

20/01/20

Models of Computation

Both sequential & computer operates on a set of instruction called algorithm. These set of instruction instructs the computer about what it has to do in each step.

Depending on instruction stream & data stream comp can be classified into 4 categories:

SISD (single instructⁿ single Data stream)

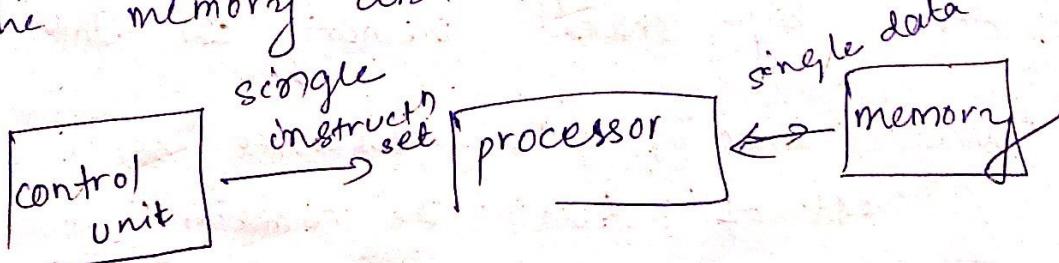
SIMD (" multiple

MISD (multiple single

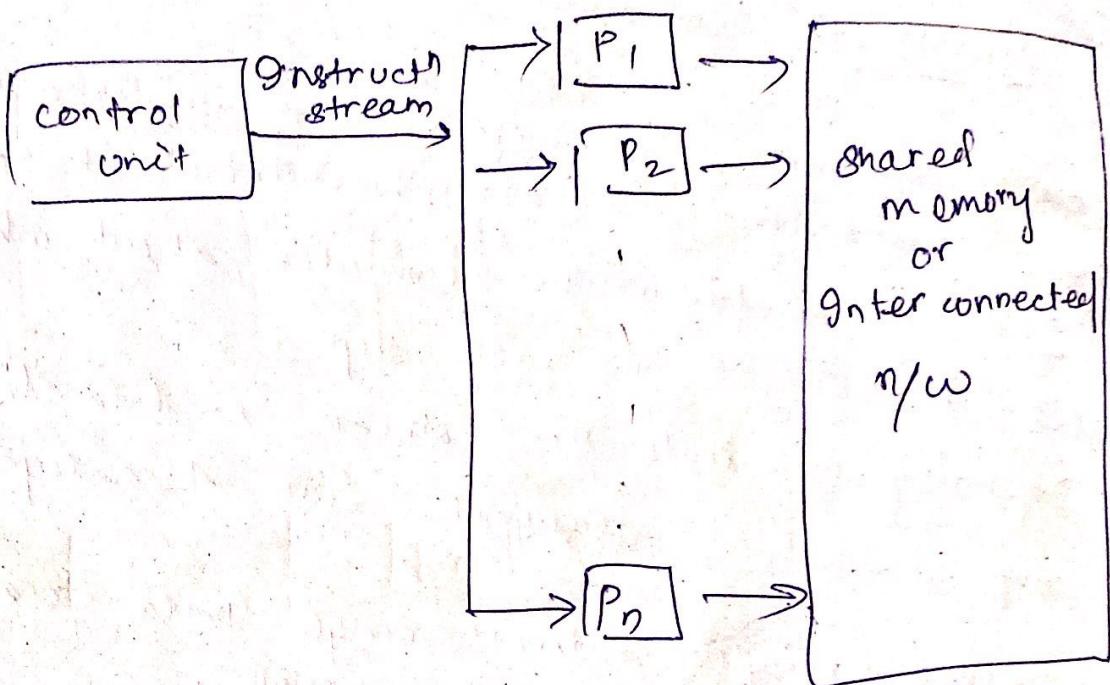
MIMD

SISD - In this type of computer single processor can receive instruction stream from control unit & again it operates on a single data stream.

During computation at each step, the processor receives one instructⁿ from control unit & operates on a single data received from the memory unit.



