Collaborative Learning
at the Interface of
Mathematics and Biology

The MMR vaccine scare and human behavior: Why does measles persist in the United Kingdom?

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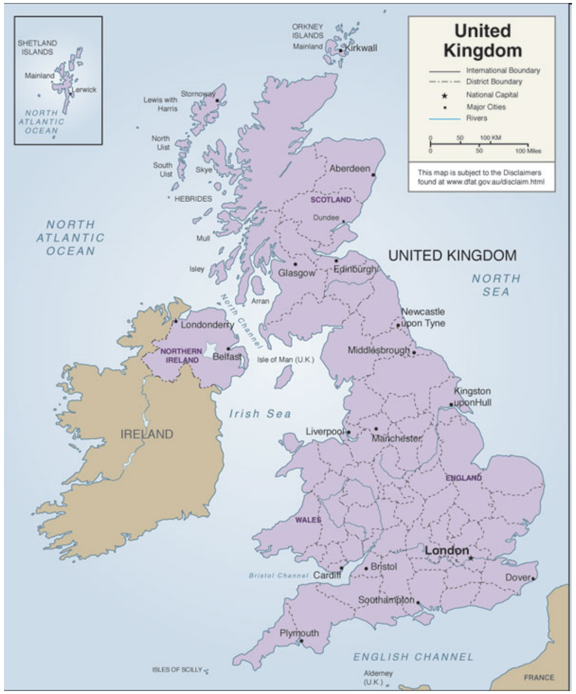
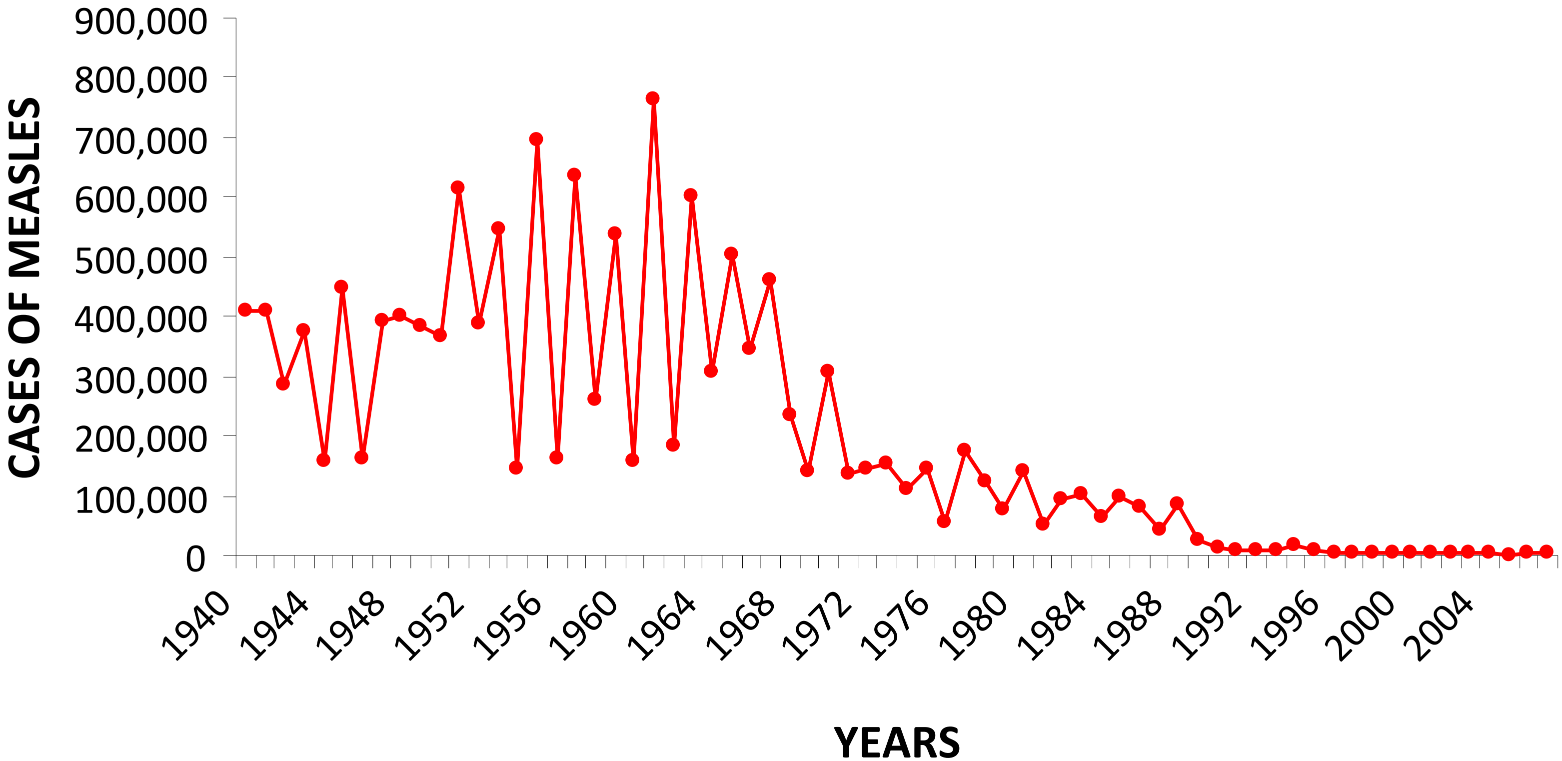
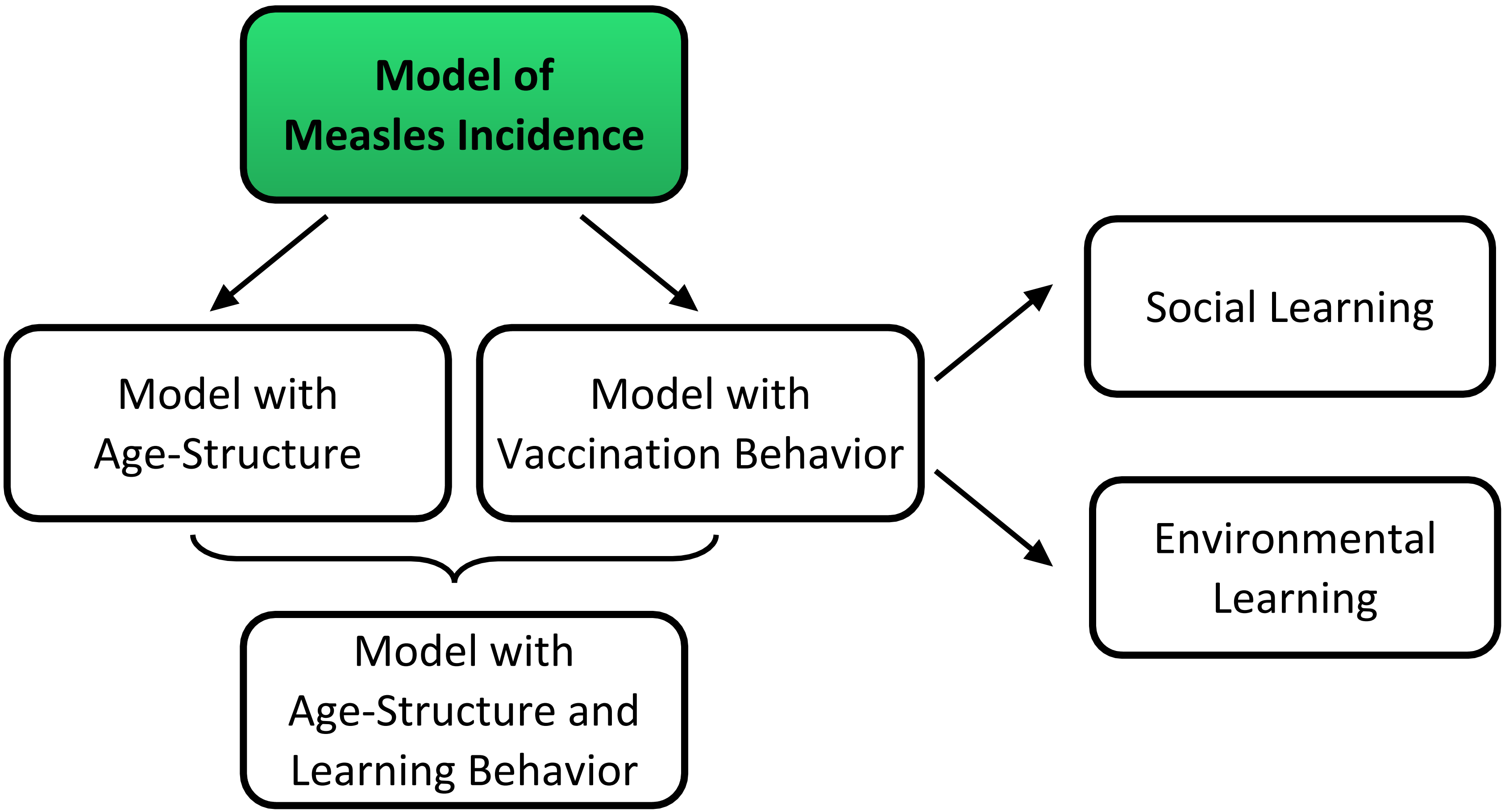


What are the consequences of voluntary vaccination on the spread of measles in the UK?

