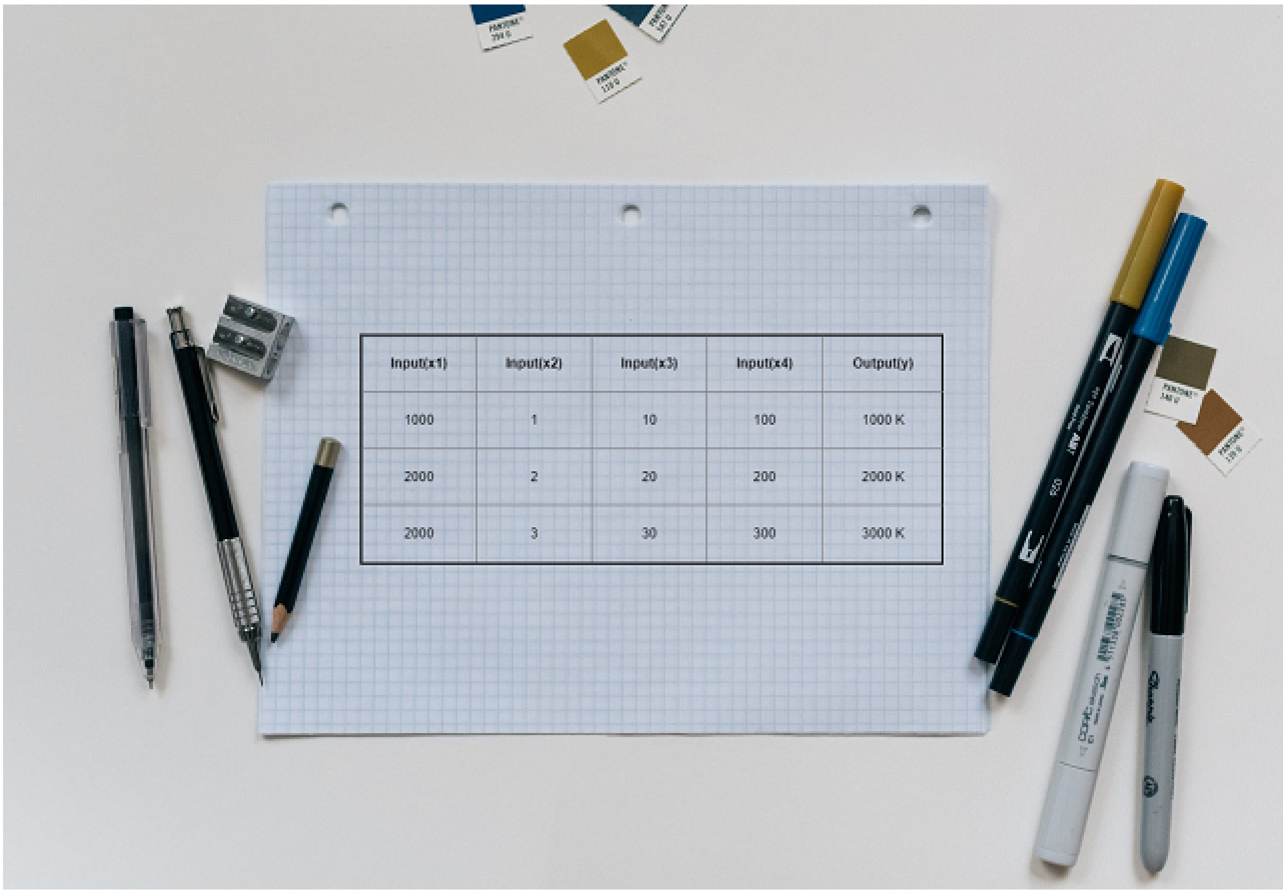
Multivariate Linear Regression From Scratch With Python



In this tutorial we are going to cover linear regression with multiple input variables. We are going to use same model that we have created in Univariate Linear Regression (https://satishgunjal.github.io/univariate_lr/) tutorial. I would recommend to read Univariate Linear Regression tutorial first. We will define the hypothesis function with multiple variables and use gradient descent algorithm. We will also use plots for better visualization of inner workings of the model. At the end we will test our model using training data.

