Technical Note with Supporting Information for Bucket Wheel Reclaimer Extrinsic Parameter Calibration

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This note provides additional information and derivations that are referred to in [1]. The main article should be read first. This technical note includes materials that could not fit within the journal page constraints. The note is organized as follows. Section I explains the plane parameter extraction method stated in Section VI-C of [1]. Section II provides derivation of eqn. (15) of [1]. Section III provides derivation of eqn. (20) of [1].

I. Plane Parameter Extraction

Standard methods for estimation of plane parameters from a set S of points on the plane can be divided into two different categories: averaging and optimization. Optimization based methods such as Principle Component Analysis (PCA) and Singular Value Decomposition (SVD) outperform averaging methods both in computational performance and quality [2].

For $p_i \in S$ the mean is

$$\bar{p}_S = \frac{1}{n}\sum_{i=1}^{n} p_i.$$

and the covariance matrix is

$$\Sigma_S = \frac{1}{n-1}\left(D - \bar{p}_S\right)^\top \left(D - \bar{p}_S\right)$$

where $D = \left[p_1, p_2, \cdots p_n\right]^\top$ and n represents the cardinality of S.

In the PCA approach [2], [3], Σ_S is decomposed into its eigenvectors and eigenvalues. The eigenvectors with the largest two eigenvalues are the directions of maximum variance of the data. The vector orthogonal to these vectors, which is also the eigenvector corresponding to the smallest eigenvalue, is the estimated normal to the plane $\hat{N}_S$. The distance parameter is $\hat{d}_S = \hat{N}_S^\top \bar{p}_S$.

II. Derivation of ${}^E\tilde{p}$

This section analyzes how the platform pose and the error in the extrinsic parameters affect the computed georectified points and thus computed plane features. This is needed to understand the tradeoffs related to segmenting each planar patch dataset into two segments.

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Given a point ${}^Lp$, the georectified E-frame point is

$$\begin{aligned}{}^Ep &= {}^ET_{EP} + {}^E_PR\left({}^PT_{PL} + {}^P_LR\;{}^Lp\right)\\ &= {}^ET_{EP} + {}^E_PR\left({}^P\hat{T}_{PL} - \delta T + {}^P_L\hat{R}(I + [\rho\times])^Lp\right) \quad (1)\end{aligned}$$

where $[\rho\times]$ is the skew-symmetric representation of vector ρ and has the form:

$$[\rho\times] \;=\; \left[\begin{array}{ccc} 0 & -\rho_3 & \rho_2 \\ \rho_3 & 0 & -\rho_1 \\ -\rho_2 & \rho_1 & 0 \end{array}\right].$$

Eqn. (1) yields,

$$\begin{aligned}{}^Ep &= {}^E\hat{p} - {}^E_PR\left(\delta T - {}^P_L\hat{R}[\rho\times]\;{}^Lp\right)\\ &= {}^E\hat{p} + {}^E\tilde{p}, \qquad (2)\end{aligned}$$

where

$$\begin{aligned}{}^E\hat{p} &\doteq {}^ET_{EP} + {}^E_PR\left({}^P\hat{T}_{PL} + {}^P_L\hat{R}\;{}^Lp\right) \text{ and}\\ {}^E\tilde{p} &= -{}^E_PR\left(\delta T - {}^P_L\hat{R}[\rho\times]\;{}^Lp\right)\\ &= -{}^E_PR\left(\delta T + {}^P_L\hat{R}[{}^Lp\times]\;\rho\right) \qquad (3)\\ &= -\left[{}^E_PR,\;\; {}^E_PR\;{}^P_L\hat{R}\;[{}^Lp\times]\right]\delta\mathbf{x} \qquad (4)\end{aligned}$$

with $\delta\mathbf{x}$ as defined in Section IV-F in [1].

III. Derivation of the Jacobian matrix $\mathbf{H}$

This section analyzes how the calibration errors affect the residuals. This analysis provides insight useful for the design of the data collection experiments.

Let the environment contain a plane with normal ${}^E\hat{N}$ and distance ${}^E\hat{d}$. Residual for any point ${}^Ep$ relative to the plane is

$$r_P = {}^E\hat{N}\cdot{}^Ep - {}^E\hat{d}.$$

Replacing ${}^Ep$ using (2), the plane residual becomes

$$r_P = {}^E\hat{N}\cdot\left({}^E\hat{p} + {}^E\tilde{p}\right) - {}^E\hat{d}$$

Using (3),

$$\begin{aligned}r_P &= {}^E\hat{N}\cdot{}^E\hat{p} - {}^E\hat{d} + {}^E\hat{N}\cdot{}^E\tilde{p}\\ &= {}^E\hat{N}\cdot{}^E\hat{p} - {}^E\hat{d} - {}^E\hat{N}^\top\,{}^E_PR\left(\delta T + {}^P_L\hat{R}[{}^Lp\times]\;\rho\right)\\ &= \hat{r}_P - {}^E\hat{N}^\top\,{}^E_PR\left(\delta T + {}^P_L\hat{R}[{}^Lp\times]\;\rho\right). \qquad (5)\end{aligned}$$

The first term of eqn. (5) is the residual for point ${}^E\hat{p}$ relative to the plane with the estimated normal ${}^E\hat{N}$ and distance ${}^E\hat{d}$.

For multiple measurements, the above equation can be written in matrix form as: $\mathbf{r}(\mathbf{x}) = \mathbf{r}(\hat{\mathbf{x}}) + \mathbf{H}\delta\mathbf{x}$, where

$$\mathbf{r}(\hat{\mathbf{x}}) = \begin{bmatrix} r_1(\hat{\mathbf{x}}) \\ \cdots \\ \cdots \end{bmatrix}$$

$$\mathbf{H} = -\begin{bmatrix} {}^E\hat{N}^\top {}^E_P R & {}^E\hat{N}^\top {}^E_P R \; {}^P_L\hat{R} \; [{}^L p_\times] \\ \vdots & \vdots \\ & \end{bmatrix}, \text{ and}$$

$$\delta\mathbf{x} = \begin{bmatrix} \delta T \\ \rho \end{bmatrix} \in \mathbb{R}^6.$$

REFERENCES

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