Group Report Submission 1

Matthew Schnese

MAE 4421

April 23, 2025

1: Introduction

The goal of this project is to model an actuator and design a compensator to control the actuator's behavior. The compensator is designed with 4 constraints in mind: The actuator cannot go past 0.05 meters, the actuator cannot accelerate faster than 1.5 meters per second, the actuator should spend as little time as possible between 0.005 and 0.035 meters, and the force from the actuator cannot exceed 12 Newtons.

2: System Analysis

The system was modeled using the bond graph method and transfer functions were found for the total force of the system and the velocity of the 2nd mass. The transfer function for the total force can be found below in equation 1.

$$TF1 = \frac{1000s^3+140s^2+375000s+25000}{s^4+12s^3+418s^2+3437s+25000} \quad [1]$$

Once found, the acceleration and position of the 2nd mass could be found by performing a derivative and integral respectively on the transfer function of velocity. This corresponds to adding an s to the numerator to take a derivative and an s to the denominator to integrate in the Laplace domain. The transfer functions for the acceleration and position of the 2nd mass can be seen below in equations 2 and 3 respectively.

$$TF2 = \frac{25000s}{s^4+12s^3+418s^2+3437s+25000} \quad [2]$$

$$TF3 = \frac{25000}{s^5+12s^4+418s^3+3437s^2+25000s} \quad [3]$$

The system was analyzed using a unit step response from 0 to 0.25 seconds since the maximum x value is 0.05 meters. The uncompensated system had similar behavior for the total force and the acceleration, with both having a large peak then beginning to reduce after around 0.2 seconds. The position of the 2nd mass showed an exponential increase, passing 0.05 meters at around 0.225 seconds. The graphs of each value with respect to time can be seen below in figure 1.

