## INTERPRETER'S CORNER

Coordinated by Rebecca B. Latimer

## Unsupervised seismic facies classification: A review and comparison of techniques and implementation

THIERRY COLÉOU, CGG-France MANUEL POUPON, Shell International E&P, Houston, Texas, U.S. KOSTIA AZBEL, Paradigm Geophysical, Houston, Texas, U.S.

I he use of automatic seismic facies classification techniques has been steadily increasing within E&P interpretation workflows over the past 10 years. It is not yet considered a standard procedure but, with the knowledge of the advantages (and limitations) of the different seismic classification methods, its role in the interpretation process as a successful hydrocarbon prediction tool is anticipated to grow.

This paper reviews and compares the unsupervised classification methods presently used in seismic facies analysis: K-means clustering, principal component analysis (PCA), projection pursuit, and neural networks (vector quantization and Kohonen self-organizing maps). The term "unsupervised" covers all classification techniques relying only on input data and not biased by the desired output. These methods are described, compared, and illustrated by case studies taken from deep offshore Louisiana, west and south Texas, onshore California, and offshore Indonesia.

Seismic data volumes are huge and consist of highly redundant data. It is now established that their analysis can be greatly optimized by applying efficient data reduction algorithms that preserve essential features of the seismic character. The objective of the facies classification process is to describe enough variability of the seismic data to reveal details of the underlying geologic features. The classification process should do this while preserving a synthesis for the seismic signal changes.

In addition to comparing the classification methods, this paper presents relevant information pertaining to data input, data analysis, and data output.

**Comparison criteria.** Seismic data, from the statistical point of view, have characteristics like continuity, redundancy, and noise that affect the behavior of the seismic classification techniques.

At first glance, these classification techniques may appear similar. However, their ability to efficiently describe changes in seismic character and their interpretability can vary significantly. These different techniques are compared in this paper for their suitability to describe seismic data in a meaningful manner directly applicable to finding hydrocarbons.

Seismic trace shape is defined by the values of the seismic samples along each trace. To best analyze and classify the traces over a geologic interval (typically a reservoir zone), it is natural to look into the "N-dimensional" crossplot defined by the number of seismic samples. For example, a reservoir expressed over a 40-ms seismic interval at a 4-ms sample rate could be described and analyzed in a 10-dimensional crossplot.

In this crossplot, each trace is represented by one point with coordinates defined by the N values of its samples. Proximity of points in the crossplot implies similar seismic shape. Conversely, separation of points implies different seismic shapes. Classification techniques look at the organization of the points within the N-dimensional crossplot (Figure 1).

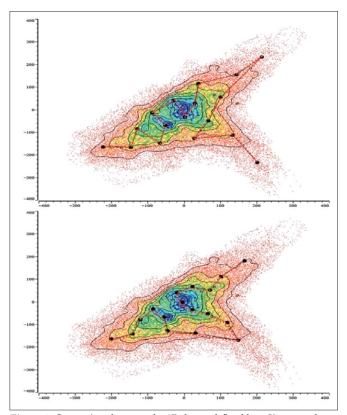


Figure 1. Comparison between the 17 classes defined by a K-means clustering technique (top) and by a self-organized neural network (bottom). Classes are indicated by black dots; red dotted lines correspond to the ordering of the classes. Note that the classes defined by K-means clustering tend to be attracted to the extreme values and are not ordered in a topological sequence. Conversely, SOM-defined classes are in a sequence, so the process is less sensitive to the number of classes and noise and generates a better-organized seismic facies map.

**Do clusters exist in seismic data?** Seismic data are highly continuous, greatly redundant, and significantly noisy. The continuity of seismic reflection data is an expression of the lateral continuity of the geology.

This *continuity* is the basis for the entire seismic data processing and interpretation sequence because it enables us to perform structural and stratigraphic interpretation. As a result, there is not a limited number of different trace shapes, but a series of intermediate shapes. Consequently, in the N-dimensional seismic crossplot, isolated clusters are unlikely while areas with a higher point concentration linked together without a significant drop of data density are common.

Redundancy in seismic data can be found both vertically and laterally, as the wavefront propagation generates some spreading of the information. Laterally, seismic data are redundant for reasons of continuity. The lateral continuity is controlled by binning and in some cases by trace interpolation.

