Rocket Nozzle Analysis

CEE 361 Matrix Structural Analysis and Finite Element Method
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Contents

1 Introduction

Nozzle design is a major component of rocket propulsion, as it can have a massive impact on performance by virtue of the area-mach number relation which governs compressible flow through a converging-diverging (CD) nozzle. Not only does the nozzle shape drive requirements for the flow properties, but it is also a major structural component of the rocket; it must essentially carry the entire weight since most of the thrust force propelling the rocket comes from the expanding flow in the combustion chamber of the bell nozzle.

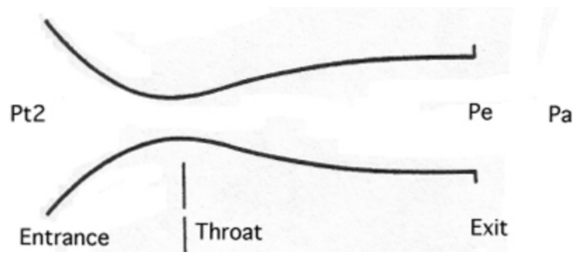

Figure 1: CD Rocket Nozzle

For these reasons, modeling the structural properties of the nozzle under load is monumentally important for the design of powerful, efficient rockets that are able to withstand immense loads in high temperature regimes. The Finite Element Method (FEM) is therefore a perfect tool to analyze rocket nozzle performance, since nozzles generally have very specific, complex geometry, and may be under constraints or loads which vary even over the course of a single flight. To that end, the work carried out in this project will be to design and carry out an experiment using the FEM implementation of CREO Parametric [1] that tests the effect of element size and edge polynomial order of elements on the accuracy of solutions.

2 Background

Rockets are governed by a series of equations, the most important being the trust equation, which relates key parameters to the force generated to propel the rocket forward.

$$T = \dot{m}V_e + A_e(p_e - p_0) \tag{1}$$

The equation for the thrust of a rocket nozzle is given in equation 1, where the e subscript references the escape values and the 0 refers to the ambient case. This shows that there exists an optimum expansion ratio to generate the maximum thrust. This occurs at the case where $p_0 = p_e$, known as perfect expansion (over expansion leads to shock waves that reduce efficiency). The bell shaped nozzle in particular is popular in the Aerospace industry, because the contour of the nozzle helps to keep the flow attached and fully expanded, even when the conditions would normally force flow in a conical nozzle to separate. The bell shaped nozzle also balances simplicity with performance. With these specifications in mind, it makes the most sense to use a bell nozzle for analysis, because it is the most applicable to actual the aerospace industry, and connects nicely to material studied in MAE426: Rocket and Air-breathing Propulsion Technology, a required class for

Mechanical and Aerospace Engineering majors at Princeton, which the authors are concurrently enrolled in.

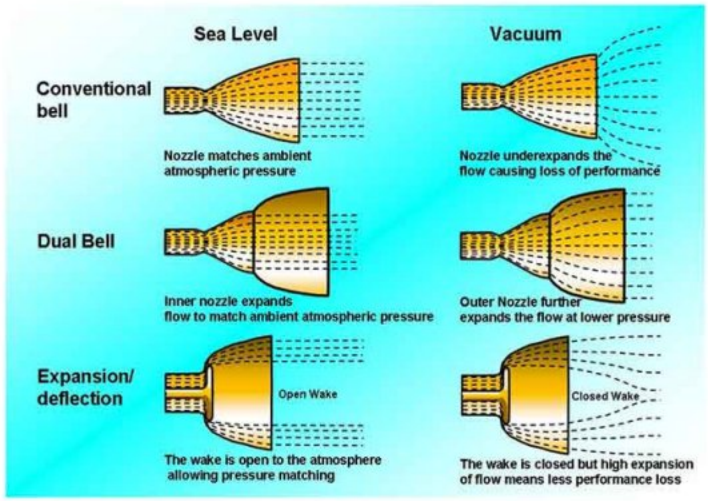

Figure 2: Different types of bell nozzles, with flow both in and out of the atmosphere

3 Methodology

The group implemented the project first by constructing a reasonable rocket nozzle mesh and assigning proper materials, displacement constraints, and forces in CREO. With these features in place, the group then carried out experimentation by testing meshes specified by imposing maximum element size constraints of 2, 1, 0.5, 0.2, 0.1, and 0.05 inches, which resulted in meshes of varying granularity. Simultaneously, the group imposed FEM analysis constraints by varying the element shape function polynomial order from 1 to 9. In this way, the group was able to compare polynomial convergence features using a range of element densities, as well as investigate some of the additional post-processing or optimization methods which CREO might use and thus better understand some of the practical and industry-standard FEM techniques.

