Q1

general solution:

$$V_1(X) = A_1 e^{-X} + B_1 e^{X} \quad 0 \le X \le L_1 \quad \text{—— A}$$
$$V_{21}(X) = A_{21} e^{-X} + B_{21} e^{X} \quad L_1 \le X \le L_{21} \quad \text{—— B}$$
$$V_{22}(X) = A_{22} e^{-X} + B_{22} e^{X} \quad L_1 \le X \le L_{22} \quad \text{—— C} \qquad (2)$$

boundary conditions:

- $$\left.\frac{dV_1}{dX}\right|_{X=0} = -(r_i\lambda_c)_1 I_{app}$$

differentiating A,

$$\frac{dV_1(X)}{dX} = -A_1 e^{-x} + B_1 e^{x} = -(r_i\lambda_c)_1 I_{app}$$

$$\left.\frac{dV_1(X)}{dX}\right|_{X=0} = -A_1 + B_1 = -(r_i\lambda_c)_1 I_{app}$$

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app}$$

- $V_{21}(L_{21}) = V_{22}(L_{22}) = 0$

By substituting to B,

$$V_{21}(L_{21}) = A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0$$

By substituting to C,

$$V_{22}(L_{22}) = A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

Nodal conditions:

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1)$$

$$V_1(L_1) = V_{21}(L_1)$$

From eq Ⓐ & Ⓑ,

$$V_1(L_1) = A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1} = V_{21}(L_1)$$

$$\underline{\underline{A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0}}$$

$$V_{21}(L_1) = V_{22}(L_1)$$

From eq Ⓑ & Ⓒ,

$$V_{21}(L_1) = A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1} = V_{22}(L_1)$$

$$\underline{\underline{A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0}}$$

$$\frac{-1}{(r_i\lambda_c)_1}\frac{dV_1}{dX}\bigg|_{X=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}}{dX}\bigg|_{X=L_1} + \frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}}{dX}\bigg|_{X=L_1} \quad \text{——— } (*)$$

From eq Ⓐ,

$$V_1(x) = A_1 e^{-x} + B_1 e^{x}$$

$$\frac{V_1(X)}{dX} = -A_1 e^{-x} + B_1 e^{x}$$

$$\frac{V_1(X)}{dX}\bigg|_{X=L_1} = -A_1 e^{-L_1} + B_1 e^{L_1} \quad \text{——— } (1')$$

From eq Ⓑ,

$$V_{21}(X) = A_{21} e^{-X} + B_{21} e^{X}$$

$$\frac{V_{21}(X)}{dX}\bigg|_{X=L_1} = -A_{21} e^{-L_1} + B_{21} e^{L_1} \quad \text{——— } (2')$$

From eq Ⓒ,

$$V_{22}(X) = A_{22} e^{-X} + B_{22} e^{X}$$

$$\frac{V_{22}(X)}{dX}\bigg|_{X=L_1} = -A_{22} e^{-L_1} + B_{22} e^{L_1} \quad \text{——— } (3')$$

By substituting 1', 2', 3' to ⊛,

$$\frac{-1\left(-A_1 e^{-L_1} + B_1 e^{L_1}\right)}{(r_i\lambda_c)_1} = \frac{-\left(-A_{21} e^{-L_1} + B_{21} e^{L_1}\right)}{(r_i\lambda_c)_{21}} - \frac{\left(-A_{22} e^{-L_1} + B_{22} e^{L_1}\right)}{(r_i\lambda_c)_{22}}$$

$$\frac{-A_1 e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1 e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i\lambda_c)_{22}} = 0$$

