Simulation of CPG's for Biped Locomotion

MAT351: Final Paper

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1 Introduction

Bipedal locomotion is a significantly complex yet surprisingly efficient means of movement. Long thought to require a high degree of feedback control, modern research has shown that periodic bipedal gaits are generated by central pattern generators (CPG) (Ijspeert, 2008). While feedback can and often does play a role in determining the parameters of the CPG when an animal moves, basic periodic gait generation requires minimal stimulus to occur.[1] This study is especially interested in exploring and validating two detailed CPG models for the purpose of developing a biomimetic electronic neural network for implementation in the bipedal robot, Roo'bot (Razin, 2011). As Roo'bot was primarily developed as a hopping robot, special attention will be payed to the parameters of the hopping gait.

Thus, the goals of the this paper are to:

1) Explore two methods of CPG modeling and determine if either are appropriate for the bipedal system.
2) Determine the full behaviors of the model(s) chosen and find appropriate conditions for gait generation
3) Discover the effects of key parameters for hop generation and determine their effect on the system response

1.1 The CPG

Central pattern generators are a widely occurring mechanism in nature. Much of the *in vitro* work on CPG's has been done on invertebrates (e.g. lampreys (Wilson, 1999; Eckhoff, 2010), cockroaches (Ghigliazza, 2004; Eckhoff, 2010), and lobsters (Ghigliazza, 2004)). These CPG's were chosen due to their ease of removal, large size, simplicity, and long "life"-span after removal from the body. All of them generate locomotor movement and are thus of interest to our study. Vertebrates also possess CPG's for locomotion,

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[1]see Rossignol et al. (2006) for a review of feedback mechanisms for gait control

which are embedded in the spinal cord (Ijspeert, 2008). The investigated of these has only just begun and a full analysis of the cell types and their parameters is currently underway (Kiehn, 2006).

In general, CPG's are built of networks of similar neurons that oscillate such that rhythmic patterns are produced. These rhythms often exhibit behaviors such as phase-locking and phase-specific stability (in- or anti-phase). They can also produce more complex behaviors such as period doubling, harmonic oscillations, and even chaos. One way to typically reduce the complexity of such systems is approaching them on the phenomenological level, such as treating them as simple oscillators which allows for the use of phase reduction techniques (Eckhoff, 2010). This type of analysis can prove extremely useful when dealing with large numbers of neurons, such as in a lamprey (Wilson, 1999; Eckhoff, 2010). The work of Golubitsky (1998) proved using number theory that for an animal with n-pairs of legs, a CPG made of $4n$ neurons is required to produce all possible gaits (Figure 1). Despite this, bipedal and quadrupedal CPG models are small enough for detailed simulations and analysis at the cellular level.
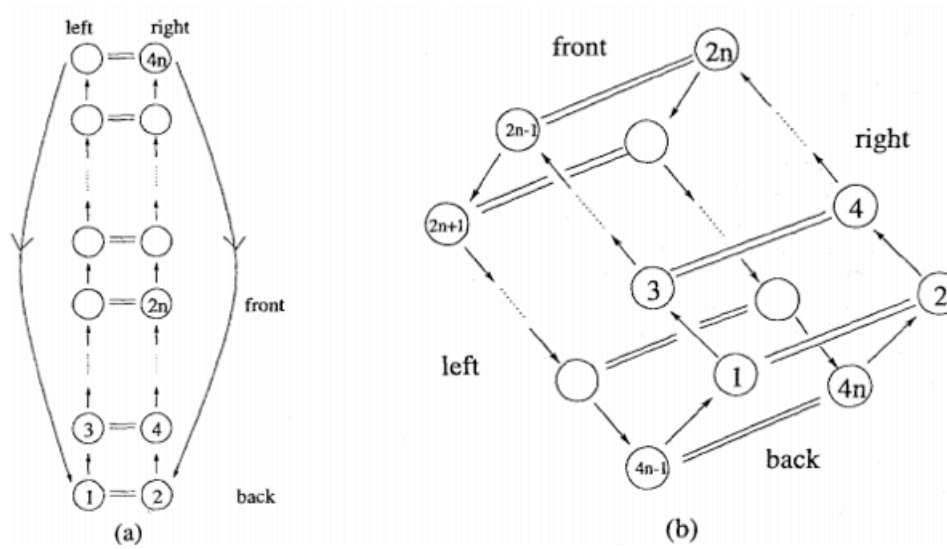

Figure 1: **The 2n-legged CPG** Neurons 1,..,2n are directly attached to the legs. (a) Depicts a 2D schematic of the 4n cell network. (b) The folded network. The upper surface is attached to the legs while the far side provides additional intraneural connections (Golubitsky, 1998).