4 Construction of The Mesh

4.1 Rao Parameterization

To construct a reasonable geometric, a parameterize of a bell nozzle was researched. In order to generate the shape of the mesh used in this project, the G.V.R Rao parameterization [2] for the optimum thrust bell nozzle cross-section was used.

The bell shaped nozzle consists of three parts, two circles, and one parabolic section. The larger circle has a radius of $1.5r_t$, where r_t is the radius of the throat. The smaller circle has radius $0.45r_t$.

The parabolic section is defined by angles, where the beginning exits the circle at angle 34.4° and the end of the nozzle is offset from horizontal by 13.2°. The length was defined by the area ratio, which was $\frac{A_e}{A_t} = 19.325$. With these parameters, the outline trajectory of the nozzle can be created, and revolved around the center axis to produce a bell shaped contour.

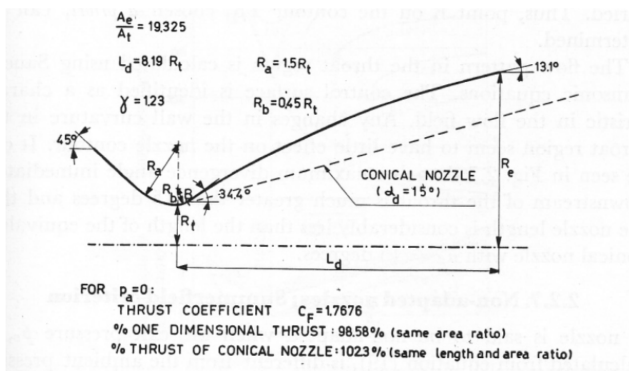

Figure 3: G.V.R Rao Bell Nozzle Design

4.2 Materials

One of the design challenges was choosing the correct material and thickness. It was decided to model the nozzle after the Space Shuttle Main Engine (SSME), it it had similar specifications to the nozzle that was designed. The material used was Niobium Alloy, which was loaded into CREO as a special file with specifications easily found online for the structural properties. A thickness of 0.1 was chosen, since it would be relatively thin compared to the overall length of the nozzle, and is reasonable for the main structure of the nozzle.

4.3 Mesh Results

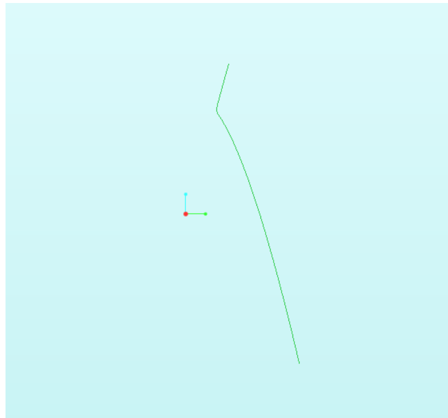

Figure 4: Sketch To Be Revolved

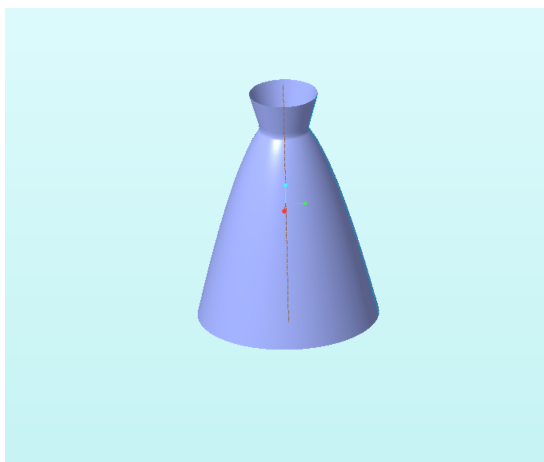

Figure 5: Revolved solid

Shown in figure 4 the trajectory that was loaded into CREO Parametric, followed in figure 5 by the revolved solid, which was actually analyzed.