Reported Measles Cases in England & Wales 1940-2007

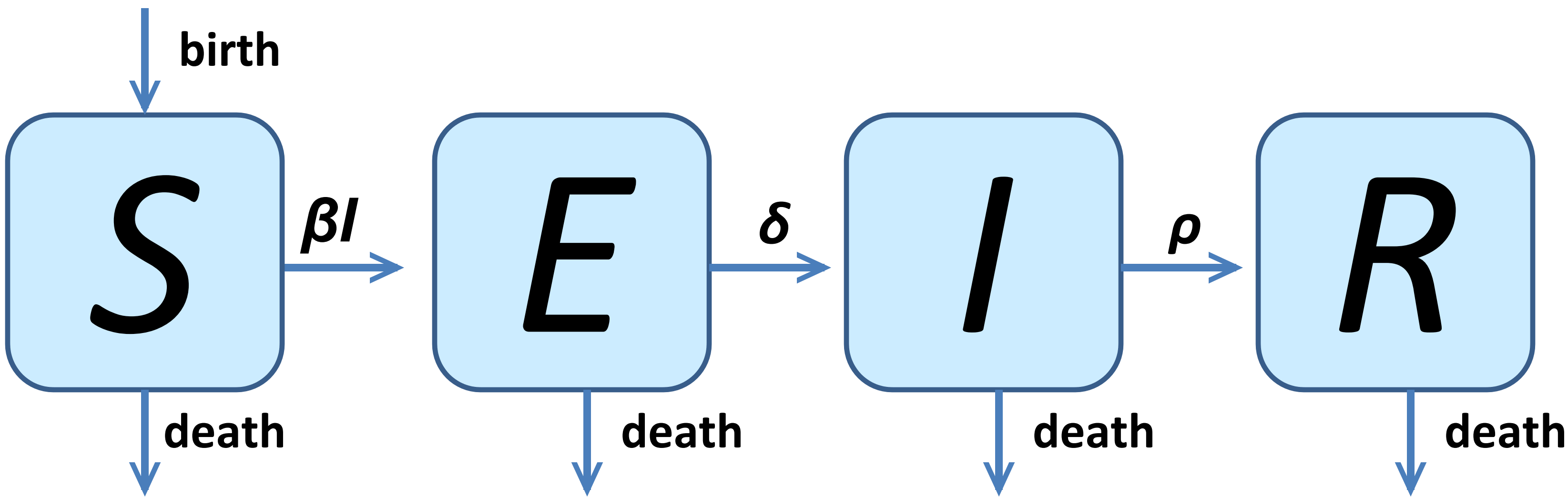


Building a model



An SEIR model for disease dynamics

Susceptible, Exposed, Infected, Recovered

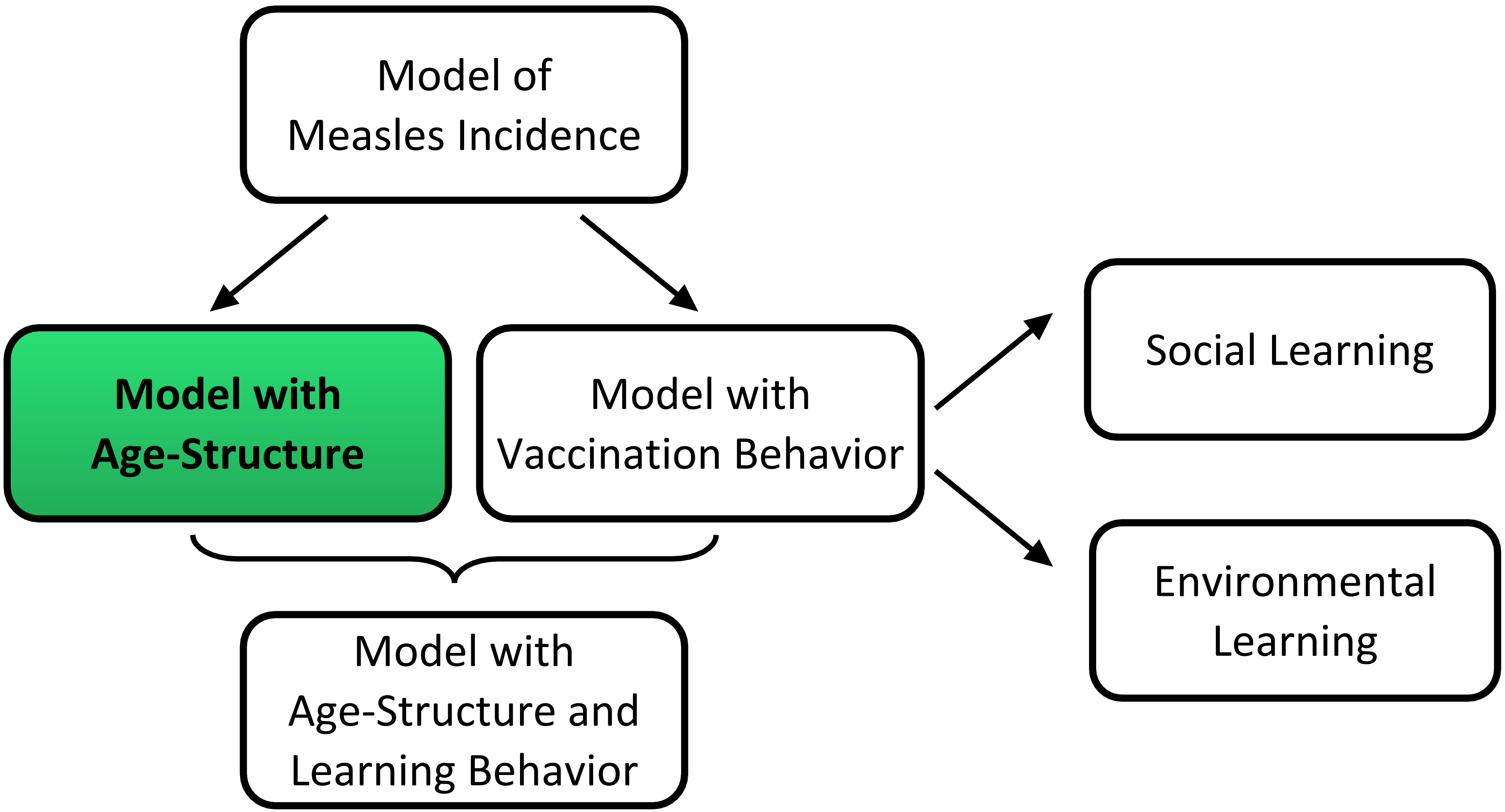


βI = contact rate x level of infection

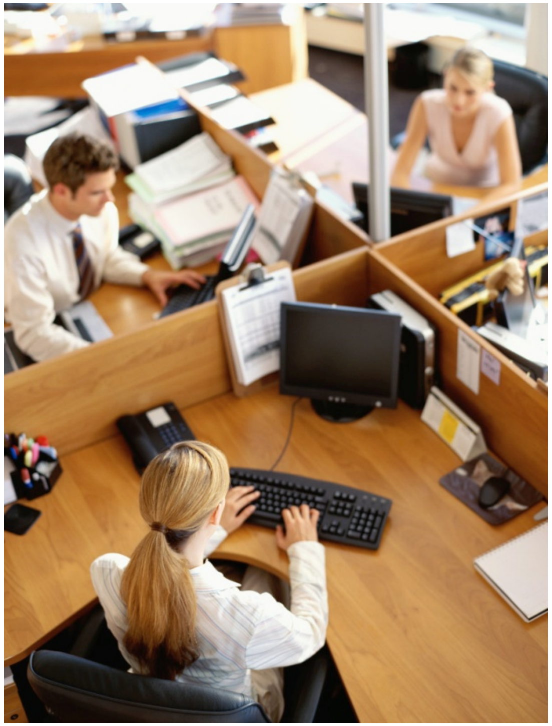
δ = latent transition rate

ρ = recovery rate

The transmission of measles is highly age-dependent

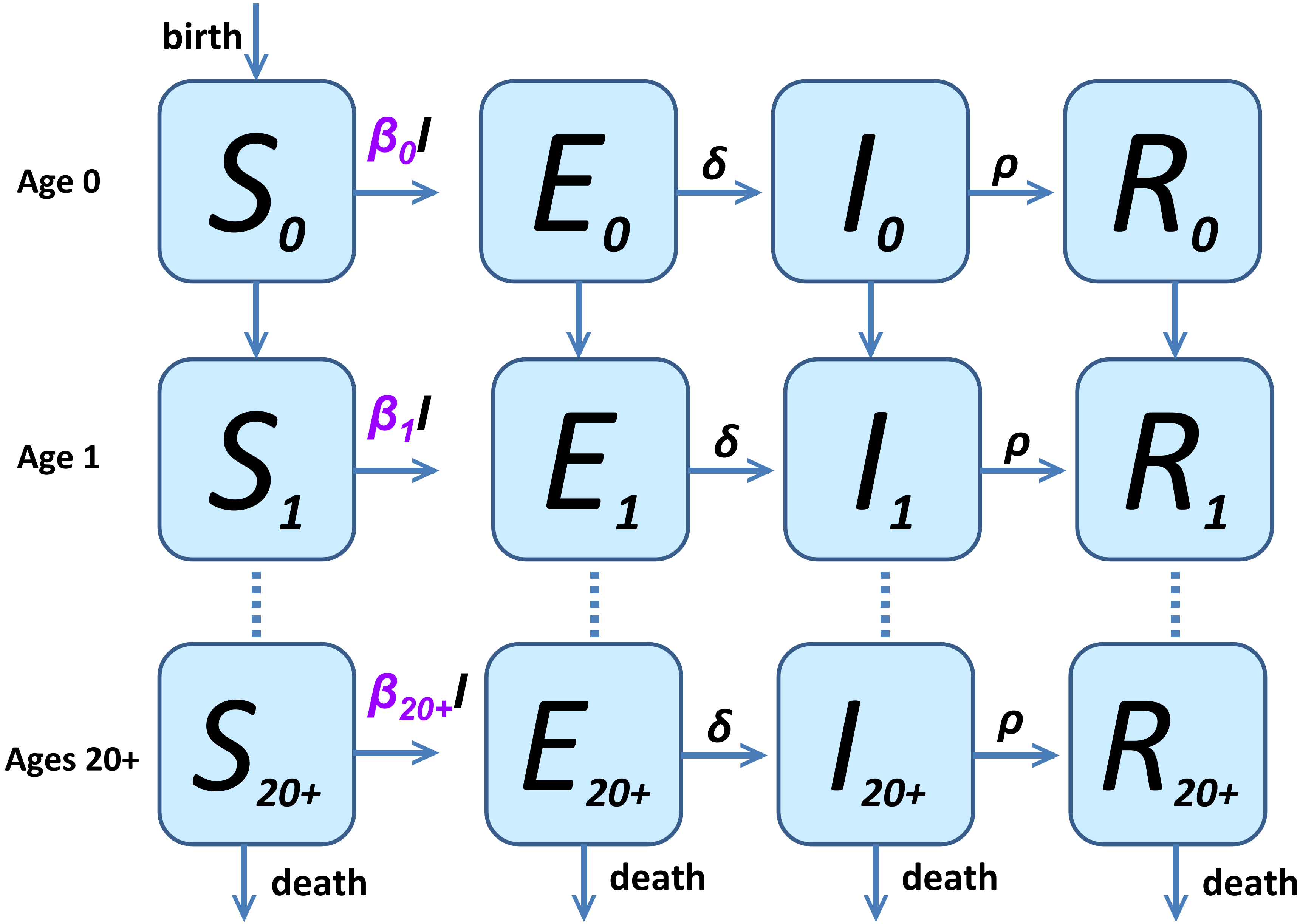


Schenzle (1984): Age-Specific Contact Rates

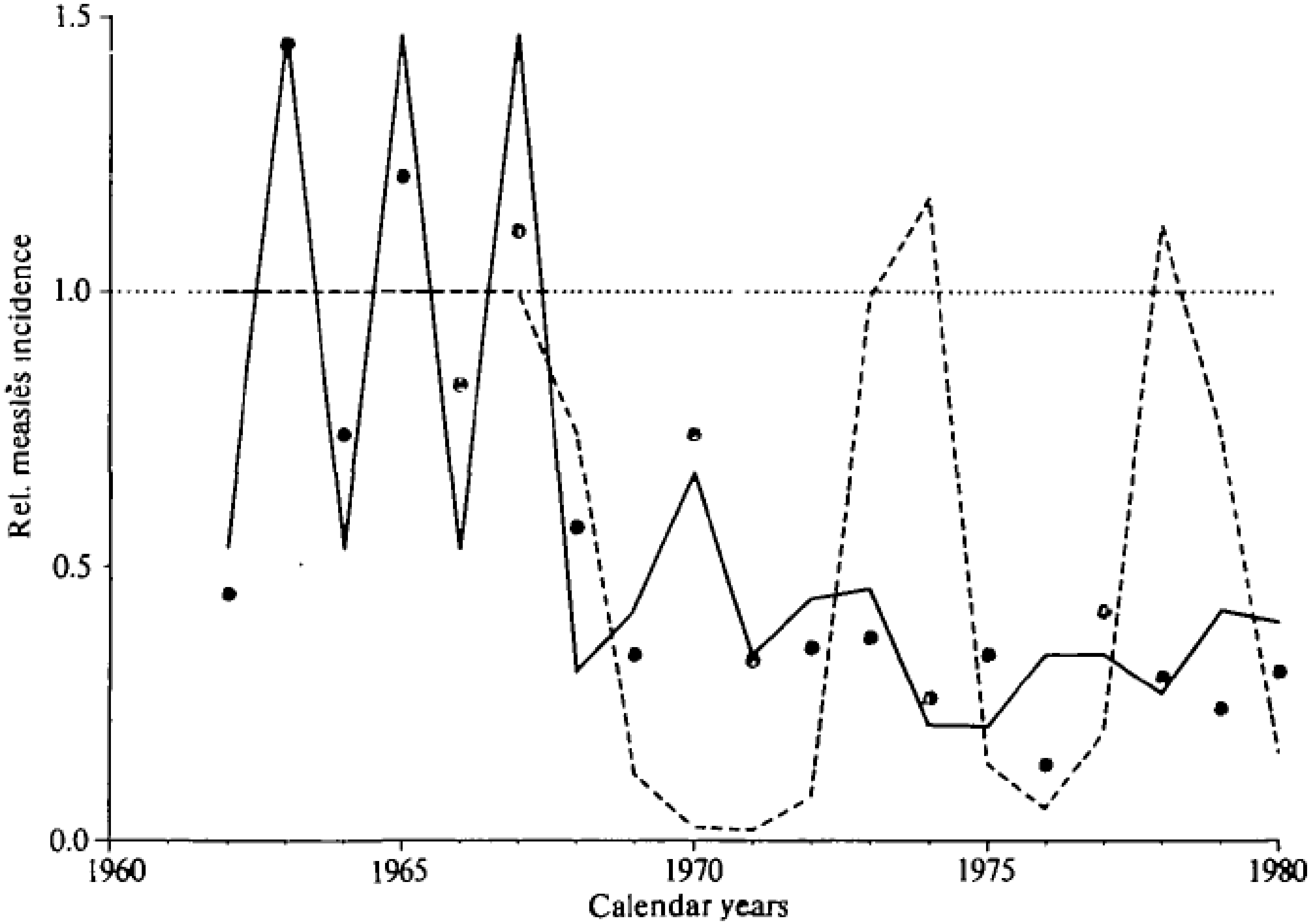


Contact rates for adults < Contact rates for children

Schenzle (1984): Age-Specific Contact Rates

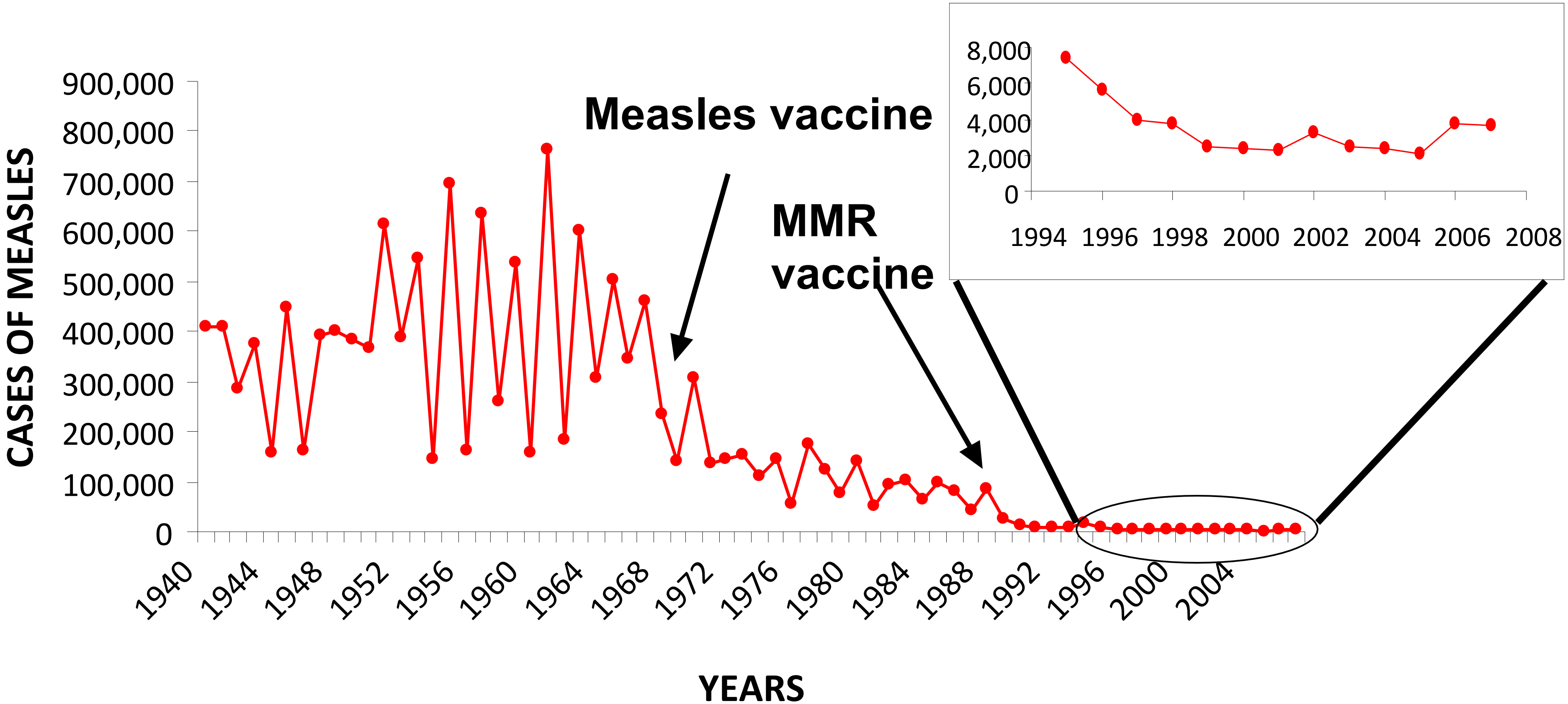


Results of Schenzle (1984)

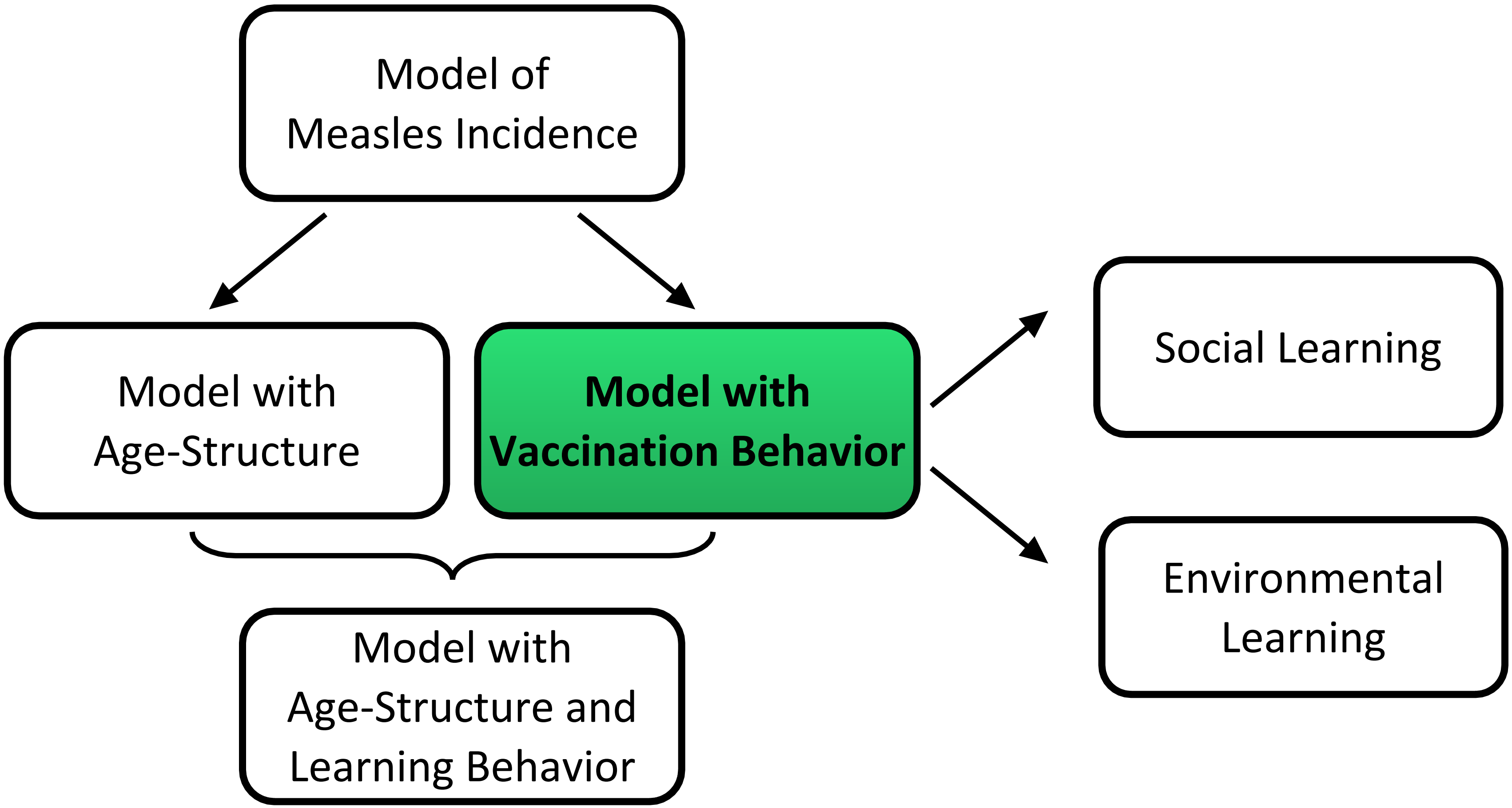


model yields a biennial measles cycle

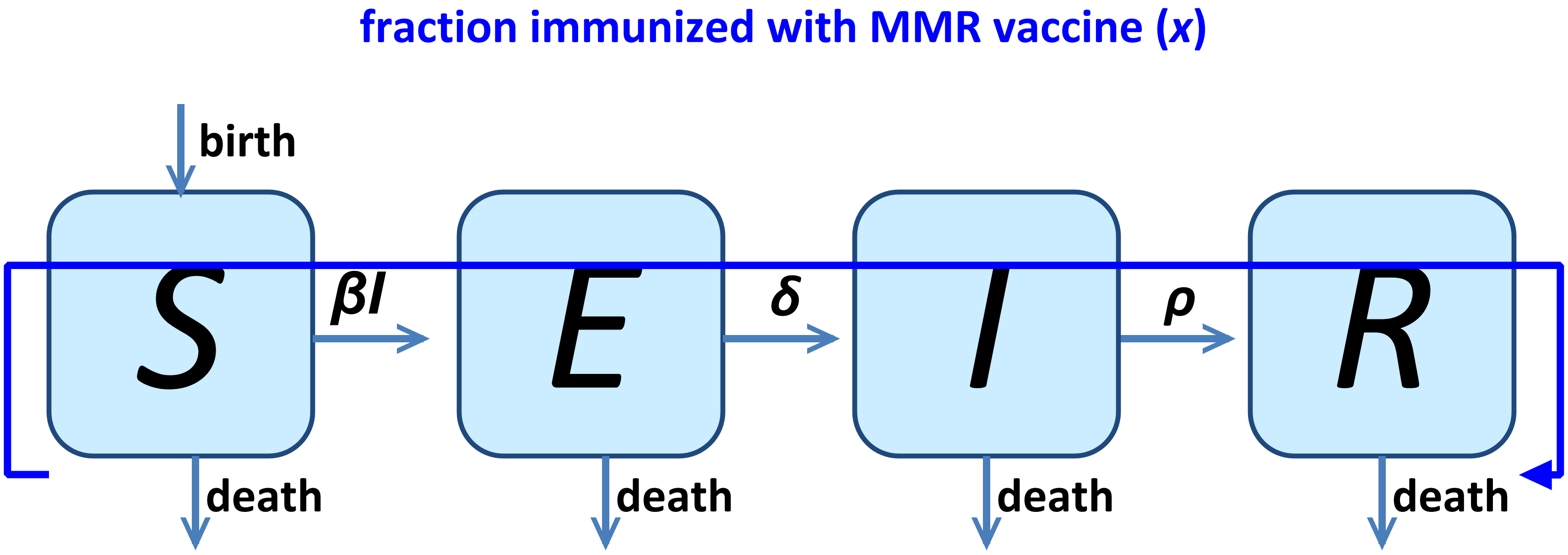
Reported Measles Cases in England & Wales 1940-2007



