

# NPTEL lectures

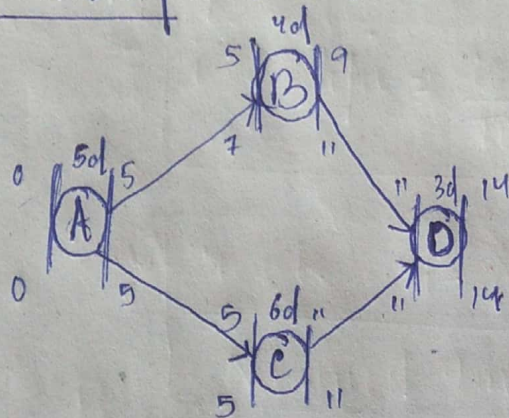
## (Week-5) Time-Cost Tradeoff (Crashing)

### 1) Fast-Tracking v/s Crashing

Rel<sup>n</sup> b/w Activity Direct Cost & Activity Duration  
Assumptions

Crashing: Reducing the duration of a project by reducing duration of activity  
↳ Also called "Time-cost tradeoff".

Example of ABCD Project.



Activity	Duration	Predecessors
A	5	-
B	4	A
C	6	A
D	3	B, C

Current duration: 14d

Expected duration: 10d

How can it be done?

- Focus on the critical activities.
- Reduce the duration of the critical activities.
- 2 ways to reduce project duration:

- Fast tracking: Performing activities in parallel.
- Crashing: Reducing the duration of activity.



\* Change of execution sequence or duration like in above 2 cases will have cost implication.

⇒ Note : Time cost trade off analysis is used to find the min<sup>m</sup> overall project cost for a specified project duration.

\* Crashing is performed by expending additional resources.

\* 2 types of duration w.r.t an activity.

→ Normal duration : Normal activity duration

→ Crash duration : Minimum duration

\* 2 types of cost :

→ Normal cost : Cost of completing activity in normal duration.

→ Crash cost : Cost of completing activity in minimum duration.

\* To reduce duration : (of an activity only)

↳ add extra resources

↳ use advanced technology

↳ Alt. construction materials (cost ↑) → premium

↳ multiple shifts demand more cost.

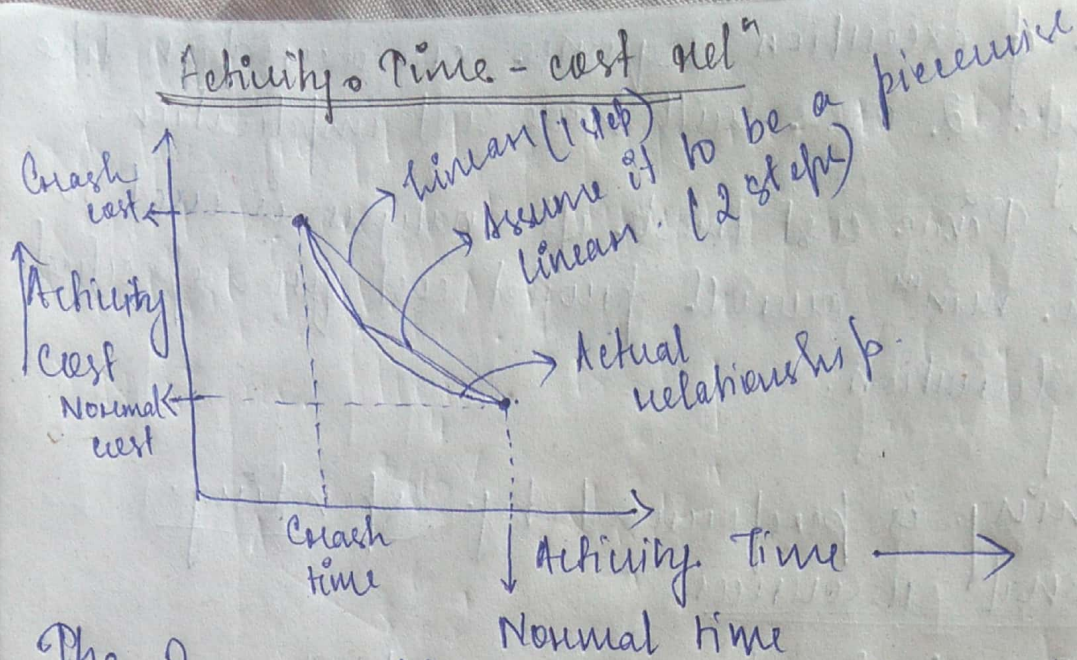
↳ overtime work (late hrs on weekends)

↳ Bring in higher capacity equipment.

