2} \text{ m}$$

9. Let  $I = \int_0^{\pi/2} \frac{5 \sin x + 3 \cos x}{\sin x + \cos x} dx \dots \dots \dots \text{(i)}$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{5 \cos x + 3 \sin x}{\cos x + \sin x} dx \dots\dots\dots (ii) \quad \left( \because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right) \quad 1\frac{1}{2} m$$

Adding (i) and (ii) 1+1 m

$$2 I = 8 \int_0^{\pi/2} 1 \cdot dx = 4\pi$$

$$\Rightarrow I = 2\pi$$

OR

$$\text{put } \log x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt \quad 1 \text{ m}$$

$$= \int e^t \left( \log t + \frac{1}{t^2} \right) dt$$

$$= \int e^t \left[ \left( \log t - \frac{1}{t} \right) + \left( \frac{1}{t} + \frac{1}{t^2} \right) \right] dt \quad 1\frac{1}{2} m$$

$$= e^t \left( \log t - \frac{1}{t} \right) + c \quad 1 m$$

$$= x \left[ \log(\log x) - \frac{1}{\log x} \right] + c \quad \frac{1}{2} m$$

10.  $[15000 \quad 15000] \begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800] \quad 2 m$

$$\Rightarrow 300 + 150x = 1800 \quad 1 m$$

$$\Rightarrow x = 10\% \quad 1 m$$

yes : compassionate or any other relevant value 1 m

11.  $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \quad 1\frac{1}{2} m$

and  $\tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}} \quad 1\frac{1}{2} m$

$$\therefore \sin \left( \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \right) = \cos \left( \cos^{-1} \frac{1}{\sqrt{1+x^2}} \right)$$

$$\Rightarrow 1+x^2 + 2x + 1 = 1+x^2 \Rightarrow x = -\frac{1}{2} \quad 1 m$$

OR

$$2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31}$$

$$= 2 \tan^{-1} \frac{3}{4} - \tan^{-1} \frac{17}{31} \quad 1 m$$

$$= \tan^{-1} \frac{24}{7} - \tan^{-1} \frac{17}{31} \quad 1 \text{ m}$$

$$= \tan^{-1} 1 = \frac{\pi}{4} \quad 1+1 \text{ m}$$

12.  $C_1 \rightarrow C_1 + C_2 + C_3 ,$

$$(a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$R_2 \rightarrow R_2 - R_1 , \quad R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because a+b+c \neq 0) \quad 2 \text{ m}$$

$$\Rightarrow -a^2 - b^2 - c^2 + ab + bc + ca = 0 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow -\frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow a = b = c$$

13.  $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A \quad 1 \text{ m}$

$$R_2 \rightarrow R_2 - 2R_1 ,$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_2 \rightarrow R_2 - 3R_3$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_1 \rightarrow R_1 + R_2, \quad R_3 \rightarrow R_3 - 2R_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A \quad (2 \text{ marks for all operations})$$

$$\therefore A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} \quad 1 \text{ m}$$

14.  $f(x) = x - |x - x^2| = |x - x(1-x)| = \begin{cases} 2x - x^2, & -1 \leq x < 0 \\ 0, & x = 0 \\ x^2, & 0 < x \leq 1 \end{cases}$  1 m

$f(x)$  being a polynomial is continuous on  $[-1, 0] \cup [0, 1]$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2x - x^2) = 0 \quad \frac{1}{2} \text{ m}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0 \quad \frac{1}{2} \text{ m}$$

$$\text{Also, } f(0) = 0$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x) \quad 1 \text{ m}$$

$\Rightarrow$  There is no point of discontinuity on  $[-1, 1]$  1 m

15. 
$$\left. \begin{array}{l} \vec{a}_1 = -\hat{i}, \quad \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k} \\ \vec{a}_2 = -2\hat{j} + \hat{k}, \quad \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k} \end{array} \right\} \quad 1 \text{ m}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k} \quad \frac{1}{2} \text{ m}$$

$$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \quad \frac{1}{2} \text{ m}$$

$$\left| \vec{b}_1 \times \vec{b}_2 \right| = \frac{7}{12} \quad 1 \text{ m}$$

$$\text{S.D.} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{\left| \vec{b}_1 \times \vec{b}_2 \right|} \right| = 2 \quad 1 \text{ m}$$