1.2 Neuronal Model

As we are interested in a neurologically valid model for bipedal locomotion, we will focus on two of the more detailed dynamical models for the neuron. The first is governed by the 4-dimensional Hodgkin-Huxley (HH) equations (as given in Eckhoff (2010)) and

the second is a 2D simplification, called the FitzHugh-Nagumo model (Eckhoff, 2010; Golubitsky, 1998).

1.2.1 Hodgkin-Huxley Model

Originally, the biped CPG modeling put forth by Pinto (2007) looked like a very attractive option for our study. However, it was brought to our attention that the model presented there for neuronal coupling in the network was not biologically realistic.[2] Therefore, we based the structure of our modeling equations on Eckhoff (2010) but retained the defined coupling parameters from Pinto (2007) to check consistency with her results as well as others (see Golubitsky (1998)).

We choose to model the internal dynamics of the HH neuron as

$$f(v_i, \bar{g}_{Na}, \bar{g}_K) = -1/C(\bar{g}_{Na}(v_i - E_{Na}) + \bar{g}_K(v_i - E_K) + g_L(v_i - E_L)), \quad \text{(1a)}$$

$$\dot{m}_i = \Phi(A_m(v_i)(1 - m_i) - B_m(v_i)), \quad \text{(1b)}$$

$$\dot{n}_i = \Phi(A_n(v_i)(1 - n_i) - B_n(v_i)), \quad \text{(1c)}$$

$$\dot{h}_i = \Phi(A_h(v_i)(1 - h_i) - B_h(v_i)), \quad \text{(1d)}$$

where the temperature correction factor $\Phi = 3^{\frac{T-6.3}{10}}$ and T is the temperature in Kelvin. The notation of Eckhoff (2010) is adopted such that $\bar{g}$ denotes a function of the gating variables $m(t), n(t)$, and $h(t)$, such that $\bar{g}_{Na} = g_{Na}m^3h$ and $\bar{g}_K = g_K n^4$. Furthermore,

$$A_m(v_i) = 0.1\frac{(25 - v_i)}{e^{\frac{25-v}{10}} - 1}, \qquad\qquad B_m(v_i) = 4e^{\frac{-v_i}{18}},$$

$$A_n(v_i) = 0.01\frac{(10 - v_i)}{e^{\frac{10-v}{10}} - 1}, \qquad\qquad B_n(v_i) = 0.125e^{\frac{-v_i}{80}},$$

$$A_h(v_i) = 0.07e^{\frac{-v_i}{20}}, \qquad\qquad B_h(v_i) = \frac{1}{1 + e^{\frac{30-v_i}{10}}}.$$

In the equations given in (1), v is the membrane voltage and m, n and h repesent gating variables for the K^+ and Na^+ ion channels in the cell membrance. Together (1) represent the internal dynamics of the neuron, and can be combined into a CPG neural network such that, for a biped (n=1)

$$\dot{v}_i = f(v_i, \bar{g}_{Na}, \bar{g}_K) + I/C + k_1(v_{i+2} - v_i) + k_2(v_m - v_i) + k_3(v_{i+\epsilon_i} - v_i) \quad \text{(2a)}$$

$$\epsilon_i = (-1)^{i+1} \qquad\qquad m = \begin{cases} i + 3 & \text{for } i = 1, 3 \\ i + \epsilon_{i+1} & \text{for } i = 2, 4 \end{cases} \quad \text{(2b)}$$