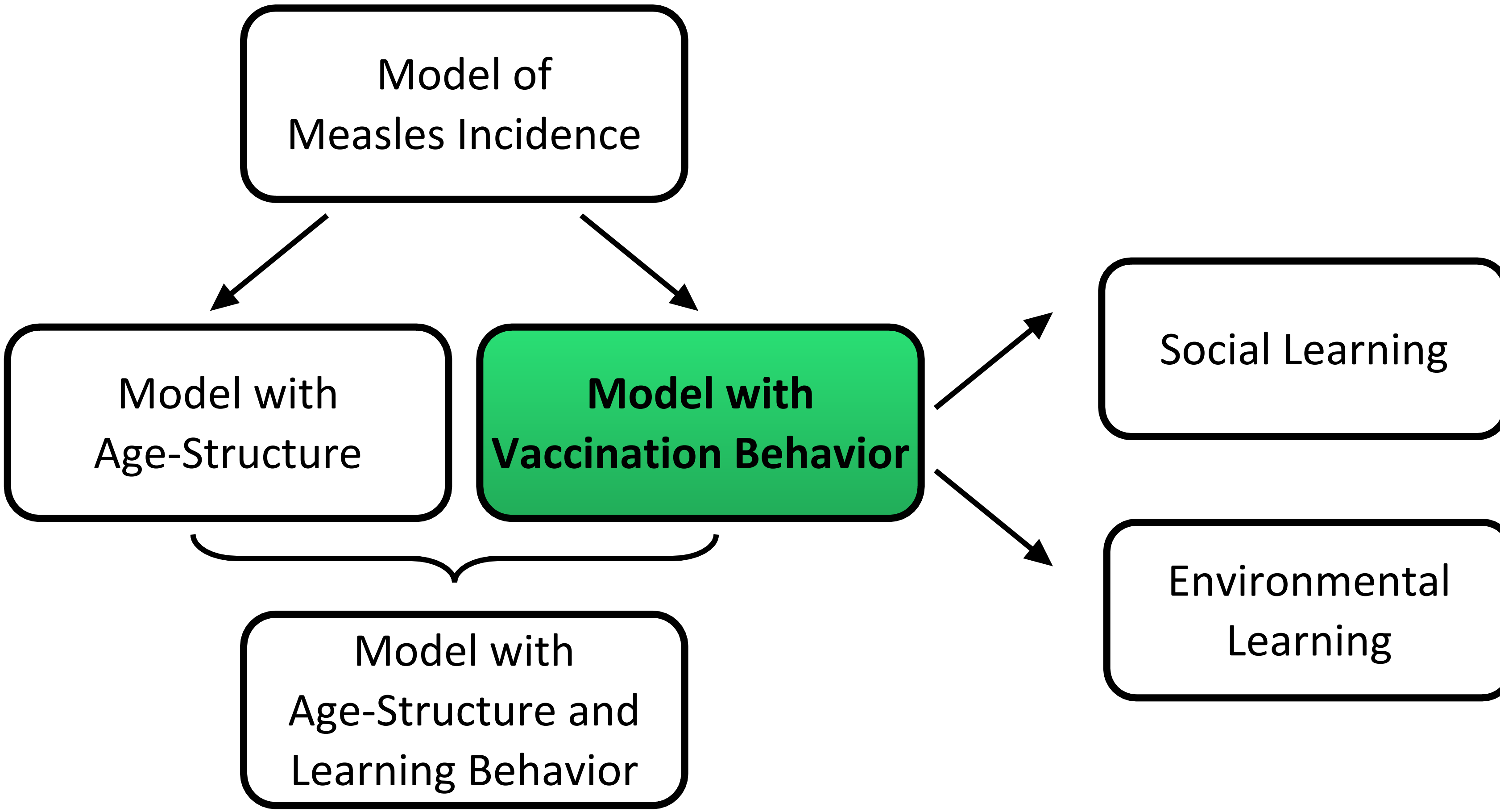
Compulsory Vaccination



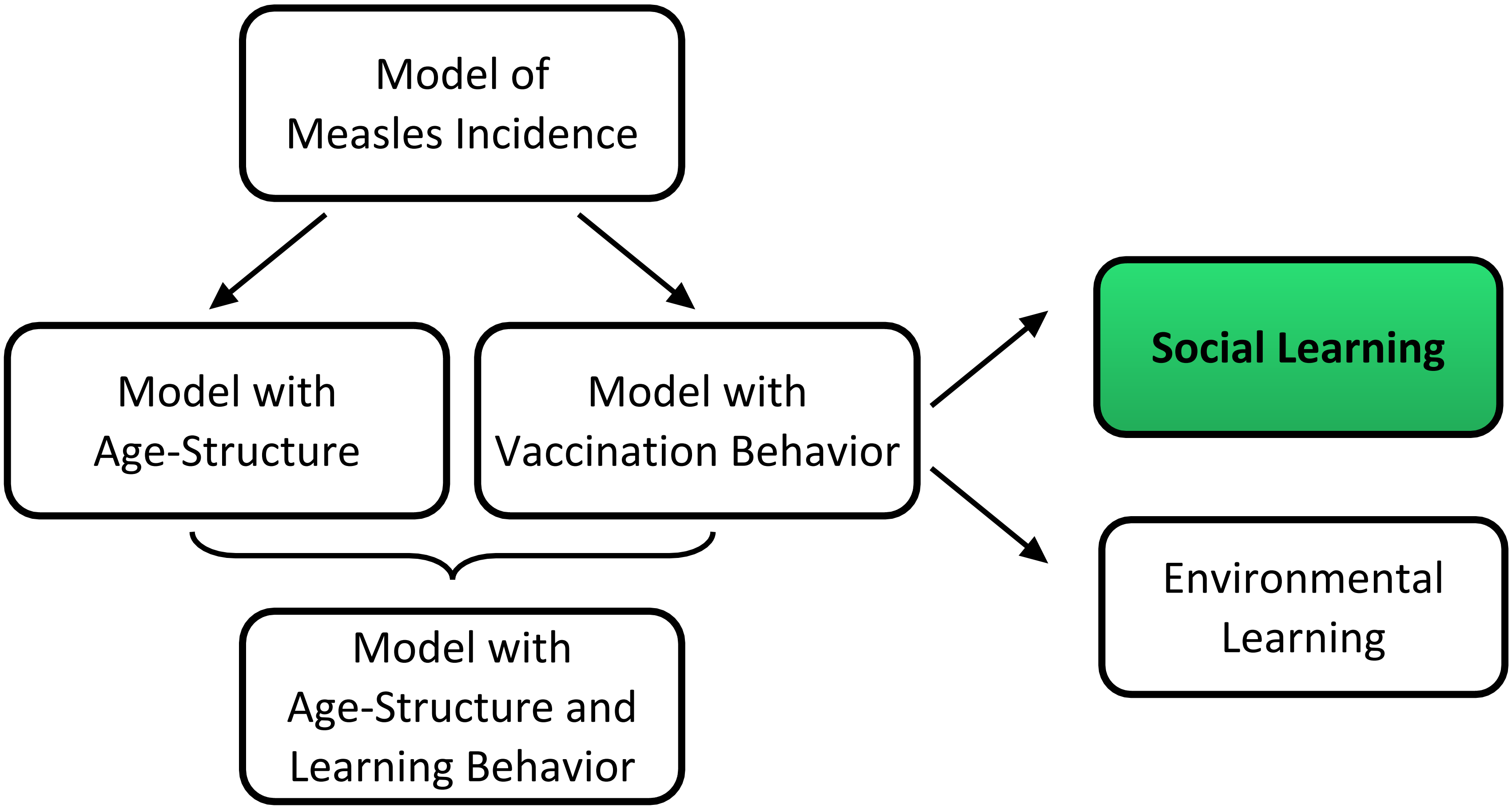
Incorporating Vaccination



Vaccination Programs



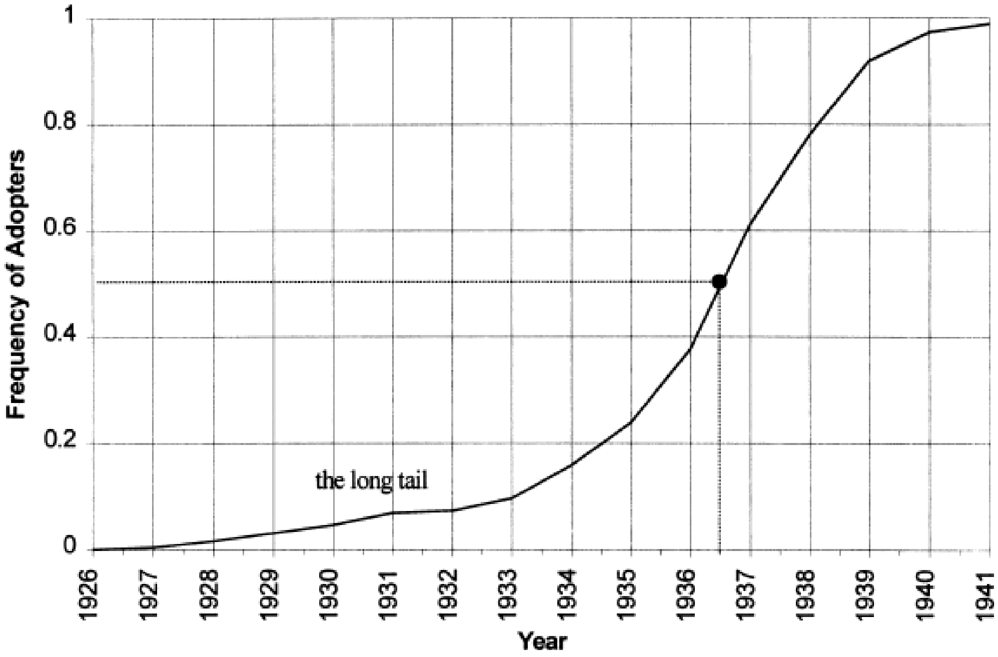
Vaccination is Voluntary in the UK



Social Learning

Henrich (2001): “Cultural Transmission and the Diffusion of Innovations”

- Innovation spread by contact between individuals is characterized by an s-shaped curve
- Potential adaptors of an idea initially resist change
- There is an exponential growth phase and a leveling off phase



Diffusion of hybrid corn seed in two Iowa farming communities

Social Learning

Bauch (2005): "Imitation Dynamics Predict Vaccination Behavior"

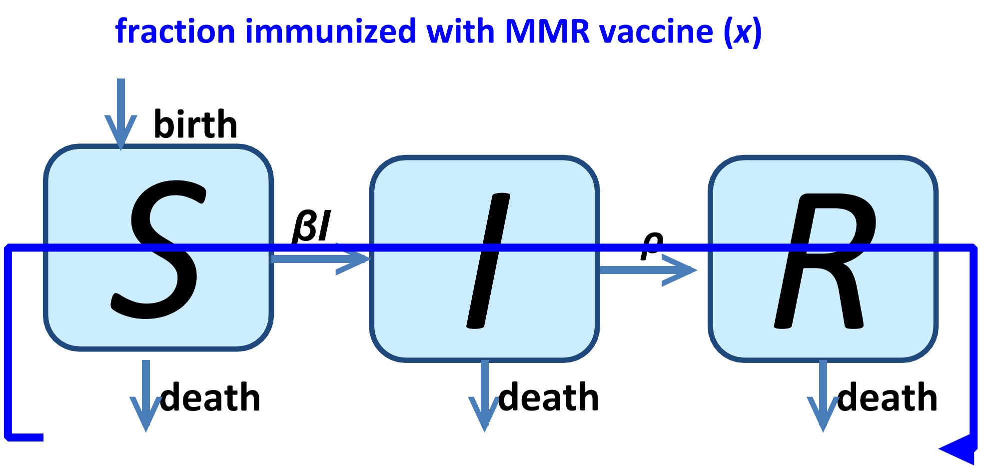
- Individuals adopt vaccination strategies according to an imitation dynamic
- Vaccination decision based on disease prevalence and perceived risk of vaccine and disease

Bauch's Model

$$dx/dt = \kappa\, x(1-x)\, [\omega\, I - 1]$$

dx /dt = change in proportion of vaccinators over time

x(1-x) = frequency at which vaccinators contact non-vaccinators and vice versa



Model parameters	
κ	Rate at which individuals sample others and switch vaccination strategies
$\frac{1}{\omega}$	Level of infection necessary before social learners can become convinced to vaccinate

Two Ways to Understand ω

$$dx/dt = \kappa\, x(1-x)\, [\omega\, I - 1]$$

$$\frac{1}{\omega} = \text{Level of infection necessary before social learners can become convinced to vaccinate}$$

$$\omega = \frac{\text{Perceived risk of infection}}{\text{Perceived risk of the vaccine}}$$

$\frac{1}{\omega}$ = Level of infection necessary before social learners can become convinced to vaccinate

If ω = **10,000** (a high value)

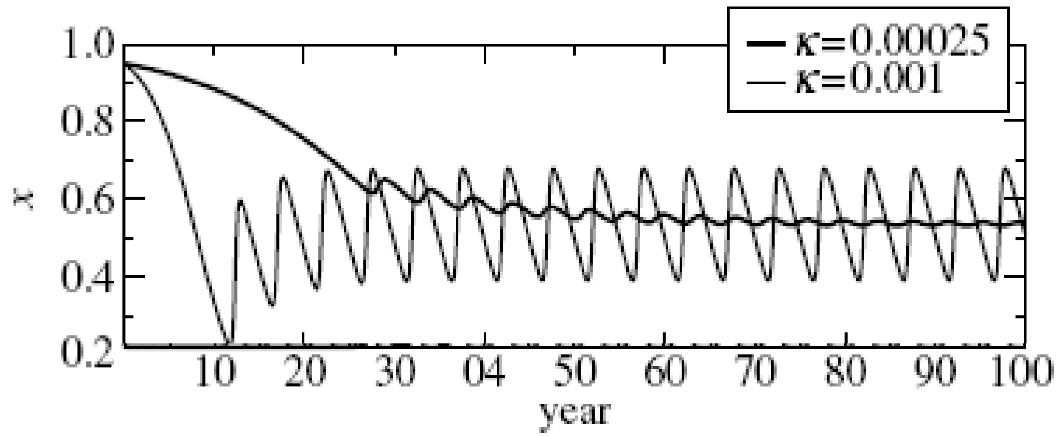
$$\frac{1}{\omega} =$$ Level of infection necessary before social learners can become convinced to vaccinate

If ω = **3,000** (a low value)

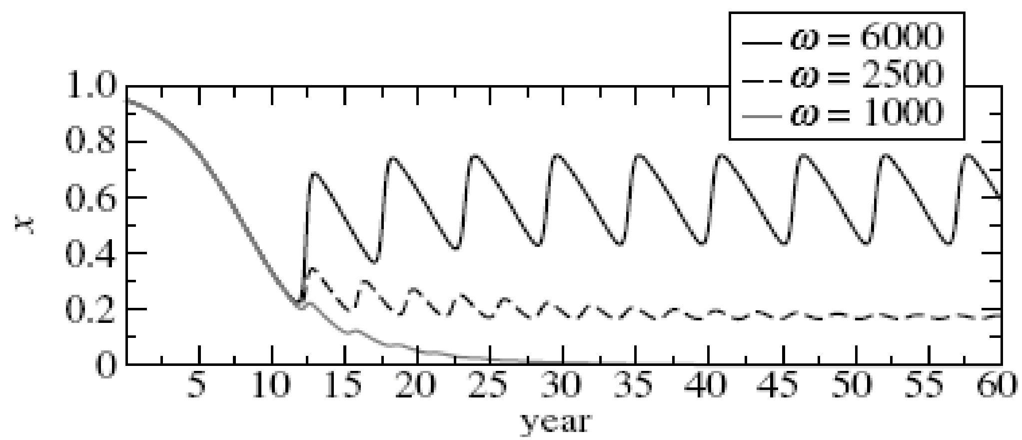
Results of Bauch (2005)

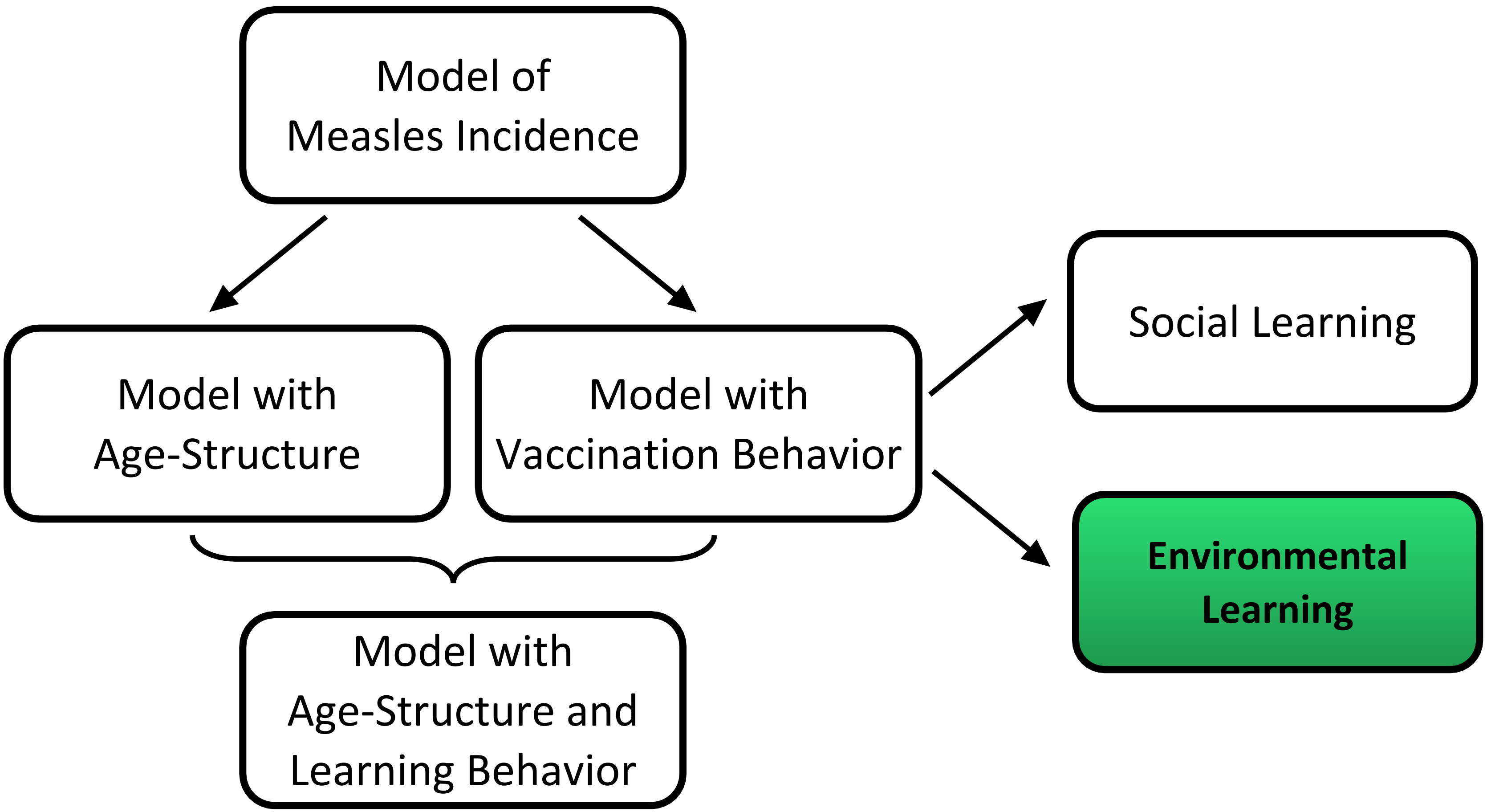
- oscillations occur in vaccine uptake when

individuals imitate each other more



Individuals react more quickly to disease prevalence

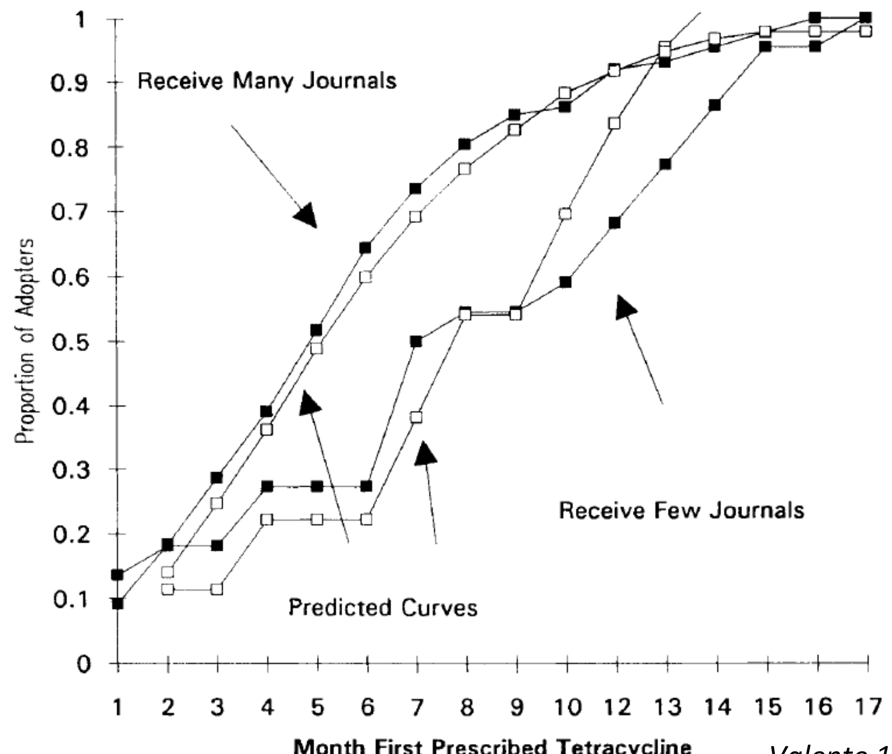



Model of
Measles Incidence
Model with
Age-Structure
Model with
Vaccination Behavior
Social Learning
Environmental
Learning
Model with
Age-Structure and
Learning Behavior