SIMD



SIMD contains

Operating computat' at each step, all processors receive a single instruction set from the control unit & operates on diff. set of data.

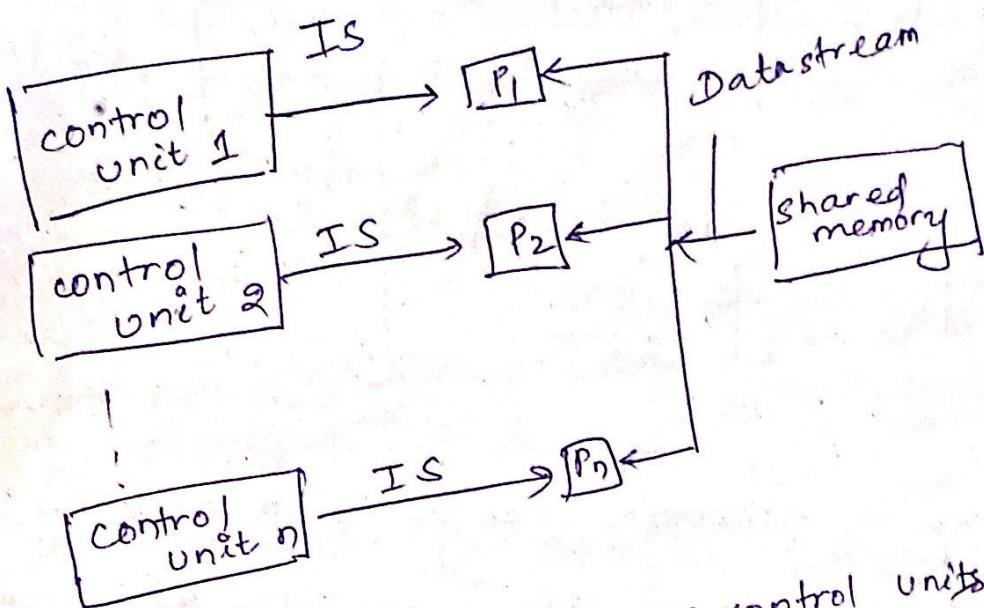
Each of the processing unit have its own local memory to store data & instruct'.

In SIMD comp., the processor need to communicate among themselves. This is done by shared memory or interconnect n/w.

While some of the processors will execute a set of instruct', the remaining processor wait for their next set of instruct'.

Instruction from control unit decides which processor will be active or inactive

MISD -



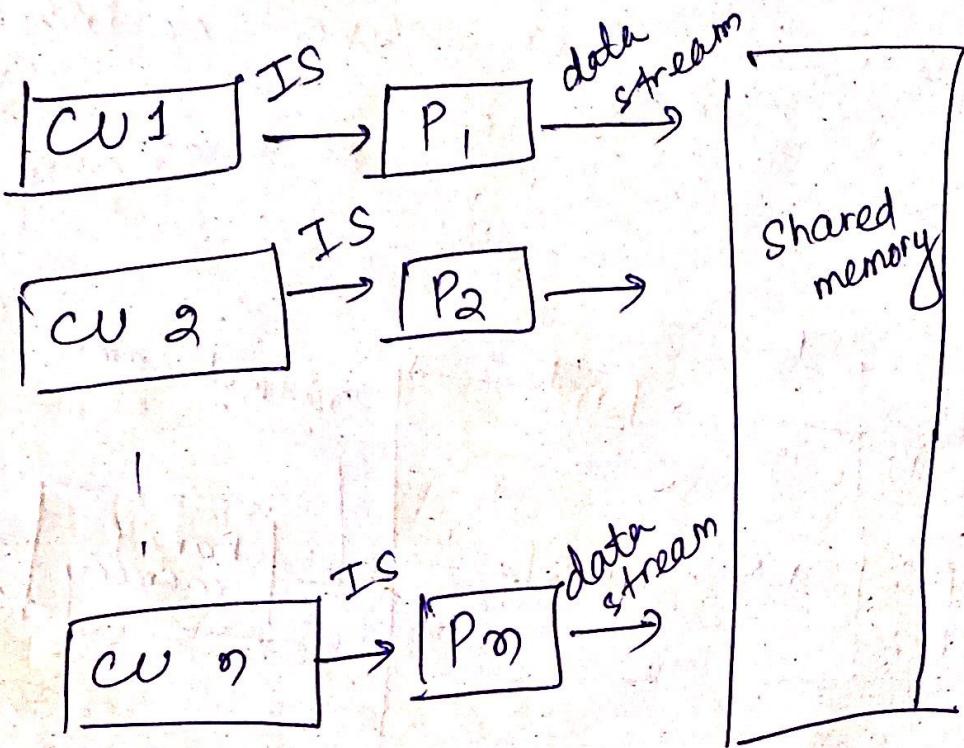
It consists of more than one control units & multiple