OR

Foot of perpendicular are  $(0, b, c)$  &  $(a, 0, c)$  1 m  
Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0 \quad 2 \text{ m}$$

$$\Rightarrow bcx + acy - abz = 0 \quad 1 \text{ m}$$

$$16. \quad p(x=2) = 9 \cdot P(x=3) \quad 1 \text{ m}$$

$$\Rightarrow {}^3C_2 p^2 q = 9 \cdot {}^3C_3 p^3 \cdot q^0 \quad 1 \text{ m}$$

$$\Rightarrow 3p^2(1-p) = 9p^3 \quad 1 \text{ m}$$

$$\Rightarrow p = \frac{1}{4} \quad 1 \text{ m}$$

OR

Let  $H_1$  be the event that red ball is drawn

$H_2$  be the event that black ball is drawn

$E$  be the event that both balls are red

$$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{5_{C_2}}{10_{C_2}} = \frac{2}{9}, \quad P(E/H_2) = \frac{3_{C_2}}{10_{C_2}} = \frac{1}{15} \quad 1 \text{ m}$$

$$P(E) = P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2) \quad 1 \text{ m}$$

$$= \frac{3}{8} \cdot \frac{2}{9} + \frac{5}{8} \cdot \frac{1}{15} = \frac{1}{8}$$
1 m

17.  $I = \int \frac{x \cos x}{\cos x + x \sin x} dx$

1 m

put  $\cos x + x \sin x = t$   
 $\Rightarrow x \cos x dx = dt$

1 m

$$= \int \frac{dt}{t}$$
1 m

$$= \log |\cos x + x \sin x| + c$$
1 m

18.  $\int \frac{x^4 dx}{(x-1)(x^2+1)} = \int \left[ (x+1) + \frac{1}{(x-1)(x^2+1)} \right] dx$

1 m

(using partial fractions)

$$= \int (x+1) dx + \frac{1}{2} \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{x+1}{x^2+1} dx$$
1½ m

$$= \frac{x^2}{2} + x + \frac{1}{2} \log |x-1| - \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + c$$
1½ m

19.  $\overrightarrow{AB} = -2\hat{i} - 5\hat{k}$

$$\overrightarrow{AC} = \hat{i} - 2\hat{j} - \hat{k}$$
1 m

$$\overrightarrow{AB} \times \overrightarrow{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$$
1 m

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{165}$$
½ m

$$\hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|}$$
1 m

$$= \frac{(-10\hat{i} - 7\hat{j} + 4\hat{k})}{\sqrt{165}} \text{ or } \frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}}$$
½ m

### SECTION - C

20.  $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

put  $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

1 m

$$\Rightarrow \frac{1+v^2}{v^3} = -\frac{dx}{x}$$

1 m

Integrating both sides

$$-\frac{1}{2v^2} + \log v = -\log x + c$$

1 m

$$\Rightarrow -\frac{x^2}{2y^2} + \log y = c$$

when  $x = 1, y = 1 \Rightarrow c = -\frac{1}{2}$

1 m

$$\Rightarrow \log y = \frac{x^2 - y^2}{2y^2}$$

½ m

when  $x = x_0, y = e \Rightarrow x_0 = \sqrt{3}e$

1½ m

OR

$$I.F = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$$

1 m

$$\therefore \frac{d}{dx} (y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x$$

1 m

$$\Rightarrow y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x dx + c$$

2 m

$$\Rightarrow y = x^3 + c \cos x$$

when  $x = \frac{\pi}{3}, y = 0; \text{ we get } c = \frac{-2\pi^3}{27}$

1 m

$$\therefore y = x^3 - \frac{2\pi^3}{27} \cos x \quad 1 \text{ m}$$

21. Equation of line is  $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} \quad 1 \text{ m}$

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$\Rightarrow 2x + y + z - 7 = 0 \dots \text{(i)} \quad 1 \text{ m}$$

general point on given line  $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$  lies on (i)  $1 \text{ m}$

$$\therefore 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\frac{2}{3} \quad 1 \text{ m}$$

$$\therefore \text{Point of intersection } \left(\frac{5}{3}, 6, -\frac{7}{3}\right) \quad 1 \text{ m}$$

22.

