CLASS –XII MATHEMATICS NCERT SOLUTIONS
Integrals
Miscellaneous Exercise

Answers

1. Let I = $\int \frac{1}{x-x^3}\,dx$

Here, $\frac{1}{x-x^3} = \frac{1}{x\left(1-x^2\right)} = \frac{1}{x(1-x)(1+x)} = \frac{A}{x}+\frac{B}{1-x}+\frac{C}{1+x}$ ....(i)

$\Rightarrow \quad 1 = A(1-x)(1+x)+Bx(1+x)+Cx(1-x)$

$\Rightarrow \quad 1 = A\left(1-x^2\right)+B\left(x+x^2\right)+C\left(x-x^2\right)$

$\Rightarrow \quad 1 = A - Ax^2 + Bx + Bx^2 + Cx - Cx^2$

Comparing the coefficients of x^2 $\quad -A + B - C = 0$ ....(ii)

Comparing the coefficients of x $\quad B + C = 0$ ....(iii)

Comparing constants $\quad A = 1$ ....(iv)

On solving eq. (ii), (iii) and (iv), we get $\quad A = 1, B = \frac{1}{2}, C = \frac{-1}{2}$

Putting these values in eq. (i), $\quad \frac{1}{x-x^3} = \frac{1}{x}+\frac{\frac{1}{2}}{1-x}+\frac{\frac{-1}{2}}{1+x}$

$\Rightarrow \quad \int \frac{1}{x-x^3}\,dx = \int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{1}{1-x}\,dx - \frac{1}{2}\int \frac{1}{1+x}\,dx$

$= \log|x| + \frac{1}{2}\frac{\log|1-x|}{-1} - \frac{1}{2}\log|1+x| + c = \frac{1}{2}\left[2\log|x| - \log|1-x| - \log|1+x|\right]+c$

$= \frac{1}{2}\left[\log|x|^2 - \left(\log|1-x| + \log|1+x|\right)\right]+c = \frac{1}{2}\left[\log|x|^2 - \log|1-x||1+x|\right]+c$

$= \frac{1}{2}\left[\log|x|^2 - \log\left|1-x^2\right|\right]+c = \frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right|+c$

2. Let I = $\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}\,dx = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}\,dx$

$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b}\,dx = \int \frac{\sqrt{x+a}-\sqrt{x+b}}{a-b}\,dx = \frac{1}{a-b}\int\left(\sqrt{x+a}-\sqrt{x+b}\right)dx$

$= \frac{1}{a-b}\left[\int (x+a)^{\frac{1}{2}}\,dx - \int (x+b)^{\frac{1}{2}}\,dx\right] = \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}(1)} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}(1)}\right]+c$

$= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right]+c = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right]+c$

3. Let I = $\int \frac{1}{x\sqrt{ax-x^2}}\,dx$ ....(i)

Putting $x = \frac{1}{t} = t^{-1} \quad \Rightarrow \quad dx = \frac{-1}{t^2}\,dt$

$\therefore$ From eq. (i), I = $\int \frac{\frac{-1}{t^2}\,dt}{\frac{1}{t}\sqrt{\frac{a}{t}-\frac{1}{t^2}}} = -\int \frac{dt}{\sqrt{at-1}} = -\int (at-1)^{\frac{-1}{2}}\,dt$

$= \frac{-(at-1)^{\frac{1}{2}}}{\frac{1}{2}\times a} + c = \frac{-2}{a}\sqrt{\frac{a}{x}-1} + c = \frac{-2}{a}\sqrt{\frac{a-x}{x}} + c$

4. Let I = $\int \frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}\,dx = \int \frac{1}{x^2\left[x^4\left(1+\frac{1}{x^4}\right)\right]^{\frac{3}{4}}}\,dx = \int \frac{1}{x^2.x^3\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}\,dx = \int \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{\frac{-3}{4}}\,dx$

Putting $1+\frac{1}{x^4} = t \quad \Rightarrow \quad -4x^{-5}\,dx = dt \quad \Rightarrow \quad \frac{1}{x^5}\,dx = \frac{-1}{4}\,dt$

$\therefore$ I = $\frac{-1}{4}\int t^{\frac{-3}{4}}\,dt = \frac{-1}{4}.\frac{t^{1/4}}{1/4} + c = -\left(1+\frac{1}{x^4}\right)^{1/4} + c$

5. Let I = $\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}\,dx$

Putting $x^{\frac{1}{6}} = t \quad \Rightarrow \quad x = t^6 \quad \Rightarrow \quad dx = 6t^5\,dt$

$\therefore$ I = $\int \frac{6t^5}{t^3+t^2}\,dt = 6\int \frac{t^5}{t^2(t+1)}\,dt = 6\int \frac{t^3}{t+1}\,dt = 6\int \frac{\left(t^3+1\right)-1}{t+1}\,dt$

$= 6\left[\int \frac{t^3+1}{t+1} - \frac{1}{t+1}\,dt\right] = 6\left[\int \left(\frac{(t+1)\left(t^2-t+1\right)}{t+1} - \frac{1}{t+1}\right)dt\right] = 6\left[\int \left(t^2-t+1-\frac{1}{t+1}\right)dt\right]$

$= 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c = 2t^3 - 3t^2 + 6t - 6\log|t+1| + c$

$= 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left|x^{\frac{1}{6}}+1\right| + c$

6. Let I = $\int \frac{5x}{(x+1)\left(x^2+9\right)}\,dx$ ....(i)

Let $\frac{5x}{(x+1)\left(x^2+9\right)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$ ....(ii)

$\Rightarrow \quad 5x = A(x^2+9)+(Bx+C)(x+1)$

$\Rightarrow \quad 5x = Ax^2+9A+Bx^2+Bx+Cx+C$

Comparing coefficients of x^2 $\quad$ A + B = 0 ....(iii)

Comparing coefficients of x $\quad$ B + C = 5 ....(iv)

Comparing constants $\quad$ 9A + C = 0 ....(v)

On solving eq. (iii), (iv) and (v), we get $\quad A = \frac{-1}{2},\ B = \frac{1}{2},\ C = \frac{9}{2}$

Putting these values of A, B and C in eq. (ii), $\quad \frac{5x}{(x+1)(x^2+9)} = \frac{\frac{-1}{2}}{x+1} + \frac{\frac{1}{2}x+\frac{9}{2}}{x^2+9}$

$\therefore$ From eq. (i), $I = \int \frac{5x}{(x+1)(x^2+9)}\,dx = \frac{-1}{2}\int\frac{1}{x+1}\,dx + \frac{1}{2}\int\frac{x}{x^2+9}\,dx + \frac{9}{2}\int\frac{1}{x^2+9}\,dx$