MISD compl. contains

Each processor has its own control units
and they share a common memory units

All the processors gets instructⁿ individually
from their own control cent & they operates
on a single stream of data. As per the
'instructⁿ' they have received from their
respective control cent.

MIMD



* Each processor has its own memory, control and they receive diff. set of instrucⁿ. from their own control unit and operates on diff. sets of data stream.

- ① Multicomputer
- ② Multidistributed sys.

P.D.S

Analysing Parallel algorithm II

We can analyze This refers to the process of determining how good an algm is that how fast, & how expensive to run & how efficient it is in its use of available resources.

It is usually evaluated using following criteria -

- ① Running time
- ② No. of processors used

- ③ Cost

① Running time - (It is essential to speed of oprn.)
It is defined as time taken by the algm to solve a problem on Net computer i.e. the time lapse from the starting of algm to the moment the algm terminates.

Counting steps

Speed up

Before actually implementing the algorithm on a computer it is necessary to conduct a theoretical analysis of the time it will require to solve the computational problem as executed by the algoⁿ at worst case.

The running time of a parallel algoⁿ usually obtained by counting 2 kinds of steps -

① Computational step

② Routing step

A comp. step is an arithmetic or logic opⁿ performed on a piece of info^m within a processor.

Routing step - The piece of info^m travels from one processor to another processor via shared memory or interconnect n/w.

Speed up is eventually a parallel algoⁿ for

a given problem

speed up = $\frac{\text{running time of a known fastest sequential algo for a particular problem}}{\text{worst case running time of parallel algo}}$

* It detects faster the speed up better the parallel algoⁿ.

$\frac{\text{Cost}}{\text{cost}}$ of the II^{el} alg^m is defined as
the product of

$$\text{cost} = \text{II}^{\text{el}} \text{ running time} * \text{no. of processors used}$$

efficiency = worst case running time of
fastest known sequential
alg^m for a problem

cost of II^{el} alg^m.

worst

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MATRIX MULTIPLICATION USING PARALLEL PROCESSORS -

single processor -

11^{el} c/m -

Algorithm

procedure MATRIXMULTI

begin

for $k=1$ to $n-1$ step 1 do

begin

for all p_{ij} , where $i \leq j$ ranges from 1 to n do

if i is greater than k

then rotate a_{ij} in the east direction

if j is greater than k then

rotate b_{ij} in south direction

end if

end.

Step 2 for all p_{ij} , where i, j lies b/w 1 to n do

compute the product of a and b , and store the result in c .

for $k=1$ to $n-1$ step 1 do for all p_{ij} , where $i \leq j$ ranges

from 1 to n

rotate a in east direction

rotate b in south direction

$$c = c + a \times b$$

end

Eg. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$A = \begin{pmatrix} 1/5 & 2/6 \\ 3/7 & 4/8 \end{pmatrix}$$

processor 1 - a_{11}, b_{11}
processor 2 - a_{12}, b_{12}
processor 3 - a_{21}, b_{21}
processor 4 - a_{22}, b_{22}

$$n^2 = 4 \text{ (no of processor)}$$

$$n = 2$$

$K = 1$ to $(2-1)$ for c .
 $a_{21} > K$, $b_{21} > K$.
 $a_{22} > K$, $b_{22} > K$.
for both $i \leq j$

$$A = \begin{pmatrix} 1 & 15 \\ 4 & 7 \\ 2 & 8 \\ 3 & 6 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 15 & 28 \\ 4x7 & 3x8 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 5 & 16 \\ 28 & 18 \end{pmatrix}$$

store the value in memory.