↳ increase manpower / equipment

⇒ In all the above scenarios, reducing cost will require additional expenditure.





The 2 assumptions are made for calculation purpose.

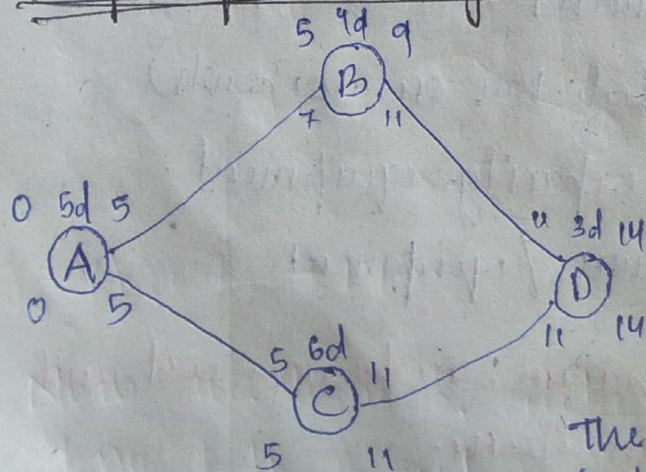
Slope of the line ~~is given~~

$$\Rightarrow \text{cost slope} = \frac{\text{Cost}}{\text{Time}}$$

$$\Rightarrow \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal Time} - \text{Crash Time}}$$

Lesson 2:

Time - cost tradeoff: ABCD example Project,  
Steps of crashing



Cost of crashing: ?  
determine minimum  
cost of completing  
project in 13, 12, ... 8 day  
Max<sup>m</sup> the project can  
be crashed to: ?

The CD given in the question  
is the least an activity that  
can be crashed.



Act	ND	Pred	CD	NE	CE	Crash days	CC/day
A	5	-	3	250	300	2	25
B	4	A	3	300	375	1	75
C	6	A	3	350	875	3	175
D	3	B, C	2	300	350	1	50

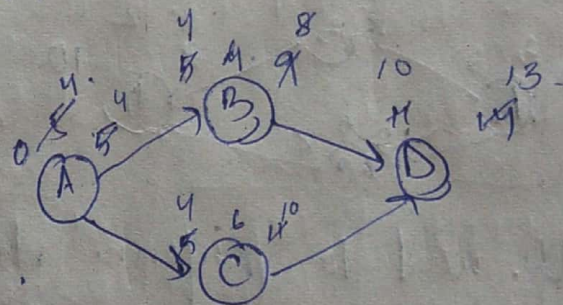
① A → C → D is the critical path.  
Find unit cost of crashing A, C, D & use it as the benchmark.

② Find Crash days  
= ND - CD. (Normal duration - Crash duration)

③ Crash cost/day =  $\frac{(CE - NE)}{(ND - CD)}$

Need not use B.

A - 25/day } choose to crash A.  
C - 175/d  
D - 50/d



→ It cost us 25/day.

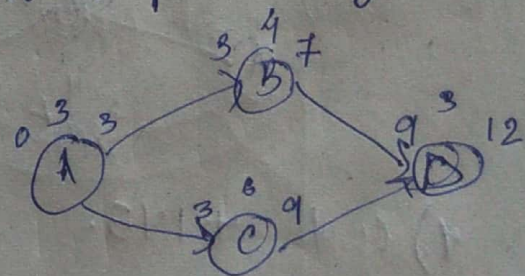
→ In 14 days, the normal cost + 25  
" 13 days, " " "

NOW FOR 12 days:

① In above diagram, the critical path is again ACD.