$= \frac{-1}{2}\log|x+1| + \frac{1}{4}\int\frac{2x}{x^2+9}\,dx + \frac{9}{2}.\frac{1}{3}\tan^{-1}\frac{x}{3}+c$

$= \frac{-1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{3}{2}\tan^{-1}\frac{x}{3}+c$

$= \frac{-1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3}+c$

7. Let $I = \int\frac{\sin x}{\sin(x-a)}\,dx = \int\frac{\sin(x-a+a)}{\sin(x-a)}\,dx = \int\frac{\sin(x-a)\cos a+\cos(x-a)\sin a}{\sin(x-a)}\,dx$

$= \int\left(\frac{\sin(x-a)\cos a}{\sin(x-a)}+\frac{\cos(x-a)\sin a}{\sin(x-a)}\right)dx = \int(\cos a+\sin a\cot(x-a))\,dx$

$= \int\cos a\,dx + \int\sin a\cot(x-a)\,dx = \cos a\int 1\,dx + \sin a\int\cot(x-a)\,dx$

$= (\cos a)x + \sin a\frac{\log|\sin(x-a)|}{1}+c = x\cos a+\sin a\log|\sin(x-a)|+c$

8. Let $I = \int\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}\,dx = \int\frac{e^{\log x^5}-e^{\log x^4}}{e^{\log x^3}-e^{\log x^2}}\,dx = \int\frac{x^5-x^4}{x^3-x^2}\,dx = \int\frac{x^4(x-1)}{x^2(x-1)}\,dx$

$\Rightarrow \quad I = \int x^2\,dx = \frac{x^3}{3}+c$

9. Let $I = \int\frac{\cos x}{\sqrt{4-\sin^2 x}}\,dx$ ....(i)

Putting $\sin x = t \quad \Rightarrow \quad \cos x = \frac{dt}{dx} \quad \Rightarrow \quad \cos x\,dx = dt$

$\therefore$ From eq. (i), $I = \int\frac{dt}{\sqrt{4-t^2}} = \sin^{-1}\left(\frac{t}{2}\right)+c = \sin^{-1}\left[\frac{1}{2}\sin x\right]+c$

10. Let I = $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x}\,dx$ ....(i)

$\Rightarrow$ I = $\int \frac{(\sin^4 x)^2 - (\cos^4 x)^2}{1-2\sin^2 x\cos^2 x}\,dx = \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{1-2\sin^2 x\cos^2 x}\,dx$

$\Rightarrow$ I = $\int \frac{\left\{(\sin^2 x)^2 - (\cos^2 x)^2\right\}\left\{(\sin^2 x)^2 + (\cos^2 x)^2\right\}}{1-2\sin^2 x\cos^2 x}\,dx$

$\Rightarrow$ I = $\int \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)\left\{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\tan^2 x\right\}}{1-2\sin^2 x\cos^2 x}\,dx$

$\Rightarrow$ I = $\int \frac{-(\cos^2 x - \sin^2 x)\left\{1-2\sin^2 x\tan^2 x\right\}}{1-2\sin^2 x\cos^2 x}\,dx = \int \frac{-\cos 2x\left\{1-2\sin^2 x\tan^2 x\right\}}{1-2\sin^2 x\cos^2 x}\,dx$

$\Rightarrow$ I = $-\int \cos 2x\,dx = \frac{-\sin 2x}{2} + c$

11. Let I = $\int \frac{1}{\cos(x+a)\cos(x+b)}\,dx$ ....(i)

$\Rightarrow$ I = $\frac{1}{\sin(a-b)}\int \frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\,dx = \frac{1}{\sin(a-b)}\int \frac{\sin[(x+a)-(x+b)]}{\cos(x+a)\cos(x+b)}\,dx$

$\Rightarrow$ I = $\frac{1}{\sin(a-b)}\int \frac{\sin(x+a)\cos(x+b) - \cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\,dx$

$\Rightarrow$ I = $\frac{1}{\sin(a-b)}\int \left(\frac{\sin(x+a)\cos(x+b)}{\cos(x+a)\cos(x+b)} - \frac{\cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\right) dx$

$\Rightarrow$ I = $\frac{1}{\sin(a-b)}\int (\tan(x+a) - \tan(x+b))\,dx$

$\Rightarrow$ I = $\frac{1}{\sin(a-b)}\left[-\log|\cos(x+a)| + \log|\cos(x+b)|\right] + c$

$\Rightarrow$ I = $\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + c$

12. Let I = $\int \frac{x^3}{\sqrt{1-x^8}}\,dx = \frac{1}{4}\int \frac{4x^3}{\sqrt{1-x^8}}\,dx$ ....(i)

Putting $x^4 = t \Rightarrow 4x^3 = \frac{dt}{dx} \Rightarrow 4x^3\,dx = dt$

$\therefore$ From eq. (i), I = $\frac{1}{4}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{4}\sin^{-1} t + c = \frac{1}{4}\sin^{-1}(x^4) + c$

13. Let I = $\int \frac{e^x}{(1+e^x)(2+e^x)}\,dx$ ....(i)

Putting $e^x = t \quad \Rightarrow \quad e^x = \dfrac{dt}{dx} \quad \Rightarrow \quad e^x\ dx = dt$

$\therefore$ From eq. (i), $I = \displaystyle\int \frac{dt}{(1+t)(2+t)} = \int \frac{1}{(t+1)(t+2)}\ dt$ ....(ii)

$\Rightarrow$ $I = \displaystyle\int \frac{(t+2)-(t-1)}{(t+1)(t+2)}\ dt = \int \left(\frac{t+2}{(t+1)(t+2)} - \frac{t-1}{(t+1)(t+2)} \right) dt$

$\Rightarrow$ $I = \displaystyle\int \left(\frac{1}{(t+1)} - \frac{1}{(t+2)} \right) dt = \log|t+1| - \log|t+2| + c$

$\Rightarrow$ $I = \log\left|\dfrac{t+1}{t+2}\right| + c = \log\left|\dfrac{e^x+1}{e^x+2}\right| + c$

14. Let $I = \displaystyle\int \frac{1}{(x^2+1)(x^2+4)}\ dx$ ....(i)

$\Rightarrow$ $I = \dfrac{1}{3}\displaystyle\int \frac{3}{(x^2+1)(x^2+4)}\ dx = \frac{1}{3}\int \frac{(x^2+4)-(x^2+1)}{(x^2+1)(x^2+4)}\ dx$

$\Rightarrow$ $I = \dfrac{1}{3}\displaystyle\int \left(\frac{1}{(x^2+1)} - \frac{1}{(x^2+4)} \right) dx = \frac{1}{3}\left[\int \frac{1}{x^2+1}\ dx - \int \frac{1}{x^2+4}\ dx \right]$