Q2

$$\mathrm{x} = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} \tag{8}$$

equations (7) can be rewritten as the following **matrix equation**

$$\mathrm{Ax} = \mathrm{b} \tag{9}$$

where

$$\mathrm{b} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

and

$$\mathrm{A} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ -e^{-L_1}/(r_i\lambda_c)_1 & e^{L_1}/(r_i\lambda_c)_1 & e^{-L_1}/(r_i\lambda_c)_{21} & -e^{L_1}/(r_i\lambda_c)_{21} & e^{-L_1}/(r_i\lambda_c)_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \end{pmatrix}$$

$$Ax = b$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ -e^{-L_1}/(r_i\lambda_c)_1 & e^{L_1}/(r_i\lambda_c)_1 & e^{-L_1}/(r_i\lambda_c)_{21} & -e^{L_1}/(r_i\lambda_c)_{21} & e^{-L_1}/(r_i\lambda_c)_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$
\begin{pmatrix}
A_1 & -B_1 & 0 & 0 & 0 & 0 \\
0 & 0 & A_{21}e^{-L_{21}} & B_{21}e^{L_{21}} & 0 & 0 \\
0 & 0 & 0 & 0 & A_{22}e^{-L_{22}} & B_{22}e^{L_{22}} \\
A_1e^{-L_1} & B_1e^{L_1} & -A_{21}e^{-L_1} & -B_{21}e^{L_1} & 0 & 0 \\
0 & 0 & -A_{21}e^{-L_1} & B_{21}e^{L_1} & -A_{22}e^{-L_1} & -B_{22}e^{L_1} \\
-A_1e^{-L_1}/(r_i\lambda_c)_1 & B_1e^{L_1}/(r_i\lambda_c)_1 & A_{21}e^{-L_1}/(r_i\lambda_c)_{21} & -B_{21}e^{L_1}/(r_i\lambda_c)_{21} & A_{22}e^{-L_1}/(r_i\lambda_c)_{22} & -B_{22}e^{L_1}/(r_i\lambda_c)_{22}
\end{pmatrix}
=
\begin{pmatrix}
(r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{pmatrix}
$$

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app}$$

$$A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0$$

$$A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$

$$A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{-L_1} = 0$$

$$A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0$$

$$\frac{-A_1e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i\lambda_c)_{22}} = 0$$