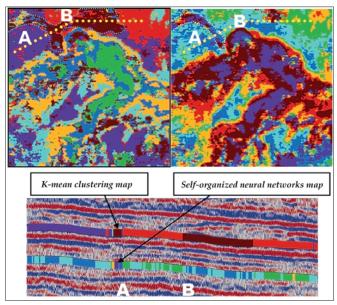


Figure 2. Comparison between a K-means clustering map (left) and an SOM (right) of a Frio Channel gas play (South Texas). Note the abrupt changes of color on the K-means clustering map compared to the ordered class colors on the SOM. The SW-E arbitrary line (yellow dotted line) highlights the interval of interest between the two color ribbons and the differences between the two seismic facies techniques. Note how the NW-SE feeder channel is classified with the same class as the main channel on the SOM (purple and white dotted outline), while it is classified with a similar class as the overbank formation on the K-means clustering map (brown and light blue dotted outline—area A). Also note in area B interleaved patterns unique to SOM corresponding to a transition in seismic trace shape; this is never found in K-means results which can only have abrupt color changes. (Data courtesy of CGG)

Vertically, redundancy is controlled by a bandwidth that is generally much less than the Nyquist frequency at a standard 4-ms sample rate. Therefore, consecutive samples along and between the seismic traces are highly correlated. The true dimensionality of the seismic data is much lower than the large number of samples would indicate. As a result, N-dimensional crossplots of seismic data exhibit significant organization generally expressed by elongated subclouds with some possible branches.

*Noise* often is a significant component of the seismic data. The processing sequence is a trade-off between resolution enhancement on one hand and continuity (or focusing improvement) and noise reduction on the other. In an N-dimensional crossplot, noise will tend to inflate the data cloud or, in the presence of subclusters, to link them and to remove their external isolation.

In such an inflated crossplot, techniques designed for data segmentation (such as K-means clustering) will position the cluster nodes as far apart as possible. This repulsion between cluster nodes makes them sensitive to noise, prevents meaningful ordering, and leads to unstable results heavily impacted by the selected number of clusters. On the other hand, techniques designed to describe the hierarchical structure of the data (such as self-organized maps) will identify the cloud branches or eventual isolated clouds and the intracluster organization of these continuous crossplots, which have been inflated by noise (Figure 2).

Some techniques commonly used for classification. The K-means clustering technique was described very early in statistical literature (McQueen, 1967). Its simplicity of implementation often makes it selected for multivariate statistical analysis. The object of K-means clustering is to iden-

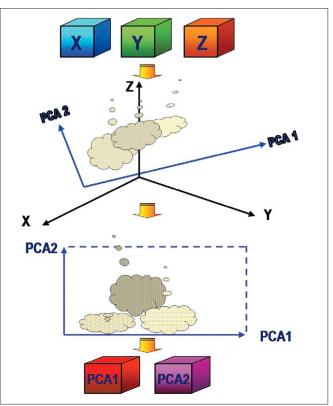
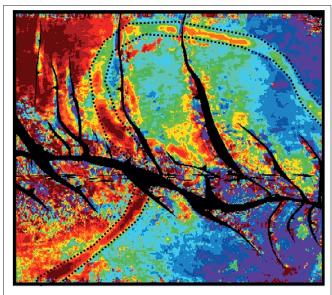


Figure 3. Schematic diagram of PCA analysis showing a three-dimensional crossplot reduced to two PCA components.

tify subclouds within the N-dimensional crossplot. Its purpose is not data reduction but data partitioning into disjointed subsets. Separation of the clusters is often based on the standardized Euclidean distance, the weights coming from normalization of the samples, or the more general Minkowski metric, among which the basic Manhattan or cityblock distance is found (see Appendix).

For K-means clustering, in its simplest versions, a priori knowledge of the number of clusters is also required. Overall, it makes for an excellent filing system but does not describe the topological properties of the seismic data. It is known to perform well if data (i.e., N-dimensional crossplot) are organized into separated compact subclouds—also described as hyperellipsoidal clusters with internal cohesion and external isolation (Cormack, 1971). That is, the classification is improved if the N-dimensional crossplot is organized into properly separated compact subclouds. It also works well where data are noise-free and with a high relative dimensionality. However, seismic data, with their continuity, low dimensionality, and high noise level do not possess the prerequisite features for successful analysis of seismic trace shape by K-means clustering. If the goal of the classification is to identify data anomalies which stand out from the background trend in the multidimensional crossplot (e.g. AVO anomalies), then K-means clustering can provide meaningful results. However, it will fail to order the classes in a progressive sequence and is sensitive to noise.