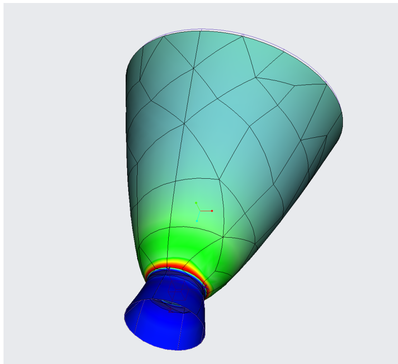

Figure 6: Max Element side length: 2 in

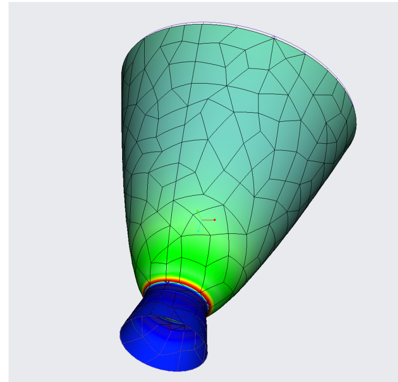

Figure 7: Max Element side length: 1 in

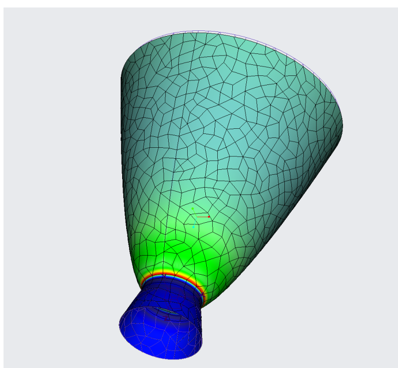

Figure 8: Max Element side length: 0.5 in

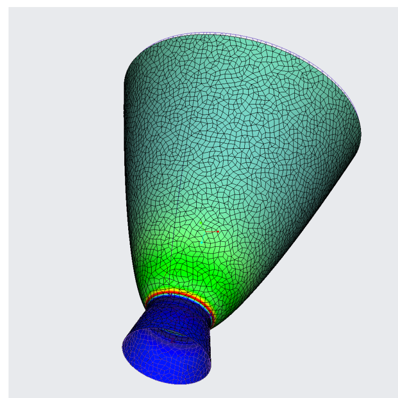

Figure 9: Max Element side length: 0.2 in

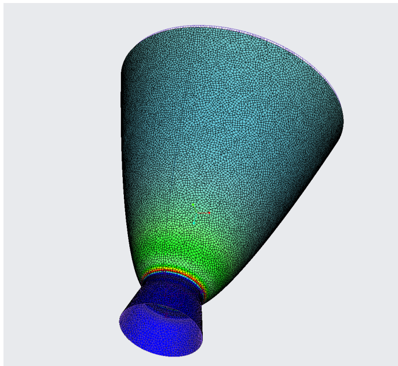

Figure 10: Max Element side length: 0.1 in

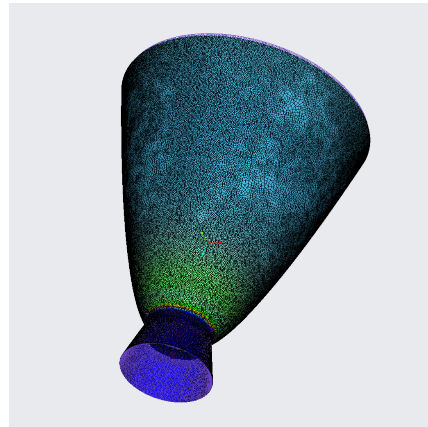

Figure 11: Max Element side length: 0.05 in

Shown in figures 7 through 11 are the different size meshes that were constructed by CREO. CREO takes the a maximum element side length and constructs both triangular and quadrilateral isoparametic elements to mesh the solid design. The shading on the figures is a representation of the stress, and it becomes clear that the location of the greater stress is refined as more elements are used, due to the smaller tiling effect, which creates a finer resolution.

5 Assumptions

To properly assess the forces on the nozzle, several assumptions were made. The first was that pressure in the nozzle is constant, throughout the whole nozzle. A value (5 MPa) that would typically be found in a rocket combustor was chosen, as this would over estimate the pressure in the nozzle, and thus produce values that are strictly greater than the actual maximum stress, while allowing for more simple and faster analysis. It was also assumed that pressure forces are the only forces in the nozzle, and that the flow does not contribute to forces. This assumption is a vast simplification given that there is a very strong flow within the nozzle, but the pressure force and changes are greater in magnitude, and by analyzing those, a general sense for the forces and deformations of the nozzle can be deduced. Since the rocket moves through space while in use, modeling the rocket forces also becomes difficult. By only looking at a rocket operating in space, we can neglect gravity and any frictional forces on the outside of the rocket. Furthermore, we can attach a body reference frame to the nozzle, that is moving through space at the same speed as the rocket. With respect to this frame, the engine is in static equilibrium which allows for finite element analysis to be conducted properly.