Environmental Learning

Henrich (2001): "Cultural Transmission and the Diffusion of Innovations"

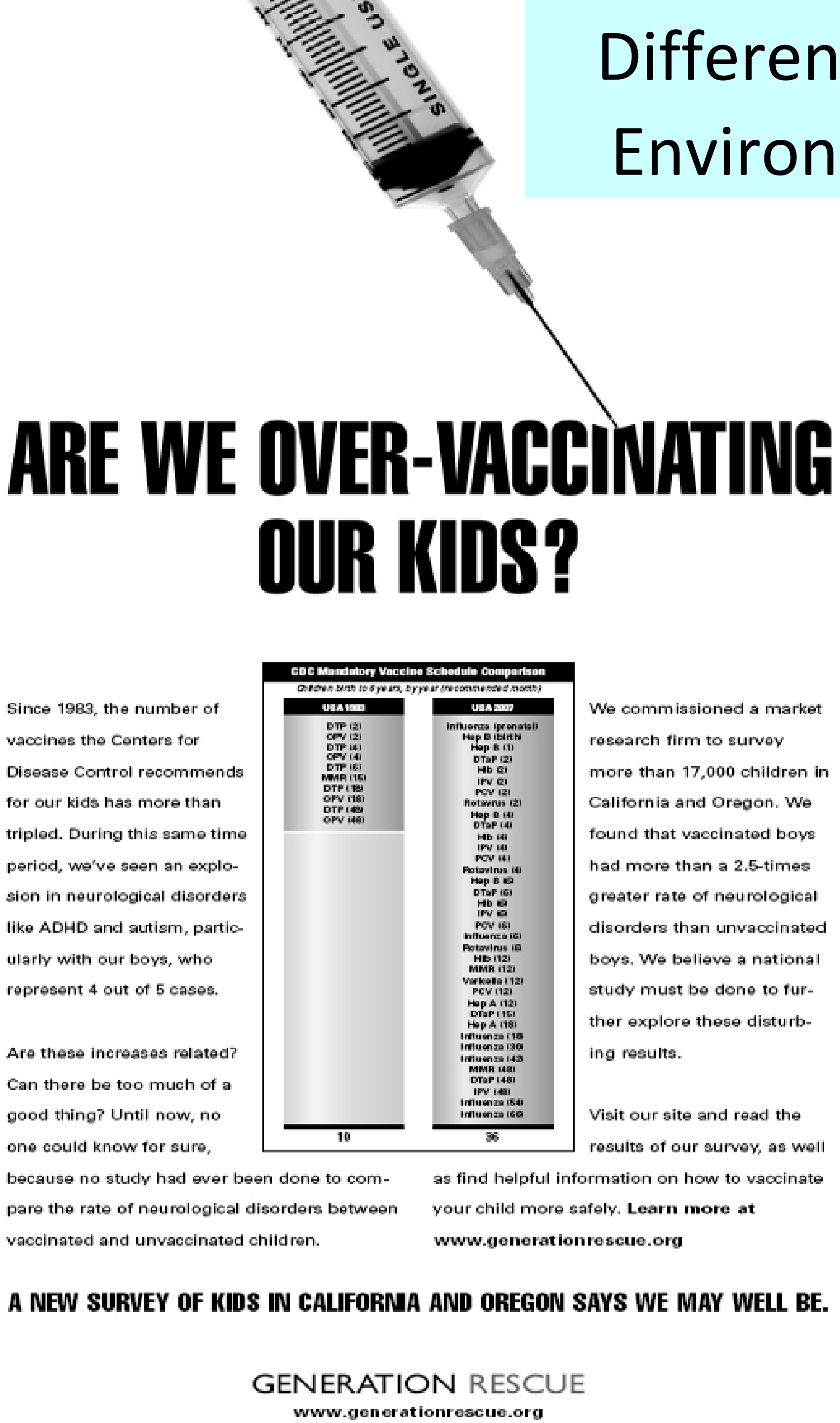
- Individual has complete knowledge of payoff gains of the innovation
- Individual chooses to adopt the innovation if it has a larger payoff



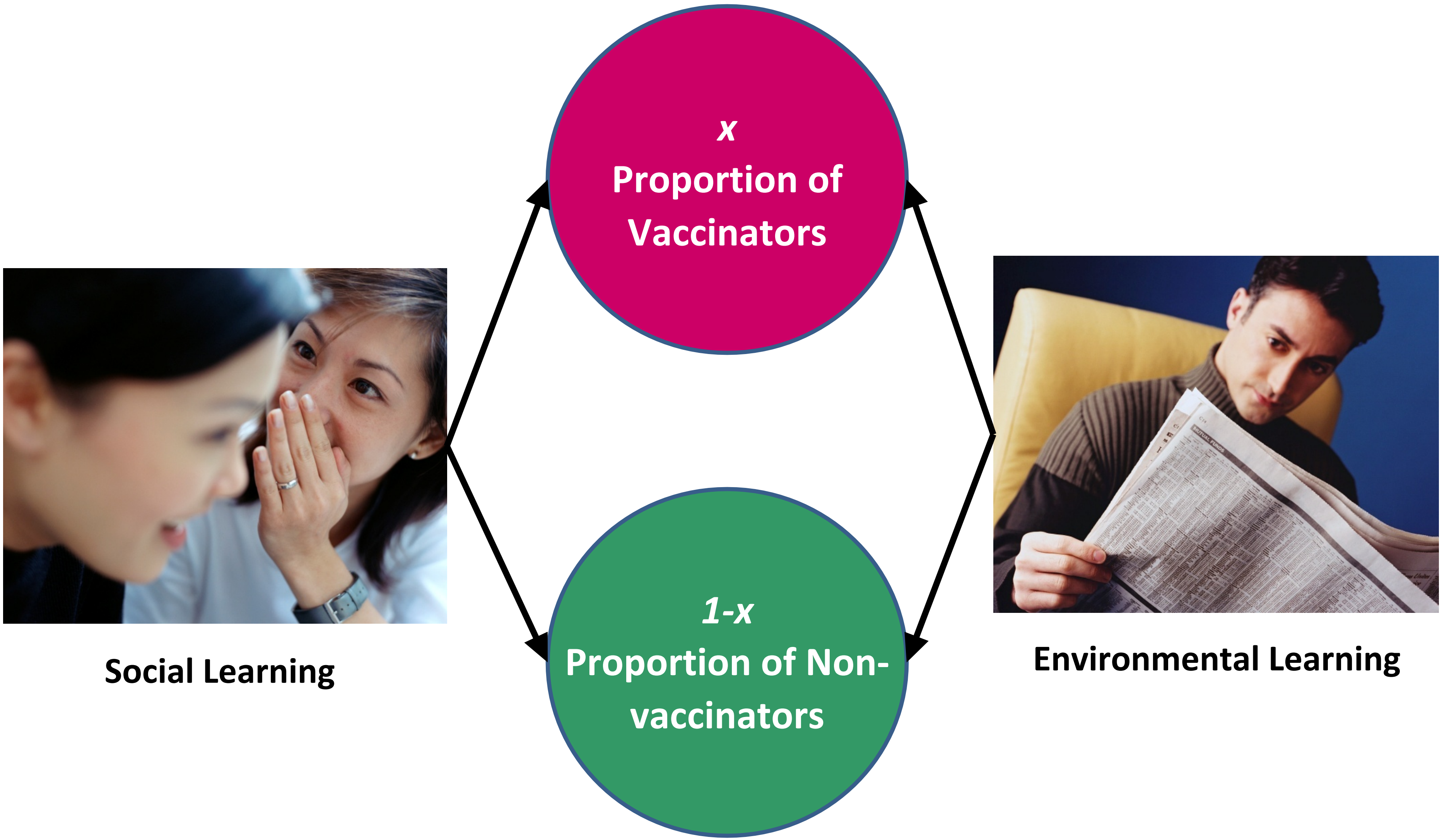
Valente 1993

Actual and predicted curves for the diffusion of tetracycline prescriptions among Illinois doctors who receive many and few journals

Different Messages from Environmental Learning



How do we model voluntary vaccination?



Incorporating Environmental Learning into Vaccination Behavior

$$dx/dt = \underbrace{L_s\, x(1-x)\, [\omega\, I - 1]}_{\text{Social learning}} + \underbrace{L_e\, (1 - x/c)]}_{\text{Environmental learning}}$$

Learning parameters	
L_s	Rate of social learning
L_e	Rate of environmental learning
c	Proportion of time that environmental learners switch to a vaccinating strategy

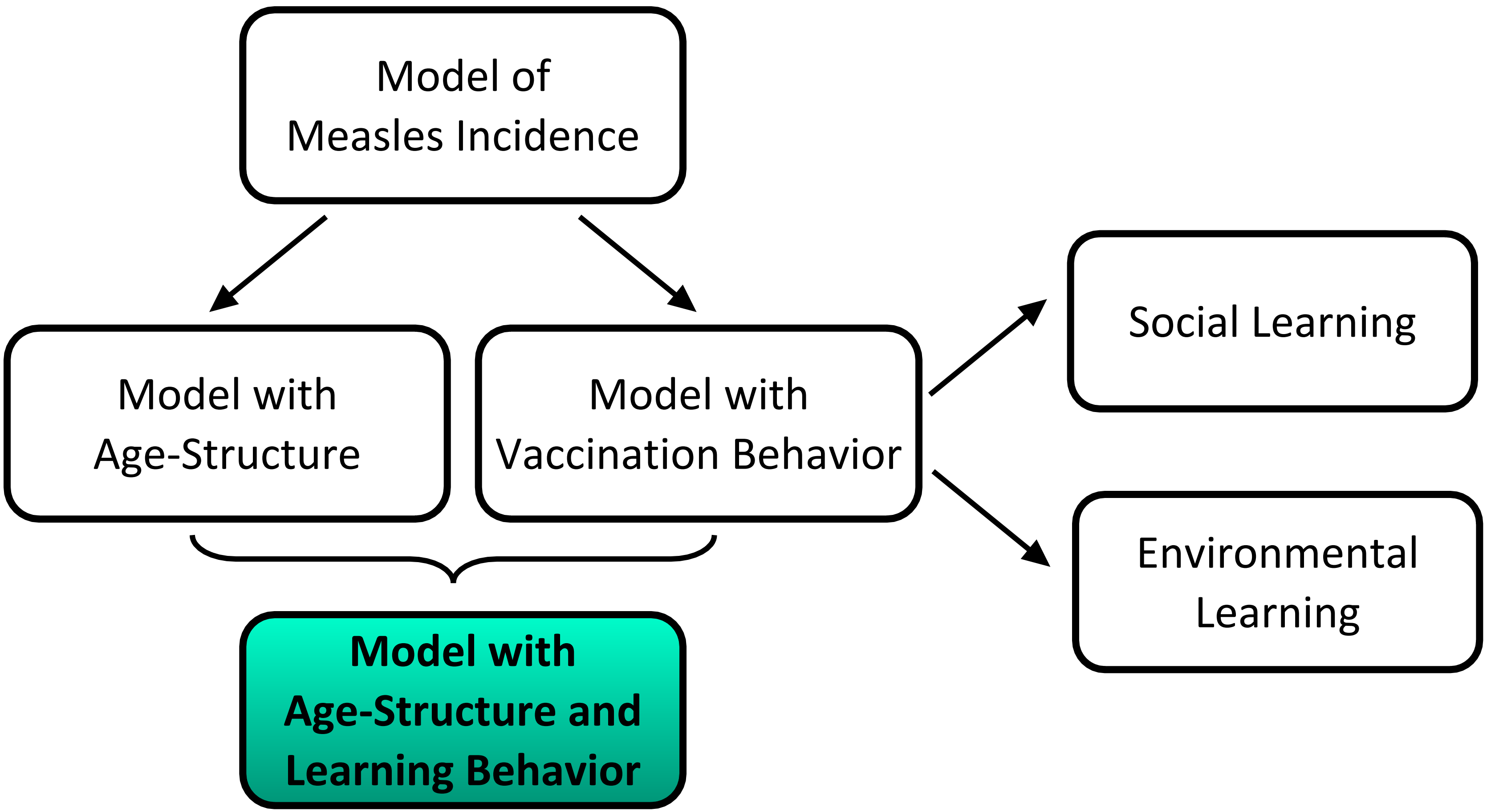
Summary of Previous Work

Age-Structure

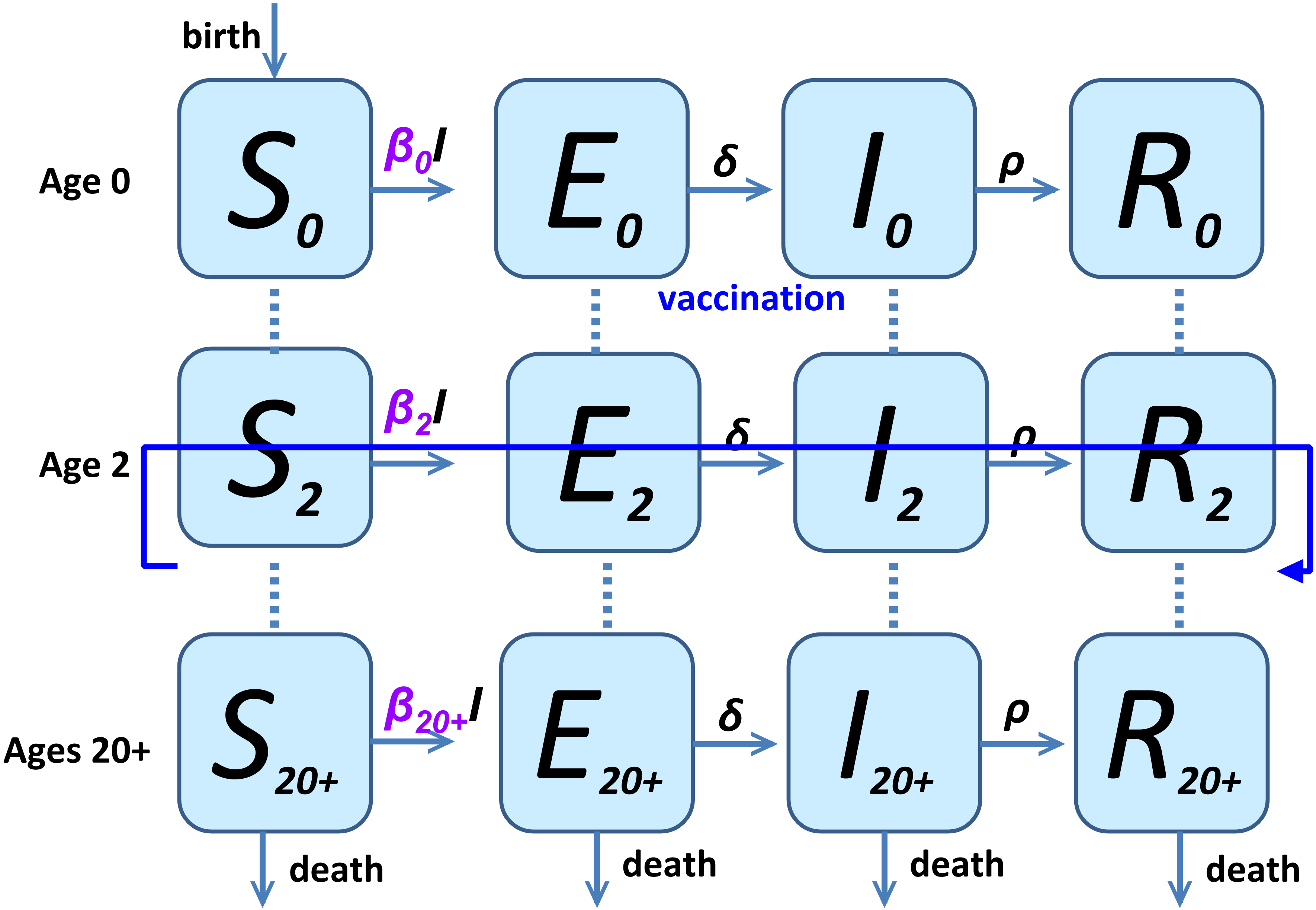
Voluntary Vaccination

- Schenzle (1984): accurately captured measles pre-vaccine incidence by incorporating age-structure but did not include voluntary vaccination
- Bauch (2005): modeled disease dynamics with vaccination behavior but did not include age-structure
- In our model we include both age-structure and voluntary vaccination

The Big Picture



Model with Age-Structure and Vaccination



Our Model

1. Age-Specific SEIR Equations

$N = 50$ million, $\nu = 666,666$, $\mu = 1/55$,
$\delta = 1/8$, $\rho = 1/5$, $\Sigma =$ summation over $j = 1, 2, \ldots, 20$
$\beta =$ matrix of age-specific contact rates.