Step 2

$$A = \begin{pmatrix} 2 & 7 & 11 & 6 \\ 3 & 5 & 4 & 8 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 14 & 6 \\ 15 & 32 \end{pmatrix}$$

store the value in memory.

$$\textcircled{1} + \textcircled{11} \Rightarrow$$

$$C_{ij} = A = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix} \quad \left[\text{Matrix multiplication using 11el} \right]$$

processor

$$\text{Q/A } A = \left[\begin{array}{cc|cc} 2 & 1 & 5 & 3 \\ 6 & 7 & 1 & 6 \\ \hline 9 & 2 & 4 & 4 \\ 3 & 6 & 7 & 2 \end{array} \right] \quad 9 \times 4$$

$$B = \left[\begin{array}{cc|cc} 6 & 1 & 2 & 3 \\ 4 & 5 & 6 & 5 \\ \hline 1 & 9 & 8 & -8 \\ 4 & 0 & -8 & 5 \end{array} \right] \quad 4 \times 4$$

$$P_0 = \begin{bmatrix} 2 & 1 \\ 6 & 7 \end{bmatrix}$$

$$P_{11} = \begin{bmatrix} 5 & 3 \\ 1 & 6 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 9 & 2 \\ 3 & 6 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 4 & 4 \\ 7 & 2 \end{bmatrix}$$

$$A = \begin{pmatrix} AP_0 | BP_0 & AP_1 | BP_1 \\ AP_2 | BP_2 & AP_3 | BP_3 \end{pmatrix} \Rightarrow A = \begin{pmatrix} AP_0 | BP_0 & AP_1 | BP_3 \\ -AP_3 | BP_2 & AP_2 | BP_1 \end{pmatrix}$$

$$A = \left[\begin{array}{cc|cc} 2 & 1 & 5 & 3 \\ 6 & 7 & 1 & 6 \\ \hline 4 & 4 & 9 & 2 \\ 7 & 2 & 4 & 0 \end{array} \right] = \left[\begin{array}{cc|cc} 6 & 1 & 8 & -8 \\ 1 & 6 & -8 & 5 \\ \hline 8 & 0 & 3 & 0 \\ 15 & 6 & -42 & 37 \end{array} \right]$$

store the value.

Step-2

$$A = \begin{pmatrix} 5 & 3 & | & 1 & 9 \\ 1 & 6 & | & 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & | & 2 & 3 \\ 6 & 4 & | & 6 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 2 & | & 6 & 1 \\ 3 & 6 & | & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 4 & | & 8 & -8 \\ 2 & 2 & | & -8 & 5 \end{pmatrix}$$

$\textcircled{1} + \textcircled{11} \Rightarrow$

$$A = \begin{pmatrix} 10 & 10 & | & 10 & 10 \\ 10 & 10 & | & 10 & 10 \end{pmatrix}$$

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case of vector, we will assign each row

in directional partitioning

One Row-wide

$$P_0 \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \times \begin{bmatrix} - \end{bmatrix}_{3 \times 1} \rightarrow \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$$

- * P_0 contains 0th row of vector A & 0th element of vector X. Similarly $P_1 \rightarrow 1^{\text{st}}$ row of vector.

- * For multiplication, P_0, P_1, P_2 will broadcast its single element of X vector to all processes. This called all to all broadcast.



- * Vector 'X' is common for all.

After multiplication, resultant matrix is stored in result matrix

Time complexity - $O(n^2)$
Matrix \times vector multiplication By using 1-D partitioning

Multiplication of an $n \times n$ matrix with an $n \times 1$ vector using

1-D row-wise partitioning for 1 row per process.