② CC/day = 25/day.

③ Hence reduce A by 1.



→ 14) NC.

13) 25

12) + 25

Cumulative = 50 (25 + 25)

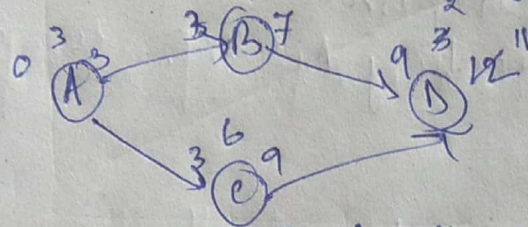


11 days

⇒ A has reached 3 days i.e. the CD. Hence can't be crashed anymore.

⇒ Critical Path - ACD

⇒ Next we choose D by 50/day. Crash D.



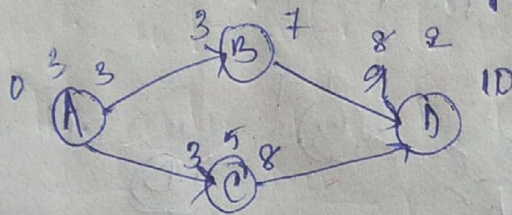
Now D reached the max<sup>m</sup> crash.

11) cost = + 50.

10 days

⇒ Critical Path stays same.

⇒ Crash C. at 175/day.



C can be crashed further by 3 days after D. But not at a time. Because at 4, there will be 2 critical paths.

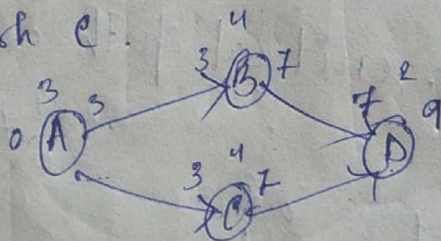
We need to perform step wise reduction.

10) cost = + 175.

9 days

Critical Path ACD

Crash C.



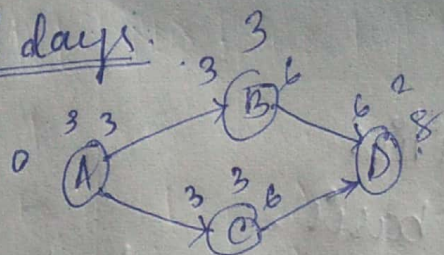
9) cost = + 175.

Now we have 2 critical paths.

Hence both B and C are crashed.



8 days.



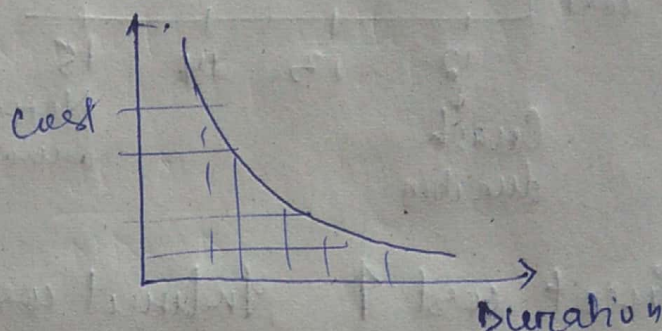
B<sub>1</sub> reached max crash limit.

$$\text{Cost} = +145 + 75 = +220.$$

Now add an attribute "Direct cost"

	<u>Direct cost</u>
14	$\Sigma NC = 1200$
13	$+25 = 1225$
12	$+25 = 1250$
11	$+50 = 1300$
10	$+175 = 1475$
9	$+175 = 1650$
8	$+250 = 1900$

Total direct cost on the project = 1900.