Introduction

In case of multivariate linear regression output value is dependent on multiple input values. The relationship between input values, format of different input values and range of input values plays important role in linear model creation and prediction. I am using same notation and example data used in Andrew Ng's Machine Learning course (https://www.coursera.org/learn/machine-learning/home/welcome)

Hypothesis Function

Our hypothesis function for univariate linear regression was

```
h(x) = theta_0 + theta_1*x_1
where x_1 is only input value
```

For multiple input value, hypothesis function will look like,

```
h(x) = theta_0 + theta_1 * x_1 + theta_2 * x_2 .....theat_n * x_n
where x_1, x_2...x_n are multiple input values
```

If we consider the house price example then the factors affecting its price like house size, no of bedrooms, location etc are nothing but input variables of above hypothesis function.

Cost Function

Our cost function remains same as used in Univariate linear regression

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

For more details about cost function please refer 'Create Cost Function' section of Univariate Linear Regression (https://satishgunjal.github.io/univariate_lr/)

Gradient Descent Algorithm

Gradient descent algorithm function format remains same as used in Univariate linear regression. But here we have to do it for all the theta values(no of theta values = no of features + 1).

$$\begin{aligned}
&\text{repeat until convergence: } \{ \\
&\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) \cdot x_0^{(i)} \\
&\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) \cdot x_1^{(i)} \\
&\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) \cdot x_2^{(i)} \\
&\cdots \\
\}
\end{aligned}$$

For more details about gradient descent algorithm please refer 'Gradient Descent Algorithm' section of Univariate Linear Regression (https://satishgunjal.github.io/univariate_lr/)

Python Code

Notations used

- m = no of training examples (no of rows of feature matrix)
- n = no of features (no of columns of feature matrix)
- x’s = input variables / independent variables / features
- y’s = output variables / dependent variables / target

Import the required libraries

- numpy : Numpy is the core library for scientific computing in Python. It is used for working with arrays and matrices.
- pandas: Used for data manipulation and analysis
- matplotlib : It’s plotting library, and we are going to use it for data visualization

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

Load the data

- We are going to use ‘multivariate_housing_prices_in_portlans_oregon.csv’ CSV file
- File contains three columns ‘size(in square feet)’, ‘number of bedrooms’ and ‘price’

```
df =
pd.read_csv('https://raw.githubusercontent.com/satishgunjal/datasets/master/multivariate_housing_price
s_in_portlans_oregon.csv')
df.head() # To get first n rows from the dataset default value of n is 5
```

	size(in square feet)	number of bedrooms	price
0	2104	3	399900
1	1600	3	329900
2	2400	3	369000
3	1416	2	232000
4	3000	4	539900

```
X = df.values[:, 0:2]  # get input values from first two columns
y = df.values[:, 2]  # get output values from last coulmn
m = len(y) # Number of training examples

print('Total no of training examples (m) = %s \n' %(m))

# Show only first 5 records
for i in range(5):
    print('x =', X[i, ], ', y =', y[i])
```

```
Total no of training examples (m) = 47

x = [2104    3] , y = 399900
x = [1600    3] , y = 329900
x = [2400    3] , y = 369000
x = [1416    2] , y = 232000
x = [3000    4] , y = 539900
```

Understand the data

- There are total 97 training examples (m= 97 or 97 no of rows)
- There are two features (two columns of feature and one of label/target/y)
- Total no of features (n) = 2 (Later we will add column of ones(x_0) to make it 3)