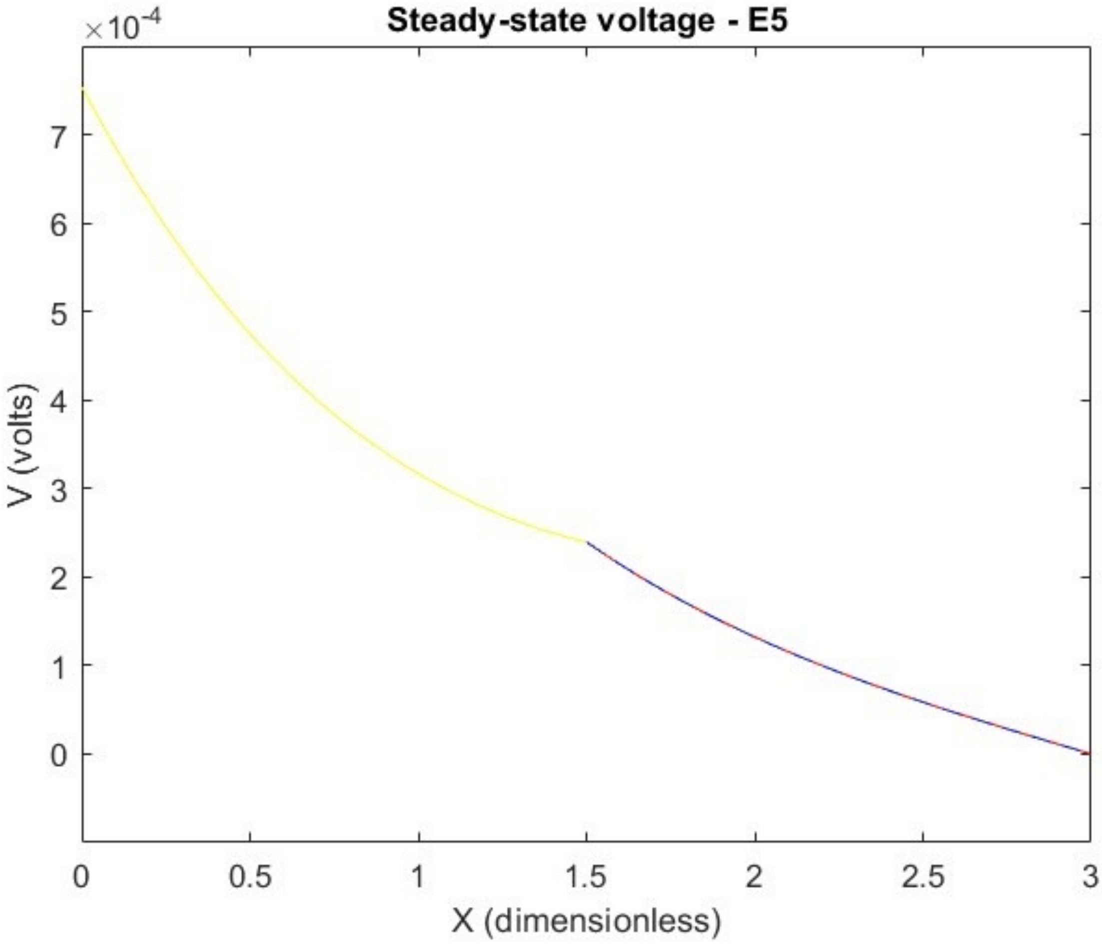
Q3

```
% electrical constants and derived quantities for typical
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;             % cm
d21 = 30e-4;            % cm
d22 = 15e-4;            % cm
%d21 = 47.2470e-4;        % E9 cm
%d22 = d21;               % E9 cm

l1 = 1.5;           % dimensionless
l21 = 3.0;          % dimensionless
l22 = 3.0;          % dimensionless

% Electrical properties of compartments

Rm = 6e3;           % Ohms cm^2
Rc = 90;            % Ohms cm
Rs = 1e6;           % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);     % Ohms
rl21 = c1*d21^(-3/2);       % Ohms
rl22 = c1*d22^(-3/2);       % Ohms


% Applied current

iapp = 1e-9;    % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22 -rl1*exp(-l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';
```

```
x =

   7.369756483719069e-04
   1.672259544622199e-05
   1.129071096533838e-03
  -2.798687438144323e-06
   1.129071096533838e-03
  -2.798687438144323e-06
```

Q4

```
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



What do you note about the steady state voltage profile in the two daughter branches?

The voltage decreases as you move away from the branch point.

Both daughter branches have the same steady state voltage profiles which can be seen from the overlapping blue and red lines.

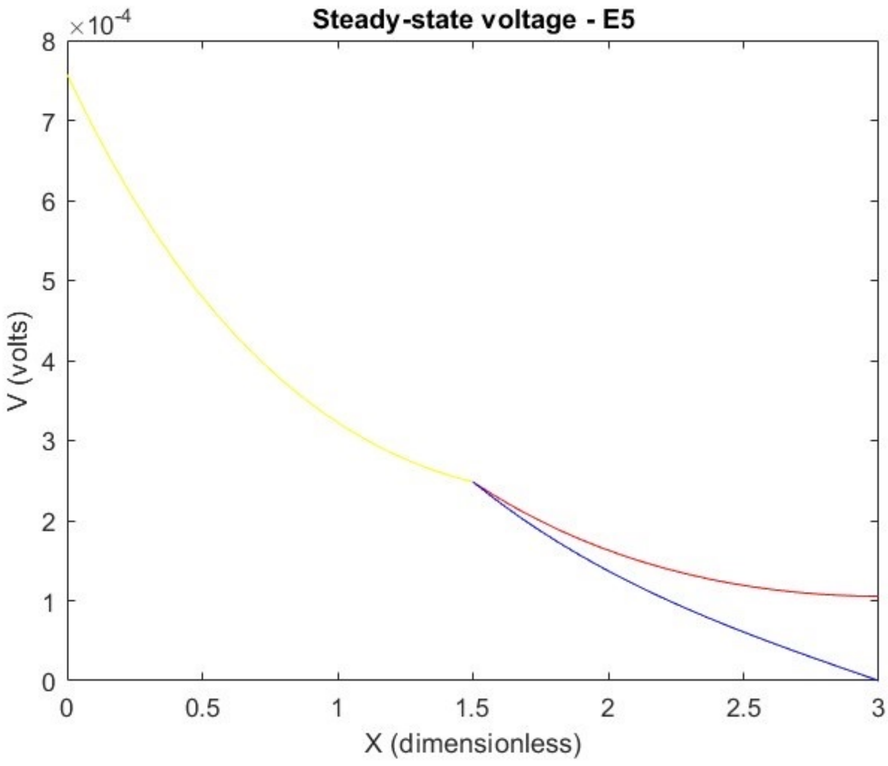
This is verified by the values we got in Q3 where $A_{21} = A_{22}$ and $B_{21} = B_{22}$

Solutions for different boundary conditions.

a)