*	0	1	2	3	4	5	6	
0	0	1	2	3	4	5	6	
1	1	2	3	4	5	6	0	
2	2	3	4	5	6	0	1	
3	3	4	5	6	0	1	2	
4	4	5	6	0	1	2	3	
5	5	6	0	1	2	3	4	
6	6	0	1	2	3	4	5	

4 m

$$\forall a \in \{0, 1, 2, 3, 4, 5, 6\}$$

$$a * 0 = a = 0 * a \Rightarrow 0 \text{ is identity} \quad 1 \text{ m}$$

$$\forall a \in \{1, 2, 3, 4, 5, 6\}$$

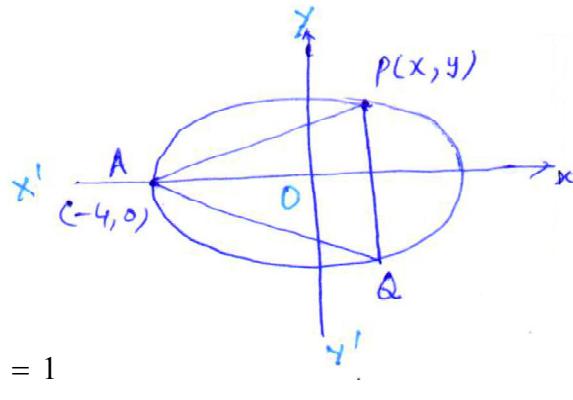
$$a * b = 0 = b * a$$

$$\Rightarrow a * (7-a) = 0 = (7-a) * a$$

$\Rightarrow (7-a)$  is inverse of  $a$

1 m

23.  $A = y(x+4)$



$$\text{Let } z = A^2 = \frac{9}{16} (16 - x^2) (x + 4)^2 \Rightarrow y^2 = \frac{9}{16} (16 - x^2) \dots\dots\dots \text{(i)}$$

1 m

$$= \frac{9}{16} (4 - x) (4 + x)^3$$

1 m

$$\frac{dz}{dx} = \frac{9}{16} (4 + x)^2 (8 - 4x)$$

1 m

$$\frac{dz}{dx} = 0 \Rightarrow x = 2$$

1 m

$$\frac{d^2z}{dx^2} = -\frac{9}{4} (4 + x)^2 + \frac{9}{8} (4 + x) (8 - 4x)$$

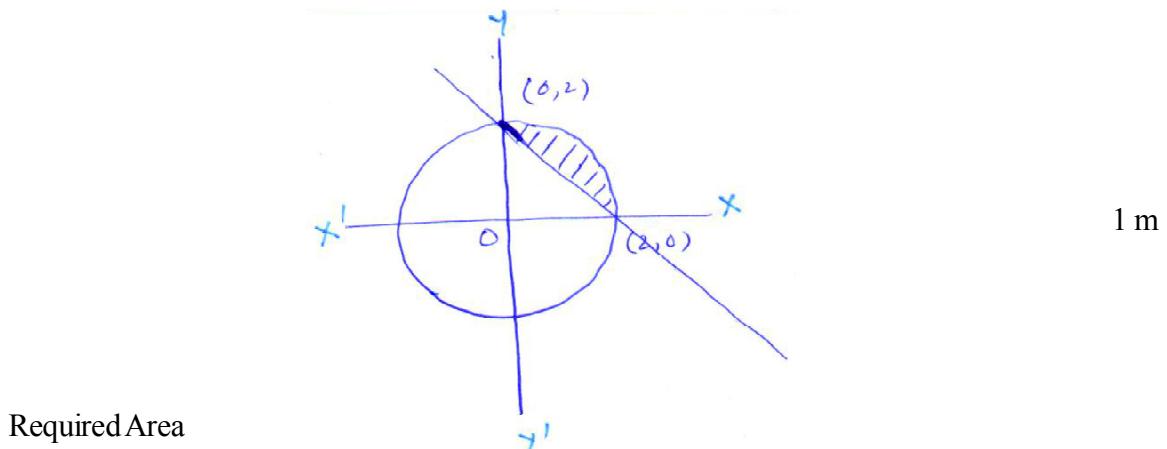
$$\left. \frac{d^2z}{dx^2} \right|_{x=2} < 0$$