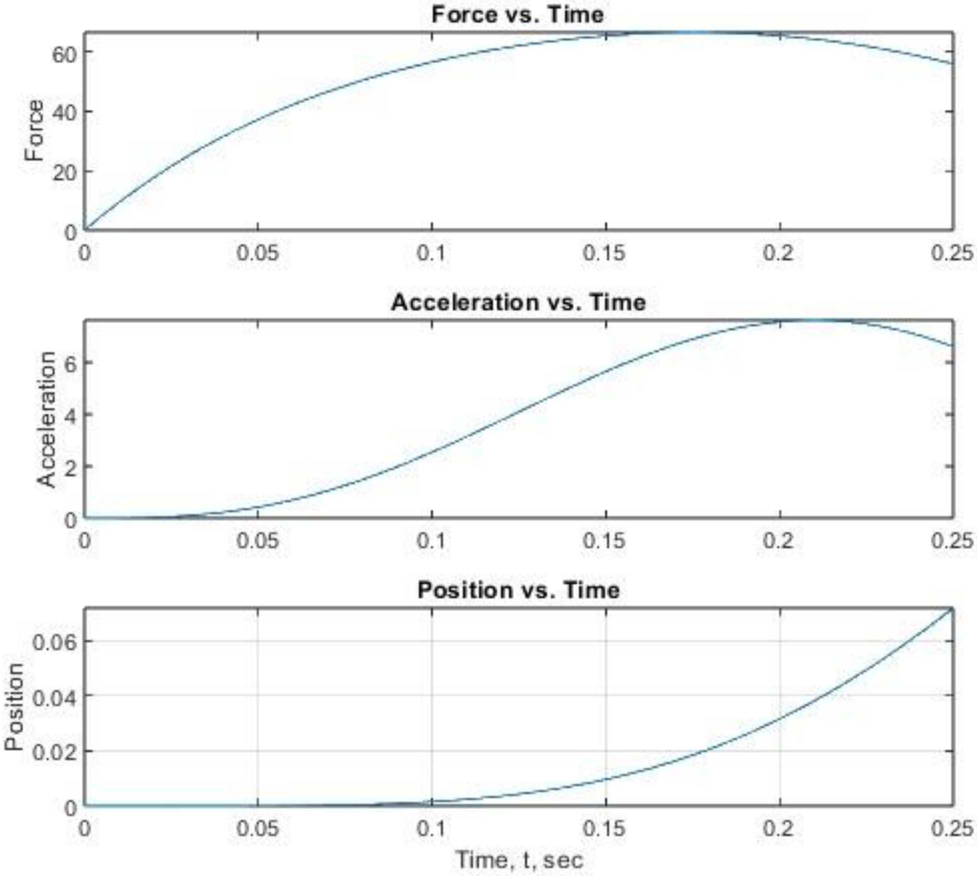
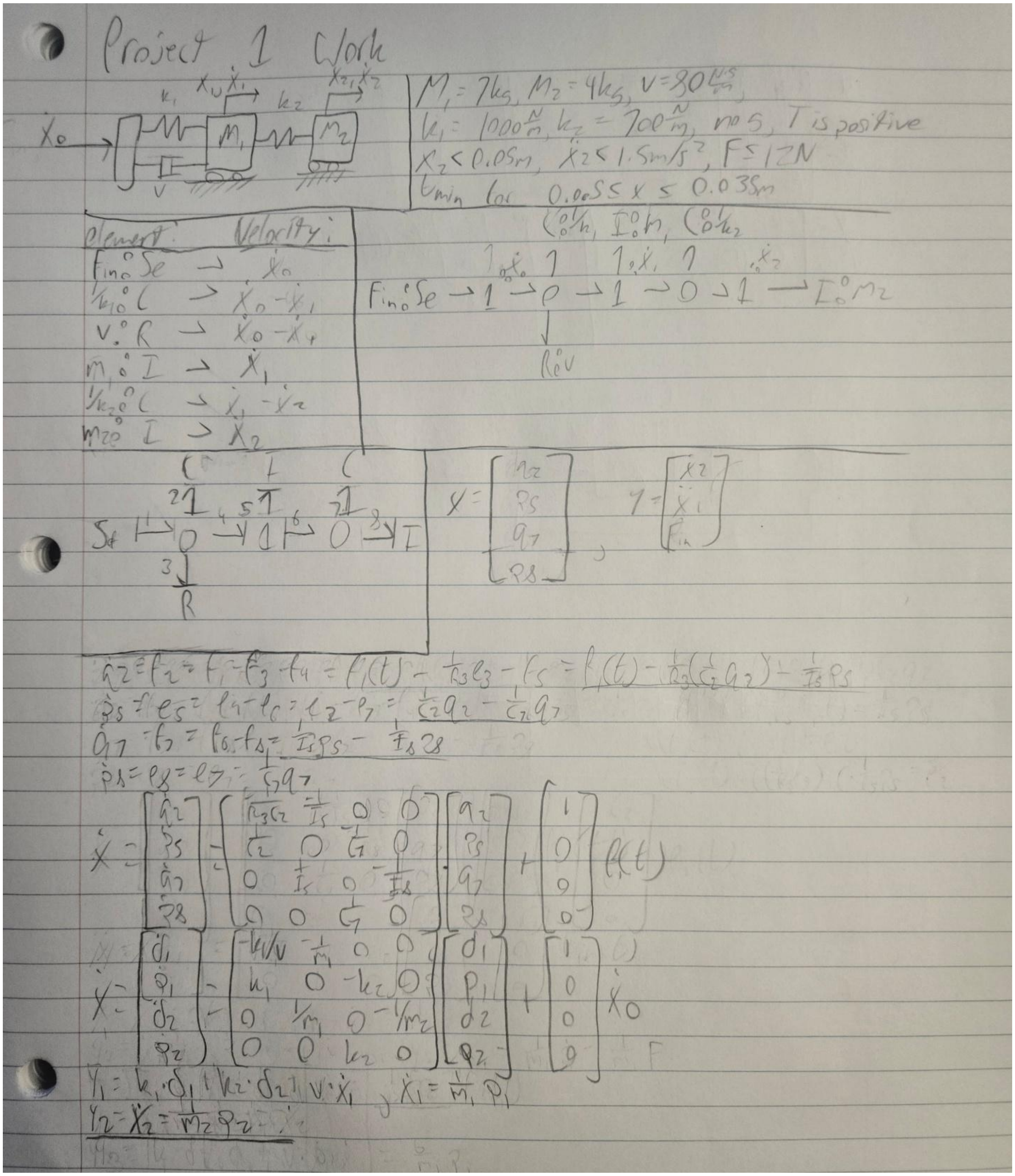


