A Discrete Fracture Network Generation and Analysis Library for Use in CAD Software Environments

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1.0 INTRODUCTION

Generally, fractures represent a space between planes (Hernqvist, 2009), ubiquitous in many natural, engineered and biological materials. A rock fracture is a discontinuity within a rock mass (Jing, 2003; Priest, 1993). This discontinuity may be because of mechanical failure (from tectonic events(Gillespie et al., 1993) or human factors such as hydraulic fracturing, tunnel evacuation (Baek et al., 2017)), chemical processes (e.g. weathering (Lachassagne et al., 2011)). The term therefore also includes faults, joints, fissures, cleavages and even discontinuities between mineral particles (Lacazette, 2001). A fracture can be on a scale of a few microns to several kilometres (e.g. faults). Fractures are essential as they play a significant role in material strength, rock block stability, and in creating pathways for fluid flow (NRC, 1996; Dverstorp, 1991). Fractures are of great research interest in various fields of studies, not limited to, geotechnical applications, reservoir engineering, waste disposal, mining engineering and earthquake studies (Berkowitz, 2002).

A fracture network is a system of fractures developed within the same rock volume. A network may involve several distinct fracture sets, which may or may not intersect (Sanderson & Nixon, 2015). These generally evolve and vary in their spatial distribution (Bear & Verruijt, 1987). Fracture network modelling and simulation is an active area of research which has received much attention in the last few years, basically due to the challenge of directly observing the detailed 3D structure of fracture networks deep in the crust (Lei, Latham, & Tsang, 2017). Fractures are complex objects in terms of their geometry and topology, occur at all scales, which makes it an exciting area of research. Direct observations of fracture networks are relatively scarce and are limited to surface outcrops (2D), tunnel wall (2D), and core drilling (1D) (Alghalandis, 2014; Maillot et al., 2016). Although seismological surveys may be able to locate 3D large-scale structures, current technology can hardly detect widespread medium and small fractures due to resolution limits (Lei et al., 2017). Increased computing software and hardware capabilities have contributed to the rapid growth in the modelling of fractured rock (Fadakar Alghalandis, 2017).

Lei et al. (2017) defined a Discrete Fracture Network (DFN) as “a computational model that explicitly represents the geometrical properties of each fracture (e.g. size, orientation, position, shape and aperture), and the topological relationships between individual fractures and fracture sets” as opposed to continuum modelling which models the entire system as one domain (Alghalandis, 2014; Neuman, 2005). DFN models provide an effective method for simulating and studying features of fractured rock (Thomas, 2019). There are few commercial and non-commercial software capable of DFN modelling. However, they are often; expensive (e.g. MVE (MVE, 2018), NAPSAC (Wheeler, 2016), FracMan (Golder, 2019), MoFrac (MoFrac, 2019)), designed for specific tasks, have little or no functionality for extensive research, closed source, and complex to use (Erhel et al., 2014; Hyman et al., 2015).

This work aims to develop an open source library for generating three-dimensional stochastic discrete fracture networks, utilising the Rhinoceros 3D commercial CAD (Computer Aided Design) application. CAD software provides a user-friendly and relatively inexpensive platform for manipulating geometry, streamlining the process for both developing and using a DFN generation library. The software offers a Python API which the library will expand on, adding functionality tailored explicitly to DFN generation and analysis. By exploiting the variety of input and output file formats available in contemporary CAD software, networks generated using the library will subsequently be applicable in many different applications. Furthermore, the library will include methods and statistical tools for examining the properties of the generated networks, including fracture intensity, cut-plane properties, network connectivity, and percolation, thereby providing a platform for examining the science of fracture networks.

2.0 LITERATURE REVIEW

2.1 Review on Discrete Fracture Network

Various modelling approaches have been employed to represent fractured rock. Xing & Sanderson (2002) categorised this approach into discrete and continuous models. The primary assumption the continuum based methods is that the computational domain is treated as a single body (Lisjak & Grasselli, 2014), as opposed to discrete methods in which each fracture is characterised within a structural domain (Rogers et al., 2015). However, hybrid models exist which involve the combination of these methods (Thomas, 2019).

