Synchrony in Coupled Neural Oscillators

Andrea Ekey, Ethan Levien, Sadik Shahidain,
Stephanie Willoughby, Taisa Kushner, Taj Sangha,Tek Chhetri

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1 Introduction

Quantitatively understanding how populations of neurons interact in the brain is one of the great challenges of computational and mathematical neuroscience, and even the simple case of two coupled neurons is not well understand. A great deal of research has been devoted to studying the dynamical systems that arise when this system is modeled. In this report we summarize a particular example of this research: the work of Lewis and Rinzel [1] which examines how inhibitory and electric coupling act to promote synchrony. We also present a simple extension of these results to a heterogeneous system.

2 LIF Neurons and the Theory of Weakly Coupled Oscillators

Lewis and Rinzel [1] based their work on a single-compartment leaky-integrate-and-fire (LIF) model. In order to account for both mutual inhibition and electrical coupling, two G-functions are used,

Mutual inhibition: $G_s(\phi) = \frac{-g_s A}{IT^2(1-\alpha)^2}((e^{-\alpha T}((T+B)(1-\alpha)-1)-(B(1-\alpha)-1))(e^{\phi T}e^{(1-\phi)T}) + (1-e^{-T})(e^{\phi T}e^{(1-\alpha)(1-\phi)T}(((1-\phi)T+B)(1-\alpha)-1)-(e^{(1-\phi)T}e^{(1-\alpha)\phi T}((\phi T+B)(1-\alpha)-1)))$

Electrical coupling:

$$G_c(\phi) = g_c\frac{2}{T}(\phi\sinh((1-\phi)T)-1-\phi)\sinh(\phi T)) + g_c\frac{\beta}{IT^2}(e^{\phi T}-e^{(1-\phi)T}), 0<\phi<1 \text{ and}$$
$$G_c(\phi) = 0, \phi = 0, 1$$

where $A = \frac{\alpha^2}{1-e^{-\alpha T}}$, $B = \frac{Te^{-\alpha T}}{1-e^{-\alpha T}}$, and $T = \ln(\frac{I}{I-1})$ and g_s, g_c represent coupling strength values which simply scale the G-functions. Using these equations and parameters presented in [1], we reproduced the figures and results they obtained in order to ourselves gain a better understanding. It was found that increasing the α parameter (synaptic kinetics) increases the range of I over which both the synchronous and anti-synchronous states are stable with only the synchronous state stable for all I when $\alpha = 0$ and an increasing region of AS/S stability as α increases.

The zeros of the G-function for the electrical synaptic connections are qualitatively the same as those for the inhibitory case, in both location and stability. This gives qualitatively similar bifurication plots versus the applied current parameter. The parameter β (spike size) shows dynamics

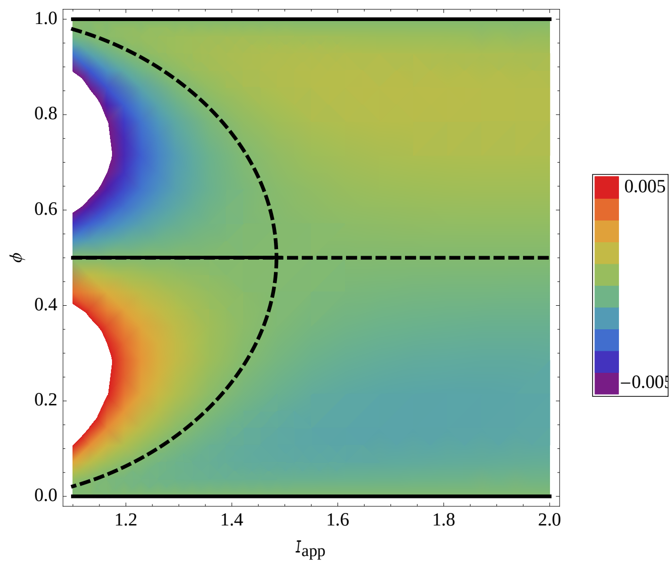

Figure 1: Bifurcation diagram for inhibitory coupling. Solid and dashed lines indicate stable and unstable states, respectively. For values less than the critical point I^*, $(I < I^*)$ both antisynchronous and synchronous states are stable, while for large $I > I^*$, we have that only the synchronous state is stable. Here $\alpha = 4$. $I^* = 1.48$.