```
A(2,:) = [0 0 -exp(-l21) exp( l21) 0 0];
% A(3,:) = [0 0 0 0 -exp(-l22) exp(-l22)];
%
% b(1) = 0;
% b(2) = rl21*iapp;
% b(3) = rl22*iapp;

x = A\b;
display(x);

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```

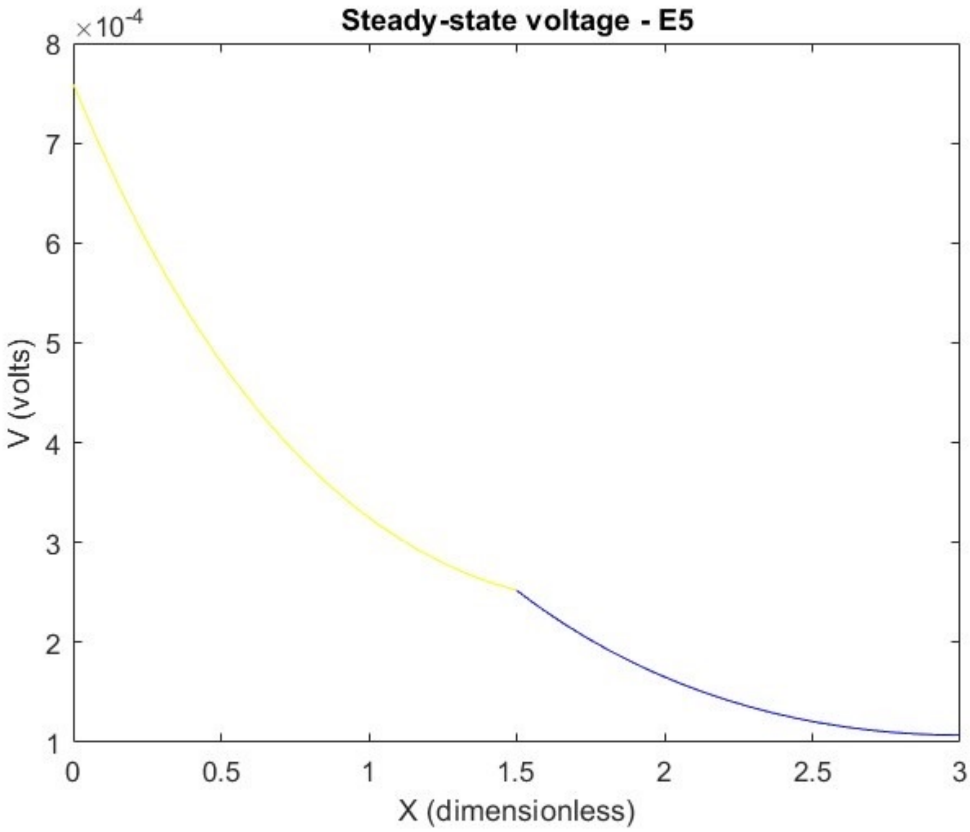
```
x =

   7.388761604453568e-04
   1.862310751967192e-05
   1.060149536692740e-03
   2.627847971668962e-06
   1.171244083412537e-03
  -2.903223821166419e-06
```



b)

```
A(2,:) = [0 0 -exp(-l21) exp( l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp( l22)];
%
% b(1) = 0;
% b(2) = rl21*iapp;
% b(3) = rl22*iapp;

x = A\b;
```