Principal component analysis (PCA) is the most widely used statistical technique for data reduction. More recently, it has become the interpreter's answer to the proliferation of 3D attribute volumes. It identifies the main elongation directions (aka principal directions) within a multiattribute crossplot, and enables a change of input space (where the principal axes are orthogonal to each other) via N-dimensional rotation. Data reduction is obtained by dropping the dimensions with the lowest variability (Figure 3). It is not truly a classification



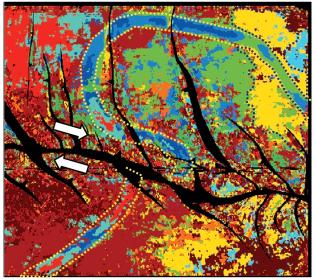


Figure 4. Comparison of an unsupervised trace shape seismic facies map without PCA (top) and with PCA (bottom) on a fluvial channel deposit from Natuna Sea, Indonesia. Note that the seismic facies map without PCA does not differentiate the channel facies from the flood plain deposits (same red class). It also tends to bias the interpretation toward a channel system not affected by the W-E strike-slip fault. On the other hand, the PCA seismic facies map differentiates the channel facies (blue class) and clearly highlights the effect of the W-E strike-slip fault in a relatively noisy area. Also, note the development of the overbank deposits to the east (yellow facies outlined by black dotted line).

technique, but is often used as a filter to reduce the seismic data dimensions prior to classifying. By reducing the noise and the redundancy of the data, it can significantly increase the quality of the resulting 2D maps or 3D volumes, particularly in noisy areas (Figure 4). Although the low dimensionality of seismic data makes PCA efficient in reducing the number of samples in the data to be classified, the size of 3D seismic surveys (in terms of their number of traces) is a limitation due to memory-intensive processes.

Projection pursuit is a classification technique to retrieve organization of the crossplots. Less sensitive to data redundancy than PCA, it does not rely on multigaussian hypotheses. It is also based on linear projections and seeks non-normally distributed structure in the seismic data. It is still sensitive to noise.

Artificial neural networks (ANN) provide several techniques for seismic classification purposes. Beyond the hype about

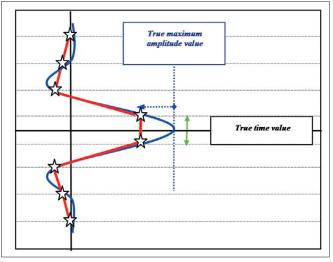


Figure 5. Comparison between a "nearest sample" approximation (red curve) and "trace shape" reconstruction (blue curve). Note that the sampling to the nearest 4-ms sample (white stars) generates a ±2-ms error ("unbiased noise") on the time pick (green arrow) and up to 25% decrease ("biased noise") from the true maximum amplitude value (blue arrow). On the other hand, trace shape reconstruction reduces the sampling noise and takes full advantage of propagation beyond the seismic sample.

ANN and their supposed reproduction of human brain behavior, they feature interesting capabilities. It should be remembered, however, that the only brain involved is the interpreter's! (See channel interpretation on Figure 4).

One interesting aspect of some neural networks is the great potential for parallelism (unlike PCA). Parallelism allows processing huge amounts of data without excessive memory requirements. There are several types of networks but only two are routinely used for seismic facies classification.

1) Vector quantization (VQ) can be viewed as unsupervised density estimation and is closely related to K-means clustering. VQ does data clustering or data segmentation without analysis of the transition between clusters or the transition within the data itself. This technique is based on a predefined number of segments and has been proposed for seismic waveform unsupervised classification. The resulting classes, like K-means clustering, tend to be uncorrelated.

In the N-dimensional crossplot, the repulsion between nodes tends to position them equidistantly. There is no obvious neighbor to a particular node, which can make the classification unstable and may further complicate the interpretation of the nodes relationships. In other words, the resulting maps are sometimes difficult to interpret due to their lack of continuity.

