Parameter Invariant Monitoring for Signal Temporal Logic

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Motivations

Challenges in monitoring Real Time Systems -

- *Partially Observable* States.
- *Partially Observable* Traces.
- *High Computation* Cost.

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Extends Parameter Invariant (PAIN) tests to -

- STL to support continuous systems
- Efficiently monitor STL online.

Example

Monitoring a Diabetic Patient

STL formula, "If patient has a meal (M), then (s)he either recieved bolus t_1 units ago, or will recieve one in t_2 units"

$$\Box_{[t_1,\infty)}\left((M > c_1) \rightarrow \Diamond_{(-t_1,t_2)}(B > c_2)\right)$$

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- *State* variables not directly Observable.
- *Output* variables affected by sensor and environment noise.
- Models are *parametric*. Estimating (*nuissance*) parameters is not feasible.

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- Rely on *probabilistic beliefs* about system state at a given point in time.

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- B (Bolus) is observable, but M (Meal) is not. Furthermore, B can be affected by noise.
- Avoid relying on exact models of the system.
- Rely on *probabilistic beliefs* about system state at a given point in time.
- Furthermore, use *maximally invariant* statistics to derive *probabilistic beliefs*

Signal Temporal Logic

Signals

Given a finite set of variables Z, signal $f : \mathbb{R}_{\geq 0} \to \mathbb{R}^{\mathsf{Z}}$ is a mapping from a point in *time* to a *valuation* of Z.

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Formulas over Signals

- An formula $\theta : \mathbb{R}^{\mathsf{Z}} \rightarrow \mathbb{R}$ maps a *valuation* to a real value.
- Function $\theta \circ f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ maps points in time to values defined by θ.
- At time $t \in \mathbb{R}_{\geq 0}$, signal f is *true* w.r.t. θ whenever $\theta(f(t)) > 0$. Conversely, f is *false* w.r.t θ w.r.t. θ whenever $\theta(f(t)) < 0$.
- Whenever, $\theta(f(t)) = 0$, value of f is *unknown* on θ at time t.
- $\theta(f(t))$, is the *robustness degree*. Larger *robustness degree* values signify greater *belief* about signal's adherence to θ.

Signal Temporal Logic

Syntax

- $\varphi ::= \top \mid \bot \mid \Theta \mid \neg\Theta \mid \varphi \wedge \varphi \mid \varphi\, \mathcal{U}_{\mathcal{I}_{\geq 0}}\, \varphi \mid \varphi \mathcal{R}_{\mathcal{I}_{\geq 0}} \varphi$
- $\Diamond_{\mathcal{I}} \varphi \equiv \top\, \mathcal{U}_{\mathcal{I}}\, \varphi$
- $\Box_{\mathcal{I}} \varphi \equiv \bot \mathcal{R}_{\mathcal{I}} \varphi$
- Law of Excluded Middle not assumed. Robustness 0 means signal neither satisfies nor not satisfies a formula. It is possible for $\varphi \vee \neg\varphi$ to not be true.
- Law of non contradiction still holds. It is not possible for both φ and $\neg\varphi$ to be true.

Semantics

Continuous Semantics

Given signal f and a point in time $r : \mathbb{R}_{\geq 0}$, Continuous time STL semantics are given inductively as -

$$f, r \models_{\text{CNT}} \top \;:=\; \infty \qquad f, r \models_{\text{CNT}} \bot \;:=\; -\infty$$

$$f, r \models_{\text{CNT}} \theta \;:=\; \theta(f(r)) \qquad f, r \models_{\text{CNT}} \neg\theta \;:=\; -\theta(f(r))$$

$$f, r \models_{\text{CNT}} \phi \vee \psi \;:=\; f, r \models_{\text{CNT}} \phi \sqcup f, r \models_{\text{CNT}} \psi$$

$$f, r \models_{\text{CNT}} \phi \wedge \psi \;:=\; f, r \models_{\text{CNT}} \phi \sqcap f, r \models_{\text{CNT}} \psi$$

where $\sqcup$ and $\sqcap$ represent sup, inf respectively.