Feature Normalization

- As you can notice size of the house and no of bedrooms are not in same range(house sizes are about 1000 times the number of bedrooms). This will have negative impact on gradient descent algorithm performance.
- In gradient descent algorithm we calculate the cost for every step. And if our input values differ by order of magnitude then results after every gradient descent step will also vary a lot.
- We can avoid this by changing the range of our input variables.
- We use below techniques to change the range of input variables
  - Feature Scaling
  - Mean Normalization
- **Feature Scaling**: In feature scaling we divide the input value by range(max - min) of input variable. By this technique we get new range of just 1.

```
x1 = x1 / s1
where,
x1 = input variable
s1 = range
```

- **Mean Normalization**: In mean normalization we subtract the average value from the input variable and then divide it by range(max - min) or by standard deviation of input variable.

```
x1 = (x1 - mu1)/s1
where,
x1 = input variable
mu1 = average value
s1 = range or standard deviation
```

Lets create function to normalize the value of input variable

```python
def feature_normalize(X):
  """
    Normalizes the features(input variables) in X.

    Parameters
    ----------
    X : n dimensional array (matrix), shape (n_samples, n_features)
        Features(input varibale) to be normalized.

    Returns
    -------
    X_norm : n dimensional array (matrix), shape (n_samples, n_features)
        A normalized version of X.
    mu : n dimensional array (matrix), shape (n_features,)
        The mean value.
    sigma : n dimensional array (matrix), shape (n_features,)
        The standard deviation.
  """
  #Note here we need mean of indivdual column here, hence axis = 0
  mu = np.mean(X, axis = 0)
  # Notice the parameter ddof (Delta Degrees of Freedom)  value is 1
  sigma = np.std(X, axis= 0, ddof = 1)  # Standard deviation (can also use range)
  X_norm = (X - mu)/sigma
  return X_norm, mu, sigma


X, mu, sigma = feature_normalize(X)

print('mu= ', mu)
print('sigma= ', sigma)
print('X_norm= ', X[:5])
```

```
mu=  [2000.68085106    3.17021277]
sigma=  [7.94702354e+02 7.60981887e-01]
X_norm=  [[ 0.13000987 -0.22367519]
 [-0.50418984 -0.22367519]
 [ 0.50247636 -0.22367519]
 [-0.73572306 -1.53776691]
 [ 1.25747602  1.09041654]]
```

Note: New mean or avearage value of normalized X feature is 0

```python
mu_testing = np.mean(X, axis = 0) # mean
mu_testing
```

```
array([3.77948264e-17, 2.74603035e-16])
```

Note: New range or standard deviation of normalized X feature is 1

```
sigma_testing = np.std(X, axis = 0, ddof = 1) # mean
sigma_testing
```

```
array([1., 1.])
```

```
# Lets use hstack() function from numpy to add column of ones to X feature
# This will be our final X matrix (feature matrix)
X = np.hstack((np.ones((m,1)), X))
X[:5]
```

```
array([[ 1.        ,  0.13000987, -0.22367519],
       [ 1.        , -0.50418984, -0.22367519],
       [ 1.        ,  0.50247636, -0.22367519],
       [ 1.        , -0.73572306, -1.53776691],
       [ 1.        ,  1.25747602,  1.09041654]])
```

Note above, we have added column of ones to X so final dimension of X is mxn i.e 97x3

Compute Cost

- function definition is same as used in Univariate Linear Regression
- numpy.dot() this function returns the dot product of two arrays. For 2-D vectors, it is the equivalent to matrix multiplication
- numpy.subtract() this function perform the element wise subtraction
- numpy.square() this function perform the element wise square

```
def compute_cost(X, y, theta):
  """
  Compute the cost of a particular choice of theta for linear regression.

  Input Parameters
  ----------------
  X : 2D array where each row represent the training example and each column represent the feature
ndarray. Dimension(m x n)
      m= number of training examples
      n= number of features (including X_0 column of ones)
  y : 1D array of labels/target value for each traing example. dimension(1 x m)

  theta : 1D array of fitting parameters or weights. Dimension (1 x n)

  Output Parameters
  -----------------
  J : Scalar value.
  """
  predictions = X.dot(theta)
  #print('predictions= ', predictions[:5])
  errors = np.subtract(predictions, y)
  #print('errors= ', errors[:5])
  sqrErrors = np.square(errors)
  #print('sqrErrors= ', sqrErrors[:5])
  #J = 1 / (2 * m) * np.sum(sqrErrors)
  # OR
  # We can merge 'square' and 'sum' into one by taking the transpose of matrix 'errors' and taking dot
product with itself
  # If your confuse about this try to do this with few values for better understanding
  J = 1/(2 * m) * errors.T.dot(errors)

  return J
```