These simplifications will introduce error in the analysis compared to how a real rocket nozzle might behave, but they were chosen with the expectation that deformations would be small. Additionally, the group preferred to evaluate the FEM results for their qualitative purposes rather than the realism of the force and constraint settings, so these simplifications made sense to make.

Finally, the assumption is made that the FEM mesh with the smallest number of elements is the true solution. This may not be the case, however it allows for analysis of the effectiveness of different size elements and polynomial orders against the correct solution.

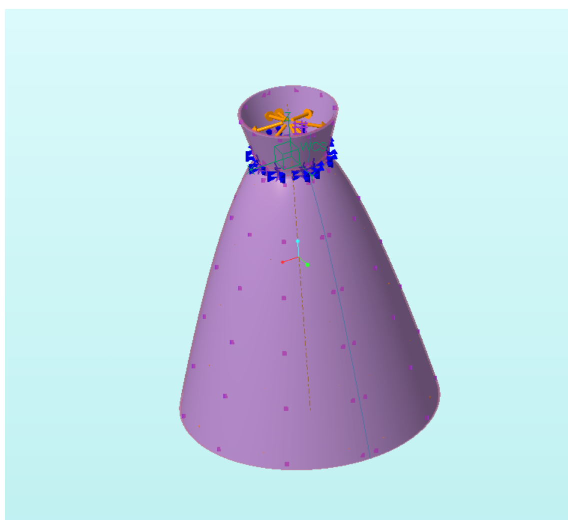

Figure 12: Simulation constraints and pressure forces

The model was analyzed as given in figure 12, where the nozzle is only constrained at the throat, and the pressure force is applied everywhere other than the throat.

6 Analysis

Analysis was completed using CREO's Simulate feature, which is capable of running FEM in 3D. This choice was made due to the complicated nature of defining the boundary conditions for a revolved solid: a flat, 2D mesh would have been unable to capture the oblique internal force which runs down the nozzle surface. As a compromise, the nozzle was modeled using CREO's "shell" feature, which represents a mesh of 2D elements organized over a 3D surface.

When each analysis (using the specified element max size and polynomial order) was completed, several metrics were recorded from CREO: the maximum principle stress, stress error, and stress error as a percentage of maximum principle stress. The stress "errors" reported from CREO are obtained by sampling the local error estimates along external edges [1], and are meant to be used as a guideline for uncertainty. Additionally, a fourth metric was computed by calculating the percent error of each maximum principle stress for each polynomial order relative to the simulations using elements with max size 0.05 inches and corresponding polynomial order. This metric is meant to provide a more holistic view on errors from each analysis, since ultimately solutions should converge if as the polynomial order and number of elements increases. The following equation gives a formal definition of this "Relative Error" metric:

$$\%\ \text{Relative Error} = \frac{\sigma_{0.05\text{ in., max}} - \sigma_{\max}}{\sigma_{0.05\text{ in., max}}} \cdot 100 \tag{2}$$

In particular, the analyses conducted here are chiefly meant to examine and validate a core principle of FEM, which is that solutions to the weak formulation of boundary value problems approach the solution to the strong formulation as the set of test functions increases, and in the case of FEM, this tenet should hold true as element count or polynomial order increase to infinity.

7 Results

7.1 Maximum Principle Stresses

Figure 13 shows the maximum principle stresses in log scale for each analysis plotted over the polynomial order. Each line represents a different element size, as shown in the legend.

The expectation that FEM should converge using an infinite number of elements is verified, in part, by these results, which show each line beginning from a varied starting point and converging on a much more clustered value. As the number of elements increases, the first-order polynomial solutions become closer and closer to this clustered value, showing that, with the polynomial order held constant, increasing element count generally yields a more accurate solution.