1 m

$$\therefore \text{Maximum value of } A = 9\sqrt{3} \text{ sq. units}$$

1 m

24.



$$= \int_0^2 \sqrt{4 - x^2} dx - \int_0^2 (2 - x) dx \quad 2 \text{ m}$$

$$= \left[ \frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 2x - \frac{x^2}{2} \right]_0^2 \quad 1+1 \text{ m}$$

$$= (\pi - 2) \text{ sq. units} \quad 1 \text{ m}$$

25. Let us consider the man invested on x

electronic and y manually operated machines

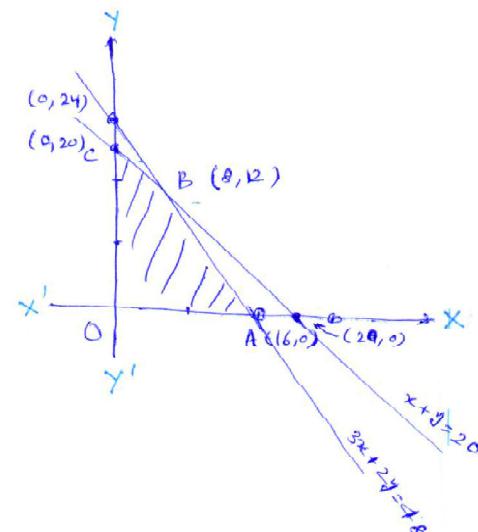
$$\text{Maximise } P = 220x + 180y \dots \text{(i)} \quad 1 \text{ m}$$

subject to

$$x + y \leq 20$$

$$3600x + 2400y \leq 57600 \Rightarrow 3x + 2y \leq 48 \quad 1\frac{1}{2} \text{ m}$$

$$x, y \geq 0$$



(1 mark for  
plotting each  
line) = 2 m

( $\frac{1}{2}$  to find the vertices  
of feasible  
region)

$$P \Big|_{A(16, 0)} = 3520 \text{ Rs.}$$

$$P \Big|_{B(8, 12)} = 3920 \text{ Rs.}$$

$$P \Big|_{C(0, 20)} = 3600 \text{ Rs.}$$

Maximum profit is Rs. 3920 at x = 8, y = 12

1 m

26. Let  $H_1$  : be the event 1, 2 appears

$H_2$  : be the event 3, 4, 5, 6 appears

1 m

$E_3$  : be the event that head appears

$$P(H_1) = \frac{2}{6} = \frac{1}{3}, \quad P(H_2) = \frac{4}{6} = \frac{2}{3}$$

1 m

$$P(E/H_1) = \frac{3}{8} \quad P(E/H_2) = \frac{1}{2}$$

1 m

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)}$$

1 m

$$= \frac{8}{11}$$

2 m

OR

Let  $H_1$  : be the event that 4 occurs

$H_2$  : be the event that 4 does not occur

1 m

$E$  : be the event that man reports 4 occurs

on a throw of dice

$$P(H_1) = \frac{1}{6}, \quad P(H_2) = \frac{5}{6}$$

1 m

$$P(E/H_1) = \frac{3}{5} \quad P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5}$$

1 m

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)}$$

1 m

$$= \frac{3}{13}$$

2 m

**QUESTION PAPER CODE 65/3/B**

**EXPECTED ANSWERS/VALUE POINTS**

**SECTION - A**

Marks

1.  $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$  (Standard form)  $\frac{1}{2}$  m

I.F. =  $\log x$   $\frac{1}{2}$  m

2.  $x = 2, y = 9$  ( $\frac{1}{2}$  for correct x or y)  
 $\therefore x + y = 11$   $\frac{1}{2}$  m

3. order 3, or degree 1  $\frac{1}{2}$  m  
 $\therefore$  Degree + order = 4  $\frac{1}{2}$  m

