Bobby Lerias

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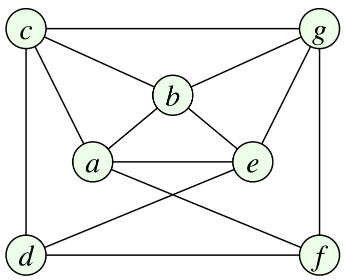
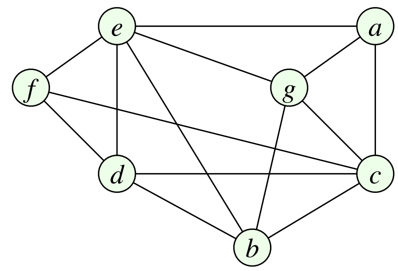
CS/MATH111 ASSIGNMENT 5

due Friday May 28, 2021 (11:59PM)

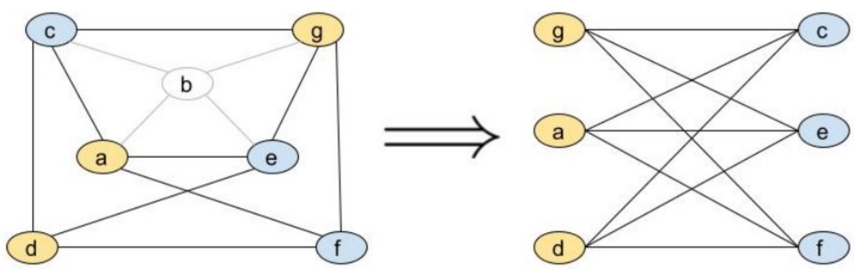
Problem 1. Determine whether the two graphs below are planar or not. To show planarity, give a planar embedding. To show that a graph is not planar, use Kuratowski's theorem.

Graph G: 

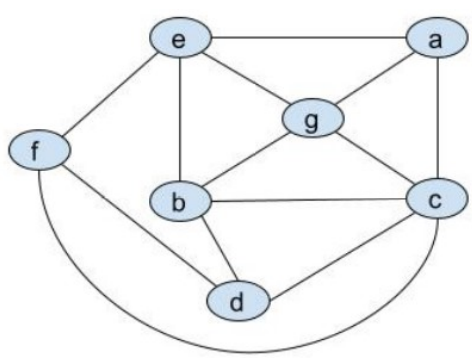
Graph H: 

Solution:

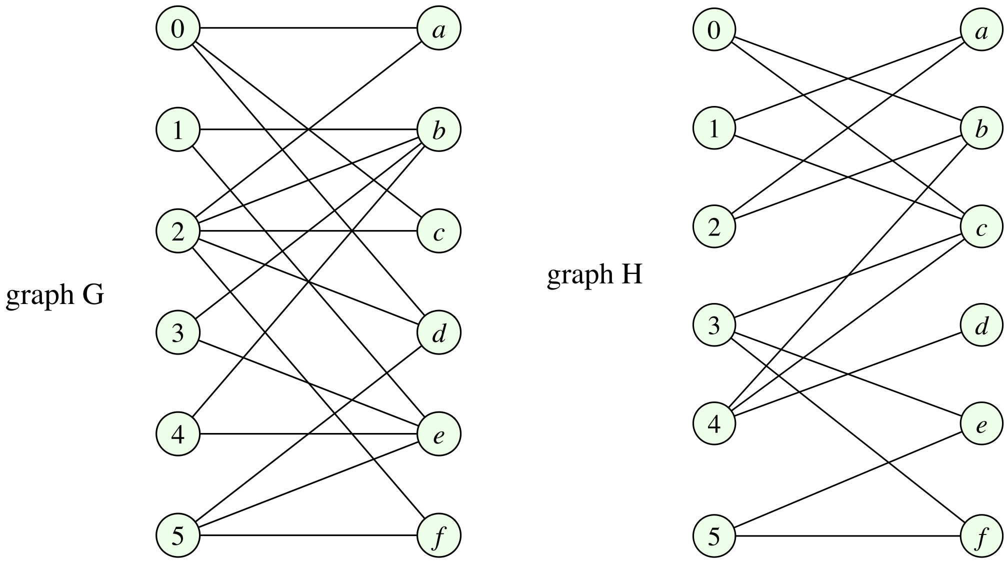
Graph G: We know Graph G to not be planar because of Kuratowski's Theorem. This theorem states that any graph is considered planar if and only if it does not contain a sub-graph that is subdivision of K_5 and $K_{3,3}$. By eliminating Vertex B and its corresponding edges, we ensured that each vertex has a degree of 3. Noting that the yellow vertices, $\{g, a, d\}$, do not share any edges and neither do the blue vertices, $\{c, e, f\}$, we can take this as a hint that a $K_{3,3}$ sub-graph exists as shown in the diagrams below:



Graph H: We know graph H to be planar because we can draw a version of it where no edge cross another edge.