Stochastic DFN studies began in the 1980s to investigate two essential topics: percolation theory and hydrogeology (Thomas, 2019). Long et al. (1982) developed methods to determine if between fractured systems should be treated as an equivalent porous (continuum) medium or as a collection of the discrete fracture flow path. They extended their work to 3-D when they modelled the steady flow of fluid in disc-shaped random network fractures (Long et al., 1985). Endo et al. (1984) and Endo (1984) performed tracer experiments and suggested that not all fractures behave like equivalent porous media. They observed that fracture systems with continuous fractures have directionally dependent hydraulic effective porosity which negates the idea of an equivalent porous medium. (Balberg et al. (1984) used stochastic networks to study the dependence of the percolation threshold of the 3-D sticks systems on aspect ratio and macroscopic anisotropy. They employed the Monte Carlo technique to determine percolation thresholds for randomly placed sticks in a domain. Andersson & Dverstorp (1987) investigated how to predict flow through a network of discrete fractures in a 3-D domain. Before the term DFN became standard, “Rock Joint Systems” was used to refer to these types of models (Dershowitz & Einstein, 1988). This technique has continuously developed afterwards with many applications in civil, environmental and reservoir engineering and other geoscience fields.

A recent review on DFN by (Lei et al., 2017) grouped DFNs into three categories:

Geological-mapped DFNs using datasets observed from outcrops such as analogue mapping, borehole imaging, aerial photographs, seismic survey. This method ensures the preservation of geological realism and fair characterisation of complex topologies (e.g. intersection, spacing, clustering, truncation and hierarchy). The limitations of this method include difficulty in building 3D structures (since most measurements are in 2D), limited feasibility for applications to underground rocks, and constraints from measurement scale and resolution (in case of seismic surveys).

Stochastic DFNs use statistical principles to create fracture datasets. Fracture properties used as input include fracture lengths, orientations, locations and shapes. Advantages of this method include simplicity and convenience, efficient fracture generation, viability for both 2-D and 3-D, applicability for various scales. However, the shortcomings of this technique involve oversimplification of fracture geometries and topologies, uncertainties in statistical parameters, negligence of physical processes.

Geo-mechanical DFNs incorporate fracture physics and fracture network evolution in simulating fracture datasets. This technique involves the use of rock and fractures mechanical properties, paleostress conditions as critical inputs. It links geometry with physical mechanisms and ensures the correlation between different fracture attributes. However, this technique is not without shortcomings such as; requiring large amounts of computational resources, difficulties in including hydrological, thermal and chemical processes into its simulation and uncertainties in input properties. Nonetheless, modern DFN studies have continuously made use of fracture data collected from geological mapping to refine the input parameters required for stochastic DFNs(Rogers et al., 2015), both as a standalone tool (Alghalandis, 2014; Mauldon, 1998; Min et al., 2004) or integrated within more complex geo-mechanical simulations(Elmo et al., 2014; Rogers, Elmo et al., 2014; Thomas, 2019), leveraging on the strengths of the three techniques to create more realistic fracture datasets.

2.2 Existing DFN tools.

There are many DFN simulation tools, both as standalone commercial software programmes and close source in-house computer codes. Among the notable standalone DFN commercial software include;

Midland Valley Exploration (MVE) fracture modelling tool uses geometrical properties such as stress and strain values and statistical properties such as curvature as proxies for intensity and orientation for DFN generation. It allows for multiple direct inputs of filed data to constrain generation of DFNs. This tool is applicable to fracture networks characterisation and provides direct output on a geocellular model, which shows the current state of the subsurface in 3-D by representing complex geological reservoir with millions of cells (MVE, 2018).