4. using  $\sin \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$   $\frac{1}{2}$  m

$\Rightarrow \theta = 0^\circ$   $\frac{1}{2}$  m

5.  $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$   $\frac{1}{2}$  m

$y = \pm \sqrt{40}$  or  $\pm 2\sqrt{10}$   $\frac{1}{2}$  m

6.  $a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma$   $\frac{1}{2}$  m  
 $= 2 a^2$   $\frac{1}{2}$  m

**SECTION - B**

7. 
$$\begin{aligned} \overrightarrow{AB} &= -2\hat{i} - 5\hat{k} \\ \overrightarrow{AC} &= \hat{i} - 2\hat{j} - \hat{k} \end{aligned} \quad \left. \right\}$$
 1 m

$\overrightarrow{AB} \times \overrightarrow{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$  1 m

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{165}$   $\frac{1}{2}$  m

$$\hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|} \quad 1\text{ m}$$

$$= \frac{(-10\hat{i} - 7\hat{j} + 4\hat{k})}{\sqrt{165}} \text{ or } \frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}} \quad \frac{1}{2}\text{ m}$$

$$8. \quad \left. \begin{array}{l} \vec{a}_1 = -\hat{i}, \quad \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k} \\ \vec{a}_2 = -2\hat{j} + \hat{k}, \quad \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k} \end{array} \right\} \quad 1\text{ m}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k} \quad \frac{1}{2}\text{ m}$$

$$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \quad \frac{1}{2}\text{ m}$$

$$\left| \vec{b}_1 \times \vec{b}_2 \right| = \frac{7}{12} \quad 1\text{ m}$$

$$\text{S.D.} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{\left| \vec{b}_1 \times \vec{b}_2 \right|} \right| = 2 \quad 1\text{ m}$$

OR

Foot of perpendicular are  $(0, b, c)$  &  $(a, 0, c)$  1 m  
 Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0 \quad 2\text{ m}$$

$$\Rightarrow bcx + acy - abz = 0 \quad 1\text{ m}$$

$$9. \quad p(x=2) = 9 \cdot P(x=3) \quad 1\text{ m}$$

$$\Rightarrow {}^3C_2 p^2 q = 9 \cdot {}^3C_3 p^3 \cdot q^0 \quad 1\text{ m}$$

$$\Rightarrow 3p^2(1-p) = 9p^3 \quad 1\text{ m}$$

$$\Rightarrow p = \frac{1}{4} \quad 1 \text{ m}$$

OR

Let  $H_1$  be the event that red ball is drawn

$H_2$  be the event that black ball is drawn

E be the event that both balls are red

$$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{5_{C_2}}{10_{C_2}} = \frac{2}{9}, \quad P(E/H_2) = \frac{3_{C_2}}{10_{C_2}} = \frac{1}{15} \quad 1 \text{ m}$$

$$P(E) = P(H_1) \cdot P(E|H_1) + P(H_2) \cdot P(E|H_2) \quad 1 \text{ m}$$

$$= \frac{3}{8} \cdot \frac{2}{9} + \frac{5}{8} \cdot \frac{1}{15} = \frac{1}{8} \quad 1 \text{ m}$$

$$10. \quad \frac{y}{x} = [\log x - \log (a + b x)] \quad \frac{1}{2} m$$

$$\Rightarrow \frac{x \frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a + bx} \quad 1 \text{ m}$$

Differentiating again,

$$x \frac{d^2y}{dx^2} = \frac{a^2}{(a + b x)^2} \quad 1 \text{ m}$$

$$x^3 \cdot \frac{d^2y}{dx^2} = \left( \frac{ax}{a+bx} \right)^2 = \left( x \frac{dy}{dx} - y \right)^2 \text{ (using (i))}$$