Age 0

$$\frac{dS_0}{dt} = \nu - \Sigma\beta_{0j}(\tau)S_0, \qquad \frac{dE_0}{dt} = \Sigma\beta_{0j}(\tau)S_0 - \delta E_0,$$
$$\frac{dI_0}{dt} = \delta E_0 - \rho I_0, \qquad \frac{dR_0}{dt} = \rho I_0$$

Age k, where $k = 1, 2, \ldots, 19$

$$\frac{dS_k}{dt} = -\Sigma\beta_{kj}(\tau)S_k, \qquad \frac{dE_k}{dt} = \Sigma\beta_{kj}(\tau)S_k - \delta E_k,$$
$$\frac{dI_k}{dt} = \delta E_k - \rho I_k, \qquad \frac{dR_k}{dt} = \rho I_k$$

Age n, where $n = 20+$

$$\frac{dS_n}{dt} = -[\Sigma\beta_{nj}(\tau) + \mu]S_n, \qquad \frac{dE_n}{dt} = \Sigma\beta_{nj}(\tau)S_n - (\delta + \mu)E_n,$$
$$\frac{dI_n}{dt} = \delta E_n - (\rho + \mu)I_n, \qquad \frac{dR_n}{dt} = \rho I_n - \mu R_n$$

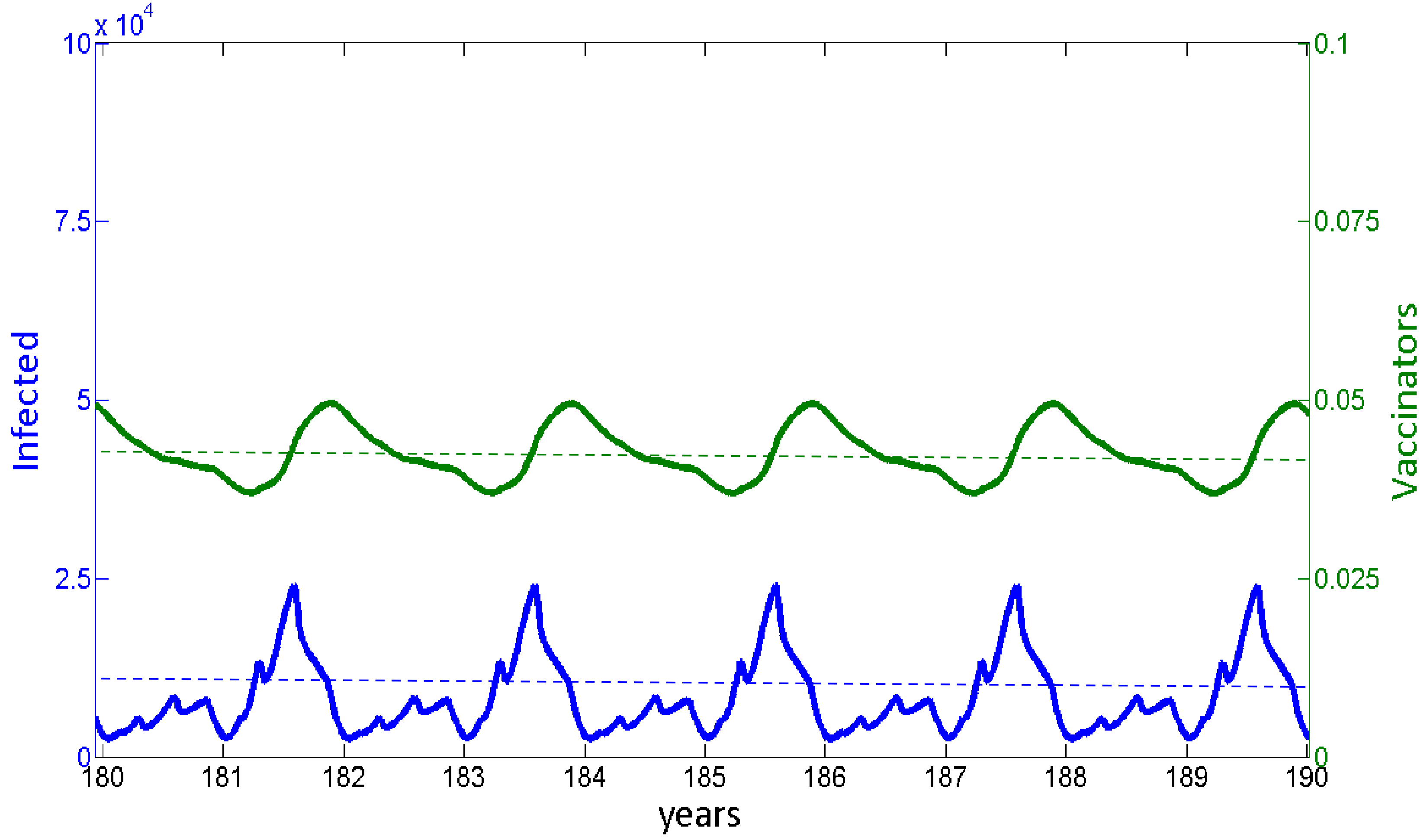
2. Vaccination Dynamics

$$\frac{dx}{dt} = L_s x(1 - x)[\omega I - 1] + L_e[1 - x/c]$$

Methods

- Fixed a population size of 50 million
- Chose constants for births, death rate, latent transition rate, recovery rate, and contact rate from measles data
- Ran simulations in the programming language MATLAB to examine both short and long term vaccination and disease dynamics
- Studied three scenarios
  - Social learning dominates
  - Mixture of social and environmental learning
  - Vaccine scare

An Example of a MATLAB Output of our Model



Measure average

Results

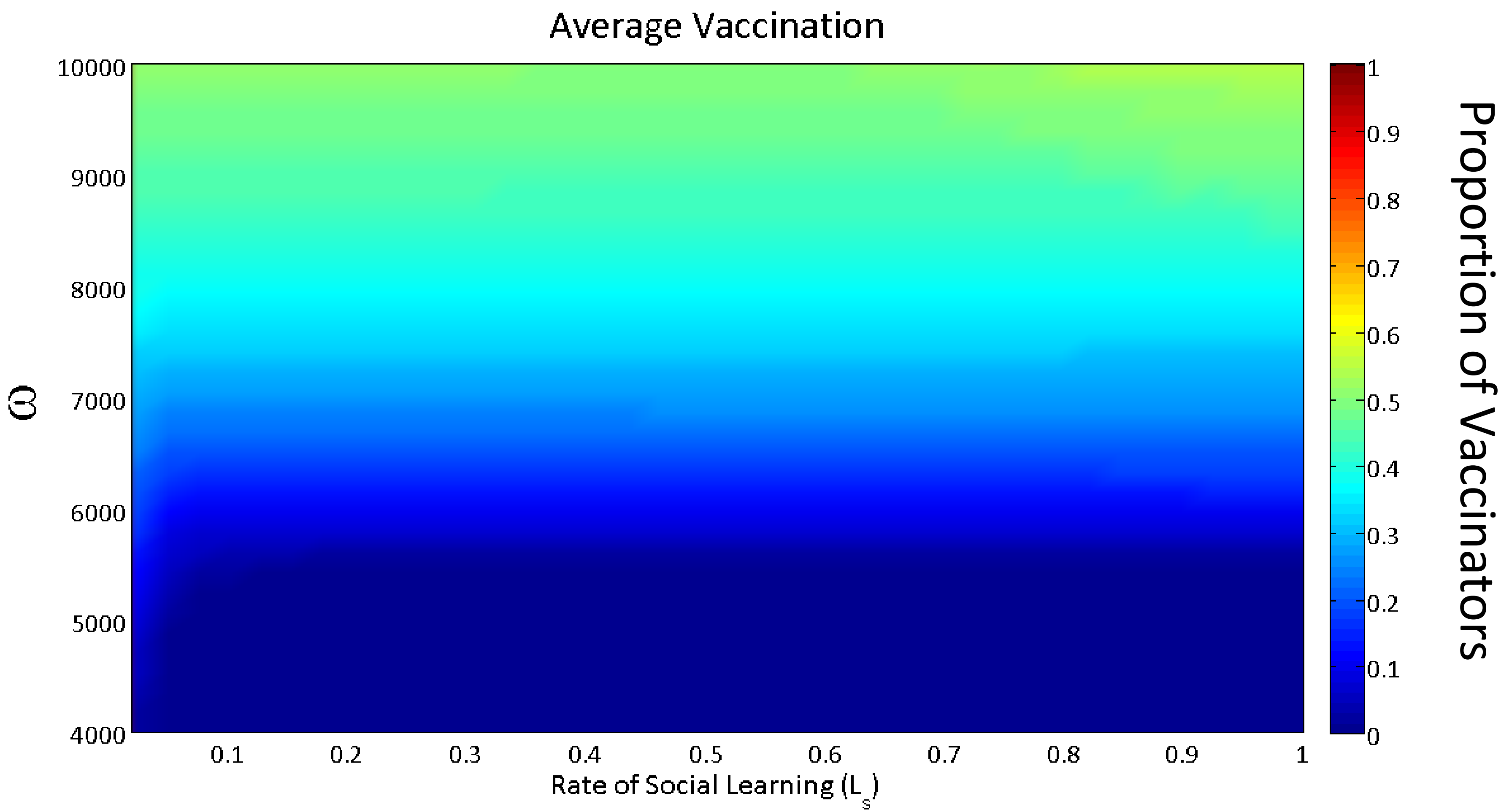
- Studied three scenarios
  - *Social learning dominates*
  - Mixture of social and environmental learning
  - Vaccine scare

Only Social Learning

$$dx/dt = L_s\, x(1-x)\, [\omega\, I - 1]$$

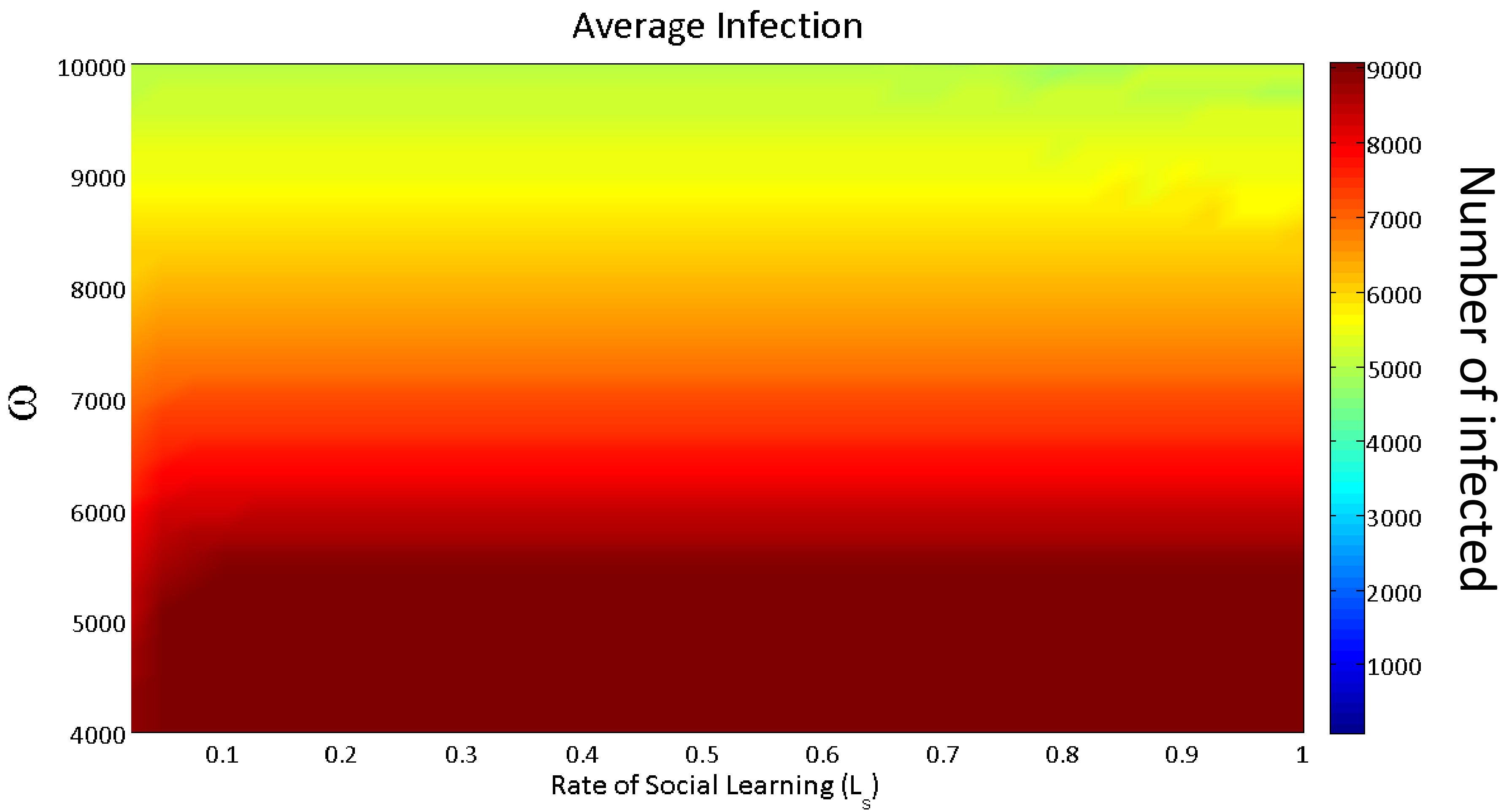
How do various parameter values influence vaccination and disease dynamics?

Effects of Social Learning on Vaccination



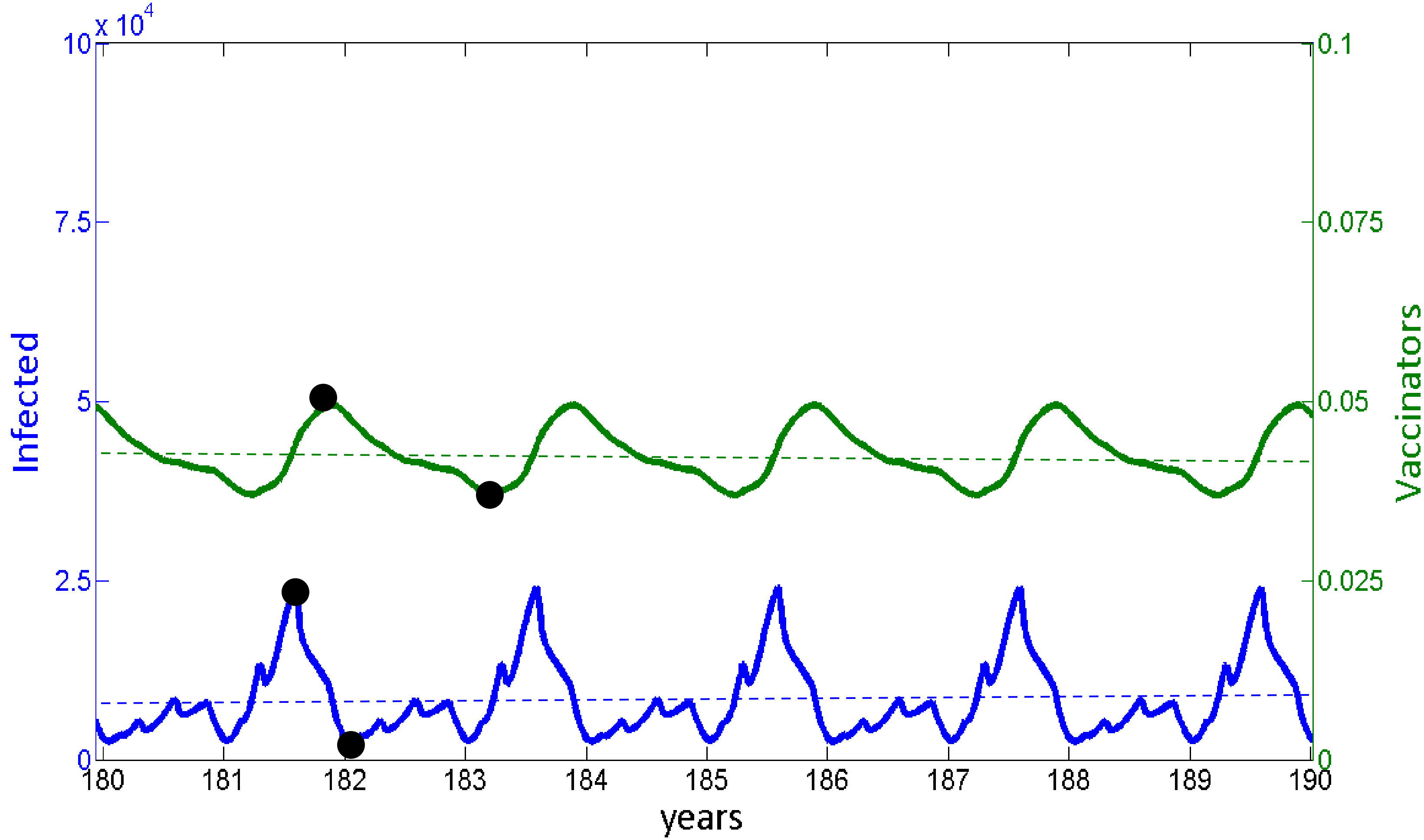
- Average vaccination increases with sensitivity to level of infection.
- Average vaccination is independent of the social learning rate.