NAPSAC generates fracture networks stochastically, ensuring fracture properties have the same statistical attributes as those of the geologically mapped field data. It employs the finite element method to simulate fluid flow in fracture networks. Users can validate model predictions against data from hydro-geological experiments (e.g. well tests). Its applications include effective permeability determination, porosity prediction, geometric and percolation analysis, tracer and salt transport modelling and transient flow modelling. It provides an output file which includes statistical properties of fracture features and pressure, flow data from generated networks, 2-D graphs of feature networks, 30D visualisation of model features (Wheeler, 2019).

GOLDER’s FracMan software suite allows for both randomly generated DFNs and DFNs guided by information from mapped structures. DFNs upscaling to equivalent porous medium (continuum) models is possible when desired, without losing details of DFNs where needed. It’s a versatile software suite applicable to various sectors including mining engineering (fragmentation assessment, stress evaluation, mining method simulation, mineral resource and hydrogeological evaluation), nuclear (groundwater and solute transport, site characterisation and evaluation, fractured rock analysis and characterisation, geotechnical design, DFN multiphase flow modelling), oil and gas (fracture modelling, well planning, geomechanics simulation, risk assessment, secondary recovery and hydraulic fracturing evaluation), civil engineering (rock fragmentation and block sizing, groundwater and tunnel inflow evaluation, in-situ stress assessment, probabilistic engineering design) (Golder, 2019).

MoFrac computer software generates DFNs leveraging user-defined fracture properties including fracture intensity, orientation, truncation rules, size, shape and undulation. The generated models are updated using geomechanical properties for realistic fracture representation. It ensures the honouring of input data during the stochastic process of fracture generation. Its applications are mainly in geomechanical and hydraulic flow modelling such as mine design, hydraulic fracturing, rock characterisation, blast optimisation, fracture mechanics, reservoir modelling and waste transport (MoFrac, 2019).

From the preceding, it is glaring the tools require users to undergo custom software training before using them. There will also be a need for support from the original developer for software updates. Moreover, these software suits are only available commercially. They are also standalone tools with limited file output formats, which implies difficulty in integrating them with other software packages. By generating DFNs using CAD software, we can produce various output file formats which usage in other software packages is possible for applications in all appropriate field of research. This project also aims to make its DFN tool easy to use in terms of network generation and analysis. Users only need to define input parameters using a text file or CAD user prompt. Also, being an open source code, there is room for its continuous development by the public and adaptation to suit different needs. Also, the python API library, which serves as the basis for this research, calls for no need of specialised training before using it.

Other hybrids, complex, industry and academic DFN simulation tools include;

SIDNUR (Erhel et al., 2014; Pichot et al., 2012) generates different realisations of DFNs following the Monte Carlo process and solves for steady-state flow in each of these networks whose statistical properties are constrained by in-situ experiments. Pencheva & Yotov (2003) proposed a “Mortar Mixed Finite Element Method”, which this tool uses to simulate flow after discretisation of the networks. The linear systems from the discretisation of DFNs are preconditioned using the Balanced Domain Decomposition proposed by (Dryja & Proskurowski, 2003). The code is parallelised to ensure speed.

Flow123d (Březina & Hokr, 2011; Březina & Exner, 2017; Březina & Stebel, 2016; Exner & Březina, 2016; Brezina, 2012; Šístek et al., 2015) combines continuum and DFN models to simulate water, solute and heat transport in fractured porous media. It consists of both steady and transient states Darcy flow hybrid solver, Finite Volume Model and discontinuous Galerkin model for solute and heat transfer.

FracSim3D (Xu et al., 2003, 2006; Dowd et al., 2009; Mardia et al., 2007; Xu & Dowd, 2010) uses marked point process, a technique which considers fracture properties as marks associated with stochastic points, to simulate 2D and 3D DFNs. Geometries and properties of fractures are modelled using probability distribution. Also, it allows for the inclusion of field samplings such as planes, windows, scanline and borehole data. This software is distinct in that it contains statistical tools such as histogram, probability plots, rose plots to analyse fracture characteristics.

