HW8 Max Schrader

March 23, 2021

1 Easy21 Task #4

Max Schrader

```
[2]: import numpy as np
from copy import copy, deepcopy
from random import random, choice, randint, uniform
import plotly.graph_objects as go
```

1.1 Creating the Objects

1.1.1 Cards

```
[3]: ACTIONS = {1: 'STICK', 0: 'HIT'}

class Cards:
    def __init__(self):
        self.distribution_range = (1, 10)

def get_card(self, first=False):
        return randint(*self.distribution_range) * 1 if (uniform(0, 1) < 2 / 3)

→or first \
        else -1 * randint(*self.distribution_range)
```

1.1.2 The Player

```
[4]: class Player:
    def __init__(self):
        self._cards = []
        self._sum = 0

    def check_bust(self):
        if self._sum < 1 or self._sum > 21:
            return True
        return False

    def clear_cards(self):
```

```
self._cards = []
self._sum = 0

def add_card(self, card):
    self._cards.append(card)
    self._sum = sum(self._cards)

def get_sum(self):
    return self._sum

def get_card(self, index):
    return self._cards[index]
```

1.1.3 The Dealer

```
[5]: class Dealer(Player):
    STICK_THRESHOLD = 17

def __init__(self):
    super(Dealer, self).__init__()

def play_strategy(self, card):
    self.add_card(card)
    if not self.check_bust():
        if self.get_sum() < self.STICK_THRESHOLD:
            return True
    return False</pre>
```

1.1.4 The Observed State

```
[6]: class State:
    def __init__(self, player_sum, dealer_first):
        self.dealer_first = dealer_first
        self.player_sum = player_sum
        self.terminal = False

    def copy(self):
        return copy(self)
```

1.1.5 The Game

```
[7]: class Easy21:
    def __init__(self, ):
        self.dealer = Dealer()
        self.player = Player()
```

```
self.cards = Cards()
       self.states = [range(1, 11), range(1, 22)]
       self.actions = ACTIONS
       self.actions_short = list(range(len(self.actions.keys())))
  def reset(self):
      self.__init__()
  def initialize game(self):
      self.dealer.clear_cards()
      self.player.clear_cards()
      self.dealer.add_card(self.cards.get_card(first=True))
      self.player.add_card(self.cards.get_card(first=True))
       s = State(self.player.get_sum(), self.dealer.get_card(0))
      return s
  def calc_reward(self):
       if self.dealer.check_bust() or (self.player.get_sum() > self.dealer.
→get_sum()):
           return 1
       elif self.player.get_sum() == self.dealer.get_sum():
           return 0
       return -1
  def step(self, action, state):
      state_1 = state.copy()
       if action == 1:
           while self.dealer.play_strategy(self.cards.get_card()):
           r = self.calc_reward()
           state_1.terminal = True
       else:
           card = self.cards.get_card()
           self.player.add card(card)
           if self.player.check_bust():
               state_1.terminal = True
               r = -1
           else:
               state_1.player_sum = self.player.get_sum()
               r = 0
       return state_1, r
```

1.2 Monte-Carlo Class (Serves as the Base Class for the Linear Function Class)

```
[8]: class MonteCarloAgent:
         def __init__(self, gym: Easy21):
             self.gym = gym()
             self.Q = np.zeros((len(self.gym.states[1]), len(self.gym.states[0]),
      →len(self.gym.actions_short)))
             self.N = deepcopy(self.Q)
             self.N0 = 100
             self.discount_factor = 1
         def calc_e(self, state: State) -> float:
             return self.NO / (self.NO + self.N[state.player_sum - 1, state.
      →dealer_first - 1].sum() * 1.)
         def get_best_action(self, state):
             rewards = self.Q[state.player_sum - 1][state.dealer_first - 1]
             max_reward = max(rewards)
             return choice([self.gym.actions_short[i] for i, reward in_
      →enumerate(rewards) if reward >= max_reward])
         def e_greedy(self, state):
             e = self.calc_e(state)
             if random() < e:</pre>
                 return choice(self.gym.actions_short)
                 return self.get_best_action(state)
         def update_q(self, history):
             for i, (s_k, a_k, r_k) in enumerate(history):
                 p_i = s_k.player_sum - 1
                 d_i = s_k.dealer_first - 1
                 G_t = sum([r_j * (self.discount_factor ** j) for j, (_, _, r_j) in_U
      →enumerate(history[i:])])
                 self.N[p_i, d_i, a_k] += 1
                 alpha = 1.0 / self.N[p_i, d_i, a_k]
                 self.Q[p_i, d_i, a_k] += alpha * (G_t - self.Q[p_i, d_i, a_k])
         def _train(self, ):
             self.gym.reset()
             s_t = self.gym.initialize_game()
             history = []
             while not s_t.terminal:
                 a_t = self.e_greedy(s_t)
                 s_t_1, r_t = self.gym.step(a_t, s_t)
                 history.append([s_t, a_t, r_t])
```

```
s_t = s_t_1
self.update_q(history)

def run(self, iterations, ):
    for _ in range(int(iterations)):
        self._train()

def get_V_star(self, ):
    player_sum = list(self.gym.states[1])
    dealer_showing = list(self.gym.states[0])
    V_star = [[max(actions) for actions in dealer] for dealer in self.Q]
    return player_sum, dealer_showing, V_star
```

```
[9]: mc_agent = MonteCarloAgent(gym=Easy21, )
[10]: mc_agent.run(iterations=int(5e5))
```