11.  $u = \sec^{-1} \left( \frac{1}{2x^2 - 1} \right) = 2 \cos^{-1} x \Rightarrow \frac{du}{dx} \frac{-2}{\sqrt{1-x^2}}$  1½ m

$$v = \sqrt{1-x^2} \Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \quad \text{1 m}$$

$$\frac{dv}{dx} \Big|_{x=\frac{1}{2}} = \frac{2}{x} = 4 \quad \text{1½ m}$$

12. Let  $I = \int_0^{\pi/2} \frac{5 \sin x + 3 \cos x}{\sin x + \cos x} dx \dots \dots \dots \text{(i)}$

$$\Rightarrow I = \int_0^{\pi/2} \frac{5 \cos x + 3 \sin x}{\cos x + \sin x} dx \dots \dots \text{(ii)} \quad \left( \because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right) \quad \text{1½ m}$$

Adding (i) and (ii) 1+1 m

$$2I = 8 \int_0^{\pi/2} 1 \cdot dx = 4\pi \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow I = 2\pi$$

OR

$$\text{put } \log x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt \quad \text{1 m}$$

$$= \int e^t \left( \log t + \frac{1}{t^2} \right) dt$$

$$= \int e^t \left[ \left( \log t - \frac{1}{t} \right) + \left( \frac{1}{t} + \frac{1}{t^2} \right) \right] dt \quad \text{1½ m}$$

$$= e^t \left( \log t - \frac{1}{t} \right) + c \quad \text{1 m}$$

$$= x \left[ \log(\log x) - \frac{1}{\log x} \right] + c \quad \frac{1}{2} \text{ m}$$

13.  $I = \int \frac{x \cos x}{\cos x + x \sin x} dx$  1 m

put  $\cos x + x \sin x = t$

$$\Rightarrow x \cos x dx = dt$$

1 m

$$= \int \frac{dt}{t}$$

1 m

$$= \log |\cos x + x \sin x| + c$$

1 m

14.  $\int \frac{x^4 dx}{(x-1)(x^2+1)} = \int \left[ (x+1) + \frac{1}{(x-1)(x^2+1)} \right] dx$  1 m

(using partial fractions)

$$= \int (x+1) dx + \frac{1}{2} \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{x+1}{x^2+1} dx$$

1½ m

$$= \frac{x^2}{2} + x + \frac{1}{2} \log|x-1| - \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + c$$

1½ m

15.  $[15000 \quad 15000] \begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800]$  2 m

$$\Rightarrow 300 + 150x = 1800$$

$$\Rightarrow x = 10\%$$

yes : compassionate or any other relevant value

1 m

16.  $\cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}$  1½ m

and  $\tan^{-1}x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$  1½ m

$$\therefore \sin\left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1} \frac{1}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow 1+x^2 + 2x + 1 = 1+x^2 \Rightarrow x = -\frac{1}{2}$$
1 m

OR

$$2\sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31}$$

$$= 2\tan^{-1} \frac{3}{4} - \tan^{-1} \frac{17}{31}$$
1 m

$$= \tan^{-1} \frac{24}{7} - \tan^{-1} \frac{17}{31}$$
1 m

$$= \tan^{-1} 1 = \frac{\pi}{4}$$
1+1 m

$$17. \quad C_1 \rightarrow C_1 + C_2 + C_3 ,$$

$$(a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$
1 m

$$R_2 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because a+b+c \neq 0)$$
2 m

$$\Rightarrow -a^2 - b^2 - c^2 + ab + bc + ca = 0$$
½ m

$$\Rightarrow -\frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$
½ m

$$\Rightarrow a = b = c$$

18.  $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$  1 m

$$R_2 \rightarrow R_2 - 2R_1,$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_2 \rightarrow R_2 - 3R_3$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_1 \rightarrow R_1 + R_2, \quad R_3 \rightarrow R_3 - 2R_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A \quad (2 \text{ marks for all operations})$$

$$\therefore A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} \quad 1 \text{ m}$$

19.  $f(x) = x - |x - x^2| = |x - x(1-x)| = \begin{cases} 2x - x^2, & -1 \leq x < 0 \\ 0, & x = 0 \\ x^2, & 0 < x \leq 1 \end{cases}$  1 m

$f(x)$  being a polynomial is continuous on  $[-1, 0] \cup [0, 1]$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2x - x^2) = 0 \quad \frac{1}{2} \text{ m}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0 \quad \frac{1}{2} \text{ m}$$