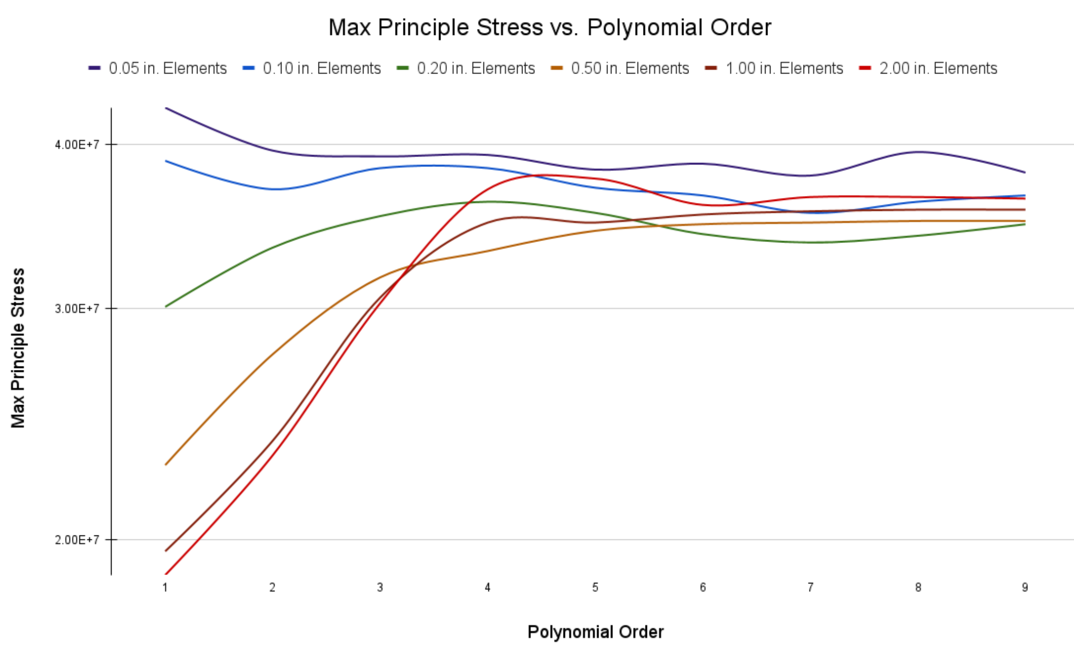

Figure 13

Additionally, Figure 14 shows the maximum principle stresses for each polynomial order plotted over the maximum element size. Just as in the previous figure, each polynomial order converges as more elements are added (element max size is decreased), and higher-order polynomials converge more quickly. This confirms the alternate realization of the original boundary value problem statement for increasing element density.

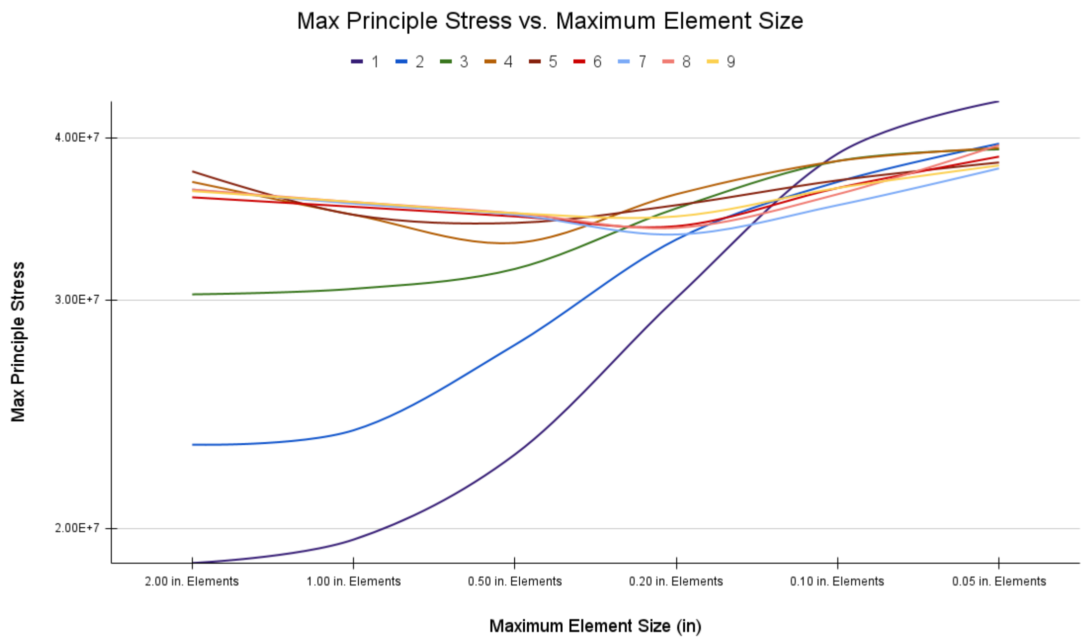

Figure 14: Maximum Principle Stress vs. Maximum Element Size. Note, Legend Entries Correspond to Polynomial Order

These results are exciting because they confirm one of the major tenets of FEM, but they also show some of the variance that one can expect, since even in this small demonstration, the distribution of first-order polynomial solutions is not aligned with the smallest element size, that is, even by choosing a small element size there is still some error.