$\Rightarrow$ $I = \dfrac{1}{3}\left[\tan^{-1} x - \dfrac{1}{2}\tan^{-1}\dfrac{x}{2} \right] + c$

15. Let $I = \displaystyle\int \cos^3 x\ e^{\log \sin x}\ dx = \int \cos^3 x \sin x\ dx = -\int \cos^3 x(-\sin x)\ dx$ ....(i)

Putting $\cos x = t \quad \Rightarrow \quad -\sin x = \dfrac{dt}{dx} \quad \Rightarrow \quad -\sin x\ dx = dt$

$\therefore$ From eq. (i), $I = -\displaystyle\int t^3\ dt = \frac{-t^4}{4} + c = \frac{-1}{4}\cos^4 x + c$

16. Let $I = \displaystyle\int e^{3\log x}(x^4+1)^{-1}\ dx = \int \frac{e^{3\log x}}{(x^4+1)}\ dx = \int \frac{e^{\log x^3}}{(x^4+1)}\ dx = \int \frac{x^3}{(x^4+1)}\ dx$

$\Rightarrow$ $I = \dfrac{1}{4}\displaystyle\int \frac{4x^3}{(x^4+1)}\ dx$

Putting $x^4 + 1 = t \quad \Rightarrow \quad 4x^3 = \dfrac{dt}{dx} \quad \Rightarrow \quad 4x^3\ dx = dt$

$\therefore$ From eq. (i), $I = \dfrac{1}{4}\displaystyle\int \frac{dt}{t} = \frac{1}{4}\log|t| + c$

$\Rightarrow$ $I = \dfrac{1}{4}\log|x^4+1| + c = \dfrac{1}{4}\log(x^4+1) + c$

17. Let $I = \displaystyle\int f'(ax+b)\{f(ax+b)\}^n\ dx = \frac{1}{a}\int f'(ax+b)^n\ af(ax+b)\ dx$ ....(i)

Putting $f(ax+b)=t \quad \Rightarrow \quad f'(ax+b)\dfrac{d}{dx}(ax+b)=\dfrac{dt}{dx}$

$\Rightarrow \quad af'(ax+b)\ dx = dt$

$\therefore$ From eq. (i), $\text{I} = \dfrac{1}{a}\int t^n\ dt = \dfrac{1}{a}.\dfrac{t^{n+1}}{n+1}+c \qquad \text{if } n\neq -1$

And $\text{I} = \dfrac{1}{a}\int t^{-1}\ dt = \dfrac{1}{a}\int \dfrac{1}{t}\ dt = \dfrac{1}{a}\log|t|+c \qquad \text{if } n=-1$

$\Rightarrow \quad \text{I} = \dfrac{\{f(ax+b)\}^{n+1}}{a(n+1)}+c \qquad \text{if } n\neq -1$

And $\text{I} = \dfrac{1}{a}\log f(ax+b)+c \qquad \text{if } n=-1$

18. Let $\text{I} = \displaystyle\int \frac{dx}{\sqrt{\sin^3 x\sin(x+\alpha)}} = \int \frac{dx}{\sqrt{\sin^3 x(\sin x\cos\alpha+\cos x\sin\alpha)}}$

$\Rightarrow \quad \text{I} = \displaystyle\int \frac{dx}{\sqrt{\sin^3 x.\sin x(\cos\alpha+\cot x\sin\alpha)}} = \int \frac{dx}{\sin^2 x\sqrt{\cos\alpha+\cot x\sin\alpha}}$

$\Rightarrow \quad \text{I} = \displaystyle\int \frac{\operatorname{cosec}^2 x\ dx}{\sqrt{\cos\alpha+\cot x\sin\alpha}}$

Putting $\cos\alpha+\cot x\sin\alpha = t \quad \Rightarrow \quad -\operatorname{cosec}^2 x\sin\alpha\ dx = dt \quad \Rightarrow \quad \operatorname{cosec}^2 x\ dx = -\dfrac{dt}{\sin\alpha}$

$\therefore \quad \text{I} = \displaystyle -\int \frac{dt}{\sin\alpha\sqrt{t}} = \frac{-1}{\sin\alpha}\int t^{\frac{-1}{2}}\ dt = \frac{-1}{\sin\alpha}.\frac{t^{1/2}}{1/2}+c$

$\Rightarrow \quad \text{I} = \dfrac{-2}{\sin\alpha}.\sqrt{\cos\alpha+\cot x\sin\alpha}+c = \dfrac{-2}{\sin\alpha}.\sqrt{\cos\alpha+\dfrac{\cos x}{\sin x}\sin\alpha}+c$

$\Rightarrow \quad \text{I} = \dfrac{-2}{\sin\alpha}.\sqrt{\dfrac{\sin x\cos\alpha+\cos x\sin\alpha}{\sin x}}+c = \dfrac{-2}{\sin\alpha}.\sqrt{\dfrac{\sin(x+\alpha)}{\sin x}}+c$

19. We know that $\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}=\dfrac{\pi}{2} \quad \Rightarrow \quad \cos^{-1}\sqrt{x}=\dfrac{\pi}{2}-\sin^{-1}\sqrt{x}$

$\therefore \quad \text{I} = \displaystyle\int \frac{\sin^{-1}\sqrt{x}\left(\frac{\pi}{2}-\sin^{-1}\sqrt{x}\right)}{\frac{\pi}{2}}\ dx = \frac{2}{\pi}\int\left(2\sin^{-1}\sqrt{x}-\frac{\pi}{2}\right)dx$

$\Rightarrow \quad \text{I} = \dfrac{4}{\pi}\displaystyle\int \sin^{-1}\sqrt{x}\ dx - \int 1\,dx = \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\ dx - x + c \qquad$ ....(i)

Putting $\sqrt{x}=\sin\theta \quad \Rightarrow \quad x=\sin^2\theta \quad \Rightarrow \quad dx = 2\sin\theta\cos\theta\ d\theta = \sin 2\theta\ d\theta$

$\therefore \quad \text{I} = \dfrac{4}{\pi}\displaystyle\int\left(\sin^{-1}(\sin\theta).\sin 2\theta\right)d\theta - x + c = \frac{4}{\pi}\int(\theta.\sin 2\theta)\ d\theta - x + c$

$\Rightarrow \quad I = \frac{4}{\pi}\int\left[\theta\left(\frac{-\cos 2\theta}{2}\right) - \int 1.\left(\frac{-\cos 2\theta}{2}\right) d\theta\right] - x + c$ [Applying product rule]