2) Self-organizing maps (SOM) are networks that provide "topological" mapping from the seismic data to the clusters (Kohonen, 1989). The sequence of clusters retrieves the internal organization of the crossplot and is similar to the principal curve and surface algorithm. A principal curve is a nonlinear generalization of a principal component. In a multivariate normal distribution, the principal curves are the same as the principal components. A one-dimensional SOM is a discretization of a principal curve. As a consequence, model traces or nodes are strongly correlated to their immediate neighbors along the principal curve. Unlike other clustering methods, the number of clusters is not critical. 1D-SOM was the first technique successfully used for seismic trace shape classification (as early as 1992 by Naamen Keskes). This technique can be extended to higher order SOM for cases where the crossplot displays a lot of branches.

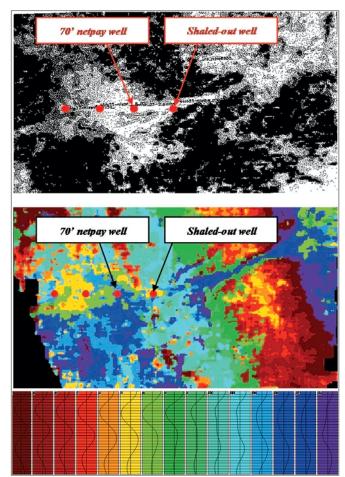


Figure 6. Comparison between a coherency slice (top, from Lanning et al.) and a SOM seismic facies map (bottom, from Poupon et al.) over a channel-fan complex (Winters Sands, California). Wells are indicated by red dots. Note that the SOM differentiates the main gas producer (70 ft of net pay in Winters Sands classified as blue trace shape) from the shaled-out well to the east classified as yellow trace shape. Trace shapes defined by the SOM are displayed at the bottom. Each horizontal line corresponds to a seismic sample. These two wells could not be separated based on coherency and/or conventional amplitude analysis (Data courtesy CGG).

SOM also include a neighborhood-size parameter for smoothness of the clusters. When reduced to zero, the algorithm is equivalent to VQ and no longer has topological ordering properties. We will address later how this topological ordering has some interesting consequences for the quantitative use of the classification results.

Trace shape reconstruction: Improving seismic facies classification. The seismic facies analysis that operates on waveform shape can be significantly improved if trace shape reconstruction is applied prior to the classification. Noise can be a dominant feature in seismic data. The vertical 4-ms sampling of the seismic traces introduces even more noise into the seismic trace shape. Trace features like peaks and troughs are not exactly located on a seismic sample for each trace. Efficient 3D propagators based on wave shape and cross correlation, enable event tracking beyond temporal seismic sample resolution. The resulting horizons usually generate sharper geometrical attribute maps, such as amplitude, dip, and azimuth.

Using these types of horizons as reference and trace shape reconstruction over the interval of interest can significantly reduce the noise level. Trace shape reconstruction is a regular resampling of the interpolated trace (using the interpreted horizon as reference) prior to classifying the trace shape.

Failure to do so affects both time and amplitude values, which are critical for trace shape analysis. It systematically increases the time error by ±2 ms (for a 4-ms sample interval) on the offset between the nearest sample and the true event location along the trace (Figure 5). More importantly, the nearest sample misses the extreme amplitude value. It is described as a "biased noise," as the amplitude of the nearest sample will always be reduced from the true maximum amplitude. The drop in amplitude will vary from trace to trace and will be a function of the time offset and the frequency content that controls the width of the cycle. As an example, a 2-ms shift at 35 Hz leads to an 8% drop in amplitude, while the same shift at 50 Hz generates a 30% amplitude reduction. For noisereduction purposes, it is always preferable to use an accurately picked event, especially when analyzing a small interval.

Multiple seismic data attributes: Expanding the information content. Single-trace poststack processing methods can produce a multiplicity of seismic attributes aiming at enhancing the interpretation. For example, a low-pass filter will facilitate the propagation along an event perturbed by noise. Similarly, instantaneous phase may enhance intrareservoir events. However, these poststack attributes are derived from the same original measurement and therefore tend to be redundant. Thus, it is no surprise if they lead to similar classification results.

Multitrace poststack processing methods, like 3D inversion, can, however, improve the classification. The noise level is significantly reduced and resolution enhanced by removing the wavelet effect. Interference areas are reduced and DHI (such as the reservoir boundaries) can be better defined after the classification process. 3D inversion, however, requires some a priori information, including a wavelet and low frequency component, which are usually derived from scarce well data. It can bias the inversion process in a positive or negative way. Band-limited trace inversion may be preferred as input to the classification.