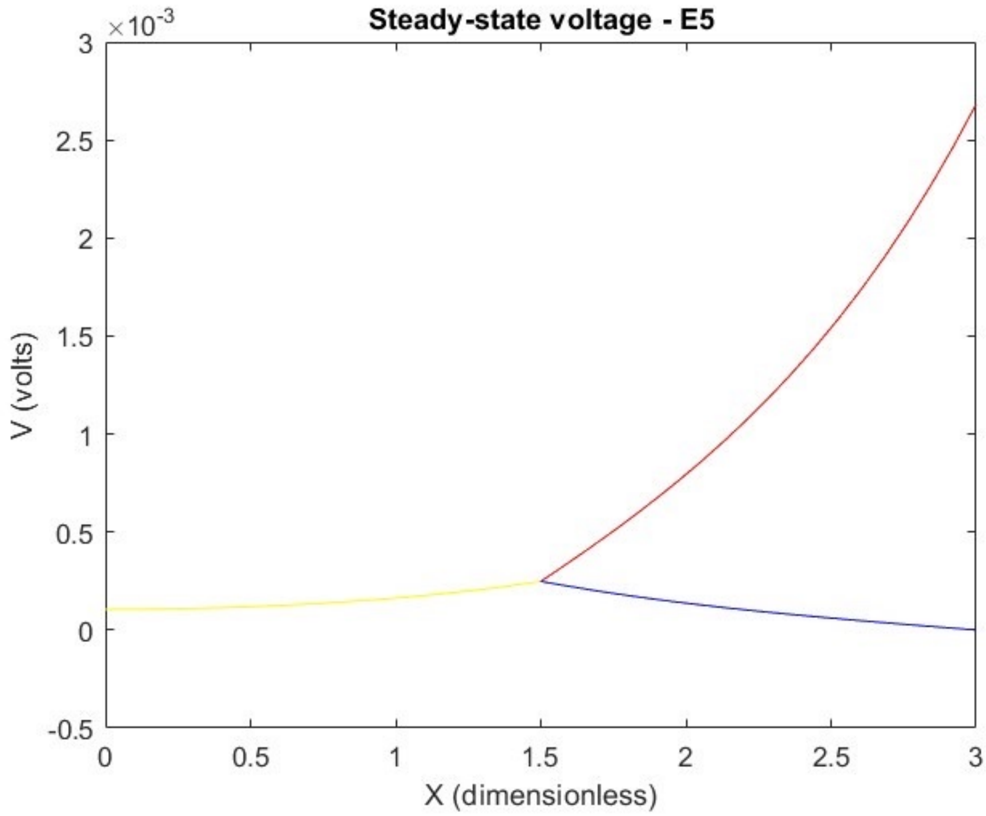
```
x =

   7.396518239155160e-04
   1.939877098983107e-05
   1.075729153962590e-03
   2.666465981888230e-06
   1.075729153962590e-03
   2.666465981888229e-06
```



c)