Step ① MATRIX A

| | | | | |
|---|---|---|---|---|
| - | - | - | - | - |
| - | - | - | - | - |
| - | - | - | - | - |
| - | - | - | - | - |
| - | - | - | - | - |

Step ②

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |

Step ③

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 | 5 |

Initial distribution of full vector among all processes by all-to-all broadcast

| | | | | |
|----|---|---|---|---|
| P0 | - | - | - | - |
| P1 | - | - | - | - |
| P2 | - | - | - | - |
| P3 | - | - | - | - |
| P4 | - | - | - | - |

Final distribution of all matrix & result vector &

Step 1 Consider the case in which the $n \times n$ matrix is partitioned among 'n' processes stores 1 complete row of the matrix.

Then $n \times 1$ vector 'x' is distributed such that each process owns 1 of the elements of the matrix & the vector for

row wise block 1 \rightarrow partitioning.

Step 2 - The initial block 1 \rightarrow partitioning.

- Process P_i initially owns X_j & $A_{1,j}, A_{2,j}, A_{3,j}, \dots, A_{n,j}$ and is responsible for computing Y_i (Resultant vector).

& - Vector 'x' is multiplied with each row of the matrix so every process needs the entire vector. Since each process starts with only one element of 'x'. An all-to-all broadcast is required to distribute all the elements to all the processes. After vector 'x' is distributed among all processes process P_i computes $Y[i] = \sum_{j=0}^{n-1} (P[i, j] \cdot x) * [j]$.

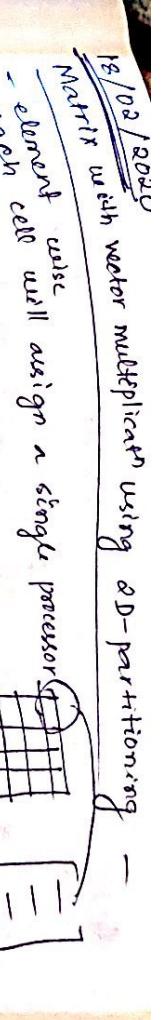
* The result vector Y is stored exactly the way the starting vector 'x' was stored.

Parallel Run Time - Starting with one vector element per process the all-to-all broadcast of the vector elements among 'n' processes requires the time $O(n)$ of any architecture.

- The multiplication of a single row of A with 'x' is also

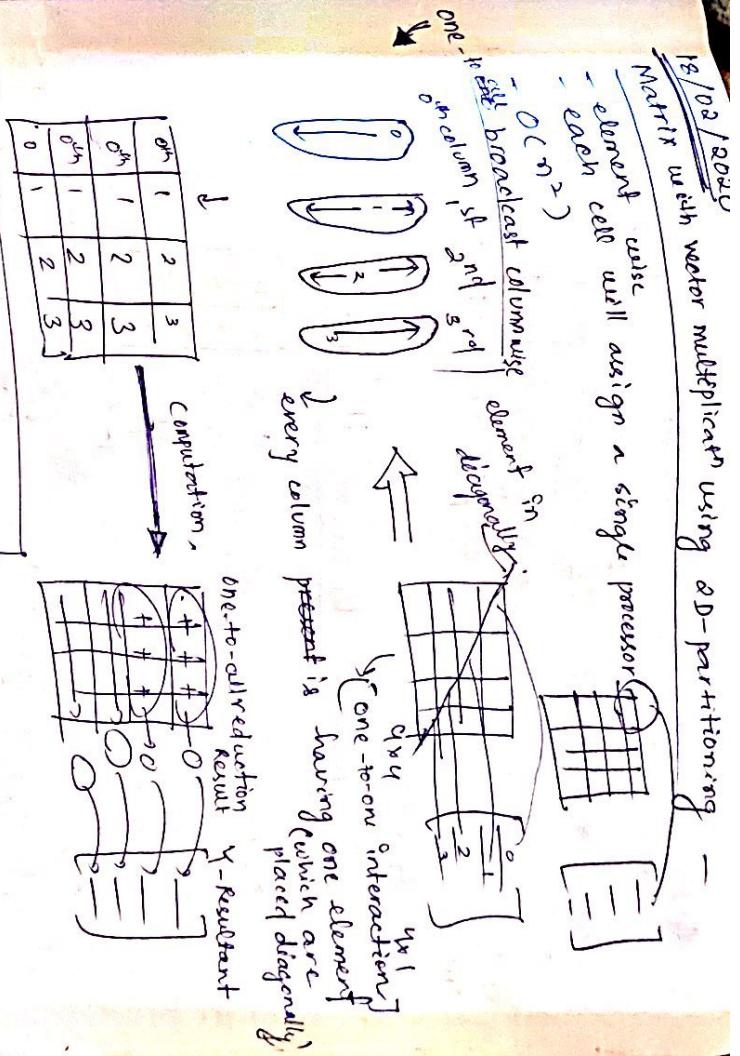
- The multiplication of a single row of A with 'x' is also performed by each process in completed by n^2 processes in time of $O(n)$. resulting in a process runtime product of $O(n^2)$. The algorithm is cost optimal.