### Crashing cost - Summary

<u>Step no</u>	<u>Project duration days.</u>	<u>Activity crashed</u>	<u>Crash cost.</u>	<u>Cumulative cost.</u>	<u>Total project Direct cost</u>
0	14	None	0	0	1200
1	13	A by 1	25	25	1225
2	12	A by 1	25	50	1250
:	:	:	:	:	:

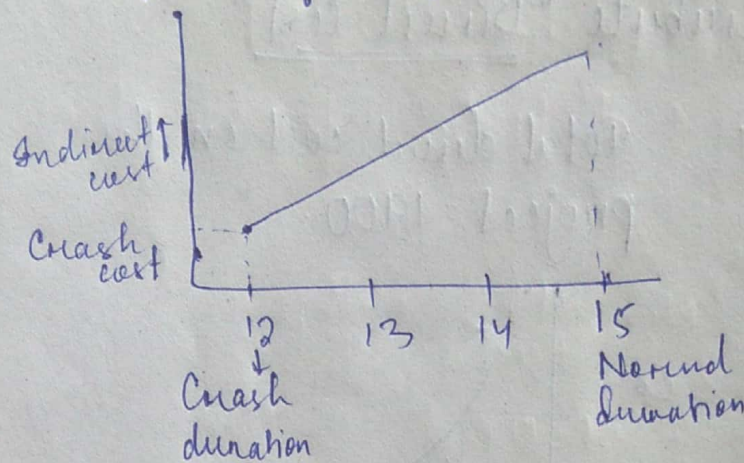
\* We can plot the max<sup>m</sup> cost graph too

\* Not used in real world. Because the linear assumption might lead to variations.



## Indirect cost

- \* Not related to any of the activities.
- \* Remain same on a daily basis.
- \*  $\Rightarrow$  If no. of days reduced, indirect cost also  $\downarrow$

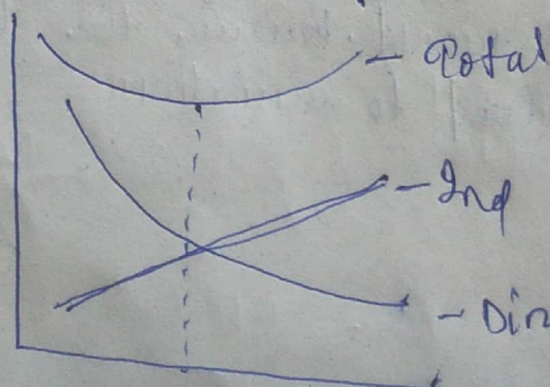


Direct cost  $\uparrow$  Indirect cost  $\downarrow$  duration  $\downarrow$

$\Rightarrow$  There must be an optimum value of both direct and indirect where project cost is min.

\* Let the Indirect cost = 100/day.

<u>Ind.</u>	<u>Dir</u>	<u>Sum</u>	
14 - 1400	1200	2600	decreases
13 - 1300	1225	2525	
12 - 1200	1250	2450	
11 - 1100	1300	2400	$\rightarrow$ minimum.
10 - 1000	1475	2475	increases
9 - 900	1650	2550	
8 - 800	1900	2700	





## List of Indirect Costs

### → Project overhead.

- Staff, manager, project engineer salary.
- Office equipment and temporary utilities like electricity, phone.

### → General overhead.

- Main office expence, rent, maintainance.
- Site equipment and vehicles.
- Lawyers and accountants.
- Advertising and social activities.

### → Contingency fee.

- Additional cost allocated for unknown events.

## Steps of crashing (from example)

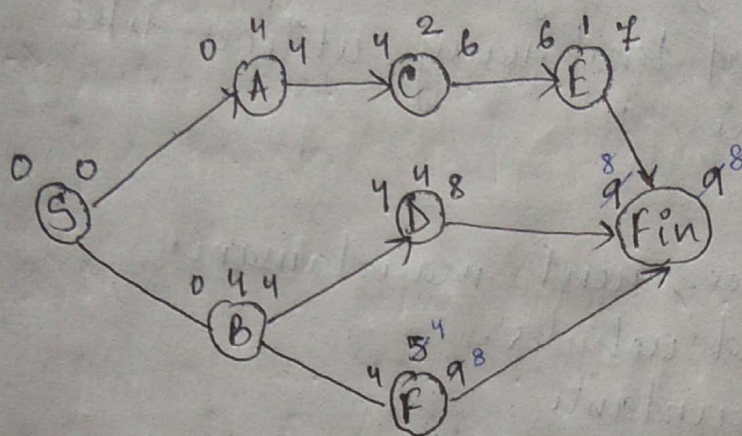
1. Identify activities on critical path(s).
2. Compare unit cost of crashing (critical) activities/combinations available for crashing.\*
3. Select activity/combinations with minimum unit cost. In case of a tie, activity which influences more paths.
4. Reduce the duration of activity/combination. (ensure no other path becomes critical if duration reduction is by more than 1 day)
5. Recalculate n/w parameters and go to step 1.