$\Rightarrow \quad I = \frac{4}{\pi}\left[-\frac{1}{2}\theta\cos 2\theta + \frac{1}{2}\int \cos 2\theta \, d\theta\right] - x + c$

$\Rightarrow \quad I = \frac{4}{\pi}\left[-\frac{1}{2}\theta\left(1-2\sin^2\theta\right) + \frac{1}{4}2\sin\theta\cos\theta\right] - x + c$

$\Rightarrow \quad I = \frac{4}{\pi}\left[-\frac{1}{2}\theta\left(1-2\sin^2\theta\right) + \frac{1}{2}\sin\theta\sqrt{1-\sin^2\theta}\right] - x + c$

Putting $\sin\theta = \sqrt{x}$

$\Rightarrow \quad I = \frac{4}{\pi}\left[-\frac{1}{2}\left(\sin^{-1}\sqrt{x}\right)(1-2x) + \frac{1}{2}\sqrt{x}\sqrt{1-x}\right] - x + c$

$\Rightarrow \quad I = \frac{4}{\pi}\left[-\frac{1}{2}(1-2x)\sin^{-1}\sqrt{x} + \frac{1}{2}\sqrt{x}\sqrt{1-x}\right] - x + c$

$\Rightarrow \quad I = -\frac{2}{\pi}(1-2x)\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x}\sqrt{1-x} - x + c$

20. Let $I = \int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

Putting $\sqrt{x} = t \Rightarrow \quad x = t^2 \Rightarrow \quad dx = 2t\,dt$

$\therefore \quad I = \int\sqrt{\frac{1-t}{1+t}}2t\,dt = 2\int t\sqrt{\frac{1-t}{1+t}}\,dt = 2\int t\sqrt{\frac{1-t}{1+t}\times\frac{1-t}{1-t}}\,dt = 2\int\frac{t(1-t)}{\sqrt{1-t^2}}\,dt$

$\Rightarrow \quad I = 2\int\frac{t-t^2}{\sqrt{1-t^2}}\,dt$ ....(i)

$\Rightarrow \quad I = 2\int\frac{(1-t^2)+t-1}{\sqrt{1-t^2}}\,dt = 2\left[\int\sqrt{1-t^2}\,dt + \int\frac{t}{\sqrt{1-t^2}}\,dt - \int\frac{1}{\sqrt{1-t^2}}\,dt\right]$

$\Rightarrow \quad I = 2\left[\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t + \int\frac{t}{\sqrt{1-t^2}}\,dt - \sin^{-1}t\right] + c$

$\Rightarrow \quad I = 2\left[\frac{1}{2}t\sqrt{1-t^2} - \frac{1}{2}\sin^{-1}t + \int\frac{t}{\sqrt{1-t^2}}\,dt\right] + c$ ....(ii)

For evaluating $\int\frac{t}{\sqrt{1-t^2}}\,dt$, putting $1-t^2 = z \quad \Rightarrow \quad -2t\,dt = dz \quad \Rightarrow \quad t\,dt = -\frac{1}{2}\,dz$

$\therefore \quad \int\frac{t}{\sqrt{1-t^2}}\,dt = \int\frac{-\frac{1}{2}z}{\sqrt{z}} = -\frac{1}{2}\int z^{\frac{1}{2}}\,dz = -\frac{1}{2}.\frac{z^{1/2}}{1/2} = -\sqrt{1-t^2}$ ....(iii)

Putting this value in eq. (ii),

$$I = 2\left[\frac{1}{2}t\sqrt{1-t^2} - \frac{1}{2}\sin^{-1}t - \sqrt{1-t^2}\right] + c$$

$\Rightarrow$ $I = t\sqrt{1-t^2} - \sin^{-1}t - \sqrt{1-t^2} + c = (t-2)\sqrt{1-t^2} - \sin^{-1}t + c$

$\Rightarrow$ $I = \left(\sqrt{x}-2\right)\sqrt{1-x} - \sin^{-1}\sqrt{x} + c$

21. Let I = $\int \frac{2+\sin 2x}{1+\cos 2x} e^x \, dx = \int e^x \frac{2+2\sin x\cos x}{2\cos^2 x} \, dx = \int e^x \left(\frac{2}{2\cos^2 x} + \frac{2\sin x\cos x}{2\cos^2 x}\right) dx$

$= \int e^x \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos x}\right) dx = \int e^x \left(\sec^2 x + \tan x\right) dx = \int e^x \left(\tan x + \sec^2 x\right) dx \left(\int e^x \left(f(x) + f'(x)\right)\right)$

$= e^x \tan x + c$

22. Let I = $\int \frac{x^2+x+1}{(x+1)^2(x+2)} \, dx$ ....(i)

Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$ ....(ii)

$\Rightarrow$ $x^2+x+1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$

$\Rightarrow$ $x^2+x+1 = A\left(x^2+3x+2\right) + B(x+2) + C\left(x^2+1+2x\right)$

$\Rightarrow$ $x^2+x+1 = Ax^2 + 3Ax + 2A + Bx + 2B + Cx^2 + C + 2Cx$

Comparing coefficients of x^2 A + C = 1 ....(iii)

Comparing coefficients of x 3A + B + 2C = 1 ....(iv)

Comparing constants 2A + 2B + C = 1 ....(v)

On solving eq. (iii), (iv) and (v), we get A = –2, B = 1, C = 3

Putting these values of A, B and C in eq. (ii), $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2}$

$\therefore$ $\int \frac{x^2+x+1}{(x+1)^2(x+2)} \, dx = \int \left(\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2}\right) dx$

$= -\int \frac{1}{x+1} \, dx + \int (x+1)^{-2} \, dx + 3\int \frac{1}{x+2} \, dx$

$= -2\log|x+1| + \frac{(x+1)^{-2+1}}{-2+1} + 3\log|x+2| + c$

$= -2\log|x+1| - \frac{1}{x+1} + 3\log|x+2| + c$

23. Let I = $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}} \, dx$ ....(i)

Putting $x = \cos 2\theta$ $\Rightarrow$ $\frac{dx}{d\theta} = -2\sin 2\theta$ $\Rightarrow$ $dx = -2\sin 2\theta \, d\theta$

And $\tan^{-1}\sqrt{\frac{1-x}{1+x}} = \tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} = \tan^{-1}\sqrt{\tan^2\theta} = \tan^{-1}(\tan\theta) = \theta$

$\therefore \quad I = \int \theta(-2\sin 2\theta\, d\theta) = -2\int \theta \sin 2\theta\, d\theta$

$= -2\left[\theta\left(\frac{-\cos 2\theta}{2}\right) - \int 1\left(\frac{-\cos 2\theta}{2}\right) d\theta\right]$ [Applying Product Rule]