Step 2/3 Matrix with vector multiplication using 2D-partitioning -

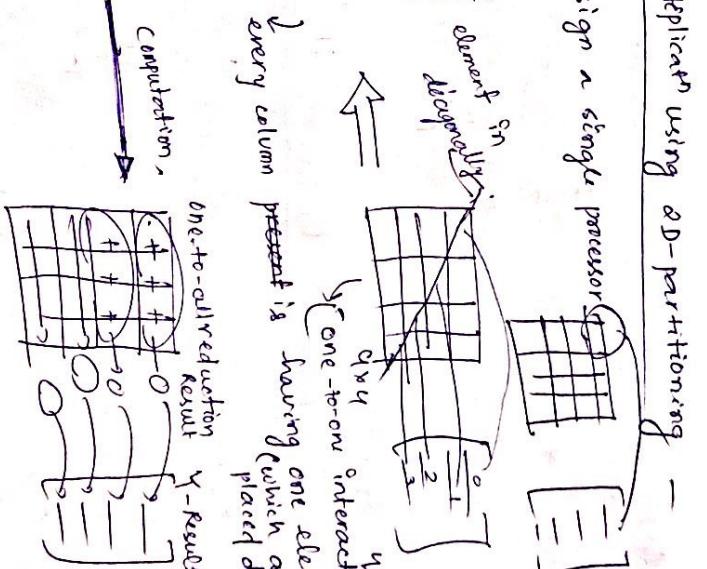


Step 3 - One-to-all broadcast columnwise element in 1st column, 2nd column, 3rd column $\rightarrow O(n^2)$

- Element cell will assign a single processor to each cell.



| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 0 | 1 | 2 | 3 |
| 0 | 1 | 2 | 3 |
| 0 | 1 | 2 | 3 |



Steps:

- ① one-to-one interaction
- ② one-to-all broadcast
- ③ computation
- ④ one-to-all reduction

Database Query Processing Using Parallel Computer

* Database Query Processing always performs multiple processing of a SQL statement. It features multiple single server process can the SQL query simultaneously to process a single SQL statement. This capability is called 'SQL query processing'. By dividing the work necessary to process a statement among multiple server processes.

Statement: Every Coordinator (Server Process) \downarrow Emp Table

The parallel query feature allows certain operations to be performed in

parallel by multiple query server processes, dispatches the execution of statement to several query servers & coordinate the results from all of the servers to send back to the user.

Query coordinator
Cserver process

select * from emp



EMP table

Query server

EMP table

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Sorting

is a process of arranging elements in a group in a particular order i.e. in ascending or in descending order.