Other multitrace attributes like coherency or semblance provide information that can also be derived from classification techniques. They measure the lateral continuity in the seismic volume or the distance between traces in the Ndimensional crossplot. Two neighboring traces in the seismic volume, with high continuity, will be neighbors in the crossplot and therefore will belong to the same cluster and have the same color in the seismic facies map. Conversely, two neighboring traces in the seismic volume with low continuity will be far apart in the crossplot and will be given different colors in the seismic facies map. Successful classification will lead to abrupt color changes when the continuity is low and it will separate continuous transitions in the seismic shape where continuity will be uniformly high. Classification brings colors to the black and white, almost binary, continuity attributes (Figure 6).

Partial stacking of multioffset data is commonly used to isolate amplitude effects as a function of reflection angle. Unlike poststack attributes, they do correspond to different measurements and provide more information than the full stack volume alone. Techniques based on multiattribute sample-to-sample comparison may have problems in synthesizing the information contained in the different substacks due to frequency content difference and possible time shifts. This will increase the noise level and the spread of the input data and may be difficult to interpret. Reconstruction of traces from each attribute volume using a reference event adjusted to each volume will provide better results.

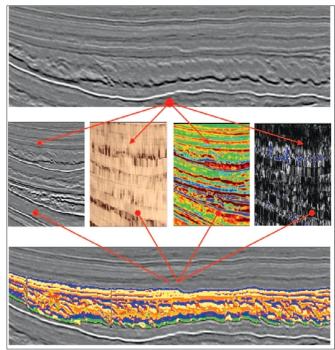


Figure 7. Multiattribute classification of slump features in the Green Canyon area, deep offshore Gulf of Mexico. Note the increased definition of the seismic facies volume (bottom) generated by combining amplitude, coherency, impedance, and parallel bedding attributes (middle) compared to the conventional amplitude stack (top). (Data courtesy of CGG).

Seismic classification inputs. Seismic data attribute volumes and maps can serve as inputs to the classification process; however, their number and type can vary significantly. Several hundred surface and volume attributes can be generated. A priori classification attempts using multiple attributes have been proposed, but there is no rule for the selection of which attribute will best illustrate rock property changes.

Most classification techniques can be used on any combination of 2D maps and 3D volumes. However, if attributes with different dimensions are used in the classification process (e.g., amplitude and impedance), some rescaling or standardization is required. On the other hand, such standardization should be avoided for attributes of the same dimension (e.g., AVO near-mid-far), as the difference in scale may be intrinsic.

The use of attributes is only limited by the number of attributes available and the hardware limitations (mostly the computer's memory). The nonexhaustiveness implies that the data set may not contain every feature from the seismic data, leading to a possible input bias. The larger the number of 2D maps, the higher the redundancy in the data set.

As some techniques are more sensitive to redundancy, there is another serious risk of input bias when dealing with multiple attribute classification. It is better to rely on techniques less affected by data redundancy (such as PCA) than to attempt to remove a priori redundant attributes that may have some valuable information not immediately apparent to the interpreter.

Some attributes contain information like structure information that is not stationary. Such attributes will easily separate clusters but they may also introduce bias in the classification results. For example, the classification of dip/azimuth maps with amplitude-based attributes (such as AVO) will likely produce a seismic facies map strongly correlated to the structural attributes and mask the fluid effect carried out by the AVO attributes.

Seismic facies classification output. Classification techniques have been applied on seismic waveform. Each trace is color coded according to the sample variations within the interval (Figure 6). The resulting seismic facies map maximizes the data compression and offers a very efficient way to synthesize the trace shape variability.

Classification has also been applied to several seismic volumes (or maps) to generate a single seismic facies volume (or map). Each sample is color coded according to the information from many input attributes. The classification of seismic volumes is less efficient in terms of data reduction and needs to be interpreted as any other 3D seismic. In addition, the resulting classification volume contains multiattribute information often helping the interpreter to visualize complex systems by increasing seismic resolution (Figure 7).

Classification of a single seismic volume, generating another seismic facies volume has also been proposed. Each sample is color coded according to the samples within a time window and to the samples of neighboring traces. It provides little data reduction, and is similar to a deconvolution without the explicit definition of an operator. The introduction of samples from neighboring traces in the input space is interesting, as it reduces the noise level similar to the multitrace inversion process. This is achieved, however, at the expense of resolution and, unlike single trace classification, is sensitive to structural changes.