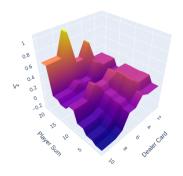
1.3 Linear Function Approximation

```
[129]: class LinearFunctionApproximation(MonteCarloAgent):
           def __init__(self, gym, Q_star=None):
               super(LinearFunctionApproximation, self).__init__(gym)
               self._e = 0.05
               self._lambda = 1
               self._alpha = 0.01
               self.Q_star = Q_star
               self._dealer_c = [[1, 4], [4, 7], [7, 10]]
               self._player_c = [[1, 6], [4, 9], [7, 12], [10, 15], [13, 18], [16, 21]]
               self._action_c = [0, 1]
               # parameters are initialize randomly
               self._shape = list(map(len, [self._dealer_c, self._player_c, self.
        →_action_c]))
               self._theta = self.calc_theta()
               self._shaped_zeros = np.zeros(self._shape)
               self._E = copy(self._shaped_zeros)
               self._param_num = np.prod(self._shape)
           def calc_theta(self):
               return np.random.randn(*self._shape) * 0.1
           def phi(self, s, a):
               d_sum, p_sum = s.dealer_first, s.player_sum
               features = copy(self._shaped_zeros)
```

```
d features = [(i, x[0] \le d sum \le x[1]) for i, x in enumerate(self.
→_dealer_c)]
       p_features = [(i, x[0] \le p_sum \le x[1]) \text{ for } i, x \text{ in enumerate(self.)}
→_player_c)]
       for i, d_inside in d_features:
           if d_inside:
               for j, p_inside in p_features:
                   if p_inside:
                       features[i, j, a] = 1
       return features
   def get best action(self, state):
       rewards = [np.dot(self.phi(state, a).flatten(), self._theta.flatten())__
→for a in self._action_c]
       max_reward = max(rewards)
       return choice([self.gym.actions_short[i] for i, reward in_
→enumerate(rewards) if reward >= max_reward])
   def calc_e(self, state=None) -> float:
       return self. e
   def compose Q(self):
       Q = np.zeros((len(self.gym.states[0]), len(self.gym.states[1]),
→len(self.gym.actions_short)))
       for i in self.gym.states[0]:
           for j in self.gym.states[1]:
               for a in self.gym.actions_short:
                   Q[i - 1, j - 1, a] = np.dot(self.phi(State(player_sum=j,_
→dealer_first=i), a).flatten(), self._theta.flatten())
       return Q
   def _step(self, log_error):
       self.gym.reset()
       s_t = self.gym.initialize_game()
       a_t = self.e_greedy(s_t)
       self._E = copy(self._shaped_zeros)
         self._theta = self.calc_theta()
       a_t_1 = a_t
       while not s_t.terminal:
           s_t_1, r_t = self.gym.step(a_t, s_t)
           phi_t = self.phi(s_t, a_t)
           q = np.dot(phi_t.flatten(), self._theta.flatten())
           if not s_t_1.terminal:
               a_t_1 = self.e_greedy(s_t_1)
               phi_t_1 = self.phi(s_t_1, a_t_1)
               q_t_1 = np.dot(phi_t_1.flatten(), self._theta.flatten())
```