ADFNE (Fadakar, 2017) is a DFN tool capable of 2D and 3D simulation. It is written in Matlab and contains functions for fracture characterisations (e.g., clustering, intersection and connectivity analyses), fluid flow applications (using Finite Difference Method) and geometric modelling (e.g., complex polygonal fracture faces).

DfnWorks (Hyman et al., 2015) is a parallelised 3D DFN tool kit which uses the Finite Volume Method to solve flow equations and applies the Lagrangian approach introduced by (Painter et al., 2012), which determines pathlines through the network, for transport simulation. It is suitable for fracture simulation in both small (mm) and large (km) scales.

ICGT (Nejati et al., 2015a, 2015b, 2016; Paluszny & Matthai, 2010; Paluszny & Matthäi, 2009; Salimzadeh, Paluszny, & Zimmerman, 2017; Salimzadeh et al., 2017; Thomas et al., 2017) is a 3D DFN simulator based on Finite Element Method. It generates fractures discretely and takes the geomechanical properties of fractures into consideration to ensure fracture growth and interactions are being implemented to simulate real networks.

Except for ADFNE, which employs a simple finite difference method to solve for fluid flow, all other tools depend on involved mathematical formulations to simulate fluid flow in fractured networks. The methods to be developed in this work require no complex mathematical formulations, other than basic statistics and algebra. The codes are closed source, except DfnWorks, precluding any form of development and adaptation by other researchers.

3.0 PROJECT PROGRESS

- Wrote a python script which allows a user to input parameters through a text file and creates a fracture domain. The user can enter the length of the sides of fracture domain, the number of circles (fractures), if the user does not want a random number and radius of fractures in meters in a text file named "DataFile".
- Wrote a script to insert fixed or randomly sized circles into a domain. The circles (fractures) are surface objects in Rhino. This representation is to ensure users can carry out intersection analysis on the fracture network generated in our domain.
- Wrote a script to make circles orientated. The orientation of the fractures, currently, are generated randomly following the uniform distribution. Each circle is an individual layer, with a layer name. This allows users to reference each layer separately. Users can also group desired fracture sets as a layer. Each layer can be visualised and analysed solely.

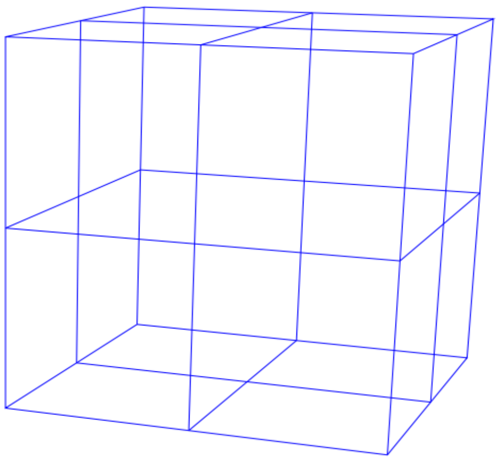

Figure 1: A 20x20x20 fracture domain

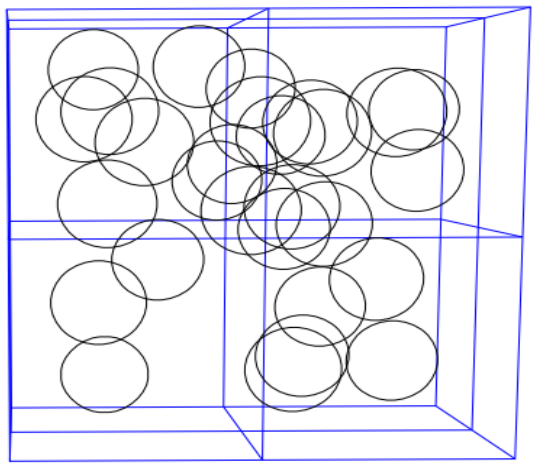

Figure 2: A fracture domain with a random number of 2m radius non-orientated circles inserted