Effects of Social Learning on Infection



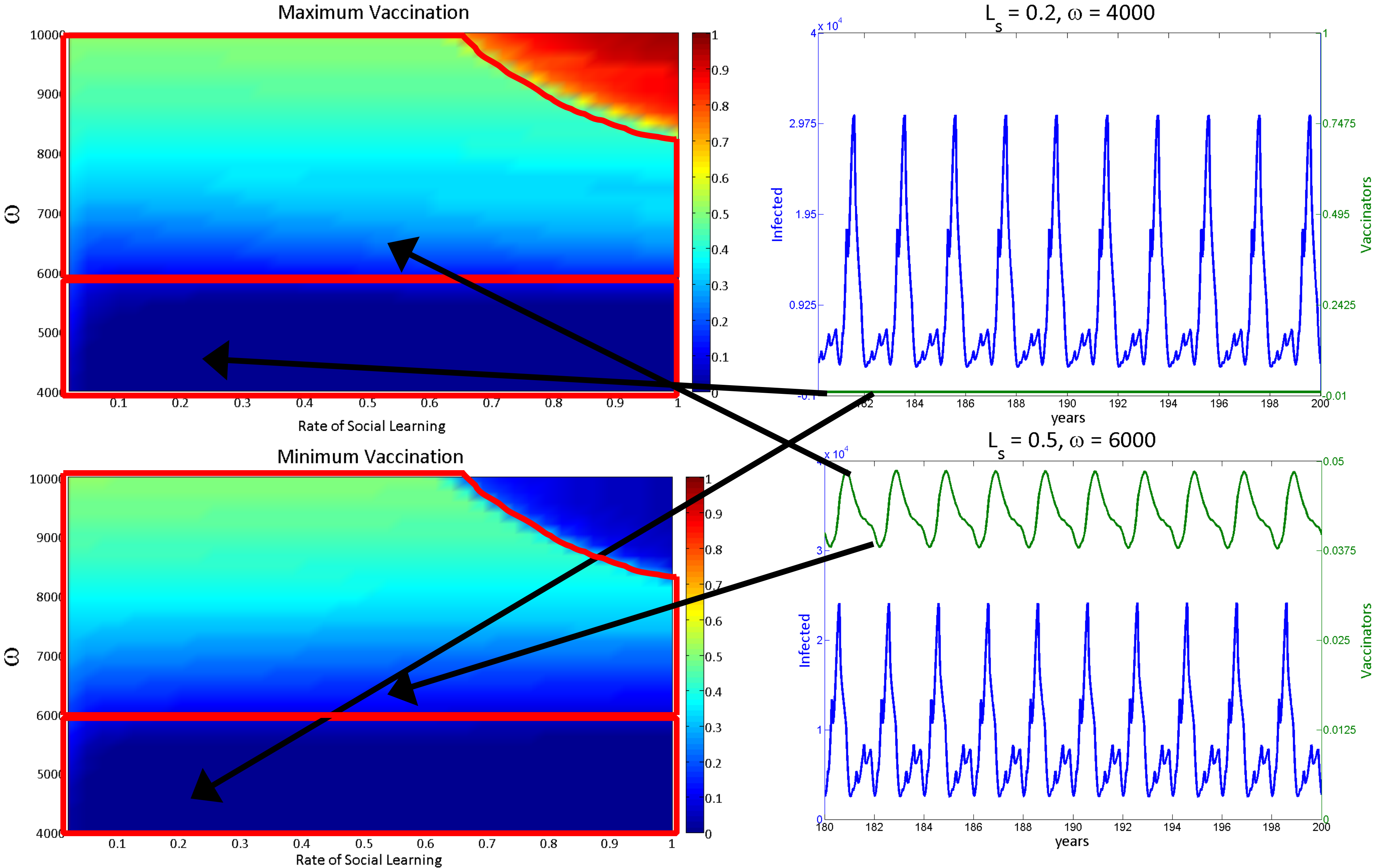
- Average infection decreases with sensitivity to level of infection.
- Average infection is independent of the social learning rate.

An Example of a MATLAB Output of our Model

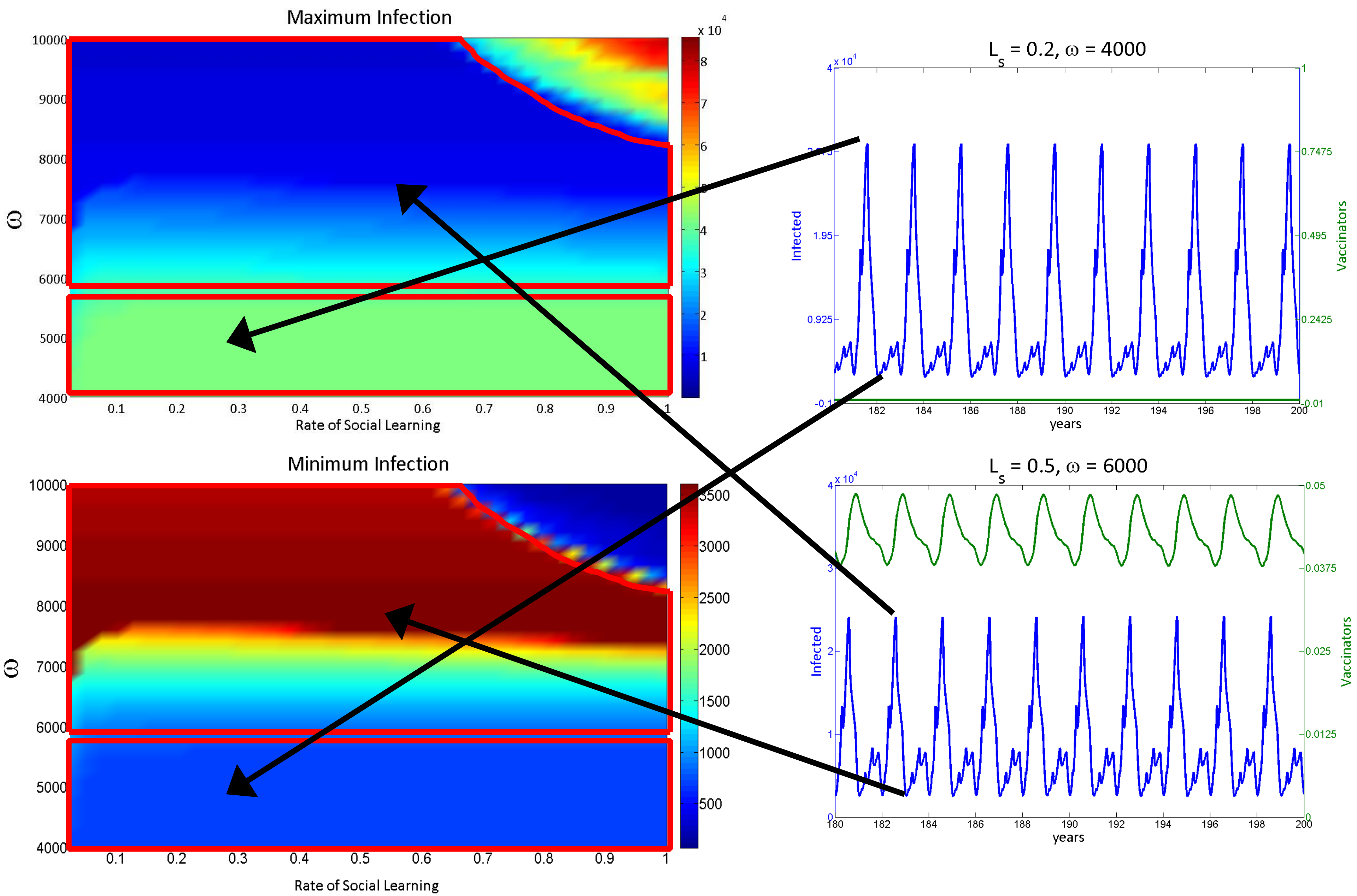


Measure average, maximum, and minimum

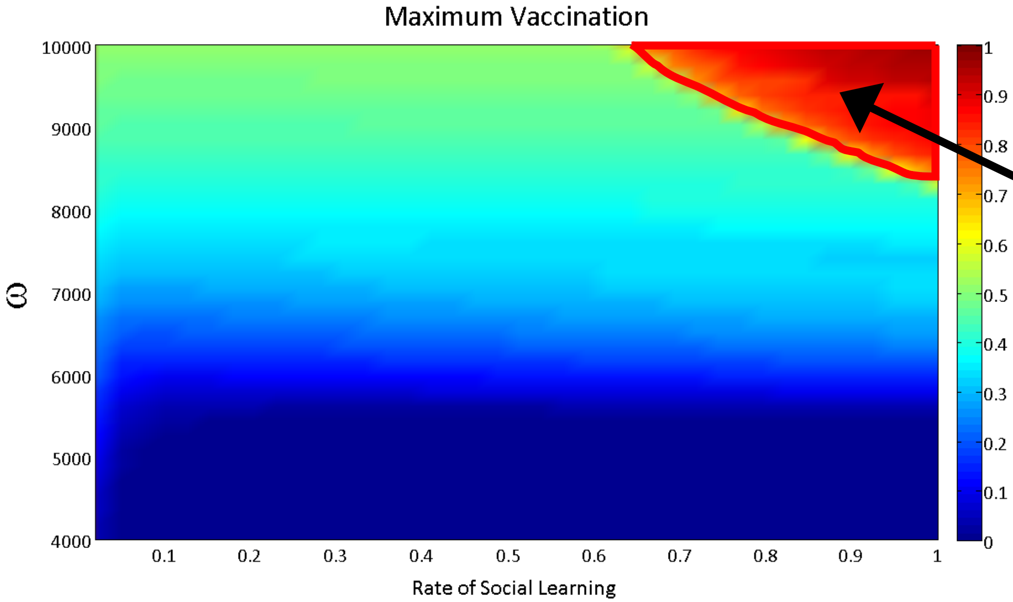
Max. and Min. Vaccination

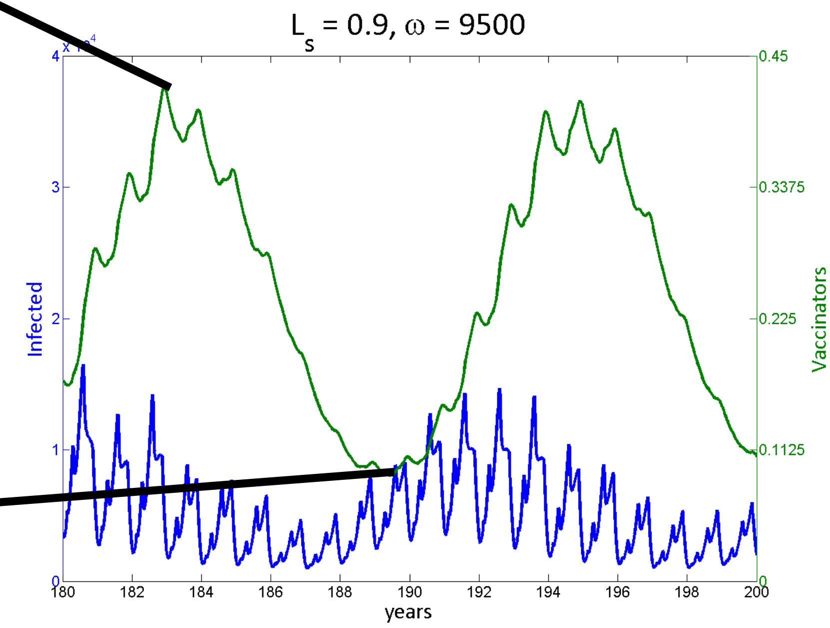


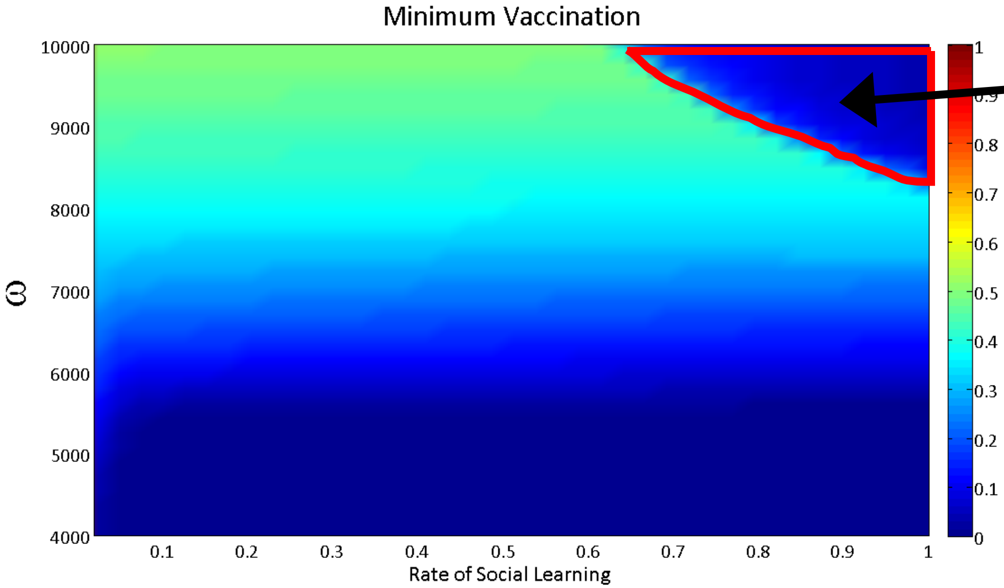
Max. and Min. Infection



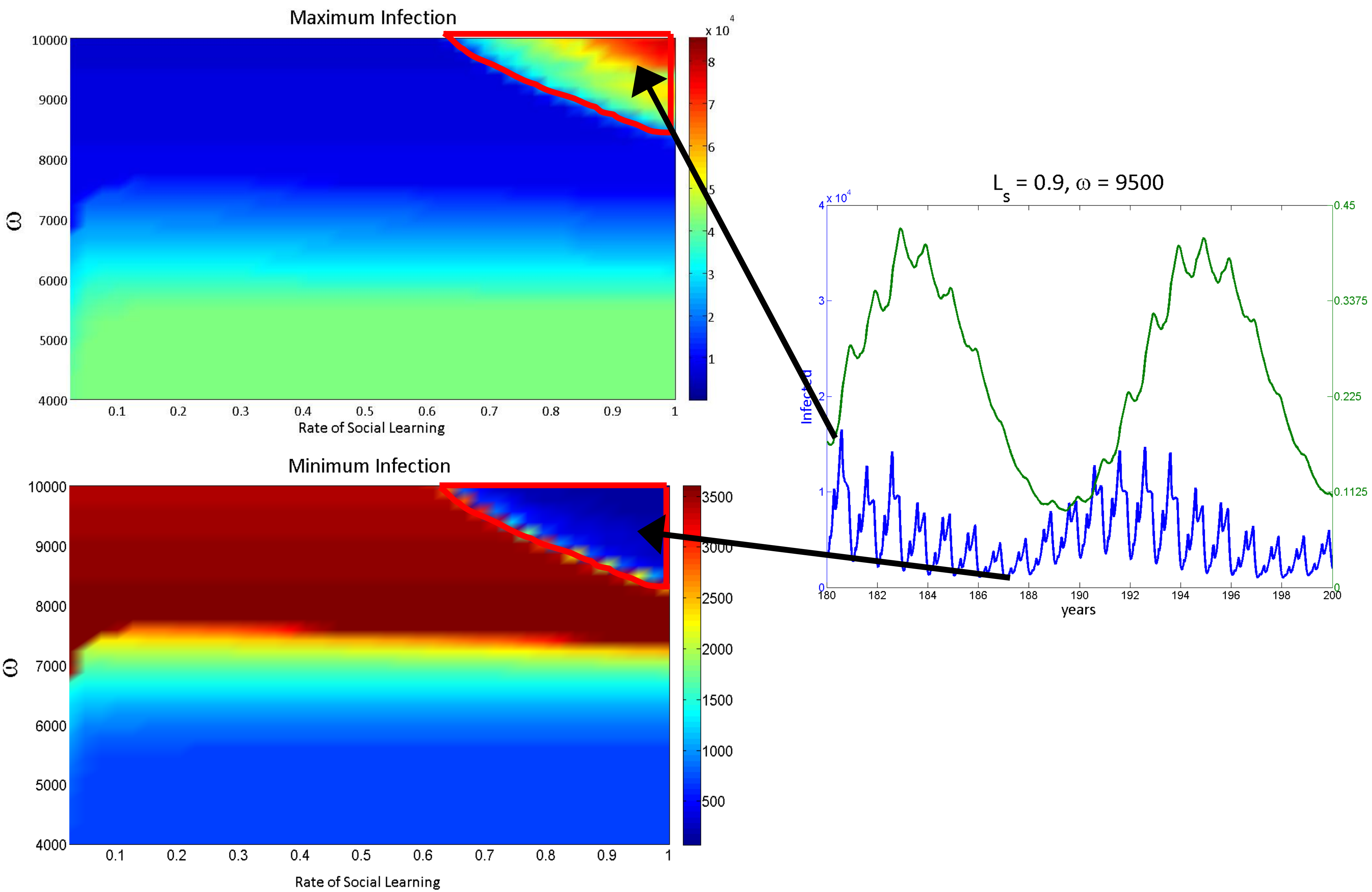
Max. and Min. Vaccination







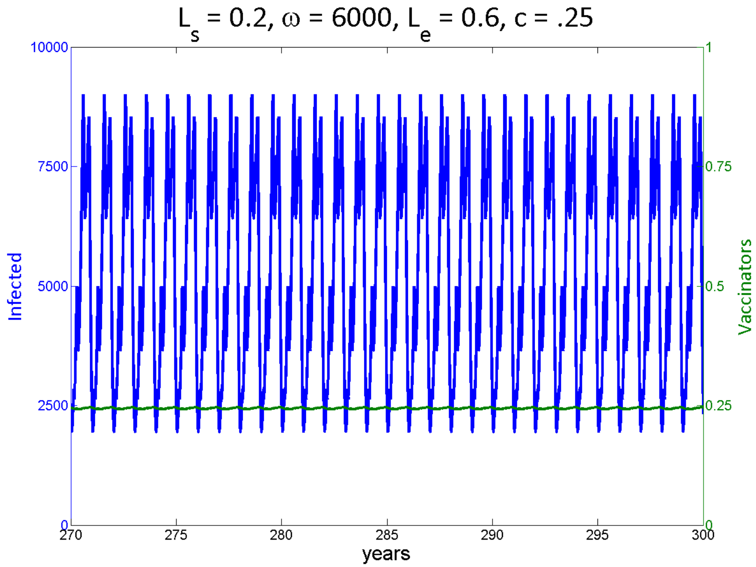
Max. and Min. Infection

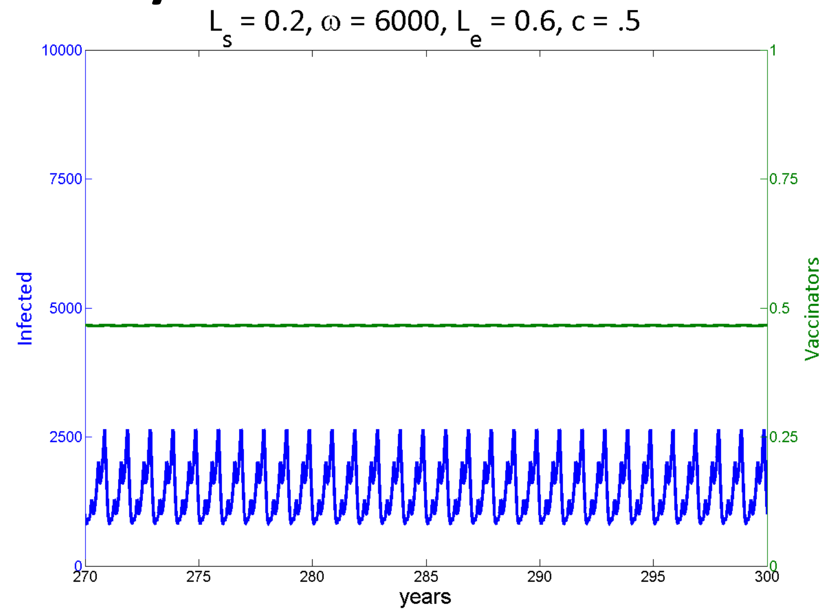


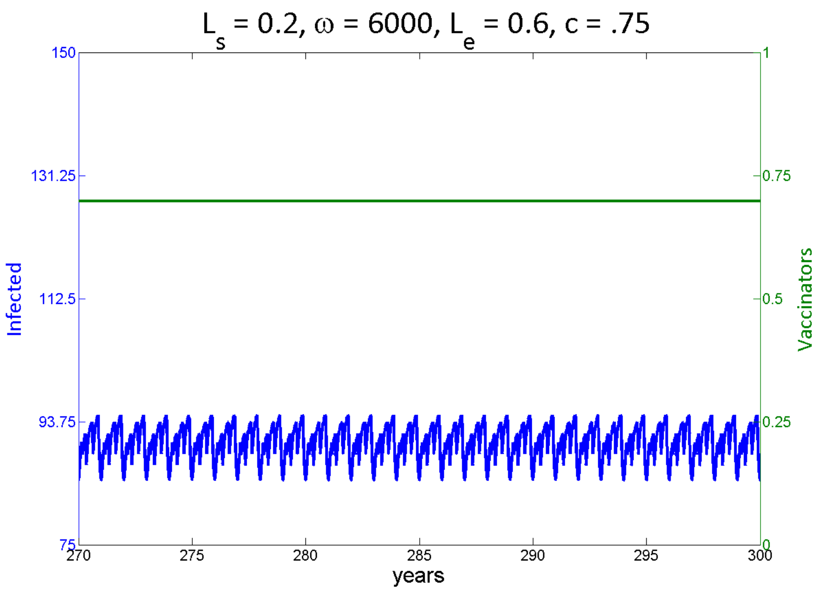
Results

- Studied three scenarios
  - Social learning dominates
  - *Mixture of social and environmental learning*
  - Vaccine scare