$= -2\left[\frac{-1}{2}\theta\cos 2\theta + \frac{1}{2}\int \cos 2\theta\, d\theta\right] = \theta\cos 2\theta - \frac{\sin 2\theta}{2} + c$

$= \theta\cos 2\theta - \frac{1}{2}\sqrt{1-\cos^2 2\theta} + c = \theta\left(\cos^{-1}x\right)x - \frac{1}{2}\sqrt{1-x^2} + c$

$= \frac{1}{2}\left[x\cos^{-1}x - \sqrt{1-x^2}\right] + c$

24. Let $I = \int \frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right) - 2\log x\right]}{x^4} dx = \int \frac{\sqrt{x^2+1}}{x^4}\left[\log\left(x^2+1\right) - \log x^2\right] dx$

$= \int \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x^4}\log\left(\frac{x^2+1}{x^2}\right) dx = \int \frac{\sqrt{\left(1+\frac{1}{x^2}\right)}}{x^3}\log\left(1+\frac{1}{x^2}\right) dx = \int \sqrt{\left(1+\frac{1}{x^2}\right)}\log\left(1+\frac{1}{x^2}\right)\frac{dx}{x^3}$

Putting $1+\frac{1}{x^2} = t \quad \Rightarrow \quad 1+x^{-2} = t \quad \Rightarrow \quad \frac{-2}{x^3}dx = dt \quad \Rightarrow \quad \frac{dx}{x^3} = -\frac{1}{2}dt$

$\therefore \quad I = -\frac{1}{2}\int \sqrt{t}\log t\, dt = -\frac{1}{2}\int (\log t).t^{\frac{1}{2}}\, dt$

$\Rightarrow \quad I = -\frac{1}{2}\left[(\log t).\frac{t^{1/2}}{1/2} - \int \frac{1}{t}.\frac{t^{3/2}}{3/2}\, dt\right]$ [Applying Product Rule]

$\Rightarrow \quad I = -\frac{1}{3}t^{3/2}\log t + \frac{1}{3}\int t^{1/2}\, dt = -\frac{1}{3}t^{3/2}\log t + \frac{1}{3}.\frac{t^{3/2}}{3/2} + c$

$\Rightarrow \quad I = \frac{2}{9}t^{3/2} - \frac{1}{3}t^{3/2}\log t + c = \frac{1}{3}t^{3/2}\left[\frac{2}{3} - \log t\right] + c = \frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\frac{2}{3} - \log\left(1+\frac{1}{x^2}\right)\right] + c$

25. Let $I = \int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right) dx = \int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx = \int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1}{2\sin^2\frac{x}{2}} - \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$

$= \int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx = \int_{\frac{\pi}{2}}^{\pi} e^x\left(-\cot\frac{x}{2} + \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right) dx$

$= \left(-e^x\cot\frac{x}{2}\right)_{\frac{\pi}{2}}^{\pi}$ $\left[\because \int e^x\left(f(x) + f'(x)\right) dx = e^x f(x)\right]$

$$= -e^{\pi}\cot\frac{\pi}{2} - \left(-e^{\frac{\pi}{2}}\cot\frac{\pi}{4}\right) = -e^{\pi(0)} + e^{\frac{\pi}{2}}(1) = e^{\frac{\pi}{2}}$$

26. Let I = $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x}\, dx$

$\Rightarrow$ I = $\int_0^{\frac{\pi}{4}} \frac{\frac{\sin x \cos x}{\cos x.\cos x.\cos^2 x}}{1+\frac{\sin^4 x}{\cos^4 x}}\, dx$ [Dividing each term by x^4]

$\Rightarrow$ I = $\int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{1+\tan^4 x}\, dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{2\tan x \sec^2 x}{1+\tan^4 x}\, dx$ ....(i)

Putting $\tan^2 x = t$ $\Rightarrow$ $2\tan x \frac{d}{dx}(\tan x) = \frac{dt}{dx}$ $\Rightarrow$ $2\tan x \sec^2 x\; dx = dt$

Limits of integration when $x = 0, t = \tan^2 x = \tan^2 0° = 0$ and when $x = \frac{\pi}{4}, t = \tan^2\frac{\pi}{4} = 1$

$\therefore$ I = $\frac{1}{2}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{2}\left(\tan^{-1} t\right)_0^1 = \frac{1}{2}\left(\tan^{-1}1 - \tan^{-1}0\right) = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$

27. Let I = $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\, dx = \int_0^{\frac{\pi}{2}} \frac{1}{1+4\tan^2 x}\, dx$ [Dividing each term by $\cos^2 x$] ....(i)

Putting $\tan x = t$ $\Rightarrow$ $\sec^2 x = \frac{dt}{dx}$ $\Rightarrow$ $dx = \frac{dt}{\sec^2 x} = \frac{dt}{1+\tan^2 x} = \frac{dt}{1+t^2}$

Limits of integration when $x = 0, t = \tan 0° = 0$ and when $x = \frac{\pi}{2}, t = \tan\frac{\pi}{2} = \infty$

$\therefore$ I = $\int_0^{\infty} \frac{1}{1+4t^2}.\frac{dt}{1+t^2} = \int_0^{\infty} \frac{1}{(1+4t^2)(1+t^2)}\, dt$ ....(ii)

$\Rightarrow$ I = $\frac{1}{3}\int_0^{\infty} \frac{3}{(1+4t^2)(1+t^2)}\, dt = \frac{1}{3}\int_0^{\infty} \frac{4(t^2+1)-(4t^2+1)}{(1+4t^2)(1+t^2)}\, dt$

$\Rightarrow$ I = $\frac{1}{3}\int_0^{\infty}\left[\frac{4(t^2+1)}{(1+4t^2)(1+t^2)} - \frac{(4t^2+1)}{(1+4t^2)(1+t^2)}\right] dt = \frac{1}{3}\left[\int_0^{\infty} 4\frac{1}{(4t^2+1)}\, dt - \int_0^{\infty}\frac{1}{(1+t^2)}\, dt\right]$

$\Rightarrow$ I = $\frac{1}{3}\left[\int_0^{\infty} 4\frac{1}{((2t)^2+1)}\, dt - \tan^{-1} t\right] = \frac{1}{3}\left[\int_0^{\infty} 4\frac{\left(\frac{1}{1}\tan^{-1}\frac{2t}{1}\right)_0^{\infty}}{2 \to \text{Coeff. of } t} - \tan^{-1} t\right] + c$

$\Rightarrow$ I = $\frac{1}{3}\left[2\left(\tan^{-1}\infty - \tan^{-1}0°\right) - \left(\tan^{-1}\infty - \tan^{-1}0°\right)\right] + c$

