$T(m) = T(m/3) + m$

$T(m) = (T(m/9) + m/3) + m$

$T(m) = (T(m/27) + m/9) + m/3 + m$

$\vdots \qquad \vdots$

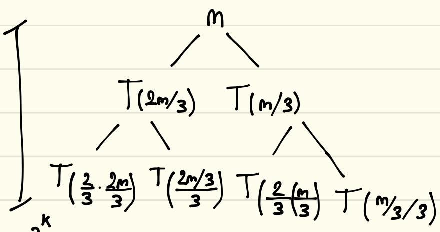
$T(m) = \left(T(m/3^K) + \frac{m}{3^{K-1}}\right) + \dots + \frac{m}{9} + \frac{m}{3} + \frac{m}{3^0}$

1º) $\sum_{i=0}^{K} (1/3)^i = m\left(\frac{1}{\frac{1}{1} - \frac{1}{3}}\right) = \dots = \frac{3m}{2} = O(m)$

TROCAR $m = 3^K$

2º) $1 + 3 + 9 + \dots + \frac{3^K}{3^{K-(K-2)}} + \frac{3^K}{3^{K-(K-1)}} + \frac{3^K}{3^{K-(K)}}$

$3^0 + 3^1 + 3^2 + \dots + 3^{K-2} + 3^{K-1} + 3^K$



$m = 3^k$

$k = \log_3 m$

m

$3m/3$

$9m/9$

$\vdots$

$\frac{3^k m}{3^k}$

$O(m \cdot \log_3 m)$

$$
\begin{aligned}
T(m) &= T(m/2) + 1 \\
&= (T(m/4) + 1) + 1 \\
&= (T(m/8) + 1) + 1) + 1 \\
&\vdots
\end{aligned}
$$

$$T(m) = (T(m/2^k)) + 1 \ldots + 1 + 1 \Rightarrow \sum_0^k 1 \Rightarrow \log m + 1$$

$$
\begin{aligned}
T(m) &= T(m/2) + m \\
&= (T(m/4) + m/2) + m \\
&= (T(m/8) + m/4) + m/2 + m \\
&\vdots \\
&= T(2^k/2^k) + T(2^k/2^{k-1}) + 2^k/2 + 2^k/2^0 \\
&= 1 + 2 + \ldots 2^{k-1} + 2^k
\end{aligned}
$$

$$T(m) = \sum_0^k 2^i \Rightarrow \frac{2^{k+1} - 1}{2-1} = \frac{2^k \cdot 2 - 1}{1} = 2m - 1$$

4.3.1) $T(m) = T(m-1) + m$

$$T(m) = m + T(m-1)$$
$$T(m) = m + (m-1 + T(m-2))$$
$$T(m) = m + (m-1 + (m-2 + T(m-3)))$$
$$T(m) = m + (m-1 + (m-2 + (m-3 + T(m-4))))$$
$$\dots$$
$$T(m) = T(m-(m-1)) + m-(m-2) + \dots + m-2 + m-1 + m$$
$$T(m) = \sum_{1}^{m} i = \frac{m(m+1)}{2} = m^2 + \frac{m}{2} \Rightarrow O(m^2)$$

4.3-2) $m = 2^K$

$$T(m) = T(m/2) + 1$$

$$T(m) = \left(T(m/4) + 1\right) + 1$$

$$T(m) = \left(T(m/8) + 1\right) + 1\right) + 1$$

$\vdots$

$$T(m) = \underbrace{\left(T(m/2^K) + 1\right) + 1\right) \ldots + 1}_{K \text{ PARCELAS}} \Rightarrow \sum_{1}^{K} 1 \Rightarrow O(\log m)$$

4.3-3) $m = 2^K$

$$T(m) = 2T(m/2) + m$$

$$= 2\left(2T(m/4) + m/2\right) + m$$

$$= 2\left(2\left(2T(m/8) + m/4\right) + m/2\right) + m$$

$\vdots$

$$= m + \frac{2m}{2} + \frac{4m}{4} + \ldots + \frac{2^K m}{2} \Rightarrow \sum_{1}^{K} m \Rightarrow O(m \log m)$$

4.3-6) $m = 2^K$ $c = 17$

$$T(m) = 2T(m/2 + c) + m$$

$$= m + 2\left(m/2 + c + 2T\left(m/4 + c/2 + c\right)\right)$$

$$= m + 2\left(m/2 + c + 2\left(m/4 + c/2 + c + 2T\left(m/8 + c/4 + c/2 + c\right)\right)\right)$$