for $i = 1, ..., 4$ and all indices are taken modulo 4. Unlike in Pinto (2007), the neurons here are coupled together by gap junctions, with coupling strengths k_j. This model for neuron coupling is accurate for fast reflexive movements and is used below by Golubitsky (1998) in the FitzHigh-Nagumo models. It is also interesting to note that even though
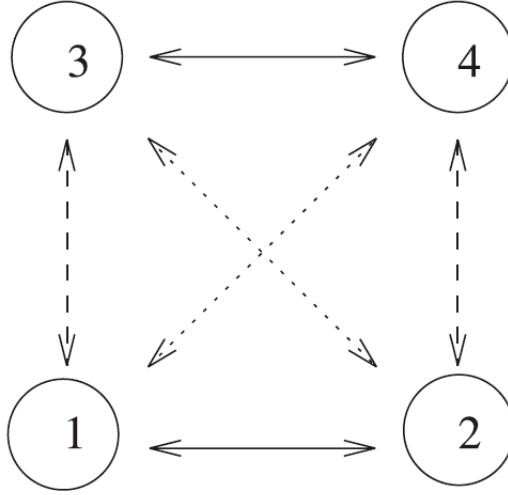
Figure 2: **The Bipedal CPG** Neurons 1 and 2 are directly attached to the legs (Pinto, 2007)

Pinto (2007) does not use gap junctions in her description of the bipedal leg motion CPG, she does include it in the biped with associated arm movement.

Examining this model a little more closely, we see that the first coupling term k_1 relates the diagonal neurons, and k_2 and k_3 relate the ipsilateral and contralateral components. As mentioned above, we understand the first $2n$ neurons as controlling the limbs.

1.2.2 The FitzHugh-Nagumo Model

As the HH equations require a large amount of computational power (the biped alone is a 16-dimensional dynamical system), a number of simplified models of the HH equations have been proposed (see (Eckhoff, 2010) for an overview). One of the systems that captures the most important aspects of the dynamics but only uses half the equations is the FitzHugh-Nagumo model. This model was originally used by Golubitsky (1998) to simulate gaits for animals with n-pairs of legs (though he validated his theory mainly with quadrapedal models).

Each neuron is modeled internally by

$$f_1(v, r) = c(v + r - v^3/3) \tag{3a}$$
$$f_2(v, r) = 1/c(v + 0.2r - 0.02) \tag{3b}$$

where v is the membrane voltage and r combines the dynamics of the gating variables m, n, and h from the HH equations. Using dynamical systems analysis, this system has been shown to behave like a neuron, as far as the basic action potential dynamics, as well

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[2]We would like to acknowledge Professor Philip Holmes for bringing this to our attention.

as exhibiting the Hopf bifurcation that gives stable limit cycles for continuous spiking activity (Eckhoff, 2010).

The CPG model for the FitzHugh-Nagumo model (Golubitsky, 1998) is

$$\dot{v}_i = f_1(v_i, r_i, c) + \alpha(v_{i-2} - v_i) + \gamma(v_{i+\epsilon} - v_i) \tag{4a}$$

$$\dot{r}_i = f_2(v_i, r_i, c) + \beta(r_{i-2} - r_i) + \delta(r_{i+\epsilon} - r_i) \tag{4b}$$

$$\epsilon_i = (-1)^{i+1} \tag{4c}$$

and once again $i = 1, ..., 4n$, with indices taken modolu $4n$.

Note that the coupling terms are defined rather differently in the two models. In HH, k_1 acts as a diagonal coupling term between neuron voltages. Here, coupling is seen between ipsilateral and contralateral neurons and their gating variables but no on the diagonals. In the general case of $2n$-legged animals, Golubitsky (1998) did not seem take into account a diagonal coupling between neurons. This was an addition by Pinto (2007) and seems to reflect reflexive interneuron behavior (Figure 3). While this behavior has not been shown for the human locomotive CPG, it is characteristic of spinal cord interneurons which connect to motoneurons and therefore are a potential source of CPG behavior.
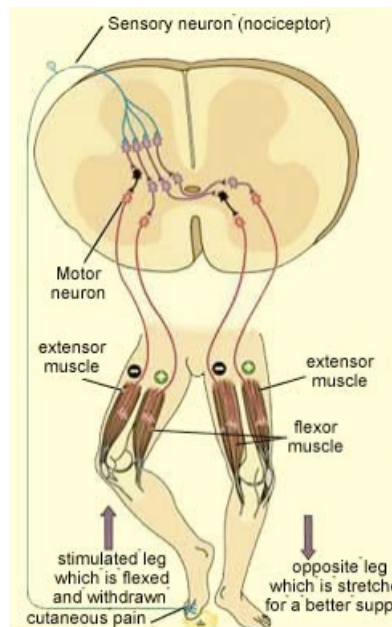

Figure 3: **The Autonomous Reflex Arc** Interneurons in the caudal spinal cord display "diagonal" coupling behavior (Dubuc, 2002).