Number of classes and class continuity. The critical issue in a classification process is the ordering and continuity of the classes. The continuous changes in seismic character will lead to different classification map textures. This can be illustrated by comparing classification with clustering techniques (which look for maximum node separation) and SOM (which looks for internal topological description).

The continuity in the chain of clusters is guaranteed when the SOM process is used. Gradual changes of seismic shape will correspond to gradual changes in color in the classification. Sudden changes will be easily identified. They will correspond to sudden jumps of colors or color dislocation on the facies map. Gradual changes of colors will typically show interleaving patterns of contiguous classes, when the trace shape oscillates between two neighboring model shapes without any significant drop in the correlation level (Figure 2 area B). When the number of classes is increased, intermediate output traces are going to be introduced, leading to an even finer description of the areas with subtle changes.

With clustering techniques, the continuity of the seismic shapes is not guaranteed and therefore continuous seismic trace changes will be represented with sudden changes of color and corresponding correlation drop. The changes of color will be the same regardless if changes in the seismic data are abrupt or continuous. With a limited number of clusters, the results will be fairly stable but the resolution will be limited. With an increased number of clusters, the results are lacking continuity. Color changes corresponding to abrupt changes in the seismic signal will not be affected, but the transitional areas will be color-coded with laterally shifting color dislocations as the number of clusters changes (Figure 8).

Some image processing techniques can help to enhance the results—in particular the edge detection algorithms designed to identify color dislocations. With K-means clustering or VQ, however, all color transitions are abrupt and all edges will be kept.

**Qualitative use of results.** The color-coded classification results need to be interpreted as any other maps or seismic volumes. Traces (or samples) with the same color should be

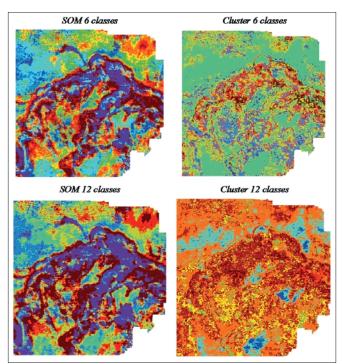


Figure 8. Sensitivity to class number comparison between SOM and K-means clustering: Note how the 6- and 12-classes SOM are similar and how K-means clustering is sensitive to the number of classes.

comparable, and traces (or samples) with different colors should be different. Classification techniques are usually location independent operators. The lateral continuity observed in the resulting map (or 3D volume) is not forced into the process but comes from the seismic data spatial organization. The spatial distribution of the color changes should be analyzed as they indicate changes in the seismic response—i.e., an expression of the underlying geologic facies. Interpretability of the results varies widely according to the technique used.

Quantitative use of results. Seismic classification results are categorical variables output as maps (or volumes). As such, the results cannot be used numerically for operations like smoothing or interpolation. Classification results cannot directly be used as secondary or explanatory variables for driving the numerical modeling of well data, which uses geostatistical techniques like cokriging or external drift. They cannot directly be translated into thickness or porosity.

Quantitative use should be restricted to individual categorical variables, defined from one class or several classes grouped into one. In practice, each facies should be interpreted and calibrated individually to rock properties.

This requires a large number of wells encompassing all possible types of lithological and structural reservoir properties. Quite often, seismic modeling is the only solution for this calibration problem.

However, these theoretical restrictions are not strictly applicable in the same way to all techniques. For SOM, the continuity of the facies model traces generated by neural networks implies that they are interpolable at least in the crossplot. This translates into a limited interpolability in the map. The ordering of the traces and the resulting gradual color changes are an illustration of that. One limitation is the loss of linearity in the relationship between the model traces when going from the model traces to the seismic traces. The presence of subclouds and branches in the seismic data crossplots introduces discontinuities within the chain of model traces