Gradient Descent Function

```
def gradient_descent(X, y, theta, alpha, iterations):
  """
  Compute cost for linear regression.

  Input Parameters
  ----------------
  X : 2D array where each row represent the training example and each column represent the feature
ndarray. Dimension(m x n)
      m= number of training examples
      n= number of features (including X_0 column of ones)
  y : 1D array of labels/target value for each traing example. dimension(m x 1)
  theta : 1D array of fitting parameters or weights. Dimension (1 x n)
  alpha : Learning rate. Scalar value
  iterations: No of iterations. Scalar value.

  Output Parameters
  -----------------
  theta : Final Value. 1D array of fitting parameters or weights. Dimension (1 x n)
  cost_history: Conatins value of cost for each iteration. 1D array. Dimansion(m x 1)
  """
  cost_history = np.zeros(iterations)

  for i in range(iterations):
    predictions = X.dot(theta)
    #print('predictions= ', predictions[:5])
    errors = np.subtract(predictions, y)
    #print('errors= ', errors[:5])
    sum_delta = (alpha / m) * X.transpose().dot(errors);
    #print('sum_delta= ', sum_delta[:5])
    theta = theta - sum_delta;

    cost_history[i] = compute_cost(X, y, theta)

  return theta, cost_history
```

Lets update the gradient descent learning parameters alpha and no of iterations

```
# We need theta parameter for every input variable. since we have three input variable including X_0
(column of ones)
theta = np.zeros(3)
iterations = 400;
alpha = 0.15;


theta, cost_history = gradient_descent(X, y, theta, alpha, iterations)
print('Final value of theta =', theta)
print('First 5 values from cost_history =', cost_history[:5])
print('Last 5 values from cost_history =', cost_history[-5 :])
```

```
Final value of theta = [340412.65957447 110631.0502787   -6649.47427067]
First 5 values from cost_history = [4.76541088e+10 3.48804679e+10 2.57542477e+10 1.92146908e+10
 1.45159772e+10]
Last 5 values from cost_history = [2.04328005e+09 2.04328005e+09 2.04328005e+09 2.04328005e+09
 2.04328005e+09]
```