1.3 Gaits

Ultimately, CPG's are important in so far as they are the source of gait generation, defined by a stable periodic orbit or limit cycle that occurs in locomotion. Within a gait the legs may behave in a synchronous or asynchronous fashion, exhibiting tendencies toward certain phase-locked states (e.g. anti-phase, in-phase). The larger the number of pairs of legs n, of course, the more gaits available to a given animal.

The main concern of Golubitsky (1998) was the number of symmetric gaits that an animal can produce. He showed that six period-1 or primary patterns exist for quadrupeds, all of which could be obtained from the $4n$ network.

Pinto (2007) also had the goal of characterizing all bipedal gaits from her HH-based CPG models. If we accept her derivation of the HH network, she was able to successfully demonstrate all four primary gaits (two-legged hop, walk, run, two-legged jump) as well as 7 other secondary gaits.
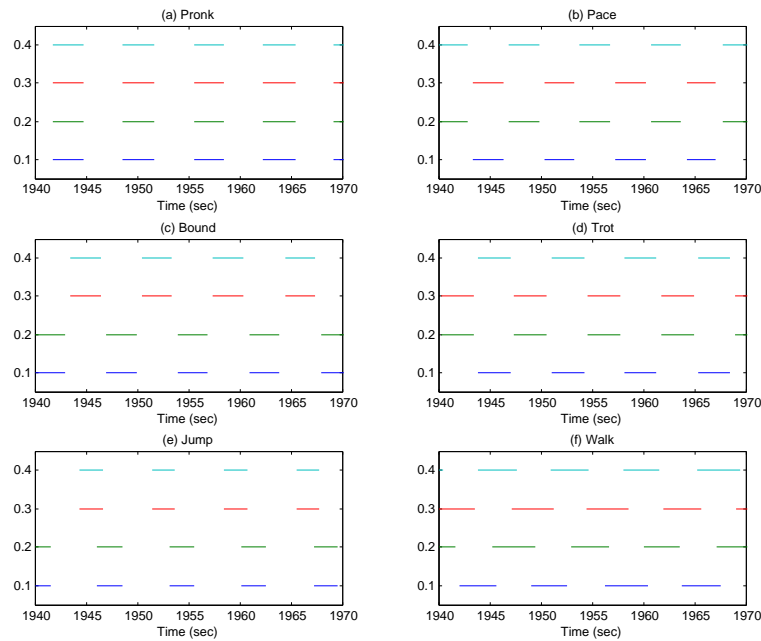

Figure 4: **Quadrupedal Stride Maps** Based on the FitzHugh-Nagumo model(Golubitsky, 1998), all six primary gaits are shown.

2 Research Methods

Simulations of both the HH and FitzHugh-Nagumo CPG's were performed with Runga-Kutta (4,5) using MATLAB's **ode45** solver. Simulations had variable time step, limited by a maximum step of 0.01 and were run for long periods of time to ensure convergence.
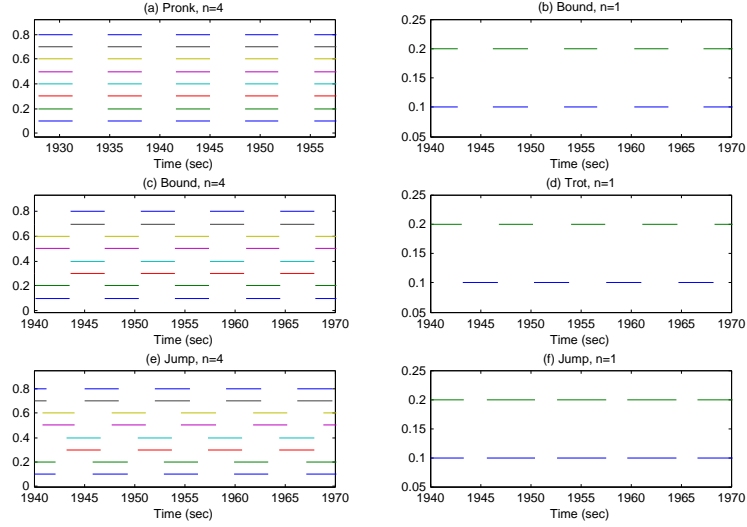
Figure 5: **Octopedal and Bipedal Stride Maps** Based on the FitzHugh-Nagumo model. Note that for the same initial conditions as quadrupeds, $2n$-legged animals tend to experience similar gaits. For bipeds (right), though, these tend to reduce to simply in-phase and anti-phase in FitzHgh-Nagumo (Golubitsky, 1998).