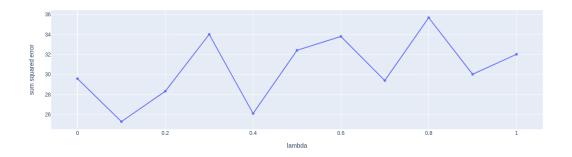
```
d = r_t + q_t_1 - q
                   else:
                       d = r t - q
                   self._E += phi_t
                   self._theta += self._alpha * d * self._E
                   self._E *= self.discount_factor * self._lambda
                   s_t = s_t_1
                   a_t = a_t_1
               if log_error:
                   return np.sum(np.square(self.Q_star - np.swapaxes(self.
        \rightarrowcompose_Q(), 0, 1)))
               return None
           def run(self, iterations, log_error=False):
               error = []
               for i in range(iterations):
                   if log_error:
                       error.append((i, self._step(log_error)))
                   self._step(log_error)
               if log_error:
                   return error
               self.Q = self.compose_Q()
           def get_V_star(self, ):
               dealer_showing = list(self.gym.states[0])
               player_sum = list(self.gym.states[1])
               V_star = [[max(actions) for actions in dealer] for dealer in self.Q]
               return player_sum, dealer_showing, V_star
[102]: If_agent = LinearFunctionApproximation(gym=Easy21, )
[103]: lf_agent.run(100000)
[104]: x, y, z = lf_agent.get_V_star()
       fig = go.Figure(data=[go.Surface(z=z, x=x, y=y, showscale=False)])
       camera = dict(
           up=dict(x=0, y=0, z=1),
           center=dict(x=0, y=0, z=0),
           eye=dict(x=-1.5, y=1.5, z=1.5)
       )
       fig.update_layout(scene_camera=camera,
                         scene=dict(yaxis_title='Dealer Card', xaxis_title='Player_

Sum', zaxis_title="V*"),
                                    margin=dict(r=20, b=10, l=10, t=10))
       fig.show()
```



1.3.1 Plotting Different λ

```
[105]: def calc_mean_error(mc_Q, sarsa_Q):
           return np.sum(np.square(mc_Q - sarsa_Q))
[127]: error = []
       lambdas = [e * .1 \text{ for } e \text{ in } range(0, 11, 1)]
       for _lambda in lambdas:
           lf_agent = LinearFunctionApproximation(gym=Easy21, )
           lf_agent._lambda = _lambda
           lf_agent.run(iterations=int(10000))
           error.append((_lambda, calc_mean_error(mc_agent.Q, np.swapaxes(lf_agent.Q,_
        \rightarrow 0, 1))))
           print(_lambda, error[-1][-1])
      0.0 29.58632969782763
      0.1 25.29116445095628
      0.2 28.331113566863216
      0.3000000000000004 34.023385363132576
      0.4 26.101176075194736
      0.5 32.42500093404073
      0.600000000000001 33.812650406530935
      0.700000000000001 29.40197275925342
      0.8 35.688091785968176
      0.9 30.030058571031514
      1.0 32.02687966594981
[128]: fig = go.Figure()
       fig.add_trace(go.Scatter(x=[e[0] for e in error], y=[e[1] for e in error]))
       fig.update_layout(xaxis_title='lambda', yaxis_title='sum squared error')
       fig.show()
```



1.3.2 Plotting the Learning Curve

[133]: error = []

```
lambdas = [e * .1 \text{ for } e \text{ in } range(0, 11, 1)]
       for _lambda in lambdas:
           lf_agent = LinearFunctionApproximation(gym=Easy21, Q_star=mc_agent.Q)
           lf_agent._lambda = _lambda
           error.append((_lambda, lf_agent.run(iterations=int(1000), log_error=True)))
             error.append((_lambda, calc_mean_error(mc_agent.Q, np.swapaxes(lf_agent.
        \hookrightarrow Q, 0, 1))))
           print(_lambda, error[-1][-1][-1])
      0.0 (999, 34.538254635252436)
      0.1 (999, 40.345285642557045)
      0.2 (999, 34.82723450370783)
      0.3000000000000004 (999, 33.7542768680677)
      0.4 (999, 34.10657223295483)
      0.5 (999, 41.40280297980871)
      0.6000000000000001 (999, 35.87625453184009)
      0.700000000000001 (999, 40.065774574823635)
      0.8 (999, 35.19697005793352)
      0.9 (999, 35.93240972600771)
      1.0 (999, 42.978300695507265)
[135]: data = []
       for _lambda, errors in error:
           data.append(go.Scatter(x=[x[0] \text{ for } x \text{ in errors}], y=[x[1] \text{ for } x \text{ in errors}],
       fig = go.Figure(data=data)
       fig.update_layout(xaxis_title='iterations', yaxis_title='sum squared error')
       fig.show()
```