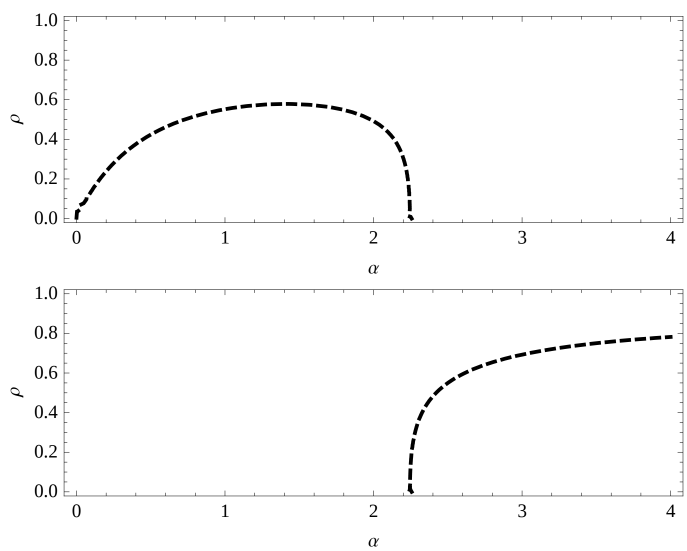

Figure 2: Dashed lines indicated changes in stability for combined electrical and inhibitory coupling in $\rho - \alpha$ parameter space for the antiphase solution when $I_{app} = 1.2$ (top) and $I_{app} = 1.3$ (bottom).

opposite to those observed for α with a large region of antisynchronous/synchronous stability when β is small and an increasing region of synchronous stability when β increases.

In order to couple the electrical and inhibitory dynamics, a new variable ρ is defined which gives the fraction of excitatory coupling, $g_c/(g_c + g_s)$. By coupling both the electrical and inhibitory dynamics, it was found that synchrony in cells is most dominant when $\rho = 1$ (given a large spike effect and fast inhibitory synapses) or when $\rho = 0$ (given slow inhibitory synapses and a small spike effect) and less so for values of ρ inbetween. Additionally, there is an observed overall increase in the region of synchronous stability given an increase in I regardless of α, β and ρ.

3 Heterogeneous Cells

One of the main limitations of the aforementioned coupled LIF model of cortical interneurons is its inability to account for heterogeneity between cells. Therefore, as an extension, we attempt to add more realism to the model by incorporating weak heterogeneity, i.e. by not assuming the two cells to be completely identical, but nearly so. To do so, we simply add an extra ϵ-small term to the governing differential equations that accounts for cells having slightly different intrinsic dynamics, as shown below

$$\frac{dX_j}{dt} = F_j(X_j) + \epsilon P(X_k, X_j) \ , \ F_j(X_j) = F(X_j) + \epsilon f_j(X_j)$$

Going through a derivation analogous to the one in [1, 2], we obtain the following

$$\frac{d\phi}{dt} = \epsilon(\Delta\omega + G(\phi))$$

where all terms are analogous to those in [1, 2] except for $\Delta\omega$, which is a new constant term, accounting for the heterogeneity. Noting that phase-locked states occur when $G(\phi) = -\Delta\omega$, if $\Delta\omega^* = \max|G(\phi)|$, then for phase-locked stable states to exist at all, $|\Delta\omega| < \Delta\omega^*$, so the dimensionless quantity $\frac{\Delta\omega^*}{\omega}$, where ω is the intrinsic frequency of the individual, homogenous cell, serves as a measure of the robustness of the system to weak heterogeneity. Figure 3 plots this quantity for the same fixed parameter values used in the bifurcation diagrams seen above and in [1]. The figure suggests that although combining electrical and inhibitory coupling may not maximize the region of synchrony, it may instead make the system more robust to heterogeneity between the intrinsic dynamics of different cells.

References

[1] Lewis, T. and Rinzel, J. (2003). *Dynamics of spiking neurons connected by both inhibitory and electrical coupling.* J. Comp. Neurosci., 14:283309.

[2] N.W. Schultheiss et al. (eds.), *Phase Response Curves in Neuroscience: Theory, 3 Experiment, and Analysis, Springer Series in Computational Neuroscience 6*, DOI 10.1007/978-1-4614-0739-3 1 Springer Science+Business Media, LLC 2012

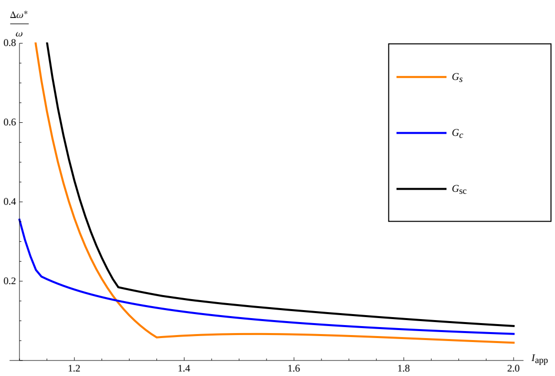

Figure 3: Sample profiles of $\Delta\omega^*/\omega$ plotted with respect to I_{app} for different coupling combinations