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$$f, r \models_{\text{CNT}} \varphi \, \mathcal{U}_{\mathcal{I}} \, \psi \; := \; \bigsqcup_{t : r + \mathcal{I}} \left(f, t \models_{\text{CNT}} \psi \sqcap \bigsqcap_{r \leq t' < t} \left(f, t' \models_{\text{CNT}} \varphi \right) \right)$$

$$f, r \models_{\text{CNT}} \varphi \, \mathcal{R}_{\mathcal{I}} \, \psi \; := \; \bigsqcap_{t : r + \mathcal{I}} \left(f, t \models_{\text{CNT}} \psi \sqcup \bigsqcup_{r \leq t' < t} \left(f, t' \models_{\text{CNT}} \varphi \right) \right)$$

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Semantics

Discrete Semantics

Given signal f and sampling function $\tau : \mathbb{N} \to \mathbb{R}_{\geq 0}$, discrete time semantics for $g := f \circ \tau : \mathbb{N} \to \mathbb{R}^{Z}$ at step $n : \mathbb{N}$ are given inductively as -

$$g, n \models_{\text{DSC}} \top \;:=\; \infty \qquad\qquad g, n \models_{\text{DSC}} \bot \;:=\; -\infty$$

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$$g, n \models_{\text{DSC}} \varphi \, \mathcal{U}_{\mathcal{I}} \, \psi \;:=\; \bigsqcup_{i : \tau^{-1}(\tau((n) + \mathcal{I})} \left(g, i \models_{\text{DSC}} \psi \sqcap \bigsqcap_{n \leq j < i} (g, j \models_{\text{DSC}} \varphi) \right)$$

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where $\sqcup$ and $\sqcap$ represent sup, inf respectively.

Strengthening STL Formulas

For STL formula φ, and $\delta : \mathbb{R}_{\geq 0}$, φ^δ (strengthening) is defined as -

- $\bot^\delta := \bot \quad \top^\delta := \top \quad p^\delta := p \quad (\neg p)^\delta := \neg p$
- $(\varphi \vee \psi)^\delta := \varphi^\delta \vee \psi^\delta \quad (\varphi \wedge \psi)^\delta := \varphi^\delta \wedge \psi^\delta$
- $(\varphi\ \mathcal{U}_{\mathcal{I}}\ \psi)^\delta := \varphi^\delta\ \mathcal{U}_{\left(\underline{\mathcal{I}}+\delta, \overline{\mathcal{I}}-\delta\right)}\ \psi^\delta$
- $(\varphi\ \mathcal{R}_{\mathcal{I}}\ \psi)^\delta := \varphi^\delta\ \mathcal{R}_{\left((\underline{\mathcal{I}}-\delta)^+, \overline{\mathcal{I}}+\delta\right)}\ \psi^\delta$

Intuitvely, strengthening slightly shortens (lengthens) intervals for $\mathcal{U}(\mathcal{R})$. Along with conditions about the signal and sampling function, strengthening allows using discrete-time STL to reason about continuous-time STL.

Discrete to Continuous Time STL

Given signal f, formula φ, *strictly increasing* sampling function τ, where $\Delta\tau = \sqcup_{n:\mathbb{N}}(\tau(n+1) - \tau(n))$, and $\delta : \mathbb{R}_{\geq 0}$, where

- $\exists \lambda : \mathbb{R}_{+}$ such that for $\theta : \Theta$, $\theta \circ f$ is λ-*Lipschitz* continuous.
- δ is sufficiently large - $\lambda \Delta\tau < \delta$.
- Sampling is sufficient - $\Delta\tau < \frac{1}{3}\min_{\mathcal{I}:\mathcal{I}_{\varphi}}(\overline{\mathcal{I}} - \underline{\mathcal{I}})$, where $\mathcal{I}_{\varphi}$ are intervals in temporal operators of φ.
- Sampling started early - $\tau(0) < \Delta\tau$

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guarantees the following hold -

- $f \circ \tau \models_{\text{DSC}} \varphi^{\Delta\tau} \implies f \models_{\text{CNT}} \varphi > 0$.
- $f \circ \tau \models_{\text{DSC}} \neg\varphi^{\Delta\tau} \implies f \models_{\text{CNT}} \varphi < 0$.

Instead of proving $f \models_{\text{CNT}} \varphi > 0$, prove $f, \tau \models_{\text{DSC}} \varphi^{\Delta\tau} > \delta$.

Partially Observable States

Say we have test functions Θ, signal f and sampling time τ. For $\theta : \Theta$, $\theta \circ f$ cannot be directly observed.