For validation of results in Golubitsky (1998) the parameter values and initial conditions given in Table 5 (see Golubitsky (1998)) were used. We expected to be able to replicate Golubitsky (1998) findings, as we used the models directly from his paper. As far as Pinto (2007) was concerned, as we had modified her models for more accurate biological systems, we hoped to reproduce qualitatively similar findings, but understandably with different initial conditions and coupling constants.

Upon running the FitzHugh-Nagumo models we found the results difficult to interpret due to high levels of rhythmic interference and decided to apply a 10th-order Butterworth lowpass filter on the CPG output (**butter** in MATLAB), to create a threshold for establishing when ground contact was established (AquaPhoenix, TM). This gait data can be found in Figures 4 and 5.

Runs of the HH-based CPG, with the modified initial conditions (see Table 2) are shown in Figures 6 and 7. In all of these simulations, $m_0 = 0.0529, n_0 = 0.3177$, and $h_0 = 0.5961$. Also, the applied current I_{appl} was fixed at $I = 100A$, and the membrane conductivity was fixed at $C = 1$. For Figure 8, the system was set on the two-legged hopping gait (see Table 2 for parameters and initial conditions) and I_{appl} was varied.
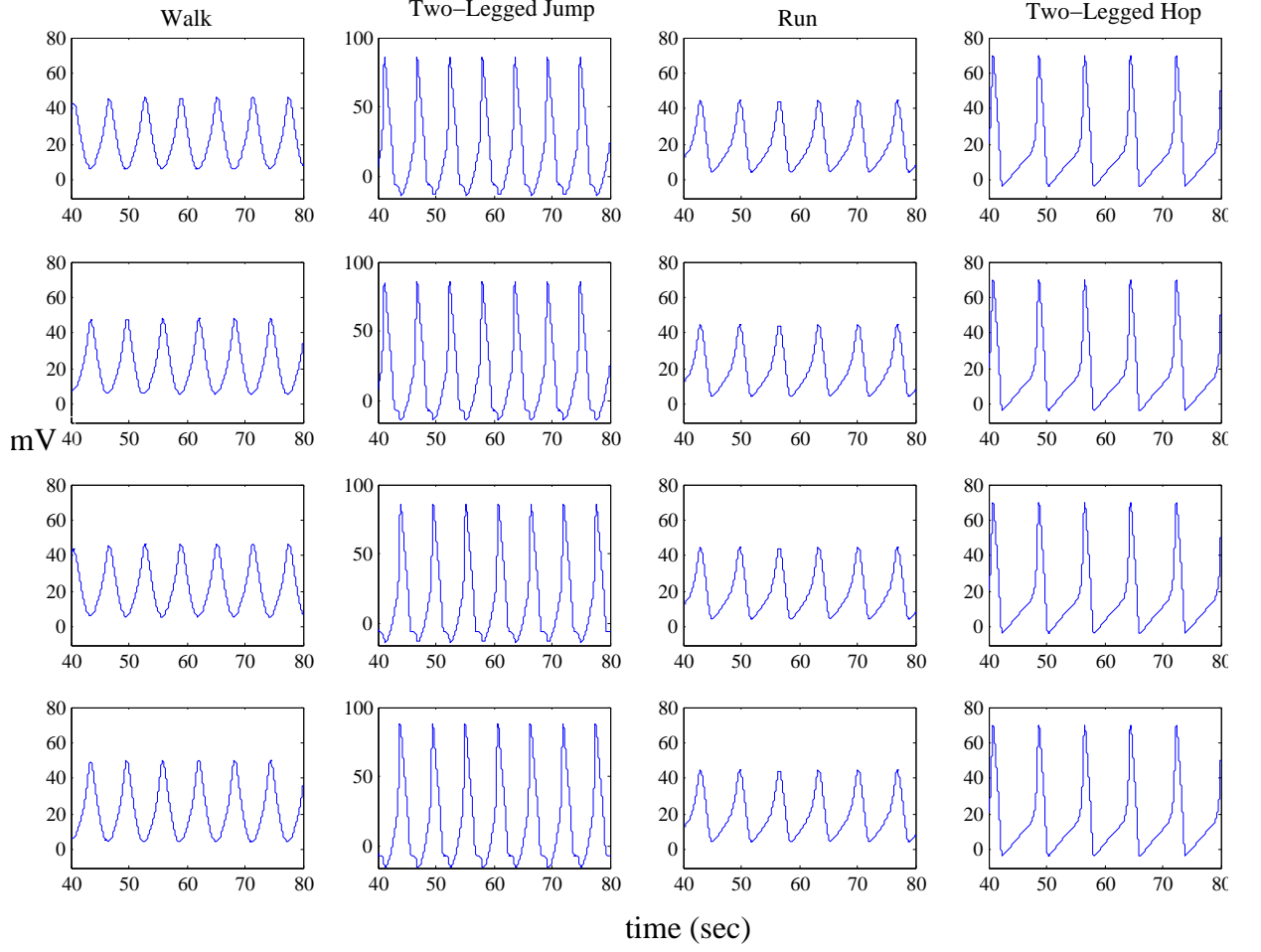
Figure 6: **Period-1 Gaits for a Biped.** Based on a $4n$ Hodgkin-Huxley CPG network. See Table 2 for parameters and initial conditions.The top two rows are the neurons that control the legs.

