$$\frac{(6.62\times10^{-34}\ \text{J.s})(1-0.5)}{} = 1.22\times10^{-12}\ \text{m} = 0.0243\ \text{Å}$$

B. Tech. – 2nd

Mathematics-II

Full Marks: 70

TIME: 3 hours

Answer any *six* questions including **Q.No.1** which is compulsory.

The figures in the right-hand margin indicate marks

1. Answer all the following questions. [2×10]

(a) Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y = 0.$$

(b) What is the difference between initial value problem and boundary value problem?

(c) When an ordinary differential equation has no singular solution?

(d) Solve the partial differential equation $u_y + u = e^{xy}$.

(e) Find the integrating factor of the differential equation

$$y\,dx + (x^2y - x)dy = 0.$$

(f) Find the center and radius of convergent of the series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(2n+2)(2n+4)} x^n.$$

(g) Find the convolution $t * e^t$ by integration.

(h) Write two assumptions for one dimensional wave equation.

(i) Find the Laplace transformation of the function

$$5e^{-at} \sin \omega t.$$

(j) Find the inverse Laplace transformation of the function

$$\frac{7}{(s-1)^3}.$$

2. a) Solve the ordinary differential equation $y' + 4y = 0$ by power series method. [5]

b) Find the general solution of the ordinary differential $y'' - 4y = 0$ by converting it to a system. [5]

3. a) Verify that the ordinary differential equation $y'' + \lambda y = 0, y'(0) = 0 = y'(\pi)$, satisfy Sturm-Liouville problems. Find the eigenvalues and eigenfunctions. [5]

b) Find the general solution of the ordinary differential equation $y'' + y = \cos x + \sec x$ by method of variation of parameter. [5]

4. a) Find two linearly independent solutions of the ordinary differential equation $y'' - 4y + y = 0$. Also find the general solution. [5]

b) Solve the initial value problem $x^2y'' - 4xy' + 6y = 0, y(1) = 2, y'(1) = 0.$ [5]

5. a) Discuss the existence and uniqueness of a solution of the initial value problem $y' = y^{\frac{1}{3}}, y(0) = 0.$ [5]

b) Find the second order ordinary differential equation for which the given functions e^{-x}, xe^{-2x} are solutions. [5]

6. a) Solve the initial value problem [5]

$y'' + 2y' + 2y = e^{-t} + 5\,\delta(t-2), y(0) = 0, y'(0) = 1.$

$$\frac{(6.62\times10^{-34}\ \text{J.s})(1-0.5)}{\ldots} = 1.22\times10^{-12}\ \text{m} = 0.0243\ \text{Å}$$

[5]

b) Solve the differential equation

$$y'' + y = r(t),\ r(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & t > 1. \end{cases}$$

7. a) Find the inverse Laplace transform of the function $\frac{s\,e^{-s}}{s^2+\omega^2}$. [5]

b) Find the type, transform to normal form and solve

$$u_{xx} - 4u_{xy} + 4u_{yy} = 0.$$ [5]

8. a) Solve the following partial differential equation

$$\frac{\partial \omega}{\partial x} + x\frac{\partial \omega}{\partial t} = x,\ \omega(x,0) = 1,\ \omega(0,t) = 1$$

by Laplace transforms. [5]

b) Find the deflection $u(x,t)$ of the vibrating string of length π and ends fixed, corresponding to zero initial velocity and initial deflection $f(x) = k(\sin x - \sin 2x)$, given $c^2 = 1$. [5]

Total Pages—5

(Set-1)

B.Tech-2nd

Mathematics-II

Full Marks : 70

Time : 3 hours

Q.No.1 is compulsory and answer any five questions from the remaining seven questions.

The figures in the right-hand margin indicate marks

1. Answer *all* parts of this questions : 2×10

(*a*) Determine the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$.

(*b*) Define the unit step function and graph the function $f(t) = u(t-2)5 \sin t$.

(*c*) Describe the level surfaces of the scalar field $f = 4x^2 + y^2 + 9z^2$.

(*Turn Over*)

(2)

(*d*) To which function the sin series of the function $f(x) = \cos x, 0 < x < \pi$ approximates in the interval $(-\pi, 0)$. Show the graph also.

(*e*) Prove that for any twice continuously differentiable scalar function f, curl (grad f) = 0.

(*f*) Find the divergence and curl of F, if F = grad $(x^3 + y^3 + z^3)$.

(*g*) Find out the greatest rate of increase of $f = x^2 + yz^2$ at the point $(1, -1, 3)$.

(*h*) Provide one example to show that partial derivative may exists at a point but the function is not differentiable at that point

(*i*) Discuss if the vectors (4, 2, 9), (3, 2, 1) and (−4, 6, 9) are linearly independent.

(*j*) Find $a \times b - b \times a$ if $a = (-3, 2, 0)$ and $b = (6, -7, 2)$.

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B.Tech-2

Format

Laplace transformation formula

2. (*a*) Find a solution of

$$(a^2 - x^2)\, y'' - 2xy' + 12y = 0$$

by series solution method. 5

(*b*) Solve the equation

$$xy'' + 2\,(1 - x)y' + (x - 2)y = 0$$

by Frobenious method. 5

3. Solve the following equations by Laplace transformation method :

(*a*) $3 \sin 2x = y\,(x) + \int_0^x (x - t)\, y(t)\, dt.$ 5

(*b*) $y'' + 5y' + 6y = 5e^{3t},\ y\,(0) = y'(0) = 0.$ 5

4. (*a*) Find the Laplace transform of

$$f(t) = 2e^{-t} \cos^2 \frac{t}{2}.$$ 5

(*b*) Find the Inverse Laplace transform of

$$\frac{s^2}{(s^2 + w^2)^2}.$$ 5

(Turn Over)

5. (a) Find the Fourier series of the function $f(x) = x^2$, $(-\pi < x < \pi)$. 5

(b) Find the Fourier series of the periodic function $f(x) = \pi x^3/2$, $(-1 < x < 1)$, $p = 2L = 2$. 5

6. (a) Sketch the function, state whether it is odd or even and find its Fourier series of

$$f(x) = \begin{cases} k & \text{if } -\pi/2 < x < \pi/2, \\ 0 & \text{if } \pi/2 < x < 3\pi/2. \end{cases}$$ 5

(b) Solve the system of equations by Laplace transform method
$y'_1 = -2y_1 + 3y_2, y'_2 = 4y_1 - y_2, y_1(0) = 4, y_2(0) = 3.$ 5

7. (a) Show that the repeated limit exists but the double limit does not when $(x, y) \to (0, 0)$, for

$$f(x) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases}$$ 5

Formula

(b) If $z = f\left[\dfrac{ny - mz}{nx - lz}\right]$, then prove that 5

$$(nx - lz)\frac{\partial z}{\partial x} + (ny - mz)\frac{\partial z}{\partial y} = 0.$$

8. (a) Expand

$f(x, y) = 21 + x - 20y + 4x^2 + xy + 6y^2$

in Taylors series about the point the $(-1, 2)$. 5

(b) Find the local maximum and the local minimum values of the function 5

$f(x, y) = 2(x^2 - y^2) - x^4 + y^4$.