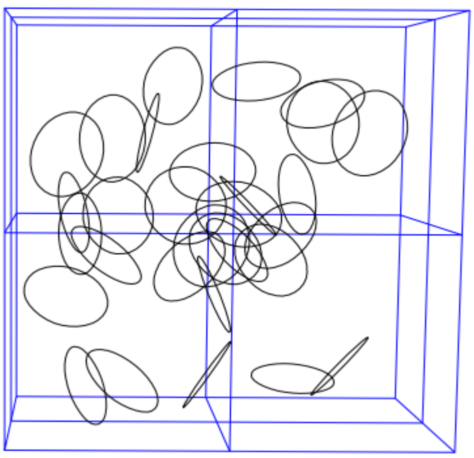

Figure 3: A fracture domain with a fixed number (30) of orientated circle inserted.

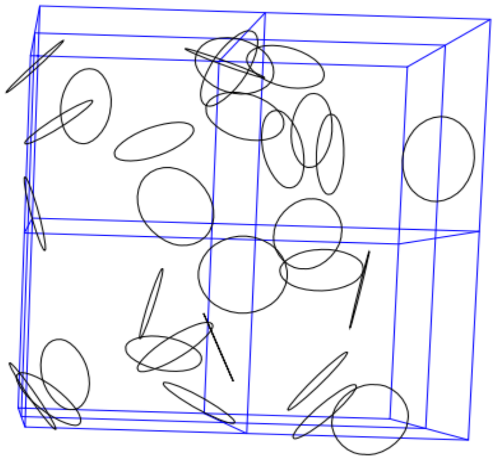

Figure 4: A fracture domain with a fixed number (30) of orientated circle inserted without constraint.

4.0 RESEARCH PLAN

The table below captures the research plan for this project. It contains columns for features required for the fracture generation and analysis library to be functional, description of the components of each feature, priority level depicting how important each feature is, and notes on the implementation details. Features with "core" priority are crucial to the aims of this research work, "medium" priority features are necessary as software best practice, add non-key performance to the library and make using the library more comfortable for users, "low" priority features are advanced and serve as accessories to the library.

Table 1: Research plan

Feature	Functionality	Priority	Implementation notes
• Methods for generating fracture networks	▪ Create fracture domain ▪ Insert fixed and random (normal distribution) number of fractures. ▪ Insert fixed and randomly (normal distribution) sized of fractures. ▪ Trim out of bounds fractures. ▪ Fracture shapes (circles, ellipse, polygon) ▪ Implement other statistical distributions for input parameters (exponential,	Core (except advanced shapes) Medium (advanced shapes)	All implemented except trimming of fractures, sophisticated shapes (ellipse and polygon) and statistical distribution. Implemented functionalities are to be improved upon as research progresses.

	lognormal, von Mises, Fisher, wrapped normal)		
• Input methods	▪ User prompt ▪ Text file ▪ GUI	Core (except GUI)	All implemented except GUI
• Methods for fracture characteristics' analysis	▪ Fracture intensity ▪ Intersection analysis ▪ Cut planes ▪ Percolation ▪ Geometric/graph analysis	Core	To be implemented
• Statistical methods for fracture analysis (histogram, graph)	▪ Volumetric fracture intensity (P_{32}) ▪ Fracture population frequency ▪ Fracture Connectivity ▪ Intersection length frequency	Core	To be implemented
• Error reporting	▪ Exceptions ▪ Bug reporting via Rhino	Medium (bug reporting via Rhino) Core (exceptions)	To be implemented
• Code parallelisation	▪ Parallelisation using the multiprocessing module	Low	To be implemented
• Documentation	▪ Auto-documentation using Sphinx	Core	To be implemented

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