$\vdots$

$$= \underbrace{m + \frac{2m}{2} + \frac{4m}{4} + \ldots + \frac{2^K m}{2^K}}_{\Downarrow} + (2c) + (2c + 4c) + (2c + 4c + 8c) + \ldots$$

$$\sum_{1}^{K} = m \Rightarrow O(\log m . m)$$

4.4-1)

$$n \longrightarrow n$$

$$n/2 \quad n/2 \quad n/2 \longrightarrow n/2$$

$$T(n/4) \quad T(n/4) \quad T(n/4) \longrightarrow n/4$$

$\log n$

$O(n \log n)$

4.4-2)

$$n^2 \longrightarrow n^2$$

$$\left(\frac{n}{2}\right)^2 \longrightarrow n^2/2$$

$$\left(\frac{n}{2}\right)^2 \longrightarrow n^2/4 \qquad \Rightarrow O(n^2 \log n)$$

$$\vdots$$

$$T\left(\left(\frac{n}{2}\right)^k\right)$$

$\log n$

4.4-3)

$$n \longrightarrow n + c$$

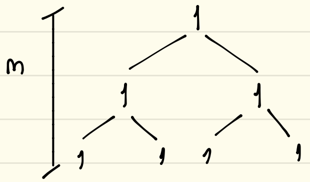
$$n/2+2 \quad n/2+2 \quad n/2+2 \quad n/2+2 \longrightarrow n/2 + c$$

$$\ldots \quad T\left(n/4 + \frac{2}{2} + 2\right) \quad \ldots \quad \ldots \longrightarrow n/4 + c$$

$\log n$

$O(\log n . n)$

4.4-4)



$$\sum_{i=0}^{m-1} = 2^i \Rightarrow 2^m - 1 \Rightarrow O(2^m)$$

4.4-5)

4.5-4) $T(m) = 4T(m/2) + m^2 \log m$

$$= m^2 \log m + 4\left(\left(m/2\right)^2 \log m/2 + 4\left(T(m/4)\right)\right)$$

$$= m^2 \log m + 4\left(\left(m/2\right)^2 \log \frac{m}{2}\right) + 16\left(\left(\frac{m}{4}\right)^2 \log \frac{m}{4} + 4\left(T(m/8)\right)\right)$$

$$= m^2 \log m + m^2 \log \frac{m}{2} + m^2 \log \frac{m}{4} + \dots$$

$$= m^2 \cdot \sum_0^k \log \frac{m}{2^i}$$

$$= m^2 \cdot \left(\sum_0^k \log m - \sum_0^k \log 2i\right) = \sum_0^k i = \frac{K(K+1)}{2}$$

$$= m^2\left(\log m \cdot \log m - \left(\frac{\log^2 m + \log m}{2}\right)\right)$$

$$= m^2\left(\log^2 m - \frac{\log^2 m}{2} + \frac{\log m}{2}\right) \Rightarrow \dots T(m) = \frac{m^2}{2} \log m(m+1)$$ //

4.5-1)

a) $T(n) = 2T(n/4) + 1$

$a = 2 \quad b = 4 \quad f(n) = 1 \Rightarrow f(n) = \Theta(1)$

CASO 1: $n^{\log_b a} = n^{\log_4 2} = n^{1/2}$

$f(n) = O(n^{\log_4 2 - \epsilon})$ com $\epsilon = 1/2 \Rightarrow T(n) = \Theta(n^{\log_4 2}) = \Theta(\sqrt{n})$

b) $T(n) = 2T(n/4) + \sqrt{n} \qquad a = 2 \quad b = 4 \quad f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_4 2} = n^{1/2}$ (CASO 2) $\Rightarrow f(n) = O(\sqrt{n}) \Rightarrow T(n) = \Theta(\sqrt{n} \log n)$

c) $T(n) = 2T(n/4) + n \qquad a = 2 \quad b = 4 \quad f(n) = n$

ANÁLOGO A b), MAS CAI CASO 3 $\Rightarrow f(n) > \sqrt{n} \Rightarrow T(n) = \Theta(n)$

d) $2T(n/4) + n^2 \qquad a = 2 \quad b = 4 \quad f(n) = n^2$

ANÁLOGO A b), MAS CAI CASO 3 $\Rightarrow f(n) > \sqrt{n} \Rightarrow T(n) = \Theta(n^2)$

4.5-3) $T(n) = T(n/2) + \Theta(1) \qquad a = 1 \quad b = 2 \quad f(n) = \Theta(1)$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$ CASO 2 $\Rightarrow f(n) = 1 \Rightarrow T(n) = \Theta(1 \log n) = \Theta(\log n)$