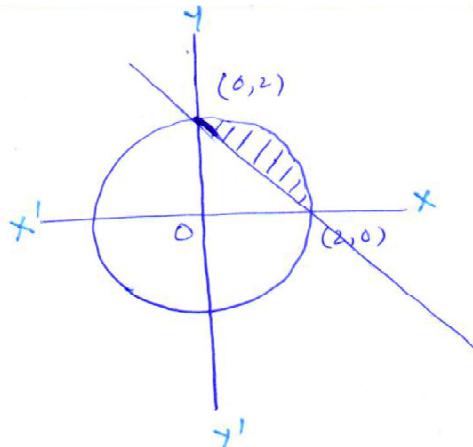
Also,  $f(0) = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x) \quad 1 \text{ m}$$

$\Rightarrow$  There is no point of discontinuity on  $[-1, 1]$  1 m

### SECTION - C

20.



1 m

Required Area

$$= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx \quad 2 \text{ m}$$

$$= \left[ \frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 2x - \frac{x^2}{2} \right]_0^2 \quad 1+1 \text{ m}$$

$$= (\pi - 2) \text{ sq. units} \quad 1 \text{ m}$$

21.  $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

$$\text{put } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \quad 1 \text{ m}$$

$$\Rightarrow \frac{1+v^2}{v^3} = - \frac{dx}{x} \quad 1 \text{ m}$$

Integrating both sides

$$-\frac{1}{2v^2} + \log v = -\log x + c \quad 1 \text{ m}$$

$$\Rightarrow -\frac{x^2}{2y^2} + \log y = c$$

$$\text{when } x = 1, y = 1 \Rightarrow c = -\frac{1}{2}$$

$$\Rightarrow \log y = \frac{x^2 - y^2}{2y^2}$$

$$\text{when } x = x_0, y = e \quad \Rightarrow \quad x_0 = \sqrt{3} e$$

1 m

1/2 m

**1½ m**

OR

$$I \int F = e^{\int \tan x \, dx} \equiv e^{\log \sec x} \equiv \sec x$$

1 m

$$\therefore \frac{d}{dx} (y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x$$

1 m

$$\Rightarrow y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x dx + C$$

$$\Rightarrow y = x^3 + c \cos x$$

when  $x = \frac{\pi}{3}$ ,  $y = 0$ ; we get  $c = \frac{-2\pi^3}{27}$

1 m

$$\therefore y = x^3 - \frac{2\pi^3}{27} \cos x$$

1 m

22. Let us consider the man invested on x electronic and y manually operated machine

1 m

subject to

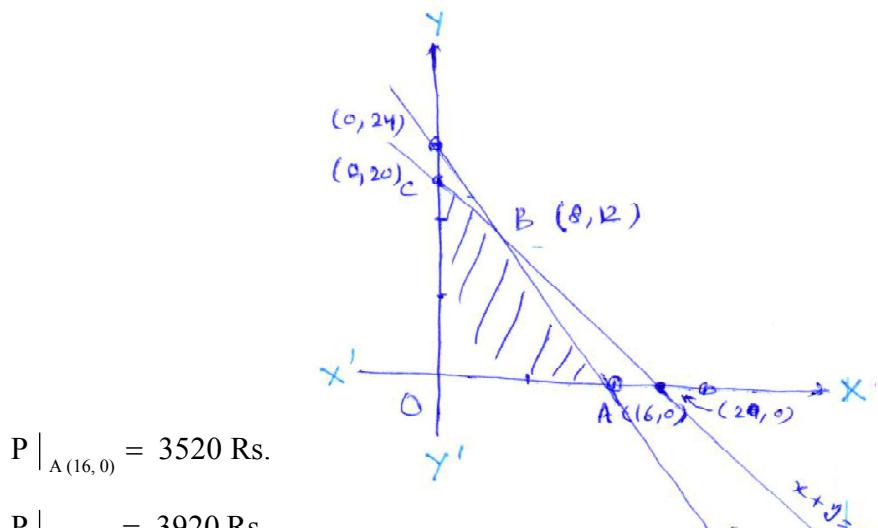
$$x + y \leq 20$$

$$3600x + 2400y < 57600 \Rightarrow 3x + 2y < 48$$

1½ m

$$x, y \geq 0$$

(1 mark for



plotting each  
line) = 2 m

( $\frac{1}{2}$  to find the vertices  
of feasible  
region)