$\Rightarrow \quad I = \frac{1}{3}\left[2\left(\frac{\pi}{2}-0\right)-\left(\frac{\pi}{2}-0\right)\right]+c = \frac{1}{3}\times\frac{\pi}{2}+c=\frac{\pi}{6}+c$

28. Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\, dx$ ....(i)

Putting $\sin x-\cos x=t \quad \Rightarrow \quad (\cos x+\sin x)\, dx=dt$

Again $(\sin x-\cos x)^2=t^2 \quad \Rightarrow \quad \sin^2 x+\cos^2 x-2\sin x\cos x=t^2 \quad \Rightarrow \quad \sin 2x=1-t^2$

Limits of integration when $x=\frac{\pi}{6}, t=\sin\frac{\pi}{6}-\cos\frac{\pi}{6}=\frac{1}{2}-\frac{\sqrt{3}}{2}=\frac{-\left(\sqrt{3}-1\right)}{2}=-\alpha$ (say)

where $\alpha=\frac{\sqrt{3}-1}{2}$ ....(ii)

when $x=\frac{\pi}{3}, t=\sin\frac{\pi}{3}-\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}-\frac{1}{2}=\frac{\left(\sqrt{3}-1\right)}{2}=\alpha$

$\therefore \quad I = \int_{-\alpha}^{\alpha}\frac{dt}{\sqrt{1-t^2}} = \left[\sin^{-1}t\right]_{-\alpha}^{\alpha} = \sin^{-1}\alpha+\sin^{-1}\alpha = 2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ [From eq. (ii)

29. Let $I = \int_0^1 \frac{1}{1+x-\sqrt{x}}\, dx = \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{\left(\sqrt{1+x}+\sqrt{x}\right)\left(\sqrt{1+x}-\sqrt{x}\right)}\, dx = \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}\, dx$

$\Rightarrow \quad I = \int_0^1\left(\sqrt{1+x}+\sqrt{x}\right) dx = \int_0^1 (1+x)^{\frac{1}{2}}\, dx+\int_0^1 (x)^{\frac{1}{2}}\, dx = \frac{\left[(1+x)^{\frac{3}{2}}\right]_0^1}{\frac{3}{2}(1)}+\frac{\left[(x)^{\frac{3}{2}}\right]_0^1}{\frac{3}{2}(1)}$

$\Rightarrow \quad I = \frac{2}{3}\left[(2)^{\frac{3}{2}}-(1)^{\frac{3}{2}}\right]+\frac{2}{3}\left[(1)^{\frac{3}{2}}-0\right] = \frac{2}{3}\left[2\sqrt{2}-1\right]+\frac{2}{3}[1-0]$

$\Rightarrow \quad I = \frac{4\sqrt{2}}{3}-\frac{2}{3}+\frac{2}{3}=\frac{4\sqrt{2}}{3}$

30. Let $I = \int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}\, dx$

Putting $\sin x-\cos x=t \quad \Rightarrow \quad (\cos x+\sin x)\, dx=dt$

Again $(\sin x-\cos x)^2=t^2 \quad \Rightarrow \quad \sin^2 x+\cos^2 x-2\sin x\cos x=t^2 \quad \Rightarrow \quad \sin 2x=1-t^2$

Limits of integration when $x=0, t=0-1=-1$ and

when $x=\frac{\pi}{4}, t=\sin\frac{\pi}{4}-\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=0$

$\therefore \quad I = \int_{-1}^{0} \frac{dt}{9+16(1-t^2)} = \int_{-1}^{0} \frac{dt}{25-16t^2} = \int_{-1}^{0} \frac{dt}{16\left(\frac{25}{16}-t^2\right)} = \frac{1}{16}\int_{-1}^{0} \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$

$\Rightarrow \quad I = \frac{1}{16} \times \left[\frac{1}{2\times\frac{5}{4}} \log\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|\right]_{-1}^{0} = \frac{1}{40}\left[\log 1 - \log \frac{1/4}{9/4}\right] = \frac{1}{40}\left[0 - \log\frac{1}{9}\right]$

$\Rightarrow \quad I = \frac{1}{40}\left[-(\log 1 - \log 9)\right] = \frac{1}{40}\log 9 = \frac{1}{40}\log 3^2 = \frac{2}{40}\log 3 = \frac{1}{20}\log 3$

31. Let I = $\int_{0}^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\ dx = \int_{0}^{\frac{\pi}{2}} 2\sin x \cos x \tan^{-1}(\sin x)\ dx$

Putting $\sin x = t \quad \Rightarrow \quad \cos x\ dx = dt$

Limits of integration when $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = \sin\frac{\pi}{2} = 1$

$\therefore \quad I = 2\int_{0}^{1} t \tan^{-1} t\ dt = 2\int_{0}^{1} \left(\tan^{-1} t\right) t\ dt$

$\Rightarrow \quad I = 2\left[\tan^{-1} t.\frac{t^2}{2} - \int \frac{1}{1+t^2}.\frac{t^2}{2}\ dt\right]$ [Applying Product Rule]

$\Rightarrow \quad I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\int \frac{(1+t^2)-1}{1+t^2}\ dt\right] = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\int\left(1-\frac{1}{1+t^2}\right) dt\right]$

$\Rightarrow \quad I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\left(t - \tan^{-1} t\right)\right] = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}t + \frac{1}{2}\tan^{-1} t\right]_0^1 + c$

$\Rightarrow \quad I = 2\left[\frac{1}{2}\left\{\left(t^2+1\right)\tan^{-1} t - t\right\}\right]_0^1 + c = \left(2\tan^{-1} 1 - 1\right) - (0-0) = 2\times\frac{\pi}{4} - 1 = \frac{\pi}{2} - 1$

32. Let I = $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\ dx = \int_0^{\pi} \frac{x\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}\ dx = \int_0^{\pi} \frac{x\sin x}{1+\sin x}\ dx$ ....(i)

$\Rightarrow \quad I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\sin(\pi - x)}\ dx = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\sin x}\ dx$ ....(ii)

Adding eq. (i) and (ii),

$2I = \int_0^{\pi} \frac{x\sin x + (\pi - x)\sin x}{1+\sin x}\ dx = \int_0^{\pi} \frac{x\sin x + \pi\sin x - x\sin x}{1+\sin x}\ dx$