7.2 Stress Errors

Figure 15 demonstrates that the error distribution for the different element sizes when plotted with different polynomial orders is multi modal. This suggests that for different sized elements and polynomial orders within a given range, there is an optimal polynomial order to generate the smallest error, although it could be difficult to predict as shown by the bumpiness of the plot. It is important to note that this error calculation is just an estimation, computed without performing higher order analysis, meaning that it is likely to be more inaccurate than the relative error results shown in Figure 17, since those results will compare many element sizes and many polynomial orders on the same problem statement.

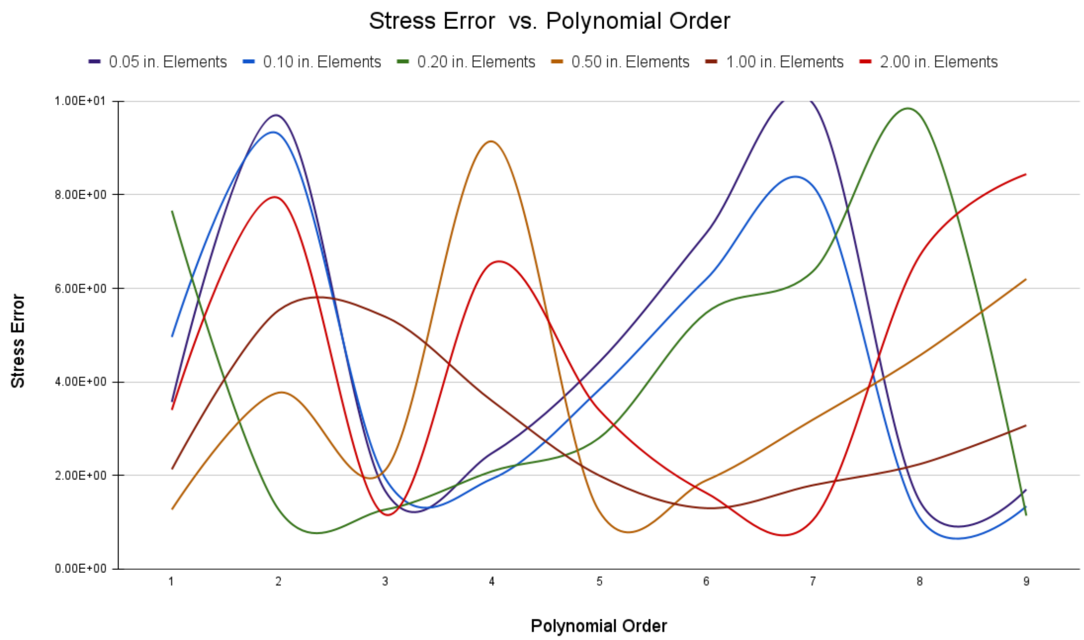

Figure 15

Figure 16, on the other hand, is computed as CREO's error estimation as a percentage of the maximum principle stress, and is much smoother and well-behaved when compared across different element sizes. This could be for several reasons, but the most straightforward explanation is just that the maximum principle stress is a more accurate measure than the error estimation, and as can be seen in Figure 13, the distribution converges on a certain value, which is almost the case here.

Another point-of-view of the results in this figure are that the optimal polynomial order for each element size is much more clear; this optimal order can be seen as the "elbow" or local minima of each line. Interestingly, this optimal polynomial order increases from about 2 for element sizes of 0.05, 0.1, and 0.2 inches, which seems to show that finer element meshes converge more quickly. Following this reasoning, the "elbows" of each line also mark a point where the accuracy diverges somewhat from the correct solution, although this divergence could also be seen as a byproduct of CREO's estimation inaccuracies carried over from the following plot.