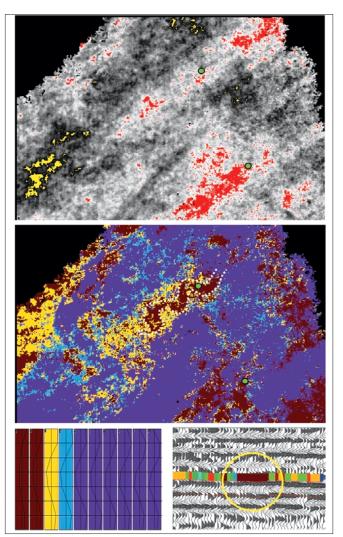


Figure 9. Comparison between an average absolute amplitude map (top figure) and a SOM facies map (central) over a meandering channel (Wolfcamp Sands, Permian, West Texas). Green dots correspond to producing wells. Note that the seismic facies map identifies a meandering channel (brown classes) not expressed in the interval amplitude map. Trace shapes and a section through the channel are shown at the bottom. (Data courtesy of Apache).

and also prevents meaningful interpolation across these discontinuities. The interpolability is then limited to the portion of the chain of model traces corresponding to the individual subclouds or branches.

With clustering techniques, the interpolability is absent in the clusters and therefore absent in the seismic facies map. The quantitative use of the results is strictly limited to individual character or categorical variables.

Quantitative use of classification results may necessitate enhancement of the lateral continuity. Smoothing procedures have been specially designed to address any seismic classification maps by enhancing lateral continuity and reducing the noise level.

**Toward supervised classification.** The seismic classification results need to be modeled so the interpreter can derive some understanding of the trace shape variations. Well control is essential but often insufficient to evaluate all aspects of the classification adequately.

Without proper statistical calibration of the results from well data, seismic modeling provides interpreters a way to improve their understanding of the possible seismic responses observed in the classification process. Experience and geologic knowledge of the reservoirs make it easier to encompass all possible variations in terms of structural and petrophysical changes and to relate the lateral variations of seismic facies to possible reservoir parameter changes. This is sometimes an ambiguous and interpretative process.

Conclusions. Seismic classification techniques are being gradually introduced into seismic interpretation workflows as new ways to look at seismic data and complement the conventional structural and amplitude analysis. These techniques have been proven successful particularly in stratigraphic plays (Figure 9), where amplitude alone fails to characterize reservoir properties.

Knowing both the interpreter's needs and the seismic data statistical properties helps in selecting a suitable technique for both data reduction and generating geologically meaningful results.

These objectives and the possible evolution toward quantitative use of the results makes hierarchical classification techniques such as SOM applied to trace shape, preferred over K-mean clustering techniques.

## Appendix 1—Keywords and definitions.

*Seismic facies:* The description and geologic interpretation of seismic reflection patterns including shapes (continuous, sigmoidal, etc.), frequency, amplitude and continuity.

*Seismic facies map:* A similarity map of actual seismic traces to a set of model traces that represents the diversity of various trace shapes present in a user-defined interval.

Clustering: Grouping similar items together to form clusters (aka nodes or classes) whose centroid or node charac-

terizes the group.

Manhattan distance-Minkowski metric: The Minkowski and Manhattan distances are:

Mink 
$$(x,y) = (\Sigma_i | x_i - y_i |^{\lambda})^{1/\lambda}$$
  
Manh  $(x,y) = \Sigma_i | x_i - y_i |$  = Mink  $(x,y)$  for  $\lambda = 1$ 

where  $x_i$  and  $y_i$  are the coordinates of two points in the "N" dimensional crossplot. The Euclidean metric is for  $\lambda = 2$ .

Suggested reading. "Seismic attribute technology for reservoir forecasting and monitoring" by Chen (*TLE*, 1997). "A review of classification" by Cormack (*Journal of the Royal Statistical Society*, 1971). "A statistical method to derive reservoir properties from seismic data," by Fournier and Derain (GEOPHYSICS, 1995). *Self-Organization and Associative Memory* by Kohonen (Springer-Verlag, 1989). "Case study of the Elkhorn Slough Field, Solano County California. Risk reduction using state-of-the-art 3D tools" by Lanning and Cambois (*SEG 1998 Expanded Abstracts*). "Some methods for classification and analysis of multivariate observations" by MacQueen (5th Berkeley Symposium on Mathematical Statistics and Probability, 1967). "Principal component analysis applied to 3D seismic data reservoir property estimation" by Scheevel and Payrazuan (SPE 1999 Annual Technical Conference).

Acknowledgments: The authors thank the staffs at CGG and Paradigm Geophysical (particularly Duane Dopkin) for reviewing this paper.

Corresponding author: tcoleou@cgg.com