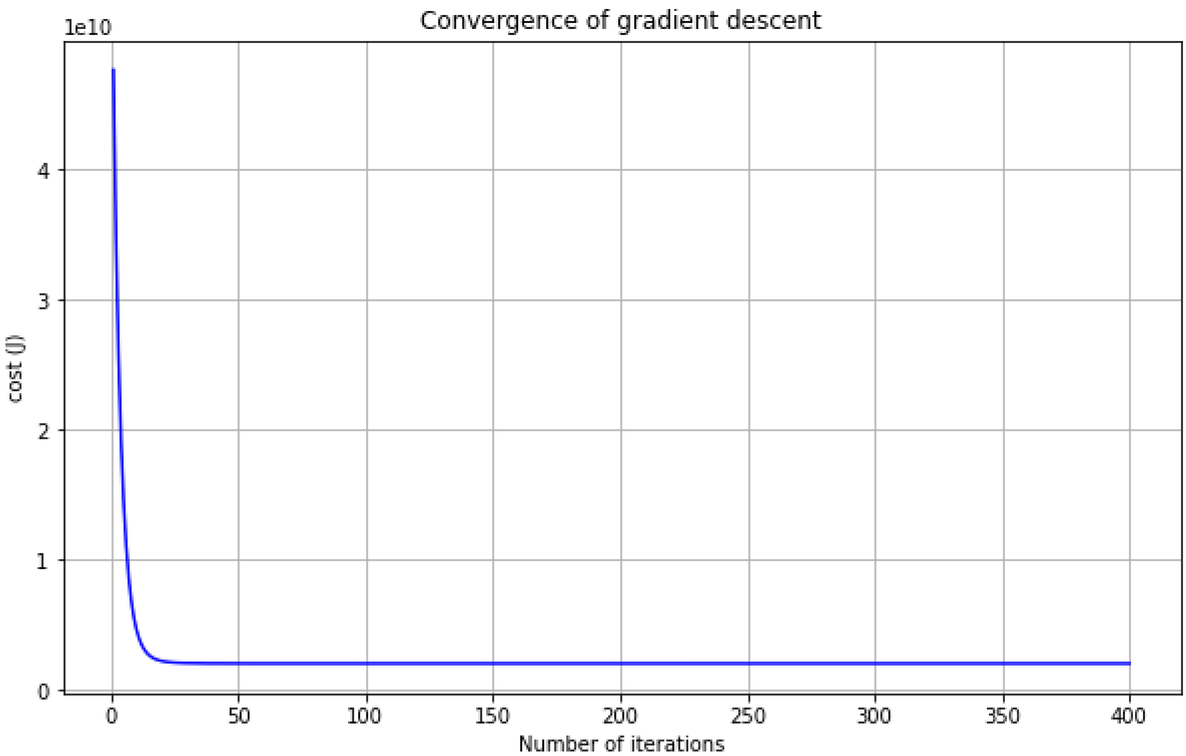
Note: 'cost_history' contains the values of cost for every step of gradient descent, and its value should decrease for every step of gradient descent

Visualization

Convergence of Gradient Descent

- cost_history contains the values of cost for every iteration performed during batch gradient descent
- If all our parameters are correct then cost should reduce for every iteration(step)
- Lets plot the values of cost against no of iterations to visualize the performance of the Gradient Descent Algorithm

```
import matplotlib.pyplot as plt
plt.plot(range(1, iterations +1), cost_history, color ='blue')
plt.rcParams["figure.figsize"] = (10,6)
plt.grid()
plt.xlabel("Number of iterations")
plt.ylabel("cost (J)")
plt.title("Convergence of gradient descent")
```