Figure 1: System behaviors to unit step input

Appendix A: Handwritten Work



Project 1 Work

$M_1 = 7kg, M_2 = 4kg, v = 80 \frac{N \cdot s}{m}$,
$k_1 = 1000 \frac{N}{m}, k_2 = 700 \frac{N}{m}$, no 5, T is positive
$x_2 < 0.05m, \ddot{x}_2 \le 1.5 m/s^2, F \le 12N$
t_{min} for $0.005 \le x \le 0.035m$

element:	Velocity:
$F_{in} \circ Se$ →	$\dot{x}_0$
$1/k_1 \circ C$ →	$\dot{x}_0 - \dot{x}_1$
$v \circ R$ →	$\dot{x}_0 - \dot{x}_4$
$m_1 \circ I$ →	$\dot{x}_1$
$1/k_2 \circ C$ →	$\dot{x}_1 - \dot{x}_2$
$m_2 \circ I$ →	$\dot{x}_2$

$$\begin{tikzcd} & & C \circ 1/k_1 & I \circ m_1 & C \circ 1/k_2 & & \\ F_{in} \circ Se \arrow[r] & 1_{\circ \dot{x}_0} \arrow[r] & 0 \arrow[r] \arrow[d] & 1_{\circ \dot{x}_1} \arrow[r] & 0 \arrow[r] & 1_{\circ \dot{x}_2} \arrow[r] & I \circ m_2 \\ & & R \circ v & & & & \end{tikzcd}$$

$$\begin{tikzcd} & C & I & C & \\ Se \arrow[r, "1"] & 0 \arrow[u, "2"] \arrow[r, "4"] \arrow[d, "3"] & 1 \arrow[u, "5"] \arrow[r, "6"] & 0 \arrow[u, "7"] \arrow[r, "8"] & I \\ & R & & & \end{tikzcd}$$

$$x = \begin{bmatrix} q_2 \\ p_5 \\ q_7 \\ p_8 \end{bmatrix}, \quad y = \begin{bmatrix} x_2 \\ \ddot{x}_1 \\ F_{in} \end{bmatrix}$$

$$\dot{q}_2 = f_2 = f_1 - f_3 - f_4 = f_1(t) - \frac{1}{R_3}e_3 - f_5 = \underline{f_1(t) - \frac{1}{R_3}\left(\frac{1}{C_2}q_2\right) - \frac{1}{I_5}p_5}$$

$$\dot{p}_5 = e_5 = e_4 - e_6 = e_2 - e_7 = \underline{\frac{1}{C_2}q_2 - \frac{1}{C_7}q_7}$$

$$\dot{q}_7 = f_7 = f_6 - f_8 = \underline{\frac{1}{I_5}p_5 - \frac{1}{I_8}p_8}$$

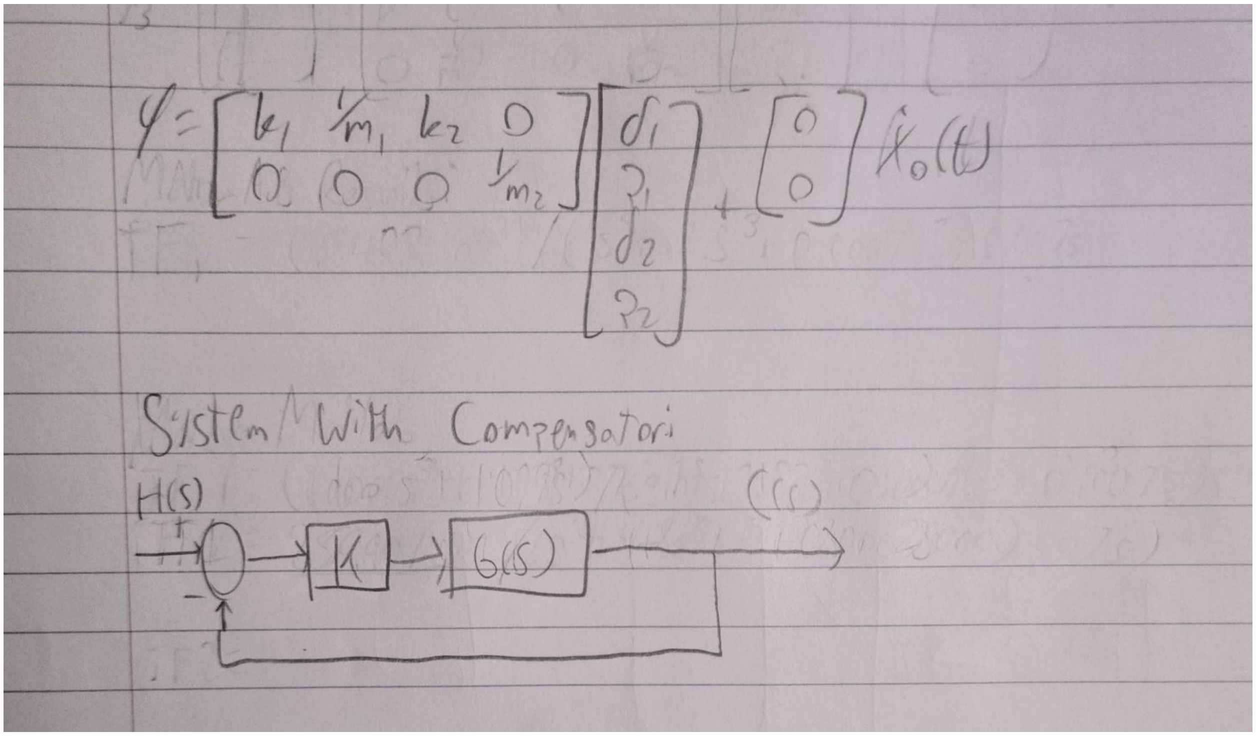
$$\dot{p}_8 = e_8 = e_7 = \frac{1}{C_7}q_7$$