Gait	v_0	$k = [k_1 k_2 k_3]$	T
Walk	[-1 -3 -1 -3]	[-1.4 -0.2 0.8]	3
Two-Legged Jump	[-1 -3 -1 -3]	[1.4 -0.2 -0.8]	10
Run	[-.1 -1 -2 -.1]	[-.5 .5 -0.8]	6.3
Two-Legged Hop	[-1 -3 -1 -3]	[-1.4 -0.2 -0.8]	10
Broken Walk	[-1 -3 -1 -3]	[1.4 -0.2 0.8]	10
One-Legged Hop	[-1 -3 -1 -3]	[-1.4 -0.2 0.8]	10
Running Skip	[-.1 -1 -.1 -.1]	[-.5 2.5 -0.8]	7
Assymetric Hop	[-.1 -1 -2 -.1]	[-.5 1.5 -0.8]	4

3 Discussion

3.1 Validation of the FitzHugh-Nagumo Model

We were able to successfully reproduce the results of Golubitsky (1998) in regards to the quadrupedal primary gaits (Figure 4). Each gait was distinctly recognizable; the only difficulty lay in attempting to define when the CPG "landed" or "took-off." The issue here was mostly higher frequency oscillations in the CPG neurons that made the signal seem "spotty". As in every case a fundamental low frequency wave was discernible as the primary feature, we chose to use a 10th- order Butterworth lowpass filter. This worked remarkably well and the signals were easily parsed after that. An arbitrary threshold for landing was chosen 30% above the signal mean, which clearly indicated the gait types.

We also found the Fitzugh-Nagumo CPG extremely easy to extend to any number of legs we wished (Figure 5). Running many legs, revealed that while there is the possibility for more gaits to exist, given the same initial conditions and parameters we get similar gaits to the quadruped.

While that CPG was remarkable at many-legged CPG modeling, it lost much of the detail when it came to the biped. It was initially hoped that since the FitzHugh-Nagumo model was so much less computationally expensive it would prove useful in modeling our CPG of interest. Instead, it was only capable of producing phase-locked gaits, either in anti-phase (walk/run), or in-phase (two-legged hop). This motivated us to pursue the more complex Hodgkin-Huxley CPG for the biped.