Different types of techniques -
1) Enumeration sort

2) Odd-even sort

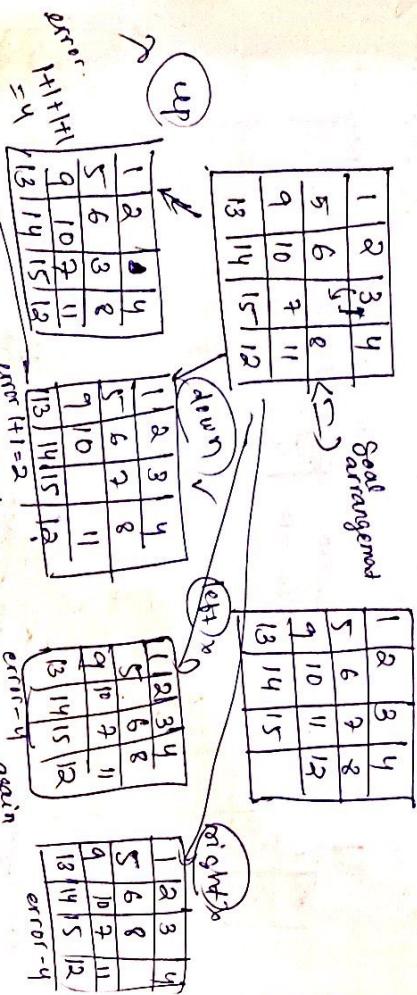
3) Parallel merge sort

4) Hyper quick sort

5) Bitonic merge sort

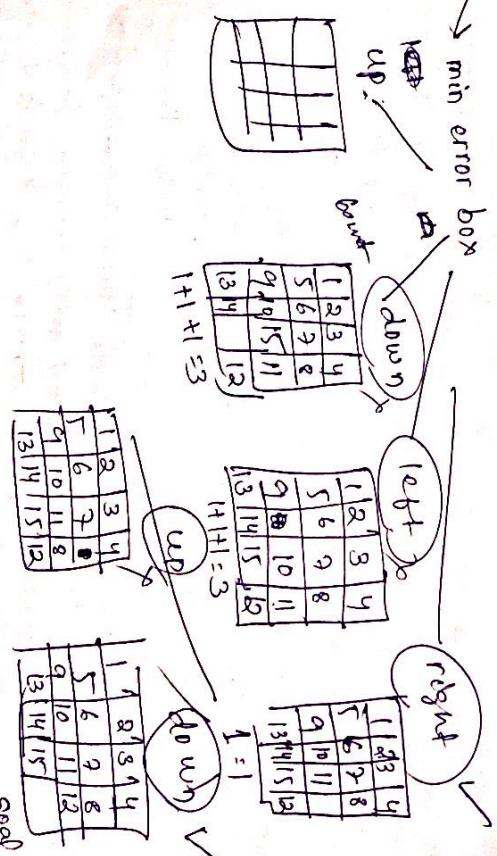
Query coordinator can breakdown the execution function into $11!$ pieces & then integrates the result produced by the query server.

15 puzzle problem by using $11!$ processors -



Branch & Bound Concept

- find min.error box - bound
other box - bound



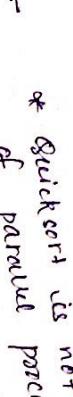
④ Quick sort

* Quicksort is not cost optimal in case of parallel processes.

& If we assign no. of processor for division process of particular array, then quick sort is cost optimal.

Implementation of quick sort &

⑤ Odd-even sort



Implementation of quick sort &

⑥ Hyper quick sort

is an hyper quick sort. 9+8 steps are -

1) divide the unsorted list among each node

2) sort each node locally from node 'o'

3) broadcast the median value

4) split each list locally, then exchange the halves across the highest dimension.

5) Repeat step 4 until the dimension reaches

5) Repeat steps 1-4 in (b) until the dimension reaches zero.