- $\mathcal{O}(\theta, f, t) = F\mu + \theta^{+}(f(t))G_0\rho_0 + \theta^{-}(f(t))G_1\rho_1 + \sigma n$
- Y is a set of *observable* variables.
- $\mu : \text{dom}(\mu) \to \mathbb{R}, \rho_0 : \text{dom}(\rho_i) \to \mathbb{R}$ for $i \in \{0, 1\}$ are *unknown nuisance* vectors.
- $F : \mathbb{R}^{Y \times \text{dom}(\mu)}$, $G_i : \mathbb{R}^{Y \times \text{dom}(\rho_i)}$ for $i \in \{0, 1\}$ are *known signal matrices*.
- σ is unkown noise multiplier, n is random noise.

Partially Observable States

- It is assumed that for each *test function* $\theta, \theta' \in \Theta$, $\theta \neq \theta' \implies Y_\theta \cup Y_{\theta'} = \emptyset$. In other words, different test functions use *disjoint observable variables*.
- $\mathcal{O}(f, t)$ - random variable corresponding to the observation of signal f at time t is *uniquely determined* by $\mathcal{O}(\theta, f, t)$ for each $\theta \in \Theta$.

Invariance to Transformations

- Given Ω, the sampling space of a probabily-space and $\mathcal{K} \subseteq \Omega \to \Omega$ a group of transformations.
- Let $\eta : \Omega \to \Omega'$ be a statistic.
- η is said to be *invariant* to group of transformations $\mathcal{K}$ iff

$$\forall \omega : \Omega, k : \mathcal{K}.\eta(\omega) = \eta(k(\omega))$$

- η is said to be *maximally invariant* to group of transformations $\mathcal{K}$ iff

$$\forall \omega, \omega' : \Omega.\eta(\omega) = \eta(\omega') \implies (\exists k : \mathcal{K}.\omega = k(\omega'))$$

Parameter Invariant Robustness Estimation of Atomic Propositions

Suppose we have

- $\Omega_{f,\tau} : \mathbb{N} \rightarrow \mathbb{R}^X$, be the space of all observations at different sample time
- A sequence $x : \Omega_{f,\tau}$ test function $\theta \in \Theta$, error and indifference bounds $\alpha', \delta' : \mathbb{R}^+$.

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One can find a algorithm $\mathcal{A}(x, \tau, \theta, \alpha', \delta')$ such that -

- $\mathcal{A}$ is *terminating* and returns value $r : \mathbb{R}$.
- $\mathbb{P}(\mid (\theta \circ f \circ \tau)(0) - r \mid > \delta') < \alpha'$
- The *Probability* that r (i.e. the *robustness degree estimate* returned by $\mathcal{A}$) is more than δ' away from the real value of $f \circ \tau \models_{\text{DSC}} \theta$ is less than α'.

Parameter Invariant Robustness Estimation of Atomic Propositions

- If *statistic* used in $\mathcal{A}$ is *Maximally Invariant*, then r, is invariant to nuissance parameters.
- A change in the *noise* in sensor measurements will not affect r.
- Use $\mathcal{A}$ to obtain estimates that are *(maximally) invariant* to nuissance parameters.

Parameter Invariant Estimation For Atomic Formulas

- Using $\mathcal{A}$, we get *robustness estimates* for atomic proposition $\theta \in \Theta$.
- Furthermore, if the estimate in $\mathcal{A}$ is derived using a *maximally invariant* test statistic, the estimate is *invariant* to *nuissance* parameters (like noise).
- For example, robustness estimate will be invariant to sensor noise.

Parameter Invariant Estimation For Atomic Formulas

- For given θ, say Y_θ it the set of *observable variables* of θ.
- Let $\mathcal{K}_{\theta,f,\tau(n)} \subseteq \mathbb{R}^{Y_\theta} \to \mathbb{R}^{Y_\theta}$ be group of transformations affecting θ.
- Let $\mathcal{K}_{f,\tau(n)} \subseteq \mathbb{R}^{Y} \to \mathbb{R}^{Y}$ be group of transformations for f at step n.
- For $v \in \mathbb{R}^Y$, $\theta \in \Theta$, function $k \in \mathcal{K}_{f,\tau(n)}$ maps $v\restriction^{Y_\theta}$ to some $k_\theta(v\restriction^{Y_\theta})$ where $k_\theta \in \mathcal{K}_{\theta,f,\tau(n)}$.
- $\mathcal{K}_{f,\tau(n)}$ is *completely determined* by $\mathcal{K}_{\theta,f,\tau(n)}$ since $\theta \neq \theta' \implies Y_\theta \cap Y_{\theta'} = \emptyset$