\* Repeat steps until activity/combinations cannot undergo further crashing.



# Lesson 3.

## Time-cost tradeoff: Exercise



Assume indirect cost of 125/day.

Act.	ND	ED	NE	CE	cc/day
A	4	3	100	125	25
B	4	3	400	400	150
C	2	1	150	300	150
D	4	1	150	600	150
E	1	0.5	200	400	400
F	5	2	200	350	50

Critical Path - A ~~S~~ → B → F → Fin.

9 days cc/day B = 150/day

F = 50/day ✓ crash.

F reduced from 5 to 4.

14 days. New 2 critical paths.

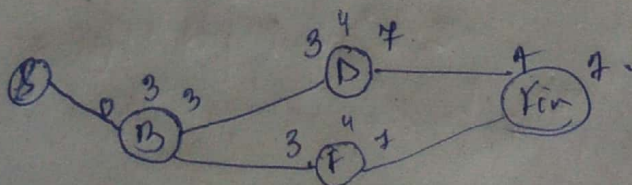
B → D → Fin.

B → F → Fin.

Crash B @ 150/day ✓ crash.

D @ 2 F @ 150 + 50 = 200/day

B crashed from 4 to 3 (Reached limit)





6 days Now we have 3 critical paths.

$$A \rightarrow C \rightarrow E = 7$$

$$B \rightarrow D = 7$$

$$B \rightarrow F = 7$$

B can't be crashed further.

Hence we can crash ~~A @ 25/day~~.

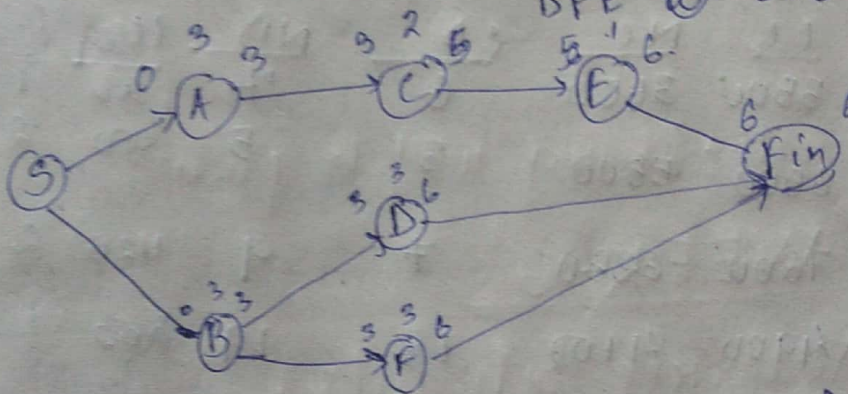
C, D, F @ 350/day

~~E @ 400/day~~

DFA @ 285 ✓

DFF @ 600

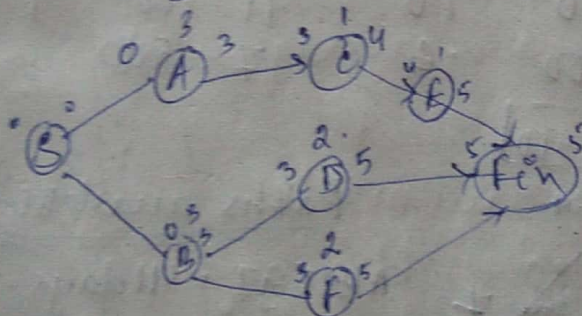
?? Why combinations?