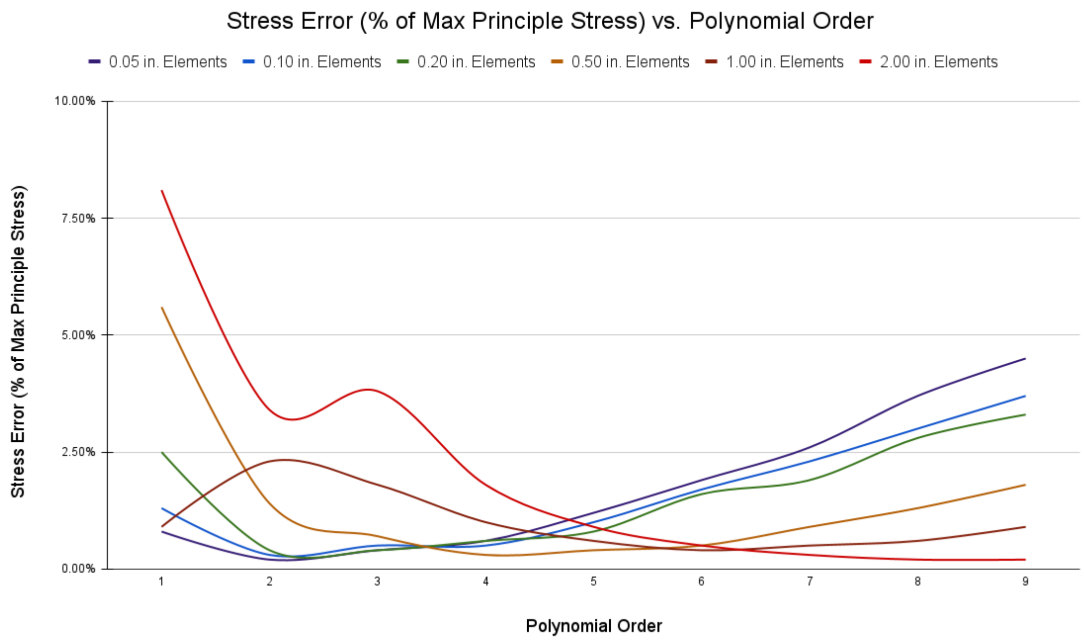
Stress Error (% of Max Principle Stress) vs. Polynomial Order
0.05 in. Elements
0.10 in. Elements
0.20 in. Elements
0.50 in. Elements
1.00 in. Elements
2.00 in. Elements
10.00%
7.50%
5.00%
2.50%
0.00%
Stress Error (% of Max Principle Stress)
1
2
3
4
5
6
7
8
9
Polynomial Order

Figure 16

7.3 Relative Error

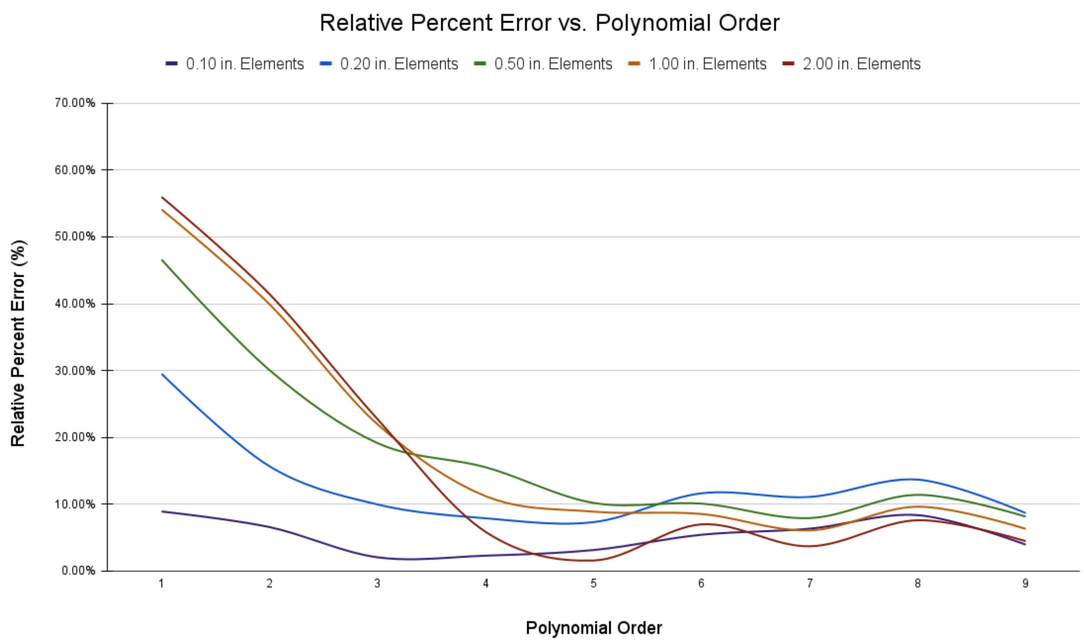

Figure 17

As discussed in Section 6, the relative error is the percent error of each maximum principle stress for each polynomial order relative to the simulations using elements with max size 0.05 inches and corresponding polynomial order. While this metric makes the assumption that the analyses with the highest element counts yield the most accurate solutions, this statement is consistent with the theoretical features of FEM, and is verified experimentally in Figure 17.