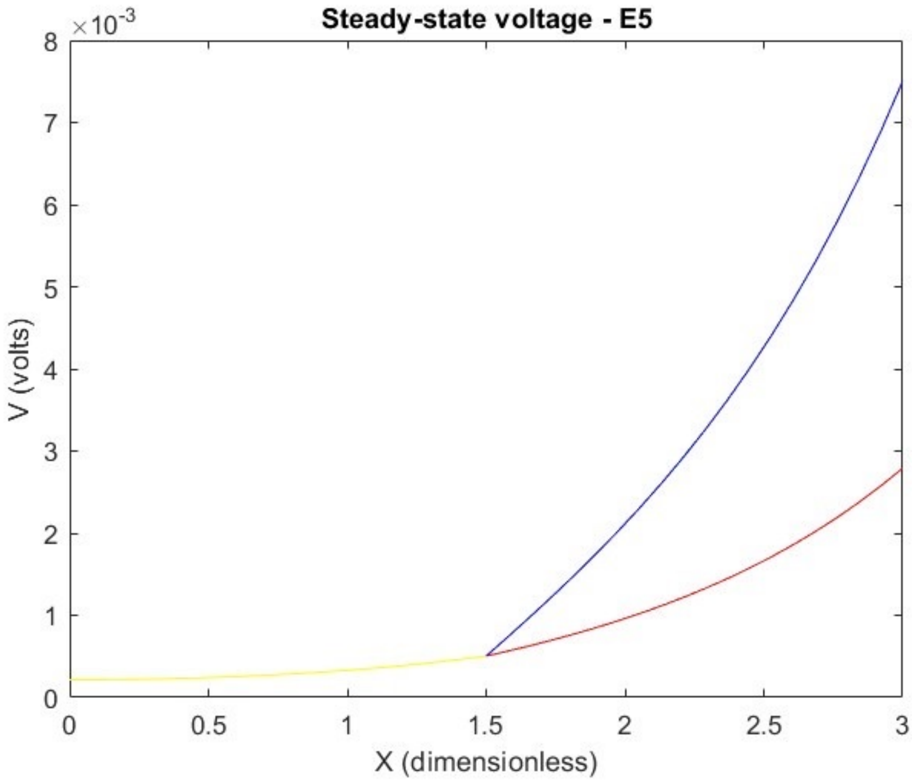
```
A(2,:) = [0 0 -exp(-l21) exp( l21) 0 0];
%A(3,:) = [0 0 0 0 -exp(-l22) exp( l22)];

b(1) = 0;
b(2) = rl21*iapp;
% b(3) = rl22*iapp;

x = A\b;
```

```
x =

    5.278173746348076e-05
    5.278173746348076e-05
   -1.651876796509386e-03
    1.376516884382506e-04
    1.171244083412537e-03
   -2.903223821166421e-06
```



d)

```
A(2,:) = [0 0 -exp(-l21) exp( l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp( l22)];

b(1) = 0;
b(2) = rl21*iapp;
b(3) = rl22*iapp;

x = A\b;

x =

    1.071148018672798e-04
    1.071148018672798e-04
   -5.605680252769466e-04
    1.403567724303581e-04
   -5.519310535929663e-03
    3.872380248268582e-04
```