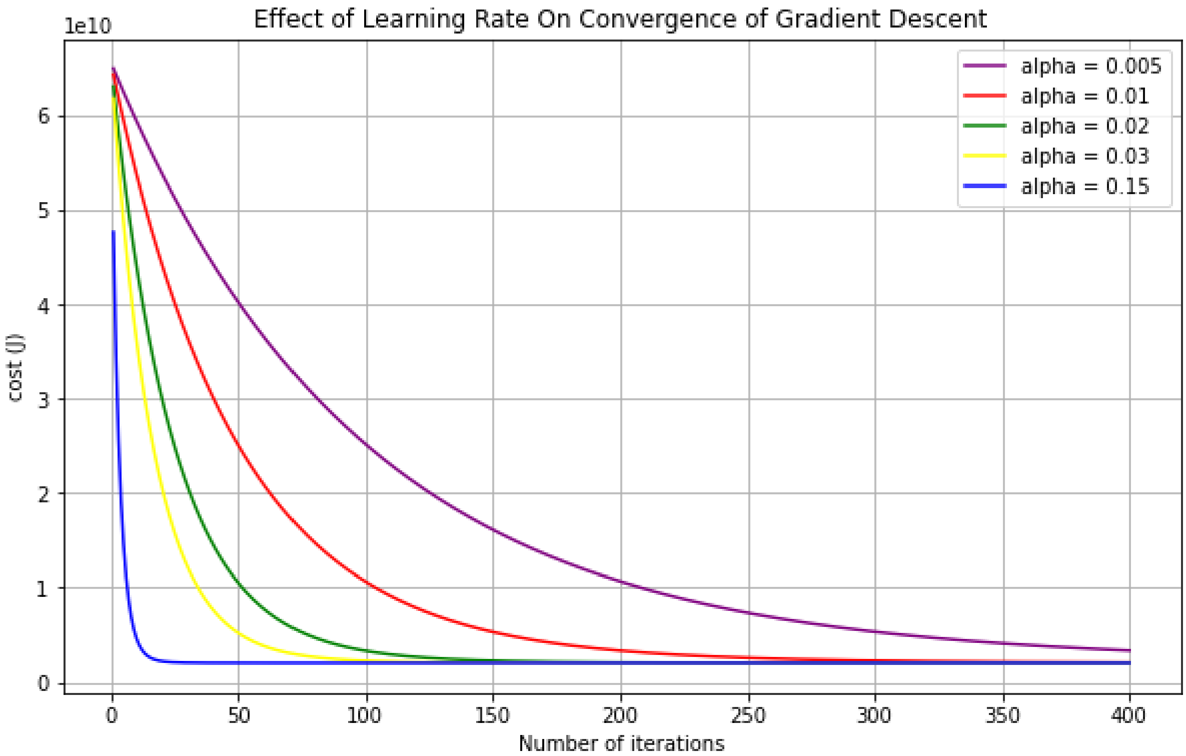


Note that curve flattened out at around 20th iteration and ramains same affter that. This is the indication of convergence of gradient descent