Parameter Invariant Estimation For Atomic Formulas

- Let $\eta_\theta : \mathbb{R}^{Y_\theta} \to \mathbb{R}$ be statistics that produces *estimates* for $\theta \in \Theta$.
- Define $\eta : \mathbb{R}^{Y} \to \mathbb{R}$ such that for $v \in \mathbb{R}^{Y}$, $\eta(v) = \eta_\theta(v\restriction^{Y_\theta})$ for $\theta \in \Theta$.
- Suppose η_θ is invariant to $\mathcal{K}_{\theta,f,\tau(n)}$ for each $\theta \in \Theta$.

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- Suppose η_θ is invariant to $\mathcal{K}_{\theta,f,\tau(n)}$ for each $\theta \in \Theta$.
- Formally, for $v : \mathbb{R}^{Y}$ and $k_\theta \in \mathcal{K}_{\theta,f,\tau(n)}$, $\eta_\theta(v\restriction^{Y_\theta}) = \eta_\theta(k_\theta(v\restriction^{Y_\theta}))$
- For $k \in \mathcal{K}_{f,\tau(n)}$, $k(v) = k_\theta(v\restriction^{Y_\theta})$.
- $\eta(k(v)) = \eta_\theta(k(v)) = \eta_\theta(k_\theta(v\restriction^{\mathbb{R}^{Y_\theta}}))$, for each $\theta \in \Theta$.
- We get $\eta(k(v)) = \eta_\theta(k_\theta(v\restriction^{Y_\theta})) = \eta_\theta(v\restriction^{Y_\theta})$ for each $\theta \in \Theta$.
- Thus, η is invariant to $\mathcal{K}_{f,\tau(n)}$

Parameter Invariant Estimation For Atomic Formulas

- We proved η is invariant to $\mathcal{K}_{f,\tau(n)}$. Moreover, if η_θ is *maximally invariant*, to $\mathcal{K}_{f,\tau(n)}$, then η is also *maximally invariant*.
- Thus, using *maximally invariant* estimates for *atomic propositions*, we get a *maximally invariant* estimate for the entire signal at a point in time.
- Recall $\mathcal{A}$ gives us an *estimate* for $\theta \in \Theta$. If $\mathcal{A}$ uses a *maximally invariant* statistic, we get a *maximally invariant* estimate for each *atomic proposition* in an STL formula.

Assumptions

Given a signal f, sampling-time function τ, STL formula φ, such that τ is strictly increasing. $g = f \circ \tau$ is *discretized signal*

- $\mathcal{M}$ is an algorithm for monitoring g against φ and -
  - needs a *complete mapping* of all *atomic propositions* in φ to *robustness degrees*.
  - takes *input* a pair (v, t), where v is a *complete mapping* of *atomic propositions* to *robustness* and t is time.
  - return interval $[a, b]$ - the inf and sup respectively of all possible robustness degrees if the observed sequence was arbitrarily extended.
- Main Algorithm (following slides) join $\mathcal{A}$, $\mathcal{M}$ to have a Parameter Invariant Monitor for STL.

Algorithm

- (**Inputs**) signal f, formula φ Lipschitz constant λ, indifference region δ and an error parameter α.
- (**Output**) accept, reject or unknown
- Has 3 functions -
    - **MAIN** - Initializes the algorithm, runs main loop, and checks termination conditions.
    - **ESTIMATE** - Uses $\mathcal{A}$ to estimate values for each atomic proposition of φ.
    - **OBSERVE** - Auxilliary function used by estimate to obtain *estimates* for atomic propositions.

Algorithm

Main Function

$x \leftarrow$ nil ▷ Sequence of observations
$n \leftarrow 0$ ▷ Current Step
function MAIN
  $\mathcal{M}_1 \leftarrow$ initialize $\mathcal{M}$ for$\varphi^{\Delta\tau}$
  $\mathcal{M}_2 \leftarrow$ Initialize $\mathcal{M}$ for$\neg(\varphi)^{\Delta\tau}$
  $[a_1, b_1] \leftarrow (-\infty, \infty)$
  $[a_2, b_2] \leftarrow (-\infty, \infty)$
  while $a_1 \leq \delta \wedge a_2 \leq \delta \wedge (\delta < b_1 \wedge \delta < b_2)$ **do**
    $(t, v) \leftarrow$ ESTIMATE
    $[a_1, b_1] \leftarrow \mathcal{M}_1(v, t)$
    $[a_2, b_2] \leftarrow \mathcal{M}_2(v, t)$
    $n \leftarrow n + 1$
if $a_1 > \delta$ **then return** accept
if $a_2 > \delta$ **then return** reject
return unknown