$= \int_0^{\pi} \frac{\pi \sin x}{1+\sin x}\ dx = \pi\int_0^{\pi} \frac{\sin x}{1+\sin x}\ dx = \pi\int_0^{\pi} \frac{(1+\sin x)-1}{1+\sin x}\ dx = \pi\int_0^{\pi}\left(1-\frac{1}{1+\sin x}\right) dx$

$$= \pi\int_0^{\pi} 1\,dx - \pi\int \frac{1}{1+\sin x}\,dx = \pi(x)_0^{\pi} - 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} = \pi(\pi) - 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin\left(\frac{\pi}{2}-x\right)}$$

$$= \pi^2 - 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x} = \pi^2 - 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{2\cos^2\frac{x}{2}} = \pi^2 - \pi\int_0^{\frac{\pi}{2}} \sec^2\frac{x}{2}\,dx$$

$$= \pi^2 - \pi\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}} = \pi^2 - 2\pi(1) = \pi(\pi-2)$$

$\Rightarrow \quad \text{I} = \pi\left(\frac{\pi-2}{2}\right)$

33. Let $\text{I} = \int_1^4 \left(|x-1|+|x-2|+|x-3|\right)\,dx$ ....(i)

If $x-1=0, x-2=0, x-3=0$ we get $x=1, x=2, x=3$ $\Rightarrow$ $x = 2, 3(1,4)$

$\therefore \quad \text{I} = \int_1^2 \left(|x-1|+|x-2|+|x-3|\right)dx + \int_2^3 \left(|x-1|+|x-2|+|x-3|\right)dx + \int_3^4 \left(|x-1|+|x-2|+|x-3|\right)dx$

$= \int_1^2 \{(x-1)-(x-2)-(x-3)\}\,dx + \int_2^3 \{(x-1)+(x-2)-(x-3)\}\,dx + \int_3^4 \{(x-1)+(x-2)+(x-3)\}\,dx$

$\Rightarrow \quad \text{I} = \int_1^2 (x-1-x+2-x+3)\,dx + \int_2^3 (x-1+x-2-x+3)\,dx + \int_3^4 (x-1+x-2+x-3)\,dx$

$\Rightarrow \quad \text{I} = \int_1^2 (4-x)\,dx + \int_2^3 (x)\,dx + \int_3^4 (3x-6)\,dx = \left(4x - \frac{x^2}{2}\right)_1^2 + \left(\frac{x^2}{2}\right)_2^3 + \left(\frac{3x^2}{2} - 6x\right)_3^4$

$\Rightarrow \quad \text{I} = (8-2) - \left(4 - \frac{1}{2}\right) + \frac{9}{2} - \frac{4}{2} + (24-24)\left(\frac{27}{2} - 18\right)$

$\Rightarrow \quad \text{I} = 6 - 4 + \frac{1}{2} + \frac{5}{2} - \left(-\frac{9}{2}\right) = 2 + \frac{1}{2} + \frac{5}{2} + \frac{9}{2} = \frac{19}{2}$

34. Let $\text{I} = \int_1^3 \frac{dx}{x^2(x+1)} = \int_1^3 \frac{1}{x^2(x+1)}\,dx$ ....(i)

Let $\frac{1}{x^2(x+1)} = \frac{\text{A}}{x} + \frac{\text{B}}{x^2} + \frac{\text{C}}{x+1}$ ....(ii)

$\Rightarrow \quad 1 = \text{A}(x)(x+1) + \text{B}(x+1) + \text{C}(x^2) \quad \Rightarrow \quad 1 = \text{A}(x^2+x) + \text{B}x + \text{B} + \text{C}x^2$

$\Rightarrow \quad 1 = \text{A}x^2 + \text{A}x + \text{B}x + \text{B} + \text{C}x^2$

Comparing coefficients of x^2 A + C + 0 ....(iii)

Comparing coefficients of x A + B = 0 ....(iv)

Comparing constants B = 1

On solving eq. (iii), (iv) and (v), we get A = –1, B = 1, C = 1

Putting these values of A, B and C in eq. (ii), $\frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}$

$\therefore \quad I = \int_1^3 \left(\frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}\right) dx = \int_1^3 \left(\frac{-1}{x}\right) dx + \int_1^3 \left(\frac{1}{x^2}\right) dx + \int_1^3 \left(\frac{1}{x+1}\right) dx$

$\Rightarrow \quad I = -\left(\log|x|\right)_1^3 + \int_1^2 x^{-2}\, dx + \left(\log|x+1|\right)_1^3 = -\left(\log|3| - \log|1|\right) + \left(\frac{x^{-1}}{-1}\right)_1^3 + \left(\log|4| - \log|2|\right)$

$\Rightarrow \quad I = -\log 3 + 0 - \left(\frac{1}{x}\right)_1^3 + \left(\log 4 - \log 2\right) = -\log 3 - \left(\frac{1}{3} - 1\right) + \left(\log 2^2 - \log 2\right)$

$\Rightarrow \quad I = -\log 3 + \frac{2}{3} + 2\log 2 - \log 2 = -\log 3 + \frac{2}{3} + \log 2 = \frac{2}{3} + \log 2 - \log 3 = \frac{2}{3} + \log\frac{2}{3}$

35. Let I = $\int_0^1 xe^x\, dx = \left(xe^x\right)_0^1 - \int_0^1 1.e^x\, dx$ [Applying Product rule]

$\Rightarrow \quad I = e - 0 - \int_0^1 e^x\, dx = e - \left(e^x\right)_0^1 = e - \left(e - e^0\right) = 1$

36. Let I = $\int_{-1}^1 x^{17} \cos^4 x\, dx$

Here $f(x) = x^{17}\cos^4 x \quad \Rightarrow \quad f(-x) = (-x)^{17}\cos^4(-x) = -x^{17}\cos^4 x = -f(x)$

$\therefore$ $f(x)$ is an odd function of x.

$\therefore \quad I = \int_{-1}^1 x^{17}\cos^4 x\, dx = 0 \quad \left[\because \int_{-a}^a f(x)\, dx = 0, \text{ if } f(x) \text{ is ann odd funtion of } x\right]$

37. Let I = $\int_0^{\frac{\pi}{2}} \sin^3 x\, dx = \int_0^{\frac{\pi}{2}} \frac{1}{4}(3\sin x - \sin 3x)\, dx = \frac{1}{4}\left[3(-\cos x) - \left(\frac{-\cos 3x}{3}\right)\right]_0^{\frac{\pi}{2}}$

$\Rightarrow \quad I = \frac{1}{4}\left[-3\cos x - \frac{1}{3}\cos 3x\right]_0^{\frac{\pi}{2}} = \frac{1}{4}\left[\left(-3\cos\frac{\pi}{2} + \frac{1}{3}\cos\frac{3\pi}{2}\right) - \left(-3\cos 0 + \frac{1}{3}\cos 0\right)\right]$

$\Rightarrow \quad I = \frac{1}{4}\left[-3\times 0 + \frac{1}{3}\times 0 + 3\times 1 - \frac{1}{3}\times 1\right] = \frac{1}{4}\left(3 - \frac{1}{3}\right) = \frac{1}{4}\times\frac{8}{3} = \frac{2}{3}$