This figure reiterates the results from the previous sections, however this time using only the max principle stresses from CREO instead of the error metric provided. Each line here, representing a simulation performed with a varying maximum element size, again converges to the common value given by the simulation with elements of max size 0.05 inches. (Unseen in this plot, since it is the "ground truth" value which all the other lines are compared to.) This plot also shows the effect of element size on convergence rate, since more granular element meshes converge more quickly, that is, they require a lower polynomial order overall in order to come closer to the ground truth value.

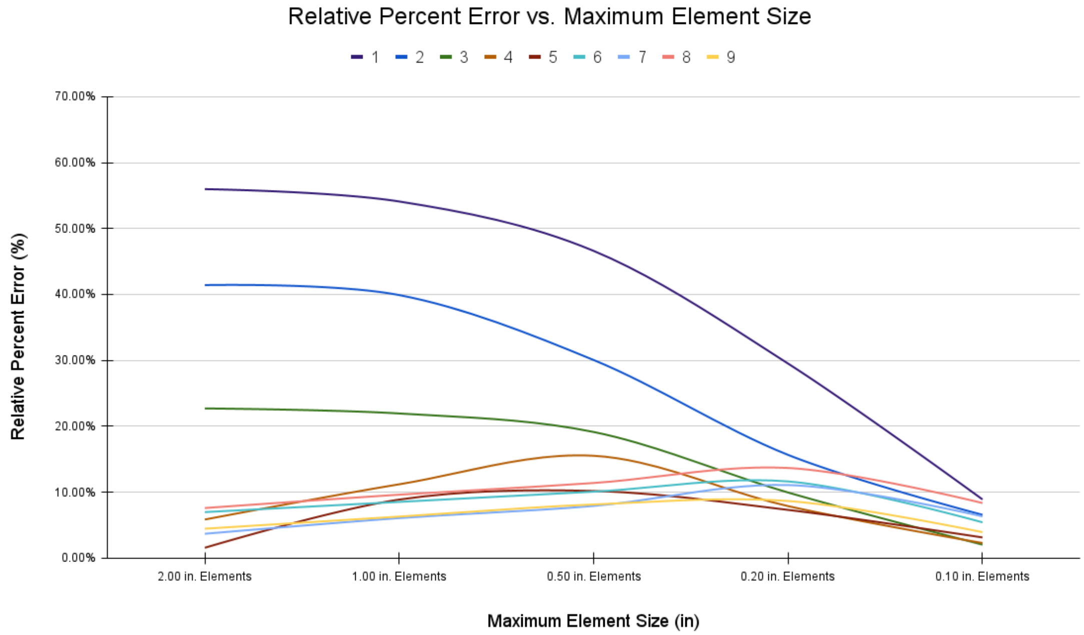

Figure 18: Relative Error vs. Maximum Element Size. Note, Legend Entries Correspond to Polynomial Order

Additionally, a similar plot is shown in Figure 18, however, with the maximum element size on the x-axis. The results here also confirm a fundamental feature of FEM, which is that analysis using higher order polynomial shape functions converges faster when compared across other factors such as element size.

8 Conclusions

The results from this project validate a fundamental principle of FEM, which is that solutions to the weak formulation of a bounday value problem approach the solution to the strong formulation as more elements are added to the mesh, or higher order polynomial shape functions are used.

Additionally, these results show exactly how quickly, when more elements are added or higher order polynomials are used, the solutions converge. If a more rigorous analysis was to be performed on the nozzle for uses in real-world situations, an experiment of this sort could prove to be very useful, since it would show how to get the most accurate results with the lowest order polynomials and least amount of elements. It stands to reason that these quantities will vary widely depending on the problem setup, depending on applied loads, displacement constraints, mesh geometry, and etc., since they all affect the modality of errors that will occur. For example, if a mesh has many tight corners, it may benefit more to use more elements rather than to increase polynomial order.

This project shows not only how useful FEM analysis can be in all disciplines of engineering and particularly in rocket propulsion, for example, in which it is infeasible and costly to test many different nozzle designs. To make the most out of the FEM analysis, however, an understanding of error and how solutions converge is essential.

References

[1] PTC Inc. Creo parametric. URL https://www.ptc.com/en/products/creo.

[2] G. V. R. RAO. Exhaust nozzle contour for optimum thrust. *Journal of Jet Propulsion*, 28(6): 377–382, 1958. doi: 10.2514/8.7324. URL https://doi.org/10.2514/8.7324.