MAT351/APC351
Topics in Mathematical Modeling

General Course Description

This course draws problems from the sciences and engineering for which mathematical models have been developed and analyzed in order to describe, understand and predict natural and man-made phenomena. Topics will change from year to year, ranging across the physical sciences and biology, and including cognitive science and neurobiology. Basic familiarity with the application field will be assumed (at high-school or freshman level), but details of specific systems and models will be provided. Model-building strategies will be described, including the level of detail and selection of appropriate mathematical 'languages.' Analytical and computational methods will be covered, and their results and implications interpreted for the applications at hand. The course will emphasise the manner in which applications motivate mathematical developments and how mathematical techniques influence the questions that science addresses.

In SPRING 2011 the topic will be
MATHEMATICAL NEUROSCIENCE.

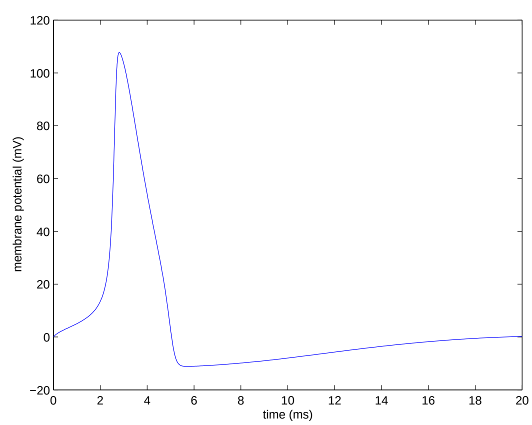

Figure 1: A mathematician's version of an action potential or spike: the currency of thought?

MAT351/APC351: MATHEMATICAL NEUROSCIENCE

• Can thinking about differential equations help us think about how we move, or even how we think?
• How random are our choices?

This course will barely reach such questions, but it will introduce some key mathematical tools used in modeling and analyzing problems and data in neuroscience. It will also provide sufficient background in neuroscience that those with little knowledge in the area can appreciate how mathematical models are constructed. The syllabus combines modeling techniques with mathematical methods including differential equations and elementary stochastic dynamical systems. After describing some of the phenomena and mathematical tools, we'll discuss cells as electrical circuits, the Hodgkin-Huxley (HH) equations that describe action potentials (spikes) in single neurons, and generalizations of HH to describe bursting neurons (e.g. in locomotion, and other rhythmic patterns). We move on to propagation of action potentials and reaction-diffusion equations (traveling waves in PDEs with one space dimension), and modeling small networks of neurons coupled by chemical synapses and electrical gap junctions. Reductions to phase oscillators and integrate-and-fire models will be discussed, and the course will also include topics such as Hopfield-Grossberg type neural nets, leaky accumulator models and drift-diffusion models of human decision making, and information theoretic approaches to the analysis of neural spike trains.

For **PACM Certificate students**, the course can provide a vehicle for independent work.

Prerequisites: Multivariable calculus and linear algebra at the level of MAT 201-202 or similar, plus some knowledge of ordinary differential equations (e.g.: APC350/CEE350 Introduction to Differential Equations or MOL 410 Introduction to Biological Dynamics), OR permission of instructor.

Class schedule: M-W 1:30-2:50pm; Location: Fine 214.

Instructor: Michael Schwemmer, Fine 212, 258-6488
mschwemm@princeton.edu Office Hours: TBA.
Assistant Instructor: Einat Fuchs, D202D E Quad, 258-9924
einat@Princeton.EDU Office Hours: TBA.
Assistant Instructor: Philip Holmes, E-quad D202B and
Fine 215, 258-2958, pholmes@math.princeton.edu Office Hours: TBA.
AI: Sam Feng, Fine 202, 258-3682
sffeng@Princeton.EDU Office Hours: TBA.

Course requirements and grading:

- ≈ 6 take-home problem assignments ($\approx 70\%$ of grade).
- Final (substantial) paper on an independent project ($\approx 20\%$ of grade).
- Class participation and oral report on final project ($\approx 10\%$ of grade).

Outline of Syllabus:

- Introduction: Neuronal properties *($\sim$ 1 week)*
- Mathematical Tools: ODEs and Numerical Methods *(1-2 weeks)*
- Models of single cells and small networks *($\approx$ 5 weeks)*
- Information theory and probabilistic approaches *($\approx$ 2.5 weeks)*
- Decisions and learning *($\approx$ 2 weeks)*
- Independent project presentations *(during exam period)*

Course Texts:

Extensive course notes will be distributed, but the following texts provide more detailed coverage, and address additional topics.

H.R. Wilson, Spikes, Decisions and Actions: Dynamical Foundations of Neuroscience, Oxford University Press, 1999. **REQUIRED TEXT**.

M.W. Hirsch, S. Smale and R.L. Devaney, Differential Equations, Dynamical Systems and an Introduction to Chaos, Academic Press/Elsevier, 2004. **OPTIONAL TEXT**. [Good for math background.]

Additional reading and sources:
For those who really want to get into the details!

F. Riecke, D. Warland, R. de Ruyter van Steveninck and W. Bialek Spikes: Exploring the Neural Code, MIT Press, 1997. [More information on probabilistic and information-theoretic methods.]

J. Keener and J. Sneyd, Mathematical Physiology, Springer Verlag, 1998. [Many details of math modeling and some math background.]

P. Dayan and L. F. Abbott, Theoretical Neuroscience; Computational and Mathematical Modeling of Neural Systems, MIT Press, 2001. [Covers similar ground to Wilson and Reicke et al., with lighter treatment of dynamics.]

G.B. Ermentrout and D.H. Terman, Mathematical Foundations of Neuroscience, Springer Verlag, 2010. [Deeper treatment of biophysics of neurons and networks, with relevant mathematical methods including stochastic ODEs.]

D. Johnston and S. Wu, Foundations of Cellular Neurophysiology, MIT Press, 1995. [Useful details on ion channels, stochastic models, etc.]

E.R. Kandel, J.H. Schwartz and T.M. Jessell, Principles of Neural Science, McGraw-Hill, 2000. [The neuroscientist's bible, 1400 pp, no math.]