Odd-even Transposition sort- It is based on the bubble sort technique. It compares 2 adjacent nos & switches them if the 1st no. > 2nd no. To get an ascending order series. The opposite case applies for a descending series. The opposite case applies for a descending series.

| order | series | odd face | odd face | odd face | odd face |
|-------|--------|----------|----------|----------|----------|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 | 8 | 8 |

processor

(acc. to index)
3 → p₁

→ p₂
2 when

p_1 & p_2 will communicate

processor

loop iteration:

- $i = 0$: printf("%d", 0);
- $i = 2$: printf("%d", 2);
- $i = 4$: printf("%d", 4);
- $i = 6$: printf("%d", 6);

Value of i printed: 3

step-2
divide even
odd

Diagram illustrating step 2 of the Sieve of Eratosthenes algorithm. The process involves two sieving steps, P2 and P3, to find prime numbers up to 100.

Step 2:

- P2 (Crossing out even numbers):** Starts with the first even number, 2, and crosses out all its multiples (4, 6, 8, 10, ...).
- P3 (Crossing out multiples of 3):** Starts with the first multiple of 3, 3, and crosses out all its multiples (6, 9, 12, 15, ...).

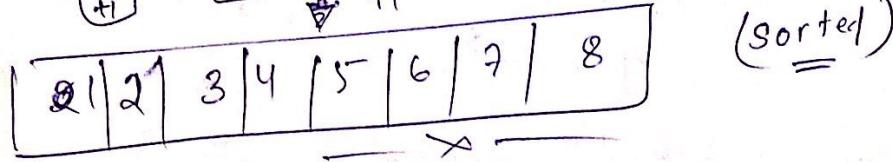
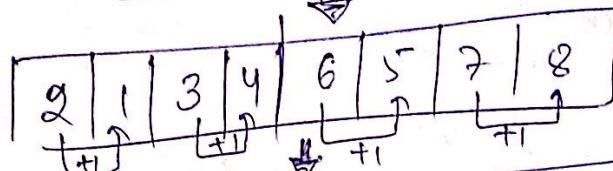
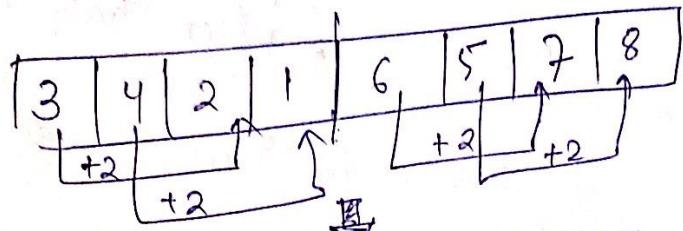
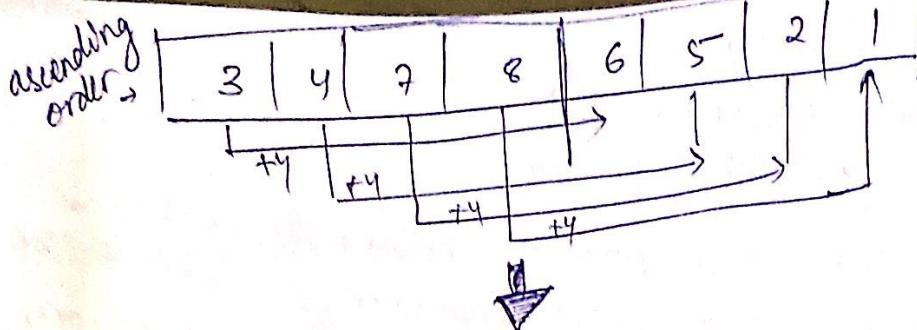
The numbers circled as prime are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

A large arrow at the bottom points to the text "sorted array".

11th Merge sort - divide & conquer
 1st divides the unsorted list.
 It just follows the parallelism principle.
 It merges sorted - It is a procedure of sorting a bitonic sequence using bitonic splits. A sequence is said to be bitonic if it gradually increases, after half position, gradually decreases.
 It gradually given below. is bitonic if there exist an index "i"
 An array given below, $\text{arr}[0, \dots, n-1]$ such that

$$0 \leq i \leq n-1$$

 where
 $x_0 < x_1 \dots < x_i \text{ and } x_i > x_{i+1} \dots > x_{n-1}$



⑨