Algorithm

Estimation Function

ESTIMATE uses $\mathcal{A}$ to calculate estimates for each atomic proposition θ.

function ESTIMATE
  $v : R^{\Theta}$
  for all $\theta \in \Theta$ **do**
    $a \leftarrow \mathcal{A}(\text{OBSERVE}, \theta, \alpha', \delta')$
    if $a - \delta' > 0$ **then**
      $v(\theta) \leftarrow a - \delta'$
    else if $a + \delta' < 0$ **then**
      $v(\theta) \leftarrow a + \delta'$
    else
      $v(\theta) \leftarrow 0$
  // Recall x is global list containing observations
  $(t, \_) \leftarrow pop(x)$
  return (t, v)

Algorithm

Observation Function

function $\textsc{observe}(n : \mathbb{N})$
  while $|x| \leq n$ **do**
    $v : \mathbb{R}^{Y}$
    $t \leftarrow$ current time
    for all $\theta \in \Theta$ **do**
      $v{\restriction}^{Y_\theta} \leftarrow$ Sensor reading for θ
    $x \leftarrow x + (t, v)$
  $(t, v) \leftarrow$ n$^{\text{th}}$ element of x
  return $(t, v{\restriction}^{Y_\theta})$

- *OBSERVE* allows *ESTIMATE* to observe multiple sensor readings at a given point in time.

Algorithm

Execution Flow

- MAIN initializes $\mathcal{M}$ for $\varphi^{\Delta\tau}$ and its negation.
- $\mathcal{M}$ is stateful. After initialization, every call to $\mathcal{M}$ with observation-time pair (v, t) makes $\mathcal{M}$ record pair.
- ESTIMATE returns *robustness degrees* for each atomic proposition, using sensor readings.
- If $\mathcal{A}$ uses a *maximally invariant* statistic to calculate *robustness estimate*, then *estimate* is invariant to noise.

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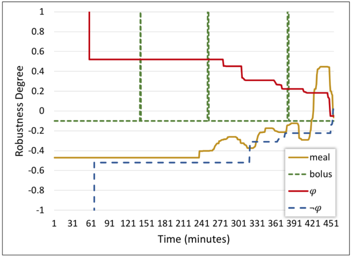
. Formula above not strictly STL (uses past time). Equivalently-

$$\Box_{[t_1,\infty)}\left(\Diamond_{(0,t_1+t_2)}(B > c_2) \vee \Box_{[t_1,t_1+t_2)}\left((M > c_1) \rightarrow \Diamond_{(0,t_2)}((B > c_2))\right)\right)$$

"At all points $r : \mathbb{R}_{\geq 0}$, either patient recieved Bolus or if meal is taken, then Bolus should be taken no later than t_2 units"

Example

Monitoring a Diabetic Patient



- $t_1 = t_2 = 30(minutes)$
- Plot shows b_1 for φ and a_2 for $\neg\varphi$
- Meal occurs at $t = 421$, but no bolus detected in $391 \leq t \leq 421$. Thus, when t slightly above 451 formula is *rejected*, as $a_2 > \delta$.
- Noisy *B (bolus)* signal, unobservable *M (meal)* variable

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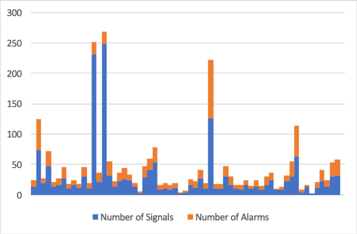

- Data for 61 patients, each monitored for different time lengths.
- Plot shows *alarms* vs *number of observations* for each patient.
- Evidence technique scales well for real world applications

Conclusions

- Introduce Parameteric Invariant Monitoring for Signal Temporal Logic
- Using *robustness estimates* for single atomic propositions, derive *robustness estimates* for entire formula.
- If *robustness estimates* for atomic propositions is *maximally invariant* then test procedure for entire formula is also *maximally invariant* to nuissance paramters.
- Demonstrate real-world applications on monitoring an STL formula with patients having $T1 - Diabetes$.