Mixture of social and environmental learning effects on dynamics

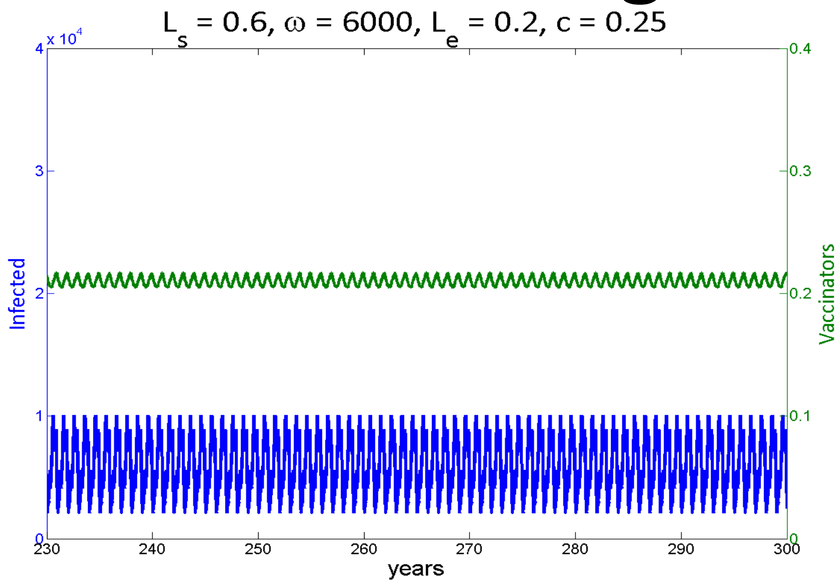


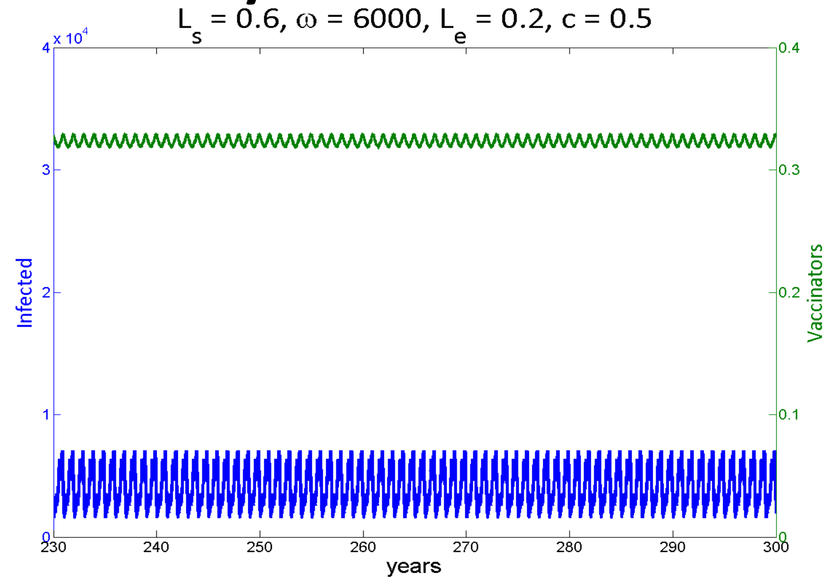


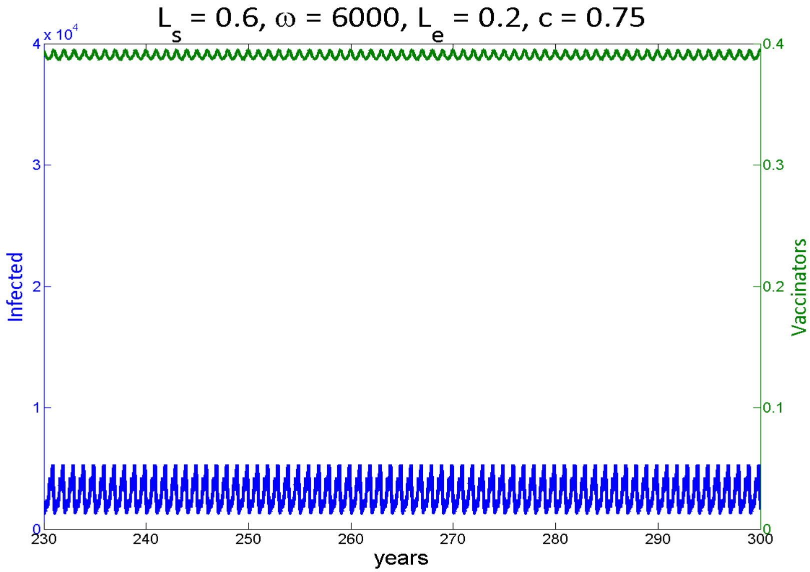


- Higher vaccination decreases maximum infection
- The average vaccination is approximately the probability a person switching to a vaccinating strategy.

Mixture of social and environmental learning effects on dynamics





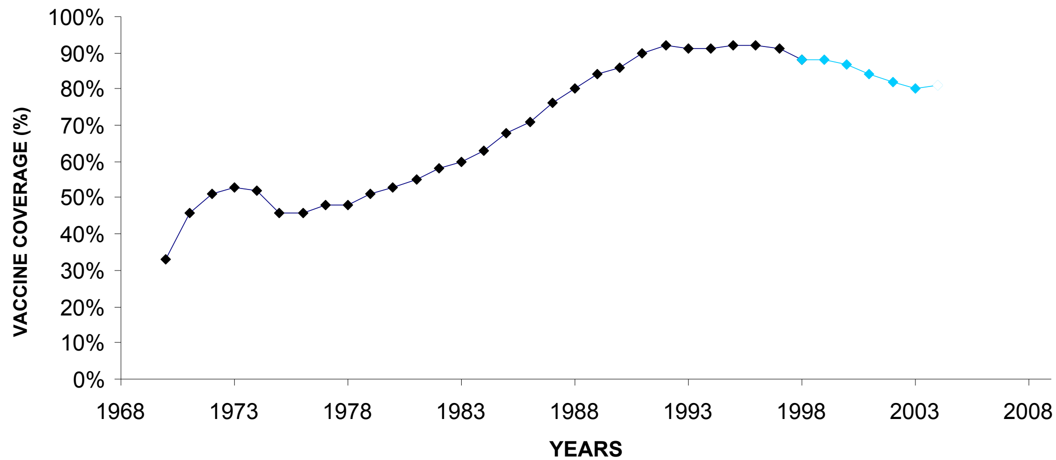


- Higher probability does not correspond to a high level of vaccinators
- Infection does not decrease as low levels.

Results

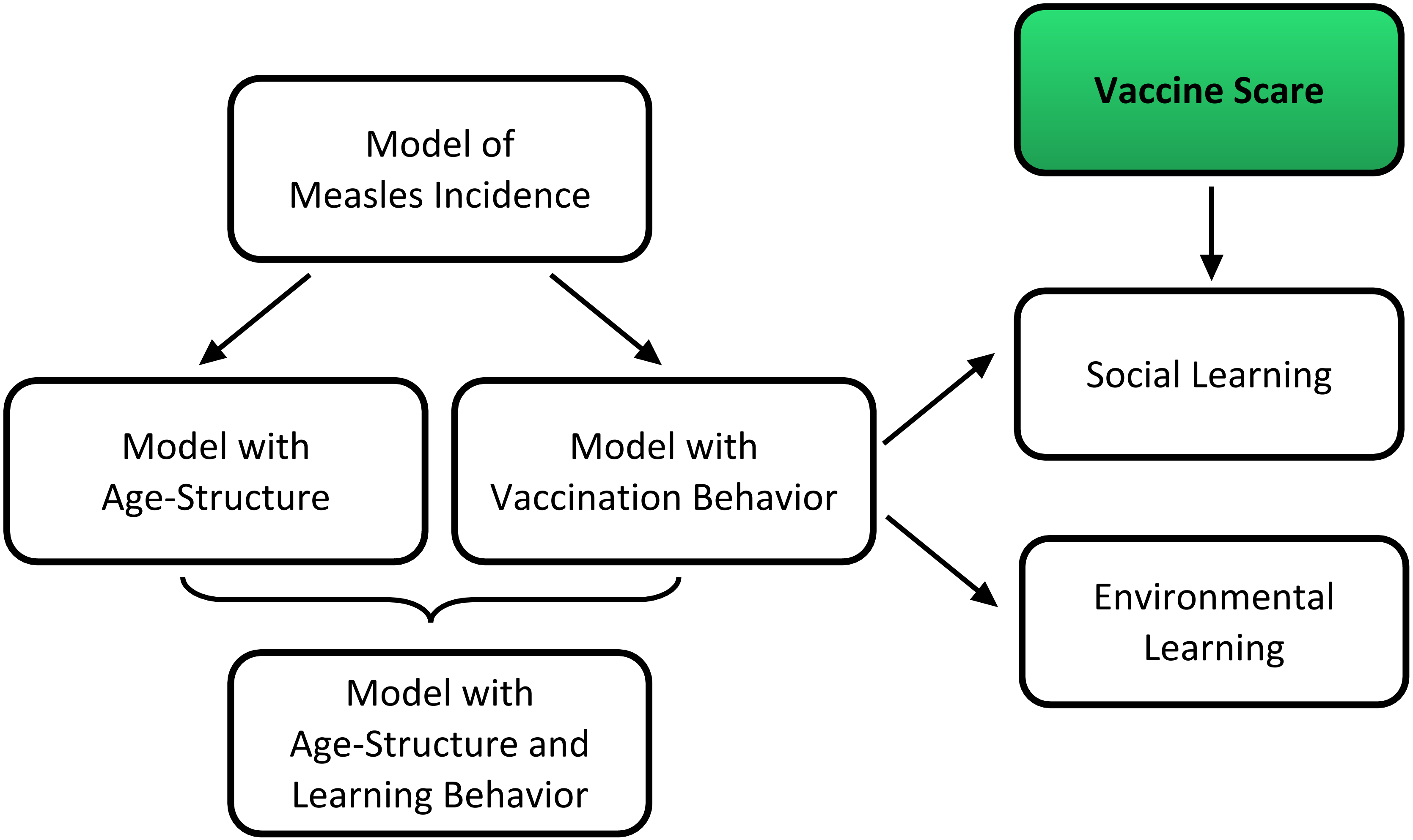
- Studied three scenarios
  - Social learning dominates
  - Mixture of social and environmental learning
  - *Vaccine scare*

Vaccine Scares

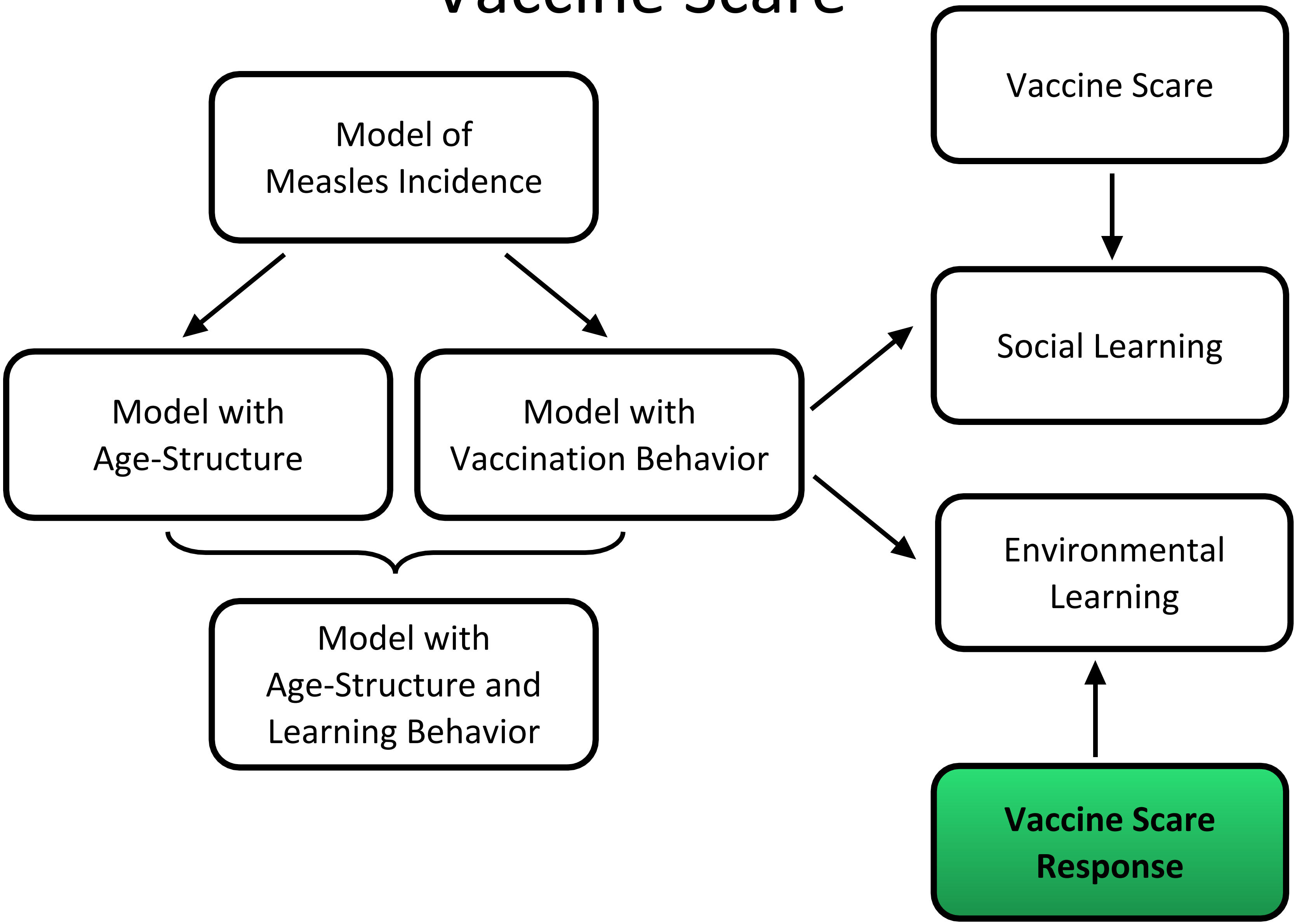


- Individuals are concerned over adverse side effects associated with the vaccine
- Low MMR vaccine uptake levels following the publication of Wakefield et al. 1998

Modeling a Vaccine Scare



Modeling a Response to a Vaccine Scare



Simulating a Vaccine Scare

Risk of vaccination increased *n*-fold for a duration of time

- People interact with each other and switch to non-vaccinating strategies due to the increased risk of the vaccine.

increase

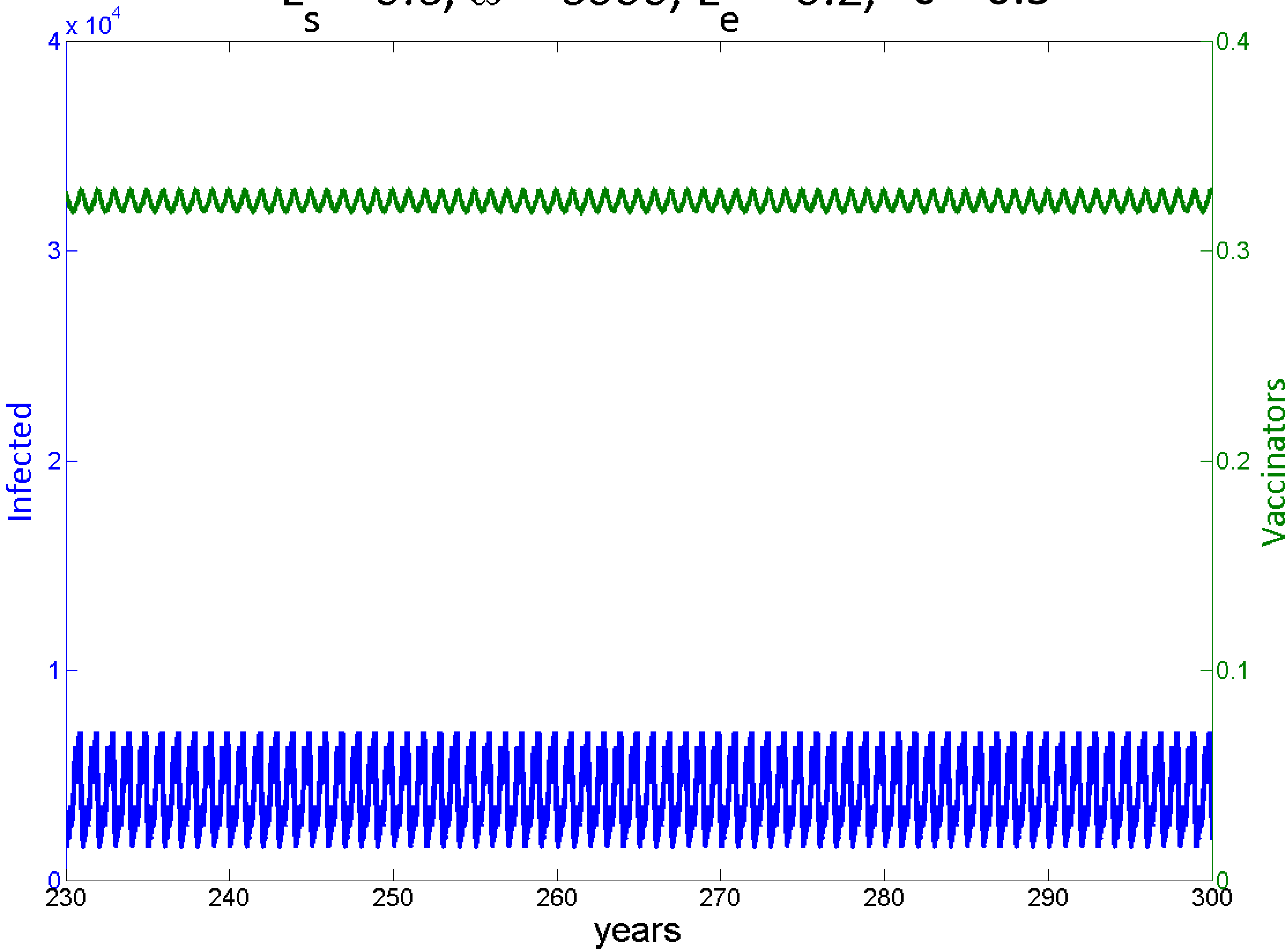
Responding to a Vaccine Scare

Counteract with environmental learning

– Public outreach campaigns encourage vaccination and the switching to vaccinating rate increases *m*-fold.