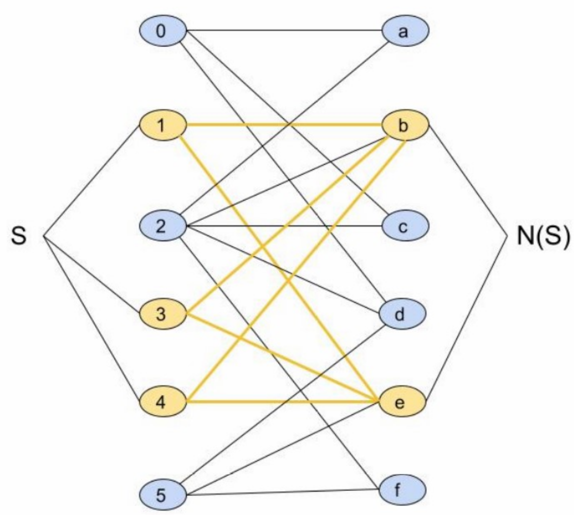
Problem 2. You are given two bipartite graphs G and H below. For each graph determine whether it has a perfect matching. Justify your answer, either by listing the edges that are in the matching or using Hall's Theorem to show that the graph does not have a perfect matching.



Solution:

Graph G: We know Graph G not to be a perfect matching bipartite graph because of Hall's Theorem. This theorem states a bipartite graph, G, is not perfectly matching when we can allocate a finite bipartite set, S, within G which has more vertices than the set of all vertices adjacent to some element of S which we can denote as the neighborhood of S or $N(S)$.

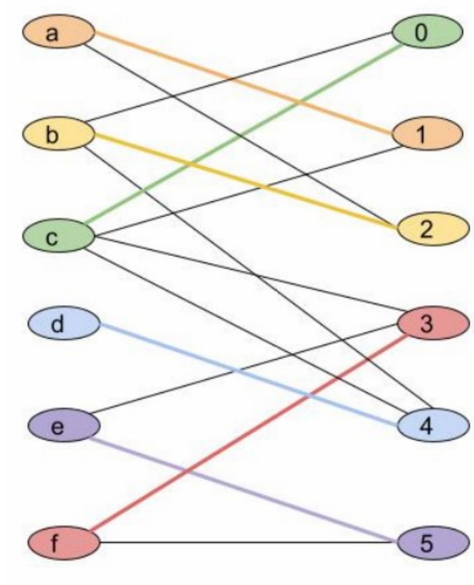
We can represent this inequality by $S \leq N(S)$.



As seen in the diagram above, vertices $\{1, 3, 4\}$ will denote S and its adjacent vertices, $\{b, c\}$, will denote the neighborhood of S or $N(S)$. Therefore, this bipartite graph is not perfectly matched given that the inequality $3 \leq 2$ does not ring true.

Graph H:

Yes. The matching is $\{(0,c),(1,a),(2,b),(3,f),(4,d),(5,e)\}$ as illustrated in the coloring of the graph below

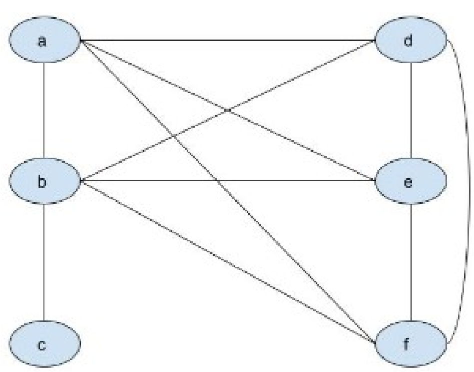


Problem 3. (a) For each degree sequence below, determine whether there is a graph with 6 vertices where vertices have these degrees. If a graph exists, draw it. If it doesn't, justify that it doesn't exist.

(a1) $4,4,4,3,3,1$.

This graph does not exist given that it goes against the Handshaking Lemma. We can prove this because $\sum_{v \epsilon V} deg(v) = 19$ which is odd

(a2) $5,4,4,4,4,1$.