3.2 Exploring Bipedal Gaits with the Hodgkin-Huxley CPG

Once we had decided on using the HH-based CPG for bipedal simulations, our goal became to qualitatively reproduce the results in Pinto (2007). We knew that we were using a different model, due to our use of the gap junction coupling in our CPG. There were also a very large number of variable parameters and initial conditions, so we decided to limit our exploration by fixing the applied current, to $I = 100A$, and the initial conditions of the gating variables $m, n,$ and h. It was also noted in early research with the model that the system was unstable for large positive values of k, T, or v_0. For very low values of T, the system also tended to slow down and eventually just stop oscillating. Thus, values of T were kept $3 < T < 10$, values of $k < 3$, and $v_0 < 0$ (see Table 2).
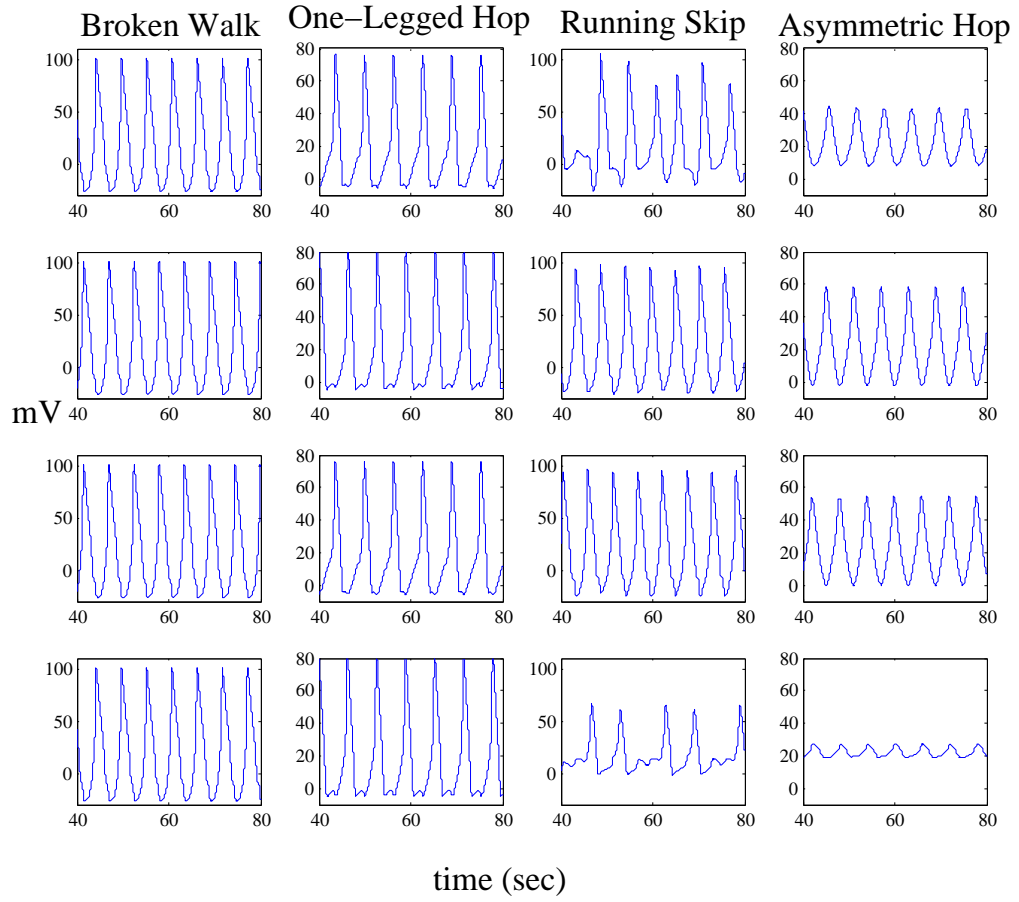
Figure 7: **Period-2 Gaits for a Biped.** Based on a $4n$ Hodgkin-Huxley CPG network. See Table 2 for parameters and initial conditions. The top two rows are the neurons that control the legs.

Even with those limitations, we were easily able to find all the primary gaits (walk,two-legged jump, run, two-legged hop) shown in Figure 6. We found that frequency over all for the primary gaits was fairly consistent, $.11 < f < .18$, but there was a significant variation in the signal amplitude. These variations became even more pronounced in the period-2 gaits (Figure 7). One of the greatest challenges for us here was on the terminology. That is because about half of the period-2 gaits are unnamed and it was hard to tell which of our results were qualitatively related to those of Pinto (2007).