$$\dot{x} = \begin{bmatrix} \dot{q}_2 \\ \dot{p}_5 \\ \dot{q}_7 \\ \dot{p}_8 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_3C_2} & \frac{-1}{I_5} & 0 & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{C_7} & 0 \\ 0 & \frac{1}{I_5} & 0 & -\frac{1}{I_8} \\ 0 & 0 & \frac{1}{C_7} & 0 \end{bmatrix} \cdot \begin{bmatrix} q_2 \\ p_5 \\ q_7 \\ p_8 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} f_1(t)$$

$$\dot{x} = \begin{bmatrix} \dot{\delta}_1 \\ \dot{p}_1 \\ \dot{\delta}_2 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} -k_1/v & -\frac{1}{m_1} & 0 & 0 \\ k_1 & 0 & -k_2 & 0 \\ 0 & 1/m_1 & 0 & -1/m_2 \\ 0 & 0 & k_2 & 0 \end{bmatrix} \begin{bmatrix} \delta_1 \\ p_1 \\ \delta_2 \\ p_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dot{x}_0$$

$Y_1 = k_1 \cdot \delta_1 + k_2 \cdot \delta_2 + v \cdot \dot{x}_1$, $\dot{x}_1 = \frac{1}{m_1} p_1$

$\underline{Y_2 = \ddot{x}_2 = \frac{1}{m_2} p_2}$



$$y = \begin{bmatrix} k_1 & 1/m_1 & k_2 & 0 \\ 0 & 0 & 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} d_1 \\ p_1 \\ d_2 \\ p_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \dot{x}_0(t)$$

System With Compensator:

Appendix B: MATLAB Code

```
clc;clear
A = [-12.5, -1/7, 0, 0; 1000, 0, -700, 0; 0, 1/7, 0, -1/4; 0, 0, 700, 0];
B = [1;0;0;0];
C1 = [1000, 1/7, 700, 0];
C2 = [0, 0, 0, 1/4];
D = [0];

%TF1
sys = ss(A,B,C1,D);
[num1, den1] = ss2tf(A,B,C1,D)

%TF2
sys = ss(A,B,C2,D);
[num2, den2] = ss2tf(A,B,C2,D)

%Plot1
sys = tf([0, 1000, 140, 3.75*10^5, 25000], [1, 12, 418, 3437, 25000]);
figure(1)
subplot (3,1,1)
t = linspace(0, .25, 1000);
ystep = step(sys, t);
plot(t, ystep)
ylabel('Force')
title('Force vs. Time')

%Plot2acceleration
sys = tf([0, 0, 0, 25000, 0], [1, 12, 418, 3437, 25000]);
subplot(3,1,2)
t = linspace(0, .25, 1000);
ystep = step(sys, t);
plot(t, ystep)
ylabel('Acceleration')
title('Acceleration vs. Time')

%Plot3position
sys = tf([0, 0, 0, 0, 0, 25000], [1, 12, 418, 3437, 25000, 0]);
subplot(3,1,3)
t = linspace(0, .25, 1000);
ystep = step(sys, t);
plot(t, ystep)
xlabel('Time, t, sec')
ylabel('Position')
title('Position vs. Time')
grid on
```