Now A and B are fully crashed.

5 days DFE @ 350 ✓ crash

DFE @ 600



Now A, B, C, F have crashed to the limit.

Cost table

Days	Direct cost	Indirect	Total cost
9	1350	125 x 9 = 1125	2475
8		1000	2400
7		875	2425
6		750	2525
5		625	2750

Draw graph



# Lesson 4

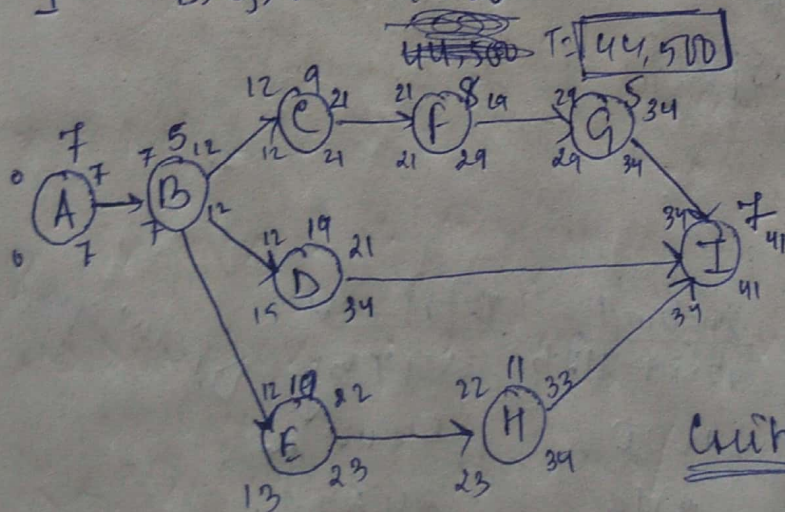
## Time Cost Trade off: Problem 3, Tabulation

### Approach

→ later we will see influence such as bonus/penalty on total project cost.

Q) Develop a new diagram and develop least cost curve for the project. Assume an indirect cost of 200/day.

Act	Preceded by	CC	NC	CD	ND	CC/day	Reduction
A	-	3500	3600	6	7	350	1
B	A	6500	5500	3	5	500	2
C	B	7200	6350	7	9	425	2
D	B	4900	4100	18	19	200	1
E	B	2200	2050	9	10	150	1
F	C	1700	1200	6	8	250	2
G	F	7000	7200	5	5	NA	0
H	E	10000	9400	10	11	600	1
I	D, G, H	4700	4500	6	7	200	1



Paths available:

ABCEFGI = 41 days.  
ABDI = 38 days.  
ABFHI = 40 days.

Critical = ABCEFGI

Red I by 1 @ 200.

I is marked. Critical Path remains same.  
All the paths reduced by 1. (New diag)



② f by 1 @ 250 (N/w diag)

③ Paths: 39 ✓  
37  
39 ✓

Now combinations / singles :

Possible combinations : CE - 545  
CH - 1025  
FE - 400  
FH - 850

on singles like A - 300 ✓  
B - 500

A by 1 @ 300 (N/w diag)

④ Same critical paths like above.

Possible comb: CE, CH, FE, FH, B.

FE by 1 @ 400 (N/w diag update)

(Continue the process above)

⇒ Sometimes become messy to draw the n/w diagrams all the time and update them.

Hence the tabulation as follows :-

→ Screenshot

Act	Paths requiring			Cost slope	Earliest crash time NS-ED	Iteration					
	nodes					1	2	3			
A B C D E F	1		0								
Initial path length	41	38	40	Iteration	Action			Iteration cost		Cum cost	
Path length	41	38	40	0						44500	



## Lesson - 5

(See again)

### Incorporating factors such as Bonus and Penalty, Problem 4.

- \* Contract may specify bonus/on penalty depending on owner's requirements.
- \* Specification can vary widely but it is typical to band in on a per day within/excess of normal duration.
- \* It's an incentive/disincentive for contractor for early/completion.
- \* Per day penalty/bonus will be given.