1 m

23. Equation of line is  $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$  1 m

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0 \quad 1 \text{ m}$$

$\Rightarrow 2x + y + z - 7 = 0 \dots \text{(i)}$  1 m

general point on given line  $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$  lies on (i) 1 m

$\therefore 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\frac{2}{3}$  1 m

$\therefore \text{Point of intersection } \left(\frac{5}{3}, 6, -\frac{7}{3}\right)$  1 m

24. Let  $H_1$  : be the event 1, 2 appears 1 m

$H_2$  : be the event 3, 4, 5, 6 appears

$E_3$  : be the event that head appears

$$P(H_1) = \frac{2}{6} = \frac{1}{3}, \quad P(H_2) = \frac{4}{6} = \frac{2}{3} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{3}{8} \quad P(E/H_2) = \frac{1}{2} \quad 1 \text{ m}$$

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)} \quad 1 \text{ m}$$

$$= \frac{8}{11} \quad 2 \text{ m}$$

OR

Let  $H_1$  : be the event that 4 occurs

$H_2$  : be the event that 4 does not occurs 1 m

$E$  : be the event that man reports 4 occurs  
on a throw of dice

$$P(H_1) = \frac{1}{6}, \quad P(H_2) = \frac{5}{6} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{3}{5} \quad P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5} \quad 1 \text{ m}$$

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)} \quad 1 \text{ m}$$

$$= \frac{3}{13} \quad 2 \text{ m}$$

25.

*	0	1	2	3	4	5	6	
0	0	1	2	3	4	5	6	
1	1	2	3	4	5	6	0	
2	2	3	4	5	6	0	1	
3	3	4	5	6	0	1	2	
4	4	5	6	0	1	2	3	
5	5	6	0	1	2	3	4	
6	6	0	1	2	3	4	5	

4 m

$$\forall a \in \{0, 1, 2, 3, 4, 5, 6\}$$

$a * 0 = a = 0 * a \Rightarrow 0$  is identity

1 m

$$\forall a \in \{1, 2, 3, 4, 5, 6\}$$

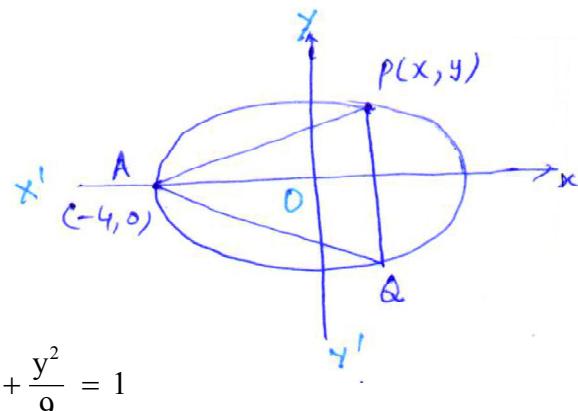
$$a * b = 0 = b * a$$

$$\Rightarrow a * (7-a) = 0 = (7-a) * a$$

$\Rightarrow (7-a)$  is inverse of  $a$

1 m

26.  $A = y(x+4)$



$$\text{Let } z = A^2 = \frac{9}{16}(16-x^2)(x+4)^2 \Rightarrow y^2 = \frac{9}{16}(16-x^2) \dots\dots\dots (i)$$

1 m

$$= \frac{9}{16}(4-x)(4+x)^3$$

1 m

$$\frac{dz}{dx} = \frac{9}{16}(4+x)^2(8-4x)$$

1 m

$$\frac{dz}{dx} = 0 \Rightarrow x = 2$$

1 m

$$\frac{d^2z}{dx^2} = -\frac{9}{4}(4+x)^2 + \frac{9}{8}(4+x)(8-4x)$$

$$\left. \frac{d^2z}{dx^2} \right|_{x=2} < 0$$

1 m

$\therefore$  Maximum value of  $A = 9\sqrt{3}$  sq. units

1 m