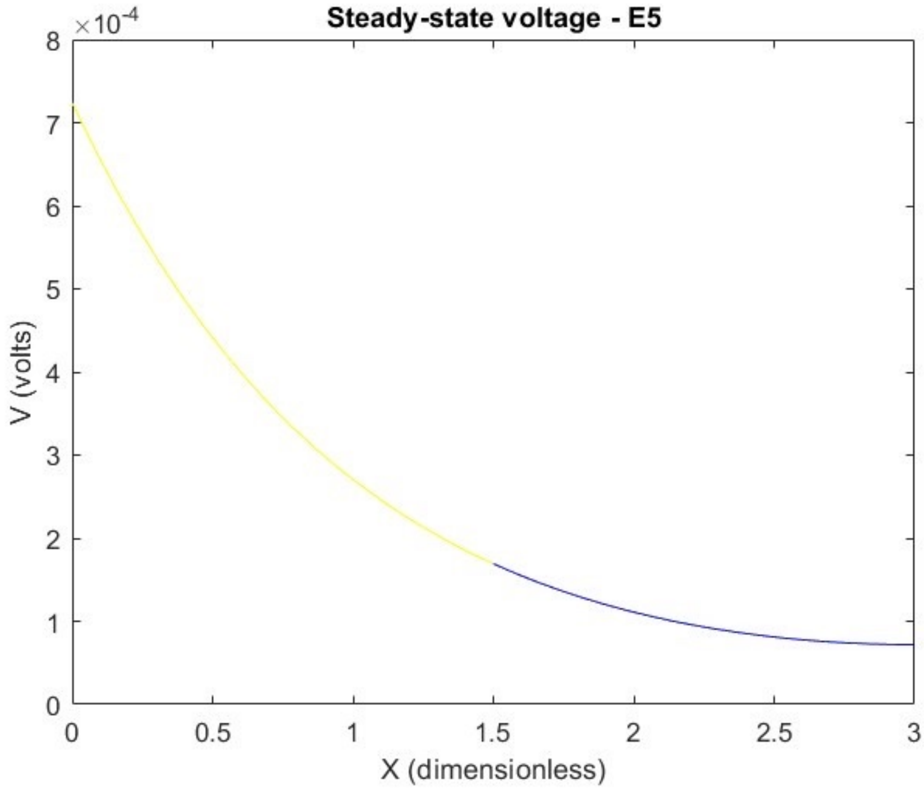
Q5

In 2(c) and 2(d) current exits from the daughter branches. Due to the current flowing through an impedence, a positive membrane potential is generated. Because of that the voltage gradient is more positive on the right hand side.

Q6

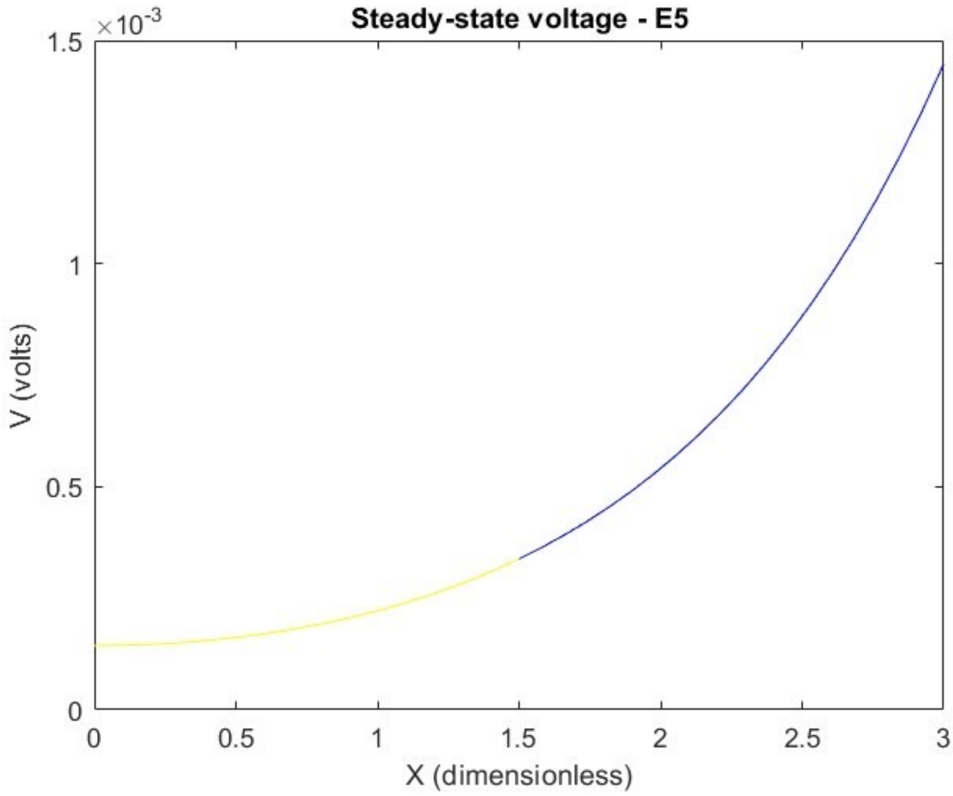
b)

```
x =

    7.220428405996924e-04
    1.789787674007517e-06
    7.220432693928202e-04
    1.789766325654747e-06
    7.220432693928203e-04
    1.789766325654747e-06
```



d)

```
x =

    7.189683523563270e-05
    7.189683523563270e-05
    7.189597764830475e-05
    7.189687793239163e-05
    7.189597764830509e-05
    7.189687793239161e-05
```



The transition from parent branch to daughter branches is smoother and has no sharp change.
The voltage profiles are equal for both daughter branches due to the equal diameter.