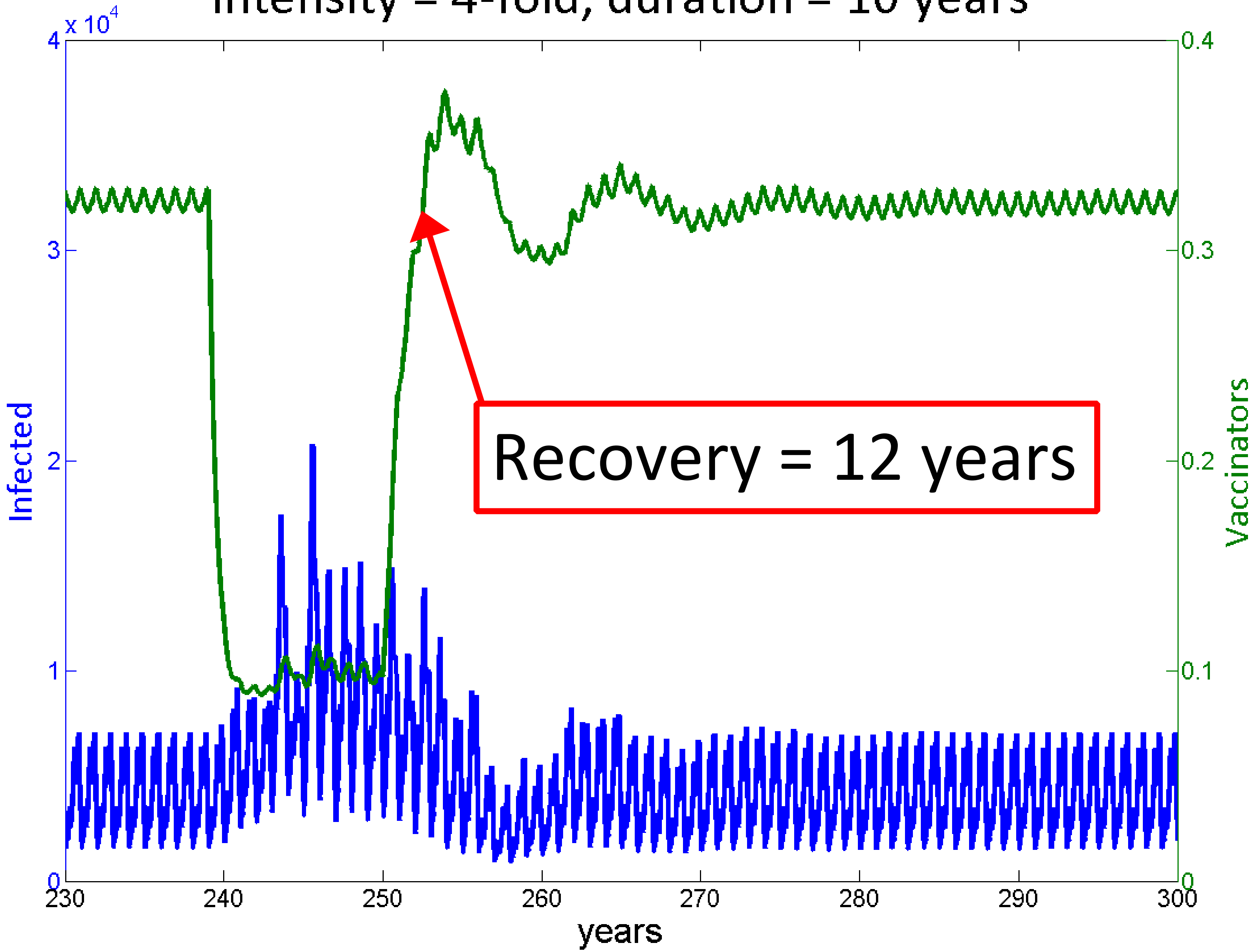
Before a scare

$L_s = 0.6$, $\omega = 6000$, $L_e = 0.2$, $c = 0.5$

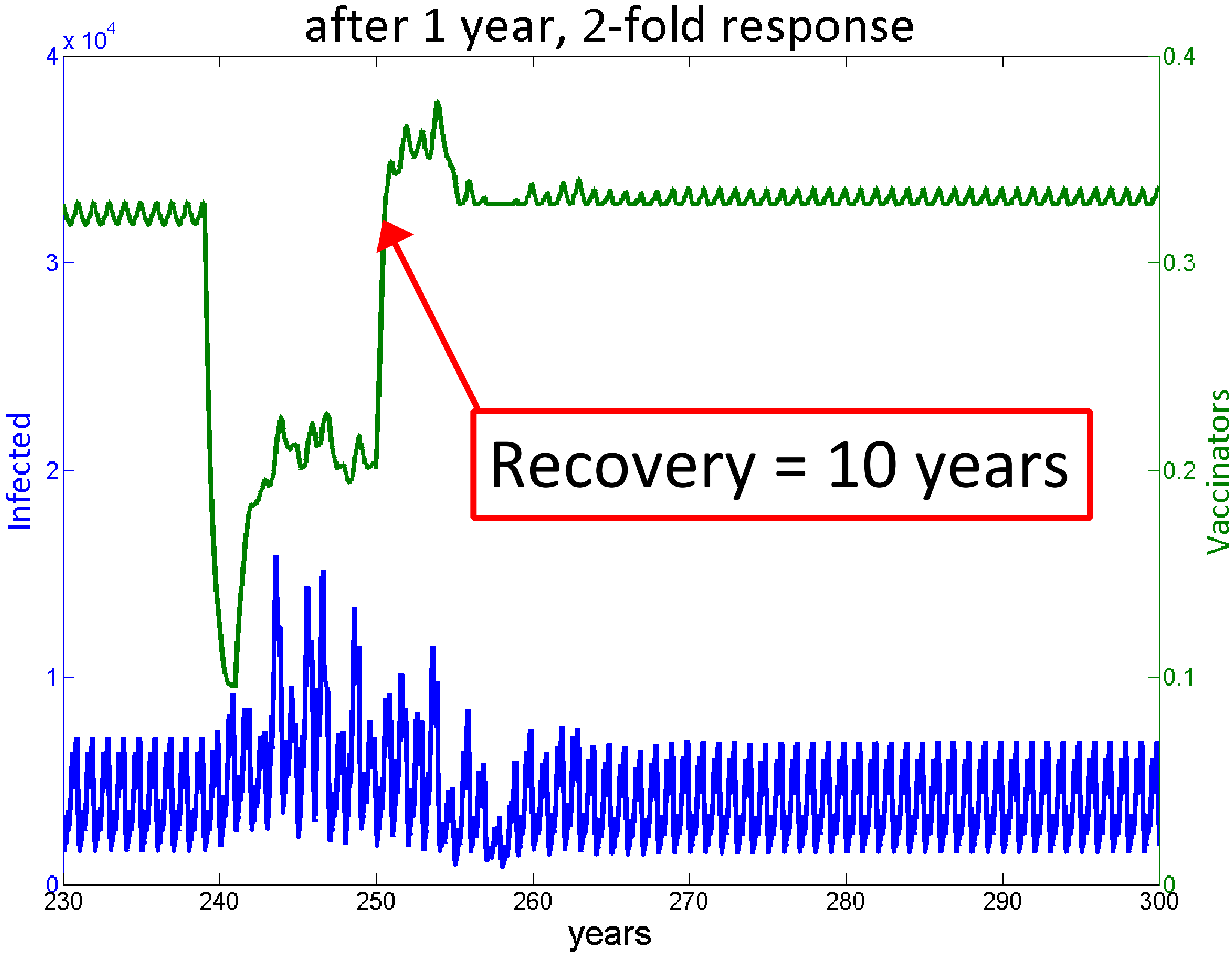


A Vaccine Scare

intensity = 4-fold, duration = 10 years

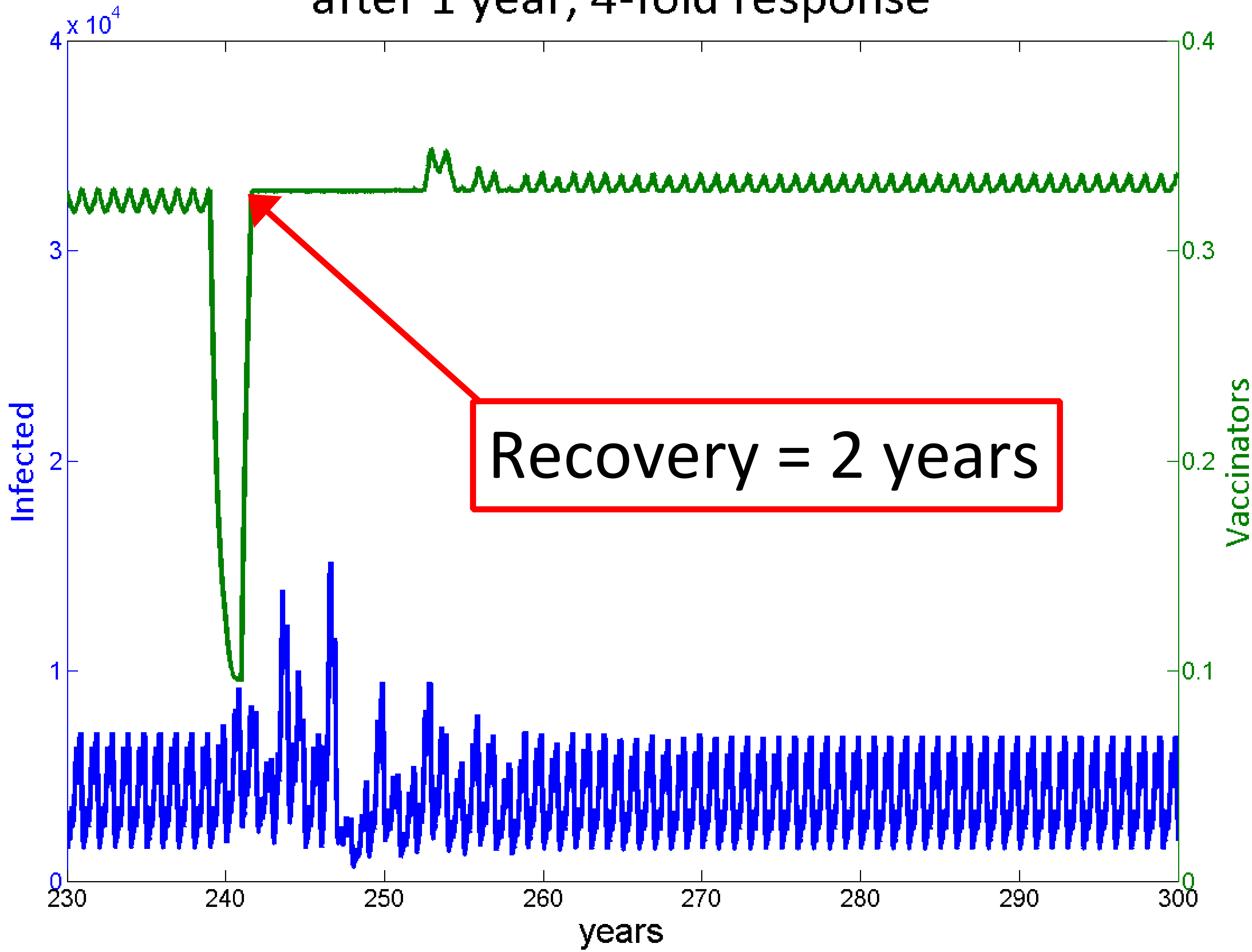


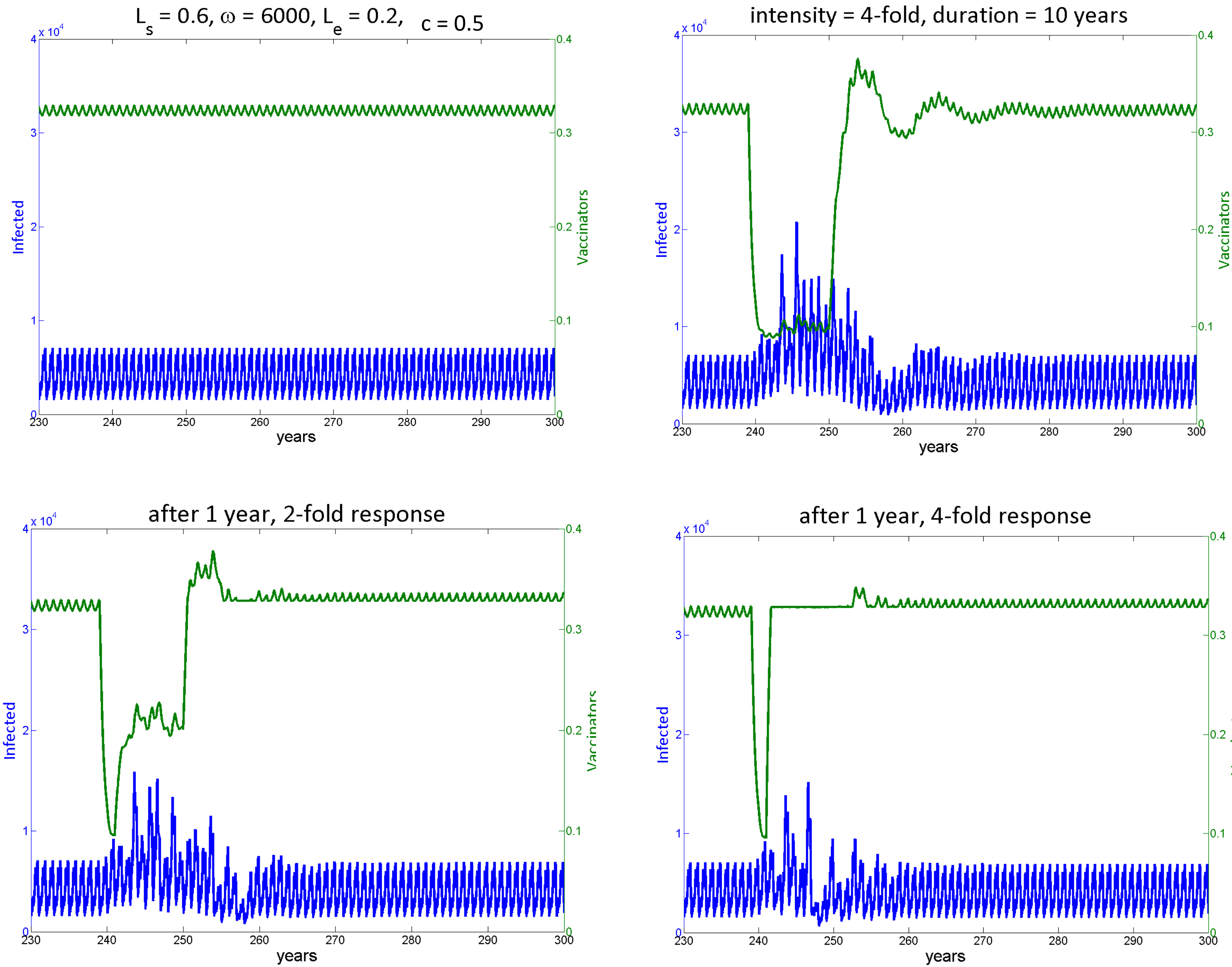
Responding to a Vaccine Scare



Responding to a Vaccine Scare

after 1 year, 4-fold response





L_s = 0.6, ω = 6000, L_e = 0.2, c = 0.5
intensity = 4-fold, duration = 10 years
after 1 year, 2-fold response
after 1 year, 4-fold response
x 10^4
Infected
Vaccinators
years
0
1
2
3
4
0.1
0.2
0.3
0.4
230
240
250
260
270
280
290
300

Summary of Vaccine Scares

- Vaccine scares
  - A lag time between vaccination rates dropping and a rise infection → due to age-dependence
  - A quick, strong response can mitigate many of the effects
    - Non-linear increase in response: twice the response causes a recovery that is five times as fast

General Conclusions

- Voluntary vaccination causes oscillations in levels of vaccinators and disease prevalence
- Social learning does not affect the average but does has an effect on epidemic maxima and minima
- Social learning can greatly decrease vaccination levels (such as in a vaccine scare)
- Environmental learning has a strong effect on the level of vaccinators
- Environmental learning can counteract vaccine scares

Future Research

- Quantitative vs. qualitative results
- Parameter values
- The non-linear relationship between vaccine scare response and recovery
- Catch-up vaccine programs

Acknowledgements

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Mathematics and Biology

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