38. Let I = $\int_0^{\frac{\pi}{4}} 2\tan^3 x\, dx = 2\int_0^{\frac{\pi}{4}} \tan x.\tan^2 x\, dx = 2\int_0^{\frac{\pi}{4}} \tan x.\left(\sec^2 x - 1\right) dx$

$\Rightarrow \quad I = 2\int_0^{\frac{\pi}{4}} \left(\tan x.\sec^2 x - \tan x\right) dx = 2\left[\int_0^{\frac{\pi}{4}} \left(\tan x.\sec^2 x\right) dx - \int_0^{\frac{\pi}{4}} \tan x\, dx\right]$ ....(i)

Let $I_1 = \int_0^{\frac{\pi}{4}} \left(\tan x.\sec^2 x\right)\, dx$

Putting $\tan x = t \quad \Rightarrow \quad \sec^2\theta\, dx = dt$

Limits of integration when $x = 0, t = \tan 0 = 0$ and when $x = \frac{\pi}{4}, t = \tan\frac{\pi}{4} = 1$

$\therefore \quad I_1 = \int_0^1 t\, dt = \left(\frac{t^2}{2}\right)_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

Putting value of I_1 in eq. (i),

$$I = 2\left[\frac{1}{2} - \int_0^{\frac{\pi}{4}} \tan x\, dx\right] = 2\left[\frac{1}{2} - \left(\log\sec x\right)_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{4}} \tan x\, dx\right] = 1 - 2\left(\log\sec\frac{\pi}{4} - \log\sec 0\right)$$

$$\Rightarrow \quad I = 1 - 2\left(\log\sqrt{2} - \log 1\right) = 1 - 2\left(\log 2^{\frac{1}{2}} - 0\right) = 1 - 2\left(\frac{1}{2}\log 2\right) = 1 - \log 2$$

39. Let $I = \int_0^1 \sin^{-1} x\, dx$

Putting $x = \sin\theta \quad \Rightarrow \quad dx = \cos\theta\, d\theta$

Limits of integration when $x = 0, \theta = 0$ and when $x = 1, \sin\theta = 1$, i.e., $\theta = \frac{\pi}{2}$

$$\therefore \quad I = \int_0^1 \sin^{-1} x\, dx = \int_0^1 \theta\cos\theta\, d\theta = \left[\theta\sin\theta\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 1.\sin\theta\, d\theta \qquad \text{[Integrating by parts]}$$

$$\Rightarrow \quad I = \left(\frac{\pi}{2} - 0\right) + \left[\cos\theta\right]_0^{\frac{\pi}{2}} = \frac{\pi}{2} + \left(\cos\frac{\pi}{2} - \cos 0\right) = \frac{\pi}{2} + (0 - 1) = \frac{\pi}{2} - 1$$

40. Given: $\int_0^1 e^{2-3x}\, dx$

Comparing $\int_a^b f(x)\, dx$ we have, $\quad a = 0, b = 1, f(x) = e^{2-3x} \quad \therefore \quad nh = b - a = 1$

Putting these values in $\int_a^b f(x)\, dx = \lim\limits_{\substack{n\to\infty \\ h\to 0}} h\left[f(a) + f(a+h) + f(a+2h) + \dots\dots + f\{a + (n-1)h\}\right]$

$$I = \int_0^1 e^{2-3x}\, dx = \lim_{\substack{n\to 0 \\ h\to 2}} h\left[e^2 + e^{2-3h} + e^{2-6h} + \dots\dots + e^{2-3(n-1)h}\right]$$

$$\Rightarrow \quad I = e^2 \lim_{\substack{n\to 0 \\ h\to 2}} h\left[\frac{e^{-3nh} - 1}{e^{-3h} - 1}\right] = e^2 \lim_{h\to 2} h\left[\frac{e^{-3} - 1}{e^{-3h} - 1}\right] = e^2\left(e^{-3} - 1\right)\lim_{h\to 2}\frac{-3h}{e^{-3h} - 1} \times \frac{1}{3}$$

$$\Rightarrow \quad I = \left(e^{-1} - e^2\right) \times 1 \times \frac{-1}{3} = \frac{1}{3}\left(e^2 - \frac{1}{e}\right)$$

41. Let I = $\int \frac{dx}{e^x + e^{-x}} = \int \frac{1}{e^x + \frac{1}{e^x}} dx = \int \frac{1}{\left(\frac{e^{2x}+1}{e^x}\right)} dx = \int \frac{e^x}{e^{2x}+1} dx$ ....(i)

Putting $e^x = t \quad \Rightarrow \quad e^x \; dx = dt$

$\therefore$ From eq. (i), I = $\int \frac{dt}{t^2+1} = \tan^{-1} t + c = \tan^{-1} e^x + c$

Therefore, option (A) is correct.

42. Let I = $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx = \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\sin x + \cos x)(\sin x + \cos x)} dx$

$\Rightarrow$ I = $\int \frac{\cos x - \sin x}{\sin x + \cos x} dx = \log|\sin x + \cos x| + c$

Therefore, option (B) is correct.

43. Given: $f(a+b-x) = f(x)$ ....(i)

Let I = $\int_a^b xf(x)\; dy$ ....(ii)

$\Rightarrow$ I = $\int_a^b (a+b-x)\, f(a+b-x)\; dx = \int_a^b (a+b-x)\, f(x)\; dx$ ....(iii)

Adding eq. (ii) and (iii),

$2I = \int_a^b (x+a+b-x)\, f(x)\; dx = \int_a^b (a+b)\, f(x)\; dx = (a+b)\int_a^b f(x)\; dx$

$\Rightarrow$ I = $\left(\frac{a+b}{2}\right)\int_a^b f(x)\; dx$

Therefore, option (D) is correct.

44. Let I = $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx = \int_0^1 \tan^{-1}\left(\frac{x+x-1}{1+x-x^2}\right) dx = \int_0^1 \tan^{-1}\left(\frac{x+(x-1)}{1-x(x-1)}\right) dx$

$\Rightarrow$ I = $\int_0^1 \tan^{-1} x + \tan^{-1}(x-1)\; dx = \int_0^1 \tan^{-1} x\; dx + \int \tan^{-1}(x-1)\; dx$

$\Rightarrow$ I = $\int_0^1 \tan^{-1} x\; dx + \int \tan^{-1}(1-x-1)\; dx = \int_0^1 \tan^{-1} x\; dx + \int \tan^{-1}(-x)\; dx$

$\Rightarrow$ I = $\int_0^1 \tan^{-1} x\; dx - \int \tan^{-1} x\; dx = 0$

Therefore, option (B) is correct.