Overall, it appears that despite her flawed model, Pinto (2007) did in fact find results that both matched her mathematical analysis of bipedal symmetric gaits and were consistent with more realistic models. Note that Golubitsky's proof that complete gait generation ss guaranteed by a $4n$-network was in fact shown for any type of coupling, synaptic or gap junction. We would also like to mention that non-convergent and chaotic gaits were also found to exist within very specific parameter setting, which are generally ignored in the literature which focuses on stable gait analysis.
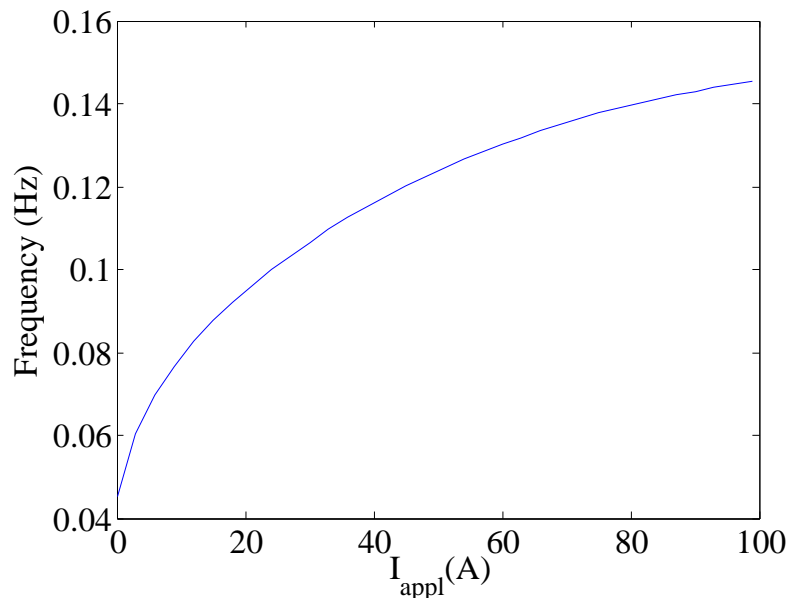

Figure 8: **Frequency vs applied current** in the HH-based CPG, for two-legged hopping.

3.3 Implications for Realization in a Hopping Robot

In addition to the findings above, we also explored the variation in frequency of the gait with changes to different parameters. It was consistently found, that while a slow and fast frequency region tended to exist (e.g. for $I_{appl} = 100$, $f_{slow} \approx 0.11$ and $f_{fast} \approx$

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0.45), the applied current controlled the fundamental frequency of the gaits. This is demonstrated in Figure 8, where the system was set in a two-legged hopping gait and I_{appl} was varied. For $I_{appl} > 0$ the system retained its gait and the only aspect that varied was its frequency. This implies that for the HH-based CPG, the applied current and frequency are (semi-)independent of the system's other parameters and dynamics.

This independence is not surprising as the frequency that an animal can hop will greatly vary on its physical structure, mass, and muscle strength (Gabriel, 1984) and having control over this independent parameter would greatly increase animal's ability to adapt. This is also good news for Roo'bot, who is optimized for a very specific frequency range as far as its physical hopping dynamics are concerned. It would appear that the Hodgkin-Huxley CPG is a strong candidate for control in biped robots but first much more will need to be understood about its various parameters.

4 Conclusion

From our work on the two CPG networks, we conclude that the FitzHugh-Nagumo is a powerful tool to simulate general results for $n \geq 2$. We found that the HH-based CPG was quiet fast, took a reasonably short time to converge, and yielded multiple interesting gaits. In general, though, the bipedal CPG favors the walk and 2-legged hop. It is especially convenient that the latter is so easily obtainable through the CPG model, as we hope to implement it in the future with Roo'bot. We also found the somewhat surprising results that the gait frequency was mostly dependent on the applied current. We anticipate future work on the system exploring the extent of the other parameters of the system, such as discovering where and when bifurcations and the onset of chaos occur. We also hope to continue extending the 2n-legged models to discover other interesting gait dynamics that occur in high-dimensions.

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