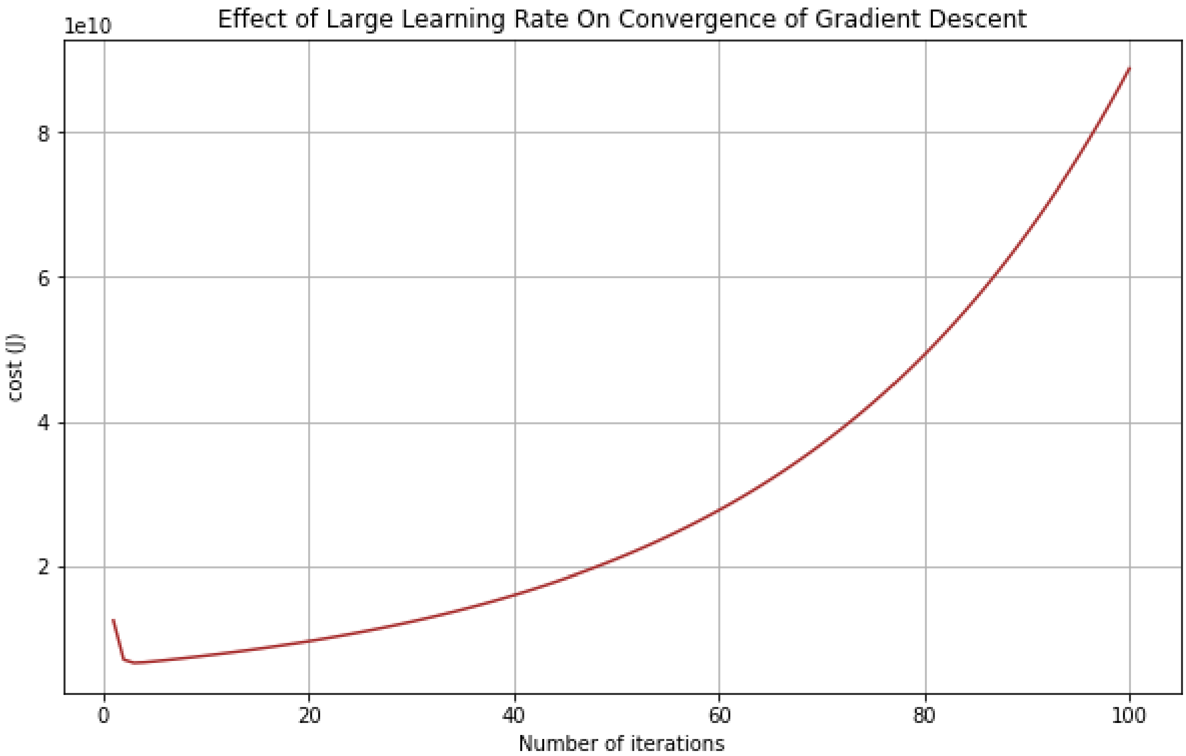
Effect Of Learning Rate On Convergence

- To check the effect of learning rate on convergence lets store the cost history for different learning rate and plot convergence plot for better visualization
- Notice the changes in the convergence curves as the learning rate changes.
- With a small learning rate(alpha = 0.005, purple line), gradient descent takes a very long time to converge to the optimal value.
- As we increase the alpha value, slope becomes sharp and gradient descent will take less time to converge
- But if the value of learning rate(alpha = 1.32, brown line) is too large then gradient descent may not decrease on every iteration, may even diverge

```
iterations = 400;
theta = np.zeros(3)

alpha = 0.005;
theta_1, cost_history_1 = gradient_descent(X, y, theta, alpha, iterations)

alpha = 0.01;
theta_2, cost_history_2 = gradient_descent(X, y, theta, alpha, iterations)

alpha = 0.02;
theta_3, cost_history_3 = gradient_descent(X, y, theta, alpha, iterations)

alpha = 0.03;
theta_4, cost_history_4 = gradient_descent(X, y, theta, alpha, iterations)

alpha = 0.15;
theta_5, cost_history_5 = gradient_descent(X, y, theta, alpha, iterations)

plt.plot(range(1, iterations +1), cost_history_1, color ='purple', label = 'alpha = 0.005')
plt.plot(range(1, iterations +1), cost_history_2, color ='red', label = 'alpha = 0.01')
plt.plot(range(1, iterations +1), cost_history_3, color ='green', label = 'alpha = 0.02')
plt.plot(range(1, iterations +1), cost_history_4, color ='yellow', label = 'alpha = 0.03')
plt.plot(range(1, iterations +1), cost_history_5, color ='blue', label = 'alpha = 0.15')

plt.rcParams["figure.figsize"] = (10,6)
plt.grid()
plt.xlabel("Number of iterations")
plt.ylabel("cost (J)")
plt.title("Effect of Learning Rate On Convergence of Gradient Descent")
plt.legend()
```




```
iterations = 100;
theta = np.zeros(3)

alpha = 1.32;
theta_6, cost_history_6 = gradient_descent(X, y, theta, alpha, iterations)

plt.plot(range(1, iterations +1), cost_history_6, color ='brown')
plt.rcParams["figure.figsize"] = (10,6)
plt.grid()
plt.xlabel("Number of iterations")
plt.ylabel("cost (J)")
plt.title("Effect of Large Learning Rate On Convergence of Gradient Descent")
```



Testing the model

- **Question: Estimate the price of a 1650 sq-ft, 3-bedroom house**

- Remember that we have normalized the data for in order to use the gradient descent algorithm. So we have to also normalize the given input data before any prediction
- Also we have to add column of ones to input data the way we have added to input feature X

```
normalize_test_data = ((np.array([1650, 3]) - mu) / sigma)
normalize_test_data = np.hstack((np.ones(1), normalize_test_data))
price = normalize_test_data.dot(theta)
print('Predicted price of a 1650 sq-ft, 3 br house:', price)
```

```
Predicted price of a 1650 sq-ft, 3 br house: 293081.46433492796
```

Conclusion

This concludes our multivariate linear regression. Now we know how to perform the feature normalization and linear regression when there are multiple input variables. In next tutorial we will use scikit-learn linear model to perform the linear regression.

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