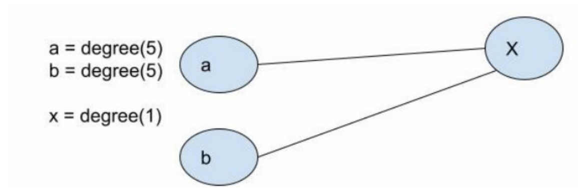


(a3) $5, 5, 3, 3, 3, 1$.

This graph does not exist because the discrepancy between the amount of high degree vertices and the lowest degree vertex. We can prove this by assigning any arbitrary values to represent the individual degrees within the sequence above and graph connection between high and low degree vertices.

$$\{5, 5, 3, 3, 3, 1\} \iff \{\text{a, b . . . x}\}$$

In general, when sketching the connectivity of a vertex, there will be a potential of $z - 1$ connections, z representing the total number of vertices. In terms of our problem, we observe that $z = 6$ and find that each vertex has a potential of $6 - 1 = 5$ connections. Given that a has a degree of 5, it will inevitably connect to vertex x. However, because vertex b has the same degree as a, it will also have to connect to vertex x violating the degree of x as shown below:



Therefore we can conclude that the sequence of degrees state above does not exist.

(b) For each degree sequence below, determine whether there is a planar graph with 6 vertices where vertices have these degrees. If a graph exists, draw it. If it doesn't, justify that it doesn't exist.

(b1) $5, 5, 4, 4, 3, 3$.

We know from Euler's Inequality: $m \leq 3n - 6 \rightarrow$ planar or non-planar

$$m > 3n - 6 \rightarrow \text{non-planar}$$

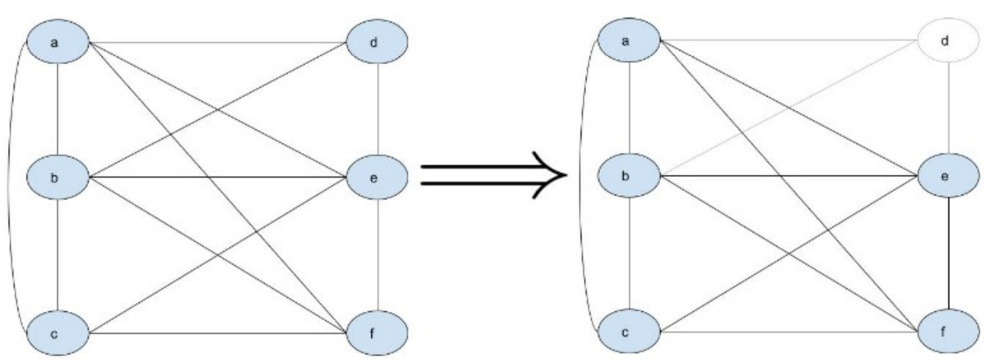
Find m and n:

$$m = \frac{5+5+4+4+3+3}{2} = \frac{24}{2} = 12$$
$$n = 6$$

Solve inequality:

$$12 \leq 3 \cdot 6 - 6$$
$$12 \leq 12$$

Because we find that $12 \leq 12$, it is indeterminate as to whether or not it is planar. With this, we now must use Kuratowski's Theorem to determine its planarity. As seen in the diagrams below, we were able to find a K_5 sub-graph causing the graph to be non-planar.



(b2) $5, 5, 4, 4, 4, 4$.

We know from Euler's Inequality: $m \leq 3n - 6 \rightarrow$ planar or non-planar

$$m > 3n - 6 \rightarrow \text{non-planar}$$

Find m and n:

$$m = \frac{5+5+4+4+4+4}{2} = \frac{26}{2} = 13$$
$$n = 6$$

Solve inequality:

$$13 > 3 \cdot 6 - 6$$
$$13 > 12$$

According to Euler's Inequality, we know that this graph will be non-planar given that $13 > 12$.

Academic integrity declaration. This homework assignment was completed in collaboration between Rodrigo Lamas and Robert Lerias from professor's Marek Chrobak CS 111 class at 3pm to 3:50pm. The sources used to complete this assignment were completely from student available tools such as the class's google drive with Lecture/discussion recordings and slides. Aside from the google drive we also consulted with T.A.s during their office hours, after discussions, and through Slack about minor concern.

Submission. To submit the homework, you need to upload the pdf file to Gradescope. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment. Remember that only LaTeX papers are accepted.